1. Introduction

A high-tech OEM strategically outsources the manufacturing of an innovative new product to a contract manufacturer (CM). Based on the strategic partnership, OEM and CM (OEM-CM) share information transparently with each other. Before the start of production (SOP), the OEM shares business information, including the maximum available number of manufacturing units and new product release time at the latest. Assuming the Bass diffusion model (BDM) is appropriate for innovative short life cycle products, OEM faces a decision problem: the capacity procurement deployment under the new product diffusion process and production ramp-up. Thus, we develop a model that provides capacity expansion decision support regarding when and how much in this research. The solution strains trade-offs between inventory holding cost, production cost, backlog cost, and manufacturing unitary relevant cost to maximize the profit span between SOP and the end of the product life cycle (EPLC).

The focus of this study was the policy of two-phase capacity deployment under demand diffusion and production ramp-up. OEM conquers the adjustment friction based on the policy to pursue the optimum profit of innovative new product introduction. Reserving the capacity from CM through contract mechanism tactically, dominant OEM programs the rational capacity deployment, and then, CM implements the production operation from SOP to the end of production. For an innovative new product, the length of production and sales is getting shorter. The operation coordination between manufacturing and marketing depends on dynamic production and demand rates. Essentially, production ramp-up speed and demand diffusion speed are the underlying factors that determine OEM’s profit.

OEM owns core capabilities: innovation, branding, and marketing. His strategic partner, CM, invests in specific and costly production equipment and organizes the ramp-up team to implement the new product’s manufacturing process. For example, CM follows the production specification and minimum production rate defined by OEM and applies for the conformance verification and qualification for starting production. The operation interface between OEM-CM is integrated into information flow and material flow, as
shown in Figure 1. The cyclic operation runs over the product life cycle (PLC), and the production rate grows with a ramp-up learning effect. The demand rate rises and declines with the diffusion effect evolving along the bell-shaped curve. The decision hierarchy level can be drawn as strategy, tactic, and operation. Order policy is based on inventory status and rules; physical material flow is driven by order policy. Information flow updates the status for each node at each period.

A good capacity strategy is based on logically reasoning sales forecasts. Under the diffusion process, a successful new product introduction depends on the appropriate production amount and opportunity capacity deployment. Van Mieghem [1] clarified that capacity size in an OEM-CM setting is a production network’s production abilities and limitations. Early this century, Cantamessa and Valentini [2] performed a study that jointly considered learning power and Bass diffusion functions. They emphasized the challenges resulting from the peak characteristic of instantaneous demand trajectory. A simple mixed-integer linear programming model was developed and then experimented with realistic data for different scenarios. Glock et al. [3] used time constant production rate and logistic curve for demand growth rate in a capacity planning dynamic model. Extending from Glock et al. [3], Kim et al. [4] lately model a multistage production planning considering worker’s learning effect and dynamic fine-tune capacity adjustment policy. Executing the ramp-up process, the production system supports the planner not only the process line balance but also the production cost. The demand rate function used by Glock et al. [3] and Kim et al. [4], a sigmoid curve, does not have a peak effect as the original Bass demand model. However, the method used in modeling labor production is inappropriate for the scenario considered in an OEM-CM production network.

Capacity planning, concerning determining sizes and timing, involves strategic and tactical issues of production-marketing coordination (Shapiro [5]). When introducing a new product, an OEM requires joint consideration of demand trajectories and CM’s production rate over the PLC. OEM markets evaluate and analyze the possible market potential of new product and their buying motivation. Customers demand products with higher functionality and quality, resulting in a rapid rate of innovation and a shorter life cycle for these products.

Subject to the contract terms, operation management is sustainable and implementable. In addition, an OEM confronts the tactic problems, including initial capacity, initial inventory, capacity adjustment, flexible capacity, and sales loss during the production ramp-up period depicted in Figure 2. Due to inherent high setup cost, high-tech OEM-CM deploys capacity in two stages. The first-stage periods start from SOP and end at the time point of capacity expansion. The second stage operates over the periods left. We conjecture that OEM confronts the challenges as follows: (1) the target hitting of market potential and outsourcing capacity procurement and allocation, (2) the substitution balance between capacity and inventory, (3) the reconfiguration timing and size of constrained capacity, (4) adjustment under the restriction of commitment, (5) allocation of the commitment and flexible capacity to conquer the resistance of capacity adjustment, and (6) sole outsourcing or dual outsourcing. Considering the resistance of capacity adjustment and the feasibility, OEM deploys outsourced capacity in two stages to balance the capacity cost, inventory cost, reconfiguration cost, and deferred cost like sales lost cost in a short time.

Heuristically, the continuous optimum constant capacity level that trade-offs the inventory and backlog is feasible. If not subject to lumpy capacity friction, it is possible to find a relative optimal capacity level that creates just enough stock for peak demand but keeps inventory as low as possible. The constant capacity level approaches the output ceiling of the production curve. Ideal cumulative production is shown as the black dot line in Figure 3(a) positioned between low production and high production. However, this option is not implementable due to a lumpy capacity constraint.

Time-dependent functions of manufacturing production rate \( o(t) \) and marketing demand rate \( d(t) \) constitute a critical supply/demand dynamic system for OEM. Joint decisions based on the dynamic changes resulting from these functions are crucial for OEM’s firm profitability. OEMs in telecommunications, electronics, semiconductors, pharmaceuticals, and automobiles are careful to assess total capacity strategically and however, tactically implement capacity deployment in time and scale.

Our paper addresses OEM’s capacity planning through procurement and deployment as a crucial issue. OEM faces a hierarchy of decisions. Decision on capacity sizing relies on an exogenous defined demand rate. Acquiring the required capacity needs a good prediction of sales peak and the timing of the peak well in advance. Through the joint consideration of demand and production rates, elaborated production and sales plans assure market potential realization. OEM-CM deploys the capacity with expensive upfront investment and introduces a new manufacturing process through a pilot run with learning before doing and improvement.

The strategic issue, i.e., “how much capacity to install?” and tactical matters, “when to launch a product and expand capacity?” are significant challenges in deciding on capacity procurement policies for achieving new product success. Decisions on “the size of capacity” and “the time to market its new product” were examined by Ho et al. [6]. Due to the mathematical intractability of optimal control theory, fixed capacity is adopted for their research. They generalized the BDM by allowing a supply constraint. They attempted to find the optimal dynamic sales plan under constrained capacity. In short, they apply optimal control theory based on a fixed capacity. In a Bass-like diffusion environment with a supply constraint, they obtain closed-form expressions of demand and sales dynamics. They and Kumar and Swaminathan [7] and Shen et al. [8] used the modified BDM to investigate the diffusion growth and sale dynamics under constrained capacity.

