Cubic Intuitionistic Fuzzy Topology with Application to Uncertain Supply Chain Management

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1. Introduction

Topological data analysis (TDA) methods are rapidly growing approaches to inferring persistent key features from possibly complex data [1]. We deal with complex issues in our daily lives due to vague and uncertain information, and if we do not use the proper modeling techniques for them, we eventually wind up with vague and unclear reasoning. For this reason, making rational and logical conclusions in the face of such imprecise and inexplicit facts is a difficult task for decision-makers. As a result, dealing with vagueness and uncertainty is a necessary part of dealing with such challenges and difficulties. Zadeh [2] initiated the notion of fuzzy set (FS) theory, which is an instantaneous extension of a crisp set. Various sets of theories and models have been developed by researchers to manage the complexity of daily life problems that include vague and uncertain information. Atanassov [3] presented the idea of an intuitionistic fuzzy set (IFS), and Atanassov [4] further initiated the notion of circular intuitionistic fuzzy sets. Yager [5, 6] introduced the concept of a Pythagorean fuzzy set (PFS), and further Yager [7] developed the notion of a q-rung orthopair fuzzy set (q-ROFS). Molodtsov [8] was the first who proposed the idea of a soft set (SS), and Zhang [9, 10] originally introduced the notion of a bipolar fuzzy set (BFS) to address bipolarity and bipolar information. Smarandache [11, 12] initiated the concept of a neutrosophic set (NS). Cuong [13] introduced the idea of a picture fuzzy set (PiFS). Gundogdu and Kahraman [14], Mahmood et al. [15], and Ashraf et al. [16] independently introduced the notion of a spherical fuzzy set (SFS). These models have a strong foothold when it comes to modeling uncertainty in a real-life complex challenges. Atanassov and Gargov [17] introduced interval-valued intuitionistic fuzzy sets. Cagman and Enginoğlu [18] proposed decision-making applications based on soft-set theory. Karaaslan and Cagman [19] introduce the parameter trees based on soft set theory and their similarity measures. Chen [20] proposed m-polar fuzzy sets (mPFS) with m membership values to address the multipolarity of objects. Jun et al. [21] developed the cubic set (CS) and its internal and external environment. But CS has some limitations, as it does not convert membership degree grades into nonmembership grades. Riaz and Hashmi [22] proposed cubic m-polar fuzzy sets and cubic m-polar fuzzy averaging...
aggregation operators for MAGDM. So, for this, Kaur and Garg [23, 24] presented the concept of a cubic intuitionistic fuzzy set by combining the concepts of IFSs, CIFSs, and IVIFs. So, CIFSs, rather than IFSs or IVIFs, is a handy technique to address information more precisely throughout the DMP. Young et al. [25] proposed cubic interval-valued intuitionistic fuzzy sets. Senapati et al. [26] introduced a cubic intuitionistic WASPAS technique. Garg and Kaur [27] suggested cubic intuitionistic fuzzy Bonferroni mean operators. Garg and Kaur [28] proposed cubic intuitionistic fuzzy TOPSIS for nonlinear programming.


Seikh et al. [47, 48] proposed the solution of matrix games with rough interval pay-offs and a defuzzification approach of type-2 fuzzy variables to solving matrix games. They developed applications of matrix games to the telecom market share problem and the plastic ban problem. Ruidas et al. [49] developed an EPQ model with stock and selling price-dependent demand and a variable production rate in an interval environment. Ruidas et al. [50] suggested an interval environment with price revision using a single-period production inventory model. Ruidas et al. [51] introduced a production-repairing inventory model considering demand and the proportion of defective items as rough intervals. Seikh and Mandal [52] proposed q-rung orthopair fuzzy Frank aggregation operators and their application in MADM with unknown attribute weights. Seikh and Mandal [53] introduced the MADM method based on 3, 4-quasirung fuzzy sets. Riaz and Farid [54] proposed the picture fuzzy aggregation approach and application to third-party logistic provider selection. Ashraf et al. [55] introduced the Maclaurin symmetric mean operator with an interval-valued picture fuzzy model. Baig et al. [56] developed new methods for enhancing resilience in developing countries for oil supply chains. Chattopadhyay et al. [57] proposed the idea of the development of a rough-MABAC-DoE-based metamodel for iron and steel supplier selection. Karamasa et al. [58] studied weighting the factors affecting logistics outsourcing. Bairagi [59] developed a novel MCDM model for warehouse location selection in supply chain management. Recently, some applications of fuzzy modeling have been developed, such as uncertain supply chains [60], medical tourism supply chains [61], sustainable plastic recycling processes [62], and pattern recognition [63].

Multicriteria group decision-making (MCGDM) is a branch of operation research in which the alternatives are evaluated by the group of decision-makers (DMs) under multiple criteria to find a ranking of alternatives and an optimal decision. It is an important aspect of MCGDM to evaluate alternatives based on their characteristics. It is extremely difficult for an individual to choose an option in a variety of situations due to inconsistencies in the data caused by human errors or a lack of knowledge. Dealing with vagueness and uncertainties in MCGDM problems is very crucial to dealing with daily life problems. For this purpose, a variety of strategies have been utilized to evaluate the stability of human decision-making by weighing a set of options against a set of criteria. The weighted product model and choice value method are well-known methods and are often utilized to rank the alternatives according to certain criteria.

The main objectives of this research work are given as follows:

1. To develop a topological structure on cubic intuitionistic fuzzy sets (CIFSs) with P-order (P-CIFT) as well as R-order (R-CIFT) and to validate some significant results and fundamental characteristics with examples. The concept of the CIFS is a strong hybrid model for modeling uncertainties with an IFS and an interval-valued IFS, simultaneously.

2. To examine various properties of the cubic intuitionistic fuzzy topology (CIFT) under P-order (R-order), such as open sets of CIFT, closed sets of CIFT, interior in CIFT, closure in CIFT, subspace of CIFT, exterior in CIFT, a frontier in CIFT, and a basis of CIFT.

3. Taking advantage of topological data analysis with cubic intuitionistic fuzzy information, we proposed two multicriteria group decision-making (MCGDM) methods.

4. To develop Algorithm 1 for a weighted product model (WPM) and Algorithm 2 for a choice value method (CVM). An application of the proposed techniques is also designed for the uncertain supply chain management.
(5) ranking of feasible alternatives is computed, and a comparative analysis of proposed methods with existing methods is also given to discuss the validity and advantage of the proposed techniques.

The remaining sections of this paper are organized as follows. In Section 2, we reviewed some fundamental concepts such as IFS, IVIFS, CIFS, operations on CIFSs, and some essential results on CIFSs. The idea of cubic intuitionistic fuzzy set topology with P-order is introduced in Section 3. We also investigated some key results on CIFSs in P-order. In Section 4, we discuss the major results of cubic intuitionistic fuzzy set topology with R-order. In Section 5, we discuss a useful application that employs the weighted product model and choice value method. The conclusion of the paper is given in Section 6.

2. Preliminaries

In this section, we study some basic concepts of IFSs, IVIFSs, CIFSs, and CIFSs. We also review some fundamental properties of CIFSs that are necessary to understand the topological structures of CIFSs.

Definition 1 (see [3]). An intuitionistic fuzzy set (IFS) in a set $k$ is described as

$$ I = \{ (\ell, \xi(\ell), \eta(\ell)) : 0 \leq \xi(\ell) + \eta(\ell) \leq 1, \ell \in k \}, \tag{1} $$

where, $\xi : k \rightarrow [0, 1]$ represents the membership function, and the nonmembership function is denoted by $\eta : k \rightarrow [0, 1]$.

Definition 2 (see [34, 35]). Let $l_1 = (\xi_1, \eta_1)$ and $l_2 = (\xi_2, \eta_2)$ be two IFSs. Then, we have the following operations on IFSs.

(i) $l_1 \leq l_2$ if $\xi_1 \leq \xi_2$ and $\eta_1 \geq \eta_2$ for all $\ell$ in $k$
(ii) $l_1 = l_2$ if $l_1 \leq l_2$ and $l_2 \leq l_1$
(iii) $l_1 + l_2 = \{ (\ell, \xi(\ell), \eta(\ell)) : \ell \in k \}$
(iv) $l_1 \| l_2 = \{ (\ell, [\xi_1 \wedge \xi_2, \vee \eta_1, \eta_2]) : \ell \in k \}$
(v) $l_1 \cap l_2 = \{ (\ell, \wedge \{ \xi_1, \xi_2 \}, \vee \{ \eta_1, \eta_2 \}) : \ell \in k \}$

In reality, it is difficult to determine the exact membership and nonmembership degrees of an element in a set. In this situation, a range of values may be a better measure to accommodate the uncertainty. For this, Atanassov and Gargov [17] introduce the idea of an interval-valued intuitionistic fuzzy number (IVIFN).

Definition 3 (see [17]). Let $k$ be a nonempty universal set. An interval-valued intuitionistic fuzzy set (IVIFS) on $k$ is defined as

$$ I = \{ (\ell, [\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)]) : \ell \in k \}, \tag{2} $$

where, $[\xi^L(\ell), \xi^U(\ell)]$ and $[\eta^L(\ell), \eta^U(\ell)]$ are the closed subintervals of $[0, 1]$ for every $\ell \in k$. For simplicity, the pair $I = ([\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ is called interval-valued intuitionistic fuzzy number (IVIFN).

