Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 9657703, 20 pages
https://doi.org/10.1155/2022/9657703

Research Article

Spherical Fuzzy Information Aggregation Based on Aczel–Alsina Operations and Data Analysis for Supply Chain

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Received 25 April 2022; Revised 21 June 2022; Accepted 6 July 2022; Published 4 October 2022

Academic Editor: Adiel T. de Almeida-Filho

Spherical fuzzy sets (SFSs) are often made up of membership, nonmembership, and hesitancy grades, and also have the advantage of accurately representing decision makers (DMs) preferences. This article proposes novel spherical fuzzy aggregation operators (AOs) based on Aczel–Alsina (AA) operations, which offer a lot of advantages when tackling real-world situations. We begin by introducing some new SFS operations, such as the Aczel–Alsina product, the Aczel–Alsina sum, the Aczel–Alsina exponent, and the Aczel–Alsina scalar multiplication. We developed many AOs namely, the "spherical fuzzy Aczel–Alsina weighted averaging (SFAAWA) operator," "spherical fuzzy Aczel–Alsina ordered weighted averaging (SFAAOWA) operator," "spherical fuzzy Aczel–Alsina hybrid averaging (SFAAHA) operator," "spherical fuzzy Aczel–Alsina weighted geometric (SFAAWG) operator," "spherical fuzzy Aczel–Alsina ordered weighted geometric (SFAAOWG) operator," and "spherical fuzzy Aczel–Alsina hybrid geometric (SFAAHG) operator." Different attributes of these operators have been defined. The idempotency, boundary, monotonicity, and commutativity of suggested averaging and geometric operators are demonstrated. Then, based on these operators, we propose a novel approach for tackling the "multi-criteria decision-making" (MCDM) problems. We use a agriculture land selection scenario to demonstrate the efficacy of our proposed approach. The outcome confirms the new technique's applicability and viability. Furthermore, sensitivity analysis and a comparison analysis between the existing approaches and the recommended technique have been provided.

1. Introduction

Decision-making problems are common in a wide range of fields, including technology, finance, and marketing. Traditionally, it has been assumed that all data on alternate access is kept as discrete integers. Because managing the imprecision and uncertainty inherent in data is crucial in real-world circumstances. There are three alternative reactions or attitudes when it comes to selection: yes, no, and refusal. However, first, the most sophisticated response is "refusal," which conventional "fuzzy sets" [1] and "intuitionistic fuzzy sets" (IFSs) [2] may not accurately represent. To address such problems, Cuong proposed the idea of "picture fuzzy set" (PFS) [3, 4]. In PFS, each component in the universe of discourse set has varying degrees of "positive membership degree" (PMD), "neutral membership degree" (N_u MD), and "negative membership degree" (N_g MD) with values ranging from $[0, 1]$.

Fuzzy clustering is a useful technique for pattern detection and information extraction from databases, and it has been used to a wide range of practical issues. Son [5] defined "distributed picture fuzzy clustering method" for PFSs. Singh [6] proposed some "correlation coefficients." The correlation coefficient is applied to clustering in PFS in this article. The benefits of proposed correlation coefficients as well as the disadvantages of existing correlation coefficients have been examined. Wei [7] proposed some new similarity measures (SMs) between PFSs, such as set-theoretic SMs, weighted set-theoretic cosine SMs, cosine SMs, weighted cosine SMs, grey SMs, and weighted grey SMs. Wei and Gao [8] introduced various innovative dice SMs for PFSs and generalized dice SMs for PFSs, indicating that dice SMs and asymmetric measures are special cases of generalized dice SMs for certain parameter values. Wei et al. [9] proposed some results about "projection model" under the PFSs, the modules of picture fuzzy number (PFN), and PF-ideal point.

Over the last few decades, there has been a strong emphasis on information fusion and the development of new AOs. AO's effectiveness and limitations have been entrenched in decision-making. AO obviously includes a number of operating rules for concatenating a finite set of fuzzy numbers into a single fuzzy number. Data aggregation is essential in decision-making, economy, administration, healthcare, technology, and intelligence areas. In terms of their functions and operating laws, numerous AOs have been established for PFSs. Wei et al. [10] proposed "picture 2-tuple linguistic AOs" with MCDM. Garg [11] proposed some weighted averaging and ordered weighted averaging operators for the aggregation of PFNs. Wei [12] presented "Hamacher" AOs for the PFS, and a realistic example of selecting an enterprise system is provided to validate the established approach and to illustrate its feasibility and efficiency. Jana et al. [13] gave the notion of picture fuzzy Dombi AOs for PFNs with MCDM applications. Tian et al. [14] defined some "picture fuzzy power Choquet ordered geometric AOs" and "picture fuzzy power shapley Choquet ordered geometric AOs" with shapley fuzzy measures-based MCDM. Wang et al. [15] proposed hotel building energy efficiency retrofit project selection under PFSs. Wang et al. [16] introduced "Muirhead mean AOs" for PFNs. Wei established TODIM method for PFSs [17], "picture 2-tuple linguistic Bonferroni mean AOs" [18], and "picture uncertain linguistic Bonferroni mean AOs" [19]. Abdullah et al. [20] proposed some new AOs based on sine trigonometric function with application. Qiyas et al. [21] defined some novel picture fuzzy AOs under the linguistic environment. Farid and Riaz [22] developed several new Einstein interactive geometric AOs for q-rung orthopair fuzzy numbers. Riaz and Farid [23] proposed some proportional distribution based spherical AOs. Farid et al. [24] introduced some AOs for the thermal power equipment supplier selection. Saha et al. [25] introduced the new hybrid hesitant fuzzy weighted AOs for MCDM that are based on Archimedean and Dombi operations. Feng et al. [26] proposed the idea of score functions related to generalized orthopair fuzzy membership grades with application. Akram et al. [27] introduced the idea of prioritized AOs for complex spherical fuzzy sets. Riaz and Farid [28] developed some fairly AOs for PFSs. Riaz et al. [29] proposed some Frank AOs for interval-valued linear Diophantine fuzzy set.

Menger [30] introduced the concept of triangle norms in his hypothesis of probabilistic metric spaces. It has been discovered that t-norms and their corresponding t-conorms are fundamental operations in fuzzy sets and structures, such as the product t-norm and probabilistic sum t-conorm [31], Einstein t-norm and t-conorm [32], and the Hamacher t-norm and t-conorm [33]. Klement et al. [34] carried out a thorough analysis of the characteristics and associated elements of triangular norms in recent years. In 1982, Aczél and Alsina [35] introduced new operations named Aczel–Alsina t-norm and Aczel–Alsina t-conorm, which place a high premium on parameter changeability. Based on the Aczel–Alsina triangular norm (AA t-norm), Wang et al. [36] devised a score level fusion technique that simultaneously increases the distance between imposters. Senapati et al. proposed Aczel–Alsina AOs for IFSs [37] and Aczel–Alsina AOs for interval-valued IFSs [38].

In everyday life, we might confront a variety of circumstances that PFS cannot resolve, such as when the sum of PMD, N_u MD, and N_g MD$\,>1$. PFS is incapable of producing an appropriate conclusion in such a case. Mahmood et al. [39], Gundogdu and Kahraman [40], and Ashraf et al. [41] separately developed the concept of SFSs in their works. SFS provides the DM with extra freedom when confronted with uncertainty in decision-making situations. Gundogdu and Kahraman developed the SF-TOPSIS [42], SF-WASPAS [43], and SF-VIKOR methods [44]. Ashraf et al. presented AOs for SFSs and the GRA method for the SF-linguistic set [45]. Zeng et al. [46] proposed a TOPSIS-based hybrid covering-based SF-rough set model. The cosine similarity measures for SFSs were proposed by Rafiq et al. [47]. Jin et al. [48] developed AOs for SFSs based on logarithmic functions. Additionally, Ashraf et al. [49] presented several Dombi AOs for SFSs with implementation to group MCDM. Jaller and Otay used [50] when they suggested SF AHP and TOPSIS for assessing efficient vehicle technology for cargo handling. Ali et al. [51] and Ashraf et al. [52] proposed some AOs for interval-valued picture fuzzy set. Kazemitash et al. [53] and Bozanic et al. [54] gave some ideas related to some different extensions of fuzzy set. For other terminologies not discussed in the paper, the readers are referred to [55–58].

In light of the foregoing, we recognize that decision-making concerns are becoming increasingly complex in reality. To select the superior alternatives to the MCDM concerns, it is significant to communicate the uncertain information in a far more beneficial approach. Furthermore, it is critical to control the relationship between input contentions. Taking each of these characteristics into account, the primary objective of this informative article is to demonstrate numerous aggregation operators under SF circumstances, referred as SF Aczel–Alsina AOs. Despite the established inventive approaches that have developed previously in this field, we have thoroughly researched every possibility to demonstrate our provided strategy, for it to outperform all previous efforts to comprehend the actual global presented concern.

The following information is included in the paper: the next section discusses some fundamental concepts relating to Aczel–Alsina triangular norms and SFSs. Section 3 summarizes the Aczel–Alsina operation laws for SFNs. In Section 4, we discuss the "spherical fuzzy Aczel–Alsina weighted averaging (SFAAWA) operator," "spherical fuzzy Aczel–Alsina ordered weighted averaging (SFAAOWA) operator," "spherical fuzzy Aczel–Alsina hybrid averaging (SFAAHA) operator," "spherical fuzzy Aczel–Alsina weighted geometric (SFAAWG) operator," "spherical fuzzy Aczel–Alsina ordered weighted geometric (SFAAOWG) operator," and "spherical fuzzy Aczel–Alsina hybrid geometric (SFAAHG) operator" as well as a few advantageous properties. In Section 5, we employ the proposed operators to develop a set of techniques for resolving MCDM difficulties in which the characteristic values are represented as SF data. Section 6 illustrates how to choose agriculture land to showcase the proposed technique. In Section 7, we analyze the effect of a parameter on the alternate raking order. In Section 8, we compare the developed method to current methods to determine the proposed technique's appropriateness. Finally, Section 9 discusses a few conclusions for future study.

2. Preliminaries

Several fundamental concepts associated with SFSs have been addressed in this section of the article.

Definition 1 (see [39–41]). A "spherical fuzzy set" (SFS) in X is defined as follows:

$$\chi = \left\{ \left\langle \breve{Y}, \kappa_{\chi}(\breve{Y}), \upsilon_{\chi}(\breve{Y}), \tau_{\chi}(\breve{Y}) \middle| \breve{Y} \in X \right\rangle \right\}, \tag{1}$$

where $\kappa_{\chi}(\breve{Y}), \upsilon_{\chi}(\breve{Y}), \tau_{\chi}(\breve{Y}) \in [0,1]$, such that $0 \leq \kappa_{\chi}^2(\breve{Y}) + \upsilon_{\chi}^2(\breve{Y}) + \tau^2\chi(\breve{Y}) \leq 1$ for all $\breve{Y} \in X$ and $\kappa_{\chi}(\breve{Y}), \upsilon_{\chi}(\breve{Y}), \tau_{\chi}(\breve{Y})$ denote degree of membership, nonmembership and hesitancy, respectively, for some $\breve{Y} \in X$.

We denote this pair as $\mathfrak{R}^{\gamma} = (\kappa_{\mathfrak{R}^{\gamma}}, \upsilon_{\mathfrak{R}^{\gamma}}, \tau_{\mathfrak{R}^{\gamma}})$, throughout this article, and called as SFN with the conditions $\kappa_{\mathfrak{R}^{\gamma}}, \upsilon_{\mathfrak{R}^{\gamma}}, \tau_{\mathfrak{R}^{\gamma}} \in [0,1]$ and $\kappa_{\mathfrak{R}^{\gamma}}^2 + \upsilon_{\mathfrak{R}^{\gamma}}^2 + \tau_{\mathfrak{R}^{\gamma}}^2 \leq 1$.