Mathematical intractability is not unusual. Nonlinear ordinary and partial differential equations do not have, in general, analytic or closed solutions. Numerical methods can integrate them, and the differential derivatives in the
differential equations can be replaced by difference approximations at a discrete set of points in space and time. Meanwhile, the size of the solution or output from the numerical method is significant; hence, table and graphical description of the solution are needed. The Laplace transform is a powerful tool for solving differential and integro-differential equations in engineering sciences. However, the solution sometimes cannot be inverted to the real domain by
the analytic method. Qiang et al. [9] used numerical methods to solve the real domain. The numerical method could generate big data, which needs graphical description. Asjad et al. [10] applied the Prabhakar fractional derivative in the transport phenomena containing nanoparticles, which was studied by Asjad et al. [10] using the Prabhakar fractional derivative. Their results were analyzed using Mathcad software. Abd El-Salam et al. [11] introduced a numerical technique for solving general nonlinear ordinary differential equations (ODEs) with variable coefficients and given conditions. The collocation method is used with rational Chebyshev (RC) functions as a matrix discretization to treat the nonlinear ODEs. The rational Chebyshev collocation (RCC) method is used to transform the problem into a system of nonlinear algebraic equations.

In this study, we will develop a numerical optimization model and solve using a CPLEX solver to study capacity policy for an OEM under production ramp-up and demand diffusion. Mathematics, numerical and graphical analysis, and computation power of this era generate jointly exciting results.

Capacity sizing and timing are crucial for OEM-CM. From the OEM’s point of view, a maximum profit objective function is set to evaluate the two-stage capacity policy. Although our dynamic planning approach resembles Glock et al. [3], our study has two significant differences. We work in a “lumpy” and “irreversible” increment production network and diffusion demand. ”Timing” is an important control factor in new product introduction success. The keys are time to market and time to volume (TTV), as defined by Terwiesch et al. [12]. Demand rate differs from production rate, resulting in the so-called nutcracker phenomenon on OEM. The evolution phenomenon for fruit flies vs. elephants in the ecosystem is a metaphor for market and volume “timing” in high-tech evolution speed, “clock speed” (Fine [13]). The Taiwanese-owned brand manufacturer, HTC, could not keep production pace with the diffusion speed of demand, resulting in the loss of the market share (Dou and Luk [14], Jim [15], Dou [16]).

In an oligopolistic market, competitors create prosperity together through the social behavior of their respective customer groups. However, the market share game boosts consumer enthusiasm and market vitality. Gu and Xu [17] deeply investigate the competitive diffusion process. The isolated social networks are linked by nodes called the “role of bridges.” Their work explains the plateau demand phenomenon of the sigmoid demand curve used by Glock et al. [3] and Kim et al. [4]. In our paper, we adopt the original BDM without considering competitive diffusion and focus on the utterly innovative product in an entire market. For the incumbent, deterring the new entrant needs speed. Trajectories of demand rate and production rate functions are vital factors. They are characterized drastically differently. For high-tech consumer durable products with a short life cycle, the demand trajectory is evident to grow to peak and then decline to end. At the same time, production after a pilot runs slowly and gradually improves to a plateau. Diffusion speed is measured by the frequency of generation alternation. Investigating indicator leading technology in the energy industry each era, demand diffusion speed takes not a short brewing time to overcome many bottlenecks to obtain development in leaps. Recently, Wang and Sun [18] formulated a diffusion model based on BDM at the macro- and micro-levels. The supply side, industry associations, breakthrough technology knowledge, the dominant side, and the government create the energy industry environment. On the client side, enterprises, divided into innovators and imitators, evaluate the leading technology by “expect utility” and “actual utility.” Their model provides the diffusion metrics for the energy industry policy by embedding two coefficients, market and policy, into BDM. Diffusion mechanism for energy industry leading technology is the topic of their research. Unlike their research scenario, our research insights into the production capacity policy that hinges on ramp-up and diffusion effects. Innovative consumer durable products are rolled out frequently to consumers through the network environment dominated by OEM, manufactured by CM.

Our model addresses the capacity deployment problems under demand diffusion and production ramp-up. In particular, the trajectories of demand and supply constitute the dynamic environment bounded by dynamic constraints. The capacity decisions are supported as follows:

(i) The opportune timing and reasonable size of the initial capacity installation and later reconfiguration

(ii) The contingent response to the impact of cost parameters on capacity deployment

(a) The new product launch timing is based on cost configuration

(b) The terminating timing of production
Five steps of innovative solutions are listed as follows:

(i) First, the demand rate function and production rate function are discretized to create the dynamic data environment for simulation.

(ii) Second, the decision support of capacity evolution is simulated. The simulation starts from SOP and ceases at EOPL. The cumulative dynamic output of demand, backlog, production, and inventory (DBXI) is obtained.

(iii) Third, a framework is built to support the co-contract of OEM-CM through decision variables, capacity size, and deployment timing.

(iv) Fourth, the phenomenon under various scenarios through graphical analysis and cost-benefit comparisons is insight.

(v) Fifth, the mathematical model to the CPLEX solver and run scenarios composed of different parameters is transmitted.

To the best of our knowledge, our model is the first one that could resolve the mismatch of peak characteristic of demand function and plateau characteristic of production function, as well as capacity lumpy characteristics. We provide the graphical illustration regarding the timing of product launching, capacity expansion and termination, and capacity expansion size. We develop an optimization model for an OEM under demand diffusion and production ramp-up.

The rest of this study is organized as follows. Section 2 details the mathematical function of BDM and the time constant model, and the concept of capacity expansion technology. Section 3 outlines the problems and assumption. Section 4 presents the model formulation. We use graphical illustration to describe the generation of data sources of model run and provide graphical illustration analysis of model run output in Section 5. Section 6 discusses the results. Section 7 concludes and suggests future research.

2. Mathematical Functions and Capacity Expansion Technology

2.1. Bass Diffusion Model. Bass [19] developed a new product growth model for consumer durable goods. Using the assumption that the timing of a consumer’s initial purchase is related to the number of previous buyers, he established a theory of timing of initial purchases of a new product. The BDM was voted as one of the “Top 10 Most Influential Papers” in the Management Science 50th anniversary (Bass [20]). Innovative and imitative behavior is proposed for the behavioral rationale; it yields good predictions of the sales peak and the timing of the peak. The basic assumption is that at each moment in time $t$, two groups of customer result in the result of the demands. One exhibits innovative behavior and the other responds to word of mouth (WOM). The parameters of the BDM were estimated using the regression model (Bass [19]) and the product-specific values obtained from the literature (Sultan, Farley, & Lehmann [21], Lilien and Bulte [22], Bayus [23]). We list the demand diffusion of the new high-tech product following the conditions of the original BDM. The conditions about the product and social behavior are summarized as follows (Bass [19] and Bass [20]): (1) durable product, (2) constant market potential, (3) no substitutability and complementarity with other products, (4) fixed price, (5) no-repeat purchase for each customer, (6) binary adopt the behavior, (7) no supply constraint, and (8) no lost sales. We used $D(t)$ to represent the number of customers who have already adopted the product. The number $m = D(t)$ is the number of customers who have not yet adopted the product, where $m$ is the market potential. Bass formulated a precise and parsimonious model as in equation (1) that the cumulative demand $D(t)$ and instantaneous demand rate, $d(t)$, as shown in equations (2) and (3).

$$d(t) = \frac{dD(t)}{dt} = p(m - D(t)) + \frac{q}{m} D(t)(m - D(t)),$$  
$$D(t) = m \frac{1 - e^{-(p+q)t}}{1 + (q/p)e^{-(p+q)t}},$$  
$$d(t) = m \frac{p(p + q)^2 e^{-(p+q)t}}{[p + (qe^{-(p+q)t})]^2},$$

where $m$ is the market potential, and $p$ and $q$ are constants representing the relative effects of mass media and WOM on the population. The constant $p$ is the probability of an initial purchase at each time interval, including $t = 0$. The magnitude of $p$ reflects the innovative diffusion effective stages’ importance on the demand and production system during the early life cycle. Principally, $p$ and $q$ depend on product type. The firm has some latitude in putting effort into strong advertising or focused actions to alter $p$ and $q$ more or less.

