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Research Article

A Nonconventional Auxiliary Information Based Robust Class of Exponential-type Difference Estimators

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This study proposes a new class of improved exponential-type difference estimators of finite population mean by using supplementary information of known median along with suitable combinations of the conventional and non-conventional measures of the auxiliary variables under simple random sampling scheme. The expressions for the mean squared error and minimum mean squared error are derived up to first order of the approximation. Six real data sets were used to assess the performance of proposed class of estimators in comparison with existing estimators. The compariosn established that the suggested class of estimators are efficient than their existing counterparts considered in this study. To further support the findings of the numerical comparison, a simulation study was carried out which also proved the superiority of the proposed class of estimators of population mean. To gauge the performance of the propsoed class of estimators when some outliers are present in the data, a robustness study was carried out which showed that the proposed estimators considerably outperform their existing counterparts in terms of lower mean squared errors.

1. Introduction

In survey sampling literature, several estimation methods such as the ratio, product, and regression are frequently used to take advantage of the supplementary information for estimation of unknown population parameters under different sampling schemes. The purpose behind utilization of auxiliary information is to improve the efficiency of the estimators of the parameters under consideration. In numerous statistical analyses, a primary goal is to estimate location parameter which is an important descriptive measure. When the observations are relatively homogeneous, the sample mean performs very well as measure of location. There are various situations in survey sampling when the distribution of the study variable is extremely skewed and sample mean becomes an inefficient measure of location. Moreover, if there are some extreme observations in the data which lies far from rest of the observations then the sample mean provides poor results. In these scenarios, an alternative measure of location known as median is usually considered as a preferred measure of location. Various authors have developed different estimators to estimate population parameters using auxiliary information under different sampling scheme. A bulk of literature is available based on utilization of supplementary variable. See, for example, Yan and Tain [1], Koyuncu [2], Subramani and Kumarapandiyan [3], Subramani and Prabavathy [4], John and Inyang [5], Singh and Pal [6], Singh et al. [7], Subramani [8], Abid et al. [9], Abbas et al. [10], Zaman [11], Zaman and Bulut [12], Yadav et al. [13]. Many researchers have used the known median of the study variable for the estimation of population mean. For example, Subramani and Prabavathy [14] and Yadav et al. [15] suggested new estimators using known median of the study variable, Irfan et al. [16] proposed some power–type ratio estimators, Shahzad et al. [17] suggested a family of exponential estimators utilizing known median of the study variable, Hafeez et al. [18] developed some new median based estimators utilizing supplementary information. In this study, a new class of exponential-type difference estimators is proposed to estimate finite population mean based on Known median of the study variable along with supplementary information of a single auxiliary variable under simple random sampling scheme.

The notations used in this study are as follows:

(i) N, Population size

(ii) n, Sample size

(iii) $f = n/N$, Sampling fraction

(iv) abr Y, Study Variables

(v) X Auxiliary variable

(vi) $\overline{Y,X}$, Population means of study variable and auxiliary variable

(vii) $\overline{y}, \overline{x}$, Sample means of study variable and auxiliary variables

(viii) $M_{Y_{0.5}} M_{X_{0.5}}$, Population median of study variable and auxiliary variable

(ix) $\widehat{M}_{Y_{0.5}} \widehat{M}_{X_{0.5}}$, sample median of study variable and auxiliary variable

(x) C_y, C_x, Coefficient of variation of the study variable and auxiliary variable

(xi) MSE (.), Mean square error of the estimator

(xii) $\widehat{\overline{Y}}_{i,}$Existing estimators of $\overline{Y}$,

(xiii) $\widehat{\overline{Y}}_{pj,}$ Proposed estimators of $\overline{Y}$

Subscript:

(i) abr i, For existing estimators

(ii) j, For existing estimators

The organization of the remaining article is as follows: Section 2 presents some existing estimators that utilize auxiliary information based on known median of the study variable to estimate finite population mean. Section 3 describes the proposed new family of exponential-type difference estimators based on known median of the study variable coupled with information on an auxiliary variable. Moreover, the theoretical minimum MSE expression of the proposed family of estimators has been derived in this Section. The performance evaluation and comparison of the suggested class of estimators based on simulation, numerical and robustness studies are presented in Section 4. Finally, Section 5 concludes the paper with a summary and concluding remarks.

2. Some Existing Estimators of Population Mean

Following are some formulas and representations that are utilized in existing and proposed median based estimators for the estimation of finite population mean.

$C_y = S_y/\overline{Y}$, $S_y^2 = 1/N - 1\sum_{i-1}^{N} (Y_i - \overline{Y})^2$, $C_{yx=\rho_{yx}} C_y C_x$, $f = n/N$, $g = 1 - f/n$, $C_x = S_x/\overline{X}$, $R = \overline{Y}/X$, $R_m = \overline{Y}/M_{Y_{0.5}}$, $S_X^2 = 1/N - 1\sum_{i-1}^{N} (X_i - \overline{X})^2$, $\text{Cov}(x, y) = 1/N - 1\sum_{i-1}^{N} (Y_i - \overline{Y})(X_i - \overline{X})$, $\rho_{xy} = \text{Cov}(x, y)/S_x S_y$, $C_{y\widehat{M}_{y_{0.5}}} = S_{y\widehat{M}_{y_{0.5}}}/\overline{Y}\widehat{M}_{y_{0.5}}$, $C_{\widehat{M}_{y_{0.5}}} = S_{\widehat{M}_{y_{0.5}}}/M_{y_{0.5}}$,

Traditionally, the sample mean, $T_1 = \overline{y}$, is used to estimate the population mean of the study variable Y. The variance of traditional mean estimator of the study variable is

$$V(T_1) = \frac{1-f}{n}\overline{Y}^2 C_y^2 \tag{1}$$

Subramani and Kumarapandiyan [3] proposed an estimator utilizing known median of auxiliary variable to estimate finite population mean as follow:

$$T_2 = \overline{y}\left(\frac{M_{X_{0.5}} + \overline{X}}{M_{X_{0.5}} + \overline{x}}\right) \tag{2}$$

The MSE of T_2 is given bellow

$$MSE(T_2) \cong g\overline{Y}^2\left(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_{yx}\right) \tag{3}$$

where. $\theta_2 = \overline{X} + M_{X_{0.5}}/\overline{X}$

Subramani and Prabavathy [14] suggested two new estimators utilizing known median of both study and auxiliary variables and mean of the auxiliary variable for the estimation of population mean. Their proposed estimators are given as

$$T_3 = \overline{y}\left(\frac{M_{X_{0.5}} M_{Y_{0.5}} + \overline{X}}{M_{X_{0.5}} \widehat{M}_{Y_{0.5}} + \overline{X}}\right),$$
$$T_4 = \overline{y}\left(\frac{\overline{X} M_{Y_{0.5}} + M_{X_{0.5}}}{\overline{X} \widehat{M}_{Y_{0.5}} + M_{X_{0.5}}}\right). \tag{4}$$

The MSE of T_3andT_4 as follows

$$\text{MSE}(T_i) \cong g\left[S_y^2 + R_m^2\theta_i^2 S_{\widehat{M}_{Y_{0.5}}}{}^2 - 2R_m\theta_i Cov\left(\overline{y}, \widehat{M}_{Y_{0.5}}\right)\right] \quad \text{for } i = 3, 4. \tag{5}$$

where $R_m = \overline{Y}/M_{Y_{0.5}}$, $\theta_3 = M_{X_{0.5}} M_{Y_{0.5}}/M_{X_{0.5}} M_{Y_{0.5}} + \overline{X}$, $\theta_4 = \overline{X} M_{Y_{0.5}}/\overline{X} M_{Y_{0.5}} + M_{X_{0.5}}$