Definition 2 (see [40]). When implementing SFNs to real-world situations, it is critical to prioritize them. For this, "score function" (SF) for SFN $\mathfrak{R}^{\gamma} = (\kappa_{\mathfrak{R}^{\gamma}}, \upsilon_{\mathfrak{R}^{\gamma}}, \tau_{\mathfrak{R}^{\gamma}})$ can be defined as follows:

$$S(\mathfrak{R}^{\gamma}) = (\kappa_{\mathfrak{R}^{\gamma}} - \tau_{\mathfrak{R}^{\gamma}})^2 - (\upsilon_{\mathfrak{R}^{\gamma}} - \tau_{\mathfrak{R}^{\gamma}})^2. \tag{2}$$

Example 1. Consider two SFNs $\mathfrak{R}_1^{\gamma} = \langle 0.236, 0.126, 0.175 \rangle$ and $\mathfrak{R}_2^{\gamma} = \langle 0.308, 0.228, 0.482 \rangle$ then by using equation (2), we get $S(\mathfrak{R}_1^{\gamma}) = 0.00119646$ and $S(\mathfrak{R}_2^{\gamma}) = -0.0341963$. As $S(\mathfrak{R}_1^{\gamma}) > S(\mathfrak{R}_2^{\gamma})$ so, we have $\mathfrak{R}_1^{\gamma} > \mathfrak{R}_2^{\gamma}$.

However, because the aforementioned function appears incapable of classifying the SFNs in a variety of conditions, it is hard to determine which one is larger $S(\mathfrak{R}_1^{\gamma}) = S(\mathfrak{R}_2^{\gamma})$. For this, an "accuracy function H" of $\mathfrak{R}^{\gamma}$ is defined as follows:

$$H(\mathfrak{R}^{\gamma}) = \kappa_{\mathfrak{R}^{\gamma}}^2 + \upsilon_{\mathfrak{R}^{\gamma}}^2 + \tau_{\mathfrak{R}^{\gamma}}^2. \tag{3}$$

Based on [40], we presented some operational rules to aggregate the SFNs.

Definition 3. Let $\mathfrak{R}_1^{\gamma} = \langle \kappa_1, \upsilon_1, \tau_1 \rangle$ and $\mathfrak{R}_2^{\gamma} = \langle \kappa_1, \upsilon_1, \tau_1 \rangle$ be two SFNs, then

$$\begin{aligned}
\mathfrak{R}_1^{\gamma^c} &= \langle \upsilon_1, \kappa_1, \tau_1 \rangle, \\
\mathfrak{R}_1^{\gamma} \vee \mathfrak{R}_2^{\gamma} &= \left\{ \begin{array}{c} \max\{\kappa_1, \kappa_2\}, \min\{\upsilon_1, \upsilon_2\}, \\ \min\left\{ \left(1 - \left((\max\{\kappa_1, \kappa_2\})^2 + (\min\{\upsilon_1, \upsilon_2\})^2 \right)\right)^{\frac{1}{2}}, \max\{\tau_1, \tau_2\} \right\} \end{array} \right\}, \\
\mathfrak{R}_1^{\gamma} \wedge \mathfrak{R}_2^{\gamma} &= \left\{ \begin{array}{c} \min\{\kappa_1, \kappa_2\}, \max\{\upsilon_1, \upsilon_2\}, \\ \max\left\{ \left(1 - \left((\min\{\kappa_1, \kappa_2\})^2 + (\max\{\upsilon_1, \upsilon_2\})^2 \right)\right)^{\frac{1}{2}}, \min\{\tau_1, \tau_2\} \right\} \end{array} \right\}, \\
\mathfrak{R}_1^{\gamma} \oplus \mathfrak{R}_2^{\gamma} &= \left\{ \begin{array}{c} \left(\kappa_1^2 + \kappa_2^2 - \kappa_1^2 \kappa_2^2\right)^{1/2}, \upsilon_1 \upsilon_2, \\ \left(\left(1 - \kappa_2^2\right)\tau_1^2 + \left(1 - \kappa_1^2\right)\tau_2^2 - \tau_1^2 \tau_2^2\right)^{1/2} \end{array} \right\}, \\
\mathfrak{R}_1^{\gamma} \otimes \mathfrak{R}_2^{\gamma} &= \left\{ \begin{array}{c} \kappa_1 \kappa_2, \left(\upsilon_1^2 + \upsilon_2^2 - \upsilon_1^2 \upsilon_2^2\right)^{1/2}, \\ \left(\left(1 - \upsilon_2^2\right)\tau_1^2 + \left(1 - \upsilon_1^2\right)\tau_2^2 - \tau_1^2 \tau_2^2\right)^{1/2} \end{array} \right\}, \\
\mathfrak{R}_1^{\gamma^{\sigma}} &= \left\{ \begin{array}{c} \mu_{A_s}^{\sigma}, \left(1 - \left(1 - \upsilon_1^2\right)^{\sigma}\right)^{1/2}, \\ \left(\left(1 - \upsilon_1^2\right)^{\sigma} - \left(1 - \upsilon_1^2 - \pi_{A_s}^2\right)^{\sigma}\right)^{1/2}, \sigma > 0 \end{array} \right\}, \text{ and} \\
\sigma \cdot \mathfrak{R}_1^{\gamma} &= \left\{ \begin{array}{c} \left(1 - \left(1 - \kappa_1^2\right)^{\sigma}\right)^{1/2}, \upsilon_1^{\sigma}, \\ \left(\left(1 - \kappa_1^2\right)^{\sigma} - \left(1 - \kappa_1^2 - \tau_1^2\right)^{\sigma}\right)^{1/2} \end{array} \right\}, \sigma > 0 \end{array} \right\}.
\end{aligned} \tag{4}$$

Definition 4 (see [40]). Let $\mathfrak{R}_1^{\gamma} = \langle \kappa_1, \upsilon_1, \tau_1 \rangle$ and $\mathfrak{R}_2^{\gamma} = \langle \kappa_2, \upsilon_2, \tau_2 \rangle$ be two SFNs and $\sigma, \sigma_1, \sigma_2 > 0$ be the real numbers, then we have,

(1) $\mathfrak{R}_1^{\gamma} \oplus \mathfrak{R}_2^{\gamma} = \mathfrak{R}_2^{\gamma} \oplus \mathfrak{R}_1^{\gamma}$

(2) $\mathfrak{R}_1^{\gamma} \otimes \mathfrak{R}_2^{\gamma} = \mathfrak{R}_2^{\gamma} \otimes \mathfrak{R}_1^{\gamma}$

(3) $\sigma(\mathfrak{R}_1^{\gamma} \oplus \mathfrak{R}_2^{\gamma}) = (\sigma\mathfrak{R}_1^{\gamma}) \oplus (\sigma\mathfrak{R}_2^{\gamma})$

(4) $(\mathfrak{R}_1^{\gamma} \otimes \mathfrak{R}_2^{\gamma})^{\sigma} = \mathfrak{R}_1^{\gamma^{\sigma}} \otimes \mathfrak{R}_2^{\gamma^{\sigma}}$

(5) $(\sigma_1 + \sigma_2)\mathfrak{R}_1^{\gamma} = (\sigma_1\mathfrak{R}_1^{\gamma}) \oplus (\sigma_2\mathfrak{R}_2^{\gamma})$

(6) $\mathfrak{R}_1^{\gamma^{\sigma_1+\sigma_2}} = \mathfrak{R}_1^{\gamma^{\sigma_1}} \otimes \mathfrak{R}_1^{\gamma^{\sigma_1}}$

2.1. Basics about t-Norm, t-Conorm, and Aczel–Alsina t-Norm

Definition 5 (see [34]). A function $\breve{\mho}^{\lambda}$: $[0,1]^2 \longrightarrow [0,1]$ is a t-norm, if for all $g, h, u \in [0,1]$, the consecutive axioms are fulfilled.

(1) $\breve{\mho}^{\lambda}(g,h) = \breve{\mho}^{\lambda}(h,g)$

(2) $\breve{\mho}^{\lambda}(g,h) \le \breve{\mho}^{\lambda}(g,u)$ if $h \le u$

(3) $\breve{\mho}^{\lambda}(g, \breve{\mho}^{\lambda}(h,u)) = \breve{\mho}^{\lambda}(\breve{\mho}^{\lambda}(g,h),u)$

(4) $\breve{\mho}^{\lambda}(g,1) = g$

These axioms are called, symmetry, monotonicity, associativity, and "1" as identity, respectively.

Definition 6 (see [34]). A function $\overline{\mathfrak{I}}$: $[0,1]^2 \longrightarrow [0,1]$ is a t-conorm, if for all $g, h, u \in [0,1]$, the consecutive axioms are fulfilled.

(1) $\overline{\mathfrak{I}}(g,h) = \overline{\mathfrak{I}}(h,g)$

(2) $\overline{\mathfrak{I}}(g,h) \le \overline{\mathfrak{I}}(g,u)$ if $h \le u$

(3) $\overline{\mathfrak{I}}(g, \overline{\mathfrak{I}}(h,u)) = \overline{\mathfrak{I}}(\overline{\mathfrak{I}}(g,h),u)$

(4) $\overline{\mathfrak{I}}(g,0) = g$

These axioms are called, symmetry, monotonicity, associativity, and "0" as identity, respectively.

Example 2. Some famous t-norms are given as follows:

(i) $\breve{\mho}^{\lambda}P(g,h) = g.\, h$ (Product t-norm)

(ii) $\breve{\mho}^{\lambda}M(g,h) = \min(g,h)$ (Minimum t-norm)

(iii) $\breve{\mho}^{\lambda}L(g,h) = \max(g+h-1, 0)$ (Lukasiewicz t-norm)

(iv) $\breve{\mho}^{\lambda}D(g,h) = \begin{cases} f, & \text{if } h = 1 \\ h, & \text{if } g = 1 \\ 0, & \text{otherwise} \end{cases}$ for all $g, h \in [0,1]$

(Drastic t-norm)

Example 3. Some famous t-conorms are given as follows:

(i) $\overline{\mathfrak{I}}P(g,h) = g + h - g.\, h$ (Probabilistic sum)

(ii) $\overline{\mathfrak{I}}M(g,h) = \max(g,h)$ (Maximum t-conorm)

(iii) $\overline{\mathfrak{I}}L(g,h) = \min(g+h, 1)$ (Lukasiewicz t-conorm)

(iv) $\overline{\mathfrak{I}}D(g,h) = \begin{cases} g, & \text{if } h = 0 \\ h, & \text{if } g = 0 \\ 1, & \text{otherwise} \end{cases}$ for all $g, h \in [0,1]$

(Drastic t-conorm)

Definition 7 (see [35]). This class of t-norm is originally proposed by Aczel–Alsina in mid-1980s under the condition of functional equations.

