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Research Article

Cubic Intuitionistic Fuzzy Topology with Application to Uncertain Supply Chain Management

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The concept of the cubic intuitionistic fuzzy set is an effective hybrid model for modeling uncertainties with an intuitionistic fuzzy set and an interval-valued intuitionistic fuzzy set, simultaneously. The primary objective of this study is to develop a topological structure on cubic intuitionistic fuzzy sets with P-order and R-order as well as to define some fundamental characteristics and significant results with illustrations. Taking advantage of topological data analysis with cubic intuitionistic information, novel multicriteria group decision-making methods are developed for an uncertain supply chain management. Algorithms 1 and 2 are proposed for extensions of the weighted product model and the choice value method towards a cubic intuitionistic fuzzy environment, respectively. A comparative analysis is also given to discuss the validity and advantages of the proposed techniques.

1. Introduction

Topological data analysis (TDA) methods are rapidly growing approaches to inferring persistent key features from possibly complex data [1]. We deal with complex issues in our daily lives due to vague and uncertain information, and if we do not use the proper modeling techniques for them, we eventually wind up with vague and unclear reasoning. For this reason, making rational and logical conclusions in the face of such imprecise and inexplicit facts is a difficult task for decision-makers. As a result, dealing with vagueness and uncertainty is a necessary part of dealing with such challenges and difficulties. Zadeh [2] initiated the notion of fuzzy set (FS) theory, which is an instantaneous extension of a crisp set. Various sets of theories and models have been developed by researchers to manage the complexity of daily life problems that include vague and uncertain information. Atanassov [3] presented the idea of an intuitionistic fuzzy set (IFS), and Atanassov [4] further initiated the notion of circular intuitionistic fuzzy sets. Yager [5, 6] introduced the concept of a Pythagorean fuzzy set (PFS), and further Yager [7] developed the notion of a q-rung orthopair fuzzy set (q-ROFS). Molodtsov [8] was the first who proposed the idea of a soft set (SS), and Zhang [9, 10] originally introduced the notion of a bipolar fuzzy set (BFS) to address bipolarity and bipolar information. Smarandache [11, 12] initiated the concept of a neutrosophic set (NS). Cuong [13] introduced the idea of a picture fuzzy set (PiFS). Gundogdu and Kahraman [14], Mahmood et al. [15], and Ashraf et al. [16] independently introduced the notion of a spherical fuzzy set (SFS). These models have a strong foothold when it comes to modeling uncertainty in a real-life complex challenges. Atanassov and Gargov [17] introduced interval-valued intuitionistic fuzzy sets. Cagman and Enginoglu [18] proposed decision-making applications based on soft-set theory. Karaaslan and Cagman [19] introduce the parameter trees based on soft set theory and their similarity measures. Chen [20] proposed m-polar fuzzy sets (mPFS) with m membership values to address the multipolarity of objects.

Jun et al. [21] developed the cubic set (CS) and its internal and external environment. But CS has some limitations, as it does not convert membership degree grades into nonmembership grades. Riaz and Hashmi [22] proposed cubic m-polar fuzzy sets and cubic m-polar fuzzy averaging aggregation operators for MAGDM. So, for this, Kaur and Garg [23, 24] presented the concept of a cubic intuitionistic fuzzy set by combining the concepts of IFSs, CFSs, and IVIFSs. So, CIFS, rather than IFSs or IVIFSs, is a handy technique to address information more precisely throughout the DMP. Young et al. [25] proposed cubic interval-valued intuitionistic fuzzy sets. Senapati et al. [26] introduced a cubic intuitionistic WASPAS technique. Garg and Kaur [27] suggested cubic intuitionistic fuzzy Bonferroni mean operators. Garg and Kaur [28] proposed cubic intuitionistic fuzzy TOPSIS for nonlinear programming.

Classical topology derives its inspiration from classical analysis and has a wide range of scientific applications. In 1968, Chang [29] proposed the concept of fuzzy topology. Coker [30] pioneered intuitionistic fuzzy topology. Olgun [31] expanded on this concept by introducing Pythagorean fuzzy topology. Topological structures on fuzzy soft sets [32] and cubic m-polar fuzzy sets [33] have robust applications in decision-making. Xu and Yager [34] and Xu [35] originated the notion of an intuitionistic fuzzy number (IFN) and their aggregation operators. Zhang and Xu [36] developed an extension of TOPSIS for Pythagorean fuzzy numbers (PyFNs). They also suggested a domestic airline MCDM application to examine the service quality of airlines. Feng et al. [37] proposed the MADM application by using a new score function for ranking alternatives with generalized orthopair fuzzy membership grades. Akram [38] initiated the concept of BFS graphs, and Akram et al. [39] suggested a hybrid decision-making framework by using aggregation operators under a complex spherical fuzzy prioritization approach. Alghamdi et al. [40] proposed some MCDM methods for bipolar fuzzy environments. Liu and Wang [41] proposed some basic operational laws of q-ROFNs and q-ROF aggregation operators. Ye [42] proposed MADM with new similarity measures based on the generalized distance of neutrosophic Z-number sets. Senapati and Yager [43] proposed WPM for Fermatean fuzzy numbers. Kahraman and Alkan [44] developed the TOPSIS method for circular intuitionistic fuzzy sets. Sinha and Sarmah [45] developed supply chain coordination using fuzzy set theory. Alshurideh et al. [46] proposed supply chain management with fuzzy-assisted human resource management.

Seikh et al. [47, 48] proposed the solution of matrix games with rough interval pay-offs and a defuzzification approach of type-2 fuzzy variables to solving matrix games. They developed applications of matrix games to the telecom market share problem and the plastic ban problem. Ruidas et al. [49] developed an EPQ model with stock and selling price-dependent demand and a variable production rate in an interval environment. Ruidas et al. [50] suggested an interval environment with price revision using a single-period production inventory model. Ruidas et al. [51] introduced a production-repairing inventory model considering demand and the proportion of defective items as rough intervals. Seikh and Mandal [52] proposed q-rung orthopair fuzzy Frank aggregation operators and their application in MADM with unknown attribute weights. Seikh and Mandal [53] introduced the MADM method based on $3, 4$-quasirung fuzzy sets. Riaz and Farid [54] proposed the picture fuzzy aggregation approach and application to third-party logistic provider selection. Ashraf et al. [55] introduced the Maclaurin symmetric mean operator with an interval-valued picture fuzzy model. Baig et al. [56] developed new methods for enhancing resilience in developing countries for oil supply chains. Chattopadhyay et al. [57] proposed the idea of the development of a rough-MABAC-DoE-based metamodel for iron and steel supplier selection. Karamasa et al. [58] studied weighting the factors affecting logistics outsourcing. Bairagi [59] developed a novel MCDM model for warehouse location selection in supply chain management. Recently, some applications of fuzzy modeling have been developed, such as uncertain supply chains [60], medical tourism supply chains [61], sustainable plastic recycling processes [62], and pattern recognition [63].

Multicriteria group decision-making (MCGDM) is a branch of operation research in which the alternatives are evaluated by the group of decision-makers (DMs) under multiple criteria to find a ranking of alternatives and an optimal decision. It is an important aspect of MCGDM to evaluate alternatives based on their characteristics. It is extremely difficult for an individual to choose an option in a variety of situations due to inconsistencies in the data caused by human errors or a lack of knowledge. Dealing with vagueness and uncertainties in MCGDM problems is very crucial to dealing with daily life problems. For this purpose, a variety of strategies have been utilized to evaluate the stability of human decision-making by weighing a set of options against a set of criteria. The weighted product model and choice value method are well-known methods and are often utilized to rank the alternatives according to certain criteria.

The main objectives of this research work are given as follows:

(1) To develop a topological structure on cubic intuitionistic fuzzy sets (CIFSs) with P-order (P-CIFT) as well as R-order (R-CIFT) and to validate some significant results and fundamental characteristics with examples. The concept of the CIFS is a strong hybrid model for modeling uncertainties with an IFS and an interval-valued IFS, simultaneously.

(2) To examine various properties of the cubic intuitionistic fuzzy topology (CIFT) under P-order (R-order), such as open sets of CIFT, closed sets of CIFT, interior in CIFT, closure in CIFT, subspace of CIFT, exterior in CIFT, a frontier in CIFT, and a basis of CIFT.

(3) Taking advantage of topological data analysis with cubic intuitionistic fuzzy information, we proposed two multicriteria group decision-making (MCGDM) methods.

(4) To develop Algorithm 1 for a weighted product model (WPM) and Algorithm 2 for a choice value method (CVM). An application of the proposed techniques is also designed for the uncertain supply chain management.

(5) ranking of feasible alternatives is computed, and a comparative analysis of proposed methods with existing methods is also given to discuss the validity and advantage of the proposed techniques.

The remaining sections of this paper are organized as follows. In Section 2, we reviewed some fundamental concepts such as IFS, IVIFS, cubic sets, CIFS, operations on CIFSs, and some essential results on CIFSs. The idea of cubic intuitionistic fuzzy set topology with P-order is introduced in Section 3. We also investigated some key results on CIFSs in p-order. In Section 4, we discuss the major results of cubic intuitionistic fuzzy set topology with R-order. In Section 5, we discuss a useful application that employs the weighted product model and choice value method. The conclusion of the paper is given in Section 6.

2. Preliminaries

In this section, we study some basic concepts of IFSs, IVIFSs, CSs, and CIFSs. We also review some fundamental properties of CIFSs that are necessary to understand the topological structures of CIFSs.

Definition 1 (see [3]). An intuitionistic fuzzy set (IFS) in a set $\Bbbk$ is described as

$$\mathbb{I} = \{(\ell, \zeta(\ell), \eta(\ell)) \colon 0 \leq \zeta(\ell) + \eta(\ell) \leq 1, \ell \in \Bbbk\}, \quad (1)$$

where, $\zeta\colon \Bbbk \longrightarrow [0,1]$ represents the membership function, and the nonmembership function is denoted by $\eta\colon \Bbbk \longrightarrow [0,1]$.

Definition 2 (see [34, 35]). Let $\mathbb{I}_1 = (\zeta_1, \eta_1)$ and $\mathbb{I}_2 = (\zeta_2, \eta_2)$ be two IFNs. Then, we have the following operations on IFNs.

(i) $\mathbb{I}_1 \subseteq \mathbb{I}_2$ if $\zeta_1 \leq \zeta_2$ and $\eta_1 \geq \eta_2$ for all $\ell \in \Bbbk$
(ii) $\mathbb{I}_1 = \mathbb{I}_2$ if $\mathbb{I}_1 \subseteq \mathbb{I}_2$ and $\mathbb{I}_2 \subseteq \mathbb{I}_1$
(iii) $\mathbb{I}_1^c = \{(\ell, \eta_1(\ell), \zeta_1(\ell)); \ell \in \Bbbk\}$
(iv) $\mathbb{I}_1 \bigcup \mathbb{I}_2 = \{(\ell, \vee\{\zeta_1, \zeta_2\}, \wedge\{\eta_1, \eta_2\}) \colon \ell \in \Bbbk\}$
(v) $\mathbb{I}_1 \cap \mathbb{I}_2 = \{(\ell, \wedge\{\zeta_1, \zeta_2\}, \vee\{\eta_1, \eta_2\}) \colon \ell \in \Bbbk\}$

In reality, it is difficult to determine the exact membership and nonmembership degrees of an element in a set. In this situation, a range of values may be a better measurement to accommodate the uncertainty. For this, Atanassov and Gargov [17] introduce the idea of an interval-valued intuitionistic fuzzy set (IVIFS).

Definition 3 (see [17]). Let $\Bbbk$ be a nonempty universal set. An interval-valued intuitionistic fuzzy set (IVIFS) on $\Bbbk$ is defined as

$$\mathbb{I} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right]\right); \ell \in \Bbbk\right\}, \quad (2)$$

where, $[\zeta^L(\ell), \zeta^U(\ell)]$ and $[\eta^L(\ell), \eta^U(\ell)]$ are the closed subintervals of $[0,1]$ for every $\ell \in \Bbbk$. For simplicity, the pair $\mathbb{I} = ([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ is called interval-valued intuitionistic fuzzy number (IVIFN).

By fusing the concept of IFS and IVIFS, Jun et al. [21] defined the cubic intuitionistic set as follows:

Definition 4 (see [21]). A cubic set $\mathbb{C}$ on a universal set $\Bbbk$ is expressed as

$$\mathbb{C} = \{\ell, C(\ell), \sigma(\ell) \colon \ell \in \Bbbk\}, \quad (3)$$

in which $C(\ell)$ is interval-valued fuzzy set and $\sigma(\ell)$ is fuzzy set on $\Bbbk$. For use of ease, this pair is referred as $\mathbb{C} = \langle C, \sigma\rangle$

Definition 5 (see [21]). For any cubic fuzzy sets $\mathbb{C}_i = \langle C_i, \sigma_i\rangle$, $i \in \Lambda$, we have

(i) P-union $\cup_P \mathbb{C}_i = \langle \vee_{i\in\Lambda} C_i, \vee_{i\in\Lambda}\sigma_i\rangle$
(ii) P-intersection $\underset{P}{\cap} \mathbb{C}_i = < \underset{i\in\Lambda}{\wedge} C_i, \underset{i\in\Lambda}{\wedge} \sigma_i >$
(iii) R-union $\cup_R \mathbb{C}_i = \langle \vee_{i\in\Lambda} C_i, \vee_{i\in\Lambda}\sigma_i\rangle$
(iv) R-intersection $\cup_P \mathbb{C}_i = \langle \vee_{i\in\Lambda} C_i, \vee_{i\in\Lambda}\sigma_i\rangle$

Definition 6 (see [23, 24]). Let $\Bbbk$ be a universal set of discourse. A cubic intuitionistic fuzzy set (CIFS) on universal set $\Bbbk$ is described as

$$\mathbb{C}_{\mathbb{I}} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right], (\zeta, \eta)\right); \ell \in \Bbbk\right\}, \quad (4)$$

in which $([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ is an IVIFS and (ζ, η) is an IFS in $\Bbbk$. For ease of use, we denote these pairs as $\mathbb{C}_{\mathbb{I}} = (C, \sigma)$, where $C = ([\zeta^L, \zeta^U], [\eta^L, \eta^U])$ and $\sigma = (\zeta, \eta)$ is known as cubic intuitionistic fuzzy number (CIFN) with the condition that $[\zeta^L, \zeta^U], [\eta^L, \eta^U] \subseteq [0,1]$, $\zeta, \eta \in [0,1]$ and $\zeta + \eta \leq 1$.

That is why the CIFS has the advantage of being capable to contain a lot more data to represent both the IVIFN and the IFN at the same time.

2.1. Operations on CIFSs

Now we review some fundamental operations of CIFSs, which have been explored in [23, 24].

