

Research Article

Modeling and Optimizing Based on OTCPN in Multi-Tier Shuttle Warehousing System

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Received 11 July 2022; Revised 1 August 2022; Accepted 5 August 2022; Published 29 August 2022

Academic Editor: Dinesh Kumar Saini

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This paper models the multi-tier shuttle warehousing system and optimizes the system structure and control strategy. This kind of system is a discrete event dynamic system because it only takes place at discrete time points, while previous warehousing system models. We propose the object-oriented timed colored Petri net to model it, to improve the accuracy of the model and avoid the problem of status explosion in the process of Petri net modeling. We propose two control strategy optimization methods: one is to balance the task assignment, and the other is to change the return time point of the buffer status information. The results of numerical experiments show that consideration of both optimization strategies can reduce the total picking time by 40% ~ 62%. Additionally, we analyze the influence of the buffer capacity. The results show that the buffer capacity can reduce the total picking time, and enough amount of buffer capacity has the same effect as control strategy optimization.

1. Introduction

Multi-tier shuttle warehouse system (MSWS) is a new automatic warehouse system to store and retrieve items stored on totes. MSWS belongs to the tier-captive autonomous vehicle-based storage and retrieval system (AVS/RS) with the tier-captive shuttles; i.e., the shuttles are captive to its dedicated tier to provide quick horizontal tote movement. Moreover, the vertical tote movement is provided by the lifter located at the beginning of the aisle. Therefore, considering high-density storage and flexibility, MSWS has been adopted by more and more logistics distribution centers. Figure 1 presents the components of MSWS. A single-deep, double-side storage rack is used to store totes. The shuttles run along the aisles to transfer the totes and only hold one tote at once. The lifters, which also hold one tote once time, are dedicated to one aisle that is divided into input lifter and output lifter. The input/output point (I/O point) is located at the first tier and combined by several roller conveyors to join the I/O lifter and transportation loop. The workstations are located along the

periphery of the transportation loop and divided into input workstations and output workstations.

MSWS is controlled by the order management system. The order management system receives orders from customers and suppliers and extracts effective information to create storage and retrieval tasks. Orders occur only at discrete time points; therefore, the MSWS conforms to a discrete event dynamic system (DEDS). Common theoretical research methods for DEDS are as follows: formal language and automata, Markov chain, queuing theory, and Petri net. There are some studies through queuing theory to the model system. Marchet et al. [1] used the open queuing network (OQN) to model the tier-captive AVS/RS and evaluate the system performance. Zou et al. [2] modeled the tier-captive AVS/RS as a fork-join queuing network (FJQN) and compared the parallel and sequential processing policy. Wang et al. [3] provided the OQN to study the storage assignment optimization. Tappia et al. [4] developed the semi-open queuing network (SOQN) to study the effect of storage system technology on order throughput times and the effect of the picking station input buffer size on order picking

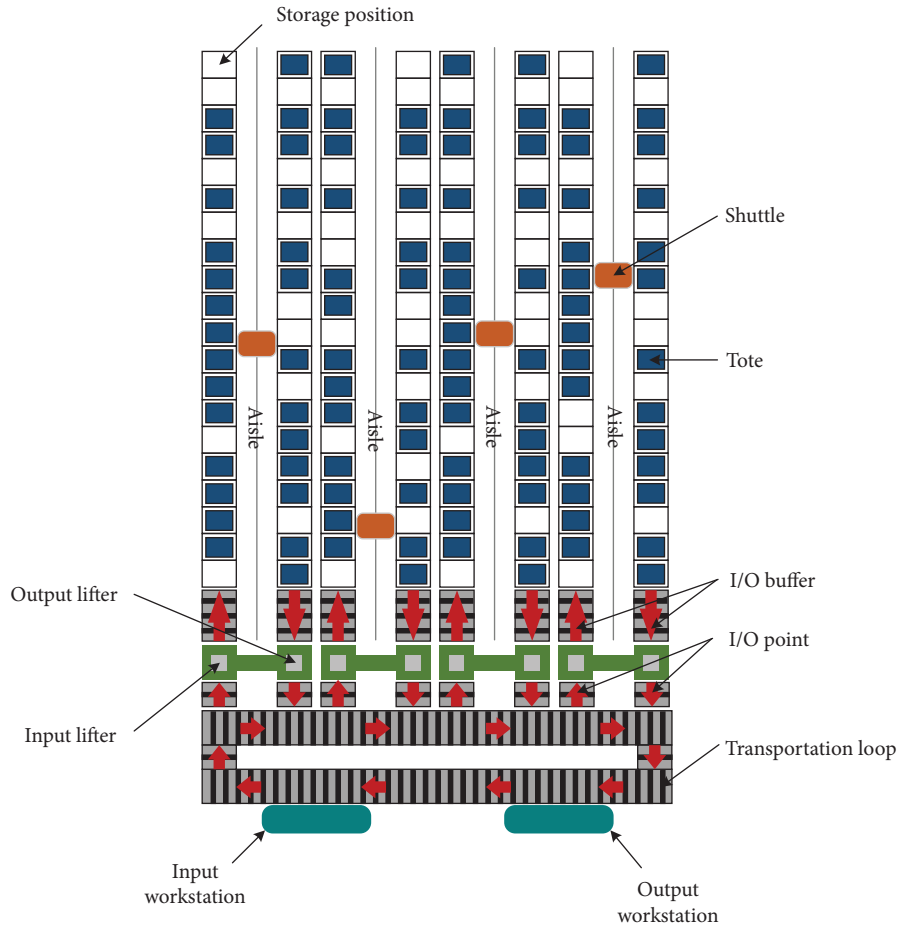


FIGURE 1: Top view of MSWS.

performance. Wu et al. [5] used the OQN to model the shuttle-based storage and retrieval system (SBS/RS) and compare the SBS/RS with the robotic order fulfillment system (ROFS) in terms of cost and system throughput. Different from the above, Lerher et al. [6, 7] proposed the travel time model to evaluate the system performance with single- and double-deep storage position. Ning et al. [8] studied the optimal solution of the tier-captive AVS/RS through an efficient simulation model. Borovinšek et al. [9] proposed a multi-objective solution including the minimization of the average cycle time of transactions, energy consumption, and total investment cost for designing the tier-captive AVS/RS. Ekren et al. [10] proposed discrete time Markov chain (DTMC) to estimate the mean and variance of travel time of AVS/RS, as well as the mean amount of energy consumption and energy regeneration per transaction for a predefined SBS/RS.

Petri net can describe the system structure visually and presents the parallel, synchronization, conflict relationship of the system. However, to the best of the knowledge, there are few studies to model the MSWS through Petri net. Qiqiang and Ran [11] and Yindi et al. [12] proposed the Petri net to model the sorting system, but the operation process of the equipment is not reflected. Tian et al. [13] studied the automated storage and retrieval system (AS/RS) which is similar to the MSWS. They developed the hierarchical colored Petri nets

and used the CPN tools to optimize the scheduling strategy. However, ordinary Petri nets are difficult to describe the operation process of complex discrete event systems. Therefore, high-level Petri nets combine object-oriented theory and give time and type constraints to make complex discrete event systems easier to understand [14]. Consequently, this paper develops the object-oriented timed colored Petri net (OTCPN) to model the MSWS and illustrates the operation process of each component. Via numerical examples and sensitivity analysis, we find that the MSWS performance is sensitive to the I/O buffer capacity and task scheduling level.

The remainder of this paper is as follows: Section 2 briefly describes the MSWS and presents the retrieval and storage transaction flow. Section 3 presents the OTCPN concept and the MSWS models. Section 4 describes the control strategy optimization method. Section 5 analyzes the optimization effect by numerical experiments. Section 6 summarizes the results, the contributions, and directions for future research.