The category $(\breve{\mho}^{\lambda^{\beth}}_{A})_{\beth\in[0,\infty]}$ of Aczel–Alsina t-norms is stated by the following equation:

$$\breve{\mho}^{\lambda^{\beth}}_{A}(g,h) = \begin{cases} \breve{\mho}^{\lambda}D(g,h), & \text{if } \beth = 0, \\ \min(g,h), & \text{if } \beth = \infty, \\ e^{-\left((-\log g)^{\beth}+(-\log h)^{\beth}\right)^{1/\beth}}, & \text{otherwise}. \end{cases} \tag{5}$$

The category $(\overline{\mathfrak{I}}^{\beth}_{A})_{\beth\in[0,\infty]}$ of Aczel–Alsina t-conorms is stated by the following equation:

$$\overline{\mathfrak{I}}^{\beth}_{A}(g,h) = \begin{cases} \overline{\mathfrak{I}}D(g,h), & \text{if } \beth = 0, \\ \max(g,h), & \text{if } \beth = \infty, \\ 1 - e^{-\left((-\log(1-g))^{\beth}+(-\log(1-h))^{\beth}\right)^{1/\beth}}, & \text{otherwise}. \end{cases} \tag{6}$$

Limiting Cases: $\breve{\mho}^{\lambda^{0}}_{A} = \breve{\mho}^{\lambda}_{D}, \breve{\mho}^{\lambda^{1}}_{A} = \breve{\mho}^{\lambda}_{P}, \breve{\mho}^{\lambda^{\infty}}_{A} = \min$, $\overline{\mathfrak{I}}^{0}_{A} = \overline{\mathfrak{I}}_{D}, \overline{\mathfrak{I}}^{1}_{A} = \overline{\mathfrak{I}}_{P}$, and $\overline{\mathfrak{I}}^{\infty}_{A} = \max$.

For every $\beth \in [0,\infty]$ the t-norm $\breve{\mho}^{\lambda^{\beth}}_{A}$ and t-conorm $\overline{\mathfrak{I}}^{\beth}_{A}$ are dual to each other. The class of Aczel–Alsina t-norms is strictly increasing and the class of Aczel–Alsina t-conorms is strictly decreasing.

3. Aczel–Alsina Operations for SFNs

In this section, we will introduce the Aczel–Alsina operations for SFNs and look at some of their basic properties.

Definition 8. Let $\mathfrak{R}^{\gamma} = (\kappa_{\mathfrak{R}^{\gamma}}, \upsilon_{\mathfrak{R}^{\gamma}}, \tau_{\mathfrak{R}^{\gamma}}), \mathfrak{R}_1^{\gamma} = (\kappa_{\mathfrak{R}_1^{\gamma}}, \upsilon_{\mathfrak{R}_1^{\gamma}}, \tau_{\mathfrak{R}_1^{\gamma}})$, and $\mathfrak{R}_2^{\gamma} = (\kappa_{\mathfrak{R}_2^{\gamma}}, \upsilon_{\mathfrak{R}_2^{\gamma}}, \tau_{\mathfrak{R}_2^{\gamma}})$ be three SFNs, $\aleph \ge 1$ and $\beth > 0$. Then, the Aczel–Alsina t-norm and t-conorm operations of SFNs are defined as follows:

(1) $$\mathfrak{R}_1^{\gamma} \oplus \mathfrak{R}_2^{\gamma} = \left\langle \sqrt{1 - e^{-\left((-\log(1-\kappa^2_{\mathfrak{R}_1^{\gamma}}))^{\aleph} + (-\log(1-\kappa^2_{\mathfrak{R}_2^{\gamma}}))^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left((-\log\upsilon^2_{\mathfrak{R}_1^{\gamma}})^{\aleph} + (-\log\upsilon^2_{\mathfrak{R}_2^{\gamma}})^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left((-\log\tau^2_{\mathfrak{R}_1^{\gamma}})^{\aleph} + (-\log\tau^2_{\mathfrak{R}_2^{\gamma}})^{\aleph}\right)^{1/\aleph}}} \right\rangle$$

(2) $$\mathfrak{R}_1^{\gamma} \otimes \mathfrak{R}_2^{\gamma} = \left\langle \sqrt{e^{-\left((-\log\kappa^2_{\mathfrak{R}_1^{\gamma}})^{\aleph} + (-\log\kappa^2_{\mathfrak{R}_2^{\gamma}})^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left((-\log(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}))^{\aleph} + (-\log(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}))^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left((-\log(1-\tau^2_{\mathfrak{R}_1^{\gamma}}))^{\aleph} + (-\log(1-\tau^2_{\mathfrak{R}_2^{\gamma}}))^{\aleph}\right)^{1/\aleph}}} \right\rangle$$

(3) $$\beth\mathfrak{R}^{\gamma} = \left\langle \sqrt{1 - e^{-\left(\beth(-\log(1-\kappa^2_{\mathfrak{R}^{\gamma}}))^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth(-\log\upsilon^2_{\mathfrak{R}^{\gamma}})^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth(-\log\tau^2_{\mathfrak{R}^{\gamma}})^{\aleph}\right)^{1/\aleph}}} \right\rangle$$

(4) $$\mathfrak{R}^{\gamma^{\beth}} = \left\langle \sqrt{e^{-\left(\beth(-\log\kappa^2_{\mathfrak{R}^{\gamma}})^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\beth(-\log(1-\upsilon^2_{\mathfrak{R}^{\gamma}}))^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\beth(-\log(1-\tau^2_{\mathfrak{R}^{\gamma}}))^{\aleph}\right)^{1/\aleph}}} \right\rangle$$

Theorem 1. *Let* $\mathfrak{R}^{\gamma} = (\kappa_{\mathfrak{R}^{\gamma}}, \upsilon_{\mathfrak{R}^{\gamma}}, \tau_{\mathfrak{R}^{\gamma}}), \mathfrak{R}_1^{\gamma} = (\kappa_{\mathfrak{R}_1^{\gamma}}, \upsilon_{\mathfrak{R}_1^{\gamma}}, \tau_{\mathfrak{R}_1^{\gamma}})$, *and* $\mathfrak{R}_2^{\gamma} = (\kappa_{\mathfrak{R}_2^{\gamma}}, \upsilon_{\mathfrak{R}_2^{\gamma}}, \tau_{\mathfrak{R}_2^{\gamma}})$ *be three SFNs, then we have* *(i)* $\mathfrak{R}^{\gamma}_{1}\oplus\mathfrak{R}^{\gamma}_{2}=\mathfrak{R}^{\gamma}_{2}\oplus\mathfrak{R}^{\gamma}_{1}$

(ii) $\mathfrak{R}^{\gamma}_{1}\otimes\mathfrak{R}^{\gamma}_{2}=\mathfrak{R}^{\gamma}_{2}\otimes\mathfrak{R}^{\gamma}_{1}$

(iii) $\beth(\mathfrak{R}^{\gamma}_{1}\oplus\mathfrak{R}^{\gamma}_{2})=\beth\mathfrak{R}^{\gamma}_{1}\oplus\beth\mathfrak{R}^{\gamma}_{2},\beth>0$

(iv) $(\beth_1+\beth_2)\mathfrak{R}^{\gamma}=\beth_1\mathfrak{R}^{\gamma}\oplus\beth_2\mathfrak{R}^{\gamma},\beth_1,\beth_2>0$

(v) $(\mathfrak{R}^{\gamma}_{1}\otimes\mathfrak{R}^{\gamma}_{2})^{\beth}=\mathfrak{R}^{\gamma^{\beth}}_{1}\otimes\mathfrak{R}^{\gamma^{\beth}}_{2},\beth>0$

(vi) $\mathfrak{R}^{\gamma^{\beth_1}}\otimes\mathfrak{R}^{\gamma^{\beth_2}}=\mathfrak{R}^{\gamma^{(\beth_1+\beth_2)}},\beth_1,\beth_2>0$

Proof. For the three SFNs $\mathfrak{R}^{\gamma}$, $\mathfrak{R}^{\gamma}_{1}$, and $\mathfrak{R}^{\gamma}_{2}$, and $\beth,\beth_1,\beth_2>0$, we can get the following equations:

(i)

$$\begin{aligned}
\mathfrak{R}^{\gamma}_{1}\oplus\mathfrak{R}^{\gamma}_{2}&=\left\langle\sqrt{1-e^{-\left(\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}+\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\left\langle\sqrt{1-e^{-\left(\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}+\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}+\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}+\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\mathfrak{R}^{\gamma}_{2}\oplus\mathfrak{R}^{\gamma}_{1}
\end{aligned}\tag{7}$$

(ii)

$$\begin{aligned}
\mathfrak{R}^{\gamma}_{1}\otimes\mathfrak{R}^{\gamma}_{2}&=\left\langle\sqrt{e^{-\left(\left(-\log\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{1-e^{-\left(\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}+\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{1-e^{-\left(\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}+\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\left\langle\sqrt{e^{-\left(\left(-\log\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}+\left(-\log\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{1-e^{-\left(\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}+\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{1-e^{-\left(\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}+\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\mathfrak{R}^{\gamma}_{2}\otimes\mathfrak{R}^{\gamma}_{1}
\end{aligned}\tag{8}$$

(iii) Using this, we get

$$\begin{aligned}
\beth(\mathfrak{R}^{\gamma}_{1}\oplus\mathfrak{R}^{\gamma}_{2})&=\beth\left\langle\sqrt{1-e^{-\left(\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}+\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\left\langle\sqrt{1-e^{-\left(\beth\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}+\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}+\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\left\langle\sqrt{1-e^{-\left(\beth\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{1}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{1}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&\oplus\left\langle\sqrt{1-e^{-\left(\beth\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}_{2}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\upsilon^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}},\sqrt{e^{-\left(\beth\left(-\log\tau^2_{\mathfrak{R}^{\gamma}_{2}}\right)^{\aleph}\right)^{1/\aleph}}}\right\rangle,\\
&=\beth\mathfrak{R}^{\gamma}_{1}\oplus\beth\mathfrak{R}^{\gamma}_{2}
\end{aligned}\tag{9}$$

(iv)

$$
\begin{aligned}
\beth_1\mathfrak{R}^{\gamma}\oplus\beth_2\mathfrak{R}^{\gamma} &= \left\langle \sqrt{1-e^{-\left(\beth_1\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_1\left(-\log \upsilon^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_1\left(-\log \tau^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&\oplus \left\langle \sqrt{1-e^{-\left(\beth_2\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_2\left(-\log \upsilon^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_2\left(-\log \tau^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{1-e^{-\left(\left(\beth_1+\beth_2\left(-\log\left(1-\kappa^2_{\mathfrak{R}^{\gamma}}\right)\right)^{\aleph}\right)\right)^{1/\aleph}}}, \sqrt{e^{-\left(\left(\beth_1+\beth_2\right)\left(-\log \upsilon^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\left(\beth_1+\beth_2\right)\left(-\log \tau^2_{\mathfrak{R}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left(\beth_1+\beth_2\right)\mathfrak{R}^{\gamma}
\end{aligned}
\tag{10}
$$

(v)

$$
\begin{aligned}
\left(\mathfrak{R}_1^{\gamma}\otimes\mathfrak{R}_2^{\gamma}\right)^{\beth} &= \left\langle \sqrt{e^{-\left(\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}+\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}+\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \right. \\
&\left. \sqrt{1-e^{-\left(\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}+\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}\,\beth} \right\rangle, \\
&= \left\langle \sqrt{e^{-\left(\beth\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}+\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}+\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \right. \\
&\left. \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}+\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{e^{-\left(\beth\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&\otimes \left\langle \sqrt{e^{-\left(\beth\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \mathfrak{R}_1^{\gamma^{\beth}}\otimes\mathfrak{R}_2^{\gamma^{\beth}}
\end{aligned}
\tag{11}
$$

(vi)

$$
\begin{aligned}
\mathfrak{R}^{\gamma^{\beth_1}} \otimes \mathfrak{R}^{\gamma^{\beth_2}} &= \left\langle \sqrt{e^{-\left(\beth_1\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth_1\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth_1\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&\otimes \left\langle \sqrt{e^{-\left(\beth_2\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth_2\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\beth_2\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{e^{-\left(\left(\beth_1+\beth_2\right)\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\left(\beth_1+\beth_2\right)\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\left(\beth_1+\beth_2\right)\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \mathfrak{R}^{\gamma^{\left(\beth_1+\beth_2\right)}}
\end{aligned}
\tag{12}
$$

□

4. Spherical Fuzzy Aczel–Alsina Aggregation Operators

In this section, we present a few SF aggregation operators by means of the Aczel–Alsina operations.

