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Research Article

Thermal Vibration of Rectangle Plate with Two-Dimensional Circular Thickness Effect

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Frequencies comprising two-dimensional circular thickness impacts are computed for the first time, for clamped rectangle plate under nonhomogeneity and temperature field. The study also comprises the two-dimensional temperature and one-dimensional density effect, on frequencies computation, utilizing the Rayleigh–Ritz technique. The objective of the study is to reduce the frequencies of the plate as well as to provide the effect of various plate parameters on frequencies. Numerical illustration and comparative study of the frequencies with other available results well authenticate the objective of the study. A comparison of frequency modes done for the clamped rectangle and square plate corresponding to tapering and aspect ratio and nonhomogeneity concludes that frequency obtained for the present study is very less in comparison with other studies done. A convergence study of the obtained results is also presented with the help of figures. The convergence study done concludes that rate of convergence in case of circular variation is faster than the other variations.

1. Introduction

Plate structure and plate parameters play an important impact on the study of vibration of plates or we can say that plate structure and plate parameters directly impact the vibrational behaviour of plate. To analyze the vibration of plates in meaningful and accurate manner, we have to take an appropriate structure and variation in plate parameters. The following literature shows bench mark research in the field of vibrations.

Fundamental frequency of isotropic and orthotropic rectangle plates [1] with fixed corners and different mass attachment has been calculated analytically by using Rayleigh's method. An analysis on cantilever, partially clamped rectangle plate [2] has been carried out by using the superposition method, and frequencies and mode shapes have been calculated. The Ritz method [3] has been applied to compute six modes of vibrations and shapes of clamped and simply supported circular plates with two-dimensional thicknesses. Two-dimensional thickness effects on natural frequency of rectangle plate [4] using boundary characteristic on the Rayleigh–Ritz method have been presented, and three-dimensional mode shapes have been computed. Circular density and Poisson's ratio effects on skew plate [5] with one-dimensional thickness have been analyzed and time period of frequencies has been computed. Natural frequencies of skew plate [6] with thickness variation have been computed on different boundary conditions. 2D effect of thickness and temperature on orthotropic rectangle plate [7] has been presented and time period, logarithmic decrement, and deflection have been computed. Vibration of Mindlin's rectangle plate [8] has been presented by using the Rayleigh–Ritz method, and natural frequencies and mode shapes have been computed on six boundary conditions. The variation method [9] has been applied to analyze the vibration of Mindlin's rectangle plate and showed the elastic restraints effect on frequencies. The Rayleigh–Ritz method with orthogonal plate function [10] has been applied to analyze the vibration of rectangle plate. The independent coordinate coupling method [11] has been applied to investigate frequency of rectangle plate with multiple cut-out in the shape of circle and rectangle. Frequency of simply supported rectangle plate [12] has been computed using the finite element method. Frequency of rectangle plate [13] with central cut-out has been investigated using the finite element method. Frequency of functionally graded circular plate [14] with thickness and temperature variation has been computed using GQM. Vibration of rectangle plate [15] with 2D thickness variation under temperature effect has been computed. Plane wave propagation has been studied [16–19] in different mediums.

The above literature shows that many researchers studied the vibration of plates with different structures with different variations in plate parameters and provides a useful data regarding the vibrational frequencies of plates with variable thickness with or without consideration of temperature and nonhomogeneity. Although the impact of one-dimensional circular variation on frequency of different plate structures [20–24] has been studied, no one studied the impact of two-dimensional circular thickness impact on vibration of rectangle plates with combination of linear density and two-dimensional temperature variation. This provides a motivation to us to study two-dimensional circular thickness impact on frequencies of rectangle plate along with no homogeneity and temperature. The impact of this study on frequency modes is shown with the help of tables which justify the objectives of the study. The comparison study of frequency modes for the present study with the available published results established the fact that due to circular variation in plate parameter (tapering), we can get the less vibrational and variation in frequency modes.

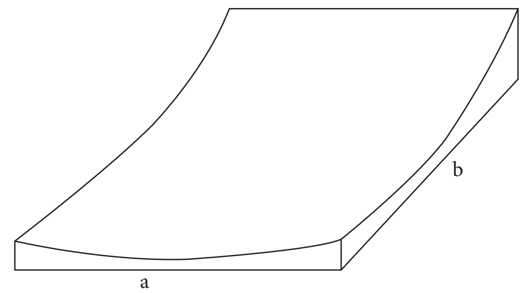

FIGURE 1: Rectangle plate with 2D circular thickness.

2. Analysis

Consider a nonhomogeneous rectangle plate with sides a and b, thickness l, and density ρ as shown in Figure 1.