Subramani and Prabavathy [4] suggested median based estimators using quartiles and their functions for the estimation of population mean, which are given bellow

$$T_5 = \overline{y}\left(\frac{M_{Y_{0.5}} + Q_1}{\widehat{M}_{Y_{0.5}} + Q_1}\right),$$

$$T_6 = \overline{y}\left(\frac{M_{Y_{0.5}} + Q_3}{\widehat{M}_{Y_{0.5}} + Q_3}\right),$$

$$T_7 = \overline{y}\left(\frac{M_{Y_{0.5}} + Q_r}{\widehat{M}_{Y_{0.5}} + Q_r}\right), \quad (6)$$

$$T_8 = \overline{y}\left(\frac{M_{Y_{0.5}} + Q_d}{\widehat{M}_{Y_{0.5}} + Q_d}\right),$$

$$T_9 = \overline{y}\left(\frac{M_{Y_{0.5}} + Q_a}{\widehat{M}_{Y_{0.5}} + Q_a}\right),$$

The mean square errors of the suggested estimators T_i are

$$MSE(T_i) \cong g\left[S_y^2 + R_m^2\theta_i^2 S_{\widehat{M}_{Y_{0.5}}}{}^2 - 2R_m\theta_i \mathrm{Cov}\left(\overline{y}, \widehat{M}_{Y_{0.5}}\right)\right] \quad \text{for } i = 5, 6, \ldots, 9. \quad (7)$$

where $R_m = \overline{Y}/M_{Y_{0.5}}$, $\theta_5 = M_{Y_{0.5}}/M_{Y_{0.5}} + Q_1$, $\theta_6 = M_{Y_{0.5}}/M_{Y_{0.5}} + Q_3$, $\theta_7 = M_{Y_{0.5}}/M_{Y_{0.5}} + Q_r\theta_8 = M_{Y_{0.5}}/M_{Y_{0.5}} + Q_d\theta_9 = M_{Y_{0.5}}/M_{Y_{0.5}} + Q_3$

Yadav et al. [15] proposed two estimators to estimate finite population mean using known median of both study and auxiliary variables, which are

$$T_{10} = K_{10}\overline{y}\left(\frac{M_{X_{0.5}}M_{Y_{0.5}} + \overline{X}}{M_{X_{0.5}}\widehat{M}_{Y_{0.5}} + \overline{X}}\right),$$

$$T_{11} = K_{11}\overline{y}\left(\frac{\overline{X}M_{Y_{0.5}} + M_{X_{0.5}}}{\overline{X}\widehat{M}_{Y_{0.5}} + M_{X_{0.5}}}\right). \quad (8)$$

The minimum MSE of T_{10} and T_{11} are as follows

$$K_i^{\text{opt}} = \frac{A_i}{B_i} \text{for} i = 10, 11,$$

$$A_i = 1 + g\left[\theta_i^2 \frac{S^2_{\widehat{M}_{Y_{0.5}}}}{M^2_{y_{0.5}}} - \theta_i \frac{\mathrm{Cov}\left(\overline{y}, \widehat{M}_{Y_{0.5}}\right)}{\overline{Y}\, M_{y_{0.5}}}\right], \quad (9)$$

$$B_i = 1 + g\left[C_y^2 + 3\theta_i^2 \frac{S^2_{\widehat{M}_{Y_{0.5}}}}{M^2_{y_{0.5}}} - 4\theta_i \frac{\mathrm{Cov}\left(\overline{y}, \widehat{M}_{Y_{0.5}}\right)}{\overline{Y}\, M_{y_{0.5}}}\right].$$

where $\theta_{10} = M_{X_{0.5}}M_{Y_{0.5}}/M_{X_{0.5}}M_{Y_{0.5}} + \overline{X}$, $\theta_{11} = \overline{X}M_{Y_{0.5}}/\overline{X}M_{Y_{0.5}} + M_{X_{0.5}}$

$$MSE_{\min}(T_i) \cong \overline{Y}^2\left[1 - \frac{A_i^2}{B_i}\right] \text{for} i = 10, 11. \quad (10)$$

Subramani [8] suggested an estimator based on known median of study variable for the estimation of population mean, which is given below

$$T_{12} = \overline{y}\left(\frac{M_{Y_{0.5}}}{\widehat{M}_{Y_{0.5}}}\right). \quad (11)$$

The MSE of the suggested estimator T_{12} is

$$MSE(T_i) \cong g\left[S_y^2 + R_m^2 S_{\widehat{M}_{Y_{0.5}}}{}^2 - 2R_m \mathrm{Cov}\left(\overline{y}, \widehat{M}_{Y_{0.5}}\right)\right]. \quad (12)$$

where $R_m = \overline{Y}/M_{Y_{0.5}}$,

Kumar et al. [19] proposed an estimator to estimate finite population mean using known median of study variables, which is defined as

$$T_{13} = \overline{y}exp\left(\frac{M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}}{M_{Y_{0.5}} + (1-a)\widehat{M}_{Y_{0.5}}}\right). \quad (13)$$

The MSE of T_{13} is

$$MSE(T_{13}) \cong g\overline{Y}^2\left[C_y^2 + \frac{C_{\widehat{M}_{y_{0.5}}}{}^2}{a^2} - \frac{2}{a}C_{y\widehat{M}_{y_{0.5}}}\right]. \quad (14)$$

Which is optimum for. $a^{\text{opt}} = C_{\widehat{M}_{y_{0.5}}}{}^2 / C_{y\widehat{M}_{y_{0.5}}}$

So, the minimum MSE of T_{13} is

$$MSE_{\min}(T_{13}) \cong g\overline{Y}^2\left[C_y^2 - \frac{C_{y\widehat{M}_{y0.5}}^2}{C_{\widehat{M}_{y0.5}}^{\ 2}}\right]. \quad (15)$$

Irfan et al. [16] introduced a class of power–type ratio estimators using both known median of the study and auxiliary variables to estimate population mean, which are

$$\begin{aligned}
T_{14} &= K_1\overline{y}\left(\frac{M_{X_{0.5}}M_{Y_{0.5}}+\overline{X}}{M_{X_{0.5}}\widehat{M}_{Y_{0.5}}+\overline{X}}\right)^{M_{X_{0.5}}M_{Y_{0.5}}/M_{X_{0.5}}M_{Y_{0.5}}+\overline{X}},\\
T_{15} &= K_2\overline{y}\left(\frac{\overline{X}M_{Y_{0.5}}+M_{X_{0.5}}}{\overline{X}\widehat{M}_{Y_{0.5}}+M_{X_{0.5}}}\right)^{\left(\overline{X}M_{Y_{0.5}}/\overline{X}M_{Y_{0.5}}+M_{X_{0.5}}\right)},\\
T_{16} &= K_3\overline{y}\left(\frac{M_{Y_{0.5}}+1}{\widehat{M}_{Y_{0.5}}+1}\right)^{\left(M_{Y_{0.5}}/M_{Y_{0.5}}+1\right)},\\
T_{17} &= K_4\overline{y}\left(\frac{M_{X_{0.5}}M_{Y_{0.5}}+R_m\overline{X}}{M_{X_{0.5}}\widehat{M}_{Y_{0.5}}+R_m\overline{X}}\right)^{M_{X_{0.5}}M_{Y_{0.5}}/M_{X_{0.5}}M_{Y_{0.5}}+R_m\overline{X}},\\
T_{18} &= K_5\overline{y}\left(\frac{\overline{X}R_mM_{Y_{0.5}}+M_{X_{0.5}}}{\overline{X}R_m\widehat{M}_{Y_{0.5}}+M_{X_{0.5}}}\right)^{\left(\overline{X}R_mM_{Y_{0.5}}/\overline{X}R_mM_{Y_{0.5}}+M_{X_{0.5}}\right)},\\
T_{19} &= K_6\overline{y}\left(\frac{M_{Y_{0.5}}+R_m}{\widehat{M}_{Y_{0.5}}+R_m}\right)^{\left(M_{Y_{0.5}}/M_{Y_{0.5}}+R_m\right)}.
\end{aligned} \quad (16)$$

