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Research Article

A Dual-Position Loop LLADRC Control Method Based on Harmonic Gear Drive

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High-resolution imaging has become a development trend and is widely used in military and civil fields. As the carrying equipment of imaging system, the speed stability of tracking turntable is the basis of high-resolution and stable imaging. At present, in the aerospace field, there are high requirements for peak power dissipation and holding torque, so flexible joints such as harmonic gear drive are mostly used to realize the function. The characteristics of flexible load have a great impact on the characteristics of motion control, which is easy to cause mechanical resonance, lead to system instability, and have a great impact on speed stability and position tracking accuracy. Therefore, it is necessary to study the servo system of flexible load. In order to solve the problems of high-precision position control and speed stability at low speed of flexible turntable with uncertain load, on the one hand, we comprehensively consider the advantages and disadvantages of semi-closed-loop and full closed-loop control and design a dual-position loop feedback control system combined with the analysis of dynamic equation to realize speed stability and high-precision position control. On the other hand, according to the requirements of the speed stability at low speed of the turntable, the tracking differentiator (TD) is designed innovatively through the language three-point interpolation subdivision and five-point pre-deduction calculation method. Finally, a dual-position loop LLADRC (language linear active disturbances rejection controller) control method based on harmonic gear drive is studied. By comparing the semi-closed loop, dual-position loop, dual-position loop LADRC (linear active disturbances rejection controller, ADRC), and dual-position loop LLADRC methods through simulation analysis, it can be shown that the double position LLADRC control method is obviously superior to other schemes in terms of rapidity, speed stability at low speed, and position tracking accuracy. The theoretical research is verified by experimental test. When the given speed is 0.1°/s, taking the pitch axis as an example, the pitch speed error is 0.0039°/s (3σ). When the maximum speed of the given curve is 20°/s and the maximum acceleration is 16°/s, the position tracking error is 0.0025° (3σ). This control method solves the problems of system instability and low-speed stability in high-precision control of turntable system based on harmonic gear drive and provides a method for high-precision control of high-resolution imaging turntable.

1. Introduction

The reason why the control of turntable with harmonic deceleration mechanism is much more complex than that of pure rigid turntable is mainly due to the following reasons: firstly, because the parameters in the dynamic equation change with the displacement of the rotating mechanism, the system is time-varying and is a strong nonlinear rigid-flexible coupling system; secondly, the nonlinearity of the system belongs to infinite dimension in theory, and the degree of freedom of the system is greater than the number of normal control variables, so the ill-conditioned characteristics are presented; thirdly, due to the extrusion deformation of the flexible wheel of the harmonic gear drive, the system presents nonminimum phase characteristics [1–3].

ADRC technology is suitable for engineering. Its essence is to determine the total disturbance of the system through the input and output of the online observation system, and estimate and compensate the total disturbance. This process is called dynamic compensation linearization of the system, which can effectively improve the anti-disturbance and parameter robustness of the control system [4]. Based on this, the active disturbance rejection controller based on the dynamic model based on the harmonic gear drive and mathematical model of the second-order LADRC (linear active disturbance rejection controller) system is studied in this paper. Based on the structure of the dual-position loop feedback control system, the design of TD is studied through the three-point interpolation subdivision of language and the five-point pre-calculation method. Therefore, we use LLADRC to design the dual-position loop active disturbance rejection control system and build the system models under four control methods in the MATLAB/Simulink simulation software. The simulation verifies that the dual-position loop control system based on LLADRC has better control performance. Finally, the control accuracy and high reliability of the control system are proved by experimental verification. This research has certain reference value in theory and practical application.

The rest of the paper is organized as follows: Section 1 discusses basic physical theory; In Section 2, the closed-loop control method based on the harmonic gear drive, ADRC principle, and main factors affecting its accuracy are introduced; in Section 3, the system dynamics equations and the research on the dual-position loop control method are introduced in detail. Section 4 introduces the proposed LLADRC control method and the research on the dual-position loop control method of LLADRC. Section 5 introduces the establishment of the simulation model and the comparison of the simulation results of the three control methods. In Section 6, the experimental results comparison of the speed stability and position tracking accuracy of the four control methods are introduced. In Section 7, conclusions are given.

2. Related Work

The position feedback of the turntable with harmonic gear drive generally includes two feedback methods: motor position feedback and harmonic gear drive output position feedback. According to the different installation positions of the feedback sensor, the control system can be divided into semi-closed-loop and full closed-loop control system. When the position sensor is installed on the motor shaft to measure the harmonic gear drive output position indirectly, it is called a semi-closed-loop system. When the sensor is installed on the output shaft of the harmonic gear drive to directly measure the displacement of the output end, the system is called a full closed-loop system. Although the existence of nonlinear factors in the harmonic drive cannot affect its output position accuracy, it can be seen from the Routh stability criterion that the value range of the proportional gain coefficient of the controller has certain limitations, and the system will be unstable if the value is not selected carefully. At present, the controller of the harmonic deceleration rotating mechanism is generally designed based on the information of the motor position feedback sensor. The semi-closed loop has high-speed stability, but it is obviously affected by nonlinear factors, which will reduce the trajectory tracking ability of the system and make the output position accuracy unable to be guaranteed. Wei et al. proposed the concept of "load stiffness" to compensate the load deformation information into the load motion, which improves the robustness and the ability to resist external disturbances, but the speed stability and position tracking accuracy still do not meet the needs of high-resolution imaging [5].