The kinetic energy and strain energy for vibration of plate are taken as follows [25]:

$$T_s = \frac{1}{2}\omega^2 \int_0^a \int_0^b \rho l \Phi^2 d\psi \, d\zeta, \tag{1}$$

$$V_s = \frac{1}{2}\int_0^a \int_0^b D_1 \left[\left(\frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + \left(\frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 + 2\nu \frac{\partial^2 \Phi}{\partial \zeta^2} \frac{\partial^2 \Phi}{\partial \psi^2} + 2(1-\nu) \left(\frac{\partial^2 \Phi}{\partial \zeta \, \partial \psi} \right)^2 \right] d\psi \, d\zeta, \tag{2}$$

where Φ is the deflection function, ω is the natural frequency, and $D_1 = El^3/12(1-\nu^2)$ is the flexural rigidity; here, E and ν are Young's modulus and Poisson's ratio of the plate.

The Rayleigh–Ritz method requires maximum kinetic energy that must be equal to maximum strain energy, i.e.,

$$L = \delta(V_s - T_s) = 0. \tag{3}$$

Using (1) and (2), we have

$$L = \frac{1}{2}\int_0^a \int_0^b D_1 \left[\left(\frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + \left(\frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 + 2\nu \frac{\partial^2 \Phi}{\partial \zeta^2} \frac{\partial^2 \Phi}{\partial \psi^2} + 2(1-\nu) \left(\frac{\partial^2 \Phi}{\partial \zeta \, \partial \psi} \right)^2 \right] d\psi \, d\zeta - \frac{1}{2}\omega^2 \int_0^a \int_0^b \rho l \Phi^2 d\psi \, d\zeta = 0. \tag{4}$$

Introducing nondimensional variables $\zeta_1 = \zeta/a$, $\psi_1 = \psi/a$ together with two-dimensional circular thickness [26] and one-dimensional linear variation in density as

$$l = l_0\left(1+\beta_1\left\{1-\sqrt{1-\zeta_1^2}\right\}\right)\left(1+\beta_2\left\{1-\sqrt{1-\frac{a^2\psi_1^2}{b^2}}\right\}\right),$$
$$\rho = \rho_0(1+m\zeta_1), \tag{5}$$

where l_0 and ρ_0 are the thickness and density, respectively, at origin. Also, β_1 and β_2 are tapering parameters and m is known as nonhomogeneity parameter, respectively.

Two-dimensional steady state temperature is assumed to be linear as [23]

$$\tau = \tau_0 (1 - \zeta_1)\left(1 - \frac{a\psi_1}{b}\right), \tag{6}$$

where τ and τ_0 denote the temperature on and at the origin, respectively. The modulus of elasticity is given by

$$E = E_0 (1 - \gamma\tau), \tag{7}$$

where E_0 is Young's modulus at $\tau = 0$ and γ is called the slope of variation.

Using equations (6), (7) becomes

$$E = E_0\left(1 - \alpha (1 - \zeta_1)\left(1 - \frac{a\psi_1}{b}\right)\right), \tag{8}$$

where $\alpha = \gamma\tau_0$, $(0 \le \alpha < 1)$ is called the thermal gradient.

Using (5) and (8) and nondimensional variables, the functional in (4) becomes

$$L = \frac{D_0}{2}\int_0^1\int_0^{b/a}\left(1 - \alpha (1 - \zeta_1)\left(1 - \frac{a\psi_1}{b}\right)\right)\left(1 + \beta_1\left\{1 - \sqrt{1 - \zeta_1^2}\right\}\right)^3\left(1 + \beta_2\left\{1 - \sqrt{1 - \frac{a^2\psi_1^2}{b^2}}\right\}\right)^3 \cdot \left\{\left(\frac{\partial^2\Phi}{\partial\zeta_1^2}\right)^2 + \left(\frac{\partial^2\Phi}{\partial\psi_1^2}\right)^2 + 2\nu\frac{\partial^2\Phi}{\partial\zeta_1^2}\frac{\partial^2\Phi}{\partial\psi_1^2} + 2(1 - \nu)\left(\frac{\partial^2\Phi}{\partial\zeta_1\partial\psi_1}\right)^2\right\} \cdot d\psi_1 d\zeta_1 - \lambda^2\int_0^1\int_0^{b/a}\left[\left(1 + \beta_1\left\{1 - \sqrt{1 - \zeta_1^2}\right\}\right)\left(1 + \beta_2\left\{1 - \sqrt{1 - \frac{a^2\psi_1^2}{b^2}}\right\}\right)(1 + m\zeta_1)\right]\Phi^2 d\psi_1 d\zeta_1 = 0, \tag{9}$$

where $D_0 = E_0 l_0^3/12\,(1 - \nu^2)$ and $\lambda^2 = \rho_0\omega^2 l_0 a^4/D_0$.

Deflection function which satisfies all the boundary conditions is taken as

$$\Phi(\zeta, \psi) = \left[(\zeta_1)^e (\psi_1)^f (1 - \zeta_1)^g\left(1 - \frac{a\psi_1}{b}\right)^h\right] \cdot \left[\sum_{i=0}^{n}\Psi_i\left\{(\zeta_1)(\psi_1)(1 - \zeta_1)\left(1 - \frac{a\psi_1}{b}\right)\right\}^i\right], \tag{10}$$

where $\Psi_i, i = 0, 1, 2 \ldots n$, are unknowns and the value of e, f, g, h can be 0, 1, and 2, corresponding to free, simply supported, and clamped condition, respectively.