Definition 7. The complement of the CIFS $\mathbb{C}_{\mathbb{I}} = (C, \sigma)$ is defined as $\mathbb{C}_{\mathbb{I}}^c = (C^c, \sigma^c)$ where $C^c = ([\eta^L(\ell), \eta^U(\ell)], [\zeta^L(\ell), \zeta^U(\ell)])$ be the complement of the IVIFS, $C = ([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ and $\sigma^c = (\eta, \zeta)$ be the complement of the IFS, $\sigma = (\zeta, \eta)$. Thus, the complement of CIFS is expressed as

$$\mathbb{C}_{\mathbb{I}}^c = \left\{\left(\ell, \left[\eta^L(\ell), \eta^U(\ell)\right], \left[\zeta^L(\ell), \zeta^U(\ell)\right], (\eta, \zeta)\right); \ell \in \Bbbk\right\}. \quad (5)$$

Definition 8. Consider two CIFSs on a universal set $\Bbbk$ is given as follow:

$$\mathbb{C}_{\mathbb{I}}^1 = \left\{\left(\ell, \left[\zeta_1^L, \zeta_1^U\right], \left[\eta_1^L, \eta_1^U\right], (\zeta_1, \eta_1)\right); \ell \in \Bbbk\right\}, \quad (6)$$

and

$$\mathbb{C}_{\mathbb{I}}^2 = \left\{\left(\ell, \left[\zeta_2^L, \zeta_2^U\right], \left[\eta_2^L, \eta_2^U\right], (\zeta_2, \eta_2)\right); \ell \in \Bbbk\right\}, \quad (7)$$

we define

Step 1. Obtain the decision matrix from the decision-makers, which indicates the alternative's $\mathbb{X}_j$, $(j = 1 \dots m)$ evaluation values on the basis of criterion $\mathbb{C}_i$, $(i = 1, \dots n)$ by $\mathbb{T}_{ji} = (C_{ji}, \sigma_{ji})$, where $C_{ji} = ([\zeta_{ji}^L, \zeta_{ji}^U], [\eta_{ji}^L, \eta_{ji}^U])$ an IVIFN and $\sigma_{ji} = (\zeta_{ji}, \eta_{ji})$ is known as a cubic intuitionistic fuzzy number. The decision-makers provide the decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ of the form.

$$\begin{array}{c} \\ \mathbb{X}_1 \\ \mathbb{X}_2 \\ \mathbb{X}_m \\ \\ \end{array} \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 & \cdots\cdots & \mathbb{C}_n \\ (C_{11}, \sigma_{11}) & (C_{12}, \sigma_{12}) & \cdots\cdots & (C_{1n}, \sigma_{1n}) \\ (C_{21}, \sigma_{21}) & (C_{22}, \sigma_{22}) & \cdots\cdots & (C_{2n}, \sigma_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (C_{m1}, \sigma_{m1}) & (C_{m2}, \sigma_{m2}) & \cdots\cdots & (C_{mn}, \sigma_{mn}) \end{bmatrix}.$$

Step 2. Then, the decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ is made normalized by a linear approach. Assume the criteria are categorized into benefit criteria $\mathbb{B}$ and cost criteria $\mathbb{K}$. The normalization of every $i \in \mathbb{B}$ is defined as
$\overline{\mathbb{T}}_{ji} = \mathbb{T}_{ji}/\max_j \mathbb{T}_{ji}$,
where $\max_j \mathbb{T}_{ji}$ is defined as
$\max_j \mathbb{T}_{ji} = ([(\max \zeta_{ji}^L, \max \zeta_{ji}^U)], ([\min \eta_{ji}^L, \min \eta_{ji}^U]), (\min \zeta_{ji}, \max \eta_{ji}))$.
Similarly, the normalization of every $i \in \mathbb{K}$ is defined as
$\overline{\mathbb{T}}_{ji} = \min_j \mathbb{T}_{ji}/\mathbb{T}_{ji}$,
where $\min_j \mathbb{T}_{ji}$ is defined as
$\min_j \mathbb{T}_{ji} = ([(\min \zeta_{ji}^L, \min \zeta_{ji}^U)], ([\max \eta_{ji}^L, \max \eta_{ji}^U]), (\max \zeta_{ji}, \min \eta_{ji}))$.
The decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ is then transformed into normalized decision matrix $\overline{M} = (\overline{\mathbb{T}}_{ji})_{m\times n}$ and is given as

$$\begin{array}{c} \\ \mathbb{X}_1 \\ \mathbb{X}_2 \\ \mathbb{X}_m \\ \\ \end{array} \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 & \cdots\cdots & \mathbb{C}_n \\ (\overline{C}_{11}, \overline{\sigma}_{11}) & (\overline{C}_{12}, \overline{\sigma}_{12}) & \cdots\cdots & (\overline{C}_{1n}, \overline{\sigma}_{1n}) \\ (\overline{C}_{21}, \overline{\sigma}_{21}) & (C_{22}, \sigma_{22}) & \cdots\cdots & (\overline{C}_{2n}, \overline{\sigma}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\overline{C}_{m1}, \overline{\sigma}_{m1}) & (\overline{C}_{m2}, \overline{\sigma}_{m2}) & \cdots\cdots & (\overline{C}_{mn}, \overline{\sigma}_{mn}) \end{bmatrix}.$$

Step 3. According to CIFS-WPM, the relative importance of j alternatives is denoted as $\mathbb{Y}_j$ and is defined as
$\mathbb{Y}_j = \prod i = 1n(\overline{\mathbb{T}}_{ji})^{wi}$,
Here, we use the operation of power rule of CIFNs and also the product operation of CIFNs.
Step 4. Find the score function of all vales $\mathbb{Y}_j$.
Step 5. Ranking of alternatives according to the score functions of $\mathbb{Y}_j$.

Algorithm 1: Weighted product model (WPM).

Step 1. Obtain the decision matrix from the decision-makers, with alternative's $\mathbb{X}_j$ evaluate on the basis of criterion $\mathbb{C}_i$ by $\mathbb{T}_{ji} = (C_{ji}, \sigma_{ji})$, where $C_{ji} = ([\zeta_{ji}^L, \zeta_{ji}^U], [\eta_{ji}^L, \eta_{ji}^U])$ an IVIFN and $\sigma_{ji} = (\zeta_{ji}, \eta_{ji})$ is known as cubic intuitionistic fuzzy number. The decision-makers provide the decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ of the form.

$$\begin{array}{c} \\ \mathbb{X}_1 \\ \mathbb{X}_2 \\ \mathbb{X}_m \\ \\ \end{array} \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 & \cdots\cdots & \mathbb{C}_n \\ (C_{11}, \sigma_{11}) & (C_{12}, \sigma_{12}) & \cdots\cdots & (C_{1n}, \sigma_{1n}) \\ (C_{21}, \sigma_{21}) & (C_{22}, \sigma_{22}) & \cdots\cdots & (C_{2n}, \sigma_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (C_{m1}, \sigma_{m1}) & (C_{m2}, \sigma_{m2}) & \cdots\cdots & (C_{mn}, \sigma_{mn}) \end{bmatrix}.$$

Step 2. The decision-makers also give weightage to the criteria, with the condition that the sum of the weights must be equal to unity. We compute the multiplication of the decision matrix with criteria weights.

$$\begin{bmatrix} (C_{11}, \sigma_{11}) & (C_{12}, \sigma_{12}) & \cdots\cdots & (C_{1n}, \sigma_{1n}) \\ (C_{21}, \sigma_{21}) & (C_{22}, \sigma_{22}) & \cdots\cdots & (C_{2n}, \sigma_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (C_{m1}, \sigma_{m1}) & (C_{m2}, \sigma_{m2}) & \cdots\cdots & (C_{mn}, \sigma_{mn}) \end{bmatrix} \begin{bmatrix} W1 \\ W2 \\ Wn \end{bmatrix}.$$

Step 3. We find the score function of each value.
Step 4. Compute the ranking of the alternatives according to their score function values.

Algorithm 2: Choice value method (CVM).

(i) (P-order) $\mathbb{C}_{\mathbb{I}}^1 \subseteq_P \mathbb{C}_{\mathbb{I}}^2$ if $[\zeta_1^L, \zeta_1^U] \subseteq [\zeta_2^L, \zeta_2^U]$, $[\eta_1^L, \eta_1^U] \supseteq [\eta_2^L, \eta_2^U]$, $\zeta_1 \leq \zeta_2$ and $\eta_1 \geq \eta_2$

(ii) (R-order) $\mathbb{C}_{\mathbb{I}}^1 \subseteq_R \mathbb{C}_{\mathbb{I}}^2$ if $[\zeta_1^L, \zeta_1^U] \subseteq [\zeta_2^L, \zeta_2^U]$, $[\eta_1^L, \eta_1^U] \supseteq [\eta_2^L, \eta_2^U]$, $\zeta_1 \geq \zeta_2$ and $\eta_1 \leq \eta_2$

(iii) (Equality) $\mathbb{C}_{\mathbb{I}}^1 = \mathbb{C}_{\mathbb{I}}^2$ if $[\zeta_1^L, \zeta_1^U] = [\zeta_2^L, \zeta_2^U]$, $[\eta_1^L, \eta_1^U] = [\eta_2^L, \eta_2^U]$, $\zeta_1 = \zeta_2$ and $\eta_1 = \eta_2$

Definition 9. For any CIFSs

$$\mathbb{C}_{\mathbb{I}}^i = \left(\left[\left[\zeta_i^L, \zeta_i^U\right], \left[\eta_i^L, \eta_i^U\right], (\zeta_i, \eta_i)\right); \ell \in \mathbb{k}\right\} i \in \Lambda, \tag{8}$$

the operations listed have been defined as follows:

(i) (P-union) $\cup_P \mathbb{C}_{\mathbb{I}}^i = \left\{([\vee_{i\in\Lambda} \zeta_i^L, \vee_{i\in\Lambda} \zeta_i^U], [\wedge_{i\in\Lambda} \eta_i^L, \wedge_{i\in\Lambda} \eta_i^U]), (\vee_{i\in\Lambda} \zeta_i, \wedge_{i\in\Lambda} \eta_i)\right\}$

(ii) (P-intersection) $\cap_P \mathbb{C}_{\mathbb{I}}^i = \left\{([\wedge_{i\in\Lambda} \zeta_i^L, \wedge_{i\in\Lambda} \zeta_i^U], [\vee_{i\in\Lambda} \eta_i^L, \vee_{i\in\Lambda} \eta_i^U]), (\wedge_{i\in\Lambda} \zeta_i, \vee_{i\in\Lambda} \eta_i)\right\}$

(iii) (R-union) $\cup_R \mathbb{C}_{\mathbb{I}}^i = \left\{([\vee_{i\in\Lambda} \zeta_i^L, \vee_{i\in\Lambda} \zeta_i^U], [\wedge_{i\in\Lambda} \eta_i^L, \wedge_{i\in\Lambda} \eta_i^U]), (\wedge_{i\in\Lambda} \zeta_i, \vee_{i\in\Lambda} \eta_i)\right\}$

(iv) (R-intersection) $\cap_R \mathbb{C}_{\mathbb{I}}^i = \left\{([\wedge_{i\in\Lambda} \zeta_i^L, \wedge_{i\in\Lambda} \zeta_i^U], [\vee_{i\in\Lambda} \eta_i^L, \vee_{i\in\Lambda} \eta_i^U]), (\vee_{i\in\Lambda} \zeta_i, \wedge_{i\in\Lambda} \eta_i)\right\}$

2.2. Some Results on CIFSs. Now we review some essential properties and results of CIFSs, which have been explored in [23, 24].

Definition 10. A CIFS

$$\mathbb{C}_{\mathbb{I}} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right], (\zeta, \eta)\right); \ell \in \mathbb{k}\right\}, \tag{9}$$

for which $([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)]) = ([0,0], [1,1])$ and $(\zeta, \eta) = (1,0)$ for all $\ell \in \mathbb{k}$ is denoted by ${}^0\mathbb{C}_{\mathbb{I}}$

Definition 11. A CIFS

$$\mathbb{C}_{\mathbb{I}} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right], (\zeta, \eta)\right); \ell \in \mathbb{k}\right\}, \tag{10}$$

for which $([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)]) = ([1,1], [0,0])$ and $(\zeta, \eta) = (0,1)$ for all $\ell \in \mathbb{k}$ is denoted by ${}^1\mathbb{C}_{\mathbb{I}}$

Definition 12. A CIFS

$$\mathbb{C}_{\mathbb{I}} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right], (\zeta, \eta)\right); \ell \in \mathbb{k}\right\}, \tag{11}$$

for which $([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)]) = ([1,1], [0,0])$ and $(\zeta, \eta) = (0,1)$ for all $\ell \in \mathbb{k}$ is denoted by ${}^0\mathbb{C}_{\mathbb{I}}$

Definition 13. A CIFS

$$\mathbb{C}_{\mathbb{I}} = \left\{\left(\ell, \left[\zeta^L(\ell), \zeta^U(\ell)\right], \left[\eta^L(\ell), \eta^U(\ell)\right], (\zeta, \eta)\right); \ell \in \mathbb{k}\right\}, \tag{12}$$

for which $([\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)]) = ([1,1], [0,0])$ and $(\zeta, \eta) = (1,0)$ for all $\ell \in \mathbb{k}$ is denoted by ${}^1\mathbb{C}_{\mathbb{I}}$

Definition 14. Let $\mathbb{C}_{\mathbb{I}} = \left\{(\ell, [\zeta^L(\ell), \zeta^U(\ell)], [\eta^L(\ell), \eta^U(\ell)], (\zeta, \eta))\right\}$ be a CIFN. The score function $\mathbb{S}(\mathbb{C}_{\mathbb{I}})$ and the accuracy function $\mathbb{A}(\mathbb{C}_{\mathbb{I}})$ on for CIFNs are defined as

For P-order

$$\mathbb{S}(\mathbb{C}_{\mathbb{I}}) = \frac{\zeta^L + \zeta^U - \eta^L - \eta^U}{2} + \zeta - \eta. \tag{13}$$

For R-order

$$\begin{aligned} \mathbb{S}(\mathbb{C}_{\mathbb{I}}) &= \frac{\zeta^L + \zeta^U - \eta^L - \eta^U}{2} + \eta - \zeta, \\ \mathbb{A}(\mathbb{C}_{\mathbb{I}}) &= \frac{\zeta^L + \zeta^U + \eta^L + \eta^U}{2} + \zeta + \eta. \end{aligned} \tag{14}$$

The ranking of CIFNs in relation to the proposed scoring function and accuracy function is determined as.

(i) $\mathbb{C}_{\mathbb{I}} < \mathbb{C}_{\mathbb{I}}^1$ if $\mathbb{S}(\mathbb{C}^{\mathbb{I}}) < \mathbb{S}(\mathbb{C}_{\mathbb{I}}^1)$,

(ii) If $\mathbb{S}(\mathbb{C}_{\mathbb{I}}) = \mathbb{S}(\mathbb{C}_{\mathbb{I}}^1)$, then $\mathbb{C}_{\mathbb{I}} < \mathbb{C}_{\mathbb{I}}^1$ if $\mathbb{A}(\mathbb{C}_{\mathbb{I}}) < \mathbb{A}(\mathbb{C}_{\mathbb{I}}^1)$

(iii) If $\mathbb{S}(\mathbb{C}_{\mathbb{I}}) = \mathbb{S}(\mathbb{C}_{\mathbb{I}}^1)$ and $\mathbb{A}(\mathbb{C}_{\mathbb{I}}) = \mathbb{A}(\mathbb{C}_{\mathbb{I}}^1)$, then $\mathbb{C}_{\mathbb{I}} = \mathbb{C}_{\mathbb{I}}^1$

Definition 15. Let $\mathbb{C}_{\mathbb{I}} = \left\{(\ell, [\zeta^L, \zeta^U], [\eta^L, \eta^U], (\zeta, \eta)); \ell \in \mathbb{k}\right\}$ and

$$\mathbb{C}_{\mathbb{I}}^i = \left\{\left(\ell, \left[\zeta_i^L, \zeta_i^U\right], \left[\eta_i^L, \eta_i^U\right], (\zeta_i, \eta_i)\right); \ell \in \mathbb{k}\right\}, (i = 1, 2), \tag{15}$$

be the CIFNs and let $p > 0$ be any real number. The basic operations on CIFs are given as

(i) $\mathbb{C}_{\mathbb{I}}^1 + \mathbb{C}_{\mathbb{I}}^2 = (([1 - \prod_{i=1}^2 (1 - \zeta_i^L), 1 - \prod_{i=1}^2 (1 - \zeta_i^U)], [\prod_{i=1}^2 \eta_i^L, \prod_{i=1}^2 \eta_i^U]), (\prod_{i=1}^2 \zeta_i, 1 - \prod_{i=1}^2 \eta_i))$

(ii) $\mathbb{C}_{\mathbb{I}}^1 \times \mathbb{C}_{\mathbb{I}}^2 = ([\prod i = 12\zeta_i^L, \prod i = 12\zeta_i^U], [1 - \prod i = 12(1 - \eta_i^L), 1 - \prod i = 12(1 - 1 - \eta_i^U)])t, n(1 - \prod i = 12\zeta_i, \prod i = 12\eta_i))$

(iii) $p\mathbb{C}_{\mathbb{I}} = (([1 - (1 - \zeta^L)^p, 1 - (1 - \zeta^U)^p], [(\eta^L)^p, (\eta^U)^p]), ((\zeta)^p, 1 - (1 - (\eta)^p))$

(iv) $\mathbb{C}_{\mathbb{I}}^p = (([(\zeta^L)^p, (\zeta^U)^p], [1 - (1 - \eta^L)^p, 1 - (1 - \eta^U)^p]), (1 - \{1 - (\zeta)^p, (\eta)^p))$

Definition 16. Let

$$\mathbb{C}_{\mathbb{I}}^i = \left\{\left(\ell, \left[\zeta_i^L, \zeta_i^U\right], \left[\eta_i^L, \eta_i^U\right], (\zeta_i, \eta_i)\right); \ell \in \mathbb{k}\right\}, (i = 1, 2), \tag{16}$$

be the CIFNs. Then, the division operator on CIFN is given as

$$\begin{aligned} \frac{\mathbb{C}_{\mathbb{I}}^1}{\mathbb{C}_{\mathbb{I}}^2} = &\left(\left(\left[\min \zeta_1^L, \zeta_2^L, \min \zeta_1^U, \zeta_2^U\right], \left[\max \eta_1^L, \eta_2^L, \max \eta_1^U, \eta_2^U\right]\right),\right. \\ &\left.\left(\max \zeta_1, \zeta_2, \min \eta_1^U, \eta_2^U\right)\right). \end{aligned} \tag{17}$$

3. Cubic Intuitionistic Topology under P-Order

In this section, we introduce the concept of a P-cubic intuitionistic fuzzy topology (P-CIFT) or a cubic intuitionistic fuzzy topology with P-order.