By fusing the concept of IFS and IVIFS, Jun et al. [21] defined the cubic intuitionistic set as follows:

Definition 4 (see [21]). A cubic set $C$ on a universal set $k$ is expressed as

$$ C = \{ (\ell, C(\ell), \sigma(\ell)) : \ell \in k \}, \tag{3} $$

in which $C(\ell)$ is interval-valued fuzzy set and $\sigma(\ell)$ is fuzzy set on $k$. For use of ease, this pair is referred as $C = (C, \sigma)$

Definition 5 (see [21]). For any cubic fuzzy sets $C_i = (C_i, \sigma_i)$, $i \in \Lambda$, we have

(i) P-union $\bigcup_{\ell}^C = \{ \langle \ell, C_{i\ell}, \sigma_{i\ell} \rangle \}
(ii) P-intersection $\bigcap_{\ell}^C = \{ \langle \ell, C_{\ell}, \sigma_{\ell} \rangle \}
(iii) R-intersection $\bigcup_{\ell}^C = \{ \langle \ell, C_{i\ell}, \sigma_{i\ell} \rangle \}
(iv) R-intersection $\bigcap_{\ell}^C = \{ \langle \ell, C_{\ell}, \sigma_{\ell} \rangle \}

Definition 6 (see [23, 24]). Let $k$ be a universal set of discourse. A cubic intuitionistic fuzzy set (CIFS) on universal set $k$ is described as

$$ C_i = \{ (\ell, [\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)], (\zeta, \eta)) : \ell \in k \}, \tag{4} $$

in which $([\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ is an IVIFS and $(\zeta, \eta)$ is an IFS in $k$. For ease of use, we denote these pairs as $C_i = (C_{i\ell}, \sigma_{i\ell})$, where $C_{i\ell} = (C_i^L, C_i^U)$, $\sigma_{i\ell} = (\eta_i, \eta_{i\ell})$ and $\sigma = (\zeta, \eta)$ is known as cubic intuitionistic fuzzy number (CIFN) with the condition that $[\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)] \subseteq [0, 1]$, $\zeta, \eta \in [0, 1]$ and $\zeta + \eta \leq 1$.

That is why the CIFS has the advantage of being capable to contain a lot more data to represent both the IVIFN and the IFN at the same time.

2.1. Operations on CIFSs. Now we review some fundamental operations on CIFSs, which have been explored in [23, 24].

Definition 7. The complement of the CIFS $C_i = (C_{i\ell}, \sigma_{i\ell})$ is defined as $C_i^c = (C_{i\ell}^c, \sigma^c_{i\ell})$ where $C_{i\ell}^c = ([\eta^L(\ell), \eta^U(\ell)], [\xi^L(\ell), \xi^U(\ell)])$ be the complement of the IVIFS, $C_{i\ell}^c = ([\xi^L(\ell), \xi^U(\ell)], [\eta^L(\ell), \eta^U(\ell)])$ and $\sigma^c_{i\ell} = (\zeta, \eta)$ be the complement of the IFS, $\sigma = (\zeta, \eta)$. Thus, the complement of CIFN is expressed as

$$ C_i^c = \{ (\ell, [\eta^L(\ell), \eta^U(\ell)], [\xi^L(\ell), \xi^U(\ell)], (\eta, \zeta)) : \ell \in k \}. \tag{5} $$

Definition 8. Consider two CIFSs on a universal set $k$ is given as follows:

$$ C_i^1 = \{ (\ell, [\xi_1^L(\ell), \xi_1^U(\ell)], [\eta_1^L(\ell), \eta_1^U(\ell)], (\zeta_1, \eta_1)) : \ell \in k \} \tag{6} $$

and

$$ C_i^2 = \{ (\ell, [\xi_2^L(\ell), \xi_2^U(\ell)], [\eta_2^L(\ell), \eta_2^U(\ell)], (\zeta_2, \eta_2)) : \ell \in k \} \tag{7} $$

we define
Step 1. Obtain the decision matrix from the decision-makers, which indicates the alternative’s $X_i$, ($j = 1 \ldots m$) evaluation values on the basis of criterion $C_j$, ($i = 1 \ldots n$) by $\mathbf{T}_j = (C_{ji}, \sigma_{ji})$, where $C_{ji} = ([c^L_{ji}, c^U_{ji}], [\eta^L_{ji}, \eta^U_{ji}])$ an IVIFN and $\sigma_{ji} = (\xi_{ji}, \eta_{ji})$ is known as a cubic intuitionistic fuzzy number. The decision-makers provide the decision matrix $M = (\mathbf{T}_j)_{mn}$ of the form.

\[
\begin{bmatrix}
X_1 & C_1 & \cdots & C_n \\
X_2 & C_2 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
X_m & C_{m1} & \cdots & C_{mn} \\
\end{bmatrix}
\]

Step 2. Then, the decision matrix $M = (\mathbf{T}_j)_{mn}$ is made normalized by a linear approach. Assume the criteria are categorized into benefit criteria $B$ and cost criteria $K$. The normalization of every $i \in B$ is defined as $\mathbf{T}_i = \mathbf{T}_i/\max \mathbf{T}_i$, where $\max \mathbf{T}_i$ is defined as

\[
\max \mathbf{T}_i = \{(\max c^L_{ji}, \max c^U_{ji}), (\min \eta^L_{ji}, \max \eta^U_{ji})\}, \quad (\min \eta^L_{ji}, \max \eta^U_{ji}), (\max c^L_{ji}, \min c^U_{ji})
\]

Similarly, the normalization of every $i \in K$ is defined as $\mathbf{T}_i = \min \mathbf{T}_i/\max \mathbf{T}_i$, where $\min \mathbf{T}_i$ is defined as

\[
\min \mathbf{T}_i = \{(\min c^L_{ji}, \min c^U_{ji}), (\max \eta^L_{ji}, \max \eta^U_{ji})\}, \quad (\max c^L_{ji}, \min c^U_{ji}), (\min \eta^L_{ji}, \max \eta^U_{ji})
\]

The decision matrix $M = (\mathbf{T}_j)_{mn}$ is then transformed into normalized decision matrix $\mathbf{M} = (\mathbf{T}_j)_{mn}$ and is given as

\[
\begin{bmatrix}
X_1 & C_1 & \cdots & C_n \\
X_2 & C_2 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
X_m & C_{m1} & \cdots & C_{mn} \\
\end{bmatrix}
\]

Step 3. According to CIFS-WPM, the relative importance of $j$ alternatives is denoted as $\gamma_j$ and is defined as $\gamma_j = \prod \mathbf{T}_j$. Here, we use the operation of power rule of CIFS and also the product operation of CIFS.

Step 4. Find the score function of all values $\gamma_j$.

Step 5. Ranking of alternatives according to the score functions of $\gamma_j$.

**Algorithm 1:** Weighted product model (WPM).

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Step 1. Obtain the decision matrix from the decision-makers, with alternative’s $X_i$ evaluate on the basis of criterion $C_i$ by $\mathbf{T}_i = (C_{ji}, \sigma_{ji})$, where $C_{ji} = ([c^L_{ji}, c^U_{ji}], [\eta^L_{ji}, \eta^U_{ji}])$ an IVIFN and $\sigma_{ji} = (\xi_{ji}, \eta_{ji})$ is known as cubic intuitionistic fuzzy number. The decision-makers provide the decision matrix $M = (\mathbf{T}_j)_{mn}$ of the form.

\[
\begin{bmatrix}
X_1 & C_1 & \cdots & C_n \\
X_2 & C_2 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
X_m & C_{m1} & \cdots & C_{mn} \\
\end{bmatrix}
\]

Step 2. The decision-makers also give weightage to the criteria, with the condition that the sum of the weights must be equal to unity. We compute the multiplication of the decision matrix with criteria weights.

\[
\begin{bmatrix}
(c_{11}, \sigma_{11}) & (c_{12}, \sigma_{12}) & \cdots & (c_{1m}, \sigma_{1m}) \\
(c_{21}, \sigma_{21}) & (c_{22}, \sigma_{22}) & \cdots & (c_{2m}, \sigma_{2m}) \\
\vdots & \vdots & \ddots & \vdots \\
(c_{m1}, \sigma_{m1}) & (c_{m2}, \sigma_{m2}) & \cdots & (c_{mn}, \sigma_{mn}) \\
\end{bmatrix} \times \begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_n \\
\end{bmatrix}
\]

Step 3. We find the score function of each value.
Step 4. Compute the ranking of the alternatives according to their score function values.