To minimize (9), it is required that

$$\frac{\partial L}{\partial \Psi_i} = 0, \quad i = 0, 1, \ldots, n. \tag{11}$$

Solving (11), we have frequency equation as

$$\left|P - \lambda^2 Q\right| = 0, \tag{12}$$

where $P = [p_{ij}]_{i,j=0,1,..n}$ and $Q = [q_{ij}]_{i,j=0,1,..n}$ are square matrix of order $(n + 1)$.

3. Numerical Illustration and Discussion

Frequencies of clamped rectangle plate with two-dimensional circular thickness, two-dimensional linear temperature, and one-dimensional density effects have been computed on various values of plate parameters and presented using tables. The value of Poisson's ratio $\nu = 0.345$ and aspect ratio $a/b = 1.5$ is taken in the calculation.

Two-dimensional circular thickness effects on frequencies of clamped rectangle plate are incorporated in Table 1 for three different values of thermal gradient α and nonhomogeneity m, i.e., $\alpha = m = 0.0, 0.4, 0.8$. Table 1 shows that the effect of two-dimensional circular thicknesses increases the frequencies for all three values of thermal gradient α and nonhomogeneity m. However, increase in values of nonhomogeneity m and thermal gradient α (i.e., from 0.0 to 0.8) decreases the frequencies of the plate. Frequencies increase in higher rate with respect to tapering β_2 in comparison to frequencies increasing with respect to tapering β_1.

Temperature along with two-dimensional circular thicknesses effect on frequencies of clamped rectangle plate for three different values of nonhomogeneity m, i.e., $m = 0.0, 0.4, 0.8$ is accumulated in Table 2. From Table 2, one can easily see that frequencies decrease with respect to increase in thermal gradient α for all the three values of nonhomogeneity m. However, higher the nonhomogeneity m results in the less frequencies. Frequencies increase with respect to increase in both the tapering β_1 and β_2 for all the three values of nonhomogeneity m. The rate of increment in frequencies corresponding to increase in both the tapering β_1 and β_2 is higher when compared with the rate of decrement in frequencies corresponding to thermal gradient α.

Table 1: Frequencies of clamped rectangle plate vs. tapering parameters β_1 and β_2.

β_1	$\beta_2 = 0.0$ λ_1	$\beta_2 = 0.0$ λ_2	$\beta_2 = 0.2$ λ_1	$\beta_2 = 0.2$ λ_2	$\beta_2 = 0.4$ λ_1	$\beta_2 = 0.4$ λ_2	$\beta_2 = 0.6$ λ_1	$\beta_2 = 0.6$ λ_2	$\beta_2 = 0.8$ λ_1	$\beta_2 = 0.8$ λ_2	$\beta_2 = 1.0$ λ_1	$\beta_2 = 1.0$ λ_2
$\alpha = m = 0$												
0.0	60.84	240.47	64.16	253.37	67.77	268.23	71.63	284.29	75.69	301.61	79.92	320.04
0.2	63.03	248.62	66.49	262.13	70.21	277.22	74.18	293.76	78.36	311.58	82.71	330.56
0.4	65.47	257.31	68.97	271.23	72.79	286.77	76.87	303.81	81.17	322.17	85.64	341.71
0.6	67.98	266.50	71.58	280.84	75.50	296.86	79.69	314.41	84.11	333.32	88.71	353.45
0.8	70.61	276.16	74.30	290.94	78.33	307.45	82.63	325.53	87.17	345.01	91.90	365.74
1.0	73.33	286.26	77.13	310.49	81.27	318.50	85.69	337.13	90.35	357.19	95.20	378.55
$\alpha = m = 0.4$												
0.0	52.69	208.25	55.80	220.65	59.17	234.45	62.74	249.50	66.48	265.67	70.36	282.82
0.2	54.71	215.47	57.90	228.22	61.36	242.40	65.03	257.87	68.87	274.48	72.85	292.11
0.4	56.85	223.18	60.13	236.29	63.67	250.87	67.43	266.77	71.38	283.85	75.47	301.97
0.6	59.11	231.34	62.46	244.82	66.10	259.82	69.96	276.17	74.01	293.74	787.21	312.37
0.8	61.46	239.91	64.90	253.79	68.63	269.21	72.59	286.03	76.75	304.10	81.06	323.27
1.0	63.91	248.88	67.44	263.15	71.26	279.02	75.32	296.33	79.58	314.91	84.01	334.63
$\alpha = m = 0.8$												
0.0	45.99	181.77	48.97	193.74	52.16	206.98	55.52	221.33	59.01	236.68	62.62	252.90
0.2	47.85	188.35	50.89	200.62	54.16	214.19	57.60	228.92	61.19	244.67	64.89	261.31
0.4	49.81	195.36	52.93	207.96	56.27	221.89	59.80	237.00	63.47	253.17	67.27	270.25
0.6	51.88	292.79	55.07	215.72	58.49	230.02	62.10	245.54	65.87	262.14	69.76	279.69
0.8	54.04	210.60	57.29	223.87	60.79	238.56	64.49	254.49	68.36	271.54	72.36	289.57
1.0	56.27	218.76	59.60	232.39	63.18	247.47	66.97	263.84	70.94	281.36	75.04	299.88

Table 2: Frequencies of clamped rectangle plate vs. thermal gradient α.

