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Research Article

Reliability Analysis of Hybrid System Using Geometric Process in Multiple Level of Constant Stress Accelerated Life Test through Simulation Study for Type-II Progressive Censored Masked Data

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Numerous studies have already been attempted to explore the reliability of systems considering mask data, though the mass of them has largely focused on basic series or parallel systems, where component failures are assumed to follow an exponential or Weibull distribution. However, most electrotonic products and systems are made up of numerous components integrated in parallel-series, series-parallel, and other bridge hybrid structures, and the number of studies in the area of accelerated life testing (ALT) employing masked data for hybrid systems is limited. In this paper, the constant-stress ALT (CSALT) is explored based on type-II progressive censoring scheme (TIIPCS) for a four-component hybrid system using geometric process (GmP). The failure times of the components of the system are assumed to follow the generalized Pareto (GP) distribution. The maximum likelihood estimate (MLE) technique is used to establish statistical inference for the model's unknown parameters under the premise that the failure reasons are unknown for the hybrid system. In addition, the asymptotic confidence intervals (ACIs) are also obtained by inverting the fisher information matrix. Finally, a simulation study is given to explain the proposed techniques and to evaluate the performance of the estimates. The performance of MLEs is assessed in terms of root mean square errors (RMSEs) and relative absolute biases (RABs), whereas the performance of ACIs is assessed in terms of their interval length (IL) and coverage probabilities (CPs). The findings show that the technique can deliver good estimation performance with small and intermediate sample sizes, and the estimates are more accurate when more failures are observed, showing the estimation method's efficiency.

1. Introduction

Typical life testing and reliability tests are supposed to look at failure time data collected in normal working conditions. However, due to a lack of testing budget and time restrictions, life data for highly reliable objects like electronics systems, electric circuits, engines and insulating materials etc., are exceptionally difficult to obtain employing typical life-testing methods. Because of its ability to provide rapid failure information to examine product design and life at a reduced cost, accelerated life testing (ALT) is extensively employed in the manufacturing industry for analyzing product reliability and projected life. ALT is a method for inferring the life expectancy of systems under normal usage circumstances based on failure data acquired under harsh conditions (e.g., vibration, voltage, temperature, pressure, and humidity) using the relationship between life characteristics and stress variables. In ALT, there are three ways that are often used to apply stress (harsh conditions): constant stress, step stress, and linearly rising stress. ALT has been examined by numerous writers using various lifespan distributions and test conditions. See Miller and Nelson [1]; Nelson [2]; Meeker et al. [3]; Guan et al. [4]; El-Din et al. [5]; Han and Bai [6]; Kamal et al. [7]; and Zhang et al. [8] as examples and for more information.

However, step-stress and progressive stress tests have the advantage of ensuring failures quickly due to increasing stress levels, but they have a significant drawback when it comes to estimating reliability. Since the majority of goods in real-life situations operate under constant stress rather than step stress or progressive stress, the model must appropriately account for the cumulative effect of increasing stresses over time. Additionally, controlling the step and increasing stresses appropriately may be challenging. Fitting these models is thus more complex than fitting a model for a constant stress test. As a result, constant stress testing is often favored over step-stress and progressive stress tests for estimating dependability. Nelson [2]; Zarrin et al. [9]; El-Din et al. [5]; and Kamal [10] provide further information on constant stress testing. We explored a CSALT that used a TIIPCS in this paper.

Most electrotonic goods and systems are made up of numerous separate components that are linked in one of two fundamental ways, such as series and parallel. More complicated systems are often a mix of these fundamental subsystems interconnected in parallel-series, series-parallel, and other bridge structures such as the four-component series-parallel hybrid system under study and can be seen in Figure 1. The reliability of these systems is dependent on the reliability of the components and subsystems. The primary task that must be performed prior to the launch of the product for usage in real life is to assess its reliability. Collecting lifetime data for reliability analysis for such complex structures is a challenging task since the component causing the system failure is not always identifiable for a variety of reasons. This phenomenon is known as masking, and data produced from such studies is referred to as masked data.

Miyakawa [11] proposed a model based on masked data for two component series system assuming the constant failure rate for components. Since then, a substantial amount of research has been conducted using mask data based on typical reliability tests and ALTs considering basic series or parallel systems, see, for example, Guess et al. [12]; Xu and Tang [13]; Zarrin et al. [14]; Xu et al. [15]; Wang et al. [16]; Cai et al. [17]; and Shi et al. [18]. Unfortunately, thus far, in the existing literature, only a few studies on ALTs that focused on hybrid systems and masked data are available. Shi et al. [19] and Shi et al. [20] derived the MLEs for hybrid system of four components under SSPALT and CSPALT, respectively, based on masked data. Xiaolin et al. [21] considered two distinct three-component hybrid systems

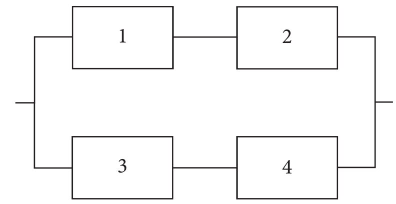

Figure 1: The parallel series hybrid system.

and derived MLEs of the modified Weibull distribution. Recently, Liu et al. [22] investigated a series system with component dependence structure and applied a nonparametric Bayesian technique for censored masked data in an ALT with the copula function. Kamal [23] investigated a three-component hybrid system for the power linear hazard rate distribution and utilized the MLE technique to estimate parameters using progressive hybrid censored masked data.

Lin Ye [24] pioneered the use of the GmP model to study repair and replacement problems. A significant number of research on system reliability and maintenance issues has demonstrated that the GmP model is an effective and simple approach for analyzing data with single or multiple trends. Lam and Zhang [25] used the GmP model to analyze a two-component series system with one repairman. Using the GmP model, Yeh [26] examined a multistate system and proposed an optimal replacement policy to decrease the long-run average cost per unit time. Zhang [27] modelled a basic repairable system with delayed repair using the GmP. Several studies have been conducted to date that employ the GmP in the investigation of ALT. Huang [28] proposed the use of the GmP model in CSALT for analysis of complete and censored exponential failure data. Under CSALT, Kamal et al. [29] extended the GmP model to evaluate complete Weibull failure data. Kamal [30] further explored the GmP model to evaluate censored Weibull failure data. Kamal et al. [31] and Mohamed et al. [32] are among others who implemented the GmP model to estimate the parameters of different distributions with different types of data under CSALT. Recently, Rahman et al. [33] computed the parameters of the Burr X distribution employing type-I censoring and the MLE approach under CSALT, assuming that lifespan comprises a GmP with increasing stress levels. Aly et al. [34] utilized the GmP model to explore the CSALT and made statistical inferences based on the MLE technique by taking into account the generalized half logistic lifespan distribution under TIIPCS.

