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Research Article

Optimal Replenishment Strategy for Inventory Mechanism with a Known Price Increase and Backordering in Finite Horizon

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For the finite horizon inventory mechanism with a known price increase and backordering, based on minimizing the inventory cost, we establish two mixed integer optimization models. By buyer's cost analysis, we present the closed-form solutions to the models, and by comparing the minimum cost of the two strategies, we provide an optimal ordering policy to the buyer. Numerical examples are presented to illustrate the validity of the model, and sensitivity analysis on major parameters is also made to show some insights to the inventory model.

1. Introduction

Generally, the prices of grain crops such as wheat, corn, and soybeans are low during the first few months after harvest, and the prices will increase thereafter till the next harvest season, and so on. In view of the situation, the buyer should decide the ordering policy so that his inventory cost is as low as possible. This problem can also be stated as that a supplier announces an impending price increase in the future, and the buyer should decide whether to purchase additional stock before the price increase. Further, since wheat, corn, and soybeans are harvested once a year, the buyer's inventory planning time is generally one year. In this paper, we consider the inventory problem with backordering in a finite horizon in which the seller announces the price will increase permanently and the buyer can have a special order before it happens.

The inventory model with known price increase has been frequently discussed in the literature. Possibly, the first work performed in this field was done by Naddor [1] and Brown [2]. They assumed that the price of goods would increase in the future and a buyer would have an opportunity to make a special order forehand. Taylor and Bradley [3] extended the model and obtained the optimal ordering strategies for situations where the price increase does not coincide within the end of an EOQ cycle. Erel [4] investigated the effects of continuous changes of the purchasing price and holding cost on the optimal order quantity and annual cost. Tersine [5] proposed an economic production quantity model under an announced price increase. Shah [6] developed a discrete-time stochastic inventory model for perishable items when the vendor announces a price increase at some future time. Huang et al. [7] considered an infinite horizon deterministic inventory model with an announced price increase which assumes that the special order is an integral multiple of the new EOQ quantity. Abad [8] considered a supply chain model in which the producer considers temporary reduction and increase in unit purchasing cost separately and the buyer places a special order in both situations. Chung et al. [9] investigated the buyer's selling policy in response to the future price increase for deteriorating items when the buyer can make a special order before the price increase happens. Ouyang et al. [10] explored the possible effects of price increases on a retailer's replenishment policy when the special order quantity is limited and the rate of deterioration of the goods is assumed to be constant. Wang et al. [11, 12] considered an inventory mechanism with a nondeterministic short-term price discount.

For the basic EOQ model with backordering of demand during stockout periods, Ghosh [13] presented an infinite horizon deterministic inventory model that handles inventory shortages under an announced price increase. Sharma [14] developed a production inventory model with partial backordering in which the suppliers can increase or temporarily decrease the prices and the buyers can make a special order. A comprehensive survey on this was made by Pentico and Drake [15]. Taleizadeh et al. [16] considered an EOQ problem with backordering under partial delayed payment.

For the finite horizon deterministic inventory model, Lev and Soyster [17] considered the inventory model on determining the optimal ordering policies based on known information about imminent price increase without shortages. Arcelus, Pakkala, and Srinivasan [18] showed the potential differences between the profit-maximizing and the cost-minimizing solutions to the deterministic finite horizon inventory problem, and Arcelus et al. [19] considered the inventory model over a finite horizon under one-time cost changes. Lev and Weiss [20] considered the extensive inventory models with cost changes for a finite horizon and an infinite horizon.

Furthermore, for the basic EOQ model that considers both price changes and backordering, Taleizadeh and Pentico et al. [21] extended classic economic order quantity (EOQ) model with partial backordering in an infinite horizon, in which the supplier announces the price will increase permanently and the retailer can have a special order before it happens. Taleizadeh et al. [22] extended the situation in [21] to probabilistic replenishment intervals further. Taleizadeh et al. [23] considered the rate of demand in the model was assumed proportional to the unit purchasing cost and partial backordering was allowed as a fixed parameter. However, the length of the inventory system operating time is infinite in these articles.

This paper considers the inventory model with backordering in finite horizon, in which a permanent price increase will take place at or before buyer's next scheduled ordering time, and the buyer may have a special order before that. For this set, to maximize the buyer's inventory profit, the buyer should make a tradeoff between enjoying the benefit of the low ordering price and bearing the increase of inventory holding cost lead by the special order. On the basis of the inventory cost analysis, we establish an optimization inventory model based on minimizing the inventory cost and derive a closed-form solution to the model. Some numerical experiments are made to illustrate the validity of the model.