α	$\beta_1 = \beta_2 = 0.0$ λ_1	$\beta_1 = \beta_2 = 0.0$ λ_2	$\beta_1 = \beta_2 = 0.2$ λ_1	$\beta_1 = \beta_2 = 0.2$ λ_2	$\beta_1 = \beta_2 = 0.4$ λ_1	$\beta_1 = \beta_2 = 0.4$ λ_2	$\beta_1 = \beta_2 = 0.6$ λ_1	$\beta_1 = \beta_2 = 0.6$ λ_2	$\beta_1 = \beta_2 = 0.8$ λ_1	$\beta_1 = \beta_2 = 0.8$ λ_2	$\beta_1 = \beta_2 = 1.0$ λ_1	$\beta_1 = \beta_2 = 1.0$ λ_2
$m = 0.0$												
0.0	60.84	240.47	66.49	262.13	72.79	286.77	79.69	314.41	87.17	345.01	95.20	378.55
0.2	59.30	234.38	64.99	256.25	71.31	281.11	78.23	308.93	85.70	339.69	93.72	373.38
0.4	57.72	228.13	63.46	250.24	69.81	275.32	76.73	303.35	84.20	334.29	92.20	368.13
0.6	56.10	221.70	61.88	244.09	68.26	269.42	75.19	297.67	82.66	328.81	90.64	362.82
0.8	54.42	215.08	60.26	237.77	66.68	263.38	73.63	291.87	81.09	323.23	89.06	357.43
$m = 0.4$												
0.0	55.54	219.51	60.67	239.06	66.39	261.30	72.66	286.24	79.46	313.85	86.75	344.10
0.2	54.14	213.96	59.30	233.70	65.05	256.14	71.33	281.25	78.11	309.01	85.40	339.40
0.4	52.69	208.25	57.90	228.22	63.67	250.87	69.96	276.17	76.75	304.10	84.01	334.63
0.6	51.21	202.38	56.47	226.61	62.26	245.48	68.56	271.00	75.35	299.11	82.60	329.79
0.8	49.68	196.34	54.99	216.84	60.82	239.98	67.13	265.72	73.91	294.03	81.15	324.89
$m = 0.8$												
0.0	51.42	203.23	56.16	221.17	61.43	241.60	67.22	264.50	73.48	289.85	80.21	317.62
0.2	50.12	198.08	54.89	216.22	60.19	236.83	65.98	259.89	72.24	285.38	78.96	313.27
0.4	48.78	192.80	53.59	211.15	58.91	231.95	64.72	255.19	70.98	280.84	77.68	308.87
0.6	47.41	187.37	52.26	205.95	57.61	226.97	63.42	250.41	69.68	276.23	76.38	304.41
0.8	45.99	181.77	50.89	200.62	56.27	221.89	62.10	245.54	68.36	271.54	75.04	299.88

Nonhomogeneity along with two-dimensional circular thickness effect on frequencies of clamped rectangle plate for three different values of thermal gradient α, i.e., $\alpha = 0.0, 0.4, 0.8$ is tabulated in Table 3. Table 3 enlightens the facts that frequencies decrease with increase in nonhomogeneity m for all three values of thermal gradient α. Higher the temperature on the plate results the less frequencies and vice versa. Here, also the rate of increment in frequencies corresponding to increase in both the tapering β_1 and β_2 is higher when compared with the rate of decrement in frequencies corresponding to nonhomogeneity m like in Table 2.

4. Convergence of Results

The results show convergence for both clamped rectangle and clamped square plate (by taking aspect ratio $a/b = 1.0$ in the present study) when the order of approximation increased for all values of plate parameters in the ranges specified. This is presented in Figures 2 and 3.

Figure 2 shows the convergence of frequency modes of clamped rectangle plate for two different set values of plate parameters, i.e., for $\alpha = \beta_1 = \beta_2 = m = 0.0$, $\nu = 0.345$, and $a/b = 1.5$ and $\alpha = \beta_1 = \beta_2 = m = 0.4, \nu = 0.345$, and

Table 3: Frequencies of clamped rectangle plate vs. nonhomogeneity m.