The MSEs of the suggested estimators $T_i = 14, 15, \ldots, 19$ are given as

$$\begin{aligned}
MSE(T_i) \cong{}& \overline{Y}^2\left[(K_i-1)^2 + gK_i^2\left\{C_y^2 + \left(2\theta_i^4-\theta_i^3\right)C_{\widehat{M}_{y0.5}}^{\ 2} - 4\theta_i^2\frac{Cov\left(\overline{y},\widehat{M}_{Y_{0.5}}\right)}{\overline{Y}M_{Y_{0.5}}}\right\}\right]\\
&- gK_i\overline{Y}^2\left[C_y^2 + \left(\theta_i^4-\theta_i^3\right)C_{\widehat{M}_{y0.5}}^{\ 2} - 2\theta_i^2\frac{Cov\left(\overline{y},\widehat{M}_{Y_{0.5}}\right)}{\overline{Y}M_{0.5}}\right].
\end{aligned} \quad (17)$$

The optimum value of K_i is

$$K_i^{\text{opt}} = \frac{A_i}{B_i}\,\text{for}\,i = 14, 15, \ldots, 19. \quad (18)$$

where

$$\begin{aligned}
A_i &= 2 + g\left[\left(\theta_i^4-\theta_i^3\right)C_{\widehat{M}_{y0.5}}^{\ 2} - 2\theta_i^2\frac{Cov\left(\overline{y},\widehat{M}_{Y_{0.5}}\right)}{\overline{Y}\,M_{y_{0.5}}}\right],\\
B_i &= 2 + 2g\left[C_y^2 + \left(2\theta_i^4-\theta_i^3\right)C_{\widehat{M}_{y0.5}}^{\ 2} - 4\theta_i^2\frac{Cov\left(\overline{y},\widehat{M}_{Y_{0.5}}\right)}{\overline{Y}M_{y_{0.5}}}\right].
\end{aligned} \quad (19)$$

and

$$MSE_{\min}(T_i) \cong \overline{Y}^2\left[1 - \frac{A_i^2}{2B_i}\right]\text{for}\,i = 14, 15, \ldots, 19. \quad (20)$$

Yadav et al. [20] suggested two estimators of population mean utilizing known median of the study variable

$$\begin{aligned}
T_{20} &= \overline{y}\left(\frac{M_{Y_{0.5}}}{\widehat{M}_{Y_{0.5}}}\right)^a,\\
T_{21} &= \overline{y}\left(\frac{M_{Y_{0.5}}}{M_{Y_{0.5}} + a\left(\widehat{M}_{Y_{0.5}} - M_{Y_{0.5}}\right)}\right).
\end{aligned} \quad (21)$$

The MSE of suggested estimators T_{20}andT_{21} are

$$MSE(T_i) = g\overline{Y}^2\left[C_y^2 + a_i^2 C_{\widehat{M}_{y0.5}}^{\ 2} - 2a_i C_{y\widehat{M}_{y0.5}}\right] \text{for} i = 20, 21. \tag{22}$$

Which is optimum for. $a_i^{\text{opt}} = C_{y\widehat{M}_{y0.5}} / C_{\widehat{M}_{y0.5}}^{\ 2}$
So, the minimum MSE of T_i is

$$MSE_{\min}(T_i) \cong g\overline{Y}^2\left[C_y^2 - \frac{C^2_{y\widehat{M}_{y0.5}}}{C_{\widehat{M}_{y0.5}}^{\ 2}}\right] \text{for} i = 20, 21. \tag{23}$$

Kumar et al. [21] suggested an estimator of population mean using known median of the study variable

$$T_{22} = \overline{y}\left[a\left(2 - \frac{M_{Y_{0.5}}}{\widehat{M}_{Y_{0.5}}}\right) + (1-a)\left(2 - \frac{M_{Y_{0.5}}}{\widehat{M}_{Y_{0.5}}}\right)\right]. \tag{24}$$

The MSE of proposed estimator T_{22} is

$$MSE(T_{22}) \cong g\overline{Y}^2\left[C_y^2 + a^2 C_{\widehat{M}_{y0.5}}^{\ 2} - 2aC_{y\widehat{M}_{y0.5}}\right]. \tag{25}$$

Which is optimum for. $a^{opt} = -C_{y\widehat{M}_{y0.5}} / C_{\widehat{M}_{y0.5}}^{\ 2}$
So, the minimum MSE of T_{22} is

$$MSE_{\min}(T_{22}) \cong g\overline{Y}^2\left[C_y^2 - \frac{C^2_{y\widehat{M}_{y0.5}}}{C_{\widehat{M}_{y0.5}}^{\ 2}}\right]. \tag{26}$$

Yadav et al. [13] suggested two estimators of population mean utilizing known median of the study variable

$$T_{23} = \overline{y}\left(\frac{M_{Y_{0.5}}}{\widehat{M}_{Y_{0.5}}}\right)\exp\left[\frac{a\left(M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}\right)}{\left(M_{Y_{0.5}} + \widehat{M}_{Y_{0.5}}\right)}\right]. \tag{27}$$

The MSE of T_{23} is

$$MSE(T_{23}) \cong g\overline{Y}^2\left[C_y^2 + a^2 C_{\widehat{M}_{y0.5}}^{\ 2} - 2aC_{y\widehat{M}_{y0.5}}\right]. \tag{28}$$

Which is optimum for. $a^{opt} = -C_{y\widehat{M}_{y0.5}} / C_{\widehat{M}_{y0.5}}^{\ 2}$
So, the minimum MSE of T_{23} is

$$MSE_{\min}(T_{23}) \cong g\overline{Y}^2\left[C_y^2 - \frac{C^2_{y\widehat{M}_{y0.5}}}{C_{\widehat{M}_{y0.5}}^{\ 2}}\right] \tag{29}$$

Hafeez et al. [18] introduced new estimators utilizing known median of the study variable for the estimation of finite population mean

$$\begin{aligned}
T_{24} &= \overline{y}\exp\left(\frac{M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}}{M_{Y_{0.5}} + \widehat{M}_{Y_{0.5}}}\right),\\
T_{25} &= \overline{y} + K\left(M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}\right),\\
T_{26} &= K_1\overline{y} + K_2\left(M_{Y_{0.5}} - \widehat{M}_{Y_{0.50.5}}\right),\\
T_{27} &= \left[K_1\overline{y} + K_2\left(M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}\right)\right]\exp\left(\frac{M_{Y_{0.5}} - \widehat{M}_{Y_{0.5}}}{M_{Y_{0.5}} + \widehat{M}_{Y_{0.5}}}\right),
\end{aligned} \tag{30}$$

The MSE of T_{24} estimator is

$$MSE(T_{24}) \cong \overline{Y}^2\left(C_y^2 + \frac{C^2_{\widehat{M}_{y0.5}}}{4} - C_{y\widehat{M}_{y0.5}}\right) \tag{31}$$

The minimum MSE of T_{25} at $K^{\text{opt}} = \text{Cov}(\overline{y}, \widehat{M}_{Y_{0.5}}) / V(\widehat{M}_{y_{0.5}})$. is given as

$$MSE_{\min}(T_{25}) \cong S_y^2\left(1 - \rho^2_{y\widehat{M}_{y0.5}}\right) \tag{32}$$