To the best of our knowledge, no work has yet addressed CSALT with TIIPCS for a hybrid system under masked conditions using the GmP model. The main objective of this study is to provide a resilient framework for the hybrid system (Figure 1) that collapsed owing to masked factors under the CSALT design. Then, utilizing TIIPC masked data and the MLE technique, we provide point and interval estimates for the unknown parameters of the GP distribution and the ratio of GmP. The remainder of the paper is structured as follows: Section 2 discusses some of the study's assumptions, test methodologies, and other significant points. The method for estimating the parameters is described in Section 3. Section 4 incorporates a simulation analysis to assess the efficiency of the estimators, as well as a discussion based on the study's results. Section 5 concluded the paper by suggesting some future research directions.

2. The Model and Test Procedure

2.1. The GP Distribution. If $\mathscr{Y}$ is a nonnegative random variable with a GP distribution, then its probability density function (PrDF) with scale parameter ξ and shape parameter σ, let us call it GP (σ, ξ), can be written as follows:

$$f(y|\sigma,\xi) = \sigma\xi(1+\xi y)^{-(\sigma+1)}, \quad y > 0, \sigma, \xi > 0. \tag{1}$$

The corresponding cumulative distribution, survival functions, and hazard rate are each provided by

$$\begin{aligned} F(y|\sigma,\xi) &= 1-(1+\xi y)^{-\sigma}, \quad y>0,\sigma,\xi>0,\\ R(y|\sigma,\xi) &= (1+\xi y)^{-\sigma}, \quad y>0,\sigma,\xi>0,\\ h(y|\sigma,\xi) &= \frac{\sigma\xi}{(1+\xi y)}, \quad y>0,\sigma,\xi>0. \end{aligned} \tag{2}$$

Definition 1. Geometric process.

A counting process $\{\mathscr{X}_n, \quad n=1,2,\ldots\}$ is a nonnegative, integer, and nondecreasing stochastic process that depicts the number of failures during a life testing experiment. If there exists a real-valued $\delta > 0$ such that the random variables $\mathscr{Z}_n = \delta^{n-1}\mathscr{X}_n, \quad n=1,2,\ldots,$ are independent and identically distributed according to a given distribution function $F(.)$, then the counting process $\{\mathscr{X}_n, \quad n=1,2,\ldots\}$ is known as a GmP, where δ is known as the ratio of GmP.

It is obvious that a GP for $\delta > 1$ is decreasing function stochastically, whereas it is an increasing function with $0<\delta<1$. As a result, the GmP can be considered as more natural way for evaluating sequential data with trends.

If occurrence $\{\mathscr{X}_1, E(\mathscr{X}_1)=\mu, \mathrm{var}(\mathscr{X}_1)=\sigma^2\}$ form GmP $\{\mathscr{X}_n, \quad n=1,2,\ldots\}$ has a PrDF $f(.)$, then it is easy to show that $\delta^{n-1}f(\delta^{n-1}y)$ will be the PrDF of $\{\mathscr{X}_n, E(\mathscr{X}_n)=\mu/\delta^{n-1}, \mathrm{var}(\mathscr{X}_n)=\sigma^2/\delta^{2(n-1)}\}$.

2.2. Assumptions. Assume we are subjected to a CSALT with k increasing levels of stress. Let $\mathscr{A}_s, \quad s=0,1,2,\ldots,k$ are k levels of stress. If $s=0$, stress is normal use condition, whereas $s=1,2,\ldots,k$ represents accelerated conditions. Consider the parallel series hybrid system explained by Figure 1. Let $\mathscr{Y}_{si}$ be the lifetime of i^{th} system at s^{th} stress and y_{si} being its observation. Also, $\mathscr{Y}_{sij}$ represents the lifetime of the j^{th} component of i^{th} system at s^{th} stress and its observed value is y_{sij}. We have the following assumptions:

(i) The test contains N systems in total and N is first split into the samples of sizes $n_1, n_2, \ldots, n_k$ such that $\sum_{s=1}^{k} n_s = N$. Now, each n_s assigned to test at a prespecified stress $\mathscr{A}_s, s=0,1,2,\ldots,k$.

(ii) The component lifetimes in the system are independent.

(iii) At any constant stress $\mathscr{A}_s, s=0,1,2,\ldots,k,$ the failure times follows GP (σ,ξ) distribution given in equation (1).

(iv) $\mathscr{Y}_{si}, s=0,1,2,\ldots,k; i=1,2,\ldots,n$ are i.i.d. at s^{th} stress.

(v) The relationship between the scale parameter ξ and the stresses $\mathscr{A}_s$ is a log-linear function defined by $\log\xi_s = \theta_0+\theta_1\mathscr{A}_s, \quad s=0,1,2,\ldots,k,$ where θ_0 and θ_1 are the unknown parameters of the relationship and their values usually depend on true nature of the items under the consideration.

(vi) The stresses are increased with an equal amount d, which means stresses are equidistant and can be explained by the relation $\mathscr{A}_s = \mathscr{A}_{s-1} + d$.

(vii) Let RVs, $\mathscr{Y}_0, \mathscr{Y}_1, \mathscr{Y}_2, \ldots, \mathscr{Y}_k$, represent the lifetimes at s^{th} stress level, and hence, the sequence $\{\mathscr{Y}_s, \quad s=0,1,2,\ldots,k\}$ constitutes a GmP with a ratio $\delta > 0$.