ADRC is proposed by Han Jingqing, a researcher of the Chinese Academy of Sciences. On the basis of inheriting the advantages of classic PID control, it is a new type of controller formed by improving the inherent defects of classic PID [4]. Its core connotation lies in the following: firstly, through tracking differentiator (TD) to arrange a suitable transition process for the given signal; secondly, the change of the internal parameters of the system, the uncertainty of the model, and the external disturbance are equivalent to a lumped disturbance, pioneeringly proposed to use nonlinear extended state observer (NLESO) to estimate and compensate the sum of disturbances acting on the system, instead of error integral control; thirdly, design nonlinear states error feedback (NLESF), so as to make the closed-loop dynamic system have better control performance. Although the nonlinear gain of ADRC makes its performance excellent, it has many defects, such as more adjustable parameters, and it is difficult to analyze performance and stability, which limits its application and theoretical research in engineering. Professor Gao Zhiqiang from Cleveland State University was the first to realize the parameter adjustment problem of ADRC and transformed ADRC into Linear Active Disturbance Rejection Controller (LADRC) by introducing the concept of bandwidth [6]. The main process is to linearize and parameterize the nonlinear gains of NLESO and NLSEF, and at the same time, the parameter configuration method of LESO and LSEF is given, which greatly reduces the design parameters of ADRC. Reference [7] analyzed the stability conditions of the LADRC; Reference [8] obtained the conclusion that the LESO observation capability is proportional to the observer bandwidth, and the physical meaning of the bandwidth is more easily accepted by engineers [9].

Rens et al. proposed to realize the stable control of LFA through ADRC, which greatly shortens the adjustment time compared with PID [10]. Oh et al. proposed an EADC method combining disturbance observer-based control (DOBC) and ADRC [11]. Oh et al. proposed an EADC method combining DOBC and ADRC [11]. Marilier and Richard proposed an ADRC for flexible link manipulator (FLM) based on fractional order control to track desired trajectory in the joint space and to cancel the link's vibrations [12], but this study has only simulation analysis and no experimental verification. Sun et al. proposed an ADRC based on a feedforward compensation unit. This method improves the observation effect of ESO and improves the control accuracy of ADRC. However, this method requires many parameters to be tuned, which is not conducive to the control of variable load models [13]. Su et al. used the ESO to estimate and compensate the nonlinear and uncertain parts of the stiffness and damping of the rigid-flexible coupled platform, but this method requires many parameters to be tuned [14–16].

2.1. Research on Dual-Position Loop Control System. Based on the mechanical characteristics of the harmonic gear drive, we firstly conduct a dynamic analysis on the front and rear ends of the harmonic mechanism of the turntable. The motor torque τ_m generated by the motor drives the harmonic deceleration mechanism to rotate, and the elastic deformation of the harmonic deceleration mechanism will generate elastic damping torque τ and then drive the load to rotate. Because the viscous friction coefficient of the actual system is very difficult to obtain and the damping effect is weak, the viscous damping and friction coefficient are approximated as μ. At this time, the dynamic motion equation of the motor is shown in

$$\begin{cases} \dot{\theta}_L = \omega_L, \\ J_L \dot{\omega}_L = \tau, \\ \dot{\theta}_m = \omega_m, \\ J_m \dot{\omega}_m + \dfrac{1}{N}\tau + \mu\omega_m = \tau_m, \\ \tau = k\left(\dfrac{\theta_m}{N} - \theta_L\right), \end{cases} \tag{1}$$

where θ_L and ω_L are the load angle and angular velocity, θ_m and ω_m are the motor angle and angular velocity, J_L is the load moment of inertia, J_m is the motor rotor moment of inertia, τ is the motor torque, μ is the motor damping coefficient, k is the stiffness of the harmonic reducer, and N is the reduction ratio of the harmonic reducer.

Transform by Laplace:

$$\begin{aligned} G_L(s) &= \frac{\theta_L}{\tau} = \frac{N\omega_a^2}{(J_m s + \mu)s\left(s^2 + \omega_n^2\right)}, \\ G_m(s) &= \frac{\theta_m}{\tau} = \frac{s^2 + \omega_a^2}{(J_m s + \mu)s\left(s^2 + \omega_n^2\right)}. \end{aligned} \tag{2}$$

Compared with the rigid connection system, there is a second-order mechanical resonance in the flexible connection system, which will lead to the vibration of motor speed and affect the stability and tracking accuracy of the system, as shown in formulas (3) and (4), and ω_n and ω_a are the system resonance frequency and anti-resonance frequency.

$$\omega_n = \sqrt{k\left(\frac{1}{J_L} + \frac{N^2}{J_m}\right)}, \tag{3}$$

$$\omega_a = \sqrt{\frac{k}{J_L}}. \tag{4}$$

Through the transfer function, a dual-position feedback control system can be derived to achieve low-speed velocity stability and high-precision position tracking control. The structure of the control system is shown in Figure 1. The motor and the harmonic gear drive form the controlled object, and a closed loop is formed by the load position loop, load speed loop, motor position loop, and motor speed loop.

2.2. Research on Active Disturbance Rejection Control Method. Considering the limitations of ADRC in practical applications, this paper takes the second-order controlled object as an example to study LADRC and has the following second-order system:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = bu + f(x_1, x_2, w, t), \\ y = x_1. \end{cases} \tag{5}$$

In this paper, x_1 and x_2 are state variables, b is the control gain, u is the control input, w is the external disturbance, $f(x_1, x_2, w, t)$ is the sum of unmodeled dynamics and internal and external disturbances of the system, and y is control target. The structure using LADRC control is shown in Figure 2.

2.2.1. Research on TD Based on Language Interpolation. The input signal and feedback signal in the actual system usually contain a lot of noise. Due to the noise amplification effect of the differentiator in the classical differential form, the extracted differential signal is often unusable due to noise pollution. TD is a kind of signal processing link, which mainly uses its signal tracking characteristics and extracting differential signal characteristics to arrange a suitable "transition process" for the signal. Especially for abrupt signal, after TD, it can give a smooth input signal and its differential, which can prevent overshoot and ensure the stability of the system. However, the output of this method is only related to the input signal at the current moment and the estimated value of the input signal, so it still has a certain phase lag and deviation.