In terms of the behavioral rationale of the innovative and imitative behaviors, Bass [19] is a pioneer in verifying that related industry historical data yields a good prediction of the sales peak and timing of the sales peak. This merit is essential for OEM capacity expansion planning.

Parameters $p$ and $q$ describe the diffusion of a product. According to the product types and associated relative magnitude between these two parameters, Figure 4 displays three distinct classes of demand curves. The dotted curves for $p$ greater or equal to $q$ are style goods and perishable items; dash-dot curves for $p$ much smaller than $q$ are imitative products, the WOM effect prevailing imitative type; and solid curves for $p$ smaller than $q$ describe innovative behavior prevailing products. We aimed at the short life cycle of innovative type products, and it is the major challenge of OEM in semiconductor and consumer electric industries.

Bass explained that analysis of the potential market and the buying motives could make it possible to guess the market potential $m$, the size of the market, and the values of $p$ and $q$. Hence, it is essential to understand three parameters. The constant $p$ is the probability of an initial purchase at each period, and the initial magnitude reflects the influence of innovators in the social system. Two purchaser groups
interact to constitute the demand rate, as shown in Figure 5. The BDM assumes that, at each moment in time, $t$, the potential market $m$, is split into two groups. One has already adopted, $D(t)$, and the other has not, $m - D(t)$.

Relaxing some limitations of original BDM, there are many variant modifications for adapting to different scenarios. For example, Anand et al. [24] explore the impact of positive and negative factors on innovative and imitative behavior over diffusion. They conceptualize these influence activities for diffusion as "infusion." Unlike BDM with time-variable only, they used two-dimensional variables including time and revenue. The infusion incurs cost variables such as reliability and quality costs combined with time. Similar to our dynamic capture intention of production rate and demand rate, their model uses interplay infusion rate and diffusion rate to construct the profit over the PLC. Demand volatility is not considered in the original BDM. From a standard Brownian motion view, diffusion is a continuous-time stochastic process. This study will assume that demand is deterministic and adopt a stationary scenario to study the mathematical intractable capacity deployment policy as the cornerstone for further research related to stochastic issues.

2.2. Time Constant Model. Many articles have defined ramp-up (e.g., Terwiesch et al. [12] and Hansen and Grunow [25]). Considering the effects of new production ramp-up and demand diffusion process on capacity planning, ramp-up is the period spanning over the first production and the attainment of target capacity. It constitutes a significant fraction of the total life cycle of a product, which renders it a substantial part of the sale period. It is critical for the successful introduction of a product to a market. The ramp-up phase is characterized by an increase in output and product quality and a reduction in unit production cost. Terwiesch and Bohn [26] discussed these improvements in production processes resulting from learning, autonomous learning, and learning by experiments. Different learning functions have been developed since Wright (1936) [27] reported the discovery of the learning phenomenon for the aircraft industry about a century ago. In contrast to the new product ramp-up production rate, the new product demand rate fluctuates rapidly. Peak demand rate near the center of life cycle vs. plateau of production rate toward the end of time horizon brings a challenge to the decision-maker of an OEM.

The outsourcing manufacturing process of customized products and complex components to CM resulted in their learning opportunity. They improve efficiency over time by optimizing the production process. Researchers and practitioners have debated the form of the learning curves. Three principal versions of learning curves, i.e., a power function, a sigmoid function, and an exponential function, have been observed often. For a detailed review of theory, models, and applications of learning functions, the reader is referred to Jaber [28].

The time constant model (TCM), an exponential function, was the most suitable in consumer electronics and semiconductor industries because the parameters are easily defined and acceptable to initiatives concerning capacity planning, delivery date, setting of performance, training standard, and system dynamic analysis (Hackett [29], Naim and Towill [30], Glock et al. [3]). The time constant function is related to three parameters, the initial production rate, the maximum production rate that could be attained, and the time constant $\tau$. A nonpeak variation function stabilizes toward the end of the time horizon in conjunction with the logistic demand curve. Glock et al. [3] conducted a production planning study. Their lot sizing model, i.e., finding the lengths of production runs that match a steadily increasing demand, concluded that optimal planning is synchronized between the production rate and demand rate curve as much as possible. Because of mathematical complexity, their model is also developed and demonstrated with numerical examples. However, their study scenario is different from a production network with a lumpy capacity increment.

This study adopts the time constant function as production rate to study the capacity policy for OEM’s ramp-up process. The time constant model is given in the exponential form:

$$o(t) = o_p - (o_p - o_i)e^{-t/\tau},$$

(4)

$$O(t) = \int_0^t o(t)dt = o_p t + \tau(o_p - o_i)(e^{-t/\tau} - 1),$$

(5)

where $o(t)$ is the production rate at time $t$, $o_i$ is the initial production rate, $o_p$ is the maximum production capacity, $t$ is time, $\tau$ is the time constant, and $O(t)$ is the cumulative production. For a shorter life cycle for high-tech consumer durable products, OEM-CM’s improvement rate with experience is essential. In Figure 6, we have displayed three production curves with $\tau = 4$, $\tau = 8$, and $\tau = 12$ over the Bass demand rate curve for $m = 3000$, $p = 0.01$, and $q = 0.35$.

This study aimed at studying the capacity procurement policy for an OEM launching innovative short life cycle products under diffusion growth in demand and learning in production. A few kinds of literature focused on capacity
planning with learning under ramp-up production and demand diffusion. Glock et al. [3] developed a production planning model for a manufacturing system that undergoes a ramp-up period with learning and diffusion in demand. They used the time constant model to characterize the learning effect. However, the demand growth sigmoid function tends to stabilize toward the end of the time horizon.

2.3. Capacity Expansion. As to the production line setup, the unitary production rate is determined by the line balance of each facility capacity. A lumpiness factor is introduced about the friction of capacity expansion (Van Mieghem [1]). The friction of capacity adjustment composes irreversibility, indivisibility (lumpy capacity), and non-convexity. When is it ready to launch a new product in the market? In marketing, it is a milestone problem consideration. From the viewpoint of marketing, managers consider economic factors such as premium price, price skimming, and competition strategy (Ozer [31], Ozer and Uncu [32]).

The SOP and launch hinge on initial production rate, learning capability, initial capacity size, initial inventory, and demand diffusion rate. Timing of SOP, launch, and capacity expansion are critical concerns of OEM. Inventory holding or excess capacity during the production ramp-up period is inevitable—the problems are caused by supply and demand uncertainty. OEM faces the trade-off between inventory cost

Figure 5: Bass demand rate (a) and Bass cumulative demand (b).