**Algorithm 2:** Choice value method (CVM).

(i) (P-order) $C_j^1 \subseteq p C_j^2$ if $[c^L_{j1}, c^U_{j1}] \subseteq [c^L_{j2}, c^U_{j2}]$, $[\eta^L_{j1}, \eta^U_{j1}] \supseteq [\eta^L_{j2}, \eta^U_{j2}]$, $\xi_{j1} \leq \xi_{j2}$ and $\eta_{j1} \geq \eta_{j2}$

(ii) (R-order) $C_j^1 \subseteq R C_j^2$ if $[c^L_{j1}, c^U_{j1}] \subseteq [c^L_{j2}, c^U_{j2}]$, $[\eta^L_{j1}, \eta^U_{j1}] \supseteq [\eta^L_{j2}, \eta^U_{j2}]$, $\xi_{j1} \geq \xi_{j2}$ and $\eta_{j1} \leq \eta_{j2}$
Definition 9. For any CIFSs
\[ C_i^j = \left( \left\{ \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right\} : \ell \in k \right) \]  
the operations listed have been defined as follows:
(i) (P-union) \( \cup_{\ell} C_i^j = \left\{ \left( \left( \bigvee_{r \in A} \zeta_i^L, \bigwedge_{r \in A} \eta_i^U \right), \left( \bigwedge_{r \in A} \zeta_i, \bigvee_{r \in A} \eta_i \right) \right) \right\} \)
(ii) (P-intersection) \( \cap_{\ell} C_i^j = \left\{ \left( \left( \bigwedge_{r \in A} \zeta_i^L, \bigvee_{r \in A} \eta_i^U \right), \left( \bigvee_{r \in A} \zeta_i, \bigwedge_{r \in A} \eta_i \right) \right) \right\} \)
(iii) (Equality) \( \setminus_{\ell} (\zeta_i, \eta_i) \)
(iv) (R-intersection) \( \cap_{\ell} C_i^j = \left\{ \left( \left( \bigwedge_{r \in A} \zeta_i^L, \bigvee_{r \in A} \eta_i^U \right), \left( \bigvee_{r \in A} \zeta_i, \bigwedge_{r \in A} \eta_i \right) \right) \right\} \)

2.2. Some Results on CIFSs. Now we review some essential results and properties of CIFSs, which have been explored in [23, 24].

Definition 10. A CIFS
\[ C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]  
for which \( (\left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U]) = ([0, 0], [1, 1]) \) and \( (\zeta_i, \eta_i) = (1, 0) \) for all \( \ell \in k \) is denoted by \( C_1 \)
\( i = 1, 2 \)

Definition 11. A CIFS
\[ C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]  
for which \( (\left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U]) = ([1, 1], [0, 0]) \) and \( (\zeta_i, \eta_i) = (0, 1) \) for all \( \ell \in k \) is denoted by \( C_1 \)
\( i = 1, 2 \)

Definition 12. A CIFS
\[ C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]  
for which \( (\left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U]) = ([1, 1], [0, 0]) \) and \( (\zeta_i, \eta_i) = (0, 1) \) for all \( \ell \in k \) is denoted by \( C_1 \)
\( i = 1, 2 \)

Definition 13. A CIFS
\[ C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]  
for which \( (\left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U]) = ([1, 1], [0, 0]) \) and \( (\zeta_i, \eta_i) = (1, 0) \) for all \( \ell \in k \) is denoted by \( C_1 \)
\( i = 1, 2 \)

Definition 14. Let \( C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \) be a CIFN. The score function \( S(C_i) \) and the accuracy function \( A(C_i) \) for CIFNs are defined as
\[ S(C_i) = \frac{\zeta^U + \zeta^L - \eta^U - \eta^L}{2} + \zeta - \eta \]  
and
\[ A(C_i) = \frac{\zeta^U + \zeta^L + \eta^L + \eta^U}{2} + \zeta + \eta \]

The ranking of CINFs in relation to the proposed scoring function and accuracy function is determined as
(i) \( C_i < C_j \) if \( S(C_i) < S(C_j) \)
(ii) If \( S(C_i) = S(C_j) \), then \( C_i = C_j \) if \( A(C_i) < A(C_j) \)
(iii) If \( S(C_i) = S(C_j) \) and \( A(C_i) = A(C_j) \), then \( C_i = C_j \)

Definition 15. Let \( C_1 = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \) and
\[ C_j = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]  
be the CINFs and let \( p > 0 \) be any real number. The basic operations on CIFSs are given as
(i) \( C_i \cap C_j = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \)
(ii) \( C_i \cup C_j = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \)
(iii) \( pC_i = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \)
(iv) \( (C_i)^\ell = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \)

Definition 16. Let \( C_i = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \) be the CINFs. Then, the division operator on CINF is given as
\[ \frac{C_i}{C_j} = \left\{ \left( \ell, \left[ \zeta_i^L, \zeta_i^U \right], [\eta_i^L, \eta_i^U], (\zeta_i, \eta_i) \right) : \ell \in k \right\} \]
\[ (\max \zeta_i, \min \eta_i) \]

3. Cubic Intuitionistic Topology under P-Order
In this section, we introduce the concept of a P-cubic intuitionistic fuzzy topology (P-CIFT) or a cubic intuitionistic fuzzy topology with P-order.

Definition 17. Consider \( k \) to be a nonempty universal set, and let \( c(k) \) to be the accumulation of all CIFSs in \( k \). If the collection \( \mathbb{T}_{C_{IP}} \) containing CIFSs satisfies the following
are PCIFOSs. From the definition of P-CIFT,

\[ 0_C^1, 1_C^1, \overline{C_1}^1 \text{ and } \overline{C_1}^1 \in T_{C_1}. \]

(2) If \((C_{ip})_i \in T_{C_i} \forall i \in \Lambda \text{ then } \cup_p (C_{ip})_i \in T_{C_p}.\]

(3) If \(C_{ip}, C_{ip}' \in T_{C_p}\) then \(C_{ip} \cap_p C_{ip}' \in T_{C_p}.\)

Then, the pair \((k, T_{C_p})\) is called cubic intuitionistic fuzzy topological space with a P-order (P-CIFT).

**Example 1.** Let \(k\) be a universal set. Then, \(c\{k\}\) be the assemblage of all P-cubic intuitionistic fuzzy sets PCIFCSs in \(k\). Consider P-order fuzzy subsets of \(c\{k\}\) given as

\[
\begin{align*}
C_{ip}^1 & = \{[0.20, 0.31], [0.41, 0.52], (0.32, 0.44)\}, \\
C_{ip}^2 & = \{[0.20, 0.31], [0.41, 0.52], (1, 0)\}, \\
C_{ip}^3 & = \{[0.20, 0.31], [0.41, 0.52], (0, 1)\}, \\
C_{ip}^4 & = \{[1, 1], [0, 0], (0.32, 0.44)\}, \\
C_{ip}^5 & = \{[0, 0], [1, 1], (0.32, 0.44)\}.
\end{align*}
\]

The union and intersection with a P-order for the above CIFs are given in Tables 1 and 2, respectively. Clearly,

\[
T_{C_{ip}} = \{0_C^1, 1_C^1, \overline{C_1}^1, 1_C^1\},
\]

and

\[
T_{C_{ip}}^2 = \{0_C^1, 1_C^1, \overline{C_1}^1, 1_C^1, C_{ip}^1, C_{ip}^2, C_{ip}^3, C_{ip}^4, C_{ip}^5\},
\]

are cubic intuitionistic topology with a P-order.

**Definition 18.** Let \(k\) be a nonempty set and \(T_{C_p} = \{C_{ip}\}_k\) where \(C_{ip}\) represent the cubic intuitionistic fuzzy subsets of universal set \(k\). Then, \(T_{C_p}\) is termed as a P-cubic intuitionistic fuzzy topology on \(k\) and it is the largest P-cubic intuitionistic fuzzy topology on \(k\) and is entitled as P-discrete cubic intuitionistic fuzzy topology.

**Definition 19.** Let \(k\) be a universal set and \(T_{C_p} = \{0_C^1, 1_C^1, \overline{C_1}^1, 1_C^1\}\) be the assemblage of cubic intuitionistic fuzzy sets. Then, \(T_{C_p}\) is termed as a P-cubic intuitionistic fuzzy topology on universal set \(k\) and is the smallest P-cubic intuitionistic fuzzy topology on \(k\) and is entitled as P-indiscrete cubic intuitionistic fuzzy topology.

**Definition 20.** The elements of a P-cubic intuitionistic fuzzy topology \(T_{C_p}\) is termed as P-cubic intuitionistic fuzzy open sets PCIFOSs in \((k, T_{C_p})\).

**Theorem 1.** If \((k, T_{C_p})\) is any P-cubic intuitionistic fuzzy topological space. Then,

(1) \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \text{ are PCIFOSs.}\)

(2) The P-union of any number of PCIFOSs is PCIFOS.

(3) The P-intersection of finite PCIFOSs is PCIFOS.

**Proof**

(1) By the Definition 4.2 of a P-cubic intuitionistic fuzzy topology (P-CIFT), \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \in T_{C_p}.\)

Hence, \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \text{ are PCIFOSs.}\)

(2) Let \((C_{ip})_i \in \Lambda\) be PCIFOSs. Then, \((C_{ip})_i \in T_{C_p}.\)

From the definition of P-CIFT

\[
\bigcup_p (C_{ip})_i \in T_{C_p}.\]

Hence, \(\bigcup_p (C_{ip})_i \in T_{C_p}\).

(3) Let \(C_{ip}^i, C_{ip}^2, \ldots, C_{ip}^n\) be PCIFOSs. Then, \((C_{ip}^i)_i, (C_{ip}^2)_i, \ldots, (C_{ip}^n)_i\) are PCIFOSs. So,

\[
\bigcap_p (C_{ip})_i \in T_{C_p}.\]

Hence, \(\bigcap_p (C_{ip})_i \in T_{C_p}\).

**Theorem 2.** If \((k, T_{C_p})\) is any P-cubic intuitionistic fuzzy topological space. Then,

(1) \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \text{ are PCIFCSs.}\)

(2) The P-intersection of any number of PCIFCSs is PCIFCS.

(3) The P-union of finite PCIFCSs is PCIFCS.