The minimum MSE of T_{26} at optimum values

$$\begin{aligned}
\text{K}_1^{\text{opt}} &= \frac{1}{1 + {}^2_{\text{Cy}}\left(1 - \rho^2_{\text{y}\widehat{\text{M}}_{\text{y}_{0.5}}}\right)},\\
\text{K}_2^{\text{opt}} &= \frac{{}^2_{\text{Sy}}\rho^2_{\text{y}\widehat{\text{M}}_{\text{y}_{0.5}}}}{1 + {}^2_{\text{Cy}}\left(1 - \rho^2_{\text{y}\widehat{\text{M}}_{\text{y}_{0.5}}}\right)}.
\end{aligned} \tag{33}$$

is

$$MSE_{min}(T_{26}) \cong \frac{\overline{Y}^2 C_y^2\left(1 - \rho^2_{y\widehat{M}_{y0.5}}\right)}{1 + C_y^2\left(1 - \rho^2_{y\widehat{M}_{y0.5}}\right)} \tag{34}$$

The minimum MSE of T_{27} at optimum values

$$K_1^{opt} = \frac{1 - 1/8C_{\widehat{M}_{y0.5}}^{\ \ 2}}{1 + {}_{Cy}^{2}\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right)},$$

$$K_2^{opt} = \left[\left(\frac{1 - 1/8C_{\widehat{M}_{y0.5}}^{2}}{1 + {}_{Cy}^{2}\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right)}\right)\left(\frac{C_{y\widehat{M}_{y0.5}}}{C_{\widehat{M}_{y0.5}}^{\ \ 2}} - 1\right) + 1/2\right]R_2, \tag{35}$$

$$MSE_{\min}(T_{27}) \cong \frac{\overline{Y}^2 C_y^2\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right)}{1 + C_y^2\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right)} - \overline{Y}^2 C_{\widehat{M}_{y0.5}}^{2}\left(4C_y^2\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right) + \frac{C_{\widehat{M}_{y0.5}}^{2}}{4}\right)16\left[1 + C_y^2\left(1 - \rho_{y\widehat{M}_{y0.5}}^{2}\right)\right].$$

3. Proposed Class of Exponential-type Difference Estimator Based on Median

The proposed class of exponential-type difference estimators for the estimation of population mean using known median of the study variable is defined as follows

$$T_{p(j)} = \left[\lambda_1\overline{y} + \lambda_2\left(M_{y_{0.5}} - \widehat{M}_{y_{0.5}}\right)\right] \cdot \exp\left[\frac{a\left(M_{y_{0.5}} - \widehat{M}_{y_{0.5}}\right)}{a\left(M_{y_{0.5}} - \widehat{M}_{y_{0.5}}\right) + 2b}\right] \text{for } j = 1, 2, \ldots, 6. \tag{36}$$

where λ_1 and λ_2 are the constants at which mean squared error is minimized and a, b be the known characteristics of the parameters of the population. The mean squared error of proposed class $T_{p(j)}$ can be obtained as follows:

To study the properties of the suggested estimators based on known median of study variable under simple random sampling scheme the error terms are defined as follows:

Let us define $e_o = \overline{y} - \overline{Y}/\overline{Y}$ and $e_1 = \widehat{M}_{Y_{0.5} - M_{Y_{0.5}}}/M_{Y_{0.5}}$ so that $\overline{y} = \overline{Y}(1 + e_0)$ and $\widehat{M}_{y_{0.5}} = M_{Y_{0.5}}(1 + e_1)$. From these definitions of e_0 and e_1, We get $E(e_0) = E(e_1) = 0$, while $E(e_0^2) = (1 - f/n)C_y^2$, $E(e_1^2) = (1 - f/n)C_{M_{Yo.5}}^2$, and $E(e_o e_1) = (1 - f/n)\rho_{yM_{Yo.5}} C_y C_{M_{Y0.5}}$ with. $C_{yM_{Y0.5}} = \mathrm{Cov}(\overline{y}, \widehat{M}_{Y_{0.5}})/\overline{Y}M_{y_{0.5}}$

The proposed class of estimators $T_{p(j)}$ can be written in the terms of $e_0$ande_1 as

$$\begin{aligned} T_{p(j)} &= \left[\lambda_1\overline{Y}(1 + e_0) + \lambda_2\left(M_{y_{0.5}} - M_{y_{0.5}}(1 + e_1)\right)\right]\exp\left[\frac{a\left(M_{y_{0.5}} - M_{y_{0.5}}(1 + e_1)\right)}{a\left(M_{y_{0.5}} - M_{y_{0.5}}(1 + e_1)\right) + 2b}\right], \\ T_{p(j)} &= \left[\lambda_1\overline{Y}(1 + e_0) - \lambda_2 M_{y_{0.5}} e_1\right]\exp\left[\frac{-aM_{y_{0.5}} e_1}{2\left(aM_{y_{0.5}} + b\right)\left(1 + aM_{y_{0.5}} e_1/2\left(aM_{y_{0.5}} + b\right)\right)}\right]. \end{aligned} \tag{37}$$

Now,

$$\left(T_{p(j)} - \overline{Y}\right) = \overline{Y}(\lambda_1 - 1) + \left(\overline{Y}e_0 - \frac{\overline{Y}\theta e_0}{2} + \frac{3}{8}Y\theta^2 e_1^2 - \frac{\overline{Y}\theta e_0 e_1}{2} - \lambda_2 M_{y_{0.5}} e_1 + \lambda_2\frac{M_{y_{0.5}}\theta e_1^2}{2}\right), \tag{38}$$

where $\theta = aM_{y_{0.5}}/aM_{y_{0.5}} + b$.

By squaring both sides and ignoring the e_i terms which are greater than two, the mean squared error is given as

$$\begin{aligned} MSE\left(T_{p(j)}\right) \cong{}& \overline{Y}^2 + \overline{Y}^2\lambda_1^2\Delta_A + \lambda_2^2\Delta_B + \lambda_1\overline{Y}^2\Delta_C \\ & - \lambda_2\overline{Y}\Delta_D + \lambda_1\lambda_2\overline{Y}\Delta_E. \end{aligned} \tag{39}$$

where

$$\Delta_A = \left[1 + g\left(C_y^2 + \frac{1}{4}\theta^2 C_{M_{y_{0.5}}}{}^2 + \frac{3}{8}\theta^2 C_{M_{y_{0.5}}}{}^2 - 2\theta C_{yM_{y_{0.5}}}\right)\right],$$

$$\Delta_B = \left(g M_{y_{0.5}}^2 C_{M_{y_{0.5}}}{}^2\right) \Delta_C = \left[g\left(\theta C_{yM_{y_{0.5}}} - \frac{3}{8}\theta^2 C_{M_{y_{0.5}}}{}^2 - 2\right)\right],$$

$$\Delta_D = \left(g\theta M_{y_{0.5}} C_{M_{y_{0.5}}}{}^2\right) \Delta_E$$

$$= \left[g\left(2\theta M_{y_{0.5}} C_{M_{y_{0.5}}}{}^2 - 2M_{y_{0.5}} C_{yM_{y_{0.5}}}\right)\right]. \quad (40)$$

The $MSE(T_{p(j)})$ is minimized for the values

$$\lambda_1^* = \left(\frac{\Delta_D \Delta_E + 2\Delta_B \Delta_C}{\Delta_E^2 - 4\Delta_A \Delta_B}\right),$$

$$\lambda_2^* = -\overline{Y}\left(\frac{\Delta_C \Delta_E + 2\Delta_A \Delta_D}{\Delta_E^2 - 4\Delta_A \Delta_B}\right). \quad (41)$$

Hence the minimum MSE of class $T_{p(j)}$ is

$$MSE_{\min}\left(T_{p(j)}\right) \cong \frac{\overline{Y}^2 \left[1 + \left(\Delta_A \Delta_D^2 + \Delta_B \Delta_C^2 + \Delta_C \Delta_D \Delta_E\right)\right]}{\left(\Delta_E^2 - 4\Delta_A \Delta_B\right)}. \quad (42)$$

4. Simulation Study

A simulation study is also conducted to evaluate the performance of existing and suggested estimators. For this purpose, the statistical programming R language is utilized. The following procedure is adopted to compute MSE of proposed estimators and their competitors.