m	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = \beta_2 = 0.2$		$\beta_1 = \beta_2 = 0.4$		$\beta_1 = \beta_2 = 0.6$		$\beta_1 = \beta_2 = 0.2$		$\beta_1 = \beta_2 = 1.0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
						$\alpha = 0.0$						
0.0	60.84	240.47	66.49	262.13	72.79	286.77	79.69	314.41	87.17	345.01	95.20	378.55
0.2	58.01	229.28	63.38	249.80	69.37	273.15	75.93	299.33	83.05	328.32	90.68	360.09
0.4	55.54	219.51	60.67	239.06	66.39	261.30	72.66	286.24	79.46	313.85	86.75	344.10
0.6	53.36	210.90	58.28	229.60	63.77	250.87	69.78	274.73	76.29	301.13	83.29	330.06
0.8	51.42	203.23	56.16	221.17	61.43	241.60	67.22	264.50	73.48	289.85	80.21	317.62
						$\alpha = 0.4$						
0.0	57.72	228.13	63.46	250.24	69.81	275.32	76.73	303.35	84.20	334.29	92.20	368.13
0.2	55.04	217.51	60.49	238.47	66.53	262.24	73.11	288.80	80.21	318.13	87.82	350.18
0.4	52.69	208.25	57.90	228.22	63.67	250.87	69.96	276.17	76.75	304.10	84.01	334.63
0.6	50.62	200.08	55.62	219.19	61.15	240.85	67.18	265.06	73.69	291.78	80.66	320.98
0.8	48.78	192.80	53.59	211.15	58.91	231.95	64.72	255.19	70.98	280.84	77.68	308.87
						$\alpha = 0.8$						
0.0	54.42	215.08	60.26	237.73	66.68	263.38	76.63	291.87	81.09	323.23	89.06	357.43
0.2	51.89	205.07	57.45	226.59	63.55	250.86	70.15	277.88	77.25	307.60	84.83	340.00
0.4	49.68	196.34	54.99	216.84	60.82	239.98	67.13	265.72	73.91	294.03	81.15	324.89
0.6	47.73	188.64	52.82	208.26	58.41	230.40	64.47	255.03	70.97	282.12	77.92	311.63
0.8	45.99	181.77	50.89	200.62	56.27	221.89	62.10	245.54	68.36	271.54	75.04	299.88

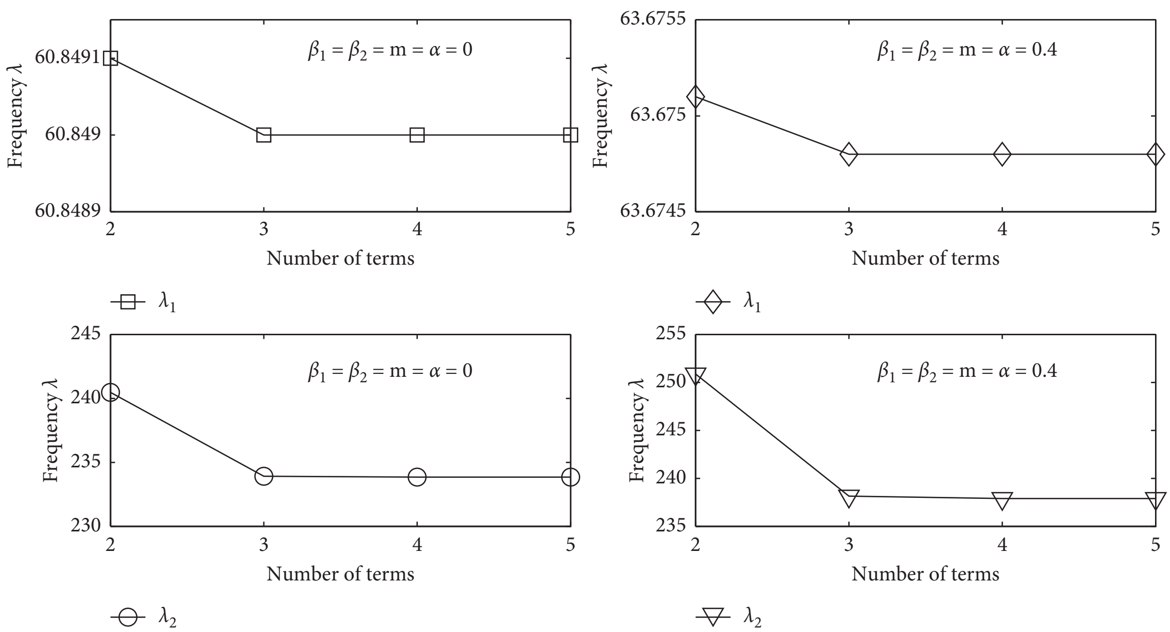

Figure 2: Convergence of frequencies for clamped rectangle plate.

$a/b = 1.5$. The first two modes of frequencies of clamped rectangle plate upto four decimal places converge in the fifth approximation.

Figure 3 shows the convergence of frequency modes of square plate for two different set values of plate parameters, i.e., for $\alpha = \beta_1 = \beta_2 = m = 0.0, \nu = 0.345$, and $a/b = 1.0$ and $\alpha = \beta_1 = \beta_2 = m = 0.4, \nu = 0.345$, and $a/b = 1.0$. The first two modes of frequencies of clamped square upto four decimal places converge in the fifth approximation.