**Proof**

(1) By the Definition 4.2 of a P-cubic intuitionistic fuzzy topology (P-CIFT), \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \in T_{C_p}.\) Hence, \(0_C^1, 1_C^1, \overline{C_1}^1\text{ and } \overline{C_1}^1 \text{ are PCIFCSs.}\)

(2) Let \((C_{ip})_i \in \Lambda\) be PCIFCSs. Then, \((C_{ip})_i \in T_{C_p}.\) From the definition of P-CIFT

\[
\bigcap_p (C_{ip})_i \in T_{C_p}.\]

Hence, \(\bigcap_p (C_{ip})_i \in T_{C_p}\).

(3) Let \(C_{ip}^1, C_{ip}^2, \ldots, C_{ip}^n\) be PCIFCSs. Then, \(C_{ip}^i, C_{ip}^2, \ldots, C_{ip}^n\) are PCIFCSs. So,
Two P-CIFTs are called comparable if
\[ \text{Definition 22.} \quad \text{The P-cubic intuitionistic fuzzy sets PCIFS, which are PCIFOSs and PCIFCSs, are entitled as P-cubic intuitionistic fuzzy clopen sets in } (k, \mathcal{T}_{CIP}). \]

**Proposition 1**

1. For every \( \mathcal{T}_{CIP} \), \( 0^n, 1^n, \overline{0}^n, \overline{1}^n \) and \( \overline{1}^n \) are P-cubic intuitionistic fuzzy clopen sets.
2. For discrete P-order cubic intuitionistic fuzzy topology, all the cubic intuitionistic subsets of \( k \) are P-cubic intuitionistic fuzzy clopen sets.
3. For in-discrete P-order cubic intuitionistic fuzzy topology, \( CIP^1, CIP^2, CIP^3 \) and \( \overline{CIP} \) are only P-cubic intuitionistic fuzzy clopen sets.

**Definition 23.** Let \( (k, \mathcal{T}_{CIP}^1) \) and \( (k, \mathcal{T}_{CIP}^2) \) be two P-CIFTs in \( k \). Two P-CIFTs are called comparable if

\[ \left( CIP^1 \right)^c, \left( CIP^2 \right)^c, \ldots, \left( CIP^3 \right)^c \in \mathcal{T}_{CIP}. \]  

From the definition of P-CIFT,
\[ \cap_p \left( \left( CIP^q \right)^c \right) \in \mathcal{T}_{CIP}. \]  

This gives \( \cap_p \left( \left( CIP^q \right)^c \right) \) is PCIFOSs, but
\[ \left( \cap_p \left( \left( CIP^q \right)^c \right) \right)^c = \left( \bigcup_p \left( CIP^q \right)^c \right). \]  

Hence, \( \cup_p \left( CIP^q \right)^c \) is PCIFCSs.

**Example 2.** Let \( k \) be a nonempty set and from Example 1
\[ \mathcal{T}_{CIP} = \{ 0^n, 1^n, \overline{1}^n, \overline{0}^n, \overline{1}^n \}, \]  

and
\[ \mathcal{T}_{CIP}^2 = \{ 0^n, 1^n, \overline{1}^n, \overline{0}^n, \overline{1}^n \}, \]  

are P-cubic intuitionistic fuzzy topologies on universal set. Then, \( \mathcal{T}_{CIP} \) is called a P-cubic intuitionistic fuzzy coarser than \( \mathcal{T}_{CIP}^2 \), and \( \mathcal{T}_{CIP}^2 \) is called a P-cubic intuitionistic fuzzy finer than \( \mathcal{T}_{CIP} \).

3.1. Subspace of CIFTp

**Definition 24.** Let \( (k, \mathcal{T}_{CIP}^k) \) be a CIFTp. Let \( Y \subseteq k \) and \( \mathcal{T}_{CIP}^k \) is a CIFTp on \( Y \) and whose PCIFOSs are
\[ CIP^k = \mathcal{T}_{CIP}^k \cap Y, \]  

where \( CIP^k \) are PCIFOSs of \( T_{CIP}^k \) and \( \mathcal{T}_{CIP}^Y \) are PCIFOSs of \( \mathcal{T}_{CIP}^k \) and \( Y \) is any P-cubic subset of PCIFS on \( Y \). Then, \( \mathcal{T}_{CIP}^k \) is called a P-cubic intuitionistic fuzzy subspace of \( \mathcal{T}_{CIP}^k \), i.e.,
Definition 25. Let \((k, \mathcal{T}_{CIP})\) be \(CIFT_{P}\) and \(CIP \in ci(k)\), the interior of \(CIP\) is expressed as \(CIP^0\) and is described as union of all \(P\)-cubic intuitionistic fuzzy open subsets contained in \(CIP\). It is the greatest \(P\)-cubic intuitionistic fuzzy open set contained in \(CIP\).

Example 4. Consider a \(P\)-cubic intuitionistic fuzzy topological space as constructed in Example 1. Let \(CIP^0 \in ci(k)\) given as

\[
CIP^0 = ([0.23, 0.39], [0.37, 0.48], (0.46, 0.33)).
\]

Then,

\[
(CIP^0)^0 = 9CIP \cup CIP^1 \cup CIP^2 \cup CIP^3 = CIP^1.
\]

Theorem 3. Let \((k, \mathcal{T}_{CIP})\) be \(CIFT_{P}\) and \(CIP \in ci(k)\). Then, \(CIP\) is open CIFS iff \(CIP^0 = CIP\).

Proof. If \(CIP\) is open CIFS, then we say that the greatest open CIFS contained in \(CIP\) is \(CIP\) itself. Thus,

\[
CIP^0 = CIP.
\]

Conversely, if \(CIP^0 = CIP\), then \(CIP^0\) is open CIFS. This implies \(CIP\) is open CIFS.

Theorem 4. Let \((k, \mathcal{T}_{CIP})\) be \(CIFT_{P}\) and \(CIP^0, CIP^1, CIP^2 \in ci(k)\). Then,

\[
(i) \quad ((CIP^0)^0)^0 = (CIP^0)^0
\]

\[
(ii) \quad CIP^0 \cap CIP \subseteq (CIP^0)^0 \cap CIP^0
\]

\[
(iii) \quad (CIP \cap CIP^0)^0 = (CIP)^0 \cap (CIP^0)^0
\]

\[
(iv) \quad (CIP^1 \cup CIP^2)^0 \subseteq CIP \cup CIP^0
\]

Proof. Proof is trivial.

Definition 26. Let \((k, \mathcal{T}_{CIP})\) be \(CIFT_{P}\) and \(CIP \in ci(k)\), the closure of \(CIP\) is expressed as \(\overline{CIP}\) and is described as the intersection of all the \(P\)-cubic intuitionistic fuzzy closed supersets of \(CIP\). It is the smallest \(P\)-cubic intuitionistic fuzzy closed superset of \(CIP\).

Example 5. Let us consider a \(P\)-cubic intuitionistic topological space as constructed in Example 1. Then, the closed CIFSs are given as

\[
\mathcal{T}_{CIP, Y} = \{CIP, \overline{CIP}, CIP, Y\}.\]
\( \left(^0 C_{ip} \right)^c = \{ [1, 1], [0, 0], (0, 1) \}, \)
\( \left(^1 C_{ip} \right)^c = \{ [0, 0], [1, 1], (1, 0) \}, \)
\( \left(\overline{C}_{ip} \right)^c = \{ [1, 1], [0, 0], (1, 0) \}, \)
\( \left(\overline{\overline{C}}_{ip} \right)^c = \{ [0, 0], [1, 1], (0, 1) \}, \)
\( \left(C_{ip}^1 \right)^c = \{ [0.41, 0.52], [0.20, 0.31], (0.44, 0.32) \}, \)
\( \left(C_{ip}^2 \right)^c = \{ [0.41, 0.52], [0.20, 0.31], (1, 0) \}, \)
\( \left(C_{ip}^3 \right)^c = \{ [0, 0], [1, 1], (0.44, 0.32) \}, \)
\( \left(C_{ip}^5 \right)^c = \{ [1, 1], [0, 0], (0.44, 0.32) \}. \)

Let \( C_{ip}^7 \in ci(k) \) given as
\[
C_{ip} = \{ [0.34, 0.50], [0.27, 0.38], (0.33, 0.41) \}.
\]

Then,
\[
\overline{C}_{ip} = \left(\overline{C}_{ip} \right)^c \cap \left(\overline{C}_{ip} \right)^c \cap \left(\overline{C}_{ip} \right)^c \cap \left(\overline{C}_{ip} \right)^c = \left(C_{ip}^5 \right)^c.
\]

**Theorem 5.** Let \( (k, T_{C_{ip}}) \) be CIFS and \( C_{ip} \in ci(k) \). Then \( \overline{C}_{ip} \) is closed CIFS iff \( \overline{C}_{ip} = C_{ip} \).

