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Research Article

Design of Prediction-Based Controller for Networked Control Systems with Packet Dropouts and Time-Delay

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A novel prediction-based controller design is proposed for networked control systems (NCSs) with stochastic packet dropouts and time-delay in their control channel. The sequence of packet dropouts, which are modelled as a Bernoulli process, is compensated by a zero-order holder (ZOH)-based module, whereas a state predictor is utilized for obtaining the predicted states at the time delayed. In view of dropout compensator and state predictor, a novel modified model predictive controller (MPC) is designed and proposed in the following procedures. Compared to cost function of a general model predictive controller, variables of states are substituted by the predicted ones as obtained from state predictor preliminarily. Then, a logical programming approach is applied to include all the possible circumstances in the prediction horizon. Consequently, the cost function is reformed as simultaneous minimax linear matrix inequalities (LMI) with constraints. As a result, toolbox YALMIP is employed in order to solve such minimax programming problem eventually. Simulation results are presented to show the feasibility and performance of proposed method.

1. Introduction

In the last few decades, there has been a strong interest in the study of networked control systems (NCSs) [1] due to their broad application in unmanned aerial vehicles, intelligent transportation systems, mobile sensor networks, cloud computing, real-time systems, etc. [2–4]. NCSs are known to achieve various attractive advantages, for example, low cost, easy installation and maintenance, and high data exchangeability. However, unreliable communication networks and limited bandwidth lead to inevitable problems such as time-delay and packet loss. There is no doubt that these factors can significantly degrade the performance of NCSs, even worse, severe instability may be incurred. For example, time-delay may occur when data are exchanged between devices shared by the network. Such delays will lead to performance degradation and system instability if the designer does not take this into account. In addition, packet loss may occur when packets are transmitted from the controller to the actuator through unreliable communication channels. Thus, it is necessary and important to study networked control systems with both time-delay and packet loss.

Research on networked control systems is generally divided into two areas:

(1) Control of network: researching and improving the intrinsic characteristics of networks, e.g., proposing new network communication protocols, network scheduling algorithms, etc.
(2) Control over network: treating existing network structures, protocols, etc., as established conditions on the basis of which reasonable control structures and control algorithms can be designed to compensate for or reduce the adverse effects on the control system due to problems such as delay and packet loss [5]. The study in this thesis is based on the latter starting point.

According to Wu and Chen research, they designed NCSs with packet loss [6]. Tan's research team demonstrated the stability of the network control system under the induced network delay [7]. In [8], it was mentioned that the optimal LQ control problem for systems with both multiplicative noise and input delay was solved by solving the forward and backward differentiation/difference methods. The problem of stability analysis of a Takagi-Sugeno fuzzy system with time-varying delays was mentioned in the study of [9]. According to Wu's research [10], a standard model predictive controller is proposed in the study, which is shown to have a fast response within 40 steps, but the robustness of this controller is not discussed in the paper. Nesic and Teel found that perturbation theory could be used to demonstrate the stability of the NCSs [11], and the stability of the transmission on the path from the sensor to the controller was demonstrated by this study. In Yang's work [12], the H-∞ controller is obtained by solving linear matrix inequalities, which has the advantage of greatly reducing the cost and the disadvantage of making the overall control system riskier. The reason is that only the most ideal state is considered and the worst case is not analysed. The wireless tracking control system for packet loss is embodied in a modified preview control proposed in paper [13]. This approach requires deterministic future information, but in real life, the future information of NCSs is random.

As shown in Figure 1, a classical control system normally includes a controller and a plant. The controller transmits the signal to the plant via a transmission channel. If unreliable communication channels are encountered, time-delay and packet loss may occur. The purpose of this thesis is to design a reasonable control strategy to reduce or compensate for the adverse effects on the control system due to time-delay and packet loss.

Then, as shown in Figure 2, the control object and the network are integrated into a new control object. In the middle of the model building process, the information of time-delay is handled by state prediction control. The sequence of packet dropouts, which are modelled as a Bernoulli process, is compensated by a zero-order holder (ZOH)-based module. Then, a logical programming approach is applied to include all the possible circumstances in the prediction horizon. Consequently, the cost function is reformed as simultaneous minimax linear matrix inequalities (LMI) with constraints. As a result, toolbox YALMIP is employed in order to solve such minimax programming problem eventually.

Model predictive control has been extensively applied in theory and practice [14]. Inspired by this, the focus of this paper is on the handling of random packet loss and time-delay in the control channel of the networked control system. Specifically, packet dropouts are compensated by a zero-order holder (ZOH)-based module. The predicted state of the time-delay is obtained by a state predictor. Given the past studies and the existing documentation, our study focused on the following areas:

(1) A novel prediction-based controller design is proposed for networked control systems (NCSs) with stochastic packet dropouts and time-delay in their control channel

(2) In view of dropout compensator and state predictor, a novel modified model predictive controller (MPC) is designed

(3) By employing the toolbox YALMIP, which incorporates all possible scenarios into the prediction horizon, the cost function is transformed into a simultaneous minimal linear matrix inequality with constraints to finally solve this minimal programming problem

(4) The validity of the presented research results is confirmed by simulation instances

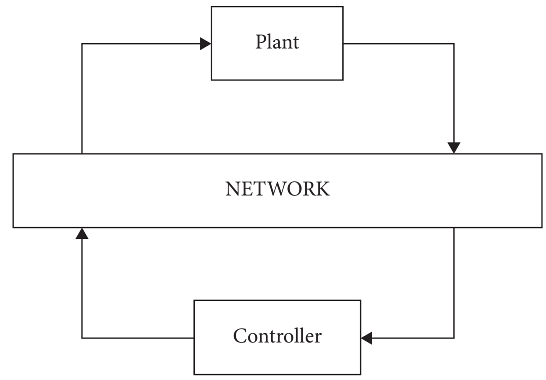


Figure 1: Sketch of networked control systems (NCSs).

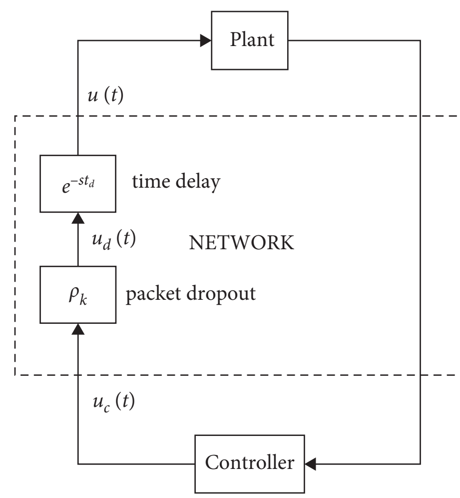


Figure 2: Networked control systems (NCSs) after reintegration.