4.1. Results Comparison. To well present the objectives of the study, a comparison of frequencies modes of clamped rectangle plate (corresponding to tapering parameters β_1 and β_2, aspect ratio a/b, and nonhomogeneity m) and clamped square plate (corresponding to tapering parameter β_1 only) with available published results is also given with the help of tables. The frequency modes obtained for clamped rectangle plate (present study) are compared with the frequency modes obtained in [15, 23, 27]. The frequency modes obtained for clamped square (present study) are compared with frequency modes obtained in [28–30].

Table 4 shows the comparison of frequencies of clamped rectangle plate (present study) with the frequencies obtained in [15, 27] corresponding to both tapering parameters β_1 and β_2, for fixed value of thermal gradient $\alpha = 0$,

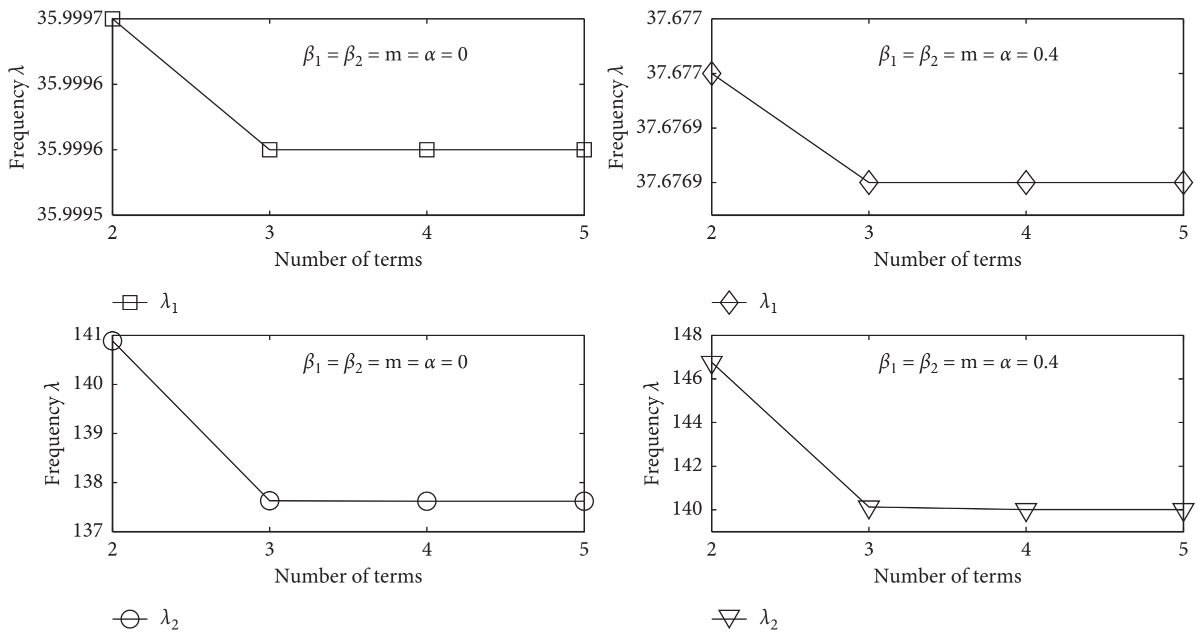

Figure 3: Convergence of frequencies for clamped square plate.

Table 4: Comparison of frequency modes λ of the present study (clamped rectangle plate) and obtained by [15, 27] versus tapering parameters β_1 and β_2 for fixed value of $\alpha = m = 0.0$ and $a/b = 1.5$.

β_1	$\beta_2 = 0.0$		$\beta_2 = 0.2$		$\beta_2 = 0.4$		$\beta_2 = 0.6$		$\beta_2 = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
	60.84	240.47	64.16	253.37	67.77	268.23	71.63	284.29	75.69	301.61
0.0	60.84[a]	240.47[a]	70.26[a]	277.75[a]	76.32[a]	302.35[a]	82.85[a]	329.48[a]	89.75[a]	359.70[a]
	64.77[b]	255.98[b]	71.60[b]	283.07[b]	79.01[b]	312.62[b]	86.48[b]	344.00[b]	94.96[b]	376.75[b]
	63.03	248.62	66.49	262.13	70.21	277.22	74.18	293.76	78.36	311.58
0.2	68.92[a]	271.43[a]	74.65[a]	294.19[a]	81.04[a]	320.15[a]	87.94[a]	348.78[a]	95.21[a]	379.61[a]
	71.40[b]	282.12[b]	78.93[b]	311.98[b]	87.09[b]	344.54[b]	95.72[b]	379.12[b]	104.66[b]	415.21[b]
	65.47	257.31	68.97	271.23	72.79	286.77	76.87	303.81	81.17	322.17
0.4	73.21[a]	287.81[a]	79.36[a]	311.85[a]	86.10[a]	339.26[a]	93.37[a]	369.47[a]	101.04[a]	425.68[a]
	78.27[b]	309.17[b]	86.53[b]	341.88[b]	95.47[b]	377.56[b]	104.91[b]	415.43[b]	114.71[b]	454.96[b]
	67.98	266.50	71.58	280.84	75.50	296.86	79.69	314.41	84.11	332.32
0.6	77.97[a]	305.20[a]	84.35[a]	330.58[a]	91.44[a]	359.50[a]	99.09[a]	391.37[a]	107.17[a]	425.68[a]
	85.34[b]	336.90[b]	94.33[b]	372.54[b]	104.07[b]	411.40[b]	114.35[b]	452.65[b]	125.02[b]	495.70[b]
	70.61	276.16	74.30	290.94	78.33	307.45	82.63	325.53	87.17	345.01
0.8	82.85[a]	323.46[a]	89.56[a]	350.23[a]	97.02[a]	380.72[a]	105.07[a]	414.32[a]	113.58[a]	450.47[a]
	92.54[b]	365.16[b]	102.29[b]	403.78[b]	112.84[b]	445.88[b]	123.98[b]	490.57[b]	135.93[b]	537.20[b]