Assumptions (i–v) are the most often used in ALT. Assumptions (vii) and (vi) may be preferable to the conventional treatment of the ALT in this discourse without adding computational complexity. Now, we examine the assumptions of the constant-stress and GmP models to demonstrate how a stochastically decreasing GmP model may be utilized as an ALT model. The following theorems show how the GmP assumption (vii) is satisfied when a life attribute and stress have a log-linear relationship assumption (v).

Theorem 1. *In ALT, if the stress increases arithmetically, i.e.,* $(\mathscr{A}_{s+1}-\mathscr{A}_s)=d; \quad s=0,1,2,\ldots,k,$ *then the sequence* $\{\xi_s, \quad s=0,1,2,\ldots,k\}$ *of life characteristic constitutes a GmP with a ratio* $e^{\theta_1 d}=\delta>0$.

Proof: Using assumption (v), we have, $\xi_s = e^{(\theta_0+\theta_1\mathscr{A}_s)}$ and $\xi_{s-1} = e^{(\theta_0+\theta_1\mathscr{A}_{s-1})}$. As a result, we can now write

$$\frac{\xi_s}{\xi_{s-1}} = e^{\theta_1(\mathscr{A}_s-\mathscr{A}_{s-1})} = e^{\theta_1 d}. \tag{3}$$

This demonstrates that the increasing stress levels generate an arithmetic sequence with a constant difference d. Let us assume $\xi_s/\xi_{s-1}=\delta>0$, which a constant ratio; therefore, $\{\xi_s, \quad s=0,1,2,\ldots,k\}$ forms a GmP with ratio $\delta>0$ which completes the proof. □

Theorem 2. *For arithmetically increasing stress level* $\mathscr{A}_s, \quad s=0,1,2,\ldots,k,$ *in CSALT, if the PrDF of* $\mathscr{Y}_0$ *is PD* (ξ) *and the sequence of RVs* $\{\mathscr{Y}_s, s=0,1,2,\ldots,k\}$ *forms a GmP with ratio* $\delta>0$, *then PrDF of* $\mathscr{Y}_s$ *can be written as*

$$f_{\mathscr{Y}_s}(y_s) = \delta^s f_{\mathscr{Y}_0}(\delta^s y_0). \tag{4}$$

Proof: From Theorem 1, we can write

$$\xi_s = \delta\xi_{s-1} = \delta^s\xi_{s-2} = \cdots = \delta^s\xi. \tag{5}$$

The PDF of the product lifespan at the s^{th} stress level may now be expressed as follows:

$$f_{\mathcal{Y}_s}(y_s) = \sigma\delta^s\xi\left(1+\delta^s\xi y\right)^{-(\sigma+1)} = \delta^s\sigma\xi\left(1+\delta^s\xi y\right)^{-(\sigma+1)} = \delta^s f_{\mathcal{Y}_0}\left(\delta^s y_0\right), \tag{6}$$

which completes the proof.

Now, from Theorems 1 and 2, it is obvious that the log linear and GmP models are equivalent in an ALT if the stress level increases arithmetically and the lifetime under each stress level forms a GmP. It is also clear from Theorem 1 and the definition of GmP that if the PrDF of $\mathcal{Y}_0$ is $f_{\mathcal{Y}_0}(y_0)$, then the PrDF of $f_{\mathcal{Y}_s}$ is determined by $\delta^s f_{\mathcal{Y}_0}(\delta^s y_0)$. As a result, it is obvious that lifetimes underneath a series of mathematically escalating levels of stress constitute a GmP with ratio $e^{\theta_1 d} = \delta > 0$. And, the life distribution at the design stress level is GP (σ, ξ), then the life distribution at the s^{th} stress level is also GP $(\sigma, \delta^s\xi)$. Now, the PrDF of the component at s^{th} stress level by using Theorem 1 can be written as

$$f\left(y|\sigma,\xi,\delta\right) = \delta^s\sigma\xi\left(1+\delta^s\xi y\right)^{-(\sigma+1)}, \quad y>0, \sigma,\xi,\delta>0. \tag{7}$$

The corresponding survival functions is provided by

$$R\left(y|\sigma,\xi\right) = \left(1+\delta^s\xi y\right)^{-\sigma}, \quad y>0, \sigma,\xi,\delta>0. \tag{8}$$

Now, we will look over masked probability in a nutshell. Let us first assume that one of the four-component failures is responsible for the system's failure. Assume that the system's failure times can be observed. Let $\mathcal{W}_{si}$ signify the set of components that might be the source of system i failure such that $\mathcal{W}_{\text{si}} \subseteq \{1,2,3,4\}$ implying that the source of system failure can only come from the smallest random subset of the set $\{1,2,3,4\}$. This implies that $\mathcal{W}_{\text{si}}$ represent a set of all the fifteen possible occurrences $\{\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,3,4\},\{1,2,3,4\}\}$ that can result in system i failure [20]. Let w_{si} be the observed values of $\mathcal{W}_{\text{si}}$, and if w_{si} looks to be composed of more than one element, the precise cause of system failure is unknown and the received life data is referred to be masked. Now, using the concept of masked probability, a theorem to obtain the reliability and density function of the system will be stated and proved. □

Definition 2. Masked probability.

Suppose that $w_{\text{si}} \epsilon \mathcal{W}_{\text{si}}$ be the masked event and $C_{\xi i}$ be the specific cause of the system i failure due to the j^{th} component. If only one element belongs to y_{si}, then the failure cause is exact; otherwise, the cause of system failure is unknown. Now, according to Wang et al. [16], the masking probability (MP) is

$$\text{MP} = P\left(\mathcal{W}_{\text{si}} = w_{\text{si}}|y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, C_{\xi i} = j\right). \tag{9}$$

In general, it is often assumed that masking mechanisms, various stress conditions, and causes of failure are statistically independent. Therefore, the expression in equation (9) for MP can be given as

$$P\left(\mathcal{W}_{\text{si}} = w_{\text{si}}|y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, C_{\xi i} = j\right) = P\left(\mathcal{W}_{\text{si}} = w_{\text{si}}|C_{\xi i} = j\right) = \Lambda. \tag{10}$$