1.1. Problem Formulation. Regardless of the architecture and sophistication of the NCSs, the study of the NCSs consisting of a number of sensors, actuators, and controllers started with each single-loop NCSs shown in Figure 1. Unlike traditional un-networked control systems, NCSs typically face two problems: packet loss and/or time-delay. Such abnormal conditions may lead to instability of the control system. In my work, we will develop separate control strategies for packet loss and time-delay faced in networked control systems. For convenience, the networked control systems (NCSs) in this study refer to the single-loop NCSs shown in Figure 1.

From Figure 2, it can be seen that the order of time-delay and packet loss can be interchanged due to the existence of the exchange law of multiplication in mathematics. Thus, the time-delay and packet loss are negligible location influencing factors. The time-delay equations are

$$u(t) = u_d(t - t_d), \tag{1}$$

where t_d is the time of delay in NCSs. Unlike packet loss, the time-delay is usually assumed to be a constant known to us in many previous studies on NCSs [15]. $u_d(t)$ is a variable, which is the control input after the packet is lost. Therefore, the control input $u_d(t)$ after packet loss is being modelled as

$$u_d(t) = \rho_k u_c(t), \tag{2}$$

$$\rho_k = \begin{cases} I, & \text{with probability of } \rho, \\ 0, & \text{with probability of } 1 - \rho, \end{cases} \tag{3}$$

where ρ_k is the probabilities of transmission that occurs in the controller-actuator channels, while $u_c(t)$ is the control input from the controller. In this study, $\rho_k = 1$ represents the successful transmission of packets from the controller to the actuator; otherwise, we model $\rho_k = 0$. Packets can be lost for a variety of reasons. In this thesis, it is reasonable and correct for us to equate the packet loss process in NCSs to a Bernoulli process. Because the characteristics of the two are very similar to each other, for both Bernoulli and packet loss processes, they have only two states, 1 and 0, representing successful transmission from the controller to the actuator or packet loss, respectively.

Although the networked control system in Figure 2 is different from a conventional control system consisting of only controller and plant, we can derive a new target plant with the help of equations (1)–(3), and the state space of this new target plant can be derived as

$$\begin{cases} \dot{x}(t) = Ax(t) + \rho_k B u_c(t - t_d), \\ y(t) = Cx(t). \end{cases} \tag{4}$$

Thus, the problem of networked control systems is transformed into a problem of nonlinear systems with delay. In this study, the control objective of our research is to model the new target plant in order to obtain better performance in nonlinear systems.

2. Methods

When modelling the entire networked control system, it is not only important to model the predictive model of the model predictive controller but also a prerequisite for using the YALMIP toolbox. Before designing the controller, attention should be paid to the sampling and discretization process. When studying the new target plant (4), it can be noted that the new plant (4) is actually a hybrid continuous and discrete system. In a typical networked control system, the data are transmitted in packets in the transmission channel, hence it is correct and appropriate to discretize the networked control systems (NCSs) during transmission towards the networked control system in Figure 1 under a sample time scale of T_s. Since the parameters A and B in (4) are known to us all along, the discrete transition functions of the time-delay of the original target plan and the control inputs at the sampling rate T_s are considered to be

$$x(k+1) = A_d x(k) + B_d U_d(k), \tag{5}$$

where A_d and B_d are corresponding discrete parameters, mark $U_d(t) = u_d(t - t_d)$.

Since predictive control involves online optimization, a considerable computational delay may be involved, and this should be taken account of. Figure 3 shows the assumptions we shall make about the timing of measurements made on the plant being controlled and the resulting control signals being applied.

The measurement interval and control update interval are assumed to be the same, with length T_s.The plant output vector is measured at time kT_s, and this measurement is labelled $y(k)$. If there is a measured disturbance, this is assumed to be measured at the same time, and this is labelled $d_m(k)$. There is then a delay of τ, which is the time taken by the predictive controller to complete its computations, after which a new control vector is produced and applied as the plant input signal. This input signal is labelled $u_d(k)$. The input signal is assumed to be held constant until it is recomputed, T_s, time units later. This sequence is repeated at time $(k+1)T_s$ and regularly thereafter.

Remark 1. In practice, process plants may have hundreds of measurements which may be taken, and/or made available, at various times during the measurement interval. If accurate modelling is required, this may have to be taken into account. Also, the computation delay τ may vary in practice, in which case the decision must be taken whether to apply the new control signal to the plant as soon as it becomes available, which probably improves the control but complicates the modelling and analysis, or whether the result should be held up until a standard interval has elapsed before applying it to the plant. It would be impossible to deal with all such eventualities here, so we will assume that all the measurements are taken synchronously, as shown in Figure 3, and that the computational delay τ is the same at each step [16].

Lemma 1 (see [16]). *In the interval of $kT_s + \tau \leq t \leq (k+1)T_s + \tau$, the following equation can be obtained:*

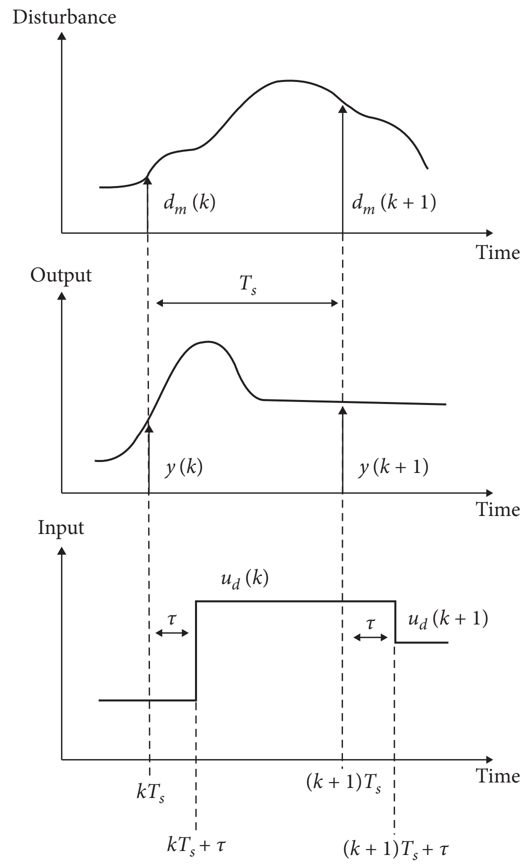


Figure 3: Assumed timing of measuring and applying signals.