4.1. Spherical Fuzzy Aczel–Alsina Averaging AOs

Definition 9. Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$, $(\phi = 1, 2, \ldots, \sqsupset)$ be an accumulation of SFNs and $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_1^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})^T$ be the weight vector (WV) of $\mathfrak{R}_{\phi}^{\gamma}$, with $\xi_{\phi}^{\zeta} > 0$ and $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$. Then, "spherical fuzzy Aczel–Alsina weighted average (SFAAWA) operator" is a mapping SFAAWA: $(L^*)^{\sqsupset} \longrightarrow L^*$, where

$$
\mathrm{SFAAWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) = \left(\xi_1^{\zeta}\mathfrak{R}_1^{\gamma} \oplus \xi_2^{\zeta}\mathfrak{R}_2^{\gamma} \oplus \ldots, \oplus \xi_{\sqsupset}^{\zeta}\mathfrak{R}_{\sqsupset}^{\gamma}\right).
\tag{13}
$$

The following theorem is obtained using Aczel–Alsina operations on SFNs.

Theorem 2. *Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ be an accumulation of SFNs, then aggregated value of them utilizing the SFAAWA operation is also an SFNs, and*

$$
\begin{aligned}
\mathrm{SFAAWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) &= \underset{\phi=1}{\overset{\sqsupset}{\oplus}}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\phi}^{\gamma}\right) \\
&= \left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,
\end{aligned}
\tag{14}
$$

where $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_2^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})$ is the WV of $\mathfrak{R}_{\phi}^{\gamma}$ s.t $\xi_{\phi}^{\zeta} > 0$ and $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$.

Proof. We can derive Theorem 2 in the following way using the mathematical induction technique.

For $\sqsupset = 2$, depend on Aczel–Alsina operations of SFNs, we obtain the following equation:

$$
\begin{aligned}
\xi_1^{\zeta}\mathfrak{R}_1^{\gamma} &= \left\langle \sqrt{1-e^{-\left(\beth_1\left(-\log\left(1-\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_1\left(-\log\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_1\left(-\log\tau^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle \text{ and} \\
\xi_2^{\zeta}\mathfrak{R}_2^{\gamma} &= \left\langle \sqrt{1-e^{-\left(\beth_2\left(-\log\left(1-\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_2\left(-\log\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\beth_2\left(-\log\tau^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.
\end{aligned}
\tag{15}
$$

Based on Aczel–Alsina operations of SFNs, we obtain the following equation:

$$\begin{aligned}
\mathrm{SFAAWA}\left(\mathfrak{R}_1^{\gamma},\mathfrak{R}_2^{\gamma}\right) &= \left\langle \sqrt{1-e^{-\left(\gimel_1\left(-\log\left(1-\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_1\left(-\log\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_1\left(-\log\tau^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\\
&\oplus\left\langle \sqrt{1-e^{-\left(\gimel_2\left(-\log\left(1-\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_2\left(-\log\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_2\left(-\log\tau^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\\
&= \sqrt{1-e^{\left(-\gimel_1\left(-\log\left(1-\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}+\gimel_2\left(-\log\left(1-\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}},\\
&\cdot\sqrt{e^{-\left(\gimel_1\left(-\log\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}+\gimel_2\left(-\log\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_1\left(-\log\tau^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}+\gimel_2\left(-\log\tau^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \text{ and}\\
&= \left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\tau^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.
\end{aligned} \tag{16}$$

Thus, it is true for $\beth = 2$.

Consider equation (14) is true for $\beth = k$, then we have the following equation:

$$\begin{aligned}
&\mathrm{SFAAWA}\left(\mathfrak{R}_1^{\gamma},\mathfrak{R}_2^{\gamma},\ldots,\mathfrak{R}_k^{\gamma}\right) =\\
&\left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\tau^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,
\end{aligned} \tag{17}$$

we will prove that equation (14) holds for $\beth = k + 1$.

$$\begin{aligned}
\mathrm{SFAAWA}\left(\mathfrak{R}_1^{\gamma},\mathfrak{R}_2^{\gamma},\ldots,\mathfrak{R}_k^{\gamma},\mathfrak{R}_{k+1}^{\gamma}\right) &= \overset{k}{\oplus}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\phi}^{\gamma}\right)\oplus\left(\xi_{k+1}^{\zeta}\mathfrak{R}_{k+1}^{\gamma}\right),\\
&= \left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\tau^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\\
&\oplus\left\langle \sqrt{1-e^{-\left(\gimel_{k+1}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{k+1}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_{k+1}\left(-\log\upsilon^2_{\mathfrak{R}_{k+1}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\gimel_{k+1}\left(-\log\tau^2_{\mathfrak{R}_{k+1}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\\
&= \left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log\tau^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.
\end{aligned} \tag{18}$$

As a result, we can conclude that equation (4.2) stands true for any $\beth$.

By applying the SFAAWA operator, we can illustrate the following features efficiently. □

Theorem 3. *If all* $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ *are equal, that is,* $\mathfrak{R}_{\phi}^{\gamma} = \mathfrak{R}^{\gamma} \forall \phi$, *then*

$$\mathrm{SFAAWA}(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) = \mathfrak{R}^{\gamma}. \tag{19}$$

Proof. Given that $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$, by equation (4.2) we get the following equation:

$$\begin{aligned}
\mathrm{SFAAWA}(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) &= \left\langle \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \tau_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{1 - e^{-\left(\left(-\log\left(1-\kappa_{\mathfrak{R}^{\gamma}}^2\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\left(-\log \upsilon_{\mathfrak{R}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\left(-\log \tau_{\mathfrak{R}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{1 - e^{\log\left(1-\kappa_{\mathfrak{R}^{\gamma}}^2\right)}}, \sqrt{e^{\log \upsilon_{\mathfrak{R}^{\gamma}}^2}}, \sqrt{e^{\log \tau_{\mathfrak{R}^{\gamma}}^2}} \right\rangle, \\
&= \left(\sqrt{\kappa_{\mathfrak{R}^{\gamma}}^2}, \sqrt{\upsilon_{\mathfrak{R}^{\gamma}}^2}, \sqrt{\tau_{\mathfrak{R}^{\gamma}}^2} \right) = \mathfrak{R}^{\gamma}.
\end{aligned} \tag{20}$$

□

Theorem 4. *Let* $\mathfrak{R}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ *be an accumulation of SFNs. Let* $\mathfrak{R}^{\gamma^-} = min(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_1^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma})$ *and* $\mathfrak{R}^{\gamma^+} = max(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma})$. *Then,*

$$\mathfrak{R}^{\gamma^-} \leq \mathrm{SFAAWA}(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) \leq \mathfrak{R}^{\gamma^+}. \tag{21}$$

Proof. Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}})$ be an accumulation of SFNs. Let $\mathfrak{R}^{\gamma^-} = \min(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) = (\kappa_{\mathfrak{R}^{\gamma}}^-, \upsilon_{\mathfrak{R}^{\gamma}}^-, \tau_{\mathfrak{R}^{\gamma}}^-)$ and $\mathfrak{R}^{\gamma^+} = \max(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) = (\kappa_{\mathfrak{R}^{\gamma}}^+, \upsilon_{\mathfrak{R}^{\gamma}}^+, \tau_{\mathfrak{R}^{\gamma}}^+)$. We have, $\kappa_{\mathfrak{R}^{\gamma}}^- = \min_{\phi}\left\{\kappa_{\mathfrak{R}_{\phi}^{\gamma}}\right\}, \upsilon_{\mathfrak{R}^{\gamma}}^- = \max_{\phi}\left\{\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right\}$,

$$\tau_{\mathfrak{R}^{\gamma}}^- = \max_{\phi}\left\{\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right\}, \kappa_{\mathfrak{R}^{\gamma}}^+ = \max_{\phi}\left\{\kappa_{\mathfrak{R}_{\phi}^{\gamma}}\right\},$$

$\upsilon_{\mathfrak{R}^{\gamma}}^+ = \min_{\phi}\left\{\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right\}$, and $\tau_{\mathfrak{R}^{\gamma}}^+ = \min_{\phi}\left\{\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right\}$ Hence, here we have the subsequent inequalities,

$$\begin{aligned}
\sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa_{\mathfrak{R}^{\gamma^-}}^2\right)\right)^{\aleph}\right)^{1/\aleph}}} &\leq \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)\right)^{\aleph}\right)^{1/\aleph}}}, \\
&\leq \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa_{\mathfrak{R}^{\gamma^+}}^2\right)\right)^{\aleph}\right)^{1/\aleph}}}, \\
\sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \upsilon_{\mathfrak{R}^{\gamma^+}}^2\right)^{\aleph}\right)^{1/\aleph}}} &\leq \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}}, \\
&\leq \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \upsilon_{\mathfrak{R}^{\gamma^-}}^2\right)^{\aleph}\right)^{1/\aleph}}},
\end{aligned} \tag{22}$$

$$\begin{aligned}
\sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \tau_{\mathfrak{R}^{\gamma^+}}^2\right)^{\aleph}\right)^{1/\aleph}}} &\leq \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \tau_{\mathfrak{R}_{\phi}^{\gamma}}^2\right)^{\aleph}\right)^{1/\aleph}}}, \\
&\leq \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi_{\phi}^{\zeta}\left(-\log \tau_{\mathfrak{R}^{\gamma^-}}^2\right)^{\aleph}\right)^{1/\aleph}}}.
\end{aligned} \tag{23}$$

Therefore, $\mathfrak{R}^{\gamma^-} \leq \mathrm{SFAAWA}(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\beth}^{\gamma}) \leq \mathfrak{R}^{\gamma^+}$ □

□

Theorem 5. *Let* $\mathfrak{R}_{\phi}^{\gamma}$ *and* $\mathfrak{R}_{\phi}^{\gamma'}$ *be two sets of SFNs, if* $\mathfrak{R}_{\phi}^{\gamma} \leq \mathfrak{R}_{\phi}^{\gamma'} \forall \phi$, *then*

$$\text{SFAAWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) \leq \text{SFAAWA}\left(\mathfrak{R}_1^{\gamma'}, \mathfrak{R}_2^{\gamma'}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma'}\right), \tag{24}$$

Now, we present "spherical fuzzy Aczel–Alsina ordered weighted averaging (SFAAOWA) operator."

Definition 10. Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ be an accumulation of SFNs. SFAAOWA operator is a mapping SFAAOWA: $(L^*)^{\sqsupset} \longrightarrow L^*$ with the corresponding WV $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_2^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})^T$ such that $\xi_{\phi}^{\zeta} > 0$, and $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$, as follows:

$$\text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) = \bigoplus_{\phi=1}^{\sqsupset}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\Upsilon(\phi)}^{\gamma}\right) = \xi_1^{\zeta}\mathfrak{R}_{\Upsilon(1)}^{\gamma} \oplus \xi_2^{\zeta}\mathfrak{R}_{\Upsilon(2)}^{\gamma} \oplus, \ldots, \oplus \xi_{\sqsupset}^{\zeta}\mathfrak{R}_{\Upsilon(\sqsupset)}^{\gamma}, \tag{25}$$

where $(\Upsilon(1), \Upsilon(2), \ldots, \Upsilon(\sqsupset))$ are the permutation of $(\phi = 1, 2, \ldots, \sqsupset)$, including $\mathfrak{R}_{\Upsilon(\phi-1)}^{\gamma} \geq \mathfrak{R}_{\Upsilon(\phi)}^{\gamma} \forall \phi = 1, 2, \ldots, \sqsupset$.