Theorem 3. *The PrDF for j independent components hybrid system failure time $\mathcal{Y}_{si}$ with MP $P(\mathcal{W}_{si} = w_{si}|C_{si} = j)$, due to masked event $y_{si} \epsilon \mathcal{W}_{si}$ at time w_{si} can be written as*

$$P\left(y_{\text{si}} < y_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, \mathcal{W}_{\text{si}} = w_{\text{si}}\right) = \sum_{j\in y_{\text{sij}}} \Lambda_{\text{si}} f_{\text{sij}}. \tag{11}$$

Proof: Following Wang et al.'s [16] steps, the failure probability $\omega_{\xi i}$ for system i which is failed because of component j at time $t_{\xi i}$ is

$$\begin{aligned} &P\left(y_{si} < \mathcal{Y}_{si} < y_{si} + dy_{si}, \mathcal{W}_{si} = w_{si}\right) \\ &= \sum_{j=1}^{4} P\left(y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, \mathcal{W}_{\text{si}} = w_{\text{si}}, C_{\xi i} = j\right) \\ &= \sum_{j=1}^{3} P\left(y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dv_{si}, C_{\xi i} = j\right) \\ &P\left(\mathcal{W}_{\text{si}} = w_{\text{si}}|y_{\text{si}} < \mathcal{Y}_{\text{si}} < v_{\text{si}} + dy_{\text{si}}, C_{\xi i} = j\right) \\ &= \sum_{j\in y_{\text{si}}} P\left(y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, C_{\xi i} = j\right) \\ &P\left(\mathcal{W}_{\text{si}} = w_{\text{si}}|y_{\text{si}} < \mathcal{Y}_{\text{si}} < y_{\text{si}} + dy_{\text{si}}, C_{\xi i} = j\right). \end{aligned} \tag{12}$$

Now, from the results in Shi et al. [19], the RF for i-th hybrid system in Figure 1 is

$$\begin{aligned} P\left(\mathcal{Y}_{si} > y_{si}\right) &= 1 - P\left(\left(\min\left(\mathcal{Y}_{\text{si}1}, \mathcal{Y}_{\text{si}2}\right)\right) \le t_{\xi i}\right) \\ &\quad P\left(\left(\min\left(\mathcal{Y}_{\text{si}3}, \mathcal{Y}_{\text{si}4}\right) \le y_{\text{si}}\right)\right. \\ &= 1 - \left[1 - R_{\text{si}1}\left(t_{\text{si}}\right)R_{\text{si}2}\left(t_{\text{si}}\right)\right] \\ &\quad \left[1 - R_{\text{si}3}\left(t_{\text{si}}\right)R_{\text{si}4}\left(t_{\text{si}}\right)\right]. \end{aligned} \tag{13}$$

Similarly, the PrDF of i-th system failure because of component j at time $t_{\xi i}$ is derived as

$$\begin{aligned} f_{\text{si}1} &= f_{s1}\left(t_{\text{si}}\right)R_{s2}\left(t_{\text{si}}\right)\left[1 - R_{s3}\left(t_{\text{si}}\right)R_{s4}\left(t_{\text{si}}\right)\right]; \\ f_{\text{si}2} &= R_{s1} f_{s2}\left(t_{\text{si}}\right)\left(t_{\text{si}}\right)\left[1 - R_{s3}\left(t_{\text{si}}\right)R_{s4}\left(t_{\text{si}}\right)\right], \\ f_{\text{si}3} &= \left[1 - R_{s1}\left(t_{\text{si}}\right)R_{s2}\left(t_{\text{si}}\right)\right] f_{s3}\left(t_{\text{si}}\right)R_{s4}\left(t_{\text{si}}\right); \\ f_{\text{si}4} &= \left[1 - R_{s1}\left(t_{\text{si}}\right)R_{s2}\left(t_{\text{si}}\right)\right]R_{s3}\left(t_{\text{si}}\right) f_{s4}\left(t_{\text{si}}\right). \end{aligned} \tag{14}$$

Now, using equations (9), (10), (14), and assumption 2, we obtained the following:

$$P\left(\mathcal{Y}_{\text{si}} < \mathcal{Y}_{\text{si}} < \mathcal{Y}_{\text{si}} + d\mathcal{Y}_{\text{si}}, \mathcal{W}_{\text{si}} = w_{\text{si}}\right) = \sum_{j\in \mathcal{Y}_{\text{si}}} \Lambda_{\text{si}} f_{\text{sij}}, \tag{15}$$

which completes the proof.

Now, by using Theorem 2 and 3, the PrDF at the s^{th} stress for the system can be written as

$$f\left(\mathcal{Y}|\xi,\sigma,\delta\right) = \delta^s\sigma\xi\left(1+\delta^s\xi y\right)^{-(2\sigma+1)}\left(1-\left(1+\delta^s\xi y\right)^{-2\sigma}\right) \cdot y>0, \sigma,\xi,\delta>0. \tag{16}$$

The corresponding survival functions is provided by

$$R(\mathscr{Y}|\xi,\sigma,\delta) = 2\left(1+\delta^s\xi y\right)^{-2\sigma} - \left(1+\delta^s\xi y\right)^{-4\sigma}, \quad y>0,\sigma,\xi,\delta>0. \tag{17}$$

□

3. Inference under TIIPCS

Suppose that we are dealing with a CSALT with $\mathscr{A}_s, s = 0,1,2,\ldots,k$ increasing stress levels. Now, a random sample of n_s, $s = 0,1,2,\ldots,k$, identical systems is exposed to test at each stress $\mathscr{A}_s$, $s = 0,1,2,\ldots,k$, and the testing is initiated on all $\mathscr{A}_s$ at the same time. Let $y_{\text{si}}, i = 1,2,\ldots,n$, $s = 0,1,2,\ldots,k$, be the observed time for failure of i^{th} system at $\mathscr{A}_s, s = 0,1,2,\ldots,k$, stress level. According to the TIIPCS, at each stress level $\mathscr{A}_s$, at the first breakdown point y_{s1}, R_{s1} systems are eliminated from the remaining (n_s-1) randomly. Likewise, at the time y_{s2} of second failure, R_{s2} systems are eliminated from the remaining (n_s-2-R_{s1}) systems, or so until the specified sample of size $m_s, s = 0,1,2,\ldots,k$ is accomplished at each stress level $\mathscr{A}_s, s = 0,1,2,\ldots,k$, and afterwards, the test is concluded by eliminating all the existing $R_{\text{sm}_s} = n_s - m_s - \sum_{i=1}^{m_s-1} R_{\text{si}}$ systems.