$$\begin{aligned}x(k+1) &= e^{A_d(T_s-\tau)}x(kT_s+\tau)+\left(\int_{kT_s+\tau}^{(k+1)T_s} e^{A_d[(k+1)T_s-\theta]}\mathrm{d}\theta\right)B_d u_d(k)\\ &= e^{A_d(T_s-\tau)}\left[e^{A_d\tau}x(k)+\Gamma_1 B_d u_d(k-1)\right]+\left(\int_0^{T_s-\tau} e^{A_d\eta}\mathrm{d}\eta\right)B_d u_d(k)\\ &= A_1 x(k)+B_1 u_d(k-1)+B_2 u_d(k),\end{aligned}\tag{6}$$

where

$$\begin{aligned}A_1 &= e^{A_d T_s},\\ B_1 &= e^{A_d(T_s-\tau)}\Gamma_1 B_d,\\ B_2 &= \int_0^{T_s-\tau} e^{A_d\eta}\mathrm{d}\eta B_d,\end{aligned}\tag{7}$$

$$\begin{aligned}\Gamma_1 &= \int_{kT_s}^{kT_s+\tau} e^{A_d(kT_s+\tau-\theta)}\mathrm{d}\theta\\ &= \int_0^{\tau} e^{A_d\eta}\mathrm{d}\eta.\end{aligned}\tag{8}$$

Remark 2. At the control input, a zero-order holder (ZOH) compensator is proposed so that when a packet is lost, the transmitted control input data do not drop to zero, but remain the same as last time, i.e.,

$$u_d(k)=\rho_k u_c(k)+(1-\rho_k)u_d(k-1).\tag{9}$$

Finally, according to equations (6) and (9), we can obtain the matrix function (10) after augmentation as

$$\begin{bmatrix} x(k+1)\\ u_d(k)\end{bmatrix}=\begin{bmatrix} A_1 & B_1+B_2(1-\rho_k)\\ 0 & 1-\rho_k\end{bmatrix}\begin{bmatrix} x(k)\\ u_d(k-1)\end{bmatrix}+\begin{bmatrix} B_2\rho_k\\ \rho_k\end{bmatrix}u_c(k).\tag{10}$$

For convenience, we specify equation (10) as

$$Z(k+1)=A_{zd}(k)Z(k)+B_{zd}(k)u_c(k),\tag{11}$$

where

$$A_{zd}(k) := \begin{bmatrix} A_1 & B_1 + B_2(1-\rho_k) \\ 0 & 1-\rho_k \end{bmatrix},$$
$$B_{zd}(k) := \begin{bmatrix} B_2\rho_k \\ \rho_k \end{bmatrix},$$
$$Z(k) := \begin{bmatrix} x(k) \\ u_d(k-1) \end{bmatrix} \quad (12)$$
$$A_1 = e^{A_d T_s},$$
$$B_1 = e^{A_d(T_s-\tau)}\Gamma_1 B_d,$$
$$B_2 = \int_0^{T_s-\tau} e^{A_d\eta}\mathrm{d}\eta B_d.$$

Definition 1 (see [17]). The closed-loop system (11) is described as being stable with the presence of constants $\alpha > 0$ and $\xi \in (0,1)$ that make

$$E\left\{\|Z(k)\|^2\right\} \le \alpha\xi^k E\left\{\|Z(0)\|^2\right\}. \quad (13)$$

For all $Z(0) \in \mathbb{R}^n, k \in I^+$.

Lemma 2 (Schur complement [18]). *For a given symmetric matrix*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21}^T & S_{22} \end{bmatrix}, \quad (14)$$

where $S_{11} \in R^{r\times r}$, the following three conditions are equivalent:

① $S < 0$.
② $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$.
③ $S_{22} < 0, S_{11} - S_{22}^{-1} S_{12}^T < 0$.

Lemma 3. *The controller (4) is given. The closed-loop system (11) is exponentially mean-square stable if there exists a positive definite matrix P satisfying*

$$\begin{bmatrix} -P & A_{zd}^T \\ A_{zd} & -P^{-1} \end{bmatrix} < 0. \quad (15)$$

Proof. Define a Lyapunov functional

$$V[Z(k)] = Z^T(k)PZ(k), \quad (16)$$

where P is a positive definite matrix. From equation (15), we can obtain

$$\begin{aligned} E\{V[Z(k+1)]|Z(k)\} &= E\left\{Z^T(k+1)PZ(k+1)\right\} \\ &\quad - Z^T(k)PZ(k) \\ &= Z^T(k)A_{zd}^T P A_{zd} Z(k) - Z^T(k)PZ(k) \\ &= Z^T(k)\Lambda Z(k), \end{aligned} \quad (17)$$

where

$$\Lambda = A_{zd}^T P A_{zd} - P. \quad (18)$$

By Schur complement, (15) means that $\Lambda < 0$, we can obtain from (17) that

$$\begin{aligned} &E\{V[Z(k+1)]|Z(k)\} - V[Z(k)] \\ &= Z^T(k)\Lambda Z(k) \\ &\le -\lambda_{\min}(-\Lambda)Z^T(k)Z(k) < -\alpha Z^T(k)Z(k), \end{aligned} \quad (19)$$

where

$$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \sigma\}, \sigma := \lambda_{\max}(P). \quad (20)$$

From (19), we have

$$\begin{aligned} &E\{V[Z(k+1)]|Z(k)\} - V[Z(k)] < -\alpha Z^T(k)Z(k) \\ &< -\frac{\alpha}{\sigma}V[Z(k)] := -\psi V[Z(k)], \end{aligned} \quad (21)$$

i.e.,

$$E\{V[Z(k+1)]|Z(k)\} \le (1-\psi)^k V[Z(0)]. \quad (22)$$

Consequently, based on the definition, we can follow from [19] and conclude that the closed-loop system (11) is exponentially mean-square stable. The proof is completed. □