Figure 6: Demand rate vs. production rate (a), cumulative demand vs. cumulative production (b), and inventory (c).
and excess capacity cost and takes actions of building up initial inventory from SOP to launch timing under capacity constrained. Considering that the capacity utilization and yield are still low at the beginning, experienced OEM-CM deliberately decides when and how many production lines are opened at SOP for accumulating ample initial inventory (satisfying the initial demand) and accumulating knowledge through learning by doing (Terwiesch and Bohn [26]).

On the other hand, the demand rate increases exponentially after OEM rolls out the new product. Analyzing the production rate and growing demand rate curve, it is noticed intuitively that the increase and the depletion of inventory are the relative rate of production to demand. In the initial periods, inventory increases at the beginning and then decreases. From the viewpoint of tactic, OEM needs support to decide the timing (when) and size (how much) of capacity expansion before the new product launch. If the initial capacity is sufficient, inventory plays the hedging role of diffusion but incurs holding cost. However, it probably is an overwhelming disaster for high-tech with a shorter and shorter life cycle. As the golden sentence of Tim Cook, Apple CEO, said, “Inventory... is fundamentally evil. You want to shorten lifecycle. Asthe golden sentence of Tim Cook, Apple CEO, said, “Inventory... is fundamentally evil. You want to shorten lifecycle. A trade-off strains between overcapacity and losing demand, and smart data generation via micro-manipulation. But in essence, complexity and uncertainty are inevitable. Nevertheless, Intel is highly rigorous in respect of capacity configuration changes. The famous philosophy and methodology, “Copy Exactly” (McDonald [36], Terwiesch and Xu [37]), is adopted globally and documented as Intel’s global norm (Intel Quality System Handbook, 2014). Their argument sustains our assumption of the feasibility that OEM-CM open new production lines to augment capacity through clones that inherit the capability and knowledge of incumbent production lines.

The conformance verification and qualification mentioned in the introduction are executed again when newly added production lines are opened. CM plays the role of the “backyard factory” of OEM. Leveraging information technology, data are transferred swiftly between function departments of OEM-CM at each node of the network. The core competence that OEM-CM reacts to the various uncertainties is a warranty of implementing production ramp-up. Although, during production ramp-up, it is feasible and operational to collect real data (time, cost, and quality-quantity) and smart data generation via micro-manipulation. But in essence, complexity and uncertainty are inevitable.

Inherently, complexity and uncertainty are unavoidable during production ramp-up. The metric dashboard displays smart data only without a preventive mechanism. OEM-CM intensifies three management capabilities: (1) resilience, (2) risk, and (3) agility. Through internalization and knowledge management, OEM-CM contracts the constant coefficient of TCM as the abstract representation of ramp-up management capability. Strengthening ramp-up management capability, OEM-CM implemented the clone of the new production line with a high degree of success. For a more detailed research vein about ramp-up management, we refer the readers to the literature (Mamaghani & Medini [38], Dombrowski et al. [39], Schmitt and Heine [40]).

Terwiesch and Bohn [26] analyzed the time resource allocation between regular production and experiment to explain the trade-off, learning, or process improvement. The knowledge is accumulated through induced learning and autonomous learning. Practice makes perfect; autonomous learning is also represented by the experience curve, a function of accumulated quantity. Induced learn boosts the output with an excellent progression after each improvement by stage. It is a function of time. They used a concise and understandable causal loop diagram to manifest the increasing and decreasing capacity time resource allocation ratio between “time for experiment” and “production.” Thus, different learning and improvement portfolios generate the next cycle’s processing capability. Capability ramp-up is an upward performance based on yield and utilization. The production rate, \(o(t)\), is the instantaneous production output rate at time \(t\). There is a plateau effect of diminishing returns to the learning source. We adopt two-phase capacity deployment to mitigate the “nutcracker” ramp-up effect that Terwiesch and Xu [37] referred to. The dual pressure of the ramp-up effect is caused by capacity friction and surging demand.

3. Problem Description and Assumption

3.1. Problem Description. A common problem confronted by an OEM decision-maker on capacity policy is the mathematical complexity of a dynamic supply/demand system. A high-tech consumer durable product is characterized by a shorter life cycle, taking some time to build up, then exponentially reaching a peak, and then exponentially decaying. It casts a crucial problem for an OEM’s capacity policy. A trade-off strains between overcapacity and losing part of demand. Either delaying the release to build initial inventory ahead of the peak period, or accepting out-of-stock orders during the peak period are options. This study addresses the following problems.

Problem 1. What is the impact of different diffusion coefficients of BDM on capacity expansion timing and sizing under a given initial capacity size at SOP?

Problem 2. What is the interaction between inventory, backlog, and lost capacity cost to optimum profits over the product life cycle?

Problem 3. High innovative diffusion and a lower initiate production rate at SOP render OEM building up inventory to substitute capacity impossible. Either capacity expansion or a new product launch delay is inevitable. What is the impact of cost (inventory holding cost, backlog cost, and
process technology cost) on the timing of new product launching?

**Problem 4.** Capacity adjustment is subject to “lumpy” increment. For the given initial capacity and launch time, the timing and sizing of integer capacity augmentation are critical to optimizing profit are investigated.

**Problem 5.** Capacity expansion considers the impact of irreversibility on the utilization of capacity. The idle capacity occurs when demand declines. To terminate the increasing inactive status, a problem is when the production closes and how much inventory needs to be built up. The implication of the termination effect is similar to the pulse phenomenon at the end of the season (Kurawarwala and Matsuo [41]).

**Problem 6.** What is the impact of the time constant parameter of TCM on the capacity expansion timing and sizing?

The mismatch of demand and production rates creates crucial capacity policy-making problems for an OEM. A “lumpy” capacity adjustment exacerbates the difficulty of OEM’s decision. OEM confronts the challenges in “capacity size,” “timing for launching,” and “capacity expansion timing” problems. Six challenges are outlined in the section introduction. In this study, flexible capacity is not considered, only with a single CM.

3.2. **Assumption.** Assume that an OEM owns the entire market of an innovative new product. The demand rate evolves according to the Bass diffusion equation \( d(t) \) as described in equations (1)–(3) with known \( p, q, \) and \( m. \) Severe global competition renders the OEM realizing TTV aggressively to return upfront investment and deter the new entrants in the market. The OEM procures the manufacturing process from a CM who owns expensive irreversible and indivisible capacity. As described in equations (4) and (5), the CM’s production rate curves have an initial base production rate setting and a target production ceiling rate to chase. OEM deploys outsourced capacity in two stages to balance the cost among capacity, inventory, reconfiguration, and backlog lost (Figure 7).

An OEM faces timing and sizing of capacity choices. Considering the resistance of capacity adjustment and the feasibility of clones, OEM strains the trade-off among different costs. OEM acquires initial capacity from CM at SOP and then captures the suitable timing to augment the capacity by “Copy Exactly.” Newly added manufacturing units commence production with the same output rate as current manufacturing units. After capacity expansion, the sales rate also jumps immediately under the zero lead time assumption.

OEM tactically delays the roll-out of new products after SOP if the innovative demand is high or the initial capacity level is low. OEM-CM executes total production between SOP and new product launch, and OEM has no marketing and no sales before the new product launch. The inventory will be accumulated and viewed as an alternative capacity and impacts the capacity expansion. OEM-CM contracts the available number of production lines that opened lines are never reduced and closed until TTV. OEM faces capacity adjustment frictions, including irreversibility and lumpiness, an integer increment restriction of the capacity unit augmentation.