Now, the obtained observed failure samples at s^{th} stress level can be written as $y_{s1} \le y_{s2} \le \cdots \le y_{m_s}$, $s = 0,1,2,\ldots,k$, and the likelihood for TIIPC data will be of the form:

$$L(\mathscr{Y}|\xi,\sigma,\delta) = \prod_{s=1}^{k}\left\{C_s\prod_{i=1}^{m_s}\left(\sum_{j\in Y_{\text{si}}}\Lambda_{\text{si}} f_{\text{sij}}\right)\left(1-F_{\mathscr{Y}_{\text{si}}}\left(y_{\text{si}}\right)\right)^{R_{\text{si}}}\right\}. \tag{18}$$

where $C_s = n_s(n_s-1-R_{s1})(n_s-2-R_{s1}-R_{s2})\ldots\sum_{i=1}^{m_s-1}R_{\text{si}}$. After swapping the values of $\sum_{j\in w_{\text{si}}}\Lambda_{\text{si}} f_{\text{sij}}$ & $F_{\mathscr{Y}_{\text{si}}}(y_{\text{si}})$ and applying log on both sides, the log likelihood $\ell = L(y_{\text{si}},\xi,\sigma,\delta)$ related to equation (18) is determined as follows:

$$\ell = \sum_{s=1}^{k}\sum_{i=1}^{m_s}\Big\{s\log\delta + \log\sigma + \log\xi - (2\sigma+1)\log(\mathfrak{B}) + \log\left(1-\mathfrak{B}^{-2\sigma}\right) + R_{\text{si}}\log\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)\Big\}, \tag{19}$$

where $\mathfrak{B} = (1+\delta^s\xi y_{\text{si}})$ and the model parameters' MLEs may now be computed using the following equations:

$$\frac{\partial\ell}{\partial\xi} = \sum_{s=1}^{k}\sum_{i=1}^{m_s}\frac{(\mathfrak{B}-1)}{\xi}\left\{\frac{1}{(\mathfrak{B}-1)} + \frac{2\sigma\mathfrak{B}^{-(2\sigma+1)}}{\left(1-\mathfrak{B}^{-2\sigma}\right)} - \frac{(1+2\sigma)}{\mathfrak{B}} + \frac{4R_{\text{si}}\sigma\left(\left(\mathfrak{B}^{-(4\sigma+1)}-\mathfrak{B}^{-(2\sigma+1)}\right)\right.}{\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)}\right\}, \tag{20}$$

$$\frac{\partial\ell}{\partial\sigma} = \sum_{s=1}^{k}\sum_{i=1}^{m_s}\left\{\frac{1}{\sigma} + \log(\mathfrak{B})\left(-2 + \frac{2\mathfrak{B}^{-2\sigma}}{\left(1-\mathfrak{B}^{-2\sigma}\right)} + \frac{4R_{si}\left(\mathfrak{B}^{-4\sigma}-\mathfrak{B}^{-2\sigma}\right)}{\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)}\right)\right\}, \tag{21}$$

$$\frac{\partial\ell}{\partial\delta} = \sum_{s=1}^{k}\sum_{i=1}^{m_s}\frac{s\sigma}{\delta}\left\{-2 + \frac{2\mathfrak{B}^{-2\sigma}}{\left(1-\mathfrak{B}^{-2\sigma}\right)} + \frac{4R_{\text{si}}\left(\mathfrak{B}^{-4\sigma}-\mathfrak{B}^{-2\sigma}\right)}{\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)}\right\}. \tag{22}$$

Because equations (20), (21), and (22) are nonlinear, obtaining closed-form solutions manually is very tough process. To seek numerical solutions to the aforementioned nonlinear system, an iterative technique such as Newton–Raphson can be employed, but we numerically obtained solutions in our paper using the Optim() function of R programming language.

The ACIs of the parameters may now be estimated using TIIPC masked data and the asymptotic characteristics of the MLEs. The ACIs can be calculated by mathematically inverting the observed Fisher information matrix. As a result, the estimated 95% two-sided ACIs for ξ, σ, and δ may now be calculated as follows:

$$\begin{aligned}&\widehat{\xi} \pm 1.96\sqrt{\operatorname{var}(\widehat{\xi})},\\ &\widehat{\sigma} \pm 1.96\sqrt{\operatorname{var}(\widehat{\sigma})},\\ &\widehat{\delta} \pm 1.96\sqrt{\operatorname{var}(\widehat{\delta})}.\end{aligned} \tag{23}$$

where $\operatorname{var}(\widehat{\xi})$, $\operatorname{var}(\widehat{\sigma})$, and $\operatorname{var}(\widehat{\delta})$ are main diagonal entries of F^{-1} and can be obtained as follows:

$$F^{-1} = \begin{bmatrix} -\dfrac{\partial^2\ell}{\partial\xi^2} & -\dfrac{\partial^2\ell}{\partial\xi\,\partial\sigma} & -\dfrac{\partial^2\ell}{\partial\xi\,\partial\delta} \\ -\dfrac{\partial^2\ell}{\partial\sigma\,\partial\xi} & -\dfrac{\partial^2\ell}{\partial\sigma^2} & -\dfrac{\partial^2\ell}{\partial\delta\,\partial\sigma} \\ -\dfrac{\partial^2\ell}{\partial\delta\,\partial\xi} & -\dfrac{\partial^2\ell}{\partial\sigma\,\partial\delta} & -\dfrac{\partial^2\ell}{\partial\delta^2} \end{bmatrix}^{-1}_{(\widehat{\xi},\widehat{\sigma},\widehat{\delta})} = \begin{bmatrix} \operatorname{var}(\widehat{\xi}) & \operatorname{covar}(\widehat{\xi}\widehat{\sigma}) & \operatorname{covar}(\widehat{\xi}\widehat{\delta}) \\ \operatorname{covar}(\widehat{\sigma}\widehat{\xi}) & \operatorname{var}(\widehat{\sigma}) & \operatorname{covar}(\widehat{\delta}\widehat{\sigma}) \\ \operatorname{covar}(\widehat{\delta}\widehat{\xi}) & \operatorname{covar}(\widehat{\sigma}\widehat{\delta}) & \operatorname{var}(\widehat{\delta}) \end{bmatrix}. \tag{24}$$