(i) Generate a random sample of size n = 3 and 5 from bivariate normal distribution.

(ii) Compute the MSE of existing and suggested estimators by using generated random sample with the help of the formulas defined in Section 2 and Section 3.

(iii) Repeat step (i) and (ii) 30000 times to obtain MSEs

(iv) Average these MSEs to obtain the values of the MSE of existing and proposed estimators.

The results of the simulation study in the terms of MSEs are Table 1presented in Table 2. The key findings from these results are summarized as:

(i) The proposed estimators are more efficient as compared to other traditional mean and other existing estimators based on median.

(ii) As value of n increases, the value of MSE decreases, and vice versa.

(iii) Among all the proposed estimators, the estimators $\mathbf{T}_{\mathbf{P}(21)}$ have smaller MSE for each choice of n.

Table 1: Proposed class of median based estimators.

Estimators	a	b
$\mathbf{T}_{\mathbf{P}(1)}$	$M_{X_{0.5}}$	$\overline{X}$
$\mathbf{T}_{\mathbf{P}(2)}$	$\overline{X}$	$M_{X_{0.5}}$
$\mathbf{T}_{\mathbf{P}(3)}$	1	1
$\mathbf{T}_{\mathbf{P}(4)}$	$\overline{Y}R_m$	$M_{X_{0.5}}$
$\mathbf{T}_{\mathbf{P}(5)}$	$M_{X_{0.5}}$	$\overline{Y}R_m$
$\mathbf{T}_{\mathbf{P}(6)}$	1	R_m
$\mathbf{T}_{\mathbf{P}(7)}$	1	TM
$\mathbf{T}_{\mathbf{P}(8)}$	1	HL
$\mathbf{T}_{\mathbf{P}(9)}$	1	DM
$\mathbf{T}_{\mathbf{P}(10)}$	R_m	TM
$\mathbf{T}_{\mathbf{P}(11)}$	R_m	HL
$\mathbf{T}_{\mathbf{P}(12)}$	R_m	DM
$\mathbf{T}_{\mathbf{P}(13)}$	ρ_{yx}	TM
$\mathbf{T}_{\mathbf{P}(14)}$	ρ_{yx}	HL
$\mathbf{T}_{\mathbf{P}(15)}$	ρ_{yx}	DM
$\mathbf{T}_{\mathbf{P}(16)}$	C_x	TM
$\mathbf{T}_{\mathbf{P}(17)}$	C_x	HL
$\mathbf{T}_{\mathbf{P}(18)}$	C_x	DM
$\mathbf{T}_{\mathbf{P}(19)}$	$C_{\widehat{M}_{Y_{0.5}}}$	TM
$\mathbf{T}_{\mathbf{P}(20)}$	$C_{\widehat{M}_{Y_{0.5}}}$	HL
$\mathbf{T}_{\mathbf{P}(21)}$	$C_{\widehat{M}_{Y_{0.5}}}$	DM

Table 2: MSEs of existing and proposed estimators with simulated data.

Existing Estimators	$n = 3$	$n = 5$	Proposed Estimators	$n = 3$	$n = 5$
$\mathbf{T}_1$	87.5013	51.1371	$\mathbf{T}_{\mathbf{P}(1)}$	69.9346	41.3588
$\mathbf{T}_2$	81.0106	47.3438	$\mathbf{T}_{\mathbf{P}(2)}$	69.9357	41.3592
$\mathbf{T}_3$	73.1874	42.7718	$\mathbf{T}_{\mathbf{P}(3)}$	69.9352	41.3590
$\mathbf{T}_4$	73.2586	42.8135	$\mathbf{T}_{\mathbf{P}(4)}$	69.9357	41.3592
$\mathbf{T}_5$	227.8969	227.8969	$\mathbf{T}_{\mathbf{P}(5)}$	69.9346	41.3588
$\mathbf{T}_6$	226.0171	226.0171	$\mathbf{T}_{\mathbf{P}(6)}$	69.9352	41.3590
$\mathbf{T}_7$	226.6818	226.6818	$\mathbf{T}_{\mathbf{P}(7)}$	69.8531	41.3295
$\mathbf{T}_8$	227.6978	227.6978	$\mathbf{T}_{\mathbf{P}(8)}$	69.8522	41.3292
$\mathbf{T}_9$	226.8375	226.8375	$\mathbf{T}_{\mathbf{P}(9)}$	69.8519	41.3291
$\mathbf{T}_{10}$	205.2060	205.206	$\mathbf{T}_{\mathbf{P}(10)}$	69.8531	41.3295
$\mathbf{T}_{11}$	205.2845	205.2845	$\mathbf{T}_{\mathbf{P}(11)}$	69.8523	41.3292
$\mathbf{T}_{12}$	228.9906	228.9906	$\mathbf{T}_{\mathbf{P}(12)}$	69.8519	41.3291
$\mathbf{T}_{13}$	71.9745	42.0629	$\mathbf{T}_{\mathbf{P}(13)}$	69.8527	41.3294
$\mathbf{T}_{14}$	70.4531	41.7661	$\mathbf{T}_{\mathbf{P}(14)}$	69.8519	41.3291
$\mathbf{T}_{15}$	70.5534	41.8309	$\mathbf{T}_{\mathbf{P}(15)}$	69.8516	41.3290
$\mathbf{T}_{16}$	70.5060	41.8004	$\mathbf{T}_{\mathbf{P}(16)}$	69.8538	41.3298
$\mathbf{T}_{17}$	70.5522	41.8302	$\mathbf{T}_{\mathbf{P}(17)}$	69.8528	41.3294
$\mathbf{T}_{18}$	70.4546	41.7671	$\mathbf{T}_{\mathbf{P}(18)}$	69.8524	41.3292
$\mathbf{T}_{19}$	70.5074	41.8012	$\mathbf{T}_{\mathbf{P}(19)}$	69.8501	41.3284
$\mathbf{T}_{20}$	71.9744	42.0630	$\mathbf{T}_{\mathbf{P}(20)}$	69.8500	41.3284
$\mathbf{T}_{21}$	71.9744	42.0630	$\mathbf{T}_{\mathbf{P}(21)}$	69.8499	41.3283
$\mathbf{T}_{22}$	71.9744	42.0630			
$\mathbf{T}_{23}$	71.9744	42.0630			
$\mathbf{T}_{24}$	230.0941	230.0941			
$\mathbf{T}_{25}$	224.3360	224.3360			
$\mathbf{T}_{26}$	204.9097	204.9097			
$\mathbf{T}_{27}$	203.0698	203.0698			

4.1. Numerical Study. For assessing the performance of the suggested class of estimators over existing estimators, we have performed a numerical study utilizing the datasets from the previous studies of the Subramani and Prabavathy [4]

Table 3: Datasets for numerical study.