Definition 17. Consider $\mathbb{k}$ to be a nonempty universal set, and let $ci(\mathbb{k})$ to be the accumulation of all CIFSs in $\mathbb{k}$. If the collection $\mathbb{T}_{\mathbb{C}_{\mathbb{I}\mathbb{P}}}$ containing CIFSs satisfies the following conditions, it is termed as a cubic intuitionistic fuzzy topology with a P-order (P-CIFT).

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}$, ${}^{1}\mathbb{C}_{\mathbb{I}}$, $\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{0}\mathbb{C}_{\mathbb{I}}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$

(2) If $(\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}} \forall i \in \Lambda$ then $\cup_p (\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$

(3) If $\mathbb{C}_{\mathbb{IP}}^1, \mathbb{C}_{\mathbb{IP}}^2 \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ then $\mathbb{C}_{\mathbb{IP}}^1 \cap_p \mathbb{C}_{\mathbb{IP}}^2 \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$

Then, the pair $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ is called cubic intuitionistic fuzzy topological space with a P-order (P-CIFT).

Example 1. Let $\mathbb{k}$ be a universal set. Then, $ci(\mathbb{k})$ be the assemblage of all P-cubic intuitionistic fuzzy sets PCIFSs in $\mathbb{k}$. Consider P-order fuzzy subsets of $ci(\mathbb{k})$ given as

$$\begin{aligned}
\mathbb{C}_{\mathbb{IP}}^1 &= \{[0.20, 0.31], [0.41, 0.52], (0.32, 0.44)\},\\
\mathbb{C}_{\mathbb{IP}}^2 &= \{[0.20, 0.31], [0.41, 0.52], (1, 0)\},\\
\mathbb{C}_{\mathbb{IP}}^3 &= \{[0.20, 0.31], [0.41, 0.52], (0, 1)\},\\
\mathbb{C}_{\mathbb{IP}}^4 &= \{[1, 1], [0, 0], (0.32, 0.44)\},\\
\mathbb{C}_{\mathbb{IP}}^5 &= \{[0, 0], [1, 1], (0.32, 0.44)\}.
\end{aligned} \tag{18}$$

The union and intersection with a P-order for the above CIFSs are given in Tables 1 and 2,, respectively.

Clearly,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}^1} = \left\{{}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}\right\}, \tag{19}$$

and

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}^2 = \left\{{}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}_{\mathbb{IP}}^1, \mathbb{C}_{\mathbb{IP}}^2, \mathbb{C}_{\mathbb{IP}}^3, \mathbb{C}_{\mathbb{IP}}^4, \mathbb{C}_{\mathbb{IP}}^5\right\}, \tag{20}$$

are cubic intuitionistic topology with a P-order.

Definition 18. Let $\mathbb{k}$ be a nonempty set and $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}} = \left\{\mathbb{C}_{\mathbb{IP}}^{\mathbb{k}}\right\}$ where $\mathbb{C}_{\mathbb{IP}}^{\mathbb{k}}$ represent the cubic intuitionistic fuzzy subsets of universal set $\mathbb{k}$. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is termed as a P-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and it is the largest P-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and is entitled as P-discrete cubic intuitionistic fuzzy topology.

Definition 19. Let $\mathbb{k}$ be a universal set and $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}} = \left\{{}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}\right\}$ be the assemblage of cubic intuitionistic fuzzy sets. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is termed as a P-cubic intuitionistic fuzzy topology on universal set $\mathbb{k}$ and is the smallest P-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and is entitled as P-indiscrete cubic intuitionistic fuzzy topology.

Definition 20. The elements of a P-cubic intuitionistic fuzzy topology $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is termed as P-cubic intuitionistic fuzzy open sets PCIFOS in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$.

Theorem 1. *If $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ is any P-cubic intuitionistic fuzzy topological space. Then,*

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are PCIFOSs

(2) The P-union of any number of PCIFOSs is PCIFOS

(3) The P-intersection of finite PCIFOSs is PCIFOS

Proof

(1) By the Definition 4.2 of a P-cubic intuitionistic fuzzy topology (P-CIFT), ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$. Hence, ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are PCIFOSs.

(2) Let $\{(\mathbb{C}_{\mathbb{IP}})_i | i \in \Lambda\}$ be PCIFOSs. Then, $(\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$. From the definition of P-CIFT

$$\underset{p}{\cup} (\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{21}$$

Hence, $\cup_p (\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is PCIFOSs.

(3) Let $\mathbb{C}_{\mathbb{IP}}^1, \mathbb{C}_{\mathbb{IP}}^2, \ldots, \mathbb{C}_{\mathbb{IP}}^n$ be PCIOSs. Then, from definition of P-CIFT

$$\underset{p}{\cap} (\mathbb{C}_{\mathbb{IP}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{22}$$

Hence, $\cap_p (\mathbb{C}_{\mathbb{IP}})_i$ is PCIFOSs. □

Definition 21. The complement of elements of P-cubic intuitionistic fuzzy open sets is termed as P-cubic intuitionistic fuzzy closed sets PCIFCSs in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$.

Theorem 2. *If $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ is any P-cubic intuitionistic fuzzy topological space. Then,*

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are PCIFCSs

(2) The P-intersection of any number of PCIFCSs is PCIFCS

(3) The P-union of finite PCIFCSs is PCIFCS

Proof

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are PCIFOSs. From the definition of P-CIFT

$${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{23}$$

Since the complement of ${}^{0}\mathbb{C}_{\mathbb{I}} = {}^{1}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}} = {}^{0}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}} = \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}} = \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$. So, ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are PCIFCSs.

(2) Let $\{(\mathbb{C}_{\mathbb{IP}})_i | i \in \Lambda\}$ be PCIFCSs. Then,

$$\left((\mathbb{C}_{\mathbb{IP}})_i\right)^c \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{24}$$

From the definition of P-CIFT,

$$\underset{p}{\cup} \left((\mathbb{C}_{\mathbb{IP}})_i\right)^c \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{25}$$

Hence, $\cup_p ((\mathbb{C}_{\mathbb{IP}})_i)^c$ is PCIFOSs, but

$$\left(\underset{p}{\cup} \left((\mathbb{C}_{\mathbb{IP}})_i\right)^c\right) = \left(\underset{p}{\cap} \left((\mathbb{C}_{\mathbb{IP}})_i\right)\right)^c. \tag{26}$$

So, $\cap_p (\mathbb{C}_{\mathbb{IP}})_i$ is PCIFCSs.

(3) Let $\mathbb{C}_{\mathbb{IP}}^1, \mathbb{C}_{\mathbb{IP}}^2, \ldots, \mathbb{C}_{\mathbb{IP}}^n$ be PCmPCSs. Then, $(\mathbb{C}_{\mathbb{IP}}^1)^c, (\mathbb{C}_{\mathbb{IP}}^2)^c, \ldots, (\mathbb{C}_{\mathbb{IP}}^n)^c$ are PCIFOSs. So,

Table 1: Union under P-order.

$\cup_p$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$
${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$
$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$
$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$
$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^2_{\mathbb{IP}}$
$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$
$\mathbb{C}^4_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$
$\mathbb{C}^5_{\mathbb{IP}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$

Table 2: Intersection under P-order.

$\cap_p$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^5_{\mathbb{IP}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$
$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$
$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
$\mathbb{C}^2_{\mathbb{IP}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^2_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$
$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^1_{\mathbb{IP}}$	$\mathbb{C}^3_{\mathbb{IP}}$	$\mathbb{C}^4_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$
$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^5_{\mathbb{IP}}$	$\mathbb{C}^5_{\mathbb{IP}}$

$$\left(\mathbb{C}^1_{\mathbb{IP}}\right)^c, \left(\mathbb{C}^2_{\mathbb{IP}}\right)^c, \ldots, \left(\mathbb{C}^n_{\mathbb{IP}}\right)^c \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{27}$$

From the definition of P-CIFT,

$$\underset{p}{\cap}\left(\left(\mathbb{C}_{\mathbb{IP}}\right)_i\right)^c \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}. \tag{28}$$

This gives $\cap_p\left(\left(\mathbb{C}_{\mathbb{IP}}\right)_i\right)^c \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is PCIFOSs, but

$$\left(\bigcap_p\left(\left(\mathbb{C}_{\mathbb{IP}}\right)_i\right)^c\right) = \left(\bigcup_p\left(\mathbb{C}_{\mathbb{IP}}\right)_i\right)^c. \tag{29}$$

Hence, $\cup_p\left(\mathbb{C}_{\mathbb{IP}}\right)_i$ is PCIFCSs. □

Definition 22. The P-cubic intuitionistic fuzzy sets PCIFSs, which are PCIFOSs and PCIFCSs, are entitled as P-cubic intuitionistic fuzzy clopen sets in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$.

Proposition 1

(1) *For every* $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$, ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ *and* $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ *are P-cubic intuitionistic fuzzy clopen sets*

(2) *For discrete P-order cubic intuitionistic fuzzy topology, all the cubic intuitionistic subsets of* $\mathbb{k}$ *are P-cubic intuitionistic fuzzy clopen sets*

(3) *For in-discrete P-order cubic intuitionistic fuzzy topology,* ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ *and* $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ *are only P-cubic intuitionistic fuzzy clopen sets*

Definition 23. Let $(\mathbb{k}, \mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}})$ and $(\mathbb{k}, \mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}})$ be two P-CIFTs in $\mathbb{k}$. Two P-CIFTs are called comparable if

$$\mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}} \subseteq_P \mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}}, \tag{30}$$

or

$$\mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}} \subseteq_P \mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}}. \tag{31}$$

If $\mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}} \subseteq_P \mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}}$ then, $\mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}}$ is called P-cubic intuitionistic fuzzy coarser than $\mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}}$ and $\mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}}$ is called P-cubic intuitionistic fuzzy finer than. $\mathbb{T}^1_{\mathbb{C}_{\mathbb{IP}}}$

Example 2. Let $\mathbb{k}$ be a nonempty set and from Example 1

$$\mathbb{T}_{\mathbb{C}^1_{\mathbb{IP}}} = \left\{{}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}\right\}, \tag{32}$$

and

$$\mathbb{T}^2_{\mathbb{C}_{\mathbb{IP}}} = \left\{{}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^1_{\mathbb{IP}}, \mathbb{C}^2_{\mathbb{IP}}, \mathbb{C}^3_{\mathbb{IP}}, \mathbb{C}^4_{\mathbb{IP}}, \mathbb{C}^5_{\mathbb{IP}}\right\}, \tag{33}$$

are P-cubic intuitionistic fuzzy topologies on universal set. Then, $\mathbb{T}_{\mathbb{C}^1_{\mathbb{IP}}} \subseteq_P \mathbb{T}_{\mathbb{C}^2_{\mathbb{IP}}}$. Hence, $\mathbb{T}_{\mathbb{C}^1_{\mathbb{IP}}}$ is called a P-cubic intuitionistic fuzzy coarser then, $\mathbb{T}_{\mathbb{C}^2_{\mathbb{IP}}}$.

3.1. Subspace of $\mathbb{CIFT}p$

Definition 24. Let $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{k}})$ be a $\mathbb{CIFT}p$. Let $\mathbb{Y} \subseteq \mathbb{k}$ and $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}}$ is a $\mathbb{CIFT}p$ on $\mathbb{Y}$ and whose PCIFOSs are

$$\mathbb{C}_{\mathbb{IP}\mathbb{Y}} = \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{k}} \underset{p}{\cap} \mathbb{Y}, \tag{34}$$

where $\mathbb{C}_{\mathbb{IP}_{\mathbb{k}}}$ are PCIFOSs of $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{k}}$, $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}}$ are PCIFOSs of $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}}$ and $\widetilde{\mathbb{Y}}$ is any P-cubic subset of PCIFS on $\mathbb{Y}$. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}}$ is called a P-cubic intuitionistic fuzzy subspace of $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{k}}$, i.e.,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}} = \left\{ \mathbb{C}_{\mathbb{IP}_{\mathbb{Y}}} : \mathbb{C}_{\mathbb{IP}_{\mathbb{Y}}} = \mathbb{C}_{\mathbb{IP}_{\Bbbk}} \underset{p}{\cap} \mathbb{Y}, \mathbb{C}_{\mathbb{IP}_{\Bbbk}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\Bbbk} \right\}. \tag{35}$$

Example 3. Let $\Bbbk$ be a nonempty set. From Example 1,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IP}}, \mathbb{C}^{2}_{\mathbb{IP}}, \mathbb{C}^{3}_{\mathbb{IP}}, \mathbb{C}^{4}_{\mathbb{IP}}, \mathbb{C}^{5}_{\mathbb{IP}} \right\}, \tag{36}$$

is a P-cubic intuitionistic fuzzy topology on $\Bbbk$.