(1) Step 1: initialize the values of parameters ξ, σ, and δ.
(2) Step 2: define the stress levels $\mathscr{A}_s$, $s = 0, 1, 2, \ldots, k$.
(3) Step 3: using the uniform (0, 1) distribution, generate k TIIPC samples of size m following the procedure given by Balakrishnan and Sandhu [35].
(4) Step 4: for each sample size and removed items, TIIPC sample data for each stress level using $(\exp(\ln(1-u)/\sigma) - 1)/\xi\delta^s$ based on the TIIPC data generated in step 3.
(5) Step 5: for each censoring scheme and stress levels, repeat the above steps for 10000 times.
(6) Step 6: compute the average MLEs of ξ, σ, and δ with their respective RABs and RMSEs.
(7) Step 7: compute ILs and CPs of ACIs.
(8) Step 8: compute the reliability estimates with RABs, RMSEs, lengths, and CPs using the MLEs of ξ, σ, and δ. obtained in previous step 6.

ALGORITHM 1

The constituents of F are determined by the computations as follows:

$$
\begin{aligned}
\frac{\partial^2 \ell}{\partial \xi^2} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\Bigg\{ -\frac{1}{\xi^2} - \frac{2\mathcal{Y}_{si}^2 \delta^{2s}\mathfrak{B}^{-2(1+\sigma)}(1+2\sigma)\sigma}{\xi^2\left(1-\mathfrak{B}^{-2\sigma}\right)} - \frac{4(\mathfrak{B}-1)^2\mathfrak{B}^{-2(1+2\sigma)}\sigma^2}{\xi^2\left(1-\mathfrak{B}^{-2\sigma}\right)^2} + \frac{(\mathfrak{B}-1)^2(1+2\sigma)}{\xi^2\mathfrak{B}^2} \\
&\qquad + R_{\mathrm{si}}\Bigg(-\frac{\left(4(\mathfrak{B}-1)\mathfrak{B}^{-1-4\sigma}\sigma - 4(\mathfrak{B}-1)\mathfrak{B}^{-1-2\sigma}\sigma\right)^2}{\xi\left(-\mathfrak{B}^{-4\sigma}+2\mathfrak{B}^{-2\sigma}\right)^2}, \\
\frac{\partial^2 \ell}{\partial \delta^2} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\frac{s(\mathfrak{B}-1)}{\delta^2}\Bigg\{\Bigg(-\frac{1}{(\mathfrak{B}-1)} + \frac{2\sigma(s-1)}{\mathfrak{B}\left(\mathfrak{B}^{2\sigma}-1\right)} - \frac{4s\delta\sigma^2\mathcal{Y}_{si}}{\mathfrak{B}^2\left(\mathfrak{B}^{2\sigma}-1\right)^2} - \frac{R_{\mathrm{si}}\log(\mathfrak{B})\left(4\mathfrak{B}^{-4\sigma}-4\mathfrak{B}^{-2\sigma}\right)^2}{\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)^2} + \frac{R_{\mathrm{si}}\left(-32\mathfrak{B}^{-4\sigma}+16\mathfrak{B}^{-2\sigma}\right)}{\left(2\mathfrak{B}^{-2\sigma}-\mathfrak{B}^{-4\sigma}\right)}\Bigg)\Bigg\} \\
\frac{\partial^2 \ell}{\partial \sigma^2} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\Bigg\{ -\frac{1}{\sigma^2} - \log(\mathfrak{B})\Bigg(\frac{8\mathfrak{B}^{-4\sigma}}{\left(1-\mathfrak{B}^{-2\sigma}\right)^2} - \frac{8\mathfrak{B}^{-2\sigma}}{\left(1-\mathfrak{B}^{-2\sigma}\right)} - \frac{\left(4(\mathfrak{B}-1)^2\mathfrak{B}^{-2-4\sigma}(1+4\sigma)\sigma - 4(\mathfrak{B}-1)^2\mathfrak{B}^{-2-2\sigma}(1+2\sigma)\sigma\right)}{\xi^2\left(-\mathfrak{B}^{-4\sigma}+2\mathfrak{B}^{-2\sigma}\right)}\Bigg)\Bigg\} \\
&\qquad + \frac{(\mathfrak{B}-s)(1+2\sigma)}{\mathfrak{B}^2} - \frac{2s\delta\sigma(1+2\sigma)\mathcal{Y}_{\mathrm{si}}}{\mathfrak{B}^2\left(\mathfrak{B}^{2\sigma}-1\right)} - \frac{\left(4R_{si}\sigma\left(-\mathfrak{B}\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)+\left(s\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}+2\sigma(\mathfrak{B}-1)\mathfrak{B}^{2\sigma}\right)\right)\right)\right)}{(\mathfrak{B}-1)\mathfrak{B}^2\left(1-2\mathfrak{B}^{2\sigma}\right)^2}\Bigg)\Bigg\}, \\
\frac{\partial^2 \ell}{\partial \xi\,\partial \sigma} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\left\{ \frac{-\left(\left(2\mathcal{Y}_{si}\delta^s\left(\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)\left(\left(2-5\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)+2\mathfrak{B}^{2\sigma}\left(\left(1-2\mathfrak{B}^{2\sigma}\right)^2+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)\sigma log(\mathfrak{B})\right)\right)\right)}{\left(\mathfrak{B}\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)^2\right)}\right\}, \\
\frac{\partial^2 \ell}{\partial \xi\,\partial \delta} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\left\{ \frac{-\left(\left(s\mathcal{Y}_{si}\delta^{-1+s}\left(\left(-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)^2+2\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)\left(2-5\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)\sigma+4\mathfrak{B}^{3\sigma}\left(\left(1-2\mathfrak{B}^{2\sigma}\right)^2+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)\sigma^2\right)\right)\right)}{\mathfrak{B}^2\left(1-2\mathfrak{B}^{2\sigma}\right)^2\left(\mathfrak{B}^{2\sigma}-1\right)^2}\right\}, \\
\frac{\partial^2 \ell}{\partial \delta\,\partial \sigma} &= \sum_{s=1}^{k}\sum_{i=1}^{m_s}\left\{ \frac{-\left(\left(2s\mathcal{Y}_{si}\delta^{-1+s}\xi\left(\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)\left(2-5\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)+2\mathfrak{B}^{2\sigma}\left(\left(1-2\mathfrak{B}^{2\sigma}\right)^2+2R_{si}\left(-1+\mathfrak{B}^{2\sigma}\right)^2\right)\sigma log(\mathfrak{B})\right)\right)\right)}{\left(\mathfrak{B}\left(1-3\mathfrak{B}^{2\sigma}+2\mathfrak{B}^{4\sigma}\right)^2\right)}\right\}.
\end{aligned}
\tag{25}
$$