Characteristics	Pop-1	Pop-2	Pop-3	Pop-4	Pop-5	Pop-6
N	34	34	34	34	80	80
n	3	3	5	5	3	3
$\bar{\mathbf{Y}}$	856.4118	856.4118	856.4118	856.4118	5182.637	5182.637
$\bar{\mathbf{X}}$	208.8824	199.4412	208.8824	199.4412	285.125	1126.463
$\mathbf{M}_{\mathbf{Y}_{0.5}}$	767.5	767.5	767.5	767.5	5105	5105
$\mathbf{M}_{\mathbf{X}_{0.5}}$	150	142.5	150	142.5	148	757.5
$\mathbf{Q}_1$	94.25	99.25	94.25	99.25	86.50	517.50
$\mathbf{Q}_3$	254.75	278.00	254.75	278.00	445.25	1693.75
$\mathbf{Q}_\mathbf{r}$	160.50	178.75	160.50	178.75	358.75	1176.25
$\mathbf{Q}_\mathbf{d}$	80.25	89.375	80.25	89.375	179.375	588.125
$\mathbf{Q}_\mathbf{a}$	174.50	188.625	174.50	188.625	265.875	1105.625
R	4.0999	4.2941	4.0999	4.2941	18.1767	4.6008
$\mathbf{R}_\mathbf{m}$	1.1158	1.1158	1.1158	1.1158	1.0152	1.0152
$\mathbf{S}_\mathbf{y}^2$	163356.41	163356.41	91690.371	91690.371	3365953.46	3365953.46
$\mathbf{S}_\mathbf{x}^2$	6884.4455	6857.8555	3864.1726	3849.2480	73061.2791	715671.523
$\mathbf{S}_{\mathbf{M}_{\mathbf{Y}_{0.5}}}^2$	101518.77	101518.77	59396.2836	59396.2836	1730215	1730215
$\mathbf{Cov}(\bar{\mathbf{y}},\widehat{\mathbf{M}}_{\mathbf{Y}_{0.5}})$	90236.294	90236.294	48074.9542	48074.9542	1198375	1198375
$\rho_{\mathbf{yx}}$	0.4491	0.4453	0.4491	0.4453	0.915	0.941
$\mathbf{Cov}(\bar{\mathbf{y}},\bar{\mathbf{x}})$	15061.401	14905.049	8453.8187	8366.0597	453752.241	1460496.58
$\mathbf{C}_\mathbf{y}^2$	0.222726	0.222726	0.125014	0.125014	0.125316	0.125316
$\mathbf{C}_\mathbf{x}^2$	0.157785	0.172408	0.08856	0.096771	0.898704	0.564001
$\mathbf{C}_{\mathbf{M}_{\mathbf{Y}_{0.5}}}^2$	0.172341	0.172341	0.172341	0.100833	0.066391	0.066391
$\mathbf{C}_{\mathbf{y}\mathbf{M}_{\mathbf{Y}_{0.5}}}$	0.137284	0.137284	0.137284	0.07314	0.045295	0.045295
$\mathbf{C}_{\mathbf{yx}}$	0.084194	0.08726	0.047257	0.048981	0.3071	0.25017
TM	162.25	89.375	162.25	89.375	206.937	931.562
HL	190.00	320.00	190.00	320.00	249.000	1040.500
DM	234.82	206.944	234.82	206.944	276.189	1150.700

and Subramani [8]. The descriptive statistics of these datasets are given in Table 3.

For comparison of the proposed and existing estimators, the PREs of the suggested estimators $T_{p(j)}$ with respect to the existing estimators T_i is calculated as

$$PRE\Big((T_i), T_{p(j)}\Big) = \frac{MSE(T_i)}{MSET_{p(j)}} * 100. \quad (43)$$

The MSEs and PREs for suggested and existing estimators are given in Table 4–6. The key findings from the results are summarized as:

(1) The performance of existing estimator T_{18} and T_{13} is comparatively better with small MSE and higher PRE among other competitor estimators of the study (cf. Table 4).

(2) The suggested family of estimators is more efficient as compared to traditional and existing estimators of the mean. The proposed estimator $\mathrm{T}_{P(18)}$ is most efficient (cf. Table 5).

(3) The suggested estimators are superior in performance with large value of PRE than the existing estimators of the study. The smallest gained value of PRE of a proposed estimator is also higher than largest achieved value of the existing estimator. The proposed class of estimators got highest PRE values of 202.80 and 175.73 percent against usual estimator of population mean for the sample size $n = 3$ and 5, respectively (cf. Table 6).

(4) The MSE tends to decrease as sample size n increases, and vice versa (cf. Table 5).

(5) The PRE of the suggested class of estimators in comparison to the existing estimator with least MSE among all existing estimators increases as n increases (cf. Table 6).

The graphical comparison of suggested estimators with the usual and other competing estimators is also presented in this study by using datasets 1 and 3 with sample size 3 and 5. Figures 1 and 2, shows that the proposed estimators have smaller MSEs as compared to traditional mean and other existing estimators considered in this study, which indicate the dominancy of proposed estimators over its competitors.

4.2. Robustness Study of the Proposed Estimators. The non-conventional measures utilized in the study such as tri-mean, decile mean, and Hodges-Lehmann are robust in the existence of outliers. Therefore, these non-conventional measures attain precise results than other conventional measures in the presence of extreme values in the data. In the present section, we assess the performance of our suggested estimator in case of outliers. For this purpose, we utilized both population 5 and 6 which have some extreme values. In Figures 3 and 4, the scatter diagrams are clearly indicating the existence of outliers in the data. We obtained MSEs of existing and suggested estimator with both dataset, which are given in Table 7 to compare the performance. It is

Table 4: Mean square error of the existing estimators.

Estimators	$n = 3$		$n = 5$	
	Population.1	Population.2	Population.3	Population.4
$\mathbf{T}_1$	49647.54	49647.54	15641.30	15641.30
$\mathbf{T}_2$	39715.69	40030.52	12512.30	12611.49
$\mathbf{T}_3$	26832.12	26831.98	9942.496	9942.433
$\mathbf{T}_4$	26845.73	26845.80	9948.538	9948.570
$\mathbf{T}_5$	84266.71	84147.89	54798.76	54675.13
$\mathbf{T}_6$	83413.70	83642.20	52826.66	52784.47
$\mathbf{T}_7$	83266.01	83175.32	53543.50	53322.41
$\mathbf{T}_8$	84643.75	84390.43	55174.30	54924.52
$\mathbf{T}_9$	83190.44	83153.46	53369.81	53221.37
$\mathbf{T}_{10}$	73465.06	73465.02	50891.44	50891.29
$\mathbf{T}_{11}$	73468.74	73468.76	50906.42	50906.50
$\mathbf{T}_{12}$	88379.07	88379.07	58356.92	58356.92
$\mathbf{T}_{13}$	25270.68	25270.68	9003.447	9003.447
$\mathbf{T}_{14}$	25270.84	25270.64	9692.592	9692.479
$\mathbf{T}_{15}$	25290.04	25290.15	9703.386	9703.444
$\mathbf{T}_{16}$	25281.99	25281.99	9698.863	9698.863
$\mathbf{T}_{17}$	25292.18	25292.27	9704.585	9704.638
$\mathbf{T}_{18}$	25266.28	25266.06	9690.024	9689.899
$\mathbf{T}_{19}$	25278.69	25278.69	9697.008	9697.008
$\mathbf{T}_{20}$	25270.68	25270.68	9003.447	9003.447
$\mathbf{T}_{21}$	25270.68	25270.68	9003.447	9003.447
$\mathbf{T}_{22}$	25270.68	25270.68	9003.447	9003.447
$\mathbf{T}_{23}$	25270.68	25270.68	9003.447	9003.447
$\mathbf{T}_{24}$	94267.17	94267.17	56534.89	56534.89
$\mathbf{T}_{25}$	83148.69	83148.69	52778.83	52778.83
$\mathbf{T}_{26}$	74682.14	74682.14	49235.79	49235.79
$\mathbf{T}_{27}$	71158.71	71158.71	47885.95	47885.95

Table 5: Mean squared error of the proposed estimators.