Lemma 4. *Given a scalar quantity $\gamma > 0$. The plant (11) is multiplying mean-square stable, suppose there exists a positive definite matrix P satisfying*

$$\begin{bmatrix} -P & 0 & A_{zd}^T P \\ 0 & -\gamma^2 I & B_{zd}^T P \\ PA_{zd} & PB_{zd} & -P \end{bmatrix} < 0. \quad (23)$$

Theorem 1. *Given a scalar quantity $\gamma > 0$. The plant (11) is incrementally mean-square stable, suppose there exist real matrices Q_1 and Q_2, positive definite matrices $S = S^T > 0$, and $Q = Q^T > 0$ such that*

$$\begin{bmatrix} -S^{-1} & * & * & * & * \\ -S^{-1} & -Q^{-1} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{zd} + B_{zd}Q_2 & A_{zd} & B_{zd} & -S & * \\ A_{zd} + B_{zd}Q_2 + Q_1 & A_{zd} & B_{zd} & -Q & -Q \end{bmatrix} < 0. \quad (24)$$

Proof. We divide P and P^{-1} as

$$P = \begin{bmatrix} R & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix},$$
$$P^{-1} = \begin{bmatrix} S & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, \quad (25)$$

where the segmentation of P and P^{-1} is suitable for that of A defined in (4), i.e., $R \in \mathbb{R}^{(n+m+p)\times(n+m+p)}, X_{12} \in$

$\mathbb{R}^{(n+m+p)\times(n+m+p)}$, $X_{22} \in \mathbb{R}^{(n+m+p)\times(n+m+p)}$, $S \in \mathbb{R}^{(n+m+p)\times(n+m+p)}$, $Y_{12} \in \mathbb{R}^{(n+m+p)\times(n+m+p)}$, $Y_{22} \in \mathbb{R}^{(n+m+p)\times(n+m+p)}$. Define

$$\begin{aligned} T_1 &= \begin{bmatrix} S & I \\ Y_{12}^T & 0 \end{bmatrix}, \\ T_2 &= \begin{bmatrix} I & R \\ 0 & X_{12}^T \end{bmatrix}. \end{aligned} \tag{26}$$

This means $PT_1 = T_2$ and $T_1^T P T_1 = T_1^T T_2$. By application of the congruence transformations $\text{diag}\{T_1, I, T_1\}$ to (23), we get

$$\begin{bmatrix} -S & * & * & * & * \\ -I & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{zd}S + B_{zd}Y_{12} & A_{zd} & B_{zd} & -S & * \\ J_1 & RA_{zd} + X_{12}B_{zd} & RB_{zd} + X_{12}B_{zd} & -I & -R \end{bmatrix} < 0, \tag{27}$$

where $J_1 = RA_{zd}S + RB_{zd}Y_{12}^T + X_{12}B_{zd}S + X_{12}A_{zd}Y_{12}^T$. We continue to use the congruence transformations $\text{diag}\{S^{-1}, I, I, I, R^{-1}\}$ to (27) and such that

$$\begin{bmatrix} -S & * & * & * & * \\ -S^{-1} & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{zd} + B_{zd}Y_{12}^T S^{-1} & A_{zd} & B_{zd} & -S & * \\ J_2 & J_3 & B_{zd} + R^{-1}X_{12}B_{zd} & -R^{-1} & -R^{-1} \end{bmatrix} < 0, \tag{28}$$

where $J_2 = A_{zd} + B_{zd}Y_{12}^T S^{-1} + R^{-1}X_{12} \quad B_{zd} + R^{-1}X_{12}A_{zd} Y_{12}^T S^{-1}$, $J_3 = A_{zd} + R^{-1}X_{12}B_{zd}$.

The changes to define the controller parameters are now as follows:

$$\begin{aligned} Q_1 &= R^{-1}X_{12}A_{zd}Y_{12}^T S^{-1}, \\ Q_2 &= R^{-1}X_{12}B_{zd}, \end{aligned} \tag{29}$$

then, let $Q = R^{-1}$, (28) becomes

$$\begin{bmatrix} -S^{-1} & * & * & * & * \\ -S^{-1} & -Q^{-1} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{zd} + B_{zd}Q_2 & A_{zd} & B_{zd} & -S & * \\ A_{zd} + B_{zd}Q_2 + Q_1 & A_{zd} & B_{zd} & -Q & -Q \end{bmatrix} < 0, \tag{30}$$

that is (24).

In addition, assume that the matrix inequality is workable, then we have $\begin{bmatrix} -S^{-1} & -S^{-1} \\ -S^{-1} & -Q^{-1} \end{bmatrix} < 0$, i.e., $\begin{bmatrix} S & I \\ I & R \end{bmatrix} > 0$. Derive $I - RS = X_{12}Y_{12}^T < 0$ directly from $XX^{-1} = I$. Thus, we can always find a square and nonsingular X_{12} and Y_{12} [20]. The proof is completed.

Because the augmentation function (11) is multiparametric and time-varying, the ordinary model prediction controller is not applicable to such models, so we design and propose a novel modified model prediction controller. In contrast to the cost function of the general model predictive controller, the state variables are replaced by the predictor variables initially obtained from the state predictor. Then, a logic programming approach is used to incorporate all possible scenarios into the prediction range. Thus, the cost function is transformed into a simultaneous minimum linear matrix inequality (LMI) with constraints. Therefore, the toolbox YALMIP [21] as well as the multiparametric programming solver MPT3 (Multi-Parameter Toolbox) [22] were used to finally solve this minimax programming problem. To begin with, the control target can be written as

$$\begin{aligned} &\arg\min_{u_c(0)} \sum_{k=0}^{M-1} \left[e^T(k)Qe(k) + \rho_k \Delta u_c^{\;T}(k) R \Delta u_c(k) \right] \\ &\quad + e^T(M)Qe(M), \end{aligned} \tag{31}$$

subject to

$$Z(k+1) = A_{zd}(k)Z(k) + B_{zd}(k)u_c(k), \tag{32}$$

$$\begin{aligned} e(k) &\in [e_{\min}, e_{\max}], \\ \Delta u_c(k) &\in [\Delta u_{c\min}, \Delta u_{c\max}], \\ \rho_k &\in \{0, 1\}, \end{aligned} \tag{33}$$

where $e(k) = y(k) - r(k)$, $r(k)$ is the reference signal for the kth time step; Q and R are positive semidefinite and positive definite weighting matrices, respectively; and $e_{\min}, e_{\max}$ and $\Delta u_{c\min}, \Delta u_{c\max}$ are the lower and upper bounds of $e(k)$ and $\Delta u_c(k)$, respectively. It is important to note that M is the prediction horizon of the model prediction controller.