2. MSWS Description

MSWS consists of six subsystems: the shuttle system, the input lifter system, the output lifter, the input workstation, the output workstation, and the order management system. As Figure 2 illustrates, the MSWS can execute both storage

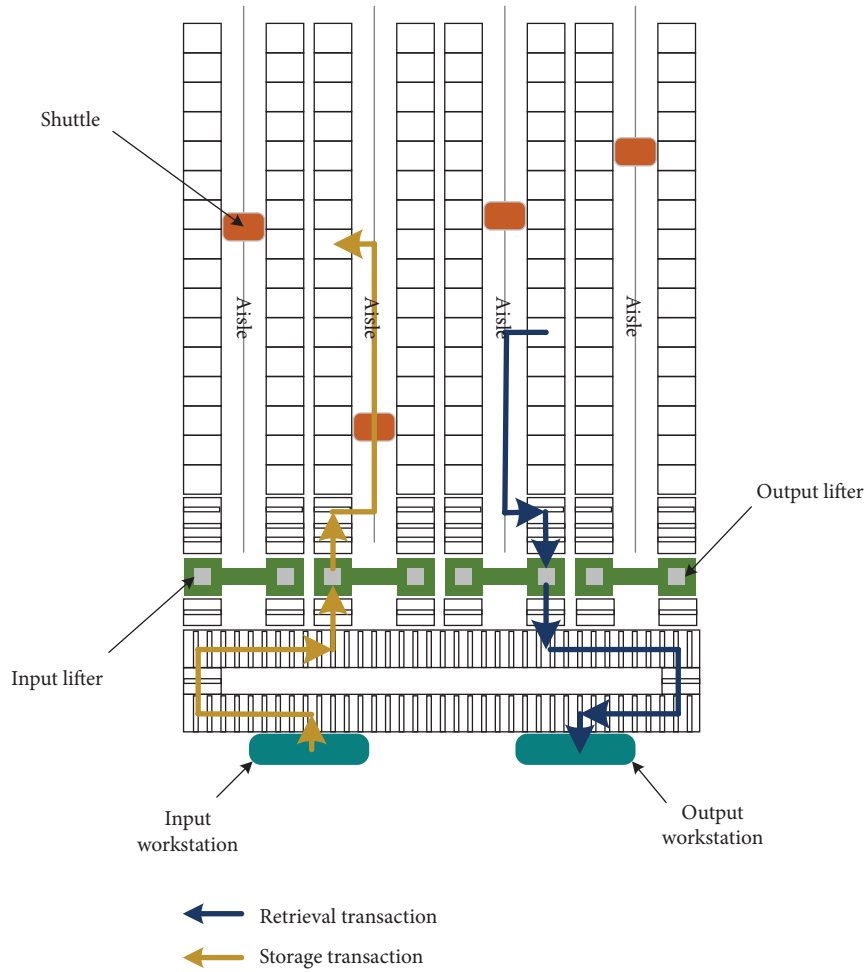


FIGURE 2: Description of the operation process in the MSWS.

and retrieval transactions concurrently. The storage transaction is initiated by the totes at the input workstation. The operators complete the packing and send the storage information to the order management system. Then, the transportation loop moves the totes to the input point. Simultaneously, the order management system sends the information to the lifter.

Subsequently, the lifter picks up the totes and drops them at the input buffer of the destination tier. Finally, the shuttle of the designated tier travels from its dwell point to the input buffer and picks up the totes. The storage transaction is completed when the totes are transferred to the destination storage position by the shuttle. Similarly, the retrieval transaction starts when the order management system sends this retrieval information to shuttles. Then, the shuttle transports the totes from their storage position to the output buffer. Then, the lifter reaches the destination tier, picks up this tote, and transfers it to the output point located at the first tier. The tote is released and conveyed to the output workstation by the transportation loop. At last, the operator picks the items from the tote according to the order.

As mentioned previously, the lifter provides vertical transaction from tier to tier, while the shuttle undertakes the horizontal transaction at the destination tier. Under the

parallel processing of transactions in the MSWS, the I/O buffer become the one factor to determine the status of the shuttles and the lifters. Namely, the input lifter is unavailable for one tier if the input buffer is occupied in this tier. Similarly, the shuttle is unavailable because the output buffer is occupied. Moreover, the shuttles and the lifters follow the first-come-first-serve (FCFS) service and point-of-service-completion (POSC) dwell point policy.

3. OTCPN Modeling for MSWS

3.1. OTCPN Concept. A basic Petri net (PN) model can be represented by a directed bipartite graph, which includes two types of nodes: places and transitions. Places $P = \{p_i\}_m$ are represented by circles and used to describe the resources and job status. Transactions $T = \{t_j\}_n$ are represented by blocks and used to describe the events and processes. These two types of nodes are linked by arrowed arcs, e.g., from the places to the transitions or from the transitions to the places, but the arc cannot connect two nodes of the same types.

The basic PN model can describe parallelism, synchronization, conflict, and causality, but it is very difficult to describe a system with high complexity only by using the basic PN model. Considering the complexity and hierarchy

of the MSWS, the object-oriented timed colored Petri net (OTCPN) combines the object Petri net (OPN), timed Petri net (TPN), and colored Petri net (CPN) to evaluate systems' performance and optimize scheduling [15].

Definition 1. An object-oriented timed colored Petri net is a tuple: $OTCPN = (OP, OT, C, E^t, I, O, N, M_0)$, where:

- (1) $OP = P \cup IMP \cup OMP$, OP is the set of places and composed of three types of finite nonempty places. P is the set of common places within the object subnet, which is used to represent the tasks and devices status in MSWS. IMP is the set of information input place between object subnets, and OMP is the set of information output place between object subnets.
- (2) $OT = T \cup G$, OT is the set of transitions and composed of T and G . T is the set of common transitions within the object subnet, which is used to represent the processing of tasks in MSWS. G is the set of condition transitions between object subnet, which determines the direction of information transfer based on the properties and status of the object.
- (3) C is the set of multiple colors. $C(P)$ represents the set of all types of colors that belongs to the token in the places P , and $C(T)$ represents the set of all types of colors that appears on the transitions T . In MSWS, the color set $Task = \{0, 1, \dots, T\}$ represents the shuttles, and the color set $Type = \{0, 1\}$ represents the task type. Therefore, $TypeTask$ is defined as the Cartesian product of $Type$ and $Task$ to represent the tasks assigned to each device.
- (4) E^t introduces the concept of time stamps so that the places P and the transitions T have time variables. In a TCPN, it represents the firing of the transitions T which takes a certain time duration and is represented as "@+time value" [16].
- (5) $I(OP, OT) = I(P, T) \cup I(OMP, G)$, $I(P, T)$ is the input function from the places P to the transitions T and represented by the colored directed arcs from P to T : $C(P) \times C(T) \rightarrow N$, $N = 0, 1, 2, \dots$. $I(OMP, G)$ is the input function from the information output places OMP to the condition transitions G and represented by the colored directed arcs from OMP to G : $C(OMP) \times C(G) \rightarrow N$, $N = 0, 1, 2, \dots$.
- (6) $O(OT, OP) = O(T, P) \cup O(G, IMP)$, $O(T, P)$ is the output function from the transitions T to the places P and represented by the colored directed arcs from T to P : $C(T) \times C(P) \rightarrow N$, $N = 0, 1, 2, \dots$. $O(G, IMP)$ is the output function from the conditions transitions G to the information input places IMP and represented by the colored directed arcs from G to IMP : $C(G) \times C(IMP) \rightarrow N$, $N = 0, 1, 2, \dots$.
- (7) N is the finite sets of all objects in the system.
- (8) M_0 is the initial marking function.