Thus, the following theorem is obtained using Aczel–Alsina operations on SFNs.

Theorem 6. *Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ be an accumulation of SFNs. SFAAOWA operator is a mapping SFAAOWA: $(L^*)^{\sqsupset} \longrightarrow L^*$ with the corresponding vector $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_2^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})^T$ such that $\xi_{\phi}^{\zeta} > 0$, and $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$. Then,*

$$\begin{aligned}\text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) &= \bigoplus_{\phi=1}^{\sqsupset}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\phi}^{\gamma}\right)\\ &= \left\langle \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\left(1-\kappa^2_{\mathfrak{R}_{\Upsilon(\phi)}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\upsilon^2_{\mathfrak{R}_{\Upsilon(\phi)}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\tau^2_{\mathfrak{R}_{\Upsilon(\phi)}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\end{aligned} \tag{26}$$

where $(\Upsilon(1), \Upsilon(2), \ldots, \Upsilon(\sqsupset))$ are the permutation of $(\phi = 1, 2, \ldots, \sqsupset)$, including $\mathfrak{R}_{\Upsilon(\phi-1)}^{\gamma} \geq \mathfrak{R}_{\Upsilon(\phi)}^{\gamma} \forall \phi = 1, 2, \ldots, \sqsupset$.

Proof. Same as Theorem 2.

By applying the SFAAOWA operator, we can illustrate the following features efficiently. □

Theorem 7. *If all $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ are equal, that is, $\mathfrak{R}_{\phi}^{\gamma} = \mathfrak{R}^{\gamma} \forall \phi$, then*

$$\text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) = \mathfrak{R}^{\gamma} \tag{27}$$

Proof. Same as Theorem 3. □

Theorem 8. *Let $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ be an accumulation of SFNs. Let $\mathfrak{R}^{\gamma^-} = min\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right)$ and $\mathfrak{R}^{\gamma^+} = min\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right)$. Then,*

$$\mathfrak{R}^{\gamma^-} \leq \text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) \leq \mathfrak{R}^{\gamma^+}. \tag{28}$$

Proof. Same as Theorem 4. □

Theorem 9. *Let $\mathfrak{R}_{\phi}^{\gamma}$ and $\mathfrak{R}_{\phi}^{\gamma'}$ be two sets of SFNs, if $\mathfrak{R}_{\phi}^{\gamma} \leq \mathfrak{R}_{\phi}^{\gamma'} \forall \phi$, then*

$$\text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) \leq \text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma'}, \mathfrak{R}_2^{\gamma'}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma'}\right). \tag{29}$$

It is self-evident that the SFAAWA operator weights only the SFNs, and that the SFAAOWA operator weights only the SFN's ordered locations. Following that, weights are used to indicate various elements of the SFAAWA and SFAAOWA operators. Nonetheless, both one and the other operators consider only one of these. To address this shortcoming, we will also demonstrate the "spherical fuzzy Aczel–Alsina hybrid averaging (SFAAHA) operator," which weights all of the given SFN and their appropriate ordered position.

Definition 11. Let $\mathfrak{R}_{\phi}^{\gamma}$ be an accumulation of SFNs. SFAAHA operator is a mapping SFAAHA: $(L^*)^{\sqsupset} \longrightarrow L^*$, s.t.

$$\text{SFAAOWA}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) = \bigoplus_{\phi=1}^{\sqsupset}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\Upsilon(\phi)}^{\ddot{\gamma}}\right) = \xi_1^{\zeta}\mathfrak{R}_{\Upsilon(1)}^{\ddot{\gamma}} \oplus \xi_2^{\zeta}\mathfrak{R}_{\Upsilon(2)}^{\ddot{\gamma}} \oplus, \ldots, \oplus \xi_{\sqsupset}^{\zeta}\mathfrak{R}_{\Upsilon(\sqsupset)}^{\ddot{\gamma}}, \tag{30}$$

where $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_2^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})^T$ is the weighting vector associated with the SFAAHA operator, with $\xi_{\phi}^{\zeta} \in [0, 1]$ and $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$; $\ddot{\mathfrak{R}}_{\phi}^{\gamma} = \sqsupset\delta_{\phi}\mathfrak{R}_{\phi}^{\gamma}, \phi = 1, 2, \ldots, \sqsupset$, $(\mathfrak{R}_{\Upsilon(1)}^{\ddot{\gamma}}, \mathfrak{R}_{\Upsilon(2)}^{\ddot{\gamma}}, \ldots, \mathfrak{R}_{\Upsilon(\sqsupset)}^{\ddot{\gamma}})$ is any permutation of a collection of the weighted SFNs $(\ddot{\mathfrak{R}}_1^{\gamma}, \ddot{\mathfrak{R}}_2^{\gamma}, \ldots \ddot{\mathfrak{R}}_{\sqsupset}^{\gamma})$, s.t. $\mathfrak{R}_{Y(\phi-1)}^{\ddot{\gamma}} \geq \mathfrak{R}_{Y(\phi)}^{\ddot{\gamma}} \cdot \delta = (\delta_1, \delta_2, \ldots \delta_{\sqsupset})^T$ is the weight vector of $\mathfrak{R}^{\gamma}_{\phi}$, with $\delta_{\phi} \in [0,1]$ and $\sum_{\phi=1}^{\beth} \delta_{\phi} = 1$, and $\beth$ is the balancing coefficient, which plays a role of balance.

Theorem 10. *Let $\mathfrak{R}^{\gamma}_{\phi}$ be the collection of SFNs. Their aggregated value by SFAAHA operator is still an SFN, and*

$$\begin{aligned}\mathrm{SFAAHA}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right) &= \mathop{\oplus}_{\phi=1}^{\beth}\left(\xi^{\zeta}_{\phi}\mathfrak{R}^{\gamma}_{\phi}\right)\\ &= \left\langle \sqrt{1-e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\left(1-\kappa^{2}_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\upsilon^{2}_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\tau^{2}_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.\end{aligned} \tag{31}$$

Proof. Same as Theorem 2. □

Theorem 11. *The SFAAWA and SFAAOWA operators are special cases of the SFAAHA operator.*

Proof. (1) Let $\xi^{\zeta} = (1/\beth, 1/\beth, \ldots, 1/\beth)^{T}$. Then,

$$\begin{aligned}\mathrm{SFAAHA}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right) &= \xi^{\zeta}_{1}\mathfrak{R}^{\dot{\gamma}}_{\beth(1)} \oplus \xi^{\zeta}_{2}\mathfrak{R}^{\dot{\gamma}}_{\beth(2)} \oplus \cdots \oplus \xi^{\zeta}_{\beth}\mathfrak{R}^{\dot{\gamma}}_{\beth(\beth)},\\ &= \frac{1}{\beth}\left(\mathfrak{R}^{\dot{\gamma}}_{\beth(1)} \oplus \mathfrak{R}^{\dot{\gamma}}_{\beth(2)} \oplus \cdots \oplus \mathfrak{R}^{\dot{\gamma}}_{\beth(\beth)}\right),\\ &= \xi^{\zeta}_{1}\mathfrak{R}^{\gamma}_{1} \oplus \xi^{\zeta}_{2}\mathfrak{R}^{\gamma}_{2} \oplus \cdots \oplus \xi^{\zeta}_{\beth}\mathfrak{R}^{\gamma}_{\beth},\\ &= \mathrm{SFAAWA}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right).\end{aligned} \tag{32}$$

(2) Let $\xi^{\zeta} = (1/\beth, 1/\beth, \ldots, 1/\beth)^{T}$. Then, $\dot{\mathfrak{R}}^{\gamma}_{\phi} = \mathfrak{R}^{\gamma}_{\phi}$ and

$$\begin{aligned}\mathrm{IFAAHA}_{\xi^{\zeta},\xi^{\zeta}}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right) &= \xi^{\zeta}_{1}\mathfrak{R}^{\dot{\gamma}}_{\beth(1)} \oplus \xi^{\zeta}_{2}\mathfrak{R}^{\dot{\gamma}}_{\beth(2)} \oplus \cdots \oplus \xi^{\zeta}_{\beth}\mathfrak{R}^{\dot{\gamma}}_{\beth(\beth)},\\ &\xi^{\zeta}_{1}\mathfrak{R}^{\dot{\gamma}}_{\beth(1)} \oplus \xi^{\zeta}_{2}\mathfrak{R}^{\dot{\gamma}}_{\beth(2)} \oplus \cdots \oplus \xi^{\zeta}_{\beth}\mathfrak{R}^{\dot{\gamma}}_{\beth\beth(\beth)},\\ &\mathrm{SFAAOWA}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right).\end{aligned} \tag{33}$$

□

4.2. Spherical Fuzzy Aczel–Alsina Geometric AOs

Definition 12. Let $\mathfrak{R}^{\gamma}_{\phi} = (\kappa_{\mathfrak{R}^{\gamma}_{\phi}}, \upsilon_{\mathfrak{R}^{\gamma}_{\phi}}, \tau_{\mathfrak{R}^{\gamma}_{\phi}})$ be an accumulation of SFNs and $\xi^{\zeta} = (\xi^{\zeta}_{1}, \xi^{\zeta}_{2}, \ldots, \xi^{\zeta}_{\beth})^{T}$ be the weight vector (WV) of $\mathfrak{R}^{\gamma}_{\phi}$, with $\xi^{\zeta}_{\phi} > 0$ and $\sum_{\phi=1}^{\beth} \xi^{\zeta}_{\phi} = 1$. Then, "spherical fuzzy Aczel–Alsina weighted geometric (SFAAWG) operator" is a mapping SFAAWG: $(L^{*})^{\beth} \longrightarrow L^{*}$, where

$$\mathrm{SFAAWG}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right) = \left(\mathfrak{R}^{\gamma^{\xi^{\zeta}_{1}}}_{1} \otimes \mathfrak{R}^{\gamma^{\xi^{\zeta}_{2}}}_{2} \otimes \ldots, \otimes \mathfrak{R}^{\gamma^{\xi^{\zeta}_{\beth}}}_{\beth}\right). \tag{34}$$

Thus, the following theorem is obtained using Aczel–Alsina operations on SFNs.

Theorem 12. *Let $\mathfrak{R}^{\gamma}_{\phi} = (\kappa_{\mathfrak{R}^{\gamma}_{\phi}}, \upsilon_{\mathfrak{R}^{\gamma}_{\phi}}, \tau_{\mathfrak{R}^{\gamma}_{\phi}})$ be an accumulation of SFNs, then aggregated value of them utilizing the SFAAWG operation is also an SFNs, and*

$$\begin{aligned}\mathrm{SFAAWG}\left(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}\right) &= \mathop{\otimes}_{\phi=1}^{\beth}\left(\xi^{\zeta}_{\phi}\mathfrak{R}^{\gamma}_{\phi}\right),\\ &= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\kappa^{2}_{\mathfrak{R}^{\gamma}_{\phi}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\left(1-\upsilon_{\mathfrak{R}^{\gamma}_{\phi}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{\beth}\xi^{\zeta}_{\phi}\left(-\log\left(1-\tau_{\mathfrak{R}^{\gamma}_{\phi}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\end{aligned} \tag{35}$$

where $\xi^{\zeta} = (\xi_1^{\zeta}, \xi_2^{\zeta}, \ldots, \xi_{\sqsupset}^{\zeta})^T$ *be the WV of* $\mathfrak{R}_{\phi}^{\gamma}$ *s.t* $\xi_{\phi}^{\zeta} > 0$ *and* $\sum_{\phi=1}^{\sqsupset} \xi_{\phi}^{\zeta} = 1$.