**Proof.** If \( C_{ip} \) is closed CIFS, then we can say that the smallest closed CIFS superset of \( C_{ip} \) is itself. Thus,
\[
\overline{C}_{ip} = C_{ip}.
\]

Conversely, if \( \overline{C}_{ip} = C_{ip} \), then \( C_{ip} \) is closed CIFS. This implies \( C_{ip} \) is closed CIFS. \( \Box \)

**Definition 27.** Let \( C_{ip} \) be a P-cubic intuitionistic fuzzy subset of \( (k, T_{C_{ip}}) \), then its boundary or frontier is defined as
\[
Fr(C_{ip}) = \overline{C}_{ip} \cap \left(\overline{\overline{C}}_{ip} \right)^c.
\]

**Definition 28.** Let \( C_{ip} \) be a P-cubic intuitionistic fuzzy subset of \( (k, T_{C_{ip}}) \), then the exterior is defined as
\[
Ext(C_{ip}) = \left(\overline{C}_{ip} \right)^c = \left(C_{ip}^5 \right)^c.
\]

**Example 6.** Consider a P-cubic intuitionistic topological space as constructed in Example 1 and \( C_{ip}^6 \) and \( C_{ip}^7 \) from Examples 4 and 5. Then,
\[
\left(C_{ip}^6 \right)^0 = C_{ip},
\]
\[
\overline{C}_{ip}^0 = \left(C_{ip}^3 \right)^c,
\]
\[
Fr(C_{ip}^6) = \left(C_{ip}^1 \right)^c,
\]
\[
Ext(C_{ip}^6) = C_{ip}^3,
\]
\[
\left(C_{ip}^7 \right)^0 = C_{ip},
\]
\[
\overline{C}_{ip}^0 = \left(C_{ip}^1 \right)^c,
\]
\[
Fr(C_{ip}^7) = \left(C_{ip}^1 \right)^c,
\]
\[
Ext(C_{ip}^7) = C_{ip}.
\]

**Theorem 6.** Let \( (k, T_{C_{ip}}) \) be CIFS and \( C_{ip} \in ci(k) \). Then,

1. \( \left(C_{ip}^6 \right)^c = \overline{C}_{ip} \)
2. \( \left(C_{ip}^7 \right)^c = \overline{C}_{ip} \)
3. \( Ext(C_{ip}^6) = C_{ip}^3 \)
4. \( Ext(C_{ip}^7) = C_{ip}^3 \)
5. \( Ext(C_{ip}) = C_{ip}^5 \)
6. \( Fr(C_{ip}) = Fr(C_{ip}) \)
7. \( Fr(C_{ip}) \cup _p C_{ip}^6 \neq C_{ip} \)

**Proof**

1. The proof is obvious.
2. The proof is obvious.
3. \( Ext(C_{ip}) = \left(\overline{C}_{ip} \right)^c \)
4. \( Ext(C_{ip}) = \left(\overline{C}_{ip} \right)^c \)
5. \( Ext(C_{ip}) \cup _p Fr(C_{ip}) \cup _p C_{ip}^6 \neq C_{ip} \)
6. \( Fr(C_{ip}) = Fr(C_{ip}) \)
7. \( Fr(C_{ip}) \cap _p C_{ip}^6 \neq C_{ip} \)

**3.3. P-Cubic Intuitionistic Fuzzy Basis**

**Definition 29.** Let \( (k, T_{C_{ip}}) \) be CIFS. Then, \( B \subseteq T_{C_{ip}} \) is called P-cubic intuitionistic fuzzy basis for \( T_{C_{ip}} \) if for every \( C_{ip} \in T_{C_{ip}} \), \( \exists B \subseteq B \) such that
\[
C_{ip} = \cup _p B.
\]
is a P-cubic intuitionistic fuzzy topology on $k$. Then,
\[
B = \left\{ 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}}, \overline{0^{C_1}}, \overline{1^{C_1}}, \overline{0^{C_1}}, \overline{1^{C_1}} \right\},
\]
(52)

is a P-cubic intuitionistic fuzzy basis for $\mathbb{T}_{C_{ip}}$.

4. Cubic Intuitionistic Topology under R-Order

In this section, we introduce the concept of an R-cubic intuitionistic fuzzy topology (R-CIFT) or a cubic intuitionistic fuzzy topology with an R-order.

**Definition 30.** Consider $k$ to be a nonempty universal set, and let $ci(k)$ to be the collection of all CIFSs in $k$. If the collection $\mathbb{T}_{C_{ir}}$ containing CIFSs satisfies the following conditions, it is termed a cubic intuitionistic fuzzy topology with an R-order (R-CIFT).

\[\begin{align*}
(1) & \quad 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \in C_{ir} \\
(2) & \quad \text{If } \{C_{ir}\}_i \in C_{ir} \forall i \in A \text{ then } \bigcup R(C_{ir}) \in C_{ir} \\
(3) & \quad \text{If } C_{ir}, C_{ir'} \in C_{ir} \text{ then } C_{ir} \cap R(C_{ir'}) \in C_{ir}
\end{align*}\]

Then, the pair $(k, \mathbb{T}_{C_{ir}})$ is termed as a cubic intuitionistic fuzzy topological space with an R-order (R-CIFT).

**Example 8.** Let $k$ be a nonempty universal set. Then, $ci(k)$ be the accumulation of all R-cubic intuitionistic fuzzy sets RCIFS in $k$. Consider the R-order fuzzy subsets of $ci(k)$ given as
\[
\begin{align*}
C_{ir}^1 &= \{0.31, 0.42\}, [0.47, 0.56], (0.29, 0.39), \\
C_{ir}^2 &= \{0.31, 0.42\}, [0.47, 0.56], (0, 1), \\
C_{ir}^3 &= \{1, 1\}, [0, 0], (0.29, 0.39), \\
C_{ir}^4 &= \{0, 0\}, [1, 1], (0.29, 0.39), \\
C_{ir}^5 &= \{0.31, 0.42\}, [0.47, 0.56], (1, 0).
\end{align*}
\]

The union and intersection with a P-order for the above CIFSs are given in Tables 3 and 4, respectively. Clearly,
\[
\begin{align*}
\mathbb{T}_{C_{ir}} &= \left\{ 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \right\},
\end{align*}
\]
(54)

and
\[
\begin{align*}
\mathbb{T}_{C_{ir}}^2 &= \left\{ 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}}, \overline{0^{C_1}}, \overline{1^{C_1}}, \overline{0^{C_1}}, \overline{1^{C_1}} \right\},
\end{align*}
\]
(55)

are the cubic intuitionistic topology with an R-order.

**Definition 31.** Let $k$ be a nonempty set, and $\mathbb{T}_{C_{ir}} = \{C_{ir}^k\}$ where $C_{ir}^k$ represent the cubic intuitionistic fuzzy subsets of the universal set $k$. Then, $\mathbb{T}_{C_{ir}}$ is termed as an R-cubic intuitionistic fuzzy topology on $k$ and is the largest R-cubic intuitionistic fuzzy topology on $k$ and is entitled as an R-discrete cubic intuitionistic fuzzy topology.

**Definition 32.** Let $k$ be a universal set and $\mathbb{T}_{C_{ir}} = \{ 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \}$ be the assemblage of cubic intuitionistic fuzzy sets. Then, $\mathbb{T}_{C_{ir}}$ is termed as an R-cubic intuitionistic fuzzy topology on the universal set $k$ and is entitled as an R-indiscrete cubic intuitionistic fuzzy topology.

**Theorem 7.** If $(k, \mathbb{T}_{C_{ir}})$ is any R-cubic intuitionistic fuzzy topological space. Then,
\[\begin{align*}
(1) & \quad 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \text{ are RCIFSs.} \\
(2) & \quad \text{The R-union of any number of RCIFSs is RCIFOS.} \\
(3) & \quad \text{The R-intersection of finite RCIFSs is RCIFOS.}
\end{align*}\]

**Proof.** The proof is trivial. □

**Definition 33.** The elements of an R-cubic intuitionistic fuzzy topology $\mathbb{T}_{C_{ir}}$ is termed as the R-cubic intuitionistic fuzzy open sets RCIFOS in $(k, \mathbb{T}_{C_{ir}})$.

**Theorem 8.** If $(k, \mathbb{T}_{C_{ir}})$ is any R-cubic intuitionistic fuzzy topological space. Then,
\[\begin{align*}
(1) & \quad 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \text{ are RCIFSs.} \\
(2) & \quad \text{The R-intersection of any number of RCIFSs is RCIFOS.} \\
(3) & \quad \text{The R-union of finite RCIFSs is RCIFOS.}
\end{align*}\]

**Proof.** The proof is trivial. □

**Definition 34.** The complement of elements of an R-cubic intuitionistic fuzzy open sets is termed as the R-cubic intuitionistic fuzzy closed sets RCIFCS in $(k, \mathbb{T}_{C_{ir}})$.

**Proposition 2**

\[\begin{align*}
(1) & \quad \text{For every } \mathbb{T}_{C_{ir}}, 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \text{ are R-cubic intuitionistic fuzzy clopen sets.} \\
(2) & \quad \text{For the discrete R-order cubic intuitionistic fuzzy topology, all the cubic intuitionistic subsets of } k \text{ are R-cubic intuitionistic fuzzy clopen sets.} \\
(3) & \quad \text{For the in-discrete R-order cubic intuitionistic fuzzy topology, } 0^{C_1}, 1^{C_1}, \overline{0^{C_1}}, \overline{1^{C_1}} \text{ are only R-cubic intuitionistic fuzzy clopen sets.}
\end{align*}\]
Definition 36. Let \( (k, T^1_{C_{IR}}) \) and \( (k, T^2_{C_{IR}}) \) be two R-CIFTs in \( k \). Two R-CIFTs are called comparable if

\[
T^1_{C_{IR}} \subseteq R T^2_{C_{IR}},
\]

or

\[
T^2_{C_{IR}} \subseteq R T^1_{C_{IR}}.
\]

If \( T^2_{C_{IR}} \subseteq R T^1_{C_{IR}} \), then, \( T^2_{C_{IR}} \) is called R-cubic intuitionistic fuzzy coarser than \( T^1_{C_{IR}} \) and \( T^1_{C_{IR}} \) is called the R-cubic intuitionistic fuzzy finer than \( T^2_{C_{IR}} \).