3.2. MSWS Model. Figure 3 presents the input workstation model. $IMP11$ means the storage tasks and items are ready.

$P11$ means the input workstation is idle. $P12$ means the operator is idle. $P13$ means the information is correct. $P14$ means the totes and items are ready to transport. $T11$ means the operator checks item information. The duration of this process is "operation_time." $T12$ means the operator binds items and totes. $T13$ means the operator puts the totes on the transportation loop. $OMP11$ means the operation is completed.

Figure 4 presents the order assignment system model. $IMP21$ means all types of tasks are in the task pool. $IMP22$ means there is an idle storage position. $IMP23$ means the order management system receives the storage tasks. $P21$ means the totes get the storage position. $P22$ means the storage task is ready. $P23$ means the retrieval task is ready.

$T21$ means the system assigns the storage position. $P23$ means the retrieval task is ready. $T21$ means the system assigns the storage position. $T23$ means the system selects the storage tasks with the condition of $ty = 0$. $T24$ means the system releases the storage tasks. $T25$ means the system selects the retrieval tasks with the condition of $ty = 1$. $T26$ means the system releases the retrieval tasks. $OMP21$ means the storage task assignment is completed. $OMP22$ means the retrieval task assignment is completed.

Figure 5 presents the input lifter model. $IMP31$ means the input lifter receives the storage tasks. $IMP32$ means there are enough input buffers to use. $P31$ means the input lifter is ready to load the totes. $P32$ means the input lifter is ready to transport the totes. $P33$ means the input lifter is ready to unload the totes. $P34$ means the input lifter is idle. $T31$ means the input lifter travels to the first tier. $T32$ means the input lifter loads the totes. $T33$ means the input lifter transports the totes to the destination tier. $T34$ means the input lifter unloads the totes. The condition of $t = b$ represents there is one buffer to hold the totes. $OMP31$ means the input lifter completes the storage task. The duration of $T31, T33$ is "operation_time" and calculated with considering the acceleration/deceleration of the lifter [5].

Figure 6 presents the shuttle model. $IMP41$ means the shuttle receives the storage tasks. $IMP42$ means the shuttle receives the retrieval tasks. $IMP43$ means there are enough output buffers to use. $P41, P42$, and $P45$ mean the shuttle is ready to transport the totes. $P43$ and $P46$ mean the shuttle is ready to unload the totes. $P44$ means the shuttle is ready to load the totes. $P47$ means the shuttle is idle. $T41$ means the shuttle travels to the input buffer. $T42$ and $T46$ mean the shuttle loads the totes. $T43$ means the shuttle transports the totes to the destination storage position. $T44$ and $T48$ mean the shuttle unloads the totes. $T45$ means the shuttle travels to the destination storage position. $T47$ means the shuttle transports the totes to the output buffer. $OMP41$ means the output buffer is idle. $OMP42$ means the shuttle completes the retrieval task. $OMP43$ means the shuttle completes the storage task. The duration of $T41, T43, T45, T47$ is "operation_time" and calculated with considering the acceleration/deceleration of the shuttle.

Figure 7 presents the output lifter model. $IMP51$ means the output lifter receives the retrieval tasks. $P51$ means the output lifter is ready to load the totes. $P52$ means the output lifter is ready to transport the totes. $P53$ means the output

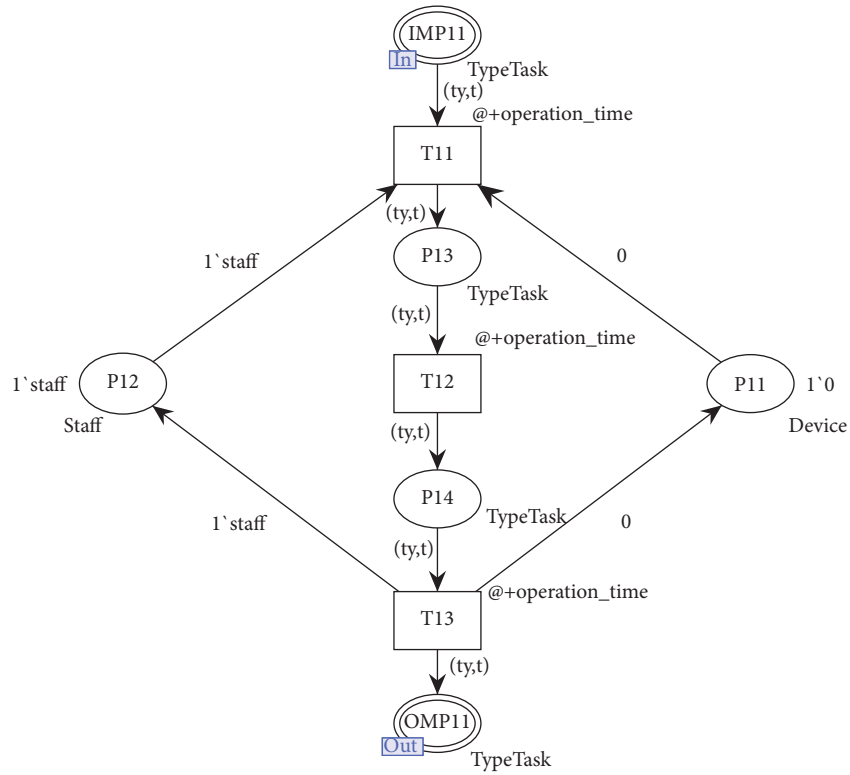


FIGURE 3: Input workstation model.

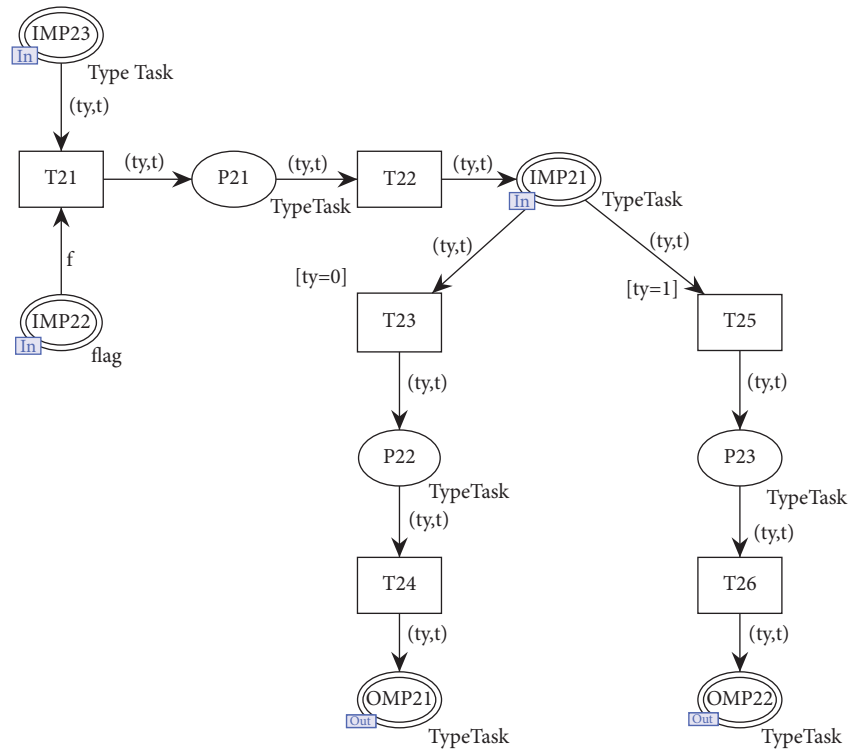


FIGURE 4: Order management system model.

lifter is ready to unload the totes. $P54$ means the output lifter is idle. $T51$ means the output lifter travels to the destination tier. $T52$ means the output lifter loads the totes. $T53$ means

the output lifter transports the totes to the first tier. $T54$ means the output lifter unloads the totes. $OMP51$ means the output buffer is idle. $OMP52$ means the output lifter