Proof. We can derive Theorem 12 in the following way using the mathematical induction technique.

For $\sqsupset = 2$, depend on Aczel–Alsina operations of SFNs, we obtain the following equation:

$$\mathfrak{R}_1^{\gamma^{\xi_1^{\zeta}}} = \left\langle \sqrt{e^{-\left(\gimel_1\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_1\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_1\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle \text{ and}$$

$$\mathfrak{R}_2^{\gamma^{\xi_2^{\zeta}}} = \left\langle \sqrt{e^{-\left(\gimel_2\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_2\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{2/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_2\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \tag{36}$$

Based on Aczel–Alsina operations of SFNs, we obtain the following equation:

$$\begin{aligned}
\mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}\right) &= \left\langle \sqrt{e^{-\left(\gimel_1\left(-\log\kappa^2_{\mathfrak{R}_1^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_1\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_1\left(-\log\left(1-\tau^2_{\mathfrak{R}_1^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&\oplus \left\langle \sqrt{e^{-\left(\gimel_2\left(-\log\kappa^2_{\mathfrak{R}_2^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_2\left(-\log\left(1-\upsilon^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{2/\aleph}}}, \sqrt{1 - e^{-\left(\gimel_2\left(-\log\left(1-\tau^2_{\mathfrak{R}_2^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle, \\
&= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\sum_{\phi=1}^{2}\xi_{\phi}^{\zeta}\left(-\log\left(1-\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.
\end{aligned} \tag{37}$$

Thus, it is true for $\sqsupset = 2$.

Consider equation (4.15) is true for $\sqsupset = k$, then we have the following equation:

$$\begin{aligned}
&\mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) \\
&= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\kappa^2_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,
\end{aligned} \tag{38}$$

we will prove that equation (4.15) holds for $\sqsupset = k + 1$.

$$\begin{aligned}
\mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_k^{\gamma}, \mathfrak{R}_{k+1}^{\gamma}\right) &= \overset{k}{\oplus}\left(\xi_{\phi}^{\zeta}\mathfrak{R}_{\phi}^{\gamma}\right)\oplus\left(\xi_{k+1}^{\zeta}\mathfrak{R}_{k+1}^{\gamma}\right)\\
&= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log \kappa_{\mathfrak{R}_{\phi}^{\gamma}}^{2}\right)\right)^{\aleph}\frac{1}{\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{\frac{1}{\aleph}}\right)}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{k}\xi_{\phi}^{\zeta}\left(-\log\left(1-\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{\frac{1}{\aleph}}\right)}} \right\rangle,\\
&\oplus\left\langle \sqrt{e^{-\left(\beth_{k+1}\left(-\log\left(\kappa_{\mathfrak{R}_{k+1}^{\gamma}}^{2}\right)\right)\right)^{\aleph}\frac{1}{\aleph}}}, \sqrt{1-e^{-\left(\beth_{k+1}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{k+1}^{\gamma}}^{2}\right)^{\aleph}\right)^{\frac{1}{\aleph}}\right)}}, \sqrt{1-e^{-\left(\beth_{k+1}\left(-\log\left(1-\tau_{k+1}^{\gamma}\right)^{\aleph}\right)\frac{1}{\aleph}\right)}} \right\rangle,\\
&= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log \kappa_{\mathfrak{R}_{\phi}^{\gamma}}^{2}\right)\right)^{\aleph}\frac{1}{\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{\frac{1}{\aleph}}\right)}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{k+1}\xi_{\phi}^{\zeta}\left(-\log\left(1-\tau_{\mathfrak{R}_{\phi}^{\gamma}}\right)^{\aleph}\right)^{\frac{1}{\aleph}}\right)}} \right\rangle.
\end{aligned} \tag{39}$$

As a result, we can conclude that equation (4.15) stands true for any $\sqsupset$.

By applying the SFAAWG operator, we can illustrate the following features efficiently. □

Theorem 13. *If all* $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ *are equal, that is,* $\mathfrak{R}_{\phi}^{\gamma} = \mathfrak{R}^{\gamma}\forall\phi$, *then*

$$\mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) = \mathfrak{R}^{\gamma}. \tag{40}$$

Proof. Given that $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$, by equation (4.15) we get the following equation:

$$\begin{aligned}
\mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right) &= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\kappa_{\mathfrak{R}_{\phi}^{\gamma}}^{2}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\left(1-\upsilon_{\mathfrak{R}_{\phi}^{\gamma}}^{2}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_{\phi}^{\zeta}\left(-\log\left(1-\tau_{\mathfrak{R}_{\phi}^{\gamma}}^{2}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,\\
&= \left\langle \sqrt{e^{-\left(\left(-\log\left(\kappa_{\mathfrak{R}^{\gamma}}^{2}\right)^{\aleph}\right)^{1/\aleph}\right)}}, \sqrt{1-e^{-\left(\left(-\log\left(1-\upsilon_{\mathfrak{R}^{\gamma}}^{2}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\left(-\log\left(1-\tau_{\mathfrak{R}^{\gamma}}^{2}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}} \right\rangle,\\
&= \left\langle \sqrt{e^{\log\kappa_{\mathfrak{R}^{\gamma}}^{2}}}\ \sqrt{1-e^{\log\left(1-\upsilon_{\mathfrak{R}^{\gamma}}^{2}\right)}}, \sqrt{1-e^{\log\left(1-\tau_{\mathfrak{R}^{\gamma}}^{2}\right)}} \right\rangle,\\
&= \left(\sqrt{\kappa_{\mathfrak{R}^{\gamma}}^{2}}, \sqrt{\upsilon_{\mathfrak{R}^{\gamma}}^{2}}, \sqrt{\tau_{\mathfrak{R}^{\gamma}}^{2}}\right) = \mathfrak{R}^{\gamma}.
\end{aligned} \tag{41}$$

□

Theorem 14. *Let* $\mathfrak{R}_{\phi}^{\gamma} = (\kappa_{\mathfrak{R}_{\phi}^{\gamma}}, \upsilon_{\mathfrak{R}_{\phi}^{\gamma}}, \tau_{\mathfrak{R}_{\phi}^{\gamma}})$ *be an accumulation of SFNs. Let*$\mathfrak{R}^{\gamma^{-}} = min\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right)$ *and* $\mathfrak{R}^{\gamma^{+}} = max\left(\mathfrak{R}_1^{\gamma}, \mathfrak{R}_2^{\gamma}, \ldots, \mathfrak{R}_{\sqsupset}^{\gamma}\right)$. *Then,*

$$\mathfrak{R}^{\gamma^-} \leq \mathrm{SFAAWG}\left(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma\right) \leq \mathfrak{R}^{\gamma^+}. \tag{42}$$

Proof. Let $\mathfrak{R}_\phi^\gamma = (\kappa_{\mathfrak{R}_\phi^\gamma}, \upsilon_{\mathfrak{R}_\phi^\gamma})$ be an accumulation of SFNs. Let $\mathfrak{R}^{\gamma^-} = \min(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma) = (\kappa^-_{\mathfrak{R}^\gamma}, \upsilon^-_{\mathfrak{R}^\gamma}, \tau^-_{\mathfrak{R}^\gamma})$ and $\mathfrak{R}^{\gamma^+} = \max(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma) = (\kappa^+_{\mathfrak{R}^\gamma}, \upsilon^+_{\mathfrak{R}^\gamma}, \tau^+_{\mathfrak{R}^\gamma})$. We have, $\kappa^-_{\mathfrak{R}^\gamma} = \min_\phi\left\{\kappa_{\mathfrak{R}_\phi^\gamma}\right\}$, $\upsilon^-_{\mathfrak{R}^\gamma} = \max_\phi\left\{\upsilon_{\mathfrak{R}_\phi^\gamma}\right\}$, $\tau^-_{\mathfrak{R}^\gamma} = \max_\phi\left\{\tau_{\mathfrak{R}_\phi^\gamma}\right\}$, $\kappa^+_{\mathfrak{R}^\gamma} = \max_\phi\left\{\kappa_{\mathfrak{R}_\phi^\gamma}\right\}$, $\upsilon^+_{\mathfrak{R}^\gamma} = \min_\phi\left\{\upsilon_{\mathfrak{R}_\phi^\gamma}\right\}$, and $\tau^+_{\mathfrak{R}^\gamma} = \min_\phi\left\{\tau_{\mathfrak{R}_\phi^\gamma}\right\}$ Hence, here, we have the subsequent inequalities,

$$\begin{gathered}
\sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\kappa^2_{\mathfrak{R}^{\gamma^+}}\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\kappa^2_{\mathfrak{R}^{\gamma\phi}}\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\kappa^2_{\mathfrak{R}^{\gamma^-}}\right)^{\aleph}\right)^{1/\aleph}}}, \\
\sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma^-}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma\phi}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^{\gamma^+}}\right)\right)^{\aleph}\right)^{1/\aleph}}},
\end{gathered} \tag{43}$$

$$\sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma^-}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma\phi}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \leq \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\tau^2_{\mathfrak{R}^{\gamma^+}}\right)\right)^{\aleph}\right)^{1/\aleph}}}. \tag{44}$$

Therefore,
$\mathfrak{R}^{\gamma^-} \leq \mathrm{SFAAWG}(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma) \leq \mathfrak{R}^{\gamma^+}$ □

Theorem 15. *Let $\mathfrak{R}_\phi^\gamma$ and $\mathfrak{R}_\phi^{\gamma'}$ be two sets of SFNs, if $\mathfrak{R}_\phi^\gamma \leq \mathfrak{R}_\phi^{\gamma'} \forall\phi$, then*

$$\mathrm{SFAAWG}\left(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma\right) \leq \mathrm{SFAAWG}\left(\mathfrak{R}_1^{\gamma'}, \mathfrak{R}_2^{\gamma'}, \ldots, \mathfrak{R}_\sqsupset^{\gamma'}\right), \tag{45}$$

Now, we present "spherical fuzzy Aczel–Alsina ordered weighted geometric (SFAAOWG) operator."

Definition 13. Let $\mathfrak{R}_\phi^\gamma = (\kappa_{\mathfrak{R}_\phi^\gamma}, \upsilon_{\mathfrak{R}_\phi^\gamma}, \tau_{\mathfrak{R}_\phi^\gamma})$ be an accumulation of SFNs. SFAAOWG operator is a mapping SFAAOWG: $(L^*)^\sqsupset \longrightarrow L^*$ with the corresponding WV $\xi^\zeta = (\xi_1^\zeta, \xi_2^\zeta, \ldots, \xi_\sqsupset^\zeta)^T$ such that $\xi_\phi^\zeta > 0$ and $\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta = 1$, as follows:

$$\mathrm{SFAAOWG}\left(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma\right) = \bigotimes_{\phi=1}^{\sqsupset}\left(\mathfrak{R}_{\Upsilon(\phi)}^{\gamma^{\xi_\phi^\zeta}}\right) = \mathfrak{R}_{\Upsilon(1)}^{\gamma^{\xi_1^\zeta}} \oplus \mathfrak{R}_{\Upsilon(2)}^{\gamma^{\xi_2^\zeta}} \oplus, \ldots, \oplus \mathfrak{R}_{\Upsilon(\sqsupset)}^{\gamma^{\xi_\sqsupset^\zeta}}, \tag{46}$$

where $(\Upsilon(1), \Upsilon(2), \ldots, \Upsilon(\sqsupset))$ are the permutation of $(\phi = 1, 2, \ldots, \sqsupset)$, including $\mathfrak{R}_{\Upsilon(\phi-1)}^\gamma \geq \mathfrak{R}_{\Upsilon(\phi)}^\gamma \forall\phi = 1, 2, \ldots, \sqsupset$.