Example 9. Let \( k \) be a nonempty set and from Example 8,

\[
T^1_{C_{IR}} = \{0_{C_I}, 1_{C_I}, 0_{C_{IR}}, 1_{C_{IR}}\},
\]

and

\[
T^2_{C_{IR}} = \{0_{C_I}, 1_{C_I}, 0_{C_{IR}}, 1_{C_{IR}}, 0_{C_{IR}}, 1_{C_{IR}}, 0_{C_{IR}}, 1_{C_{IR}}\},
\]

are R-cubic intuitionistic fuzzy topologies on the universal set. Then, \( T^1_{C_{IR}} \subseteq R T^2_{C_{IR}} \). Hence, \( T^1_{C_{IR}} \) is called the R-cubic intuitionistic fuzzy coarser than \( T^2_{C_{IR}} \).

4.1. Subspace of CIFTs

Definition 37. Let \( (k, T_{C_{IR}}) \) be a CIFT. Let \( \forall \subseteq k \) and \( T_{C_{IR}} \) is a CIFT on \( \forall \) and whose RCIFOSs are

\[
C_{IR} = T_{C_{IR}} \cap \forall,
\]

where \( C_{IR} \) are RCIFOSs of \( T_{C_{IR}} \) and \( T_{C_{IR}} \) are RCIFOSs of \( T_{C_{IR}} \) and \( \forall \) is any R-cubic subset of RCIFS on \( \forall \). Then, \( T_{C_{IR}} \) is called the R-cubic intuitionistic fuzzy subspace of \( T_{C_{IR}} \) i.e.,

\[
T_{C_{IR}} = \{C_{IRY} : C_{IR} = C_{IR} \cap \forall, C_{IR} \in T_{C_{IR}}\},
\]

\( (k, T_{C_{IR}}) \) is an R-cubic intuitionistic fuzzy topology on \( k \).

Now, consider any R-cubic fuzzy subset on \( \forall \) such that \( \forall \subseteq k \) is

\[
\forall = \{0.27, 0.38, 0.52, 0.67, 0.34, 0.28\}.
\]

Also,
<table>
<thead>
<tr>
<th>Criteria</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(C_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathbb{X}_1)</td>
<td>([0.17, 0.24], [0.36, 0.43], (0.56, 0.32))</td>
<td>([0.20, 0.28], [0.29, 0.31], (0.27, 0.20))</td>
<td>([0.18, 0.21], [0.21, 0.32], (0.39, 0.22))</td>
<td>([0.20, 0.37], [0.21, 0.43], (0.54, 0.23))</td>
</tr>
<tr>
<td>(\mathbb{X}_2)</td>
<td>([0.19, 0.22], [0.39, 0.42], (0.59, 0.40))</td>
<td>([0.27, 0.34], [0.33, 0.40], (0.43, 0.21))</td>
<td>([0.24, 0.30], [0.30, 0.39], (0.50, 0.20))</td>
<td>([0.32, 0.40], [0.19, 0.51], (0.52, 0.30))</td>
</tr>
<tr>
<td>(\mathbb{X}_3)</td>
<td>([0.20, 0.29], [0.40, 0.51], (0.81, 0.13))</td>
<td>([0.31, 0.52], [0.42, 0.50], (0.72, 0.17))</td>
<td>([0.31, 0.39], [0.18, 0.40], (0.67, 0.14))</td>
<td>([0.14, 0.63], [0.24, 0.50], (0.70, 0.18))</td>
</tr>
<tr>
<td>(\mathbb{X}_4)</td>
<td>([0.31, 0.37], [0.36, 0.49], (0.50, 0.36))</td>
<td>([0.18, 0.33], [0.28, 0.52], (0.40, 0.32))</td>
<td>([0.23, 0.40], [0.24, 0.51], (0.50, 0.33))</td>
<td>([0.08, 0.74], [0.32, 0.40], (0.46, 0.42))</td>
</tr>
<tr>
<td>(\mathbb{X}_5)</td>
<td>([0.40, 0.48], [0.51, 0.60], (0.52, 0.30))</td>
<td>([0.29, 0.41], [0.30, 0.39], (0.48, 0.40))</td>
<td>([0.40, 0.47], [0.38, 0.60], (0.56, 0.29))</td>
<td>([0.13, 0.64], [0.40, 0.47], (0.53, 0.37))</td>
</tr>
<tr>
<td>(\mathbb{X}_6)</td>
<td>([0.29, 0.38], [0.27, 0.42], (0.60, 0.27))</td>
<td>([0.40, 0.51], [0.41, 0.50], (0.47, 0.38))</td>
<td>([0.32, 0.38], [0.24, 0.72], (0.43, 0.32))</td>
<td>([0.42, 0.50], [0.37, 0.53], (0.53, 0.27))</td>
</tr>
</tbody>
</table>
4.2. Interior, Closure, Frontier, and Exterior of RCIFSs

**Definition 38.** Let \((k, \mathcal{T}_{CIR})\) be \(\text{CIFT} r\) and \(C_{IR} \in ci(k)\), the interior of \(C_{IR}\) is expressed as \(C_{IR}^0\) and is described as a union of all the R-cubic intuitionistic fuzzy open subsets contained in \(C_{IR}\). It is the greatest R-cubic intuitionistic fuzzy open set contained in \(C_{IR}\).

**Example 11.** Consider an R-cubic intuitionistic fuzzy topological space as constructed in Example 8. Let \(C_{IR}^6 \in \text{ci}(k)\) given as

\[
C_{IR}^6 = ([0.38, 0.46], [0.45, 0.51], (0.26, 0.40)).
\]

Then,

\[
(C_{IR}^6)^0 = C_{IR}^1 \cup C_{IR}^0 \cup C_{IR}^2 \cup C_{IR}^3 \cup C_{IR}^4 \cup C_{IR}^5 = C_{IR}^1.
\]

**Theorem 9.** Let \((k, \mathcal{T}_{CIR})\) be \(\text{CIFT} r\) and \(C_{IR} \in ci(k)\). Then, \(C_{IR}\) is open CIFS if \(C_{IR}^0 = C_{IR}\).

**Proof.** The proof is trivial. \(\Box\)

**Theorem 10.** Let \((k, \mathcal{T}_{CIR})\) be \(\text{CIFT} p\) and \(C_{IR}^0, C_{IR}^1 \in \text{ci}(k)\). Then,

\[
(i) \quad (C_{IR})^0 = C_{IR}^0
\]

\[
(ii) \quad C_{IR}^0 \cup C_{IR}^1 \Rightarrow (C_{IR}^0)_{C_{IR}^0} \cup (C_{IR}^1)_{C_{IR}^0}
\]

\[
(iii) \quad (C_{IR}^0 \cap C_{IR}^1)^0 = (C_{IR}^0)^0 \cap (C_{IR}^1)^0
\]

\[
(iv) \quad (C_{IR}^0 \cup C_{IR}^1)^0 = (C_{IR}^0)^0 \cup (C_{IR}^1)^0
\]

**Proof.** Proof is trivial. \(\Box\)

**Definition 39.** Let \((k, \mathcal{T}_{CIR})\) be \(\text{CIFT} r\) and \(C_{IR} \in \text{ci}(k)\), the closure of \(C_{IR}\) is expressed as \(C_{IR}\) and is described as the intersection of all the R-cubic intuitionistic fuzzy closed supersets of \(C_{IR}\). It is the smallest R-cubic intuitionistic fuzzy closed superset of \(C_{IR}\).