Assume that the combination of initial capacity size and new product launch time is determined and given. The heuristic global search from the two dimensions of space set (initial capacity size and new product launch time) will be a future research extension. We develop a model to support capacity deployment decisions under given members and explore the impact of different diffusion parameters on capacity deployment.

This study assumes that the waiting queue does not detract from the WOM effect for a hot-selling durable product. The unmet demand is fully reserved and back-ordered, waiting for future fulfillment until the EPLC. An excellent brand image always accompanies hot-selling products, and customers are patient and brand loyal. No lost customer occurrence before the phase-out of a new product is an assumption. The backlog at the end is cut off as lost sales. The original BDM is adopted, and TTV is a key performance metric of OEM market potential realization. For exploring the capacity policy of an OEM in this study, the standard BDM is appropriate as the base for diffusive demand forecast. Regarding the interaction between sales, production, and demand, OEMs that sell the hot-selling product follow the myopic policy of Ho et al. [6] and plan production each period based on the trade-off between inventory cost and backlog and capacity lost cost. Production ceases at once upon the cumulative quantity hitting the market potential.

For a high-tech new product, the capacity supply restriction of CM is outlined as below factors: learning effect, production level at SOP, and plateau production level. In this study, \( r, \) time constant production rate function, is a function of time. It is assumed to manifest a ramp-up capability combination of the synergy of yield and utilization. The process technology cost per production line (manufacturing unit) opened includes patent cost and coding cost. OEM incurs capacity expansion costs, including engineering, system integration, and consultant.

Sales price per unit, production cost per unit, backlog cost per unit per period, holding cost per unit per period, ramp-up cost per line, sales lost cost, and capacity lost cost per unit are assumed nonnegative and constant over the PLC. The discount rate, \( \theta, \) is used to convert the maximum profit of OEM over the horizon period to the present value.

**4. Model**

4.1. **Model Description.** Consider an OEM who has developed and designed a new high-tech short life cycle product. The OEM has exclusive access to its specific product market. Marketing searching, advertising, and analyzing the potential market and buying motives, the demand rate increases first and declines after a peak over time according to the Bass diffusion equation \( d(t) \) as described in equation (1)
with known \( p, q, \) and \( m \). Global competitions render the OEM desiring to exploit opportunities given by design and market. The OEM procures the manufacturing process from a CM who owns an expensive irreversible and indivisible capacity networking system. The CM’s unitary production rate curves \( o(t) \), as described in equation (4), represent the dynamic capacity per production line with lumpy characteristics. OEM anticipates demand rate with parameters \( m, p, \) and \( q \). The CM who provided vital and lumpy adjusted increment capacity with an initial capacity output of each manufacturing unit, \( o_i \), and ramp-up target capacity output of each manufacturing unit, \( o_p \), was selected. These functions are equations (1) to (5).

4.2. Model Formulation. For developing the model, the parameters are summarized in Table 1. The indices and variables are listed as follows.

Decision variables are as follows:

(i) \( x(t) \) nonnegative production quantity at period \( t \)
(ii) \( s(t) \) nonnegative sales quantity at period \( t \)
(iii) \( k_n(t) \) number of augmented production lines

\[ k_n(t) = 0, 1, \ldots, (K_{\text{max}} - K_{\text{ini}}) \]

Variables are as follows:

(i) \( I(t) \) inventory at period \( t \)
(ii) \( B(t) \) cumulative backlog quantity up to period \( t \)
(iii) \( X(t) \) cumulative production quantity up to period \( t \)
(iv) \( cl(t) \) nonnegative capacity loss quantity at period \( t \)

\( \sum_{t=0}^{T} \theta(t) \left( \frac{K_{\text{ini}} + k_n(t)}{K_{\text{max}}} \right) R(t) \)

Objective is as follows:

\[ \max_{x(t), s(t), k_n(t)} \left( \sum_{t=0}^{T} \theta(t) \left( \frac{K_{\text{ini}} + k_n(t)}{K_{\text{max}}} \right) R(t) \right) \]

\[ -rc \cdot U(t) \cdot k_n(t) \cdot (T + 1 - t) - \theta(t) \left( slc \cdot \left( m - \sum_{t=0}^{T} s(t) \right) \right), \]
subject to

\[
I(t-1) + x(t) - s(t) = I(t) \quad t = 1, 2, \ldots, T, \\
B(t-1) + d(t) - s(t) = B(t) \quad t = 1, 2, \ldots, T, \\
X(t) = \sum_{t'=0}^{t} x(t') X(t) \leq m \quad t = 1, 2, \ldots, T, \\
S(t) = \sum_{t'=0}^{t} s(t') S(t) \leq m \quad t = 1, 2, \ldots, T, \\
I(t) = X(t) - S(t) + I_{ini} \quad t = 1, 2, \ldots, T, \\
I(t) = \sum_{t'=0}^{t} x(t') - \sum_{t'=0}^{t} s(t') + I_{ini} \quad t = 1, 2, \ldots, T, \\
D(t+1) = D(t) + d(t+1) \quad t = 0, 1, \ldots, T-1, \\
S(t+1) = S(t) + s(t+1) \quad t = 0, 1, \ldots, T-1, \\
X(t+1) = X(t) + x(t+1) \quad t = 0, 1, \ldots, T-1, \\
I(t+1) = I(t) + x(t+1) - s(t+1) \quad t = 0, 1, \ldots, T-1, \\
\sum_{t=0}^{T} U(t) = 1, \\
\sum_{t=0}^{T} Z(t) = 1,
\]

\[
R(t) = 1 - \sum_{t'=0}^{T} Z(t') \quad t = 0, 1, \ldots, T, \\
k(t) = \left( \sum_{t'=0}^{t} U(t') \right) + \left( \sum_{t'=0}^{T} Z(t') \right) \quad t = 0, 1, \ldots, T, \\
x(t) \leq \left( 1 - \sum_{t'=0}^{t} Z(t') \right) * M \quad t = 0, 1, \ldots, T, \\
k_{n}(t) \leq \left( \sum_{t'=0}^{t} U(t') \right) * (K_{max} - K_{ini}) \quad t = 0, 1, \ldots, T, \\
kn(t-1) \leq kn(t) \quad t = 1, 2, \ldots, T, \\
x(t) \leq o(t) * (K_{ini} + k_{n}(t)) \quad t = 0, 1, \ldots, T, \\
k_{n}(t+1) \leq (kt(t+1) - kn(t)) \\
\quad - Z(t+1) * M + k_{n}(t) \quad t = 0, 1, \ldots, T, \\
O(t) = \sum_{t'=0}^{T} (o(t') * K_{ini} + k_{n}(t')) \quad t = 0, 1, \ldots, T, \\
cl(t) = o(t) * (K_{ini} + k_{n}(t)) - x(t) \quad t = 0, 1, \ldots, T, \\
K(t) = (K_{ini} + k_{n}(t)) * R(t) \quad t = 0, 1, \ldots, T, \\
B(t) = 0; S(t) = 0; x(t) = 0 \quad t \leq t_{lch} - 1,
\]