It is worth pointing out that since the result of optimization (31) cannot be easily implemented, we use an efficient method from [23], which is to rewrite it as min ε,

$$\begin{aligned} &\sum_{k=0}^{M-1} \left[e^T(k)Qe(k) + \rho_k \Delta u_c^{\;T}(k) R \Delta u_c(k) \right] \\ &\quad + e^T(M)Qe(M) < \varepsilon, \end{aligned} \tag{34}$$

where ε is a representational variable. Thus, in this thesis, we transform the optimization problem (31) into an LMI, as illustrated in equation (34). A wide range of research has been published in academia on how to deal with the LMI problem. The Riccati equation is typically used to deal with unconstrained problems [24], while constrained problems are usually handled with Schur complement expansions to higher order LMIs, which are then solved by the MATLAB toolbox [25].

However, in this study, the prediction model (32) under the networked control system has uncertainty and stochasticity, so dealing with optimization problems cannot be handled in the usual way. From the point of view of robustness, this study includes all possible states of the prediction model (32) in the future. Hence, [26, 27]

$$A_{zd}(k)|_{\rho_k=1} = \begin{bmatrix} A_1 & B_1 \\ 0 & 0 \end{bmatrix},$$
$$A_{zd}(k)|_{\rho_k=0} = \begin{bmatrix} A_1 & B_1 + B_2 \\ 0 & I \end{bmatrix},$$
$$B_{zd}(k)|_{\rho_k=1} = \begin{bmatrix} B_2 \\ I \end{bmatrix},$$
$$B_{zd}(k)|_{\rho_k=0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (35)$$

In addition, the uncertain variables $\beta_0(k)$ and $\beta_1(k)$ are introduced as

$$\begin{cases} \beta_0(k) = 0, \\ \beta_1(k) = 1, \end{cases} \rho_k = 1, \qquad \begin{cases} \beta_0(k) = 1, \\ \beta_1(k) = 0, \end{cases} \rho_k = 0. \quad (36)$$

Therefore, in order to make equation (12) supported by YALMIP, we transform equation (11) using the format of linear parameter variation (LPV) [28].

$$Z(k+1) = \beta_1(k)A_{zd}(k)|_{\rho_k=1}Z(k) + \beta_0(k)A_{zd}(k)|_{\rho_k=0} \cdot Z(k) + \beta_1(k)B_{zd}(k)|_{\rho_k=1}u_c(k) + \beta_0(k)B_{zd}(k)|_{\rho_k=0}u_c(k). \quad (37)$$

Special attention should be paid to the fact that the influence of the prediction range and LPV format may allow the order of the linear matrix inequality to grow rapidly, making the solution computationally costly and infeasible. In this study, we use a dynamic planning of discrete systems method with the aim of reducing the computational cost and increasing the speed of the solution. The dynamic programming method for discrete systems is used by decomposing the planning problem into subproblems and then deriving the solution of the original problem from the solution of the subproblems [29].

Programming problems for dynamic $x_{i+1} = f(x_i, u_i)$ with cost function,

$$J_0(x, U_0) = \sum_{i=0}^{M-1} l(x_i, u_i) + l_f(x_M), \quad U_0 = \{u_0, u_1, \ldots, u_{M-1}\}, \quad (38)$$

where l_f is the final cost and l is the running cost. The partial cost function is defined as

$$J_i(x, U_i) = \sum_{i=0}^{M-1} l(x_j, u_j) + l_f(x_M), \quad U_i = \{u_i, u_{i+1}, \ldots, u_{M-1}\}. \quad (39)$$

Therefore, the value function can be reduced as

$$V(x, i) = \min_{U_i} J_i(x, U_i), \quad (40)$$

when $i = 0, V(x, 0) = \min_{U_0} J_0(x, U_0)$ is the ultimate goal of programming problem. It should be noted that the value function (38) can be treated as the Bellman equation [30]. Thus,

$$V(x, i) = \min_{U_i}[l(x_i, u_i) + V(x, i+1)] = \min_{U_i}[l(x_i, u_i) + V(f(x_i, u_i), i+1)]. \quad (41)$$

In summary, in this research, the running cost is $e^T(k)Qe(k) + {u_c}^T(k)Ru_c(k)$ and the final cost is $e^T(M)Qe(M)$. To solve the MPC programming problem, the method of solving the partial value function is utilized in this thesis. This method is solved by solving $V(x, i)$ recursively from $i = M - 1$ to $i = 0$. u_0 is the end result of the control input, which is gotten by solving the final goal $V(x, 0)$.

Now, we use the YALMIP toolbox to solve this minimal programming problem. Algorithm 1 can be summarized as follows.

The structural block diagram of the closed-loop control system proposed in this article is shown in Figure 4.

3. Results and Discussion

To verify the performance of the approach, a simulation example is given. This example is based on a fixed-wing aircraft model. In this simulation experiment, the slope response and sinusoidal response of the attitude are investigated to exhibit the feasibility and robustness of the method. Compared with our previous study [26, 27], the treatment of time-delay in this study has better predictability and robustness.

3.1. Example. Consider a MIMO fixed-wing aircraft system as a target plant for networked control systems (NCSs) with a state space of [27]

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.0151 & -60.565 & 0 & -32.174 \\ -0.0001 & -1.3411 & 0.9929 & 0 \\ 0.00018 & 43.254 & -0.8694 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -2.516 & -13.136 \\ -0.1689 & -0.2514 \\ -17.251 & -1.5766 \\ 0 & 0 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t), \end{cases} \quad (42)$$

Step1: define decision variables;
Step2: define the objective function;
Step3: defining constraints;
Step4: set relevant parameters such as solver;
Step5: start solving;