Now, consider any P-cubic fuzzy subset on $\Bbbk$ such that $\mathbb{Y} \subseteq \Bbbk$ is

$$\mathbb{Y} = \{[0.98, 0.23], [0.46, 0.61], (0.27, 0.49)\}. \tag{37}$$

Since,

$$\begin{aligned}
\mathbb{Y} \overset{0}{\underset{p}{\cap}} \mathbb{C}_{\mathbb{I}} &= \{[0,0], [1,1], (0.27, 0.49)\} \\
&= \overrightarrow{\mathbb{C}_{\mathbb{IP}}}, \\
\mathbb{Y} \overset{1}{\underset{p}{\cap}} \mathbb{C}_{\mathbb{I}} &= \{[0.98, 0.23], [0.46, 0.61], (0,1)\} \\
&= \widetilde{\mathbb{C}}_{\mathbb{IP}}, \\
\mathbb{Y} \cap_{p} \overline{{}^{0}\mathbb{C}_{\mathbb{I}}} &= \{[0,0], [1,1], (0,1)\} \\
&= \mathbb{C}'_{\mathbb{IP}}, \\
\mathbb{Y} \cap_{p} \overline{{}^{1}\mathbb{C}_{\mathbb{I}}} &= \{[0.98, 0.23], [0.46, 0.61], (0.27, 0.49)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{p}{\cap} \mathbb{C}^{1}_{\mathbb{IP}} &= \{[0.98, 0.23], [0.46, 0.61], (0.27, 0.49)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{p}{\cap} \mathbb{C}^{2}_{\mathbb{IP}} &= \{[0.98, 0.23], [0.46, 0.61], (0.27, 0.49)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{p}{\cap} \mathbb{C}^{3}_{\mathbb{IP}} &= \{[0.98, 0.23], [0.46, 0.61], (0,1)\} \\
&= \widetilde{\mathbb{C}}_{\mathbb{IP}}, \\
\mathbb{Y} \underset{p}{\cap} \mathbb{C}^{4}_{\mathbb{IP}} &= \{[0.98, 0.23], [0.46, 0.61], (0.27, 0.49)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{p}{\cap} \mathbb{C}^{5}_{\mathbb{IP}} &= \{[0,0], [1,1], (0.27, 0.49)\} \\
&= \overrightarrow{\mathbb{C}}_{\mathbb{IP}}.
\end{aligned} \tag{38}$$

Then,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\mathbb{Y}} = \left\{ \overrightarrow{\mathbb{C}_{\mathbb{IP}}}, \widetilde{\mathbb{C}_{\mathbb{IP}}}, {}'\mathbb{C}_{\mathbb{IP}}, \mathbb{Y} \right\}, \tag{39}$$

is a P-cubic intuitionistic fuzzy relative topology of $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}\Bbbk}$

3.2. Interior, Closure, Frontier and Exterior of PCIFSs

Definition 25. let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ be $\mathbb{CIFT}p$ and $\mathbb{C}_{\mathbb{IP}} \in ci(\Bbbk)$, the interior of $\mathbb{C}_{\mathbb{IP}}$ is expressed as $\mathbb{C}^{0}_{\mathbb{IP}}$ and is described as union of all P-cubic intuitionistic fuzzy open subsets contained in $\mathbb{C}_{\mathbb{IP}}$. It is the greatest P-cubic intuitionistic fuzzy open set contained in $\mathbb{C}_{\mathbb{IP}}$.

Example 4. Consider a P-cubic intuitionistic fuzzy topological space as constructed in Example 1. Let $\mathbb{C}^{6}_{\mathbb{IP}} \in ci(\Bbbk)$ given as

$$\mathbb{C}^{6}_{\mathbb{IP}} = \{[0.23, 0.39], [0.37, 0.48], (0.46, 0.33)\}. \tag{40}$$

Then,

$$\left(\mathbb{C}^{6}_{\mathbb{IP}}\right)^{0} = \overline{{}^{0}\mathbb{C}_{\mathbb{I}}} \underset{p}{\cup} \mathbb{C}^{1}_{\mathbb{IP}} \underset{p}{\cup} \mathbb{C}^{3}_{\mathbb{IP}} \underset{p}{\cup} \mathbb{C}^{5}_{\mathbb{IP}} = \mathbb{C}^{1}_{\mathbb{IP}}. \tag{41}$$

Theorem 3. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ *be* $\mathbb{CIFT}\ p$ *and* $\mathbb{C}_{\mathbb{IP}} \in ci(\Bbbk)$. *Then,* $\mathbb{C}_{\mathbb{IP}}$ *is open CIFS iff* $\mathbb{C}^{0}_{\mathbb{IP}} = \mathbb{C}_{\mathbb{IP}}$.

Proof. If $\mathbb{C}_{\mathbb{IP}}$ is open CIFS, then we say that the greatest open CIFS contained in $\mathbb{C}_{\mathbb{IP}}$ is $\mathbb{C}_{\mathbb{IP}}$ itself. Thus,

$$\mathbb{C}^{0}_{\mathbb{IP}} = \mathbb{C}_{\mathbb{IP}}. \tag{42}$$

Conversely, if $\mathbb{C}^{0}_{\mathbb{IP}} = \mathbb{C}_{\mathbb{IP}}$, then $\mathbb{C}^{0}_{\mathbb{IP}}$ is open CIFS. This implies $\mathbb{C}_{\mathbb{IP}}$ is open CIFS.

Theorem 4. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ *be* $\mathbb{CIFT}p$ *and* $\mathbb{C}^{1}_{\mathbb{IP}}, \mathbb{C}^{2}_{\mathbb{IP}} \in ci(\Bbbk)$. *Then,*

(i) $((\mathbb{C}_{\mathbb{IP}})^{0})^{0} = (\mathbb{C}_{\mathbb{IP}})^{0}$

(ii) $\mathbb{C}^{1}_{\mathbb{IP}} \subseteq_{p} \mathbb{C}^{2}_{\mathbb{IP}} \Rightarrow (\mathbb{C}^{1}_{\mathbb{IP}})^{0} \subseteq_{p} (\mathbb{C}^{2}_{\mathbb{IP}})^{0}$

(iii) $(\mathbb{C}^{1}_{\mathbb{IP}} \cap_{p} \mathbb{C}^{2}_{\mathbb{IP}})^{0} = (\mathbb{C}^{1}_{\mathbb{IP}})^{0} \subseteq_{p} (\mathbb{C}^{2}_{\mathbb{IP}})^{0}$

(iv) $(\mathbb{C}^{1}_{\mathbb{IP}} \cup_{p} \mathbb{C}^{2}_{\mathbb{IP}})^{0} \supseteq_{p} (\mathbb{C}^{1}_{\mathbb{IP}})^{0} \cup_{p} (\mathbb{C}^{2}_{\mathbb{IP}})^{0}$

Proof. Proof is trivial.

Definition 26. let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ be $\mathbb{CIFT}p$ and $\mathbb{C}_{\mathbb{IP}} \in ci(\Bbbk)$, the closure of $\overline{\mathbb{C}_{\mathbb{IP}}}$ is expressed as $\overline{\mathbb{C}_{\mathbb{IP}}}$ and is described as the intersection of all the P-cubic intuitionistic fuzzy closed supersets of $\mathbb{C}_{\mathbb{IP}}$. It is the smallest P-cubic intuitionistic fuzzy closed superset of $\mathbb{C}_{\mathbb{IP}}$.

Example 5. Let us consider a P-cubic intuitionistic topological space as constructed in Example 1. Then, the closed CIFSs are given as

$$\begin{aligned}
\left({}^{0}\mathbb{C}_{\mathbb{I}}\right)^{c} &= \{[1,1],[0,0],(0,1)\},\\
\left({}^{1}\mathbb{C}_{\mathbb{I}}\right)^{c} &= \{[0,0],[1,1],(1,0)\},\\
\left(\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}\right)^{c} &= \{[1,1],[0,0],(1,0)\},\\
\left(\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}\right)^{c} &= \{[0,0],[1,1],(0,1)\},\\
\left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c} &= \{[0.41,0.52],[0.20,0.31],(0.44,0.32)\},\\
\left(\mathbb{C}^{2}_{\mathbb{IP}}\right)^{c} &= \{[0.41,0.52],[0.20,0.31],(0,1)\},\\
\left(\mathbb{C}^{3}_{\mathbb{IP}}\right)^{c} &= \{[0.41,0.52],[0.20,0.31],(1,0)\},\\
\left(\mathbb{C}^{4}_{\mathbb{IP}}\right)^{c} &= \{[0,0],[1,1],(0.44,0.32)\},\\
\left(\mathbb{C}^{5}_{\mathbb{IP}}\right)^{c} &= \{[1,1],[0,0],(0.44,0.32)\}.
\end{aligned} \tag{43}$$

Let $\mathbb{C}^{7}_{\mathbb{IP}} \in ci(\Bbbk)$ given as

$$\mathbb{C}^{7}_{\mathbb{IP}} = \{[0.34,0.50],[0.27,0.38],(0.33,0.41)\}. \tag{44}$$

Then,

$$\begin{aligned}
\overline{\mathbb{C}^{7}_{\mathbb{IP}}} &= \left(\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}\right)^{c} \underset{p}{\cap} \left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c} \underset{p}{\cap} \left(\mathbb{C}^{3}_{\mathbb{IP}}\right)^{c} \underset{p}{\cap} \left(\mathbb{C}^{5}_{\mathbb{IP}}\right)^{c}\\
&= \left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c}.
\end{aligned} \tag{45}$$

Theorem 5. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ *be* $\mathbb{CIFT}\,p$ *and* $\mathbb{C}_{\mathbb{IP}} \in ci(\Bbbk)$. *Then* $\overline{\mathbb{C}_{\mathbb{IP}}}$ *is closed CIFS iff* $\overline{\mathbb{C}_{\mathbb{IP}}} = \mathbb{C}_{\mathbb{IP}}$.

Proof. If $\mathbb{C}_{\mathbb{IP}}$ is closed CIFS, then we can say that the smallest closed CIFS superset of $\mathbb{C}_{\mathbb{IP}}$ is $\mathbb{C}_{\mathbb{IP}}$ itself. Thus,

$$\overline{\mathbb{C}_{\mathbb{IP}}} = \mathbb{C}_{\mathbb{IP}}. \tag{46}$$

Conversely, if $\overline{\mathbb{C}_{\mathbb{IP}}} = \mathbb{C}_{\mathbb{IP}}$, then $\overline{\mathbb{C}_{\mathbb{IP}}}$ is closed CIFS. This implies $\mathbb{C}_{\mathbb{IP}}$ is closed CIFS. □ □

Definition 27. Let $\mathbb{C}_{\mathbb{IP}}$ be a P-cubic intuitionistic fuzzy subset of $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$, then its boundary or frontier is defined as

$$Fr\left(\mathbb{C}_{\mathbb{IP}}\right) = \overline{\mathbb{C}_{\mathbb{IP}}} \underset{p}{\cap} \overline{\left(\mathbb{C}_{\mathbb{IP}}\right)^{c}}. \tag{47}$$

Definition 28. Let $\mathbb{C}_{\mathbb{IP}}$ be a P-cubic intuitionistic fuzzy subset of $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$, then the exterior is defined as

$$\mathrm{Ext}\left(\mathbb{C}_{\mathbb{IP}}\right) = \left(\overline{\mathbb{C}_{\mathbb{IP}}}\right)^{c} = \left(\mathbb{C}^{c}_{\mathbb{IP}}\right)^{0}. \tag{48}$$

Example 6. Consider a P-cubic intuitionistic topological space as constructed in Example 1 and $\mathbb{C}^{6}_{\mathbb{IP}}$ and $\mathbb{C}^{7}_{\mathbb{IP}}$ from Examples 4 and 5. Then,

$$\begin{aligned}
\left(\mathbb{C}^{6}_{\mathbb{IP}}\right)^{0} &= \mathbb{C}^{1}_{\mathbb{IP}},\\
\overline{\mathbb{C}^{6}_{\mathbb{IP}}} &= \left(\mathbb{C}^{3}_{\mathbb{IP}}\right)^{c},\\
Fr\left(\mathbb{C}^{6}_{\mathbb{IP}}\right) &= \left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c},\\
Ext\left(\mathbb{C}^{6}_{\mathbb{IP}}\right) &= \mathbb{C}^{3}_{\mathbb{IP}},\\
\left(\mathbb{C}^{7}_{\mathbb{IP}}\right)^{0} &= \mathbb{C}^{1}_{\mathbb{IP}},\\
\overline{\mathbb{C}^{7}_{\mathbb{IP}}} &= \left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c},\\
Fr\left(\mathbb{C}^{7}_{\mathbb{IP}}\right) &= \left(\mathbb{C}^{1}_{\mathbb{IP}}\right)^{c},\\
Ext\left(\mathbb{C}^{7}_{\mathbb{IP}}\right) &= \mathbb{C}^{1}_{\mathbb{IP}}.
\end{aligned} \tag{49}$$

Theorem 6. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ *be* $\mathbb{CIFT}p$ *and* $\mathbb{C}_{\mathbb{IP}} \in ci(\Bbbk)$. *Then,*

(1) $(\mathbb{C}^{0}_{\mathbb{IP}})^{c} = \overline{(\mathbb{C}^{c}_{\mathbb{IP}})}$

(2) $(\overline{\mathbb{C}_{\mathbb{IP}}})^{c} = (\mathbb{C}^{c}_{\mathbb{IP}})^{0}$

(3) $Ext(\mathbb{C}^{c}_{\mathbb{IP}}) = \mathbb{C}^{0}_{\mathbb{IP}}$

(4) $Ext(\mathbb{C}_{\mathbb{IP}}) = (\mathbb{C}^{c}_{\mathbb{IP}})^{0}$

(5) $Ext(\mathbb{C}_{\mathbb{IP}}) \cup_{p} Fr(\mathbb{C}_{\mathbb{IP}}) \cup_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq {}^{1}\mathbb{C}_{\mathbb{IP}}$

(6) $Fr(\mathbb{C}_{\mathbb{IP}}) = Fr(\mathbb{C}^{c}_{\mathbb{IP}})$

(7) $Fr(\mathbb{C}_{\mathbb{IP}}) \cap_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq {}^{0}\mathbb{C}_{\mathbb{IP}}$

Proof

(1) The proof is obvious.

(2) The proof is obvious.

(3) $\mathrm{Ext}(\mathbb{C}^{c}_{\mathbb{IP}}) = (\overline{\mathbb{C}^{c}_{\mathbb{IP}}})^{c}$.
$\mathrm{Ext}(\mathbb{C}^{c}_{\mathbb{IP}}) = ((\mathbb{C}^{c}_{\mathbb{IP}})^{c})^{0}$.
$\mathrm{Ext}(\mathbb{C}^{c}_{\mathbb{IP}}) = \mathbb{C}^{0}_{\mathbb{IP}}$.

(4) $\mathrm{Ext}(\mathbb{C}_{\mathbb{IP}}) = (\overline{\mathbb{C}_{\mathbb{IP}}})^{c}$.
$\mathrm{Ext}(\mathbb{C}_{\mathbb{IP}}) = (\mathbb{C}^{c}_{\mathbb{IP}})^{0}$.

(5) $\mathrm{Ext}(\mathbb{C}_{\mathbb{IP}}) \cup_{p} Fr(\mathbb{C}_{\mathbb{IP}}) \cup_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq {}^{1}\mathbb{C}_{\mathbb{IP}}$. By Example 13, we can see that $\mathrm{Ext}(\mathbb{C}^{6}_{\mathbb{IP}}) \cup_{p} Fr(\mathbb{C}^{6}_{\mathbb{IP}}) \cup_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq {}^{1}\mathbb{C}_{\mathbb{I}}$.

(6) $Fr(\mathbb{C}^{c}_{\mathbb{IP}}) = \overline{(\mathbb{C}^{c}_{\mathbb{IP}})} \cap_{p} \overline{((\mathbb{C}^{c}_{\mathbb{IP}})^{c})} Fr(\mathbb{C}^{c}_{\mathbb{IP}}) = \overline{(\mathbb{C}^{c}_{\mathbb{IP}})} \cap_{p} \overline{(\mathbb{C}_{\mathbb{IP}})} = Fr(\mathbb{C}_{\mathbb{IP}})$.

(7) $Fr(\mathbb{C}_{\mathbb{IP}}) \cap_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq \mathbb{C}_{\mathbb{IP}}$. From Example 13, we can see that $Fr(\mathbb{C}^{6}_{\mathbb{IP}}) \cap_{p} \mathbb{C}^{0}_{\mathbb{IP}} \neq {}^{0}\mathbb{C}_{\mathbb{I}}$. □

3.3. P-Cubic Intuitionistic Fuzzy Basis

Definition 29. Let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ be $\mathbb{CIFT}p$. Then, $\mathbb{B} \subseteq \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ is called P-cubic intuitionistic fuzzy basis for $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$ if for every $\mathbb{C}_{\mathbb{IP}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$, $\exists \mathscr{B} \in \mathbb{B}$ such that

$$\mathbb{C}_{\mathbb{IP}} = \underset{p}{\cup} \mathscr{B}. \tag{50}$$

Example 7. From Example 1,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IP}}, \mathbb{C}^{2}_{\mathbb{IP}}, \mathbb{C}^{3}_{\mathbb{IP}}, \mathbb{C}^{4}_{\mathbb{IP}}, \mathbb{C}^{5}_{\mathbb{IP}} \right\}, \quad (51)$$

is a P-cubic intuitionistic fuzzy topology of $\mathbb{k}$. Then,

$$\mathbb{B} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IP}}, \mathbb{C}^{2}_{\mathbb{IP}}, \mathbb{C}^{3}_{\mathbb{IP}}, \mathbb{C}^{4}_{\mathbb{IP}}, \mathbb{C}^{5}_{\mathbb{IP}} \right\}, \quad (52)$$

Is a P-cubic intuitionistic fuzzy basis for $\mathbb{T}_{\mathbb{C}_{\mathbb{IP}}}$.