**Example 12.** Let us consider an R-cubic intuitionistic topological space as constructed in Example 8. Then, the closed CIFSs are given as
Table 7: Normalized decision matrix from DMs.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>([0.17, 0.21], [0.36, 0.43], (0.56, 0.32))</td>
<td>([0.17, 0.21], [0.36, 0.43], (0.27, 0.20))</td>
<td>([0.17, 0.21], [0.36, 0.43], (0.39, 0.22))</td>
<td>([0.17, 0.21], [0.36, 0.43], (0.54, 0.23))</td>
</tr>
<tr>
<td>$X_2$</td>
<td>([0.19, 0.22], [0.39, 0.51], (0.59, 0.40))</td>
<td>([0.19, 0.22], [0.39, 0.51], (0.43, 0.21))</td>
<td>([0.19, 0.22], [0.39, 0.51], (0.50, 0.20))</td>
<td>([0.19, 0.22], [0.39, 0.51], (0.52, 0.30))</td>
</tr>
<tr>
<td>$X_3$</td>
<td>([0.20, 0.29], [0.40, 0.51], (0.81, 0.13))</td>
<td>([0.31, 0.52], [0.42, 0.50], (0.81, 0.13))</td>
<td>([0.31, 0.52], [0.18, 0.40], (0.81, 0.13))</td>
<td>([0.14, 0.63], [0.24, 0.50], (0.81, 0.13))</td>
</tr>
<tr>
<td>$X_4$</td>
<td>([0.08, 0.33], [0.36, 0.52], (0.50, 0.36))</td>
<td>([0.08, 0.33], [0.36, 0.52], (0.40, 0.32))</td>
<td>([0.08, 0.33], [0.36, 0.52], (0.50, 0.33))</td>
<td>([0.08, 0.33], [0.36, 0.52], (0.46, 0.42))</td>
</tr>
<tr>
<td>$X_5$</td>
<td>([0.40, 0.48], [0.51, 0.60], (0.56, 0.29))</td>
<td>([0.29, 0.41], [0.30, 0.39], (0.56, 0.29))</td>
<td>([0.40, 0.47], [0.38, 0.60], (0.56, 0.29))</td>
<td>([0.13, 0.64], [0.40, 0.47], (0.56, 0.29))</td>
</tr>
<tr>
<td>$X_6$</td>
<td>([0.29, 0.38], [0.41, 0.72], (0.60, 0.27))</td>
<td>([0.29, 0.38], [0.41, 0.72], (0.47, 0.38))</td>
<td>([0.29, 0.38], [0.41, 0.72], (0.43, 0.32))</td>
<td>([0.29, 0.38], [0.41, 0.72], (0.53, 0.27))</td>
</tr>
</tbody>
</table>
Table 8: Relative importance and score function.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>CIFN-WPM values</th>
<th>Score values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$([0.16, 0.20], [0.34, 0.41], (0.9980, 0.00001))$</td>
<td>0.8029</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$([0.18, 0.21], [0.38, 0.49], (0.9993, 0.00002))$</td>
<td>0.7592</td>
</tr>
<tr>
<td>$X_3$</td>
<td>$([0.21, 0.45], [0.29, 0.46], (0.9999, 0.000007))$</td>
<td>0.9548</td>
</tr>
<tr>
<td>$X_4$</td>
<td>$([0.07, 0.31], [0.34, 0.50], (0.9990, 0.00007))$</td>
<td>0.6294</td>
</tr>
<tr>
<td>$X_5$</td>
<td>$([0.25, 0.49], [0.39, 0.50], (0.9996, 0.00002))$</td>
<td>0.9245</td>
</tr>
<tr>
<td>$X_6$</td>
<td>$([0.28, 0.34], [0.39, 0.71], (0.9994, 0.00004))$</td>
<td>0.7593</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(0 C^c) & = \{[1, 1], [0, 0], (0, 1)\}, \\
(1 C^c) & = \{[0, 0], [1, 1], (1, 0)\}, \\
(0 C^c) & = \{[1, 1], [0, 0], (0, 0)\}, \\
(1 C^c) & = \{[0, 0], [1, 1], (0, 1)\}, \\
(C^6_0) & = \{[0.47, 0.56], [0.31, 0.42], (0.39, 0.29)\}, \\
(C^5_0) & = \{[0.47, 0.56], [0.31, 0.42], (1, 0)\}, \\
(C^1_0) & = \{[0, 0], [1, 1], (0.39, 0.29)\}, \\
(C^5_0) & = \{[1, 1], [0, 0], (0.39, 0.29)\}, \\
(C^6_0) & = \{[0.47, 0.56], [0.31, 0.42], (0, 1)\}.
\end{align*}
\]

Let $C^7_0 \in CI(k)$ given as
\[
C^7_0 = \{[0.30, 0.37], [0.48, 0.61], (0.38, 0.24)\}.
\]

Then,
\[
\overline{C^6_0} = (\overline{C^7_0}) \cap_R (C^5_0) = (C^5_0)^c.
\]

Theorem 12. Let $(k, T_{CI})$ be CIFT and $C^i_{IR} \in CI(k)$. Then,

\begin{align*}
(1) \quad & (C^6_0)^c = (C^5_0)^c, \\
(2) \quad & (C^5_0)^c = (C^0_0)^c, \\
(3) \quad & Ext(C^6_0) = C^6_0, \\
(4) \quad & Ext(C^5_0) = C^5_0, \\
(5) \quad & Ext(C^6_0) \cup_R Fr(C^5_0) \cup_R Fr(C^0_0) = C^i_{IR}, \\
(6) \quad & Fr(C^6_0) = Fr(C^5_0), \\
(7) \quad & Fr(C^5_0) \cap_R C^0_0 = 0_{IR}.
\end{align*}

Proof. The proof is trivial. \qed

4.3. R-Cubic Intuitionistic Fuzzy Basis

Definition 42. Let $(k, T_{CI})$ be CIFT. Then $B \subseteq T_{CI}$ is called an R-cubic intuitionistic fuzzy basis for $T_{CI}$ if for every $C^i_{IR} \in T_{CI}$, $\exists \emptyset \in B$ such that
\[
C^i_{IR} = \cup_R \emptyset.
\]

Example 14. From Example 8,
\[
T_{CI} = \{0 C^c, 1 C^c, 2 C^c, 3 C^c, 4 C^c, 5 C^c, 6 C^c\},
\]
is an R-cubic intuitionistic fuzzy topology of $k$. Then,
\[
B = \{1 C^c, 2 C^c, 3 C^c, 4 C^c, 5 C^c\},
\]
is an R-cubic intuitionistic fuzzy basis for $T_{CI}$.
Table 9: Cubic intuitionistic decision matrix from DMs.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁</td>
<td>(0.17, 0.24), (0.36, 0.43), (0.56, 0.32)</td>
<td>(0.20, 0.28), (0.29, 0.31), (0.27, 0.20)</td>
<td>(0.18, 0.21), (0.21, 0.32), (0.39, 0.22)</td>
<td>(0.20, 0.37), (0.21, 0.43), (0.54, 0.23)</td>
</tr>
<tr>
<td>X₂</td>
<td>(0.19, 0.22), (0.39, 0.42), (0.59, 0.40)</td>
<td>(0.27, 0.34), (0.33, 0.40), (0.43, 0.21)</td>
<td>(0.24, 0.30), (0.30, 0.39), (0.50, 0.20)</td>
<td>(0.32, 0.40), (0.19, 0.51), (0.52, 0.30)</td>
</tr>
<tr>
<td>X₃</td>
<td>(0.20, 0.29), (0.40, 0.51), (0.81, 0.13)</td>
<td>(0.31, 0.52), (0.42, 0.50), (0.72, 0.17)</td>
<td>(0.31, 0.39), (0.18, 0.40), (0.67, 0.14)</td>
<td>(0.14, 0.63), (0.24, 0.50), (0.70, 0.18)</td>
</tr>
<tr>
<td>X₄</td>
<td>(0.31, 0.37), (0.36, 0.49), (0.50, 0.36)</td>
<td>(0.18, 0.33), (0.28, 0.52), (0.40, 0.32)</td>
<td>(0.23, 0.40), (0.24, 0.51), (0.50, 0.33)</td>
<td>(0.08, 0.74), (0.32, 0.40), (0.46, 0.42)</td>
</tr>
<tr>
<td>X₅</td>
<td>(0.40, 0.48), (0.51, 0.60), (0.52, 0.39)</td>
<td>(0.29, 0.41), (0.30, 0.39), (0.48, 0.40)</td>
<td>(0.40, 0.47), (0.38, 0.60), (0.56, 0.29)</td>
<td>(0.13, 0.64), (0.40, 0.47), (0.53, 0.37)</td>
</tr>
<tr>
<td>X₆</td>
<td>(0.29, 0.38), (0.27, 0.42), (0.60, 0.27)</td>
<td>(0.40, 0.51), (0.41, 0.50), (0.47, 0.38)</td>
<td>(0.32, 0.38), (0.24, 0.72), (0.43, 0.32)</td>
<td>(0.42, 0.50), (0.37, 0.53), (0.53, 0.27)</td>
</tr>
</tbody>
</table>
Recent research has looked at information systems as useful data repositories and enterprises. Integration of technologies throughout the corporation and connecting all business areas by developing an enterprise information system (EIS) is crucial. Managers can use data and telecommunications to enhance information flow, reduce costs, and maintain or alter current processes and practices. Companies must analyze both internal and external processes for the production and exchange of products and services to be more efficient and competitive. The managers will be able to evaluate the value of actions for each process to determine how to boost the value among these operations that form a supply chain from supplier to business to dealer to customer. The level of integration among suppliers, business associates, and buyers is independent of their geographical location, determining the value chain's competitiveness or supply chain's effectiveness.

The construction of an integrated organizational system capable of information sharing, resources, and services in the supply chain is central to the digital supply chain management paradigm. To gain and maintain competitive advantages, companies use digital information and communications networks to standardize manufacturing processes, reduce cycle time, increase the effectiveness of procurement procedures and logistical support, reduce production costs, and increase customer satisfaction, among other things. Supply chain management based on the Internet allows a company to streamline its supply chain, increase speed, reduce costs, and be more adaptable. It can also increase consumer and supplier communications as well as smooth the ongoing flow of goods along the supply chain.

Supplier selection is highly essential in supply chain management. The objective is to locate a supplier who can offer the best products and services for the lowest price. Proper supplier selection delivers a high profit and quality level. In this strategic collaboration, the supplier is viewed as a significant element of the business. Because of the increasing focus on sustainability, identifying these providers has become more challenging. Environmental studies, often known as sustainability studies, have become increasingly popular around the world. Identifying these suppliers has become increasingly difficult as a result of the rapidly increasing emphasis on sustainability. Many methodologies for sustainable supply chain selection have been developed.

To determine the most suitable supplier selection, MCGDM techniques can be used successfully. In this section, the suggested model is used to determine the selection of appropriate suppliers for fast-moving consumer products, with the goal of selecting the best supplier among various possibilities. Several criteria have been established based on expert opinions to evaluate supplier choices. In this study, X1, X2, X3, X4, X5, and X6 are examined as possible fast-moving customers goods suppliers using the four criteria established.