Algorithm 1

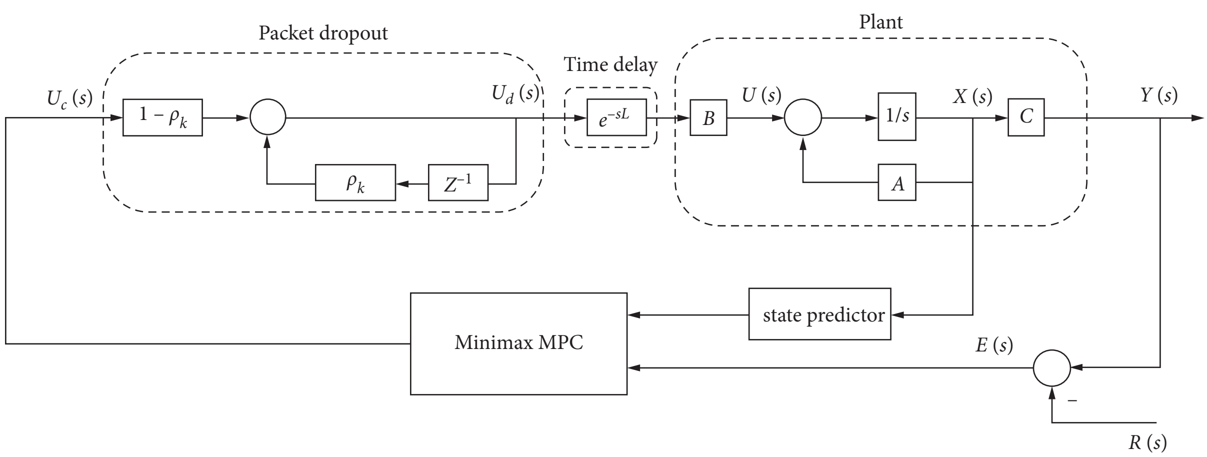


Figure 4: Block diagram of the structure of the closed-loop control system.

The fixed-wing aircraft model is shown in Figure 5. The control inputs $u(t)$ to the system are the elevator and flaperon, which have an angular range of ±25° and ±75°, respectively, and the output $y(t)$ of the system is the altitude. In this study, the sampling period, also referred to as the transmission interval, is 1 second.

The slope response results of the NCSs with the target plant (42) in Figure 2 are shown in Figure 6, which is adjusted by a traditional model predictive controller. The time-delay of the control input networks is 1.5 s and the dropout rates are 0.1.

As shown in Figure 6, the delays and randomness in the NCS lead to severe instability in the studied flight system. To substantiate the advantages of the proposed method, Figure 6 illustrates the simulation results using a proposition under more demanding conditions, where the delay time is 2.5 s and the dropout rate is 0.25.

Figure 7(a) shows the ramp response of the system (42) with the mentioned method for different delay times. The results are shown in the simulation graph. When the delay time of the networked control system increases from 1.5 s to 2.5 s, the entire networked control system remains stable, thus reflecting the stability and robustness of the proposed method. At the same time, the results in Figure 7(b) show the slope response of NCSs at different packet loss rates.

Similarly, Figure 7(b) also demonstrates that the networked control system is well compensated by the ZOH for the control inputs after packet loss occurs, thus enabling the control system to ensure good stability. The results show that the whole control system remains approximately stable when the dropout rate increases from 0.1 to 0.25.

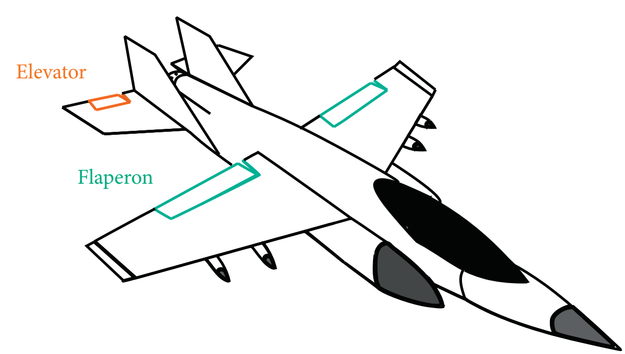


Figure 5: Sketch of fixed-wing aircraft model.

Figures 7(c) and 7(d) show the packet loss timing diagrams for packet loss rate of 0.1 and 0.25, respectively, when $\rho_k = 0$ packets are lost, vice versa.

Figure 8 demonstrates the sine response curves with different delay times for the same packet dropout rate by the proposed method. The results in Figure 8 show that when the packet dropout rate rises to 0.25, the overall control system remains stable while the performance decreases slightly.

Finally, to illustrate the superiority of the method in this paper, the method based on Pade approximation in the literature [27] is compared with the prediction-based control method proposed in this paper, as well as with the traditional model prediction control method. In addition, all three control methods are compared under the condition of dropout rate which is 1 and delayed time is 1 s.

The simulation results are shown in Figure 9. As can be seen from Figure 9, after comparing with other methods, the