4. Cubic Intuitionistic Topology under R-Order

In this section, we introduce the concept of an R-cubic intuitionistic fuzzy topology (R-CIFT) or a cubic intuitionistic fuzzy topology with an R-order.

Definition 30. Consider $\mathbb{k}$ to be a nonempty universal set, and let $ci(\mathbb{k})$ to be the collection of all CIFSs in $\mathbb{k}$. If the collection $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ containing CIFSs satisfies the following conditions, it is termed a cubic intuitionistic fuzzy topology with an R-order (R-CIFT).

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}$, ${}^{1}\mathbb{C}_{\mathbb{I}}$, $\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{0}\mathbb{C}_{\mathbb{I}}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$

(2) If $(\mathbb{C}_{\mathbb{IR}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}} \forall i \in \Lambda$ then $\cup_R (\mathbb{C}_{\mathbb{IR}})_i \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$

(3) If $\mathbb{C}^{1}_{\mathbb{IR}}, \mathbb{C}^{2}_{\mathbb{IR}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ then $\mathbb{C}^{1}_{\mathbb{IR}} \cap_R \mathbb{C}^{2}_{\mathbb{IR}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$

Then, the pair $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ is termed as a cubic intuitionistic fuzzy topological space with an R-order (R-CIFT).

Example 8. Let $\mathbb{k}$ be a nonempty universal set. Then, $ci(\mathbb{k})$ be the accumulation of all R-cubic intuitionistic fuzzy sets RCIFSs in $\mathbb{k}$. Consider the R-order fuzzy subsets of $ci(\mathbb{k})$ given as

$$\begin{aligned}
\mathbb{C}^{1}_{\mathbb{IR}} &= \{[0.31, 0.42], [0.47, 0.56], (0.29, 0.39)\},\\
\mathbb{C}^{2}_{\mathbb{IR}} &= \{[0.31, 0.42], [0.47, 0.56], (0, 1)\},\\
\mathbb{C}^{3}_{\mathbb{IR}} &= \{[1, 1], [0, 0], (0.29, 0.39)\},\\
\mathbb{C}^{4}_{\mathbb{IR}} &= \{[0, 0], [1, 1], (0.29, 0.39)\},\\
\mathbb{C}^{5}_{\mathbb{IR}} &= \{[0.31, 0.42], [0.47, 0.56], (1, 0)\}.
\end{aligned} \quad (53)$$

The union and intersection with a P-order for the above CIFSs are given in Tables 3 and 4,, respectively.

Clearly,

$$\mathbb{T}_{\mathbb{C}^{1}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}} \right\}, \quad (54)$$

and

$$\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IR}}, \mathbb{C}^{2}_{\mathbb{IR}}, \mathbb{C}^{3}_{\mathbb{IR}}, \mathbb{C}^{4}_{\mathbb{IR}}, \mathbb{C}^{5}_{\mathbb{IR}} \right\}, \quad (55)$$

are the cubic intuitionistic topology with an R-order.

Definition 31. Let $\mathbb{k}$ be a nonempty set, and $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}} = \left\{ \mathbb{C}^{\mathbb{k}}_{\mathbb{IR}} \right\}$ where $\mathbb{C}^{\mathbb{k}}_{\mathbb{IR}}$ represent the cubic intuitionistic fuzzy subsets of the universal set $\mathbb{k}$. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ is termed as an R-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and it is the largest R-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and is entitled as an R-discrete cubic intuitionistic fuzzy topology.

Definition 32. Let $\mathbb{k}$ be a universal set and $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}} \right\}$ be the assemblage of cubic intuitionistic fuzzy sets. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ is termed as an R-cubic intuitionistic fuzzy topology on the universal set $\mathbb{k}$ and is the smallest R-cubic intuitionistic fuzzy topology on $\mathbb{k}$ and is entitled as an R-indiscrete cubic intuitionistic fuzzy topology.

Definition 33. The elements of an R-cubic intuitionistic fuzzy topology $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ is termed as the R-cubic intuitionistic fuzzy open sets RCIFOS in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$.

Theorem 7. *If $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ is any R-cubic intuitionistic fuzzy topological space. Then,*

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are RCIFOSs.

(2) The R-union of any number of RCIFOSs is RCIFOS.

(3) The R-intersection of finite RCIFOSs is RCIFOS.

Proof. The proof is trivial. □

Definition 34. The complement of elements of an R-cubic intuitionistic fuzzy open sets is termed as the R-cubic intuitionistic fuzzy closed sets RCIFCSs in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$.

Theorem 8. *If $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ is any R-cubic intuitionistic fuzzy topological space. Then,*

(1) ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are RCIFCSs.

(2) The R-intersection of any number of RCIFCSs is RCIFCS.

(3) The R-union of finite RCIFCSs is RCIFCS.

Proof. The proof is trivial. □

Definition 35. The R-cubic intuitionistic fuzzy sets RCIFSs, which are RCIFOSs and RCIFCSs, are entitled as the R-cubic intuitionistic fuzzy clopen sets in $(\mathbb{k}, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$.

Proposition 2

(1) For every $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$, ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are R-cubic intuitionistic fuzzy clopen sets.

(2) For the discrete R-order cubic intuitionistic fuzzy topology, all the cubic intuitionistic subsets of $\mathbb{k}$ are R-cubic intuitionistic fuzzy clopen sets.

(3) For the in-discrete R-order cubic intuitionistic fuzzy topology, ${}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$ and $\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$ are only R-cubic intuitionistic fuzzy clopen sets.

Table 3: Union under R-order.

$\cup_R$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$
$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$
$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$
$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$
$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$
$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$
$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$
$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$

Table 4: Intersection under R-order.

$\cap_R$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$
${}^{1}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{1}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$
$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
$\mathbb{C}^{1}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
$\mathbb{C}^{2}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\overline{{}^{0}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{2}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
$\mathbb{C}^{3}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\overline{{}^{1}\mathbb{C}_{\mathbb{I}}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{1}_{\mathbb{IR}}$	$\mathbb{C}^{3}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$
$\mathbb{C}^{4}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	$\mathbb{C}^{4}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$
$\mathbb{C}^{5}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	$\mathbb{C}^{5}_{\mathbb{IR}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$	${}^{0}\mathbb{C}_{\mathbb{I}}$

Definition 36. Let $(\Bbbk, \mathbb{T}^{1}_{\mathbb{C}_{\mathbb{IR}}})$ and $(\Bbbk, \mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}})$ be two R-CIFTs in $\Bbbk$. Two R-CIFTs are called comparable if

$$\mathbb{T}^{1}_{\mathbb{C}_{\mathbb{IR}}} \subseteq_R \mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}, \tag{56}$$

or

$$\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}} \subseteq_R \mathbb{T}^{1}_{\mathbb{C}_{\mathbb{IR}}}. \tag{57}$$

If $\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}} \subseteq_R \mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}$ then, $\mathbb{T}^{1}_{\mathbb{C}_{\mathbb{IR}}}$ is called R-cubic intuitionistic fuzzy coarser than $\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}$ and $\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}$ is called the R-cubic intuitionistic fuzzy finer than. $\mathbb{T}^{1}_{\mathbb{C}_{\mathbb{IR}}}$

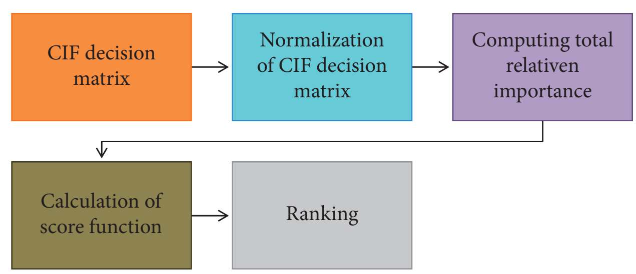


Figure 1: Flow chart of CIF WPM.

Example 9. Let $\Bbbk$ be a nonempty set and from Example 8,

$$\mathbb{T}_{\mathbb{C}^{1}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}} \right\}, \tag{58}$$

and

$$\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IR}}, \mathbb{C}^{2}_{\mathbb{IR}}, \mathbb{C}^{3}_{\mathbb{IR}}, \mathbb{C}^{4}_{\mathbb{IR}}, \mathbb{C}^{5}_{\mathbb{IR}} \right\}, \tag{59}$$

are R-cubic intuitionistic fuzzy topologies on the universal set. Then, $\mathbb{T}_{\mathbb{C}^{1}_{\mathbb{IR}}} \subseteq_R \mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}$. Hence, $\mathbb{T}_{\mathbb{C}^{1}_{\mathbb{IR}}}$ is called the R-cubic intuitionistic fuzzy coarser then, $\mathbb{T}^{2}_{\mathbb{C}_{\mathbb{IR}}}$.

4.1. Subspace of $\mathbb{CIFT}r$

Definition 37. Let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk})$ be a $\mathbb{CIFT}r$. Let $\mathbb{Y} \subseteq \Bbbk$ and $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}}$ is a $\mathbb{CIFT}r$ on $\mathbb{Y}$ and whose RCIFOSs are

$$\mathbb{C}_{\mathbb{IR}\mathbb{Y}} = \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk} \underset{R}{\cap} \mathbb{Y}, \tag{60}$$

where $\mathbb{C}_{\mathbb{IR}_k}$ are RCIFOSs of $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk}$, $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}}$ are RCIFOSs of $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}}$ and $\widetilde{\mathbb{Y}}$ is any R-cubic subset of RCIFS on $\mathbb{Y}$. Then, $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}}$ is called the R-cubic intuitionistic fuzzy subspace of $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk}$ i.e.,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}} = \left\{ \mathbb{C}_{\mathbb{IR}\mathbb{Y}} \colon \mathbb{C}_{\mathbb{IR}\mathbb{Y}} = \mathbb{C}_{\mathbb{IR}\Bbbk} \underset{R}{\cap} \mathbb{Y}, \mathbb{C}_{\mathbb{IR}\Bbbk} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk} \right\}. \tag{61}$$

Example 10. Let $\Bbbk$ be a nonempty set. From Example 8,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}} = \left\{ {}^{0}\mathbb{C}_{\mathbb{I}}, {}^{1}\mathbb{C}_{\mathbb{I}}, \overline{{}^{0}\mathbb{C}_{\mathbb{I}}}, \overline{{}^{1}\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^{1}_{\mathbb{IR}}, \mathbb{C}^{2}_{\mathbb{IR}}, \mathbb{C}^{3}_{\mathbb{IR}}, \mathbb{C}^{4}_{\mathbb{IR}}, \mathbb{C}^{5}_{\mathbb{IR}} \right\}, \tag{62}$$

is an R-cubic intuitionistic fuzzy topology on $\Bbbk$.

Now, consider any R-cubic fuzzy subset on $\Bbbk$ such that $\mathbb{Y} \subseteq \Bbbk$ is

$$\mathbb{Y} = \{[0.27, 0.38], [0.52, 0.67], (0.34, 0.28)\}. \tag{63}$$

Also,

Table 5: Cubic intuitionistic decision matrix from DMs.

Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$
$\mathbb{X}_1$	$([0.17, 0.24], [0.36, 0.43], (0.56, 0.32))$	$([0.20, 0.28], [0.29, 0.31], (0.27, 0.20))$	$([0.18, 0.21], [0.21, 0.32], (0.39, 0.22))$	$([0.20, 0.37], [0.21, 0.43], (0.54, 0.23))$
$\mathbb{X}_2$	$([0.19, 0.22], [0.39, 0.42], (0.59, 0.40))$	$([0.27, 0.34], [0.33, 0.40], (0.43, 0.21))$	$([0.24, 0.30], [0.30, 0.39], (0.50, 0.20))$	$([0.32, 0.40], [0.19, 0.51], (0.52, 0.30))$
$\mathbb{X}_3$	$([0.20, 0.29], [0.40, 0.51], (0.81, 0.13))$	$([0.31, 0.52], [0.42, 0.50], (0.72, 0.17))$	$([0.31, 0.39], [0.18, 0.40], (0.67, 0.14))$	$([0.14, 0.63], [0.24, 0.50], (0.70, 0.18))$
$\mathbb{X}_4$	$([0.31, 0.37], [0.36, 0.49], (0.50, 0.36))$	$([0.18, 0.33], [0.28, 0.52], (0.40, 0.32))$	$([0.23, 0.40], [0.24, 0.51], (0.50, 0.33))$	$([0.08, 0.74], [0.32, 0.40], (0.46, 0.42))$
$\mathbb{X}_5$	$([0.40, 0.48], [0.51, 0.60], (0.52, 0.30))$	$([0.29, 0.41], [0.30, 0.39], (0.48, 0.40))$	$([0.40, 0.47], [0.38, 0.60], (0.56, 0.29))$	$([0.13, 0.64], [0.40, 0.47], (0.53, 0.37))$
$\mathbb{X}_6$	$([0.29, 0.38], [0.27, 0.42], (0.60, 0.27))$	$([0.40, 0.51], [0.41, 0.50], (0.47, 0.38))$	$([0.32, 0.38], [0.24, 0.72], (0.43, 0.32))$	$([0.42, 0.50], [0.37, 0.53], (0.53, 0.27))$

Table 6: $\max_j \mathbb{T}_{ji}$ and $\min_j \mathbb{T}_{ji}$ values.

	$\max_j \mathbb{T}_{ji}$	$\min_j \mathbb{T}_{ji}$
$\mathbb{X}_1$	$([0.20, 0.37], [0.21, 0.31], (0.50, 0.20))$	$([0.17, 0.21], [0.36, 0.43], (0.27, 0.32))$
$\mathbb{X}_2$	$([0.32, 0.40], [0.19, 0.39], (0.59, 0.20))$	$([0.19, 0.22], [0.39, 0.51], (0.43, 0.40))$
$\mathbb{X}_3$	$([0.31, 0.63], [0.18, 0.40], (0.81, 0.13))$	$([0.14, 0.29], [0.42, 0.50], (0.67, 0.18))$
$\mathbb{X}_4$	$([0.31, 0.74], [0.24, 0.40], (0.50, 0.32))$	$([0.08, 0.33], [0.36, 0.52], (0.40, 0.42))$
$\mathbb{X}_5$	$([0.40, 0.64], [0.30, 0.39], (0.56, 0.29))$	$([0.13, 0.41], [0.51, 0.60], (0.48, 0.40))$
$\mathbb{X}_6$	$([0.42, 0.51], [0.24, 0.42], (0.60, 0.27))$	$([0.29, 0.38], [0.41, 0.72], (0.43, 0.38))$

$$
\begin{aligned}
\mathbb{Y} \overset{0}{\underset{R}{\cap}} \mathbb{C}_{\mathbb{I}} &= \{[0,0], [1,1], (1,0)\} \\
&= \overrightarrow{\mathbb{C}_{\mathbb{IR}}}, \\
\mathbb{Y} \overset{1}{\underset{R}{\cap}} \mathbb{C}_{\mathbb{I}} &= \{[0.27, 0.38], [0.52, 0.67], (1,0)\} \\
&= \widetilde{\mathbb{C}_{\mathbb{IR}}}, \\
\mathbb{Y} \cap_R \overline{{}^0\mathbb{C}_{\mathbb{I}}} &= \{[0,0], [1,1], (0.34, 0.28)\} \\
&= \mathbb{C}'_{\mathbb{IR}}, \\
\mathbb{Y} \cap_R \overline{{}^1\mathbb{C}_{\mathbb{I}}} &= \{[0.27, 0.38], [0.52, 0.67], (1,0)\} \\
&= \widetilde{\mathbb{C}_{\mathbb{IR}}}, \\
\mathbb{Y} \underset{R}{\cap} \mathbb{C}^1_{\mathbb{IR}} &= \{[0.27, 0.38], [0.52, 0.67], (0.34, 0.28)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{R}{\cap} \mathbb{C}^2_{\mathbb{IR}} &= \{[0.27, 0.38], [0.52, 0.67], (0.34, 0.28)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{R}{\cap} \mathbb{C}^3_{\mathbb{IR}} &= \{[0.27, 0.38], [0.52, 0.67], (0.34, 0.28)\} \\
&= \mathbb{Y}, \\
\mathbb{Y} \underset{R}{\cap} \mathbb{C}^4_{\mathbb{IR}} &= \{[0,0], [1,1], (0.34, 0.28)\} \\
&= \mathbb{C}'_{\mathbb{IR}}, \\
\mathbb{Y} \underset{R}{\cap} \mathbb{C}^5_{\mathbb{IR}} &= \{[0.27, 0.38], [0.52, 0.67], (1,0)\} \\
&= \widetilde{\mathbb{C}_{\mathbb{IR}}}.
\end{aligned} \tag{64}
$$

Then,

$$
\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\mathbb{Y}} = \left\{\overrightarrow{\mathbb{C}_{\mathbb{IR}}}, \widetilde{\mathbb{C}_{\mathbb{IR}}}, {}'\,\mathbb{C}_{\mathbb{IR}}, \mathbb{Y}\right\}, \tag{65}
$$

is an R-cubic intuitionistic fuzzy relative topology of $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}\Bbbk}$

4.2. Interior, Closure, Frontier, and Exterior of RCIFSs

Definition 38. let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ be $\mathbb{CIFT}r$ and $\mathbb{C}_{\mathbb{IR}} \in ci(\Bbbk)$, the interior of $\mathbb{C}_{\mathbb{IR}}$ is expressed as $\mathbb{C}^0_{\mathbb{IR}}$ and is described as a union of all the R-cubic intuitionistic fuzzy open subsets contained in $\mathbb{C}_{\mathbb{IR}}$. It is the greatest R-cubic intuitionistic fuzzy open set contained in $\mathbb{C}_{\mathbb{IR}}$.