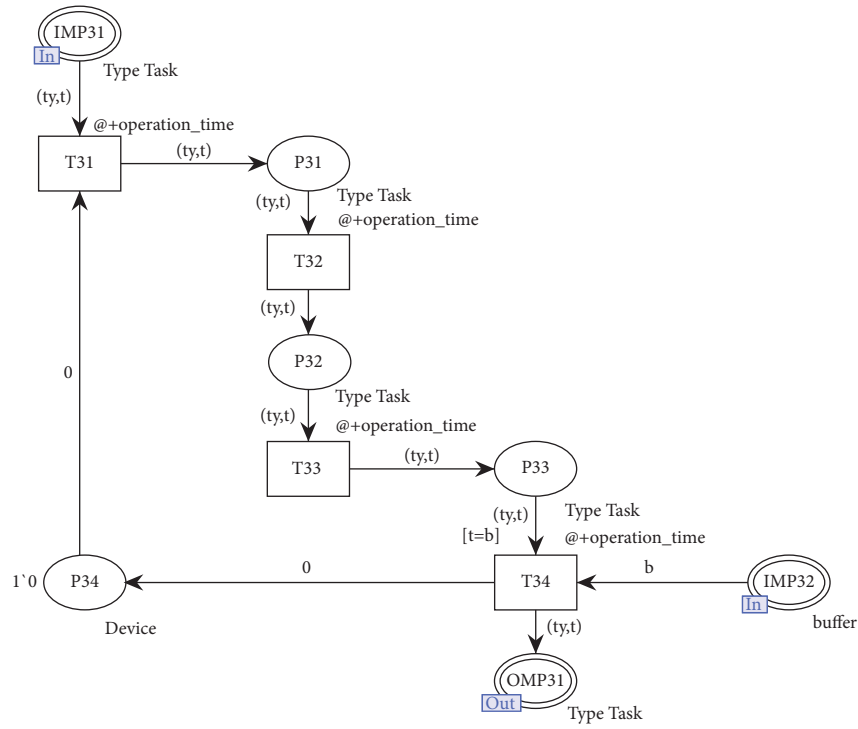


FIGURE 5: Input lifter model.

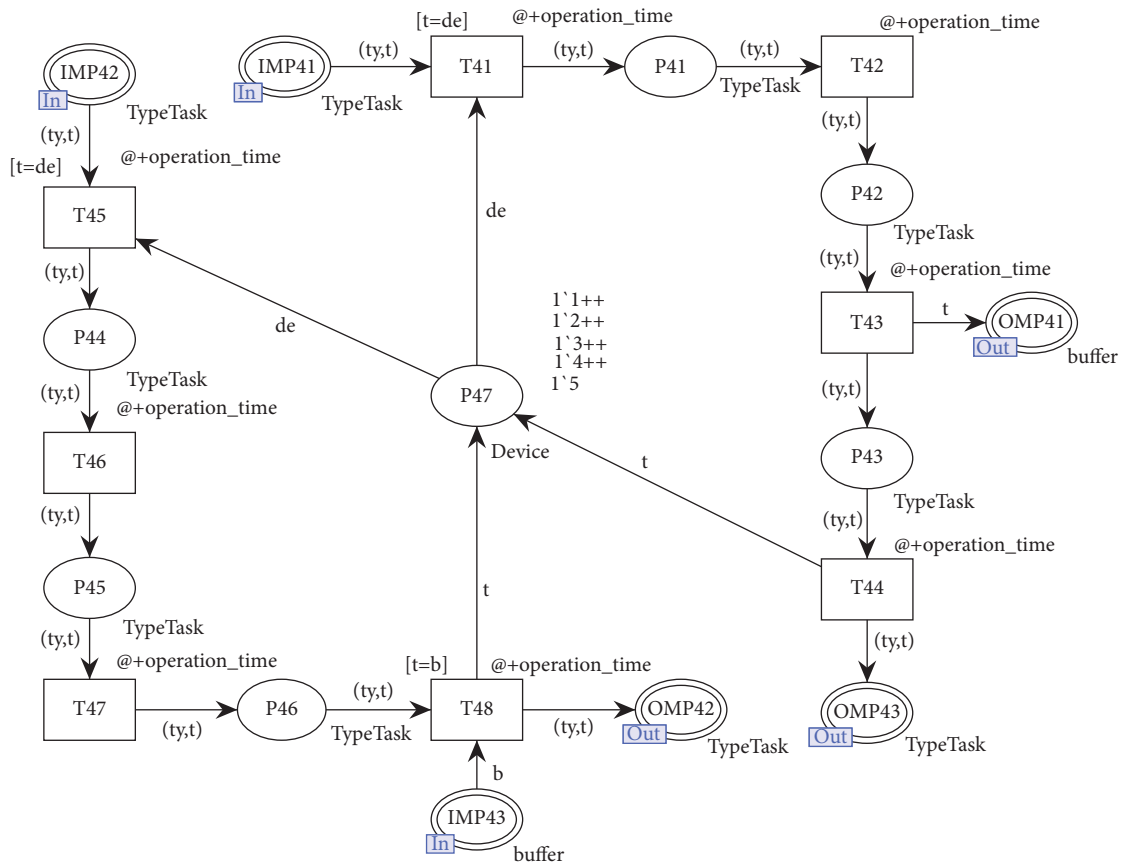


FIGURE 6: Shuttle model.

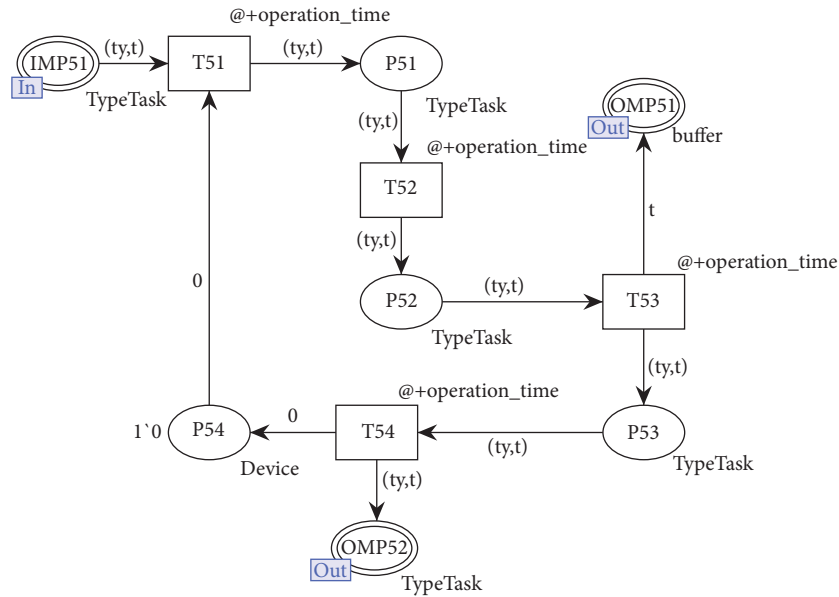


FIGURE 7: Output lifter model.

completes the retrieval task. The duration of T_{51}, T_{53} is “operation_time” and calculated with considering the acceleration/deceleration of the lifter.

Figure 8 presents the output workstation model. IMP_{61} means the output workstation receives the retrieval task. P_{61} means the operator is idle. P_{62} means the input workstation is idle. P_{63} means the information is correct. T_{61} means the operator checks item information. T_{62} means the operator picks the items by orders. OMP_{61} means the operation is completed.

Figure 9 presents the MSWS model. G_{12} means the input workstation transmits storage task information to the order management system. G_{23} means the order management system transmits storage task information to the input lifter. G_{24} means the order management system transmits retrieval task information to the shuttle. G_{34} means the input lifter transmits storage task information to the shuttle. G_{43} means the shuttle transmits the input buffer information to the input lifter. G_{45} means the shuttle transmits the retrieval task information to the output lifter and the storage position information to the order management system. G_{54} means the output lifter transmits the output buffer information to the shuttle. G_{56} means the output lifter transmits the retrieval task information to the output workstation.