(a): value obtained by [15]; (b): value obtained by [27].

nonhomogeneity $m = 0$, and aspect ratio $a/b = 1.5$. From Table 4, it could be easily identified that for both the increasing value of tapering parameters β_1 and β_2, the frequencies of the present study are very less when compared with frequencies obtained in [15, 27].

Table 5 shows the comparison of frequencies of clamped rectangle plate (present study) with the frequencies obtained in [15, 27] corresponding to aspect ratio a/b, for fixed value of nonhomogeneity $m = 0$ and variable value of tapering parameters β_1 and β_2 and thermal gradient α, i.e., $\beta_1 = \beta_2 = \alpha = 0.0, 0.2, 0.4, 0.6, 0.8$. Here also, one could easily analyze that the frequencies obtained in the present study are very less when compared with frequencies obtained in [15, 27] with the increasing value of aspect ratio a/b.

A comparison of frequencies of clamped rectangle plate (present study) with the frequencies obtained by [23, 27] corresponding to nonhomogeneity m is depicted in Table 6, for fixed value of tapering parameters $\beta_1 = \beta_2 = 0.0$, thermal gradient $\alpha = 0.0$, and aspect ratio $a/b = 1.5$. Table 6 enlightens that the frequencies obtained in the present study are less when compared with the frequencies obtained in [23, 27] for increasing value of nonhomogeneity m.

Table 7 displays the comparison of frequency modes square plates (by taking aspect ratio $a/b = 1.0$ in the present study) with the frequencies obtained by [28–30] corresponding to one-dimensional tapering (by taking another tapering $\beta_2 = 0.0$) parameter β_1 only, for fixed value of

Table 5: Comparison of frequency modes λ of the present study (clamped rectangle plate) and obtained by [15, 27] versus aspect ratio for fixed value of $m = 0$.

a/b	$\beta_1 = \beta_2 = \alpha = 0.0$		$\beta_1 = \beta_2 = \alpha = 0.2$		$\beta_1 = \beta_2 = \alpha = 0.4$		$\beta_1 = \beta_2 = \alpha = 0.6$		$\beta_1 = \beta_2 = \alpha = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.5	24.73	98.50	26.30	105.05	28.25	112.97	30.43	122.25	32.81	132.88
	26.24(a)	104.94(a)	28.96(a)	115.80(a)	32.20(a)	129.50(a)	35.82(a)	145.99(a)	39.72(a)	165.16(a)
	26.21(b)	104.85(b)	31.26(b)	125.09(b)	37.18(b)	149.03(b)	43.92(b)	176.57(b)	51.41(b)	207.60(b)
1.0	35.99	140.88	38.45	150.02	41.30	161.06	44.50	173.98	47.99	188.77
	38.35(a)	150.10(a)	42.35(a)	165.27(a)	47.10(a)	184.37(a)	52.41(a)	207.31(a)	58.14(a)	233.97(a)
	38.32(b)	149.97(b)	45.69(b)	178.84(b)	54.36(b)	212.88(b)	64.23(b)	251.93(b)	75.20(b)	295.86(b)
1.5	60.84	240.47	64.99	256.25	69.81	245.32	75.19	297.67	81.09	323.23
	60.85(a)	256.20(a)	71.57(a)	282.38(a)	79.58(a)	315.37(a)	88.55(a)	355.05(a)	98.21(a)	401.16(a)
	64.77(b)	255.98(b)	77.23(b)	305.32(b)	91.87(b)	363.57(b)	108.54(b)	430.50(b)	127.06(b)	505.84(b)
2.0	98.52	394.01	105.23	420.21	113.01	451.89	121.72	489.03	131.25	531.53
	104.96(a)	419.78(a)	115.86(a)	463.20(a)	128.80(a)	518.02(a)	143.29(a)	583.98(a)	158.90(a)	660.66(a)
	104.87(b)	419.43(b)	125.05(b)	500.37(b)	148.74(b)	596.12(b)	175.70(b)	706.28(b)	205.66(b)	830.44(b)
2.5	148.09	596.97	158.18	636.97	169.87	685.39	182.96	742.15	197.28	807.11
	157.78(a)	636.02(a)	174.16(a)	702.29(a)	193.58(a)	786.05(a)	215.34(a)	886.90(a)	238.79(a)	1004.13(a)
	157.65(b)	635.48(b)	187.97(b)	758.20(b)	223.58(b)	903.56(b)	264.08(b)	1070.93(b)	309.10(b)	1259.62(b)

(a): value from [15]; (b): value from [27].