| \( t \) | Index of period \( t = 0, 1, \ldots, T \) |
| \( T \) | The number of periods between SOP and the EPLC |
| \( \theta \) | Discount rate \( 0 < \theta \leq 1 \) |
| \( \pi \) | Sales price per unit |
| \( \rho \) | Production cost per unit |
| \( h \) | Holding cost per unit per period |
| \( b \) | Backlog cost per unit per period |
| \( clc \) | Capacity lost cost |
| \( ptc \) | Process technology cost per production line opened (manufacturing unit) |
| \( rc \) | The process cost includes patent cost and coding cost |
| \( scl \) | Capacity augmentation ramp-up and clone cost of each new production line |
| \( K_{ini} \) | Sale loss unit cost at terminal period \( T \) |
| \( K_{max} \) | Number of production lines opened at SOP |
| \( t_{lch} \) | Number of periods of launch at the latest in the contract of OEM-CM |
| \( L_{min} \) | Maximum commitment number of the production line in the contract of OEM-CM |
| \( l_{ini} \) | Period of launch at the latest in the contract of OEM-CM |
| \( m \) | Initial inventory. \( l_{ini} = 0 \) |
| \( M \) | Market potential over the new product life |
| \( M \) | Big number |
The objective function (6) represents the profit over the time span from SOP to EPLC under the given numbers of an initial number of production lines and the new product launch time. During the span, the joint decision variables $x(t)$, $s(t)$, and $K_p(t)$ interact over time from SOP to EPLC. The model system dynamic is driven by two trajectories, the time constant production curve and the Bass instantaneous demand curve. The decision sets $\{x(1), x(2), \ldots, x(T)\}$, $\{s(1), s(2), \ldots, s(T)\}$, and $\{K(1), K(2), \ldots, K(T)\}$ are production plan, sales plan, and capacity plan (production line opened plan) separately. Based on OEM's capacity policy, the profit of each period is calculated by the sum of sales revenue, $n_s(t)$, and the negative cost items (production cost, $px(t)$, inventory holding cost, $h_l(t)$, backlog cost, $bB(t)$, process technology cost, $ptc* K(t)$, new lines opened ramp-up and clone cost, capacity lost cost, and terminal sales lost cost). $K(t)$ is defined in expression (28). Constraint (7), constraint (11), and constraint (12) are inventory dynamic status equations. Constraint (8) represents the dynamic backlog equation. Constraints (9) and (10) set market potential as the target quantity of TTV. The dynamic evolution of demand, sales, production, and inventory are listed from constraint (13) to constraint (16). Constraint (17) restricts capacity expansion and must be executed once only during the PLC. Constraint (18) represents that production ceasing occurs only once during the PLC. Constraint (19) identifies the running status of each period. Constraint (20) defines three-phase statuses of production lines (0: initialization, 1: expansion, and 2: ceasing), and the value of $kt(t)$ varies in integer range from 0 to 2. Constraint (21) terminates production at once upon receiving the ceasing production signal of $Z(t) = 1$. Constraint (22) restricts the feasible range of the expanding number of production lines opened. The characteristic of capacity irreversibility is enforced in constraint (23). The feasible production output of each period is not greater than the capacity upper bound that is calculated dynamically in constraint (24). With this restriction, the production plan of each period never exceeds the total effective capacity output. Constraint (25) keeps the numbers of production lines constant after expansion. Constraint (26) expresses available cumulative capacity up to period $t$. Constraint (27) is the expression of capacity loss in each period. In objective function, it is in combination with the production line’s running status, $R(t)$. Constraint (29) meets the regulation of no marketing and no sales before a new product launch. Constraint (30) regulates full production between SOP and new product launch. Constraint (31) initializes the values of each variable. The nonnegative constraints are listed in constraint (32).

Two binary status codes and capacity change constraints map OEM-CM’s contract mechanism to implement the two-phase capacity deployment policy (Figure 8).

5. Experiment and Results

This section will present the model’s results based on the key issues. To provide a comprehensive description of the model run, the data sources are described first and shown in Figures 8 to 11. The results of the model run are organized in Figures 12 to 21.

5.1. Data Sources and CPLEX Solver. The BDM parameters $(p, q, m)$ depend on product type; we selected two main settings for our numerical experiments. One is (1) (0.02 0.4 3000); (2) (0.03 0.4 3000); and (3) (0.05 0.4 3000) for examining the important factor of innovative parameter $p$. The other is (1) (0.03 0.3 3000); (2) (0.03 0.4 3000); and (3) (0.03 0.5 3000) for investigating the effect of imitation parameter $q$. Our base setting, i.e., (0.03 0.4 3000), was used by Kumar and Swaminathan [7] as a given through their numeric study.

We run the model on IBM ILOG CPLEX Optimization Studio (20.1.0.0). Each period’s supply and demand data are generated through the difference calculation of cumulative value for a small enough time interval. Each period of the PLC 30 periods is subdivided into 10, hence corresponding to 300 periods in sequence. The smaller 300 subperiods satisfy tactic decision-making in this numeric study. Figure 9 shows the data sources of our model. They are composed of demand increment quantity and supply increment quantity of each period. In such a way, we could efficiently generate source data with relatively small bias. Due to both functions having a corresponding proved cumulative function, it is feasible to acquire the precise increment for each period. The appropriate period is characterized by different industries and varies from several days to several weeks.

To generate a discrete increment of demand and production (as shown in Figure 9), we convert a complex
Demand increment of each period

\[ \Delta D_{29} = D(29) - D(28) = 22.475 \]

\[ \Delta D_{28} = D(28) - D(27) = 21.902 \]

Demand Increment Curve

Capacity increment of each period

\[ \Delta O_{28} = O(28) - O(27) = 6.498 \]

\[ \Delta O_{29} = O(29) - O(28) = 6.454 \]

Capacity Increment Curve

Cumulative time constant curve

\[ o(t) = o_i + \tau (t - u_i) \left( e^{-\tau t} - 1 \right) \]

Figure 8: Relation between binary code \((U, Z)\) and related expression variable \((kt, R)\).

Figure 9: (a) (Left half-panel) demand \(d\) (left bottom), accumulative demand \(D\) (left top), demand increment for period \(t\) \(D_t = D(t) - D(t-1)\) (top right), table for \(\Delta D\) (right bottom) (b) (right half-panel) as in (a) but for production.
nonlinear problem to a linear problem. By giving the Bass parameters \( p, q, m \), time constant parameters \( (r, a, o) \), and the number of periods, the data conversion operation is accurate, complete, and accessible. These quantities are presented in Figure 10. The underpinnings of the optimum solution, the dynamic result of demand, production, backlog, and inventory are shown in Figure 11.

5.2. Results. The model is designed for finding the optimum capacity expansion timing and sizing. The model can find the optimal solution given the initial capacity unit and parameters. After the model is run experimentally, the results are divided into several groups according to different situations and are shown as follows:

Figures 12 to 14 address the impact of Bass parameters on expansion timing, sizing, and termination timing. Figures 15 to 18 discuss how unit inventory and backlog cost affect capacity deployment.

Figure 19 examines how the initial capacity unit number affects the impact of expansion timing and total profits. Figure 20 explores the impact of the time constant parameter on capacity expansion.