The TD designed in this paper uses the Lagrange three-point interpolation subdivision + five-point pre-push calculation method, so that the target angle can output a smooth input angle after passing through the tracking differentiator and reduce the phase lag through five-point pre-push. Lagrange three-point prediction subdivision calculation method is shown in formula (6), where $y[0]$, $y[1]$, and $y[2]$ are the angle values to be interpolated, $c[n]$ is the value of the nth subdivision point, $n[1]$ is the number of interpolation between $y[0]$ and $y[1]$, and $n[2]$ is the number of interpolation between $y[0]$ and $y[2]$. While the target

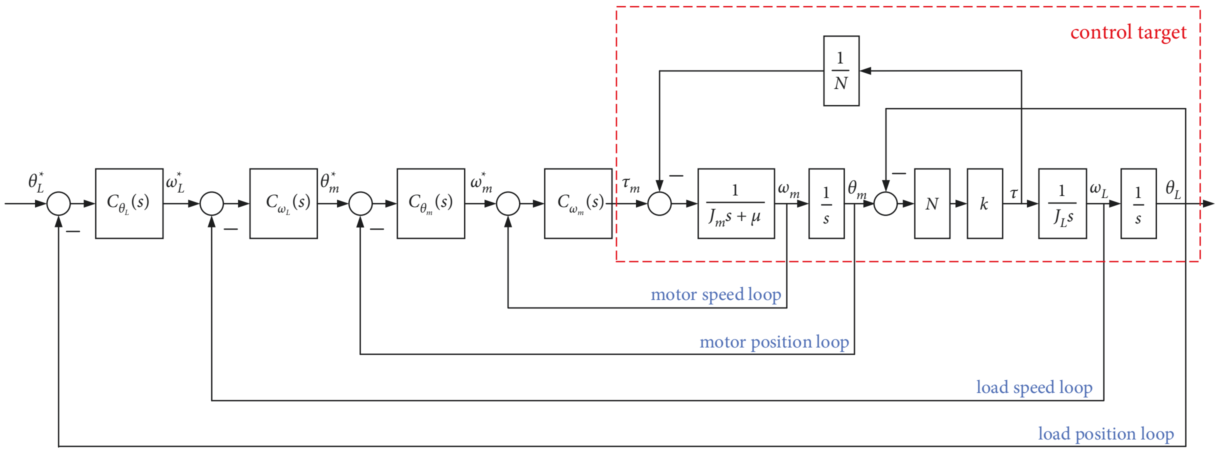


Figure 1: Structure of dual-position loop control system based on harmonic gear drive.

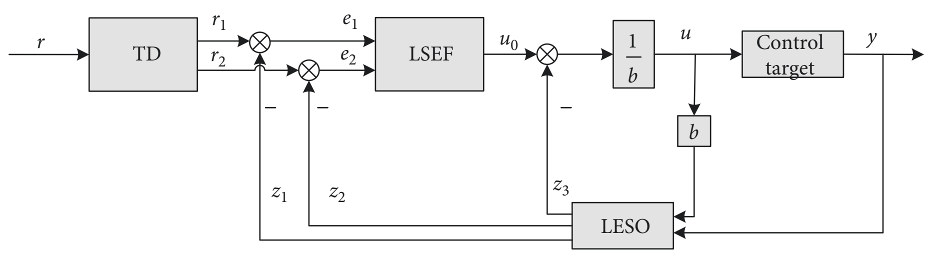


Figure 2: Schematic diagram of LADRC control for second-order objects.

angle is smoothly subdivided through interpolation, we also use the 5-point prediction calculation method in control, as shown in formula (8); that is, the newly calculated 5 $c[n]$ points can be predicted to obtain a latest target point through the 5-point prediction calculation formula. This method can ensure the original TD function, effectively reduce the tracking delay and deviation of the input curve, and improve the tracking accuracy of the system.

$$
\begin{aligned}
y[n] = {} & \frac{(n - N[1]) \times (n - N[2])}{(N[0] - N[1]) \times (N[0] - N[2])} \times y[0] + \frac{(n - N[0]) \times (n - N[2])}{(N[1] - N[0]) \times (N[1] - N[2])} \times y[1] \\
& + \frac{(n - N[0]) \times (n - N[1])}{(N[2] - N[0]) \times (N[2] - N[1])} \times y[2],
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
c[n] = {} & y[0] + \frac{(n - N[0]) \times (n - N[2])}{(N[1] - N[0]) \times (N[1] - N[2])} \times (y[1] - y[0]) \\
& + \frac{(n - N[0]) \times (n - N[1])}{(N[2] - N[0]) \times (N[2] - N[1])} \times (y[2] - y[0]).
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
A[n] = {} & \frac{((c[4] - c[1]) - (c[5] - c[1]) + (c[2] - c[1]))}{16} \\
& + \frac{(2 \times ((c[5] - c[1]) - (c[2] - c[1])) + ((c[5] - c[1]) - (c[3] - c[1])))}{4} \\
& + (c[5] - c[1]) - (c[2] - c[1]) + c[1].
\end{aligned} \tag{8}
$$

2.2.2. Research on Second-Order Linear Extended State Observer (LESO). ESO is the core of ADRC technology. It can not only observe the internal and external disturbances in real time according to the information of system output and input, in which the ADRC regards the uncertainty of the system model as internal disturbance and external disturbance as external disturbance. The two together constitute the "total disturbance" of the system, which is observed by the extended state observer. At the same time, it can estimate the disturbance of the system and compensate the control signal of the controlled object with the observed value; LSEF linearly combines the estimation error between the state variable outputs by TD and ESO and forms a control variable together with the disturbance compensation of ESO, so as to improve the performance of the control system.

The real-time value of internal and external disturbances of the system is estimated, compensation is given in the feedback, and the influence of disturbance is eliminated by the compensation method; thus, it has the effect of anti-disturbance.