Table 6: Comparison of frequency modes λ of the present study (clamped rectangle plate) and obtained by [23, 27] versus nonhomogeneity m for fixed value of $a/b = 1.5$.

	$m = 0.0$		$m = 0.2$		$m = 0.4$		$m = 0.6$		$m = 0.8$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
$\beta_1 = \beta_2 = \alpha = 0.0$	60.84	240.47	58.01	229.28	55.54	219.51	53.36	210.90	51.42	203.23
	64.77(a)	255.98(a)	63.99(a)	252.89(a)	63.32(a)	250.25(a)	62.32(a)	248.03(a)	62.30(a)	246.21(a)
	64.77(b)	255.98(b)	65.73(b)	255.98(b)	66.99(b)	264.80(b)	68.65(b)	271.48(b)	70.87(b)	280.54(b)

(a): values obtained by [27]; (b): values obtained by [23].

Table 7: Comparison of frequency modes λ of the present study (for square plate) and obtained by [28–30] versus tapering β_1.

	$\beta_1 = 0.0$		$\beta_1 = 0.2$		$\beta_1 = 0.4$		$\beta_1 = 0.6$		$\beta_1 = 0.8$		$\beta_1 = 1.0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
$m = \alpha = 0.0$	35.99	140.88	37.64	147.07	39.43	153.87	41.34	161.24	43.35	169.14	45.44	177.51
	35.99(a)	140.88(a)	38.65(a)	150.95(a)	41.57(a)	162.18(a)	44.71(a)	174.40(a)	48.02(a)	187.47(a)	51.48(a)	201.25(a)
	35.99(b)	140.88(b)	39.74(b)	155.52(b)	43.74(b)	171.07(b)	47.85(b)	187.31(b)	52.12(b)	204.07(b)	56.48(b)	221.24(b)
	35.99(c)	140.88(c)	40.03(c)	156.63(c)	45.05(c)	176.19(c)	51.23(c)	200.33(c)	58.74(c)	229.90(c)	67.73(c)	265.77(c)

(a): values obtained by [28]; (b): values obtained by [29]; (c): values obtained by [30].

thermal gradient $\alpha = 0.0$ and nonhomogeneity $m = 0.0$. Table 7 shows that the frequency modes of clamped square plate are less in comparison with frequencies obtained in [28–30].

5. Conclusions

Frequency modes of clamped rectangle plate comprise the effect of plate parameters specially the effect of two-dimensional circular thickness effect is investigated. From numerical illustration and comparison, authors would like to record the following facts:

(1) Frequency modes are less in case of two-dimensional circular thickness (present study) when compared with two-dimensional parabolic thickness variation [15] and two-dimensional linear thickness variation [27]. Frequency modes of the present study and obtained in [15] coincide at $\beta_1 = \beta_2 = 0.0$ (Table 4).

(2) Frequency modes of the present study are less when compared with the frequency modes obtained in [15, 27] corresponding to aspect ratio a/b (Table 5).

(3) Aspect ratio's effect in two-dimensional circular thicknesses (present study) gives less frequencies in comparison with aspect ratio's effect in parabolic thickness [15] and aspect ratio's effect in linear thickness [27] (Table 5).

(4) Frequency modes are less in case of linear density (present study) when compared with linear Poisson's ratio [27] and exponential Poisson's ratio [23] (Table 6).

(5) Frequency modes of clamped square plate are less in case of one-dimensional circular thickness when compared with one-dimensional parabolic thickness [28], one-dimensional linear thickness [29], and one-dimensional exponential thickness [30]. The frequencies of clamped square plate (present study) and obtained in [28–30] coincide at $\beta_1 = 0.0$ (Table 7).

(6) Tapering β_2 dominates the frequencies more in comparison with tapering β_1 (Table 1).

(7) Thickness parameters affect increment in frequencies (Table 1) while nonhomogeneity and temperature affects decrement in frequencies (Tables 2 and 3).

The points 1 to 2 well present that the frequencies can be minimized by choosing appropriate variation in plate parameter and desired frequencies can be obtained. Also, the points 6 and 7 show that variation in frequencies can be easily controlled by choosing appropriate variation in plate parameters.

Data Availability

The research data used to support the findings of this study are currently under embargo, while the research findings are commercialized. Requests for data, 6 months after the publication of this article, will be considered by the corresponding authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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