4. Scheduling Strategy Optimization

MSWS is a discrete event system with multiple devices (e.g., shuttles, I/O lifters, and I/O workstations) working in coordination and parallel. The most direct way to improve the system efficiency is to increase the number of devices or operating speed, i.e., to increase the throughputs per unit time. However, this will greatly increase equipment investment costs, or it may not be possible due to the mechanical and electrical structural constraints. Therefore, without changing the number and

performance of the equipment, and without increasing the cost, scheduling strategy optimization is the better choice to increase the parallel working time of equipment and reduce the total time of order picking. This study considers the task balance strategy as the task assignment optimization (TA-opt); i.e., the order management system evenly allocates storage tasks and retrieval tasks according to the number of shuttles and lifters. Through TA-opt, we can increase the parallel operation rate of tasks and reduce the total picking time.

Through the OTCPN analysis, the lifters and the shuttles are occupied by one task for too long time. Therefore, this paper proposes an information scheduling optimization (IS-opt) to reduce the occupied time of the equipment. From the input lifter model (see Figure 5), there are no tokens in the place of P_{34} if IMP_{32} has no matched tokens; i.e., if there is no free position in the input buffer of one tier, the input lifter will always be occupied and cannot fulfill other storage tasks. This is bound to increase the total time of storage tasks. Therefore, the IS-opt of the input lifter is as shown in Figure 10: IMP_{32} , which means there are enough input buffers to use, is changed to one of the enabling conditions. That means the input lifter needs to check whether there is enough free position in the input buffer of the destination tier before starting the storage tasks.

Also in the shuttle model (see Figure 6), the IMP_{43} , as one of the conditions for releasing the shuttles, increases the time occupied by the shuttles. For this reason, the shuttles cannot fulfill other storage tasks. Therefore, the optimization strategy is to let IMP_{43} as one of the enabling conditions for T_{45} as shown in Figure 11; i.e., when the shuttles fulfill the retrieval tasks, it needs to check whether there is enough free position in the output buffer. If no, it needs to suspend the current task and execute other tasks that meet the execution conditions first.

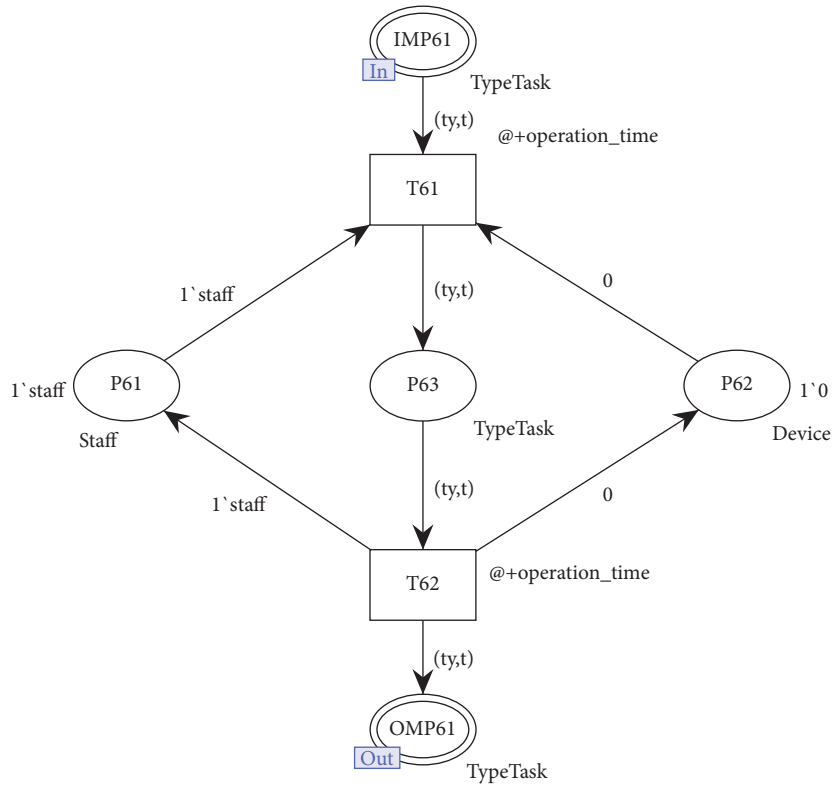


FIGURE 8: Output workstation model.

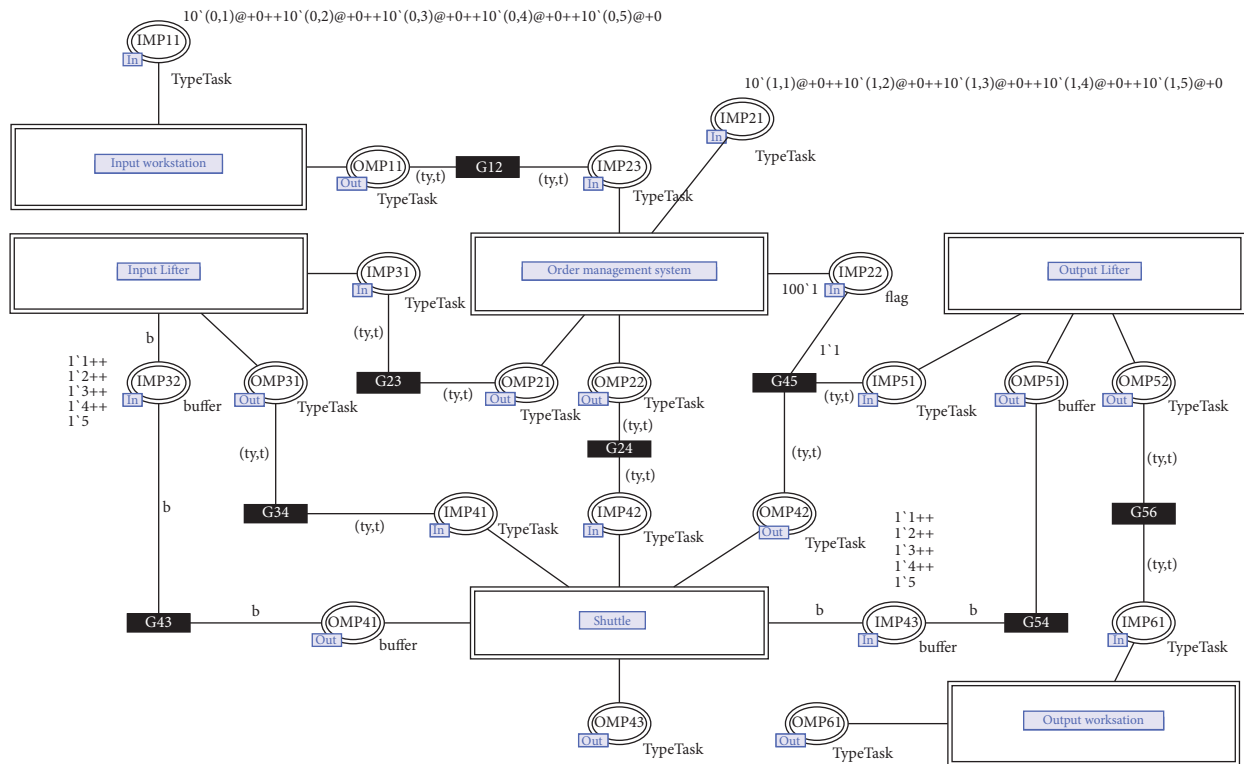


FIGURE 9: The OTCPN model of MSWS.