The remainder of the paper is organized as follows. Section 2 presents the assumptions of our model and notations used in the subsequent analysis. Section 3 considers an inventory model with backordering and without price increase in a finite planning horizon and give the optimal solution which will be used in solving the proposed model. In Section 4, we derive a global optimal solution for our concerned inventory models. We derive a solving algorithm and thus provide an optimal replenishment policy to the buyer in Section 5. Numerical experiments on sensitivity analysis are given in Section 6 to show the validity of the models. The conclusions and some extensions are given in the last section.

2. Notations, Assumptions, and Problem Formulation

First, we present the notations and assumptions used in this paper (Table 1).

The followings are the assumptions imposed on the concerned inventory model.

Assumption 1. For the inventory system, we assume that

(1) The time horizon is finite;
(2) The leading time for each order is zero;
(3) Shortage and backordered are allowed except for the last replenishment cycle;
(4) The fixed ordering cost and the unit backordering cost for regular order are, respectively, the same as those for special order;
(5) The selling price does not change;
(6) The increase of purchasing price occurs at or before the retailer's next ordering time, which means that there is no opportunity for a regular replenishment before the price increase;
(7) When price increases, it will last to the end of the inventory mechanism;
(8) The stock level is zero at the beginning and end of the inventory system.

Without loss of generality, we assume that the holding cost per item per unit time is proportional with the purchasing cost. Then in the following, we take $h = h_0 + 0.4(c - c_0)/c_0$.

From the assumptions on the concerned models, we know that the planning horizon consists of two stages $[0, t_0]$ and $(t_0, T]$, where the ordering price c_0 in the first stage is strictly less than $c(>c_0)$ in the second stage. When the supplier announces the increase of the purchasing price at t_0, there are two possible strategies by the buyer: the buyer places a special order under the current lower price before it increases to cut the cost, or the buyer does not place a special order if the remaining stock level at t_0 is too high or small price increase.

Strategy 1. A special order is not placed at t_0.

If the buyer does not place a special order at t_0, he will use a "new" inventory mechanism with backordering in a finite horizon based on the increased price after the current price cycles end.

Strategy 2. A special order is placed at t_0.

If the buyer adopts this strategy, he will use a "new" inventory mechanism with backordering in a finite horizon based on the increased price after special order cycle ends too.

TABLE 1: Notation.

Parameters	Descriptions
λ	Demand rate
A	Fixed order cost
c_0	Regular purchasing price
c	Purchasing price after increase
h_0	Regular holding cost per item per unit time
h	Holding cost per item per unit time under increased price
w	Backorder cost per unit item per unit time
T	Planning horizon of the inventory system
t_0	The time when purchasing price changes
q_0	Remaining stock at time t_0
Q_0	Order size before t_0
B_0	Maximum shortage level for a normal order at the current price before t_0
Decision variables	
Q_s	Special order size
Q	Order size after t_0 when the special order is not placed
Q'	Order size after special order
B_s	Maximum shortage level for a special order at the current price
B	Maximum shortage level for an order at the new price after t_0 when the special order is not placed
B'	Maximum shortage level for an order at the new price when a special order is placed
Other variables	
F	Inventory cost over interval $[t_0, T]$ without special order
F_s	Inventory cost over interval $[t_0, T]$ with special order

Since we suppose the unit selling price does not change and the stock level is zero at the beginning and the end of the inventory system, so the method we used is computing the difference of the minimal total costs under two strategies and choosing a strategy with the least cost to save more costs.

In order to make the solution process of solving this problem clearer, we will first provide a solution method for an inventory model with backordering in finite horizon without price increase in Section 3 then exploit these conclusions for giving a solution method to the problem in Section 4.

3. Solution Method for Inventory Model with Backordering in a Finite Horizon

In this section, we mainly consider the inventory mechanism with backordering in a finite horizon of length T and without purchasing price variation. Also, for the need discussed in the next section, we assume that the initial stock is q, and the stock is zero at the end of the inventory mechanism. See Figures 1 and 2.