Example 11. Consider an R-cubic intuitionistic fuzzy topological space as constructed in Example 8. Let $\mathbb{C}^6_{\mathbb{IR}} \in ci(\Bbbk)$ given as

$$
\mathbb{C}^6_{\mathbb{IR}} = \{[0.38, 0.46], [0.45, 0.51], (0.26, 0.40)\}. \tag{66}
$$

Then,

$$
\begin{aligned}
\left(\mathbb{C}^6_{\mathbb{IR}}\right)^0 &= {}^0\mathbb{C}_{\mathbb{I}} \underset{R}{\cup} \mathbb{C}^1_{\mathbb{IR}} \underset{R}{\cup} \mathbb{C}^4_{\mathbb{IP}} \underset{R}{\cup} \mathbb{C}^5_{\mathbb{IR}} \\
&= \mathbb{C}^1_{\mathbb{IR}}.
\end{aligned} \tag{67}
$$

Theorem 9. *Let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ be $\mathbb{CIFT}$ r and $\mathbb{C}_{\mathbb{IR}} \in ci(\Bbbk)$. Then, $\mathbb{C}_{\mathbb{IR}}$ is open CIFS iff $\mathbb{C}^0_{\mathbb{IR}} = \mathbb{C}_{\mathbb{IR}}$.*

Proof. The proof is trivial. □

Theorem 10. *Let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IP}}})$ be $\mathbb{CIFT}p$ and $\mathbb{C}^1_{\mathbb{IP}}, \mathbb{C}^2_{\mathbb{IP}} \in ci(\Bbbk)$. Then,*

(i) $((\mathbb{C}_{\mathbb{IR}})^0)^0 = (\mathbb{C}_{\mathbb{IR}})^0$
(ii) $\mathbb{C}^1_{\mathbb{IR}} \subseteq_R \mathbb{C}^2_{\mathbb{IR}} \Rightarrow (\mathbb{C}^1_{\mathbb{IR}})^0 \subseteq_R (\mathbb{C}^2_{\mathbb{IR}})^0$
(iii) $(\mathbb{C}^1_{\mathbb{IR}} \cap_R \mathbb{C}^2_{\mathbb{IR}})^0 = (\mathbb{C}^1_{\mathbb{IR}})^0 \subseteq_R (\mathbb{C}^2_{\mathbb{IR}})^0$
(iv) $(\mathbb{C}^1_{\mathbb{IR}} \cup_R \mathbb{C}^2_{\mathbb{IR}})^0 \supseteq_R (\mathbb{C}^1_{\mathbb{IR}})^0 \cup_R (\mathbb{C}^2_{\mathbb{IR}})^0$

Proof. Proof is trivial. □

Definition 39. let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ be $\underline{\mathbb{CIFT}}r$ and $\mathbb{C}_{\mathbb{IR}} \in ci(\Bbbk)$, the closure of $\mathbb{C}_{\mathbb{IR}}$ is expressed as $\overline{\mathbb{C}_{\mathbb{IR}}}$ and is described as the intersection of all the R-cubic intuitionistic fuzzy closed supersets of $\mathbb{C}_{\mathbb{IR}}$. It is the smallest R-cubic intuitionistic fuzzy closed superset of $\mathbb{C}_{\mathbb{IR}}$.

Example 12. Let us consider an R-cubic intuitionistic topological space as constructed in Example 8. Then, the closed CIFSs are given as

TABLE 7: Normalized decision matrix from DMs.

Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$
$\mathbb{X}_1$	$([0.17, 0.21], [0.36, 0.43], (0.56, 0.32))$	$([0.17, 0.21], [0.36, 0.43], (0.27, 0.20))$	$([0.17, 0.21], [0.36, 0.43], (0.39, 0.22))$	$([0.17, 0.21], [0.36, 0.43], (0.54, 0.23))$
$\mathbb{X}_2$	$([0.19, 0.22], [0.39, 0.51], (0.59, 0.40))$	$([0.19, 0.22], [0.39, 0.51], (0.43, 0.21))$	$([0.19, 0.22], [0.39, 0.51], (0.50, 0.20))$	$([0.19, 0.22], [0.39, 0.51], (0.52, 0.30))$
$\mathbb{X}_3$	$([0.20, 0.29], [0.40, 0.51], (0.81, 0.13))$	$([0.31, 0.52], [0.42, 0.50], (0.81, 0.13))$	$([0.31, 0.39], [0.18, 0.40], (0.81, 0.13))$	$([0.14, 0.63], [0.24, 0.50], (0.81, 0.13))$
$\mathbb{X}_4$	$([0.08, 0.33], [0.36, 0.52], (0.50, 0.36))$	$([0.08, 0.33], [0.36, 0.52], (0.40, 0.32))$	$([0.08, 0.33], [0.36, 0.52], (0.50, 0.33))$	$([0.08, 0.33], [0.36, 0.52], (0.46, 0.42))$
$\mathbb{X}_5$	$([0.40, 0.48], [0.51, 0.60], (0.56, 0.29))$	$([0.29, 0.41], [0.30, 0.39], (0.56, 0.29))$	$([0.40, 0.47], [0.38, 0.60], (0.56, 0.29))$	$([0.13, 0.64], [0.40, 0.47], (0.56, 0.29))$
$\mathbb{X}_6$	$([0.29, 0.38], [0.41, 0.72], (0.60, 0.27))$	$([0.29, 0.38], [0.41, 0.72], (0.47, 0.38))$	$([0.29, 0.38], [0.41, 0.72], (0.43, 0.32))$	$([0.29, 0.38], [0.41, 0.72], (0.53, 0.27))$

Table 8: Relative importance and score function.

Alternatives	CIFN-WPM values	Score values
$\mathbb{X}_1$	$([0.16, 0.20], [0.34, 0.41], (0.9980, 0.00001))$	0.8029
$\mathbb{X}_2$	$([0.18, 0.21], [0.38, 0.49], (0.9993, 0.00002))$	0.7592
$\mathbb{X}_3$	$([0.21, 0.45], [0.29, 0.46], (0.9999, 0.0000007))$	0.9548
$\mathbb{X}_4$	$([0.07, 0.31], [0.34, 0.50], (0.9990, 0.00007))$	0.6294
$\mathbb{X}_5$	$([0.25, 0.49], [0.39, 0.50], (0.9996, 0.00002))$	0.9245
$\mathbb{X}_6$	$([0.28, 0.34], [0.39, 0.71], (0.9994, 0.00004))$	0.7593

$$\begin{aligned}
\left({}^0\mathbb{C}_{\mathbb{I}}\right)^c &= \{[1,1], [0,0], (0,1)\},\\
\left({}^1\mathbb{C}_{\mathbb{I}}\right)^c &= \{[0,0], [1,1], (1,0)\},\\
\left(\overline{{}^0\mathbb{C}_{\mathbb{I}}}\right)^c &= \{[1,1], [0,0], (1,0)\},\\
\left(\overline{{}^1\mathbb{C}_{\mathbb{I}}}\right)^c &= \{[0,0], [1,1], (0,1)\},\\
\left(\mathbb{C}^1_{\mathbb{IP}}\right)^c &= \{[0.47, 0.56], [0.31, 0.42], (0.39, 0.29)\},\\
\left(\mathbb{C}^2_{\mathbb{IP}}\right)^c &= \{[0.47, 0.56], [0.31, 0.42], (1, 0)\},\\
\left(\mathbb{C}^3_{\mathbb{IP}}\right)^c &= \{[0,0], [1,1], (0.39, 0.29)\},\\
\left(\mathbb{C}^4_{\mathbb{IP}}\right)^c &= \{[1,1], [0,0], (0.39, 0.29)\},\\
\left(\mathbb{C}^5_{\mathbb{IP}}\right)^c &= \{[0.47, 0.56], [0.31, 0.42], (0, 1)\}.
\end{aligned} \tag{68}$$

Let $\mathbb{C}^7_{\mathbb{IR}} \in ci(\Bbbk)$ given as

$$\mathbb{C}^7_{\mathbb{IR}} = \{[0.30, 0.37], [0.48, 0.61], (0.38, 0.24)\}. \tag{69}$$

Then,

$$\begin{aligned}
\overline{\mathbb{C}^7_{\mathbb{IR}}} &= \left(\overline{{}^0\mathbb{C}_{\mathbb{I}}}\right)^c \underset{R}{\cap} \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c\\
&= \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c.
\end{aligned} \tag{70}$$

Theorem 11. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ *be* $\mathbb{CIFT}$ *r and* $\mathbb{C}_{\mathbb{IR}} \in ci(\Bbbk)$*. Then,* $\mathbb{C}_{\mathbb{IR}}$ *is closed CIFS iff* $\overline{\mathbb{C}_{\mathbb{IR}}} = \mathbb{C}_{\mathbb{IR}}$..

Proof. The proof is trivial. □

Definition 40. Let $\mathbb{C}_{\mathbb{IR}}$ be an R-cubic intuitionistic fuzzy subset of $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$, then its boundary or frontier is defined as

$$Fr\left(\mathbb{C}_{\mathbb{IR}}\right) = \overline{\mathbb{C}_{\mathbb{IR}}} \underset{R}{\cap} \overline{\left(\mathbb{C}_{\mathbb{IR}}\right)^c}. \tag{71}$$

Definition 41. Let $\mathbb{C}_{\mathbb{IR}}$ be an R-cubic intuitionistic fuzzy subset of $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$, then the exterior is defined as

$$\begin{aligned}
\operatorname{Ext}\left(\mathbb{C}_{\mathbb{IR}}\right) &= \left(\overline{\mathbb{C}_{\mathbb{IR}}}\right)^c\\
&= \left(\mathbb{C}^c_{\mathbb{IR}}\right)^0.
\end{aligned} \tag{72}$$

Example 4.13. Consider an R-cubic intuitionistic topological space as constructed in Example 8 and $\mathbb{C}^6_{\mathbb{IR}}$ and $\mathbb{C}^7_{\mathbb{IR}}$ from Examples 11 and 12. Then,

$$\begin{aligned}
\left(\mathbb{C}^6_{\mathbb{IR}}\right)^0 &= \mathbb{C}^1_{\mathbb{IR}},\\
\overline{\mathbb{C}^6_{\mathbb{IP}}} &= \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c,\\
Fr\left(\mathbb{C}^6_{\mathbb{IR}}\right) &= \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c,\\
\operatorname{Ext}\left(\mathbb{C}^6_{\mathbb{IR}}\right) &= \mathbb{C}^5_{\mathbb{IR}},\\
\left(\mathbb{C}^7_{\mathbb{IR}}\right)^0 &= \mathbb{C}^4_{\mathbb{IR}},\\
\overline{\mathbb{C}^7_{\mathbb{IR}}} &= \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c,\\
Fr\left(\mathbb{C}^7_{\mathbb{IR}}\right) &= \left(\mathbb{C}^5_{\mathbb{IR}}\right)^c,\\
\operatorname{Ext}\left(\mathbb{C}^7_{\mathbb{IR}}\right) &= \mathbb{C}^5_{\mathbb{IR}}.
\end{aligned} \tag{73}$$

Theorem 12. *Let* $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ *be* $\mathbb{CIFT}r$ *and* $\mathbb{C}_{\mathbb{IR}} \in ci(\Bbbk)$*. Then,*

(1) $(\mathbb{C}^0_{\mathbb{IR}})^c = \overline{(\mathbb{C}^c_{\mathbb{IR}})}$

(2) $(\overline{\mathbb{C}_{\mathbb{IR}}})^c = (\mathbb{C}^c_{\mathbb{IR}})^0$

(3) $Ext\,(\mathbb{C}^c_{\mathbb{IR}}) = \mathbb{C}^0_{\mathbb{IR}}$

(4) $Ext\,(\mathbb{C}_{\mathbb{IR}}) = (\mathbb{C}^c_{\mathbb{IR}})^0$

(5) $Ext\,(\mathbb{C}_{\mathbb{IR}}) \cup_R Fr\,(\mathbb{C}_{\mathbb{IR}}) \cup_R \mathbb{C}^0_{\mathbb{IR}} \neq {}^1\mathbb{C}_{\mathbb{IR}}$

(6) $Fr\,(\mathbb{C}_{\mathbb{IR}}) = Fr\,(\mathbb{C}^c_{\mathbb{IR}})$

(7) $Fr\,(\mathbb{C}_{\mathbb{IR}}) \cap_R \mathbb{C}^0_{\mathbb{IR}} \neq {}^0\mathbb{C}_{\mathbb{IR}}$

Proof. The proof is trivial. □

4.3. R-Cubic Intuitionistic Fuzzy Basis

Definition 42. Let $(\Bbbk, \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}})$ be $\mathbb{CIFT}r$. Then $\mathbb{B} \subseteq \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ is called an R-cubic intuitionistic fuzzy basis for $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$ if for every $\mathbb{C}_{\mathbb{IR}} \in \mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$, $\exists \mathscr{B} \in \mathbb{B}$ such that

$$\mathbb{C}_{\mathbb{IR}} = \underset{R}{\cup} \mathscr{B}. \tag{74}$$

Example 14. From Example 8,

$$\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}} = \left\{{}^0\mathbb{C}_{\mathbb{I}}, {}^1\mathbb{C}_{\mathbb{I}}, \overline{{}^0\mathbb{C}_{\mathbb{I}}}, \overline{{}^1\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^1_{\mathbb{IR}}, \mathbb{C}^2_{\mathbb{IR}}, \mathbb{C}^3_{\mathbb{IR}}, \mathbb{C}^4_{\mathbb{IR}}, \mathbb{C}^5_{\mathbb{IR}}\right\}, \tag{75}$$

is an R-cubic intuitionistic fuzzy topology of $\Bbbk$. Then,

$$\mathbb{B} = \left\{{}^1\mathbb{C}_{\mathbb{I}}, \overline{{}^1\mathbb{C}_{\mathbb{I}}}, \mathbb{C}^1_{\mathbb{IR}}, \mathbb{C}^2_{\mathbb{IR}}, \mathbb{C}^3_{\mathbb{IR}}, \mathbb{C}^4_{\mathbb{IR}}, \mathbb{C}^5_{\mathbb{IR}}\right\}, \tag{76}$$

is an R-cubic intuitionistic fuzzy basis for $\mathbb{T}_{\mathbb{C}_{\mathbb{IR}}}$.