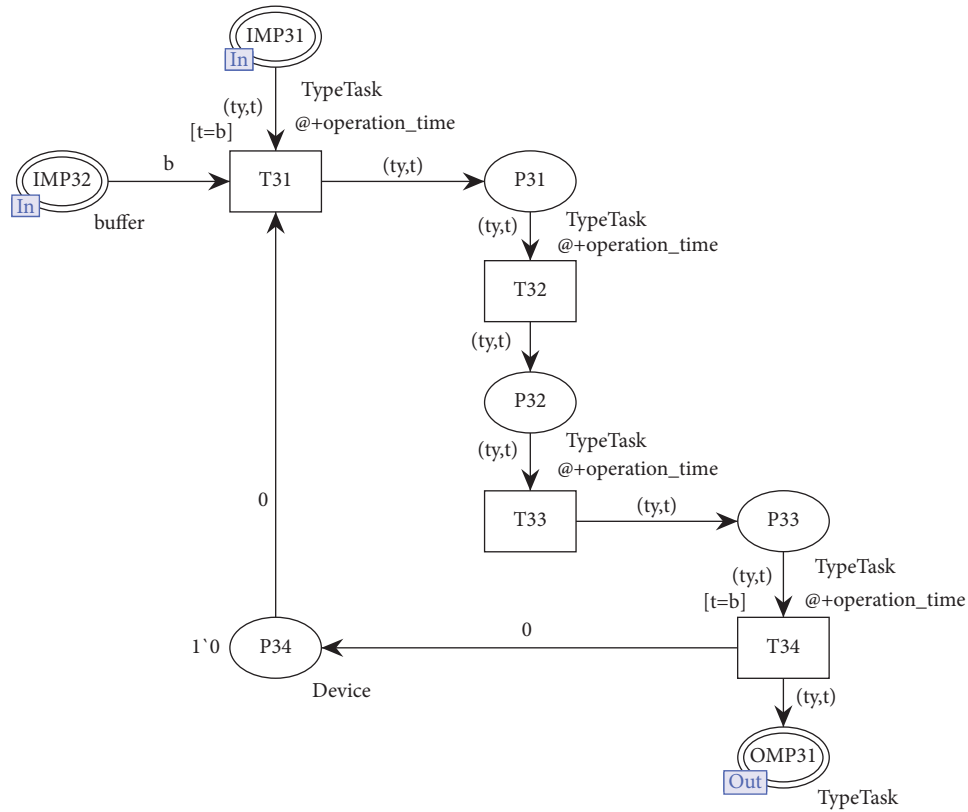


FIGURE 10: IS-opt for the input lifter.

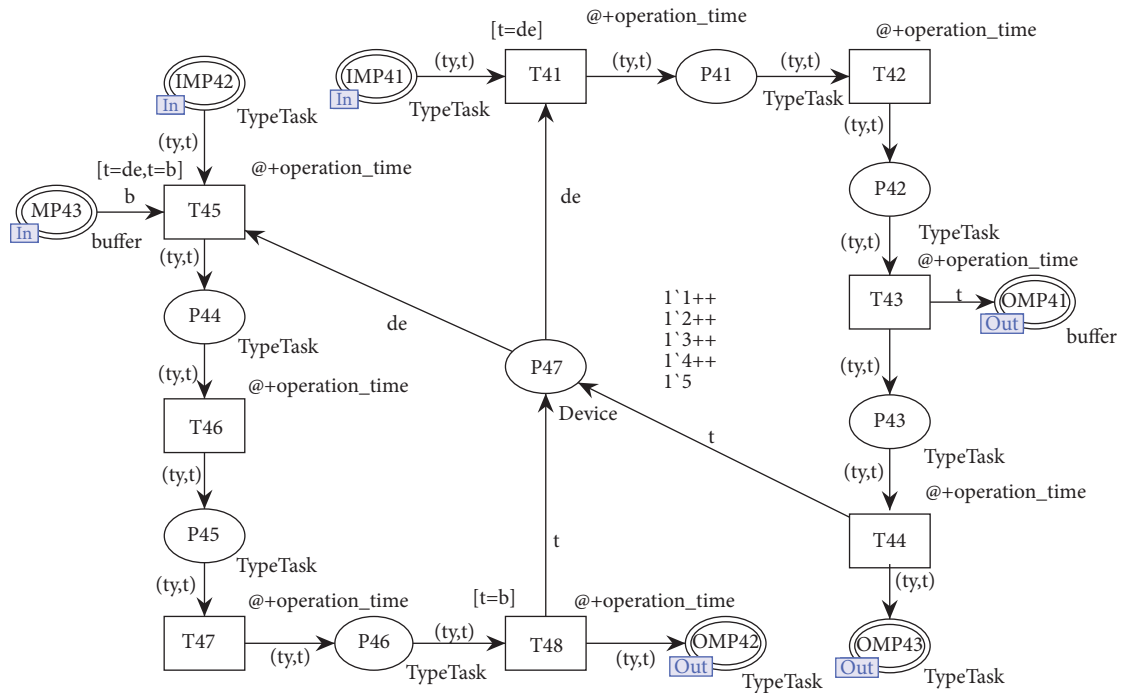


FIGURE 11: IS-opt for the shuttle.

TABLE 1: Parameters of the MSWS.

Parameters	Notation	Value
Width of a unit storage position	W_s	0.5 m
Depth of a unit storage position	D_s	0.7 m
Height of a unit storage position	H_s	0.5 m
Maximum velocity of shuttles	v_s	2 m/s
Maximum velocity of lifters	v_l	3 m/s
Acceleration/deceleration rate of shuttles	a_l	1 m/s ²
Acceleration/deceleration rate of lifters	a_l	3 m/s ²
Fixed time required for the shuttle, lifter to load or unload the tote	t_l	2 s
Fixed time required for the I/O workstation operation	t_w	15 s

TABLE 2: Comparison of the total picking time (T_{total}) under TA-opt and IS-opt.

Tier	Column	NO-opt	TA-opt	IS-opt	TA-opt + IS-opt
T = 5	C = 30	926.19	637.17	688.44	430.65
	C = 60	1228.55	763.98	750.64	461.70
	C = 90	1492.35	882.27	904.25	554.35
	C = 120	1697.25	999.66	954.70	632.75
	C = 150	1967.31	1169.38	1085.70	734.92
T = 7	C = 30	1024.11	939.88	711.30	586.41
	C = 60	1073.26	1015.20	823.15	594.49
	C = 90	1300.96	1185.26	974.34	635.53
	C = 120	1441.50	1272.07	1111.62	684.82
	C = 150	1603.75	1372.90	1279.63	774.12
T = 9	C = 30	1309.25	1235.46	755.76	748.65
	C = 60	1385.67	1336.51	789.61	754.27
	C = 90	1520.07	1478.24	879.46	777.04
	C = 120	1721.10	1585.83	1023.06	808.86
	C = 150	1891.16	1664.27	1149.85	855.72
T = 11	C = 30	1528.06	1438.08	911.52	905.52
	C = 60	1852.00	1721.46	963.75	915.17
	C = 90	1976.43	1873.07	1077.20	922.79
	C = 120	2234.68	2027.14	1244.16	940.08
	C = 150	2280.20	2018.44	1422.51	990.49
T = 13	C = 30	2003.48	1817.92	1192.24	1067.31
	C = 60	2237.61	2145.90	1327.79	1075.82
	C = 90	2514.35	2176.27	1480.56	1074.65
	C = 120	2612.89	2289.99	1631.91	1097.09
	C = 150	2825.74	2530.31	1863.11	1103.45
T = 15	C = 30	2234.74	2081.13	1225.99	1129.87
	C = 60	2391.40	2062.92	1265.52	1232.53
	C = 90	2642.82	2450.26	1250.27	1249.27
	C = 120	2897.73	2660.22	1353.61	1239.08
	C = 150	2998.06	2944.18	1470.10	1266.13