For this setting, suppose that k orders are placed in the $[t_0, T]$ with order sizes $Q_1, Q_2, \cdots, Q_k$, and the maximum shortage levels at replenishment cycles are $B_1, B_2, \cdots, B_k$, respectively. Since the demand rate is invariant, shortage is fully backordered, and the stock is zero at the end of the inventory mechanism $(i.e. B_k = 0)$, so

$$Q_1 + Q_2 + \cdots + Q_k = \lambda T - q. \quad (1)$$

From the knowledge of inventory management [24], we can obtain the total operating cost over the planning horizon T

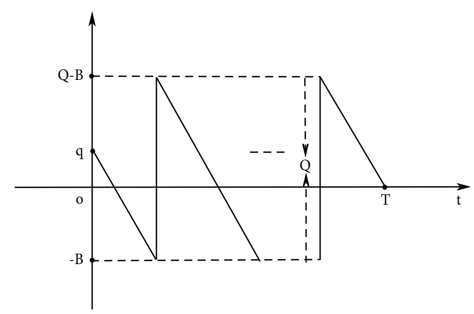

FIGURE 1: Ordering policy for $q \geq 0$.

$$\begin{aligned} &F(k, Q_1, \ldots, Q_k, B_1, \ldots, B_{k-1}) \\ &= kA + c\sum_{i=1}^{k} Q_i + \frac{h}{2\lambda}(Q_1 + q)^2 \\ &\quad + \frac{h}{2\lambda}\sum_{i=2}^{k}(Q_i - B_{i-1})^2 + \frac{w}{2\lambda}\sum_{i=1}^{k-1} B_i^2 \\ &= c(\lambda T - q) + kA + \frac{h}{2\lambda}(Q_1 + q)^2 \\ &\quad + +\frac{h}{2\lambda}\sum_{i=2}^{k}(Q_i - B_{i-1})^2 + \frac{w}{2\lambda}\sum_{i=1}^{k-1} B_i^2. \end{aligned} \quad (2)$$

To minimize the buyer's operating cost, we only need to determine the optimal order times k, the sizes $Q_i \, (i = 1, \ldots, k)$, and the maximum shortage levels $B_i \, (i = 1, \ldots, k-1)$, which can be formulated as the following optimization problem:

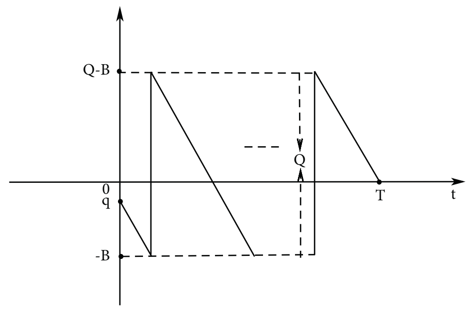

Figure 2: Ordering policy for $q < 0$.

$$\begin{aligned}&\min_{k,Q_i,B_i} F(k,Q_1,\ldots,Q_k,B_1,\ldots,B_{k-1})\\ &\text{s.t.}\quad Q_1+Q_2+\cdots+Q_k=\lambda T-q,\\ &Q_i\ge 0, i=1,\ldots,k,\\ &B_i\ge 0, i=1,\ldots,k-1,\\ &k \text{ is } a \text{ positive integer.}\end{aligned} \quad (3)$$

This is a nonlinear mixed integer optimization problem. For this problem, we have the following conclusion.

Theorem 1. *For an inventory model with backordering in finite horizon, i.e., inventory model (3), all the optimal maximum shortage levels are the same except the last one, and all the optimal ordering sizes are the same except the first one, i.e.,*

$$\begin{aligned}Q_2&=\cdots=Q_k\triangleq Q,\\ B_1&=\cdots=B_{k-1}\triangleq B,\\ Q_1&=Q-B-q.\end{aligned} \quad (4)$$

Furthermore, the optimal ordering size, maximum shortage level, and the optimal order time are, respectively,

$$\begin{aligned}Q&=\frac{(w+h)\lambda T}{kw+(k-1)h},\\ B&=\frac{h\lambda T}{kw+(k-1)h},\\ k&=\left\lfloor T\sqrt{\frac{w\lambda h}{2A(w+h)}}+\frac{h}{w+h}\right\rfloor\\ &\text{or}\left\lceil T\sqrt{\frac{w\lambda h}{2A(w+h)}}+\frac{h}{w+h}\right\rceil,\end{aligned} \quad (5)$$

and the minimum inventory cost within the horizon $[0,T]$ is

$$F=c(\lambda T-q)+kA+\frac{wh\lambda T^2}{2[kw+(k-1)h]}. \quad (6)$$

Proof. For completeness, we include the proof of the lemma in Appendix A.