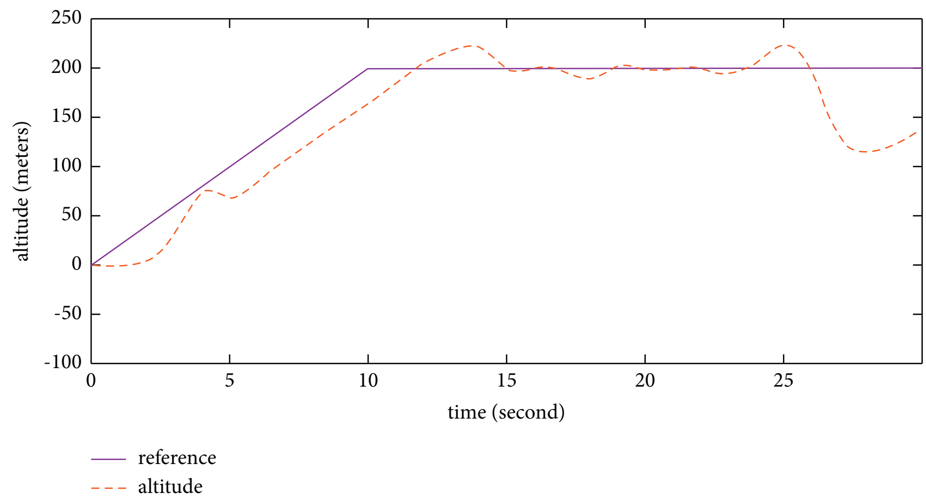
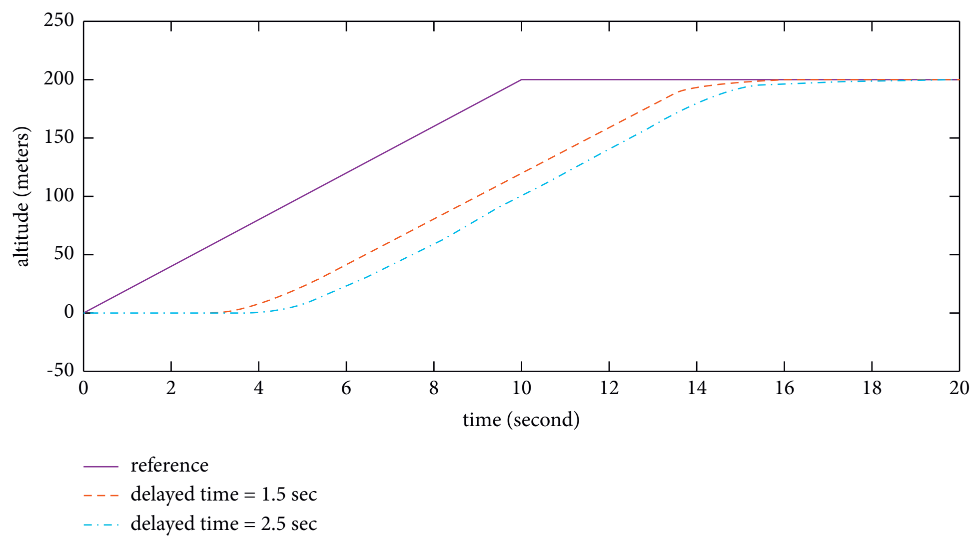
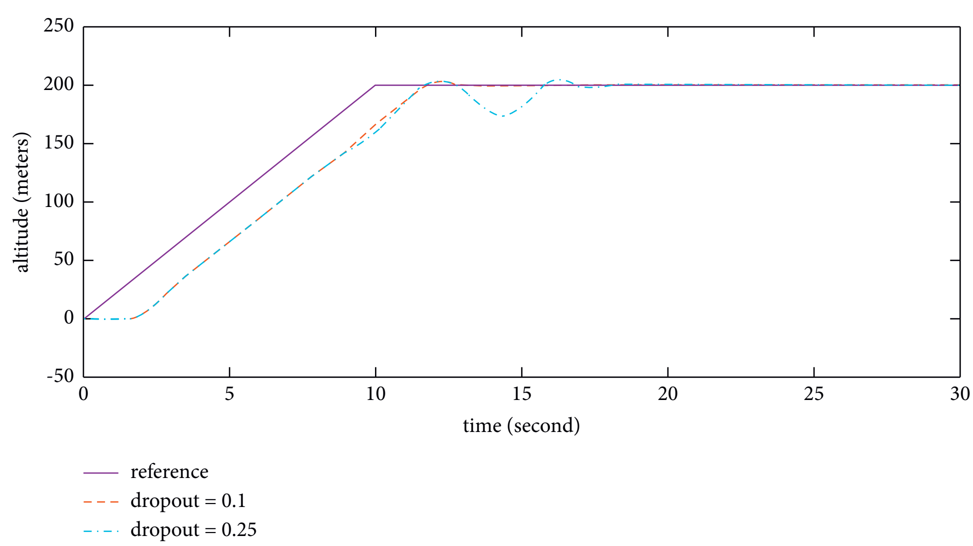


Figure 6: Slope response of conventional model predictive controllers to NCSs plants (42).

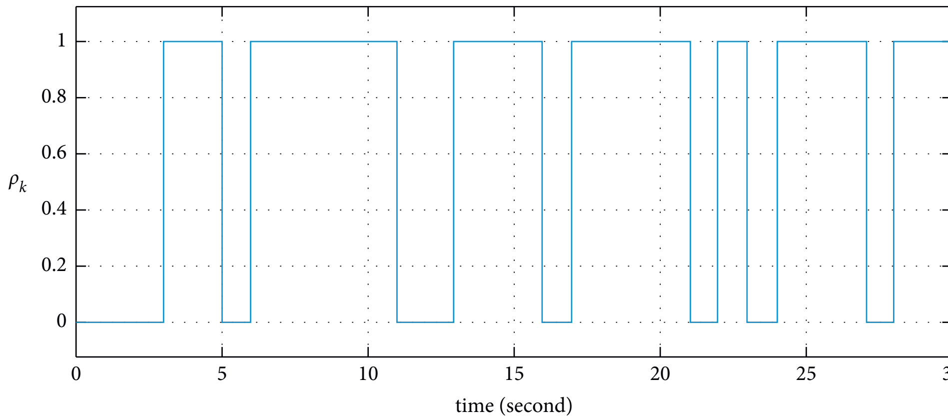


(a)

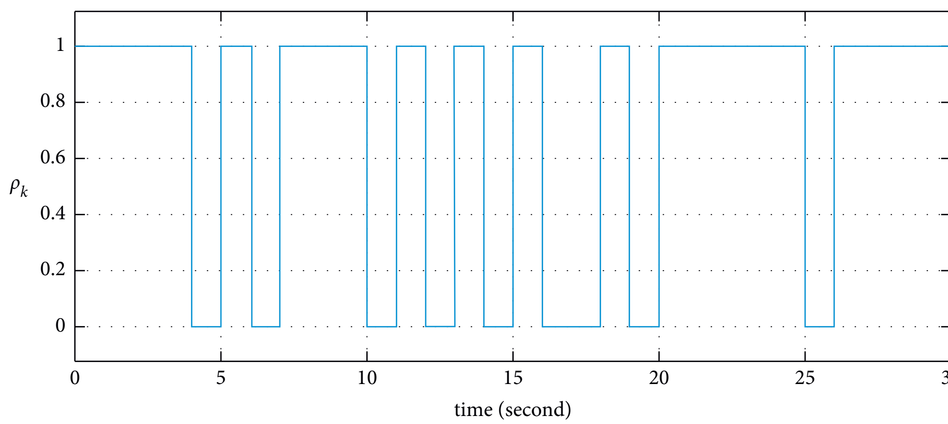


(b)

Figure 7: Continued.