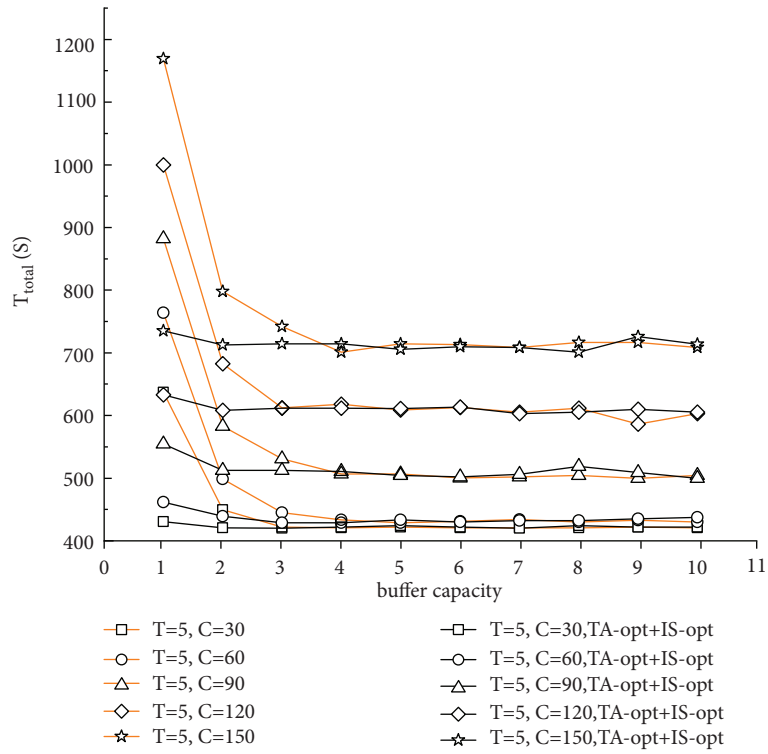
5. Numerical Experiments

5.1. Simulation of MSWS. We develop numerical experiments based on OTCPN models and consider one aisle with T tier in MSWS. The parameters of MSWS come from Table 1, and this data source is the actual device parameters of a device supplier.

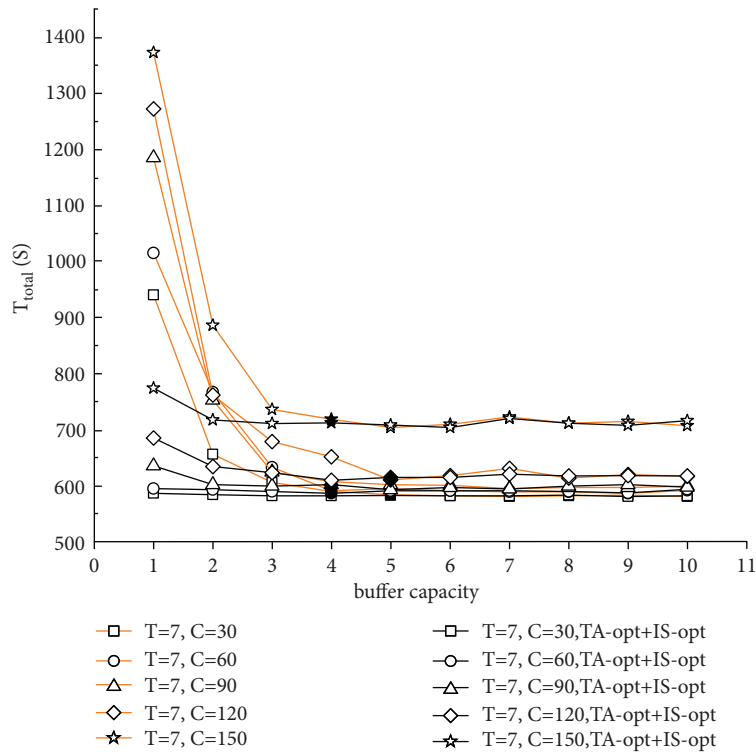
5.2. Comparison of TA-Opt and IS-Opt. This experiment is to verify the optimization effect of TA-opt and IS-opt. Therefore, the I/O buffer is set to 1 to eliminate the effect of buffer capacity. 120 scenarios are used to verify the optimization based on the variation of the number of tiers (T)

and the number of columns (C). In order to eliminate the bias, the experiment was repeated 50 times. The average value of the total operation time (T_{total}) is used.

Table 2 shows the result of total picking time T_{total} . The NO-opt column as the baselines without TA-opt and IS-opt demonstrates the superiority of the optimization. The TA-opt column shows the results with task assignment optimization, and the IS-opt column shows the results with information scheduling optimization. The result shows that the optimization efficiency of TA-opt is 1.8% ~ 40%, and IS-opt can improve 20% ~ 53%. The total picking time can be reduced by 40% ~ 62% when the TA-opt and IS-opt are used simultaneously.

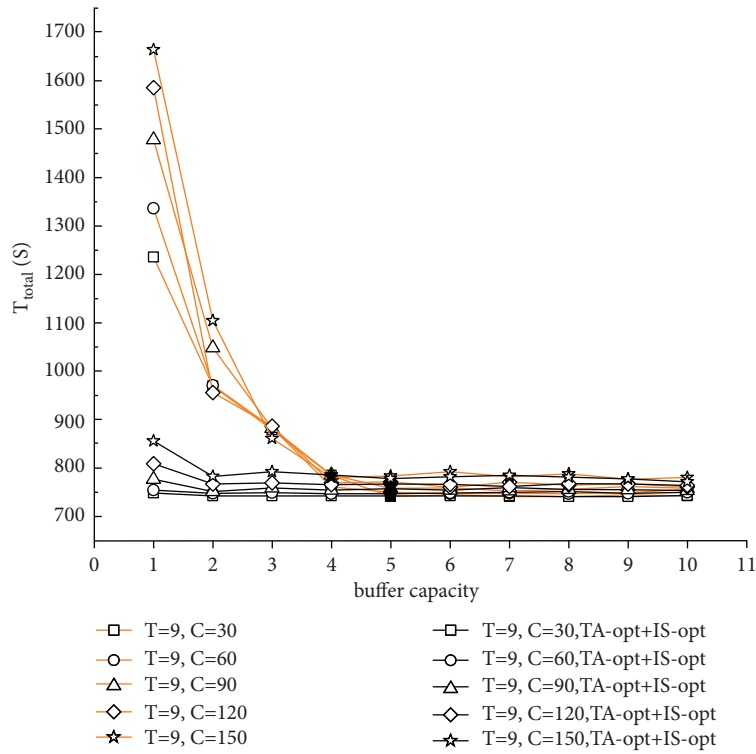


(a)

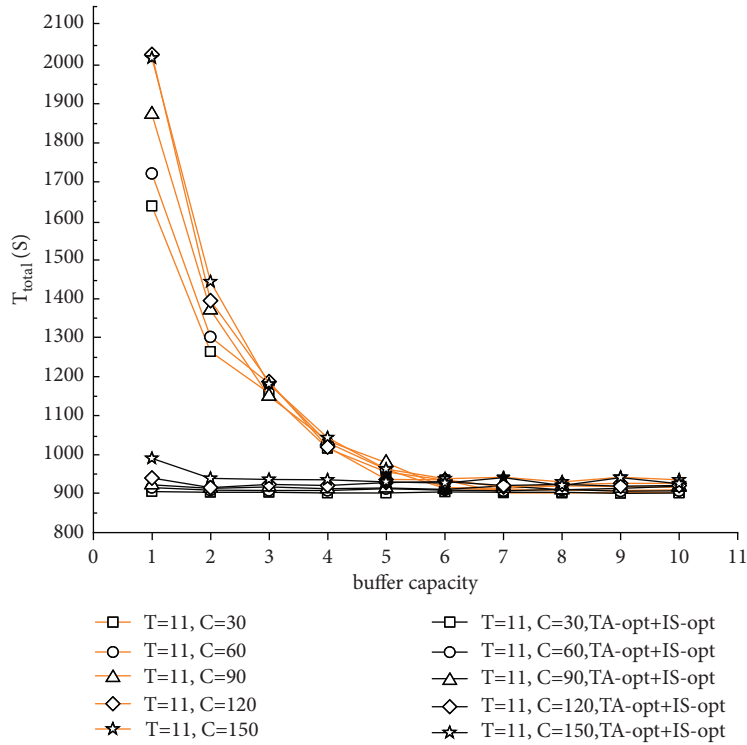


(b)

FIGURE 12: Continued.