Estimators	$n = 3$		$n = 5$	
	Population.1	Population.2	Population.3	Population.4
$\mathbf{T}_{\mathbf{P}(1)}$	24567.500	24567.500	8911.906	8911.906
$\mathbf{T}_{\mathbf{P}(2)}$	24567.730	24567.740	8911.936	8911.936
$\mathbf{T}_{\mathbf{P}(3)}$	24567.640	24567.640	8911.923	8911.923
$\mathbf{T}_{\mathbf{P}(4)}$	24567.760	24567.760	8911.939	8911.939
$\mathbf{T}_{\mathbf{P}(5)}$	24567.440	24567.440	8911.899	8911.899
$\mathbf{T}_{\mathbf{P}(6)}$	24567.600	24567.600	8911.918	8911.918
$\mathbf{T}_{\mathbf{P}(7)}$	24524.790	24524.130	8906.486	8906.401
$\mathbf{T}_{\mathbf{P}(8)}$	24519.450	24520.560	8905.806	8905.948
$\mathbf{T}_{\mathbf{P}(9)}$	24513.570	24516.400	8905.058	8905.418
$\mathbf{T}_{\mathbf{P}(10)}$	24528.270	24527.650	8906.928	8906.848
$\mathbf{T}_{\mathbf{P}(11)}$	24523.200	24524.260	8906.283	8906.418
$\mathbf{T}_{\mathbf{P}(12)}$	24517.580	24520.290	8905.568	8905.913
$\mathbf{T}_{\mathbf{P}(13)}$	24494.490	24493.300	8902.629	8902.478
$\mathbf{T}_{\mathbf{P}(14)}$	24487.960	24488.940	8901.798	8901.922
$\mathbf{T}_{\mathbf{P}(15)}$	24481.300	24484.090	8900.949	8901.305
$\mathbf{T}_{\mathbf{P}(16)}$	24489.410	24490.410	8900.479	8900.602
$\mathbf{T}_{\mathbf{P}(17)}$	24482.910	24486.050	8899.692	8900.068
$\mathbf{T}_{\mathbf{P}(18)}$	24476.370	24481.240	8898.925	8899.493
$\mathbf{T}_{\mathbf{P}(19)}$	24491.240	24490.400	8900.812	8900.708
$\mathbf{T}_{\mathbf{P}(20)}$	24484.720	24486.040	8900.011	8900.171
$\mathbf{T}_{\mathbf{P}(21)}$	24478.130	24481.230	8899.226	8899.592

TABLE 6: PRE's of existing and proposed estimators with respect to usual estimator of mean.

Existing Estimators	Pop-1 $n=3$	Pop-3 $n=5$	Proposed Estimators	Pop-1 $n=3$	Pop-3 $n=5$
$\mathbf{T}_1$	100.00	100.00	$\mathbf{T}_{\mathbf{P}(1)}$	202.08	175.51
$\mathbf{T}_2$	125.01	125.01	$\mathbf{T}_{\mathbf{P}(2)}$	202.08	175.51
$\mathbf{T}_3$	185.03	157.32	$\mathbf{T}_{\mathbf{P}(3)}$	202.09	175.51
$\mathbf{T}_4$	184.94	157.22	$\mathbf{T}_{\mathbf{P}(4)}$	202.08	175.51
$\mathbf{T}_5$	58.92	28.54	$\mathbf{T}_{\mathbf{P}(5)}$	202.09	175.51
$\mathbf{T}_6$	59.52	29.61	$\mathbf{T}_{\mathbf{P}(6)}$	202.09	175.51
$\mathbf{T}_7$	59.63	29.21	$\mathbf{T}_{\mathbf{P}(7)}$	202.44	175.62
$\mathbf{T}_8$	58.65	28.35	$\mathbf{T}_{\mathbf{P}(8)}$	202.48	175.63
$\mathbf{T}_9$	59.68	29.31	$\mathbf{T}_{\mathbf{P}(9)}$	202.53	175.65
$\mathbf{T}_{10}$	67.58	30.73	$\mathbf{T}_{\mathbf{P}(10)}$	202.41	175.61
$\mathbf{T}_{11}$	67.58	30.73	$\mathbf{T}_{\mathbf{P}(11)}$	202.45	175.62
$\mathbf{T}_{12}$	56.18	26.80	$\mathbf{T}_{\mathbf{P}(12)}$	202.50	175.64
$\mathbf{T}_{13}$	196.46	173.73	$\mathbf{T}_{\mathbf{P}(13)}$	202.69	175.69
$\mathbf{T}_{14}$	196.46	161.37	$\mathbf{T}_{\mathbf{P}(14)}$	202.74	175.71
$\mathbf{T}_{15}$	196.31	161.19	$\mathbf{T}_{\mathbf{P}(15)}$	202.80	175.73
$\mathbf{T}_{16}$	196.37	161.26	$\mathbf{T}_{\mathbf{P}(16)}$	202.73	175.73
$\mathbf{T}_{17}$	196.30	161.17	$\mathbf{T}_{\mathbf{P}(17)}$	202.78	175.75
$\mathbf{T}_{18}$	196.50	161.41	$\mathbf{T}_{\mathbf{P}(18)}$	202.84	175.77
$\mathbf{T}_{19}$	196.40	161.30	$\mathbf{T}_{\mathbf{P}(19)}$	202.72	175.73
$\mathbf{T}_{20}$	196.46	173.72	$\mathbf{T}_{\mathbf{P}(20)}$	202.77	175.74
$\mathbf{T}_{21}$	196.46	173.72	$\mathbf{T}_{\mathbf{P}(21)}$	202.82	175.76
$\mathbf{T}_{22}$	196.46	173.72			
$\mathbf{T}_{23}$	196.46	173.72			
$\mathbf{T}_{24}$	52.67	27.67			
$\mathbf{T}_{25}$	59.71	29.64			
$\mathbf{T}_{26}$	66.48	31.76			
$\mathbf{T}_{27}$	69.77	32.66			

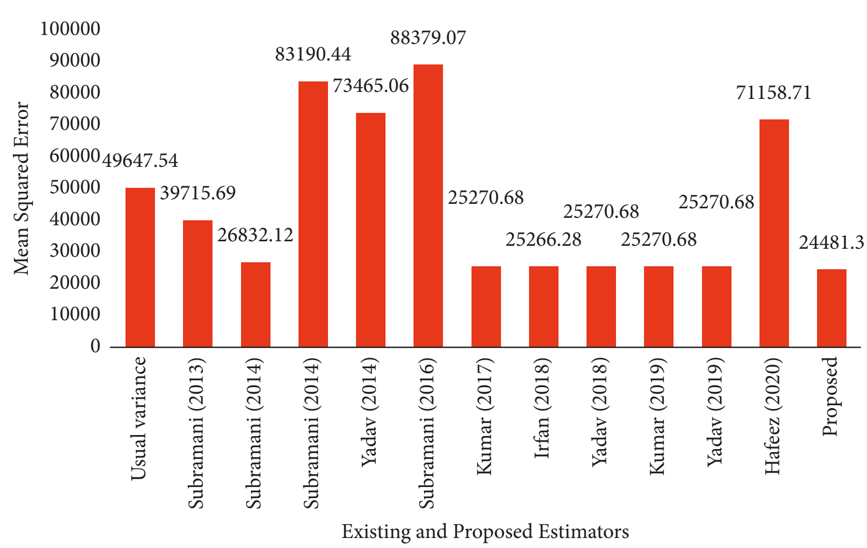

FIGURE 1: Mean squared error of the proposed and existing estimators for population-1 with $n=3$.