The unknown part of the system is equivalent to a lumped disturbance, and then, it is expanded to a new state:

$$x_3 = f(t). \tag{9}$$

Then, the nonlinear system can be transformed into a linear system:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = bu + x_3, \\ \dot{x}_3 = \dot{f}(t), \\ y = x_1. \end{cases} \tag{10}$$

For this system, a state observer can be designed as follows:

$$\begin{cases} e = z_1 - y, \\ \dot{z}_1 = z_2 - \beta_1 e, \\ \dot{z}_2 = z_3 - \beta_2 \mathrm{fal}(e, 0.5, \delta) + bu, \\ \dot{z}_3 = -\beta_3 \mathrm{fal}(e, 0.25, \delta), \end{cases} \tag{11}$$

where β_1, β_2, and β_3 are the adjustable parameters of ESO, respectively, and the fal function expression is as follows:

$$\mathrm{fal}(e, \alpha, \delta) = \begin{cases} \dfrac{e}{\delta^{a-1}}, & |e| \le \delta, \\ |e|^a \mathrm{sign}(e), & |e| > \delta. \end{cases} \tag{12}$$

Here, a is the interval length of linear segment; when $a < 1$, function has small error and large gain; and large error and small gain. Therefore, as long as the total disturbance is not infinite and the appropriate observer gain is selected, ESO can effectively estimate each state of the system.

Linearize ESO system into LESO, as shown in

$$\begin{cases} e(t) = z_1(t) - x_1(t), \\ \dot{z}_1(t) = z_2 - 3\omega_0 e(t), \\ \dot{z}_2(t) = z_3 - 3\omega_0^2 e(t) + bu, \\ \dot{z}_3(t) = -\omega_0^3 e(t), \end{cases} \tag{13}$$

where z_1 and z_2 are the estimated values of x_1 and x_2, respectively. b and ω_0 become the adjustable parameters of LESO, and the physical meaning is clear.

2.2.3. Disturbance Compensation and Linear Error Feedback Control Law (LSEF). After ESO estimates the disturbance of the system, it can perform disturbance compensation. The compensation method is shown in

$$\begin{cases} \varepsilon_1 = r_1 - z_1, \\ \varepsilon_2 = r_2 - z_2, \\ u = \dfrac{u_0 - z_3}{b}, \\ u_0 = k_1\varepsilon_1 + k_2\varepsilon_2, \end{cases} \tag{14}$$

where u is the output of the controller and u_0 is the output of LESF. For adjustable parameters a and b, the corresponding relationship is shown in

$$\begin{cases} k_1 = \omega_c^2, \\ k_2 = 2\omega_c. \end{cases} \tag{15}$$

Based on the research of double position loop and LLADRC, we propose a high-precision dual-position loop LLADRC control method of PMSM motor based on harmonic reduction mechanism. The control block diagram is shown in Figure 3. In the flexible joint control system, the turntable is the controlled object, u is the given torque of PMSM motor and drives the motor shaft displacement, and the motor shaft displacement drives the load displacement through harmonic reduction mechanism to realize the tracking task. Therefore, the original LLADRC system is transformed into two series subsystems composed of motor and load, namely, load subsystem and motor subsystem, and then TD, LESO, and LSEF are designed, respectively, according to the active disturbance rejection control principle.

2.3. Simulation Research. This paper is simulated in MATLAB/Simulink environment. The simulation block diagram is shown in Figure 4.

2.3.1. Simulation Parameters. The pole assignment of the two LESO estimators in LADRC is $\omega_{0L} = 20$ rad/s, respectively, to make it fast enough. The target angle input frequency in TD_L is 40 Hz, the output angle after interpolation and prediction is 200 Hz, the target angle input frequency in TD_M is 200 Hz, and the output angle after interpolation and prediction is 1000 Hz. System simulation parameters are shown in Table 1.

2.3.2. Speed Stability Test at Low Speed. Based on the above simulation conditions and parameter settings, the speed overshoot and speed stationarity of load output under three control methods (dual-position loop, dual-position loop LADRC, and dual-position loop LLADRC) can be obtained. Under the three control methods, the comparison curve of load speed with time and overshoot is shown in Figure 5. Figures 5(b)–5(d) show the comparison of speed stability of the three control methods, and the comparison results are shown in Table 2.

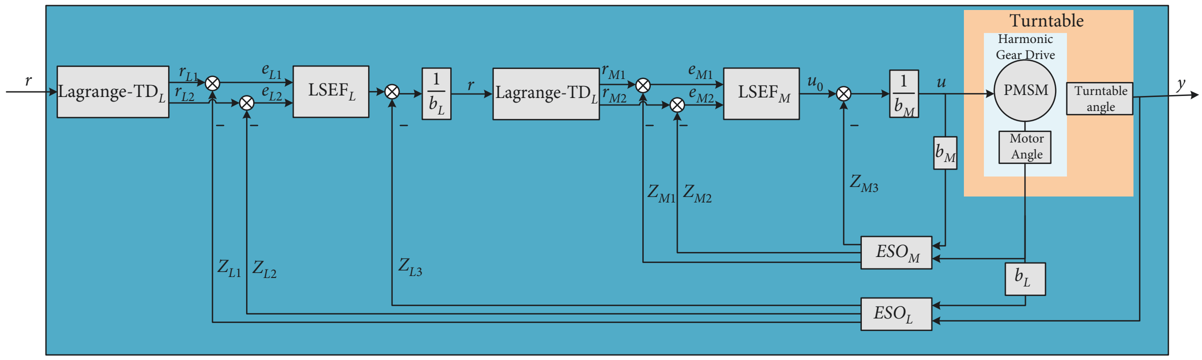


Figure 3: Structure diagram of double position ring LLADRC control system.