Thus, the following theorem is obtained using Aczel–Alsina operations on SFNs.

Theorem 16. *Let $\mathfrak{R}_\phi^\gamma = (\kappa_{\mathfrak{R}_\phi^\gamma}, \upsilon_{\mathfrak{R}_\phi^\gamma}, \tau_{\mathfrak{R}_\phi^\gamma})$ be an accumulation of SFNs. SFAAOWG operator is a mapping SFAAOWG: $(L^*)^\sqsupset \longrightarrow L^*$ with the corresponding vector $\xi^\zeta = (\xi_1^\zeta > 0, \xi_2^\zeta > 0, \ldots, \xi_\sqsupset^\zeta)^T$ such that $\xi_\phi^\zeta > 0$ and $\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta = 1$. Then,*

$$\begin{aligned}
\mathrm{SFAAOWG}\left(\mathfrak{R}_1^\gamma, \mathfrak{R}_2^\gamma, \ldots, \mathfrak{R}_\sqsupset^\gamma\right) &= \bigotimes_{\phi=1}^{\sqsupset}\left(\mathfrak{R}_\phi^{\gamma^{\xi_\phi^\zeta}}\right), \\
&= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\kappa^2_{\mathfrak{R}^\gamma_{\Upsilon(\phi)}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\upsilon^2_{\mathfrak{R}^\gamma_{\Upsilon(\phi)}}\right)\right)^{\aleph}\right)^{1/\aleph}}}, \right. \\
&\qquad \left. \sqrt{1-e^{-\left(\sum_{\phi=1}^{\sqsupset}\xi_\phi^\zeta\left(-\log\left(1-\tau^2_{\mathfrak{R}^\gamma_{\Upsilon(\phi)}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle,
\end{aligned} \tag{47}$$

where $(\Upsilon(1), \Upsilon(2), \ldots, \Upsilon(\beth))$ *are the permutation of* $(\phi = 1, 2, \ldots, \beth)$, *including* $\mathfrak{R}^{\gamma}_{\Upsilon(\phi-1)} \geq \mathfrak{R}^{\gamma}_{\Upsilon(\phi)} \forall \phi = 1, 2, \ldots, \beth$.

Proof. Same as Theorem 12.

By applying the SFAAOWG operator, we can illustrate the following features efficiently. □

Theorem 17. *If all* $\mathfrak{R}^{\gamma}_{\phi} = (\kappa_{\mathfrak{R}^{\gamma}_{\phi}}, \upsilon_{\mathfrak{R}^{\gamma}_{\phi}}, \tau_{\mathfrak{R}^{\gamma}_{\phi}})$ *are equal, that is,* $\mathfrak{R}^{\gamma}_{\phi} = \mathfrak{R}^{\gamma} \forall \phi$, *then*

$$\text{SFAAOWG}(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}) = \mathfrak{R}^{\gamma}. \tag{48}$$

Proof. Same as Theorem 13. □

Theorem 18. *Let* $\mathfrak{R}^{\gamma}_{\phi} = (\kappa_{\mathfrak{R}^{\gamma}_{\phi}}, \upsilon_{\mathfrak{R}^{\gamma}_{\phi}}, \tau_{\mathfrak{R}^{\gamma}_{\phi}})$ *be an accumulation of SFNs. Let* $\mathfrak{R}^{\gamma^-} = min(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth})$ *and* $\mathfrak{R}^{\gamma^+} = max(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth})$. *Then,*

$$\mathfrak{R}^{\gamma^-} \leq \text{SFAAOWG}(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}) \leq \mathfrak{R}^{\gamma^+}. \tag{49}$$

Proof. Same as Theorem 14. □

Theorem 19. *Let* $\mathfrak{R}^{\gamma}_{\phi}$ *and* $\mathfrak{R}^{\gamma'}_{\phi}$ *be two sets of SFNs, if* $\mathfrak{R}^{\gamma}_{\phi} \leq \mathfrak{R}^{\gamma'}_{\phi} \forall \phi$, *then*

$$\text{SFAAOWG}(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}) \leq \text{SFAAOWG}\left(\mathfrak{R}^{\gamma'}_{1}, \mathfrak{R}^{\gamma'}_{2}, \ldots, \mathfrak{R}^{\gamma'}_{\beth}\right). \tag{50}$$

It is self-evident that the SFAAWG operator weights only the SFNs, and that the SFAAOWG operator weights only the SFN's ordered locations. Following that, weights are used to indicate various elements of the SFAAWG and SFAAOWG operators. Nonetheless, both one and the other operators consider only one of these. To address this shortcoming, we will also demonstrate the "spherical fuzzy Aczel–Alsina hybrid geometric (SFAAHG) operator," which weights all of the given SFN and their appropriate ordered position.

Definition 14. Let $\mathfrak{R}^{\gamma}_{\phi}$ be an accumulation of SFNs. SFAAHG operator is a mapping SFAAHG: $(L^*)^{\beth} \longrightarrow L^*$, s.t.

$$\begin{aligned}\text{SFAAHG}(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}) &= \underset{\phi=1}{\overset{\beth}{\oplus}} \left(\mathfrak{R}\gamma^{\ddot{\xi}^{\zeta}_{\phi}}_{\Upsilon(\phi)} \right) \\ &= \xi^{\zeta}_{1} \mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(1)} \oplus \xi^{\zeta}_{2} \mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(2)} \oplus, \ldots, \oplus \xi^{\zeta}_{\beth} \mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\beth)}\end{aligned} \tag{51}$$

where $\xi^{\zeta} = (\xi^{\zeta}_{1}, \xi^{\zeta}_{1}, \ldots, \xi^{\zeta}_{\beth})^T$ is the weighting vector associated with the SFAAHG operator, with $\xi^{\zeta}_{\phi} \in [0, 1]$ and $\sum_{\phi=1}^{\beth} \xi^{\zeta}_{\phi} = 1$; $\ddot{\mathfrak{R}}^{\gamma}_{\phi} = \beth \delta_{\phi} \mathfrak{R}^{\gamma}_{\phi}, \phi = 1, 2, \ldots, \beth$, $(\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(1)}, \mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(2)}, \ldots, \mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\beth)})$ is any permutation of a collection of the weighted SFNs $(\ddot{\mathfrak{R}}^{\gamma}_{1}, \ddot{\mathfrak{R}}^{\gamma}_{2}, \ldots \ddot{\mathfrak{R}}^{\gamma}_{\beth})$, s.t. $\mathfrak{R}^{\gamma^{\cdot\cdot}}_{y(\phi-1)} \geq \mathfrak{R}^{\ddot{\gamma}}_{y(\phi)}$. $\delta = (\delta_1, \delta_2, \ldots \delta_{\beth})^T$ is the weight vector of $\mathfrak{R}^{\gamma}_{\phi}$, with $\delta_{\phi} \in [0, 1]$ and $\sum_{\phi=1}^{\beth} \delta_{\phi} = 1$, and $\beth$ is the balancing coefficient, which plays a role of balance.

Theorem 20. *Let* $\mathfrak{R}^{\gamma}_{\phi}$ *be the collection of SFNs. Their aggregated value by SFAAHG operator is still an SFN, and*

$$\begin{aligned}\text{SFAAHG}(\mathfrak{R}^{\gamma}_{1}, \mathfrak{R}^{\gamma}_{2}, \ldots, \mathfrak{R}^{\gamma}_{\beth}) &= \underset{\phi=1}{\overset{\beth}{\otimes}} \left(\mathfrak{R}^{\gamma^{\xi^{\zeta}_{\phi}}}_{\phi} \right), \\ &= \left\langle \sqrt{e^{-\left(\sum_{\phi=1}^{\beth} \xi^{\zeta}_{\phi}\left(-\log\kappa^2_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)^{\aleph}\right)^{1/\aleph}}}, \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi^{\zeta}_{\phi}\left(-\log\left(1 - \upsilon^2_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \sqrt{1 - e^{-\left(\sum_{\phi=1}^{\beth} \xi^{\zeta}_{\phi}\left(-\log\left(1 - \tau^2_{\mathfrak{R}^{\ddot{\gamma}}_{\Upsilon(\phi)}}\right)\right)^{\aleph}\right)^{1/\aleph}}} \right\rangle.\end{aligned} \tag{52}$$

Proof. Same as Theorem 12. □

Theorem 21. *The SFAAWG and SFAAOWG operators are special cases of the SFAAHG operator.*

5. MCDM Approach Based on Proposed Aczel–Alsina AOs

With the assistance of suggested AOs, we investigate MCDM problems. Consider the set of possible alternatives $\Lambda^{\delta} = \left\{\Lambda^{\delta}_{1}, \Lambda^{\delta}_{2}, \ldots, \Lambda^{\delta}_{m}\right\}$ and $\beth^{\daleth} = \{\beth^{\daleth}_{1}, \beth^{\daleth}_{2}, \ldots, \beth^{\daleth}_{n}\}$ is the collection of criterion. DMs gave their opinion matrix $D = (\mathscr{P}_{ij})_{m\times n}$, where $\mathscr{P}_{ij}$ is given for the alternatives $\Lambda^{\delta}_{i} \in \Lambda^{\delta}$ with respect to the criteria $\beth^{\daleth}_{j} \in \beth^{\daleth}$ in the form of SFNs. SFN decision matrix denoted by $D = (\mathscr{P}_{ij})_{m\times n}$.The proposed operators will be applied to the MCDM, which will include the points listed in Algorithm 1.

6. Numerical Example

This section provides an illustration of how to apply the proposed strategy to the land selection for agriculture purpose.

6.1. Explanation of the Problem. Due to the COVID-19 pandemic's consequences, the E-commerce phenomenon is accelerating, having a huge influence on global supply chains. Thus, logistics management tasks have been elevated in importance in practically every organization that transports physical commodities. There are several methods for

Step 1: acquire the declaration matrix $D = (\mathscr{P}_{ij})_{m\times n}$ in the format of SFNs from the DM.
Step 2: the declaration matrix's attributes are classified into two types, such as the cost form parameter (τ_c) and the benefit form parameter (τ_b). If all parameters are of the same form, no normalization is required. However, because the MCDM contains parameters of multiple forms, the D matrix has been converted to a normalization matrix using the normalization formula, $Y(\breve{F}_{ij})$.

$$\breve{F}_{ij} = \begin{cases} \mathscr{P}_{ij}^{c}; & j \in \tau_c, \\ \mathscr{P}_{ij}; & j \in \tau_b. \end{cases}$$

where $\mathscr{P}_{ij}^{c}$ show the compliment of $\mathscr{P}_{ij}$.
Step 3: using one of the proposed operators to evaluate combined evaluations of the alternatives. One can use geometric operators also instead of averaging operators.
$\breve{F}_i = \mathrm{SFAAWA}(\breve{F}_{i1}, \breve{F}_{i2}, \ldots, \breve{F}_{in})$,
$\breve{F}_i = \mathrm{SFAAOWA}(\breve{F}_{i1}, \breve{F}_{i2}, \ldots, \breve{F}_{in})$,
or $\breve{F}_i = \mathrm{SFAAHA}(\breve{F}_{i1}, \breve{F}_{i2}, \ldots, \breve{F}_{in})$.
Step 4: calculate the combined score for all alternative assessments.
Step 5: alternatives are ranked first by their scoring function, and the best one can be chosen.