Figure 21 addresses the necessities of either capacity expansion or launching delay for a highly innovative new product.

Figures 20 and 21 discuss the impact of capacity loss and process technology cost on profit.

For making the result description clear, we express the parameters as tuple unified communication. We use \((p, q, m)\) to express the Bass diffusion parameter set \((r, a, o)\) as the time constant parameters and \((h, b, clc, ptc, rc)\) for the cost parameter (holding cost, backlog cost, capacity lost cost, process technology cost, and ramp-up clone cost). As in Section 5.1, \((8.50, 100)\) is the base of time constant parameters, and \((0.2, 0.5, 1.5)\) is the base of the cost parameter. Two groups of \( (p, q, m) \) used in Figures 12 to 14 are listed as follows: SET1, the first set of three demand curve, is \{\((0.02, 0.3, 4.800), (0.03, 0.4, 3.000), \) and \((0.05, 0.4, 3.000)\}\), and SET2, the second set of three demand curve, is \{\((0.03, 0.3, 3.000), (0.03, 0.4, 3.000), \) and \((0.03, 0.5, 3.000)\}\).

Observation from Figures 12 to 14 is as follows.

The highest demand peak is found for each set’s strongest \( p (q) \). The most significant innovation coefficient, the highest the size of capacity expansion needed, and the earliest the timing of expansion occurred. High capacity applied and shorter tail curve render the termination of production also earlier.

Figure 12(b) depicts the corresponding two-stage accumulative capacity curves. The expansion timing and expansion size were resolved easily through the model. Three pairs of numbers \((14, 238.8), (29, 511.6), \) and \((47, 884.0)\) indicate the expansion time and available capacity at that time.

Although not so dramatic, the highest imitation coefficient exhibits the highest peak and earliest expansion time in SET2, as shown in Figure 13.

OEM-CM ceases the production as soon as accumulative production quantity reaches the total demand of PLC. This time point is the termination time of capacity supply. The impact of Bass parameters SET1/SET2 on termination time is presented in Figures 14(a) and 14(b). As the tail of the Bass curve occurs earlier and lower for higher \( p \), the termination timing occurs earlier for SET1, as shown in Figure 14(a). The termination time point for higher \( q \) also occurs earlier for SET2 (Figure 14(b)).

From Figures 12(b), 13(b), and 14, we observed that the optimum termination timing reacts more sensitive to the value of the imitation parameter \( q \). However, the optimum expansion timing reacts more sensitive to the value of the innovation parameter \( p \).

Observation from Figures 15 to 18 is as follows.

Figures 15 to 18 examine how holding cost and backlog cost affect capacity deployment. The impact of the unit holding cost on termination time is examined by comparing Figures 15(a) and 15(b), where the Bass demand curve is based on our base parameter set \((0.03, 0.40, 3.000)\). The cost parameter is \((0.5, 0.2, 1.5)\) for Figure 15(a) and \((0.2, 1.1, 5)\) for Figure 15(b). OEM-CM initiates three-line capacity with a time constant \((8, 50, 100)\). OEM-CM produced fully in a
short interval from period 145 to period 147 to build inventory to fill the remaining demand in the triangular of the very right side. The contract is terminated at period 148 in Figure 15(a). Due to unit holding costs being lower and unit backlog costs being higher for Figure 15(b), OEM-CM builds a higher inventory (area \( I_a \) and \( I_b \)) during the demand rising phase. The stock provides shipment replenishment (area \( R \)) under the peak to decrease backlog. The small backlog (\( B \)) is back-flushed by ample capacity when demand is lower than the available capacity supply. Considering the increasing capacity lost cost, the termination of the capacity contract ceases at period 130 earlier than that of (a). The corresponding trade-off between inventory holding cost, backlog cost, and capacity lost cost in Figures 15(a) and 15(b) is depicted in Figures 16(a) and 16(b). The inventory offsets backlog is evident in the model's numerical output value, as shown in Figure 17.

Capacity deployment is based on a cost trade-off. When the cost parameter is \((0.2 1 1 1 5)\), the model suppresses backlog and deploys with more inventory for shipment replenishment, as shown in Figure 16(b). Low inventory cost plays a substitute for expensive capacity. As shown in Figure 17, the numerical value is 93, and the sum of \( I_a \) and \( I_b \) provides replenishment shipment for the period from period...

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**Table 1:**

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**Figure 11:** Optimum demand, production, backlog, inventory (DXBI) analysis and capacity deployment.

**Figure 12:** Capacity deployment: (a) capacity expansion in case of Bass parameter SET1, where solid curves represent demand and dashed line represents available capacity. (b) Spotlight view of capacity expansion from cumulative capacity level.
Figure 13: Capacity deployment: (a) capacity expansion in case of Bass parameter SET2, where solid curves represent demand and dashed line represents available capacity. (b) Spotlight view of capacity expansion from cumulative capacity level.

Figure 14: (a) Termination effect in case of Bass parameter SET1, where available dashed line represents capacity and tower represents final output. (b) Termination effect in case of Bass parameter SET2.

Figure 15: (a) Chart illustrates the impact of cost parameters on the termination time of capacity. Cost parameter (0.5 0.2 1 1 5). (b) Chart illustrates the impact of cost parameters on termination time of capacity. Cost parameter (0.2 1 1 1 5).
Figure 16: (a) Chart illustrates the trade-off between inventory holding cost, backlog cost, and capacity lost cost. Cost parameter (0.5 0.2 1 1 5). (b) Chart illustrates the trade-off between inventory holding cost, backlog cost, and capacity lost cost. Cost parameter (0.2 1 1 1 5).

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\[ I_a = 34.351 \]

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\[ I_b = 59.182 \]
\[ I_a + I_b = 93.533 \]

Figure 17: (a) Model’s numerical output: demand, production, sales, and inventory for the period 9 through 37. (b) Model’s numerical output: demand, production, sales, and replenishment for the period 38 through 68 under the cost parameter (0.2 1 1 1 5).
slower learning capability prevails in saving backlog costs. The scenario of a Ramp cost, capacity lost cost, and process technology cost of \( \tau \) equals than a scenario with a slower learning capability (\( \tau \) better learning capability (\( \tau \) all cost factors and revenue, the product life cycle profit for a better learning capability (\( \tau \) suppressed and executed earlier. Figure 20(b) illustrates a constant parameter. However, the capacity expansion size is constant ramp-up is faster than that under high time expansion timing and sizing is presented in Figure 20. The impact of the time constant parameter on the expansion timing, sizing, and profit? We experiment with three scenarios with an initial capacity of one, two, and three lines. All three scenarios are performed under the Bass base parameter setting (0.03 0.40 3000), time constant setting (8, 50, 100), and cost setting (0.5 0.2 1 1). Figure 19 shows the earlier the optimal expansion timing for the smaller initial capacity. The optimal expansion timing is 14 for one unit, 21 for two units, and 29 for three units. The capacity level for all three scenarios at the second stage reached four units. Termination time is 148 for all three scenarios. The optimal profits have been calculated; they are 46205 (one initial unit), 47830 (two initial units), and 49249 (three initial units). Profits are higher for higher initial capacity. Different profits resulted from holding costs, backlog, and capacity loss trade-offs.