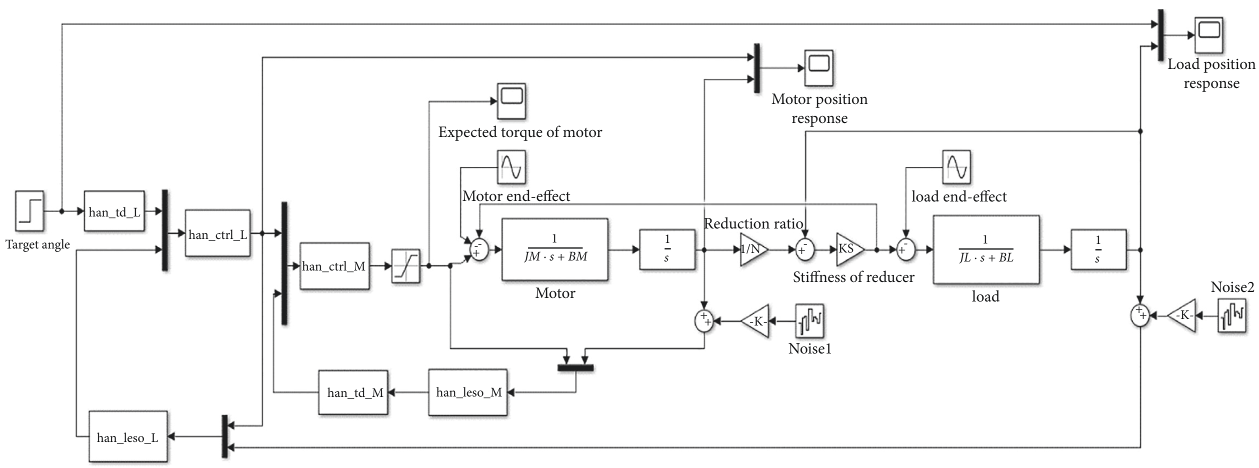


Figure 4: Simulink model of two-position loop LLADRC based on harmonic gear drive.

Table 1: System simulation parameters.

System model parameters	Value
Load inertia $J_L/(\mathrm{kg} * \mathrm{m}^2)$	0.1
Motor moment of inertia $J_M/(\mathrm{kg} * \mathrm{m}^2)$, $J_M/(kg * m^2)$	0.006
Motor pole pairs P	16
Stator inductance L_d/mH	6.25
Stator inductance L_q/mH, L_q/mH	13
Stator resistance R/Ω	6.07
Permanent magnet flux φ_1/Wb	0.2527
Damping coefficient $B/(\mathrm{N} * \mathrm{m} * \mathrm{s})$	0.01
Reduction ratio N	80
Torsional stiffness $\mathrm{N} * \mathrm{m/rad}$	57×10^3

It can be seen from Table 2 that the rise time of load speed under the three control methods is almost the same. The overshoot of the system controlled by dual-position loop LADRC is about 9%, while the overshoot of the system controlled by dual-position loop is about 2.1%. The speed stability of the system controlled by dual-position loop LLADRC is 0.0023°/s (3σ), which is obviously better than the other two control methods.

2.3.3. Position Tracking Accuracy Test. Through the simulation test, the comparison curve of load angle with time and overshoot under the four control methods is shown in Figure 6(a), and Figures 6(b)–6(d) show the comparison of positioning accuracy of the three control methods. The comparison results are shown in Table 3.

It can be seen from Table 3 that the rise time of load angle under the three control methods has little difference. The

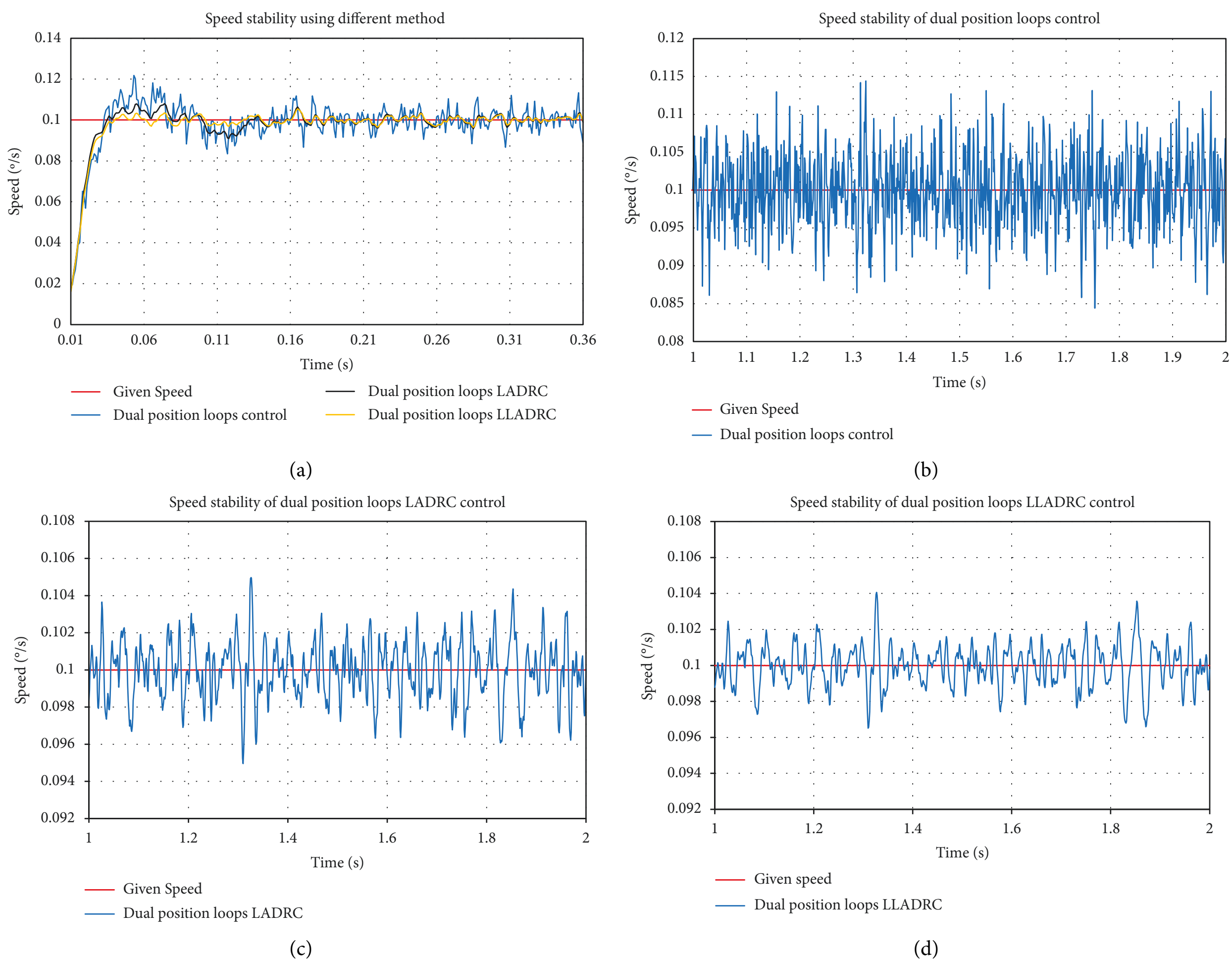


Figure 5: Comparison of speed dynamic performance simulation test of three control methods. (a) Step response of control methods; (b) dual-position loop control speed error; (c) dual-position loop LADRC control speed error; (d) dual-position loop LLADRC control speed error.