4. Simulation Study and Discussion

A Monte-Carlo simulation uses the R package carried out to examine the performance of the suggested techniques for estimating the parameters of the GP distribution based on the CSALT under TIIPCS for hybrid systems using GmP. We select initial values of parameter $(\xi = 0.8, \sigma = 0.5, \delta = 1.1)$ with various sample combinations $(n_s, m_s) = (40, 20)$, (50, 25), (60, 30), (70, 35), (80, 40), (90, 45), (100, 50), (110, 55), (120, 60), and (130, 65). Under the TIIPCS, the four levels of constant stress are considered as: the normal stress level $\mathscr{A}_0 = 5$; $\mathscr{A}_1 = 10$, $\mathscr{A}_2 = 15$, and $\mathscr{A}_3 = 20$. Additionally, we describe two distinct test censoring schemes (CS) (i) $R_{s1}, R_{s2}, \ldots, R_{s(m-1)} = (n_s - m_s)/m_s$ and $R_{sm} = 0$; (ii) $R_{s1}, R_{s2}, \ldots, R_{s(m-1)} = 1$ and $R_{sm} = n_s - 2.25m_s + 1$. Under four different constant-stress levels, TIIPC samples are created with various combinations of n_s, m_s, and the test schemes. To accommodate for the masking impact, 15% of the simulated system lifetime data is removed from all the simulated system failures. We derive average RABs and RMSEs for point estimates, as well as average confidence ILs of 95% ACIs with associated CPs, for each test scheme. Moreover,

Table 1: The MLEs, RMSEs, and RABs of the parameters with true values of parameters $\xi = 0.8, \sigma = 0.5, \delta = 1.1$, and 15% masking.

n, m	CS	ξ			σ			δ		
		MLE	RMSE	RAB	MLE	RMSE	RAB	MLE	RMSE	RAB
40, 20	1	6.70872	3.48434	0.37987	0.17639	0.01815	0.08097	1.00983	0.14779	0.1164
50, 25	1	6.58008	3.0252	0.34308	0.17583	0.01639	0.0736	1.0082	0.13027	0.10233
60, 30	1	6.49486	2.73192	0.31463	0.17549	0.01497	0.06767	1.0056	0.1198	0.09456
70, 35	1	6.43676	2.50032	0.29154	0.1749	0.01363	0.06211	1.00657	0.11088	0.08759
80, 40	1	6.3333	2.25387	0.27141	0.17486	0.01295	0.05873	1.00864	0.10381	0.08167
90, 45	1	6.32515	2.07271	0.25346	0.17477	0.01195	0.05437	1.00545	0.09704	0.07663
100, 50	1	6.30859	1.96555	0.24063	0.17465	0.01129	0.05155	1.00404	0.09172	0.07308
110, 55	1	6.29448	1.87339	0.2291	0.17437	0.01077	0.04891	1.00462	0.08701	0.06872
120, 60	1	6.27696	1.77539	0.21945	0.17431	0.01045	0.04758	1.00366	0.08378	0.0669
130, 65	1	6.24094	1.66062	0.20712	0.17425	0.00977	0.04447	1.00419	0.07984	0.06338
40, 20	2	3.43475	1.92971	0.41046	0.20131	0.02906	0.10791	1.29854	0.19853	0.12048
50, 25	2	6.21753	3.00922	0.35999	0.22651	2.04973	0.2724	1.00761	0.13504	0.10609
60, 30	2	5.01443	2.23527	0.33352	0.20304	0.58332	0.12237	1.09482	0.1325	0.09595
70, 35	2	6.06881	2.50164	0.31086	0.21741	1.22705	0.21315	1.00672	0.11487	0.0905
80, 40	2	3.2682	1.31908	0.30879	0.1991	0.02313	0.08682	1.29195	0.14047	0.0862
90, 45	2	6.06618	2.21213	0.27792	0.19603	0.02042	0.07917	1.00339	0.10027	0.07967
100, 50	2	4.93844	1.74497	0.26947	0.20443	0.8286	0.12644	1.08741	0.10398	0.0761
110, 55	2	5.99213	2.02393	0.26291	0.19558	0.01971	0.07714	1.00576	0.09243	0.07323
120, 60	2	3.21844	1.11995	0.2702	0.22703	1.44499	0.26244	1.29465	0.48778	0.07875
130, 65	2	5.9579	1.8565	0.24311	0.19602	0.01958	0.07479	1.00217	0.08499	0.06757

Table 2: The ACI lengths (ACIL) and CPs of 95% ACIs with true values of parameters $\xi = 0.25, \sigma = 1.25, \delta = 1.1$, and 15% masking.