Observation from Figure 19 is as follows. How does the initial capacity selection affect the expansion timing, sizing, and profit? We experiment with three scenarios with an initial capacity of one, two, and three lines. All three scenarios are performed under the Bass base parameter setting (0.03 0.40 3000), time constant setting (8, 50, 100), and cost setting (0.5 0.2 1 1). Figure 19 shows the earlier the optimal expansion timing for the smaller initial capacity. The optimal expansion timing is 14 for one unit, 21 for two units, and 29 for three units. The capacity level for all three scenarios at the second stage reached four units. Termination time is 148 for all three scenarios. The optimal profits have been calculated; they are 46205 (one initial unit), 47830 (two initial units), and 49249 (three initial units). Profits are higher for higher initial capacity. Different profits resulted from holding costs, backlog, and capacity loss trade-offs.

Observation from Figure 20 is as follows.

The impact of the time constant parameter on the expansion timing and sizing is presented in Figure 20. The capacity expansion analysis was conducted for the time constant parameters (8 50 100) versus (4 50 100). For better learning capability (\( \tau = 4 \)), the capacity (orange curve) expands to three lines at period 9. In the other one with learning capability (\( \tau = 8 \)), the capacity (purple curve) expands to four lines at period 14. Production under low time constant ramp-up is faster than that under high time constant parameter. However, the capacity expansion size is suppressed and executed earlier. Figure 20(b) illustrates a higher backlog quantity and a smaller capacity lost quantity for a better learning capability (\( \tau = 4 \)) scenario. Considering all cost factors and revenue, the product life cycle profit for a better learning capability (\( \tau = 4 \)) scenario is 46688, 482 more than a scenario with a slower learning capability (\( \tau = 8 \)). Ramp cost, capacity lost cost, and process technology cost of the scenario with a better learning capability are lower than those with a slower learning capability. The scenario of a slower learning capability prevails in saving backlog costs.
process technology cost of 100, the optimum launching time is 50, the capacity expansion time is 69, and the termination time is 168. Comparing scenarios a and b, delaying launch needs to consider the higher holding cost. Hence, the launch time is earlier at 9. In scenario b, holding cost is higher than backlog cost; hence, it is fine to delay to time 18. The maximum occurred at 18 with 47,278, higher than that for scenarios 46,430 at 9. The impact of process technology is evident in the scenario of the right panel. The launching time is 50—the expansion capacity from one to three at time 69. The launch time, expansion time, expansion size, maximum profit, maximum profit occurring time, and termination time are six elements in a tuple. They are (9 26 4 46430 9 156), (18 36 4 47278 18 148), and (50 69 3 9276 50 168). These results suggest that process technology cost plays an important factor in maximum profit.

Observation from Figure 21(c-1) is as follows.
There are several local optimizations appear along the curve. The curve in Figure 21(c-1) is not smooth, caused by a very high batch cost (ptc = 100 per line). In contrast to Figures 21(a-1) and 21(b-1), the nonconvex phenomena are not significant when the batch cost (ptc = 1 per line) is so tiny. Nonconvex is also one type of friction of capacity adjustment [1].

6. Discussion and Conclusion

OEM develops a new product with a short life cycle and strategically outsources the manufacturing to CM. OEM-CM faces the effects of interaction between demand diffusion and supply ramp-up. Nonlinear trajectories and capacity friction such as lumpiness and irreversibility constrain the feasible
solution region. A two-phased capacity deployment is adopted to approach the optimization. The data source is generated from two proven cumulative functions, cumulative Bass demand function, and cumulative time constant function, by discretizing the short horizontal span into 300 small intervals and then inputting it into a model that runs on CPLEX. The experiment shows that the model provides decision support for OEM managers in practice. The complicated and tangled cost factors are considered dynamically and weighed to optimize the whole PLC.

Our model is a novel approach for a dynamical demand/supply economic system. Optimal control theory theoretically could provide an analytic solution for multistage decision processes involving nonlinear differential equations. However, the mismatch of a peak of demand diffusion growth and plateau of production with learning ramp-up and irreversible and indivisible constraints render it mathematically intractable. The discrete numerical model can resolve the key issues: timing and sizing of capacity deployment for new product launching.

The following paragraphs will describe the model’s management implications, novelty, and limitations.

6.1. Management Implication and Insight

(1) Two-phase capacity deployment is achievable with three coherent policy hierarchy levels (strategy, tactic, and operation) under demand diffusion and production ramp-up.

(2) Two critical components of a capacity decision (timing and size) make up the mix of capacity deployment decision options. The complexity comes from the period set and the line number set. The complexity of decision options increases with the number of decision points in the product life cycle, and incorporating a heuristic search algorithm would be an attempt.

(3) Discretizing the nonlinear constraint function of time supports the decision model to calculate the optimal solution in the dynamic interactive simulation environment. The finer the discrete subdivision of the nonlinear process, the more helpful it is to create a more realistic simulation environment. The data environment creation is a prerequisite for simulation. Also, an extension to stochastic environments would be a more in-depth exploration.

(4) In a dynamic environment where demand diffusion and production ramp-up exist, the model developed in this study allows OEMs to weigh the interplay between time, cost, and volume to formulate capacity deployments that can be implemented.

6.2. The Novelty

(1) A parsimonious scalable model that supports OEM in evaluating capacity policies from the product lifecycle perspective is created.

(2) Two research streams of demand diffusion and production ramp-up are joined for developing a model that capacity deploy decision support of production network.

(3) Capacity adjustments are heuristically modeled using two-phase capacity deployments to coordinate supply and demand and accommodate capacity adjustments.

(4) A new and feasible method for creating and validating a data environment through discretization for the Bass demand and time constant functions is developed and contributed to a future research avenue related to trajectories of supply and demand.

(5) The ultimate effect of a single-generation new product is discovered. This will be the launch reference for the next generation of products.

6.3. Limitation. There are five significant limitations in this study:

(1) A single generation is considered only. The model needs an enhancement to fit two-generation or multigeneration product demand diffusion.

(2) The model is assumed to be deterministic without considering stochastic.

(3) Original Bass demand model is adopted without considering the impact of market-mix factors.

(4) The flexible capacity and interruption risk are not considered in this model.

(5) The lead time of supply and demand is not considered.

The product life cycle is getting shorter. However, information technology capabilities are more robust, and data analysis responds quickly. Our work sheds light on the fact that numerical methods are feasible and applicable when parameter predictions are empirical and dynamic functions are parsimonious. Other research streams related to mathematical programming, operation management, marketing management, and intelligent manufacturing are leveraged. This new synergy application turns complex and cumbersome mathematical calculations and equations into practical applications.

7. Conclusion

In this study, we provide some research results based on a heuristic optimization model for an OEM who faces the challenge of the mismatch of characteristics between the demand trajectory and the capacity trajectory. By applying mathematical and graphical analysis and the mighty digital computation power of this era, leverage of optimization language tool, we shed light on the capacity deployment and decision support of an OEM.

Our work developed the model and solved the “When” and “How much” problems of capacity deployment. Modestly relaxing some restrictions, the following extensions are worthy of future research:
References


[32] Ö. Özer and O. Uncu, “Competing on time: an integrated framework to optimize dynamic time-to-market and