(c)



(d)

FIGURE 12: Continued.

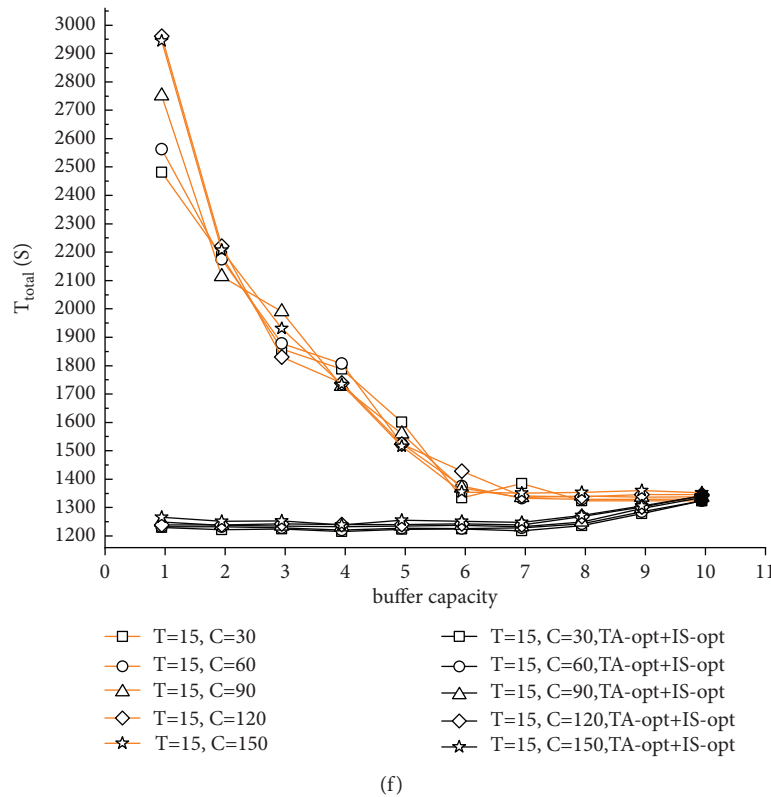
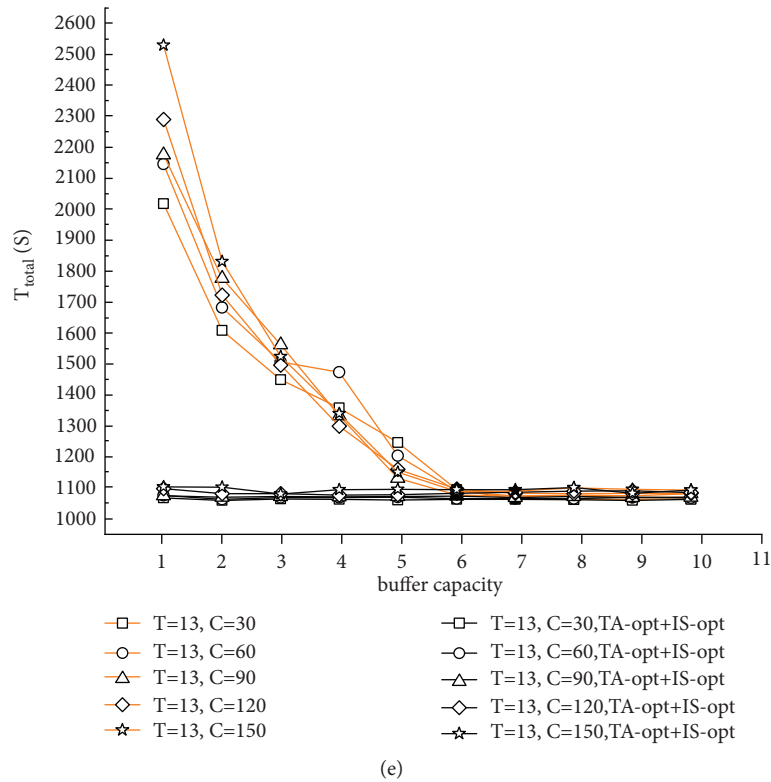


FIGURE 12: Comparison of the total picking time (T_{total}) with the buffer capacity. (a) $T = 5$. (b) $T = 7$. (c) $T = 9$. (d) $T = 11$. (e) $T = 13$. (f) $T = 15$.

5.3. Analyzing the Buffer Capacity with TA-Opt and IS-Opt. This experiment is to verify the effect of buffer capacity with TA-opt and IS-opt. Therefore, the I/O buffer is set to 1 to 10. There are 600 scenarios with the variation of the number of

tiers (T), the number of columns (C), and the buffer capacity. In order to eliminate the bias, the experiment was repeated 50 times. The average value of the total operation time (T_{total}) is used.

The result is shown in Figure 12. It is obvious that the total operation time can be reduced by expanding the buffer capacity. However, for a low rack system, the optimization effect is low or even almost loses the optimization ability, when the buffer capacity increases to a certain amount. For example, as is shown in Figure 12(a), the data show that T_{total} does not change significantly. When buffer capacity is greater than 4, that is because 4 capacities are enough to balance the efficiency of the lifters and the shuttles. On the other hand, Figure 12 shows that the optimization effect of TA-opt and IS-opt is greater with small buffer capacity, while the optimization effect of TA-opt and IS-opt is not obvious with large buffer capacity. Moreover, the expansion of buffer capacity has no obvious effect on the optimization of TA-opt and IS-opt.

6. Conclusion

This paper models the multi-tier shuttle warehouse system (MSWS) with tier-captive shuttles in which the shuttles can only serve the dedicated tier and process the horizontal movement of the totes. We proposed the object-oriented timed colored Petri net (OTCPN) to model the MSWS, and it reduces the modeling difficulty and complexity and provides a reference for other complex warehouse system modeling.

We build the OTCPN simulation model and analyze the influence of system structure and control strategy. Therefore, we carry out numerical experiments to evaluate the total picking time with the task assignment, the information schedule, and the buffer capacity. The result shows that the task assignment with the task balance strategy (TA-opt) can reduce the total picking time, and the improvement is 1.8% ~ 40%. The information schedule (IS-opt) means checking the buffer capacity before releasing the task, so that it reduces the device occupancy time. The improvement in total picking time is 20% ~ 53%. Considering both TA-opt and IS-opt, the total picking time can be reduced by 40% ~ 62%.

For the system structure, we analyze the I/O buffer capacity. The result shows that expanding the buffer capacity can reduce the total picking time. However, an unlimited increase in buffer capacity does not result in significant optimizations. The appropriate capacity of buffers to balance the efficiency of the lifters and shuttles can reduce the total picking time. Moreover, the result shows the buffer capacity has no obvious effect on the optimization of TA-opt and IS-opt.

In future work, it is interesting to model other warehouse systems with OTCPN. We can also study the deterministic relationship between buffer capacity and different system sizes in different warehouse systems to guide system planning and design.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by Shandong Provincial Natural Science Foundation (ZR2019MA022).

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