n, m	CS	ξ		σ		δ	
		ACIL	ACICP	ACIL	ACICP	ACIL	ACICP
40, 20	1	13.65862	0.9555	0.07114	0.952	0.57933	0.9519
50, 25	1	11.8588	0.9537	0.06424	0.9506	0.51067	0.9528
60, 30	1	10.70912	0.9543	0.05869	0.9551	0.46961	0.9488
70, 35	1	9.80127	0.9552	0.05343	0.9537	0.43463	0.9511
80, 40	1	8.83517	0.9543	0.05075	0.9509	0.40693	0.9523
90, 45	1	8.12502	0.9553	0.04685	0.951	0.38039	0.9516
100, 50	1	7.70497	0.9566	0.04427	0.9512	0.35956	0.9525
110, 55	1	7.34368	0.9555	0.04223	0.9492	0.34109	0.9509
120, 60	1	6.95955	0.9572	0.04097	0.9516	0.32842	0.952
130, 65	1	6.50964	0.957	0.03828	0.9503	0.31296	0.9507
40, 20	2	7.56447	0.9549	0.11391	0.9575	0.77825	0.9543
50, 25	2	11.79616	0.9525	8.03492	0.9998	0.52935	0.9509
60, 30	2	8.76226	0.9545	2.28661	0.9999	0.51941	0.9504
70, 35	2	9.80642	0.9554	4.81004	0.9997	0.4503	0.9536
80, 40	2	5.17081	0.953	0.09067	0.9578	0.55063	0.9479
90, 45	2	8.67153	0.9545	0.08006	0.9571	0.39307	0.9512
100, 50	2	6.84028	0.9551	3.24812	0.9999	0.40759	0.9509
110, 55	2	7.9338	0.9552	0.07726	0.9547	0.36234	0.9519
120, 60	2	4.39021	0.9553	5.66437	0.9996	1.91208	0.9996
130, 65	2	7.27749	0.9543	0.07676	0.959	0.33316	0.9508

the reliability estimates are also derived at normal use conditions with their RABs and RMSEs and the limits of ACIs with ILs and CPs. The simulation process is carried out in accordance with the following Algorithm 1.

The numerical results of MLEs, as well as their RMSE and RABs, are produced and shown in Table 1, whereas ACIs, together with their ILs and CPs, are represented in Table 2. Table 3 provides the reliability estimates with RABs, RMSEs, ILs, and CPs.

The computed values of the MLEs of the parameters, as well as their related RMSEs and RABs, based on the simulation study have been reported in Table 1. The ILs and CPs of ACIs for parameters have been presented in Table 2. System reliability based on MLEs, along with its RMSEs and RABs, has been discussed in Table 3. The ACIs for system reliability with their respective ILs and CPs are also presented in Table 3. The reported results show that the estimations are a little bit biased but with relatively small values for RMSEs and RABs. It

Table 3: The MLEs; RMSEs, RABs, ACI lower limit (ACILL), ACI upper limit (ACIUL), ACI lengths (ACIL), and CPs of system reliability with true values of parameters $\xi = 0.25, \sigma = 1.25, \delta = 1.1$, and 15% masking.

n, m	CS	System reliability						
		MLE	RMSE	RAB	ACILL	ACIUL	ACIL	ACICP
40, 20	1	0.62567	0.21307	0.28267	0.20806	1.04329	0.83523	0.98125
50, 25	1	0.67784	0.24200	0.29848	0.20353	1.15215	0.94862	0.95000
60, 30	1	0.65119	0.22714	0.29236	0.20599	1.09639	0.89039	0.95833
70, 35	1	0.65099	0.23644	0.30799	0.18756	1.11441	0.92685	0.96071
80, 40	1	0.66129	0.23260	0.29490	0.20540	1.11719	0.91179	0.95313
90, 45	1	0.67352	0.23086	0.28330	0.22104	1.12600	0.90496	0.96389
100, 50	1	0.67226	0.23033	0.28385	0.22082	1.12371	0.90289	0.96500
110, 55	1	0.67856	0.22242	0.27308	0.24262	1.11450	0.87187	0.96591
120, 60	1	0.66649	0.22538	0.28703	0.22474	1.10823	0.88349	0.96875
130, 65	1	0.65194	0.24156	0.31267	0.17848	1.12540	0.94692	0.96538
40, 20	2	0.59660	0.24597	0.35438	0.11451	1.07870	0.96419	0.98750
50, 25	2	0.56007	0.25623	0.38835	0.05786	1.06229	1.00444	0.99000
60, 30	2	0.58104	0.24350	0.34330	0.10377	1.05831	0.95454	0.97083
70, 35	2	0.58293	0.25797	0.37707	0.07731	1.08856	1.01125	0.98571
80, 40	2	0.61975	0.24727	0.33457	0.13509	1.10440	0.96930	0.96875
90, 45	2	0.61590	0.25139	0.34115	0.12318	1.10862	0.98543	0.96667
100, 50	2	0.59913	0.25099	0.35560	0.10719	1.09108	0.98388	0.98500
110, 55	2	0.64288	0.23529	0.30145	0.18172	1.10405	0.92233	0.95455
120, 60	2	0.57630	0.25784	0.37765	0.07094	1.08166	1.01071	0.97917
130, 65	2	0.61382	0.25105	0.33730	0.12177	1.10588	0.98411	0.96346

is noteworthy to highlight that the estimates are fairly stable and, more importantly, are coming closer to the true values as sample sizes increase. Furthermore, Table 2 shows that ACIs are attaining quite high coverage probabilities and are typically greater than 0.94 in all sample scenarios, although the length of ACIs decreases as sample size increases. A similar trend can be seen in Table 3 for the system reliability estimations. We can also see from all these tables that CS (i) performs better than CS (ii) in all sample cases.

5. Conclusion

In this article, we employed GmP to investigate the CSALT for the hybrid system in the presence of TIIPC censored masked data. Considering that the failure times of components of the hybrid system follow the GP distribution, the MLE approach was utilized to get point and interval estimates of unknown parameters as well as the system's reliability. RMSEs and RABs were computed to evaluate the efficiency of point estimates of the parameters. The observed Fisher information matrix was developed and utilized to generate the 95% ACIs for the parameters. Moreover, to assess the accuracy of the ACIs of the parameters, the ILs and CPs for parameters, as well as for the reliability estimates, were also addressed. Based on the stated results, the estimates were found to be generally consistent, with relatively small values for both RMSEs and RABs in all sample cases. ACIs were also determined to be of reasonable quality, with high CPs and precise length. The reliability estimates, like the parameter estimates, followed a consistent pattern across all sample combinations. Furthermore, in all sample circumstances, the estimates under CS (i) fared better than the estimates under CS (ii). In terms of future research options, the current study could be broadened to investigate the scenario with a more complicated hybrid system using other censoring strategies based on masked data and taking into account different failure time distributions. Instead of using MLE, the Bayesian approach for parameter estimation might be explored.

Data Availability

All the data used are in included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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