Table 2: Speed dynamic performance comparison.

System model parameters	Overshoot	Control accuracy (°/s)
Dual-position loop	22%	0.012
Dual-position loop LADRC	9%	0.0025
Dual-position loop LLADRC	2.1%	0.0023

overshoot of the system controlled by dual-position loop ADRC is about 1.2%, while the overshoot of the system controlled by dual-position loop LADRC is about 0.8%. The control accuracy of the system controlled by dual-position loop LLADRC is 0.000022° (3σ), which is obviously better than the other two control methods.

2.4. Experimental Verification. Based on the research and analysis of double position loop LLADRC control, we built an experimental platform, as shown in Figure 7. As shown in Figure 7(a), the turntable is only equipped with angle measurement feedback at the load end, so there is no angle feedback at the motor output end. Figure 7(b) is equipped with angle measuring feedback at the motor output to extract the motor speed feedback. Figure 7(c) has angle measurement feedback at both motor and turntable ends, so it can be used for dual-position loop control test.

2.4.1. Speed Stability Test at Low Speed. Considering that there is no difference between LLADRC and LADRC in terms of speed stability during uniform tracking in theoretical analysis and simulation tests, here we test semi-closed-loop, full closed-loop, dual-position loop and dual-position LLADRC, respectively.

The comparison of the speed stability test of the four control methods is shown in Figure 8. It can be seen from Figure 8(a) that the semi-closed-loop control

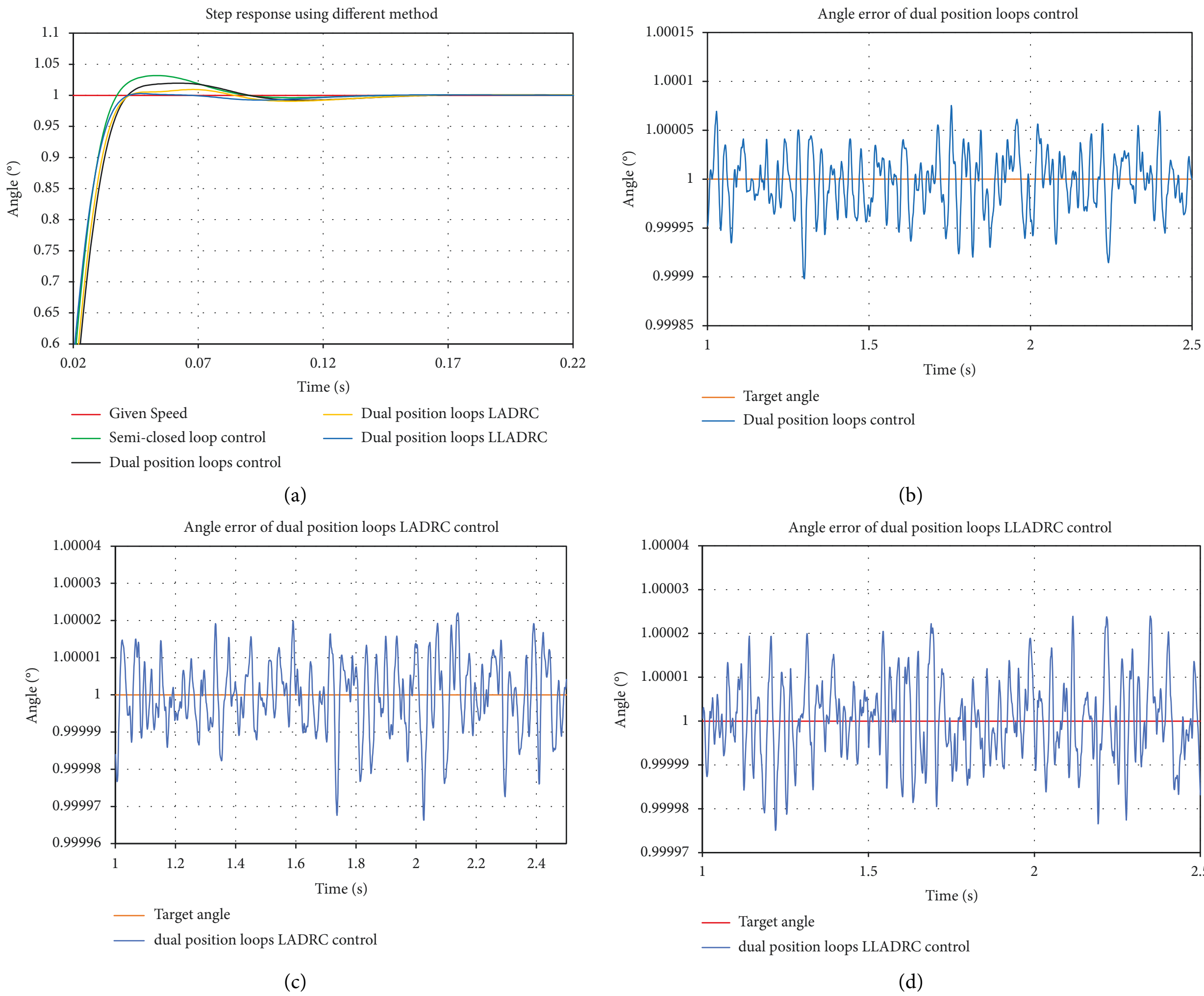


Figure 6: Comparison of position dynamic performance simulation test of three control methods. (a) Step response of control methods; (b) dual-position loop control position error; (c) dual-position loop LADRC control position error; (d) dual-position loop LLADRC control position error.