Table 9: Cubic intuitionistic decision matrix from DMs.

Criteria	$\mathbb{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$
$\mathbb{X}_1$	$([0.17, 0.24], [0.36, 0.43], (0.56, 0.32))$	$([0.20, 0.28], [0.29, 0.31], (0.27, 0.20))$	$([0.18, 0.21], [0.21, 0.32], (0.39, 0.22))$	$([0.20, 0.37], [0.21, 0.43], (0.54, 0.23))$
$\mathbb{X}_2$	$([0.19, 0.22], [0.39, 0.42], (0.59, 0.40))$	$([0.27, 0.34], [0.33, 0.40], (0.43, 0.21))$	$([0.24, 0.30], [0.30, 0.39], (0.50, 0.20))$	$([0.32, 0.40], [0.19, 0.51], (0.52, 0.30))$
$\mathbb{X}_3$	$([0.20, 0.29], [0.40, 0.51], (0.81, 0.13))$	$([0.31, 0.52], [0.42, 0.50], (0.72, 0.17))$	$([0.31, 0.39], [0.18, 0.40], (0.67, 0.14))$	$([0.14, 0.63], [0.24, 0.50], (0.70, 0.18))$
$\mathbb{X}_4$	$([0.31, 0.37], [0.36, 0.49], (0.50, 0.36))$	$([0.18, 0.33], [0.28, 0.52], (0.40, 0.32))$	$([0.23, 0.40], [0.24, 0.51], (0.50, 0.33))$	$([0.08, 0.74], [0.32, 0.40], (0.46, 0.42))$
$\mathbb{X}_5$	$([0.40, 0.48], [0.51, 0.60], (0.52, 0.30))$	$([0.29, 0.41], [0.30, 0.39], (0.48, 0.40))$	$([0.40, 0.47], [0.38, 0.60], (0.56, 0.29))$	$([0.13, 0.64], [0.40, 0.47], (0.53, 0.37))$
$\mathbb{X}_6$	$([0.29, 0.38], [0.27, 0.42], (0.60, 0.27))$	$([0.40, 0.51], [0.41, 0.50], (0.47, 0.38))$	$([0.32, 0.38], [0.24, 0.72], (0.43, 0.32))$	$([0.42, 0.50], [0.37, 0.53], (0.53, 0.27))$

Table 10: Score values.

Alternatives	Score values
$\mathbb{X}_1$	0.1125
$\mathbb{X}_2$	0.1705
$\mathbb{X}_3$	0.5490
$\mathbb{X}_4$	0.0725
$\mathbb{X}_5$	0.1435
$\mathbb{X}_6$	0.204

5. Multicriteria Group Decision-Making

The weighted product model (WPM) is a renowned and widely used MCGDM approach for evaluating a set of choices using a set of criteria. Each choice is contrasted to the others by calculating a number of ratios, one per choice criterion. Every ratio is multiplied by the proportional weight of the criterion in consideration. For the selection of one or more options from the set of alternatives based on a number of criteria is a fundamental task in MCGDM problems. Let us consider m alternatives, n criteria with weighted vectors, with the condition that the sum of the weights will be one, for an MCGDM problem in a cubic intuitionistic fuzzy set domain.

Figure 1 shows the flow chart of WPM.

5.1. Application to Uncertain Supply Chain Management. Communication and information technologies are affecting every area of the industrial sector at a rapid pace. In reality, it would be difficult to pinpoint an organization that does not use or is not touched by information and communications technologies in some way. In many cases, if technology is not employed appropriately, the firm's survival is jeopardized. Companies nowadays use technology to boost productivity, streamline operations, and form electronic conglomerates. Advanced technologies and electronic systems are radically altering how businesses operate and stay competitive. Many businesses are making strategic technology investments to obtain and maintain a competitive advantage in their industry. Management teams must use technology throughout the organization to enhance information flow, reduce cost, streamline operations, provide product variety, formulate connections with suppliers, and reduce response times to customers' needs to gain a competitive advantage through the use of information and communications technology.

Administrators and top executives should be associated with the development of enterprise-wide information systems (EIS), which should take into account such matters as computer hardware and software and infrastructure facilities, online systems, digital applications, electronic commerce, and alterations to current processes and practices. Managers can integrate data and telecommunications technologies throughout the corporation and connect all business areas by developing an enterprise broad information systems plan. Enterprise-wide integration of technology enables firms to allow consumers to get timely access to the information they need to make informed decisions. Recent research has looked at information systems as useful tools for integrating systems like enterprise resource planning, knowledge management, e-commerce, electronic markets, and supply chain management (SCM) to enhance organizational profit and efficiency.

Companies must analyze both internal and external processes for the production and exchange of products and services to be more efficient and competitive. The managers will be able to evaluate the value of actions for each process to determine how to boost the value among these operations that form a supply chain from supplier to business to dealer to customer through the evaluation of these processes. The level of integration among suppliers, business associates, and buyers, independent of their geographical location, determines the value chain's or supply chain's effectiveness.

The construction of an integrated organizational system capable of information sharing, resources, and services in the supply chain is central to the digital supply chain management paradigm. To gain and maintain competitive advantages, companies use digital information and communications networks to standardize manufacturing processes, reduce cycle time, increase the effectiveness of procurement procedures and logistical support, reduce production costs, and increase customer satisfaction, among other things. Supply chain management based on the Internet allows a company to streamline its supply chain, increase speed, reduce costs, and be more adaptable. It can also increase consumer and supplier communications as well as smooth the ongoing flow of goods along the supply chain.

Supplier selection is highly essential in supply chain management. The objective is to locate a supplier who can offer the best products and services for the lowest price. Proper supplier selection delivers a high profit and quality level. In this strategic collaboration, the supplier is viewed as a significant element of the business. Because of the increasing focus on sustainability, identifying these providers has become more challenging. Environmental studies, often known as sustainability studies, have become increasingly popular around the world. Identifying these suppliers has become increasingly difficult as a result of the rapidly increasing emphasis on sustainability. Many methodologies for sustainable supply chain selection have been developed.

To determine the most suitable supplier selection, MCGDM techniques can be used successfully. In this section, the suggested model is used to determine the selection of appropriate suppliers for fast-moving consumer products, with the goal of selecting the best supplier among various possibilities. Several criteria have been established based on expert opinions to evaluate supplier choices. In this study, $\mathbb{X}1$, $\mathbb{X}2$, $\mathbb{X}3$, $\mathbb{X}4$, $\mathbb{X}5$ and $\mathbb{X}6$ are examined as possible fast-moving customers goods suppliers using the four criteria established.

5.2. CIF Weighted Product Model. The proposed method is used to choose the best supplier among six alternatives. These alternatives are weighed against four criteria $\mathbb{C}1 =$ price, $\mathbb{C}2 =$ quality, $\mathbb{C}3 =$ performance, and $\mathbb{C}4 =$ delivery, derived from thorough expert opinions. A group of decision-makers has been assembled to assess the suppliers using the recommended methodology. Six decision-makers $\mathbb{D}1$, $\mathbb{D}2$, $\mathbb{D}3$, $\mathbb{D}4$, $\mathbb{D}5$, and $\mathbb{D}6$ were chosen, consisting of supplier experts and expert academics on multicriteria decision-making in a fuzzy environment:,,,

Step 1. Consider the decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ given by the decision-makers in the form of CIFNs on the basis of the cubic intuitionistic fuzzy linguistic scale to evaluate suppliers in accordance with established objectives and criteria is given in Table 5.

Step 2. With the help of the linear approach, we normalize the matrix $M = (\mathbb{T}_{ji})_{m\times n}$. We divide the criteria into two subsets, benefit criteria $\mathbb{B}$ and cost criteria $\mathbb{K}$. Here, $\mathbb{X}_3$ and $\mathbb{X}_6$ belong to benefit criteria $\mathbb{B}$, and the remaining others belong to cost criteria $\mathbb{K}$. For this, first, we find $\max_j \mathbb{T}_{ji}$ and $\min_j \mathbb{T}_{ji}$, which are given in Table 5. We normalized the decision matrix by utilizing the 1st and 2nd equations in Algorithm 1 and this is given in Table 6.

Normalized decision matrixes from DMs are expressed in Table 7.

Steps 3 and 4. We find the relative importance of all alternatives by utilizing the 3rd equation in Algorithm 1, and then, we calculate their score function as given in Table 8.

Step 5. Rank the alternatives according to the score function, and the final ranking is

$$\mathbb{X}_3 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_4. \tag{77}$$

As we can see that $\mathbb{X}_3$ is the most appropriate supplier among the six alternatives with the best of qualities of all criteria.

5.3. CIF Choice Value Method.

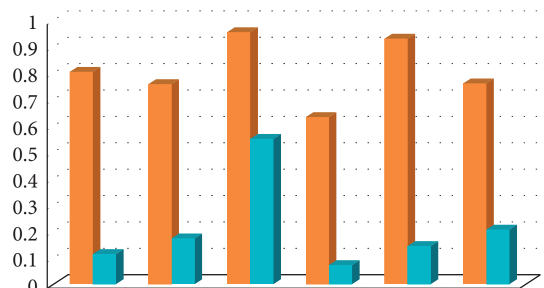
The choice value method is a renowned and widely used MCGDM basis for evaluating a set of choices using a set of criteria. Each choice is contrasted to the others by calculating a number of ratios, one per choice criterion. Every ratio is multiplied by the proportional weight of the criterion in consideration. A fundamental task in MCGDM problems is the selection of one or more options from the set of alternatives based on a number of criteria. Let us consider m alternatives, n criteria with weighted vectors, with the condition that the sum of weights will be one, for an MCGDM problem in a cubic intuitionistic fuzzy set domain.,

5.4. MCDGM Application

Step 1. Consider the decision matrix $M = (\mathbb{T}_{ji})_{m\times n}$ given by the decision-makers in the form of CIFNs given in Table 9.

Step 2. Decision-makers gives the weights to the four criteria as $\mathbb{W}1 = 0.18$, $\mathbb{W}2 = 0.24$, $\mathbb{W}3 = 0.26$, and $\mathbb{W}4 = 0.32$ with $\sum \mathbb{W}i = 1$

$$\begin{pmatrix}
\begin{pmatrix}[0.17,0.24],[0.36,0.43],\\(0.56,0.32)\end{pmatrix}\begin{pmatrix}[0.20,0.28],[0.29,0.31],\\(0.27,0.20)\end{pmatrix}\\
\begin{pmatrix}[0.18,0.21],[0.21,0.32],\\(0.39,0.22)\end{pmatrix}([0.20,0.37],[0.21,0.43],(0.54,0.23))\\
\begin{pmatrix}[0.19,0.22],[0.39,0.42],\\(0.59,0.40)\end{pmatrix}\begin{pmatrix}[0.27,0.34],[0.33,0.40]\\,(0.43,0.21)\end{pmatrix}\\
([0.24,0.30],[0.30,0.39],(0.50,0.20))([0.32,0.40],[0.19,0.51],(0.52,0.30))\\
([0.20,0.29],[0.40,0.51],(0.81,0.13))\begin{pmatrix}[0.31,0.52],[0.42,0.50],\\(0.72,0.17)\end{pmatrix}\\
([0.31,0.39],[0.18,0.40],(0.67,0.14))([0.14,0.63],[0.24,0.50],(0.70,0.18))\\
\begin{pmatrix}[0.31,0.37],[0.36,0.49],\\(0.50,0.36)\end{pmatrix}([0.18,0.33],[0.28,0.52],(0.40,0.32))\\
([0.23,0.40],[0.24,0.51],(0.50,0.33))\\
([0.08,0.74],[0.32,0.40],(0.46,0.42))\\
([0.40,0.48],[0.51,0.60],(0.52,0.30))\\
([0.29,0.41],[0.30,0.39],(0.48,0.40))([0.40,0.47],[0.38,0.60],(0.56,0.29))\\
([0.13,0.64],[0.40,0.47],(0.53,0.37))\\
([0.29,0.38],[0.27,0.42],(0.60,0.27))([0.40,0.51],[0.41,0.50],(0.47,0.38))\\
([0.32,0.38],[0.24,0.72],(0.43,0.32))([0.42,0.50],[0.37,0.53],(0.53,0.27))
\end{pmatrix}\begin{pmatrix}0.18\\0.24\\0.26\\0.32\end{pmatrix}$$



	X1	X2	X3	X4	X5	X6
CIF-WPM	0.8029	0.7592	0.9548	0.6294	0.9245	0.7593
CIF-CVM	0.1125	0.1705	0.549	0.0725	0.1435	0.204

Figure 2: Ranking of feasible alternatives.

$$\begin{pmatrix} ([0.032,0.048],[0.832,0.859],(0.900,0.067))+([0.052,0.075],[0.742,0.754],(0.730,0.052)) \\ +([0.050,0.059],[0.666,0.743],(0.782,0.062)) \\ +([0.068,0.137],[0.606,0.763],(0.821,0.080)) \\ ([0.037,0.043],[0.844,0.855],(0.909,0.087))+([0.072,0.094],[0.766,0.802],(0.816,0.055)) \\ +([0.068,0.088],[0.731,0.782],(0.835,0.056))+([0.116,0.150],[0.587,0.806],(0.811,0.107)) \\ ([0.039,0.059],[[0.847,0.885]],(0.962,0.024))+([0.085,0.161],[0.812,0.846],(0.924,0.043)) \\ +([0.091,0.120],[0.640,0.788],(0.901,0.038))+([0.047,0.272],[0.633,0.801],(0.892,0.061)) \\ ([0.064,0.079],[0.832,0.879],(0.882,0.077))+([0.046,0.091],[0.736,0.854],(0.802,0.088)) \\ +([0.065,0.124],[0.690,0.839],(0.835,0.098))+([0.026,0.350],[0.694,0.745],(0.779,0.159)) \\ ([0.087,0.111],[0.885,0.912],(0.888,0.062))+([0.078,0.118],[0.749,0.797],(0.888,0.115)) \\ +([0.124,0.152],[0.777,0.875],(0.860,0.085))+([0.043,0.278],[0.745,0.785],(0.816,0.137)) \\ ([0.059,0.082],[0.790,0.855],(0.912,0.055))+([0.115,0.157],[0.807,0.846],(0.834,0.108)) \\ +([0.095,0.116],[0.690,0.918],(0.802,0.095))+([0.159,0.198],[0.727,0.816],(0.840,0.095)) \end{pmatrix} \tag{78}$$

$$\begin{pmatrix} ([0.187,0.284],[0.249,0.367],(0.421,0.236)) \\ ([0.263,0.327],[0.277,0.432],(0.502,0.272)) \\ ([0.238,0.494],[0.278,0.472],(0.714,0.156)) \\ ([0.186,0.523],[0.293,0.469],(0.460,0.361)) \\ ([0.294,0.519],[0.383,0.499],(0.522,0.344)) \\ ([0.366,0.451],[0.278,0.537],(0.512,0.309)) \end{pmatrix}.$$

Step 3. We compute the score values for each alternative. The score values are expressed in Table 10.

Step 4. Rank the alternatives according to their score values.

$$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_4. \tag{79}$$

As a result, $\mathbb{X}_3$ is best supplier among six alternatives with qualities of all criteria.

5.5. Comparative Analysis. This paper describes techniques for dealing with the cubic intuitionistic situation. We compare our two cubic intuitionistic strategies that are

Table 11: Comparative analysis and ranking of alternatives.