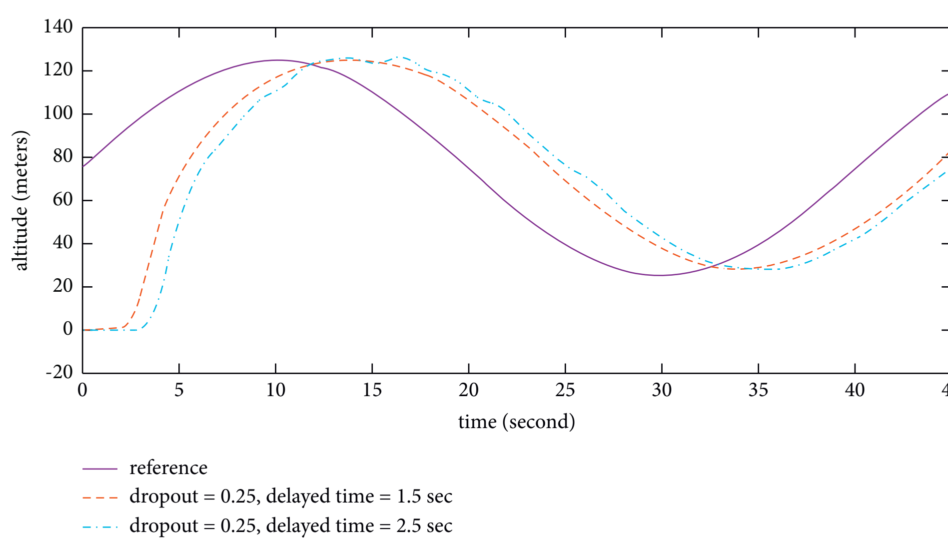


(c)



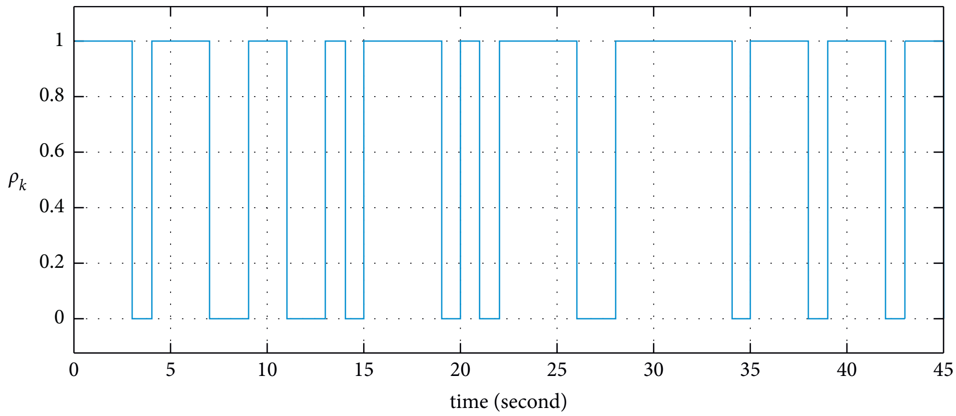
(d)

FIGURE 7: Ramp responses of networked control target plant (42) by a novel modified model predictive controller. (a) Waveforms of altitude under variant delayed times. (b) Waveforms of altitude under variant packet dropouts. (c) When the dropout rate is 0.1, the value of ρ_k. (d) When the dropout rate is 0.25, the value of ρ_k.



(a)

FIGURE 8: Continued.



(b)

Figure 8: Sine responses of networked control target plant (42) by a novel modified model predictive controller. (a) Waveforms of altitude under variant delayed times with the same dropout rate. (b) When the dropout rate is 0.25, the value of ρ_k.

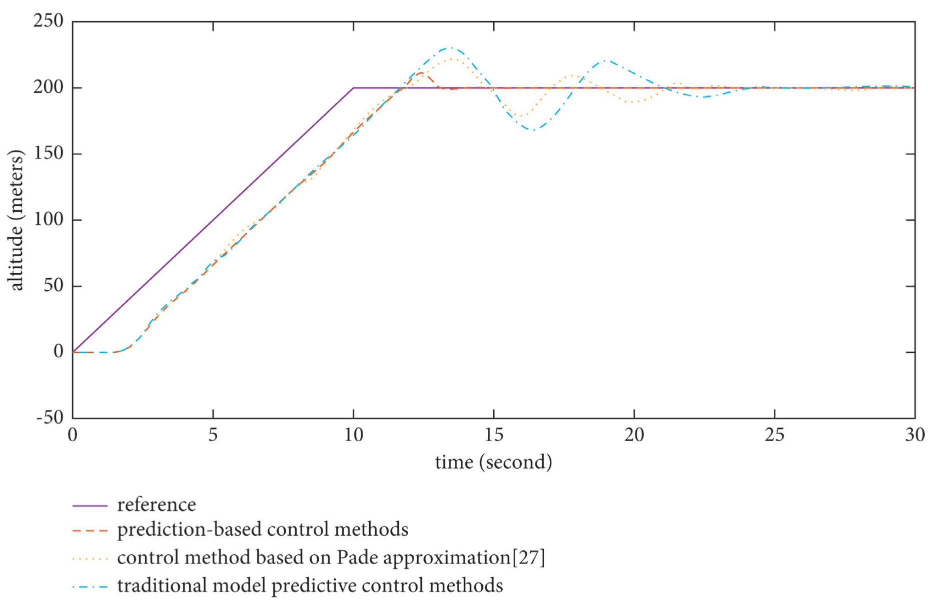


Figure 9: Comparison with different control methods.

proposed method in this paper makes the response speed and tracking accuracy of the networked control systems significantly improved.

4. Conclusions

A novel prediction-based controller design is proposed for the case of random packet dropouts and time-delay in the control channel of a networked control system. The sequence of packet dropouts is modelled as a Bernoulli process, which is compensated by a zero-order holder (ZOH)-based module, and a state predictor is used to obtain the predicted state of the delay time. Considering the dropout compensator and the state predictor, we design and propose a novel modified model prediction controller. In contrast to the cost function of the general model prediction controller, the state variables are replaced by the predictor variables obtained initially from the state predictor. Then, a logic programming approach is used to incorporate all possible scenarios into the prediction range. Thus, the cost function is transformed into a simultaneous minimum linear matrix inequality with constraints. Therefore, the toolbox YALMIP is employed in order to finally solve this minimal programming problem. Simulation results show the feasibility and performance of the proposed approach. The propositions in this study exhibit outstanding stability and are advantageous in reducing the solution time and enhancing the robustness of network control systems with delay and dropout. For future research, we plan to further increase the speed of the controller.

Data Availability

No data were used to be support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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