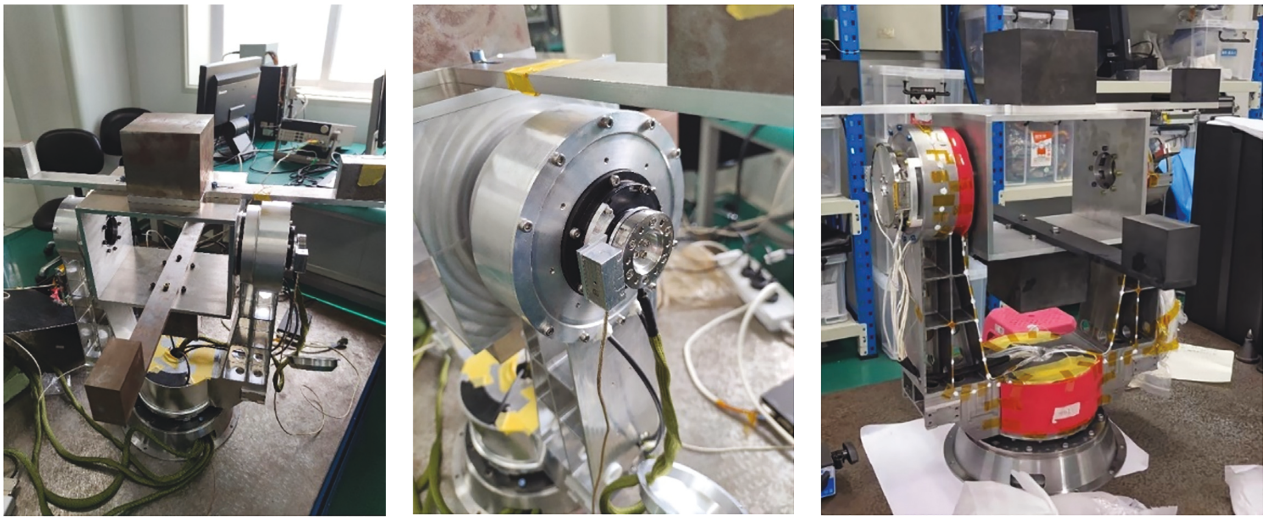
Table 3: Position dynamic performance comparison.

System model parameters	Overshoot (%)	Control accuracy (°)
Dual-position loop	2	0.0001
Dual-position loop LADRC	1.2	0.000029
Dual-position loop LLADRC	0.8	0.000022

superimposes the cogging-related fluctuation on the speed due to the uncertainty of the rear end of the load, so the speed fluctuates. It is larger and can be seen to be affected by torque disturbance. As can be seen from Figure 8(b), although there is no harmonic cogging-related fluctuation in the speed feedback curve of full closed-loop control, the control is unstable due to the lack of feedback from the motor output, and there is a slight resonance phenomenon, so the speed stability is poor. As can be seen from Figure 8(c), the speed feedback curve of dual-position loop control has neither harmonic cogging-related fluctuation nor resonance, but the stability and accuracy of speed will be affected due to the influence of motor torque fluctuation, winding torque, and other disturbances on the system. As can be seen from Figure 8(d), the speed feedback curve controlled by the dual-position loop LLADRC has neither harmonic cogging-related fluctuation nor resonance.

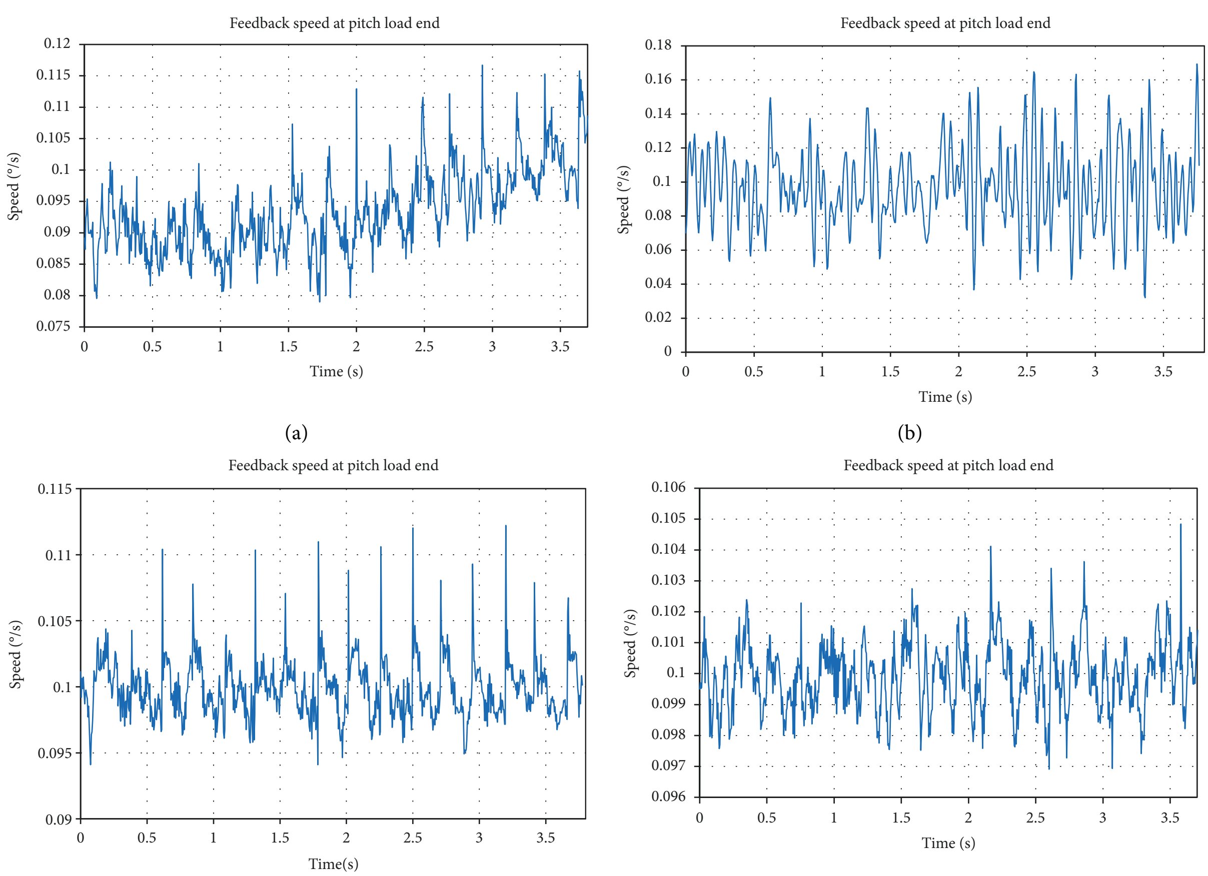
The RMS value of uniform speed error of the four control methods is shown in Table 4. The RMS value of uniform speed error of the optimized dual-position loop LLADRC control method is less than 0.0039°/s (3σ).