### Table 10: Score values.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Score values</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.1125</td>
</tr>
<tr>
<td>X2</td>
<td>0.1705</td>
</tr>
<tr>
<td>X3</td>
<td>0.5490</td>
</tr>
<tr>
<td>X4</td>
<td>0.0725</td>
</tr>
<tr>
<td>X5</td>
<td>0.1435</td>
</tr>
<tr>
<td>X6</td>
<td>0.204</td>
</tr>
</tbody>
</table>

#### 5. Multicriteria Group Decision-Making

The weighted product model (WPM) is a renowned and widely used MCGDM approach for evaluating a set of choices using a set of criteria. Each choice is contrasted to the others by calculating a number of ratios, one per choice criterion. Every ratio is multiplied by the proportional weight of the criterion in consideration. For the selection of one or more options from the set of alternatives based on a number of criteria is a fundamental task in MCGDM problems. Let us consider m alternatives, n criteria with weighted vectors, with the condition that the sum of the weights will be one, for an MCGDM problem in a cubic intuitionistic fuzzy set domain.

Figure 1 shows the flow chart of WPM.

#### 5.1. Application to Uncertain Supply Chain Management

Communication and information technologies are affecting every area of the industrial sector at a rapid pace. In reality, it would be difficult to pinpoint an organization that does not use or is not touched by information and communications technologies in some way. In many cases, if technology is not employed appropriately, the firm's survival is jeopardized. Companies nowadays use technology to boost productivity, streamline operations, and form electronic conglomerates.

Advanced technologies and electronic systems are radically altering how businesses operate and stay competitive. Many businesses are making strategic technology investments to obtain and maintain a competitive advantage in their industry. Management teams must use technology throughout the organization to enhance information flow, reduce cost, streamline operations, provide product variety, formulate connections with suppliers, and reduce response times to customers' needs to gain a competitive advantage through the use of information and communications technology.

Administrators and top executives should be associated with the development of enterprise-wide information systems (EIS), which should take into account such matters as computer hardware and software and infrastructure facilities, online systems, digital applications, electronic commerce, and alterations to current processes and practices. Managers can integrate data and telecommunications technologies throughout the corporation and connect all business areas by developing an enterprise-wide information system plan. Enterprise-wide integration of technology enables firms to allow consumers to get timely access to the information they need to make informed decisions. Recent research has looked at information systems as useful tools for integrating systems like enterprise resource planning, knowledge management, e-commerce, electronic markets, and supply chain management (SCM) to enhance organizational profit and efficiency.

Companies must analyze both internal and external processes for the production and exchange of products and services to be more efficient and competitive. The managers will be able to evaluate the value of actions for each process to determine how to boost the value among these operations that form a supply chain from supplier to business to dealer to customer through the evaluation of these processes. The level of integration among suppliers, business associates, and buyers, independent of their geographical location, determines the value chain's or supply chain's effectiveness.

### 5.2. CIF Weighted Product Model

The proposed method is used to choose the best supplier among six alternatives. These alternatives are weighted against four criteria C1 = price, C2 = quality, C3 = performance, and C4 = delivery, derived from thorough expert opinions. A group of decision-makers has been assembled to assess the
suppliers using the recommended methodology. Six decision-makers D1, D2, D3, D4, D5, and D6 were chosen, consisting of supplier experts and expert academics on multicriteria decision-making in a fuzzy environment.

Step 1. Consider the decision matrix \( M = (\mathcal{T}_{ji})_{mon} \) given by the decision-makers in the form of CIFNs on the basis of the cubic intuitionistic fuzzy linguistic scale to evaluate suppliers in accordance with established objectives and criteria is given in Table 5.

Step 2. With the help of the linear approach, we normalize the matrix \( M = (\mathcal{T}_{ji})_{max} \). We divide the criteria into two subsets, benefit criteria B and cost criteria K. Here, \( X_3 \) and \( X_6 \) belong to benefit criteria B and the remaining others belong to cost criteria K. For this, first, we find \( \max_j \mathcal{T}_{ij} \) and \( \min_j \mathcal{T}_{ij} \), which are given in Table 5. We normalized the decision matrix by utilizing the 1st and 2nd equations in Algorithm 1 and this is given in Table 6.

Normalized decision matrices from DMs are expressed in Table 7.

Steps 3 and 4. We find the relative importance of all alternatives by utilizing the 3rd equation in Algorithm 1, and then, we calculate their score function as given in Table 8.

Step 5. Rank the alternatives according to the score function, and the final ranking is

\[
X_3 \succ X_5 \succ X_4 \succ X_2 \succ X_1
\]

(77)

As we can see that \( X_3 \) is the most appropriate supplier among the six alternatives with the best of qualities of all criteria.

5.3. CIF Choice Value Method. The choice value method is a renowned and widely used MCGDM basis for evaluating a set of choices using a set of criteria. Each choice is contrasted to the others by calculating a number of ratios, one per choice criterion. Every ratio is multiplied by the proportional weight of the criterion in consideration. A fundamental task in MCGDM problems is the selection of one or more options from the set of alternatives based on a number of criteria. Let us consider \( m \) alternatives, \( n \) criteria with weighted vectors, with the condition that the sum of weights will be one, for an MCGDM problem in a cubic intuitionistic fuzzy set domain.

5.4. MCDGM Application

Step 1. Consider the decision matrix \( M = (\mathcal{T}_{ji})_{mon} \) given by the decision-makers in the form of CIFNs given in Table 9.

Step 2. Decision-makers gives the weights to the four criteria as \( \mathcal{W}_1 = 0.18, \mathcal{W}_2 = 0.24, \mathcal{W}_3 = 0.26, \) and \( \mathcal{W}_4 = 0.32 \) with \( \sum \mathcal{W}_i = 1 \).
Step 3. We compute the score values for each alternative. The score values are expressed in Table 10.

Step 4. Rank the alternatives according to their score values.

$$X_3 > X_6 > X_2 > X_5 > X_1 > X_4.$$  \hspace{1cm} (79)

As a result, $X_3$ is the best supplier among six alternatives with qualities of all criteria.

5.5. Comparative Analysis. This paper describes techniques for dealing with the cubic intuitionistic situation. We compare our two cubic intuitionistic strategies that are
already in use. If we use CIF-WPM to assemble the alternatives, they are ranked as
\[ X_3 \succ X_5 \succ X_6 \succ X_2 \succ X_1 \succ X_4. \quad (80) \]

On the other side, when we use the technique of choice value method, the ranking of alternatives is
\[ X_3 \succ X_6 \succ X_2 \succ X_5 \succ X_1 \succ X_4. \quad (81) \]

Based on these findings, it seemed that the ranking of the \( X_3 \) alternative was the same as that produced by the suggested cubic intuitionistic procedures. The rest of the alternatives have been altered, as can be seen. As a result, we concluded that in the case of only IVIFSs, the best choice matches with the indicated one; however, the other alternatives are altered, resulting in numerous decisions. As a result, this CIS condition improves the application range of the membership and nonmembership intervals by considering IFS membership values in line with it.

Figure 2 shows the bar chart of ranking of feasible alternatives by using the WPM and CVM methods. The comparison analysis of the proposed CIF-WPM and CIF-CVM with other existing techniques is expressed in Table 11.

### 6. Conclusion

A cubic intuitionistic fuzzy set is an effective method for dealing with various uncertainties in multicriteria group decision-making (MCGDM) settings. A cubic set is a two-component system that would be used to describe data with a fuzzy interval and a fuzzy number. The notion of cubic intuitionistic fuzzy sets (CIFS) is a strong hybrid model of IFSs and IVIFSs. A CIFS has two components, one indicating the IVIFS and the other indicating the IFS. A CIFS is a new fuzzy model for data analysis, computational intelligence, neural computing, soft computing, and others. The idea of cubic hesitant fuzzy topology defined on CIFS can be utilized to seek solutions to various problems of information analysis, information fusion, big data, and decision analysis.

Main findings in this manuscript are as follows:

1. We introduced the concepts of “P-cubic intuitionistic fuzzy topology” as well as “R-cubic intuitionistic fuzzy topology.” Topological structures provide robust approaches for data analysis and decision analysis under an uncertain environment.

2. Certain properties of CIF topology under P(R)-order are explored, and their related results are elaborated with illustrations.

3. The notions of CIF-open set, CIF-closed set, CIF-frontier, CIF-dense set, and CIF-basis are investigated with a corresponding example.

4. Algorithms 1 and 2 are proposed for extension of the weighted product model and the choice value method, respectively.

5. The symmetry of optimal decisions is analyzed by computations with Algorithms 1 and 2. The numerical values of alternatives are very close by using Algorithm 1. However, the numerical values of alternatives have a clear difference when using Algorithm 2.

6. An application of proposed methods named CIF-WPM and CIF-CVM towards uncertain supply chain management is presented.

7. To discuss the advantages, flexibility, and validity of proposed methods, a comparison analysis is also expressed.

For forthcoming analysis, due to the flexibility of CIF topology towards data analysis and information analysis, one can extend this work to develop new MCDM techniques with CIF-VIKOR, CIF-AHP, and CIF-aggregation operators.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Authors’ Contributions

Muhammad Riaz was responsible for conceptualization, formal analysis, and supervision. Khadija Akmal conducted methodology, formal analysis, review, and editing. Yahya Almalki performed investigation, supervision, and funding acquisition. Daud Ahmad took part in investigation, methodology, review, and editing. All authors made a significant scientific contribution to the research in the manuscript. All authors read and approved the final manuscript.

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References