We will exploit the conclusion of Theorem 1 repeatedly in the remainder of this paper. For convenience, the optimal ordering policy given in Theorem 1 for the inventory mechanism with backordering in the finite horizon and without purchasing price variation is denoted by "BEOQ" in the subsequent sections. □

4. Solution Method for Inventory Model with a Known Price Increase and Backordering in Finite Horizon

For this inventory system, there are two ordering strategies according to whether a special order is placed or not at t_0, as shown in Figures 3 and 4. If the buyer places a special order at t_0, the special order cycle will end at t_2, and the inventory cycles based on the current lower price end at t_1 if the buyer does not place a special order. It is easy to calculate that $t_1=t_0+q_0+B_0/\lambda$, $t_2=t_0+q_0+Q_s+B_s/\lambda$. For convenience, we denote $T_0=T-t_0, T_1=T-t_1$, and $T_2=T-t_2$.

Since shortages are allowed, two possible scenarios may occur at t_0: the price increase for the item occurs when there is still inventory or there is a shortage, that is:

Scenario 1: $q_0\ge 0$, and Scenario 2: $q_0<0$.

In the subsequent analysis, we will first discuss the minimum inventory costs of the two strategies, then compare the difference in the two minimum cost, and finally, choose the optimal ordering strategy.

4.1. Strategy 1: A Special Order Is Not Placed at t_0.

For this strategy, when the inventory cycles under the current price end at t_1, the buyer will continue using a new BEOQ order policy till the end of the inventory system, so that the inventory cost over the interval $[t_0,T]$ consists of inventory cost over two intervals $[t_0,t_1)$ and $[t_1,T]$.

Scenario 1. $q_0\ge 0$

Under this scenario, the price increases when there is no shortage, and the remaining stock q_0 at t_0 will be postponed to the second stage. By the knowledge of inventory control, we can calculate that the inventory cost during $[t_0,t_1)$ is

$$f_1=\frac{h_0}{2\lambda}q_0^2+\frac{w}{2\lambda}B_0^2. \quad (7)$$

Since the stock level is $-B_0$ at t_1, the inventory system will run a new "BEOQ" order policy during the horizon $[t_1,T]$ under the new price c, and the stock level is $-B_0$ and

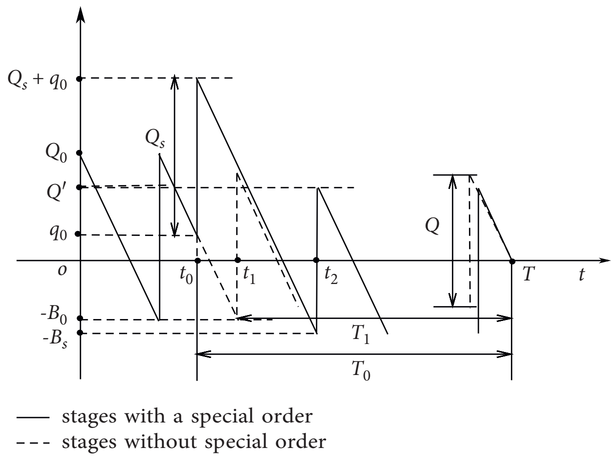

FIGURE 3: Ordering policy for $q_0 \geq 0$.

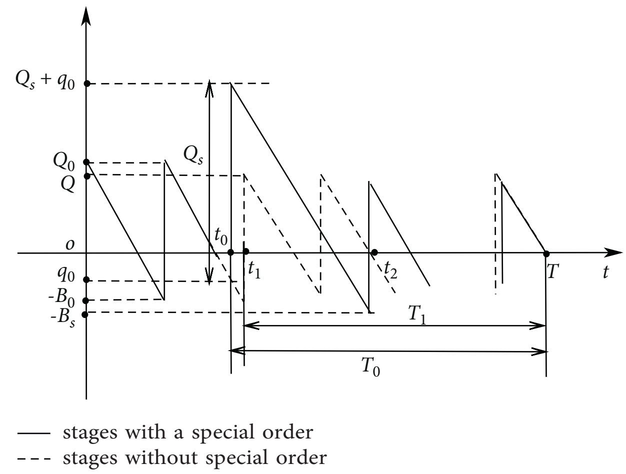

FIGURE 4: Ordering policy for $q_0 < 0$.

zero at the beginning and end of the planning horizon, respectively; see Figures 3 and 4.