2.4.2. Position Tracking Accuracy Test. In order to verify the position tracking accuracy and stability of LLADRC, we



(a) (b) (c)

Figure 7: Physical diagram of experimental turntable. (a) The full closed-loop experimental turntable with the encoder at the load end to extract the angle; (b) a semi-closed-loop experimental turntable equipped with a motor end encoder to extract the speed; (c) dual-position loop experimental turntable with load end angle feedback and motor end angle feedback.

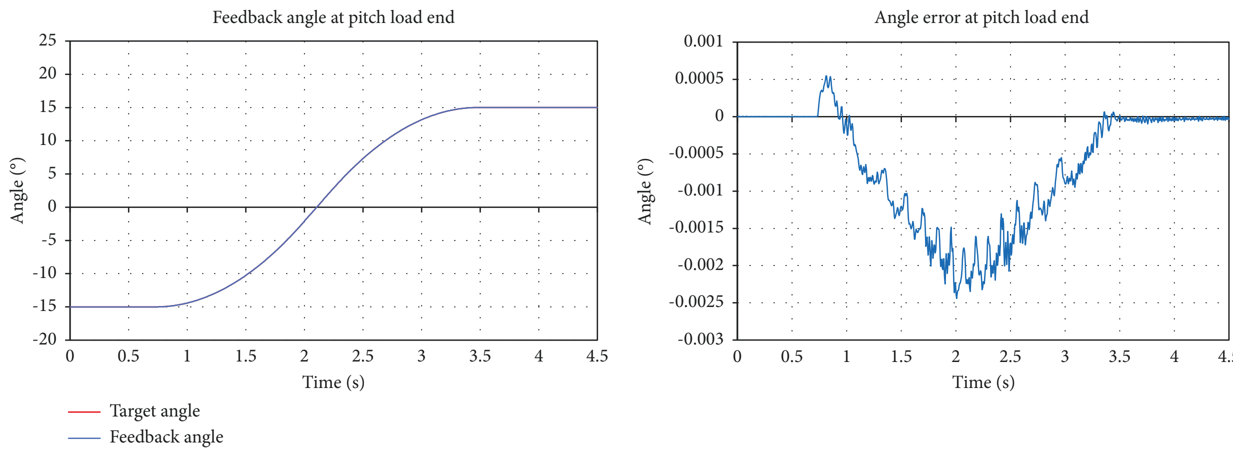


(a) (b) (c) (d)

Figure 8: Test and comparison of speed stability of four control methods at 0.1°/s. (a) Semi-closed-loop control; (b) full closed-loop position control; (c) dual-position loop control; (d) dual-position loop LLADRC control.

Table 4: RMS value of speed.

	RMS value of speed (°/s)
Semi-closed loop	0.0197
Full closed-loop position	0.0685
Dual-position loop	0.007
Dual-position loop LLADRC	0.0039



(a) (b)

Figure 9: Position tracking accuracy test during high-speed maneuver. (a) Pitch axis position setting curve and tracking feedback curve; (b) position tracking error.

collect the feedback angle at the load end when the maximum speed of the given curve of the pitch axis is 20°/s°/s and the maximum acceleration is 16°/s^2. The test curve is shown in Figure 9. It can be seen from figure a that the system has good tracking characteristics. As can be seen from figure B, the position tracking error is 0.0025° (3σ) and the positioning accuracy is 0.0001° (3σ).

3. Conclusion

Aiming at the high-precision control system of the two-dimensional turntable based on the harmonic gear drive, this paper proposes a dual-position loop feedback control to suppress the position fluctuation caused by the harmonic flexible mechanism and improve the speed stability at low speed. And according to the TD in LADRC, we propose an optimized calculation method using Lagrange three-point interpolation subdivision + five-point prediction, which reduces the phase lag and differential deviation of the input curve and suppresses overshoot compared to the original TD. Finally, this paper studies the optimized LLADRC control method in dual-position loop to improve the low-speed stability and position tracking accuracy of the system. The simulation results show that compared with the three control methods before optimization (semi-closed-loop, dual-position loop, and dual-position loop LADRC), the method in this paper effectively reduces the system overshoot and suppresses the mechanical resonance caused by the flexible mechanism. The system has higher speed stability and position tracking accuracy. After experimental tests, when the given speed is 0.1°/s, the speed stability reaches 0.0039°/s (3σ), which is 94% higher than that of the dual-position loop and 44% higher than that of the dual-position loop LADRC. In addition, when the dual-position loop LLADRC has a given curve with a maximum speed of 20°/s and a maximum acceleration of 16°/s^2, the position tracking error reaches 0.0025° (3σ), and the bit accuracy is 0.0001° (3σ). Compared with the other three control methods, the dual-position loop LLADRC effectively improves the low-speed speed stability and position tracking accuracy of the turntable based on the harmonic gear drive. Compared with the first two control methods, the fluctuation related to harmonic flexible cogging is obviously suppressed; compared with the dual-position loop control, other disturbances such as motor torque fluctuation are also significantly suppressed. The controller exhibits good parameter robustness and resistance to external disturbances.

Data Availability

The figures and tables used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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