Algorithm 1: A decision making process based on SFAAWA, SFAAOWA, SFAAHA, is proposed in Algorithm 1.

businesses to acquire a comparative edge via the out scoured of logistics management processes in today's diversified and incredibly quickly world. Exporters, distributors, and businesses with distribution networks have all demonstrated that turning to third-party logistics (3PL) providers benefits them. 3PL is a term that refers to the process through which a company outsources its warehouse and transportation activities. A 3PL organization can provide stock control, cross-docking, the door of the house distribution, and packaging material. The market for third-party logistics services has accelerated its expansion as a result of the E-commerce boom and expanded reverse logistics activities. The E-commerce trend includes faster, more dependable delivery, increased inventory turnover, and goods staged in forwarding sites near clients. There has been a large surge of 3PL firms offering a range of services to assist in maintaining this very sophisticated supply chain. 3PLs are frequently requested for assistance with E-commerce fulfilment, warehousing, and delivery facilities, and 3PLs invest in technology for both client service and internal usage. Due to the current worldwide problem, the COVID-19 pandemic, the function of E-commerce has been enhanced and expedited.

Due to the features of multidimensional decision-making difficulties, 3PL selection may well be considered a complex MCDM challenge, given the presence of statistical, interpersonal, and numerous factors in the natural decision-making phase. Given the critical nature of sustainable third-party logistics providers, there is a dearth of studies on the 3PL selection challenge in emerging economies. The 3PL sector is growing at a breakneck pace due to the rise of the e-commerce sector. Indeed, the requirement for 3PL services is projected to grow as brands and distributors seek to focus exclusively on their core industries. As a result, they frequently outsource logistical services. In a nutshell, analyzing and choosing optimum third-party logistics providers is a critical component of any business's long-term goals.

Consider a corporation that is looking for the best 3PL provider. Following prescreening, five 3PLs have been identified for further consideration $\Lambda_{\eta}^{\delta}(\eta = 1, 2, \ldots, 5)$. You must choose between the following four characteristics: (1) $\sqsupset_1^{\daleth}$ = financial stability, (2) $\sqsupset_2^{\daleth}$ = reliability and delivery time, (3) $\sqsupset_3^{\daleth}$ = reputation, and (4) $\sqsupset_4^{\daleth}$ = green operation. The DM distributes the attribute weight in the following way: $\xi^{\zeta} = (0.20, 0.30, 0.25, 0.25)^T$. Table 1 evaluates the five candidates $\Lambda_{\eta}^{\delta}(\eta = 1, 2, \ldots, 5)$.

6.2. Proposed Method Based on SFAAWA Operator. To select the best agriculture land $\Lambda^{\delta}\eta(\eta = 1, 2, \ldots, \upsilon)$, we utilize the SFAAWA operator to construct an MCDM premise with SF information, which is usually evaluated as follows:

Step 1: declaration matrix $D = (\mathscr{P}_{ij})_{m\times n}$ in the format of SFNs from the DM given in Table 1.

Step 2: there is no cost type attributes in the four characteristics: (1) $\sqsupset_1^{\daleth}$ = financial stability, (2) $\sqsupset_2^{\daleth}$ = reliability and delivery time, (3) $\sqsupset_3^{\daleth}$ = reputation, and (4) $\sqsupset_4^{\daleth}$ = green operation. So, there is no need of normalization in Table 1.

Step 3: consider that $\aleph = 4$. We take the SFAAWA operator to calculate the alternative values K_{η} of five applicants $\Lambda_{\eta}^{\delta}(\eta = 1, 2, \ldots, 5)$, $\breve{F}_1 = (0.23595, 0.0331817, 0.0329636)$, $\breve{F}_2 = (0.27865, 0.214317, 0.234073)$ $\breve{F}_3 = (0.308017, 0.19699, 0.300857)$, $\breve{F}_4 = (0.352767, 0.189726, 0.152533)$, and $\breve{F}_5 = (0.23513, 0.181907, 0.237248)$.

Step 4: evaluate the score functions $S(\breve{F}_{\eta})(\eta = 1, 2, \ldots, 5)$ of the SFNs $\breve{F}_{\eta}(\eta = 1, 2, \ldots, 5)$ as $S(\breve{F}_1) = 0.0412034$, $S(\breve{F}_2) = 0.00159675$, $S(\breve{F}_3) = -0.0107371$, $S(\breve{F}_4) = 0.0387103$, and $S(\breve{F}_5) = -0.00305816$.

Step 5: classify all five candidates according to the value of the SFN's score functions as $\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_5^{\delta} > \Lambda_3^{\delta}$. Λ_5^{δ} is chosen as the most suitable agriculture land.

Table 1: Spherical fuzzy decision matrix.

	$\beth_1^{\gimel}$	$\beth_2^{\gimel}$	$\beth_3^{\gimel}$	$\beth_4^{\gimel}$
Λ_1^{δ}	(0.173, 0.144, 0.108)	(0.243, 0.050, 0.203)	(0.253, 0.011, 0.113)	(0.223, 0.133, 0.009)
Λ_2^{δ}	(0.333, 0.244, 0.143)	(0.143, 0.223, 0.333)	(0.228, 0.151, 0.418)	(0.233, 0.433, 0.243)
Λ_3^{δ}	(0.368, 0.263, 0.273)	(0.133, 0.208, 0.543)	(0.233, 0.150, 0.218)	(0.263, 0.223, 0.431)
Λ_4^{δ}	(0.218, 0.463, 0.133)	(0.248, 0.413, 0.243)	(0.413, 0.150, 0.138)	(0.163, 0.443, 0.133)
Λ_5^{δ}	(0.141, 0.152, 0.463)	(0.268, 0.456, 0.163)	(0.118, 0.413, 0.258)	(0.213, 0.113, 0.433)

Table 2: Score functions for different values of $\aleph$.

$\aleph$	$S(\breve{F}_1)$	$S(\breve{F}_2)$	$S(\breve{F}_3)$	$S(\breve{F}_4)$	$S(\breve{F}_5)$	Ordering
4	0.0412034	0.00159675	-0.0107371	0.0387103	-0.00305816	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
10	0.0508129	0.0142563	0.00071527	0.059049	0.0000415248	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
15	0.053227	0.020284	0.00521721	0.0648227	0.00229417	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
20	0.0545411	0.023817	0.00785135	0.0678921	0.00366547	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
40	0.0568068	0.0296565	0.0124697	0.0727274	0.00595754	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
60	0.0576824	0.0317262	0.0141957	0.0744342	0.00677951	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
80	0.0581399	0.0327826	0.0150882	0.0753273	0.00720298	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$
100	0.0584175	0.0334233	0.0156325	0.0758855	0.0074612	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$

Table 3: Comparison of proposed AOs with some exiting operators.

Authors	AOs	Ranking of alternatives	The optimal alternative
Ashraf and Abdullah [59]	SFEWA	$\Lambda_1^{\delta} > \Lambda_2^{\delta} > \Lambda_5^{\delta} > \Lambda_4^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}
	SFEWG	$\Lambda_1^{\delta} > \Lambda_2^{\delta} > \Lambda_5^{\delta} > \Lambda_3^{\delta} > \Lambda_4^{\delta}$	Λ_1^{δ}
Jin et al. [48]	LSFWA	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$	Λ_1^{δ}
	LSFWG	$\Lambda_1^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_4^{\delta} > \Lambda_5^{\delta}$	Λ_1^{δ}
Ashraf et al. [49]	SFDWA	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta} > \Lambda_5^{\delta}$	Λ_1^{δ}
	SFDWG	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_5^{\delta} > \Lambda_2^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}
Ashraf et al. [41]	SFNWAA	$\Lambda_1^{\delta} > \Lambda_2^{\delta} > \Lambda_4^{\delta} > \Lambda_5^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}
	SFNWGA	$\Lambda_1^{\delta} > \Lambda_5^{\delta} > \Lambda_2^{\delta} > \Lambda_4^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}
Proposed	SFAAWA	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_4^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}
	SFAAWG	$\Lambda_1^{\delta} > \Lambda_4^{\delta} > \Lambda_2^{\delta} > \Lambda_4^{\delta} > \Lambda_3^{\delta}$	Λ_1^{δ}

7. Sensitively Analysis

To highlight the influence of different parameter $\aleph$ magnitudes, we use different parameter $\aleph$ inside our discussed methodologies to classify the options. The ordering effects of the options based on the SFAAWA operator are reported in Table 2. It is clear that as the magnitude of $\aleph$ for the SFAAWA operator increases, the score values of the alternatives gradually increase, but the optimum alternative remains constant, implying that the proposed techniques have the property of isotonicity, and the DMs can choose the appropriate value based on their preferences.

Additionally, we can observe that even though the value of $\aleph$ is varied throughout the demonstration, the results produced from the alternatives appear to be consistent, confirming the proposed operator's robustness.

8. Comparative Analysis

This section compares proposed AOs to several existing AOs. We equate our findings by solving the data with preexisting AOs and get a comparable optimum solution. This demonstrates the AO's long-term viability and efficacy. The technique outlined here is more practical and superior to a number of previously published AOs. We validate our optimal solution by running it through numerous current operators. We receive the same optimal decision, demonstrating the validity of our proposed AOs. Comparison of proposed AOs with some exiting operators is given in Table 3.

9. Conclusion

The main contribution of the work is outlined as follows:

(1) The aim of this paper is to present a novel idea about the operational laws and the AOs for the aggregation of SFNs. For this we first extended the Aczel–Alsina t-norm and t-conorm to SF contexts, then established and assessed a number of innovative operational principles for SFNs. The fundamental properties of the proposed laws are discussed in detail.

(2) Based on the proposed laws, we define several AOs to aggregate the SF information, namely the "spherical fuzzy Aczel–Alsina weighted averaging (SFAAWA) operator," "spherical fuzzy Aczel–Alsina ordered

weighted averaging (SFAAOWA) operator," "spherical fuzzy Aczel–Alsina hybrid averaging (SFAAHA) operator," "spherical fuzzy Aczel–Alsina weighted geometric (SFAAWG) operator," "spherical fuzzy Aczel–Alsina ordered weighted geometric (SFAAOWG) operator," and "spherical fuzzy Aczel–Alsina hybrid geometric (SFAAHG) operator." The basic axioms of the operators are also satisfied with the proposed work.

(3) The proposed operators have been applied to MCDM approach with SF data, and a numerical model illustrating the decision-making technique has been provided.

(4) To highlight the influence of parameter $\aleph$ in the decision-making process, we also added the sensitivity analysis. Moreover, we equate our findings by solving the data with preexisting AOs and get a comparable optimum solution. This demonstrates the AO's long-term viability and efficacy.

We will gradually apply the aforementioned operators and techniques to a variety of multiple applications, including network analysis, risk assessment, cognitive science, reinforcement learning, signal processing, and many domains in ambiguous contexts. Furthermore, the current study does not take into account the interrelationships between the pairs of attributes during the aggregation process, which we will do in future projects. Furthermore, we will attempt to develop some more generalized information metrics in order to better realize the information in our everyday lives.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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