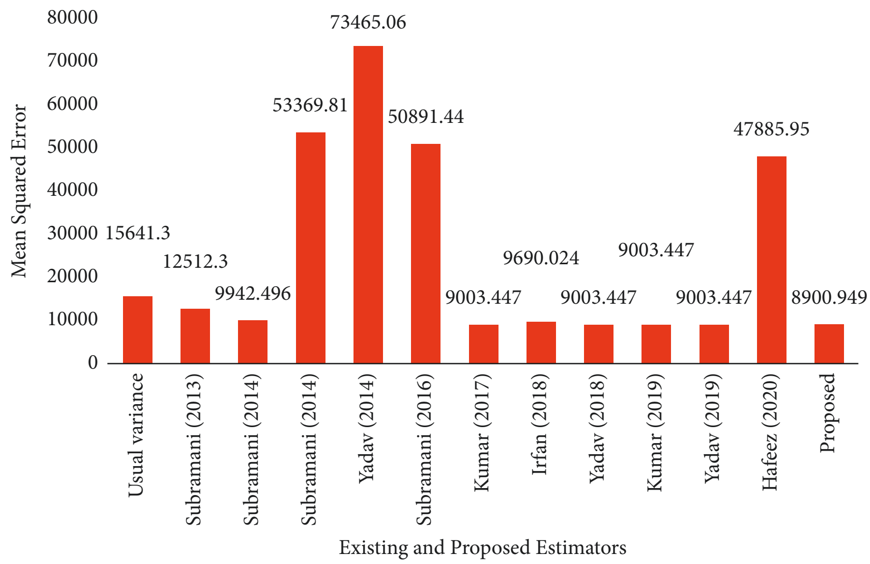

Figure 2: Mean squared error of the proposed and existing estimators of population-3 with $n = 5$.

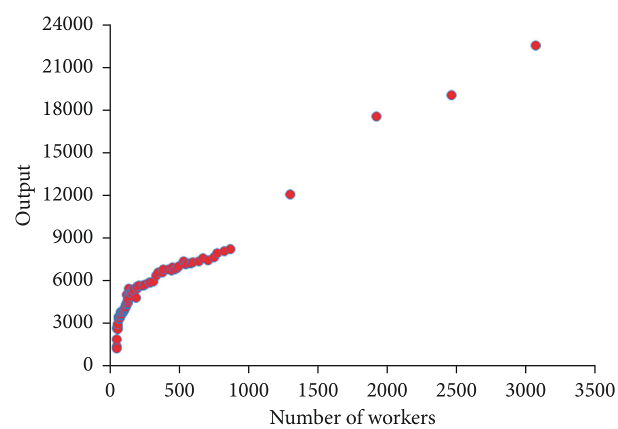

Figure 3: Scatter diagram between study and auxiliary variables of Population-5.

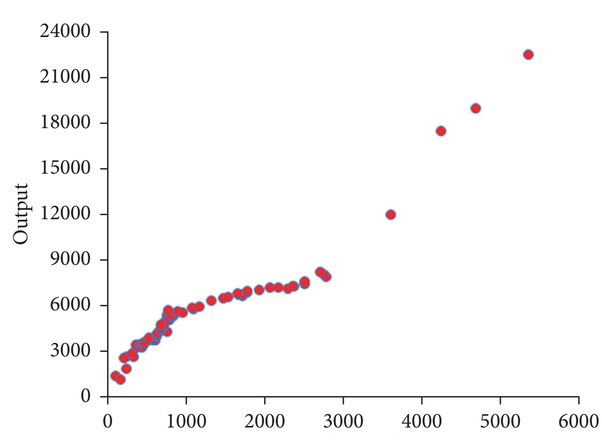

Figure 4: Scatter diagram between study and auxiliary variables of Population-6.

Table 7: MSEs of existing and proposed estimators in the presence of outliers.

Existing Estimators	n = 3 Population-5	Population-6	Proposed Estimators	n = 3 Population-5	Population-6
$\mathbf{T}_1$	1079910	1079910	$\mathbf{T}_{\mathbf{P}(1)}$	791577.3	791577.6
$\mathbf{T}_2$	952162.7	239478.2	$\mathbf{T}_{\mathbf{P}(2)}$	791578.3	791578.2
$\mathbf{T}_3$	871244.7	871275.9	$\mathbf{T}_{\mathbf{P}(3)}$	791578.0	791578.0
$\mathbf{T}_4$	871344.8	871333.9	$\mathbf{T}_{\mathbf{P}(4)}$	791578.3	791578.2
$\mathbf{T}_5$	2697608	2626793	$\mathbf{T}_{\mathbf{P}(5)}$	791577.3	791577.6
$\mathbf{T}_6$	2636557	2544339	$\mathbf{T}_{\mathbf{P}(6)}$	791578.0	791578.0
$\mathbf{T}_7$	2649272	2566306	$\mathbf{T}_{\mathbf{P}(7)}$	791436.9	791048.1
$\mathbf{T}_8$	2679581	2617952	$\mathbf{T}_{\mathbf{P}(8)}$	791410.0	791001.1
$\mathbf{T}_9$	2664264	2570760	$\mathbf{T}_{\mathbf{P}(9)}$	791393.0	790955.9
$\mathbf{T}_{10}$	2364926	2364965	$\mathbf{T}_{\mathbf{P}(10)}$	791438.9	791054.3
$\mathbf{T}_{11}$	2365052	2365038	$\mathbf{T}_{\mathbf{P}(11)}$	791412.4	791007.6
$\mathbf{T}_{12}$	2715995	2715995	$\mathbf{T}_{\mathbf{P}(12)}$	791395.6	790962.8
$\mathbf{T}_{13}$	813613.9	813613.9	$\mathbf{T}_{\mathbf{P}(13)}$	791424.5	791022.6
$\mathbf{T}_{14}$	831680.9	831731.2	$\mathbf{T}_{\mathbf{P}(14)}$	791395.5	790974.0
$\mathbf{T}_{15}$	831842.0	831824.4	$\mathbf{T}_{\mathbf{P}(15)}$	791377.1	790927.6
$\mathbf{T}_{16}$	831786.9	831786.9	$\mathbf{T}_{\mathbf{P}(16)}$	791429.6	790920.8
$\mathbf{T}_{17}$	831842.9	831825.6	$\mathbf{T}_{\mathbf{P}(17)}$	791401.4	790867.1
$\mathbf{T}_{18}$	831677.6	831728.6	$\mathbf{T}_{\mathbf{P}(18)}$	791383.6	790816.3
$\mathbf{T}_{19}$	831785.2	831785.2	$\mathbf{T}_{\mathbf{P}(19)}$	791106.9	790346.5
$\mathbf{T}_{20}$	813613.9	813613.9	$\mathbf{T}_{\mathbf{P}(20)}$	791032.8	790287.7
$\mathbf{T}_{21}$	813613.9	813613.9	$\mathbf{T}_{\mathbf{P}(21)}$	790987.9	790236.0
$\mathbf{T}_{22}$	813613.9	813613.9			
$\mathbf{T}_{23}$	813613.9	813613.9			
$\mathbf{T}_{24}$	2595164	2595164			
$\mathbf{T}_{25}$	2535939	2535939			
$\mathbf{T}_{26}$	2317166	2317166			
$\mathbf{T}_{27}$	2277016	2277016			

observed that proposed estimator have smaller values of MSE as compared to the MSE of the usual mean estimator and other competing estimators considered in this study. This comparison shows that the suggested estimators are more precise in the presence of extreme observations in the data. The estimator $\mathbf{T}_{\mathbf{P}(21)}$ is most efficient among all existing and suggested estimators with least MSE values with both datasets.

5. Summary and Conclusion

A class of median based exponential-type difference estimators under SRS scheme have been proposed utilizing auxiliary information on non-conventional measures. After empirical and graphical study, it is found that suggested class of estimators are more efficient in term of MSE as compared to usual sample mean and other existing estimators considered in this study. The PREs also reveal the dominancy of suggested estimators over traditional and competing estimators. A simulation study based on generated simple random sample with samples of size $n = 3$, and 5 also showed that the proposed median based estimator of the study variable are more efficient with smaller MSE than usual mean and other existing estimators. The present study can also be extended to the other sampling schemes such as stratified random sampling, systematic sampling, and ranked set sampling to improve efficiency of the mean estimators.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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