Methods	Ranking of alternatives	Top alternative
CIF-TOPSIS (Garg and Kaur [23])	$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_4$	$\mathbb{X}_3$
CIF-WASPAS (Senapati et al. [26])	$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_1 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_4$	$\mathbb{X}_3$
Frank AO (Seikh and Mandal [52])	$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_4$	$\mathbb{X}_3$
CIF-WPM (Algorithm 1)	$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_4$	$\mathbb{X}_3$
CIF-CVM (Algorithm 2)	$\mathbb{X}_3 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_4$	$\mathbb{X}_3$

already in use. If we use CIF-WPM to assemble the alternatives, they are ranked as

$$\mathbb{X}_3 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_4. \tag{80}$$

On the other side, when we use the technique of choice value method, the ranking of alternatives is

$$\mathbb{X}_3 \succ \mathbb{X}_6 \succ \mathbb{X}_2 \succ \mathbb{X}_5 \succ \mathbb{X}_1 \succ \mathbb{X}_4. \tag{81}$$

Based on these findings, it seemed that the ranking of the $\mathbb{X}_3$ alternative was the same as that produced by the suggested cubic intuitionistic procedures. The rest of the alternatives have been altered, as can be seen. As a result, we concluded that in the case of only IVIFSs, the best choice matches with the indicated one; however, the other alternatives are altered, resulting in numerous decisions. As a result, this CIS condition improves the application range of the membership and nonmembership intervals by considering IFS membership values in line with it.

Figure 2 shows the bar chart of ranking of feasible alternatives by using the WPM and CVM methods.

The comparison analysis of the proposed CIF-WPM and CIF-CVM with other existing techniques is expressed in Table 11.

6. Conclusion

A cubic intuitionistic fuzzy set is an effective method for dealing with various uncertainties in multicriteria group decision-making (MCGDM) settings. A cubic set is a two-component system that would be used to describe data with a fuzzy interval and a fuzzy number. The notion of cubic intuitionistic fuzzy sets (CIFS) is a strong hybrid model of IFSs and IVIFSs. A CIFS has two components, one indicating the IVIFS and the other indicating the IFS. A CIFS is a new fuzzy model for data analysis, computational intelligence, neural computing, soft computing, and others. The idea of cubic hesitant fuzzy topology defined on CIFS can be utilized to seek solutions to various problems of information analysis, information fusion, big data, and decision analysis.

Main findings in this manuscript are as follows:

(1) We introduced the concepts of "P-cubic intuitionistic fuzzy topology" as well as "R-cubic intuitionistic fuzzy topology." Topological structures provide robust approaches for data analysis and decision analysis under an uncertain environment.

(2) Certain properties of CIF topology under P(R)-order are explored, and their related results are elaborated with illustrations.

(3) The notions of CIF-open set, CIF-closed set, CIF-closure, CIF-interior, CIF-exterior, as well as CIF-frontier, CIF-dense set, and CIF-basis are investigated with a corresponding example.

(4) Algorithms 1 and 2 are proposed for extension of the weighted product model and the choice value method, respectively.

(5) The symmetry of optimal decisions is analyzed by computations with Algorithms 1 and 2. The numerical values of alternatives are very close by using Algorithm 1. However, the numerical values of alternatives have a clear difference when using Algorithm 2.

(6) An application of proposed methods named CIF-WPM and CIF-CVM towards uncertain supply chain management is presented.

(7) To discuss the advantages, flexibility, and validity of proposed methods, a comparison analysis is also expressed.

For forthcoming analysis, due to the flexibility of CIF topology towards data analysis and information analysis, one can extend this work to develop new MCDM techniques with CIF-VIKOR, CIF-AHP, and CIF-aggregation operators.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Muhammad Riaz was responsible for conceptualization, formal analysis, and supervision. Khadija Akmal conducted methodology, formal analysis, review, and editing. Yahya Almalki performed investigation, supervision, and funding acquisition. Daud Ahmad took part in investigation, methodology, review, and editing. All authors made a significant scientific contribution to the research in the manuscript. All authors read and approved the final manuscript.

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References

[1] F. Chazal and B. Michel, "An introduction to topological data analysis: fundamental and practical aspects for data scientists," *Frontiers in Artificial Intelligence*, vol. 4, pp. 1–28, 2021.

[2] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[3] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[4] K. T. Atanassov, "Circular intuitionistic fuzzy sets," *Journal of Intelligent and Fuzzy Systems*, vol. 39, no. 5, pp. 5981–5986, 2020.

[5] R. R. Yager, "Pythagorean fuzzy subsets," *Proceedings of the Joint IFSA World Congress and NAFIPS Annual Meeting*, pp. 57–61, 2013.

[6] R. R. Yager, "Pythagorean membership grades in multi-criteria decision making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958–965, 2014.

[7] R. R. Yager, "Generalized orthopair fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2017.

[8] D. Molodtsov, "Soft set theory-first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[9] W. R. Zhang, "Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis," in *Proceedings of the NAFIPS/IFIS/NASA94. Proceedings of the 1st International Joint Conference of the North American Fuzzy Information Processing Society Biannual Conference. The Industrial Fuzzy Control and Intellige*, pp. 305–309, San Antonio, TX, USA, December 1994.

[10] W. R. Zhang, "Bipolar fuzzy sets," *Proceedings of IEEE Inrenational Conference on Fuzzy Systems*, pp. 835–840, 1998.

[11] F. Smarandache, *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, DE, USA, 1999.

[12] F. Smarandache, "Neutrosophic set- a generalization of the intuitionistic fuzzy set," *International Journal of Pure and Applied Mathematics*, vol. 24, no. 3, pp. 287–297, 2005.

[13] B. C. Cuong, "Picture fuzzy sets," *Journal of Computer Science and Cybernetics*, vol. 30, no. 4, pp. 409–420, 2014.

[14] F. Kutlu Gündoğdu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 1, pp. 337–352, 2019.

[15] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Computing & Applications*, vol. 31, no. 11, pp. 7041–7053, 2019.

[16] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, and T. Mahmood, "Spherical fuzzy sets and their applications in multi-attribute decision making problems," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 3, pp. 2829–2844, 2019.

[17] K. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[18] N. Cagman and S. Enginoglu, "Soft set theory and uni–int decision making," *European Journal of Operational Research*, vol. 207, no. 2, pp. 848–855, 2010.

[19] F. Karaaslan and N. Cagman, "Parameter trees based on soft set theory and their similarity measures," *Soft Computing*, vol. 26, no. 10, pp. 4629–4639, 2022.

[20] J. Chen, S. Li, S. Ma, and X. Wang, "m-m-Polar fuzzy sets: an extension of bipolar fuzzy sets," *The Scientific World Journal*, vol. 2014, Article ID 416530, 8 pages, 2014.

[21] Y. B. Jun, C. S. Kim, and K. O. Yang, "Cubic sets," *Annals of Fuzzy Mathematics and Informatics*, vol. 4, no. 1, pp. 83–98, 2012.

[22] M. Riaz and M. R. Hashmi, "MAGDM for agribusiness in the environment of various cubic m-polar fuzzy averaging aggregation operators," *Journal of Intelligent and Fuzzy Systems*, vol. 37, no. 3, pp. 3671–3691, 2019.

[23] H. Garg and G. Kaur, "Extended TOPSIS method for multi-criteria group decision-making problems under cubic intuitionistic fuzzy environment," *Scientia Iranica*, vol. 27, pp. 1–18, 2018.

[24] H. Garg and G. Kaur, "Novel distance measures for cubic intuitionistic fuzzy sets and their applications to pattern recognitions and medical diagnosis," *Granular Computing*, vol. 5, pp. 1–16, 2018.

[25] B. J. Young, S. Seok-Zun, and K. J. Seon, "Cubic interval-valued intuitionistic fuzzy sets and their application in BCK/BCI-Algebras," *Axioms*, vol. 7, pp. 1–17, 2018.

[26] T. Senapati, R. R. Yager, and G. Chen, "Cubic intuitionistic WASPAS technique and its application in multi.criteria decision.making," *Journal of Ambient Intelligence and Humanized Computing*, vol. 12, no. 9, pp. 8823–8833, 2021.

[27] G. Kaur and H. Garg, "Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment," *Entropy*, vol. 20, pp. 65–16, 2018.

[28] H. Garg and G. Kaur, "TOPSIS based on nonlinear-programming methodology for solving decision-making problems under cubic intuitionistic fuzzy set environment," *Computational and Applied Mathematics*, vol. 38, no. 3, pp. 114–119, 2019.

[29] C. L. Chang, "Fuzzy topological spaces," *Journal of Mathematical Analysis and Applications*, vol. 24, no. 1, pp. 182–190, 1968.

[30] D. Coker, "An introduction to intuitionistic fuzzy topological spaces," *Fuzzy Sets and Systems*, vol. 88, no. 1, pp. 81–89, 1997.

[31] M. Olgun, M. Ünver, and S. Yardimci, "Pythagorean fuzzy topological spaces," *Complex & Intelligent Systems*, vol. 5, no. 2, pp. 177–183, 2019.

[32] N. Cagman, S. Karatas, and S. Enginoglu, "Soft topology," *Computers & Mathematics with Applications*, vol. 62, no. 1, pp. 351–358, 2011.

[33] M. Riaz, K. Akmal, Y. Almalki, and S. A. Alblowi, "Cubic m-polar fuzzy topology with multi-criteria group decision-making," *AIMS Mathematics*, vol. 7, no. 7, pp. 13019–13052, 2022.

[34] Z. S. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *International Journal of General Systems*, vol. 35, no. 4, pp. 417–433, 2006.

[35] Z. S. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1179–1187, 2007.

[36] X. L. Zhang and Z. S. Xu, "Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets," *International Journal of Intelligent Systems*, vol. 29, no. 12, pp. 1061–1078, 2014.

[37] F. Feng, Y. Zheng, B. Sun, and M. Akram, "Novel score functions of generalized orthopair fuzzy membership grades with application to multiple attribute decision making," *Granular Computing*, vol. 7, no. 1, pp. 95–111.

[38] M. Akram, "Bipolar fuzzy graphs," *Information Sciences*, vol. 181, no. 24, pp. 5548–5564, 2011.

[39] M. Akram, A. Khan, J. C. R. Alcantud, and G. Santos-Garcia, “A hybrid decision-making framework under complex spherical fuzzy prioritized weighted aggregation operators,” *Expert Systems*, vol. 38, no. 6, pp. 1–24.

[40] M. A. Alghamdi, N. O. Alshehri, and M. Akram, “Multi-criteria decision-making methods in bipolar fuzzy environment,” *International Journal of Fuzzy Systems*, vol. 20, no. 6, pp. 2057–2064, 2018.

[41] P. Liu and P. Wang, “Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making,” *International Journal of Intelligent Systems*, vol. 33, no. 2, pp. 259–280, 2018.

[42] J. Ye, “Similarity measures based on the generalized distance of neutrosophic Z-number sets and their multi-attribute decision making method,” *Soft Computing*, vol. 25, no. 22, pp. 13975–13985.

[43] T. Senapati and R. R. Yager, “Some new operations over f fuzzy numbers and application of f fuzzy WPM in multiple criteria decision making,” *Informatica*, vol. 30, no. 2, pp. 391–412, 2019.

[44] C. Kahraman and N. Alkan, “Circular intuitionistic fuzzy TOPSIS method with vague membership functions: supplier selection application context,” *Notes on Intuitionistic Fuzzy Sets*, vol. 27, no. 1, pp. 24–52, 2021.

[45] S. Sinha and S. P. Sarmah, “An application of fuzzy set theory for supply chain coordination,” *International Journal of Management Science and Engineering Management*, vol. 3, no. 1, pp. 19–32, 2008.

[46] M. T. Alshurideh, B. Al Kurdi, H. M. Alzoubi et al., “Fuzzy assisted human resource management for supply chain management issues,” *Annals of Operations Research*, pp. 1–19, 2021.

[47] M. R. Seikh, S. Dutta, and D. F. Li, “Solution of matrix games with rough interval pay-offs and its application in the telecom market share problem,” *International Journal of Intelligent Systems*, vol. 36, no. 10, pp. 6066–6100, 2021.

[48] M. R. Seikh, S. Karmakar, and O. Castillo, “A novel defuzzification approach of type-2 fuzzy variable to solving matrix games: an application to plastic ban problem,” *Iranian Journal of Fuzzy Systems*, vol. 18, no. 5, pp. 155–172, 2021.

[49] S. Ruidas, M. R. Seikh, and P. K. Nayak, “An EPQ model with stock and selling price dependent demand and variable production rate in interval environment,” *International Journal of System Assurance Engineering and Management*, vol. 11, no. 2, pp. 385–399, 2020.

[50] S. Ruidas, M. R. Seikh, P. K. Nayak, and B. Sarkar, “A single period production inventory model in interval environment with price revision,” *International Journal of Algorithms, Computing and Mathematics*, vol. 5, no. 1, pp. 7–20, 2019.

[51] S. Ruidas, M. R. Seikh, and P. K. Nayak, “A production-repairing inventory model considering demand and the proportion of defective items as rough intervals,” *Operational Research*, vol. 22, no. 3, pp. 2803–2829.

[52] M. R. Seikh and U. Mandal, “Q-rung orthopair fuzzy Frank aggregation operators and its application in multiple attribute decision-making with unknown attribute weights,” *Granular Computing*, vol. 7, no. 3, pp. 709–730, 2022.

[53] M. R. Seikh and U. Mandal, “Multiple attribute decision-making based on 3, 4-quasirung fuzzy sets,” *Granular Computing*, pp. 1–14, 2022.

[54] M. Riaz and H. M. A. Farid, “Picture fuzzy aggregation approach with application to third-party logistic provider selection process,” *Reports in Mechanical Engineering*, vol. 3, no. 1, pp. 318–327, 2022.

[55] A. Ashraf, K. Ullah, A. Hussain, and M. Bari, “Interval-valued picture fuzzy Maclaurin symmetric mean operator with application in multiple attribute decision-making,” *Reports in Mechanical Engineering*, vol. 3, no. 1, pp. 301–317, 2022.

[56] M. M. U. Baig, Y. Ali, and O. U. Rehman, “Enhancing resilience of oil supply chains in context of developing countries,” *Operational Research in Engineering Sciences: Theory and Applications*, vol. 5, no. 1, pp. 69–89, 2022.

[57] R. Chattopadhyay, P. P. Das, and S. Chakraborty, “Development of a rough-MABAC-DoE-based Metamodel for supplier selection in an iron and steel industry,” *Operational Research in Engineering Sciences: Theory and Applications*, vol. 5, no. 1, pp. 20–40, 2022.

[58] C. Karamasa, E. Demir, S. Memis, and S. Korucuk, “Weighting the factors affecting logistics outsourcing,” *Decision Making: Applications in Management and Engineering*, vol. 4, no. 1, pp. 19–33, 2021.

[59] B. Bairagi, “A novel MCDM model for warehouse location selection in supply chain management,” *Decision Making: Applications in Management and Engineering*, vol. 5, no. 1, pp. 194–207, 2022.

[60] M. Riaz, Y. Almalki, S. Batool, and S. Tanveer, “Topological structure of single-valued neutrosophic hesitant fuzzy sets and data analysis for uncertain supply chains,” *Symmetry*, vol. 14, no. 7, pp. 1382–1424, 2022.

[61] M. Riaz, D. Pamucar, A. Habib, and N. Jamil, “Innovative bipolar fuzzy sine trigonometric aggregation operators and SIR method for medical tourism supply chain,” *Mathematical Problems in Engineering*, vol. 2022, Article ID 4182740, 17 pages, 2022.

[62] M. Riaz, D. Pamucar, A. Habib, and M. Riaz, “A new TOPSIS approach using cosine similarity measures and cubic bipolar fuzzy information for sustainable plastic recycling process,” *Mathematical Problems in Engineering*, vol. 2021, Article ID 4309544, 18 pages, 2021.

[63] M. Ali, I. Deli, and F. Smarandache, “The theory of neutrosophic cubic sets and their applications in pattern recognition,” *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 4, pp. 1957–1963, 2016.