For this scenario, when the special order is not placed at t_0, we denote the inventory cost during this period from t_0 to the end by F, which is a function of order quantity $Q_1, \ldots, Q_n$ and maximum shortage level $B_1, \ldots, B_{n-1}$ for an order after t_0. Using an approach similar to that used and according to formula (2) in Section 3, then

$$\begin{aligned} g(n, Q_1, \ldots, Q_n, B_1, \ldots, B_{n-1}) &= nA + c\sum_{i=1}^{n} Q_i + \frac{h}{2\lambda}(Q_1 - B_0)^2 \\ &\quad + \frac{h}{2\lambda}\sum_{i=2}^{n}(Q_i - B_{i-1})^2 + \frac{w}{2\lambda}\sum_{i=1}^{n-1} B_i^2 \\ &= c(\lambda T + B_0) + nA + \frac{h}{2\lambda}(Q_1 - B_0)^2 \\ &\quad + \frac{h}{2\lambda}\sum_{i=2}^{n}(Q_i - B_{i-1})^2 + \frac{w}{2\lambda}\sum_{i=1}^{n-1} B_i^2. \end{aligned} \tag{8}$$

So the ordering policy during the horizon $[t_1, T]$ of the concerned model can be formulated as the following optimization model:

$$\begin{aligned} &\min_{n, Q_i, B_i} g(n, Q_1, \ldots, Q_n, B_1, \ldots, B_{n-1}), \\ &\text{s.t.} \quad Q_1 + Q_2 + \cdots + Q_n = \lambda T_1 + B_0, \\ &Q_i \geq 0, i = 1, \ldots, n, \\ &B_i \geq 0, i = 1, \ldots, n-1, \\ &n \text{ is a positive integer.} \end{aligned} \tag{9}$$

According to formulas (5) and (6) of Theorem 1 in Section 3, we know that all the optimal ordering sizes are the same except the first one, all the optimal maximum shortage levels are the same, and the optimal ordering size, maximum shortage level, and the optimal order time over the interval $[t_1, T]$, respectively, are

$$\begin{aligned} Q &= \frac{(w+h)\lambda T_1}{nw + (n-1)h}, \\ B &= \frac{h\lambda T_1}{nw + (n-1)h}, \end{aligned} \tag{10}$$

$$\begin{aligned} n &= \left\lfloor T_1 \sqrt{\frac{w\lambda h}{2A(w+h)}} + \frac{h}{w+h} \right\rfloor \triangleq n_1 \quad or \\ &\quad \left\lceil T_1 \sqrt{\frac{w\lambda h}{2A(w+h)}} + \frac{h}{w+h} \right\rceil \triangleq n_2, \end{aligned} \tag{11}$$

and the minimum inventory cost during the interval $[t_1, T)$ is

$$g_1 = c(\lambda T_1 + B_0) + n_1 A + \frac{wh\lambda T_1^2}{2[n_1 w + (n_1 - 1)h]}, \tag{12}$$

or

$$g_2 = c(\lambda T_1 + B_0) + n_2 A + \frac{wh\lambda T_1^2}{2[n_2 w + (n_2 - 1)h]}. \tag{13}$$

If $g_1 \leq g_2$, then the optimal ordering times and the minimum inventory cost during the interval $[t_1, T)$ are $n^* = n_1$ and g_1, respectively; otherwise, they are $n^* = n_2$ and g_2. Denote the minimum inventory cost during horizon $[t_1, T)$ by

$$g^* = c(\lambda T_1 + B_0) + n^* A + \frac{wh\lambda T_1^2}{2[n^* w + (n^* - 1)h]}. \tag{14}$$

In summary, under strategy 1, if the remaining inventory stock at t_0 is nonnegative, the minimum inventory cost over the horizon $[t_0, T]$ is

$$F^* = f_1 + g^* = c(\lambda T_1 + B_0) + n^* A + \frac{h_0}{2\lambda} q_0^2 + \frac{w}{2\lambda} B_0^2 + \frac{wh\lambda T_1^2}{2[n^* w + (n^* - 1)h]}. \quad (15)$$

Scenario 2. $q_0 < 0$.

Under this scenario, the stock position at t_0 is exhausted; therefore the inventory cost during $[t_0, t_1)$ needs to subtract holding costs (see Figure 4), so that the inventory cost during $[t_0, t_1)$ is

$$f_2 = \frac{w}{2\lambda} B_0^2 - \frac{h_0}{2\lambda} q_0^2. \quad (16)$$

Yet, if there is a shortage at t_0, again using an approach similar to that used in Section 3, the minimum inventory cost during the interval $[t_1, T)$ can still be expressed as g^*; thus the minimum inventory cost over the horizon $[t_0, T]$ is

$$F^* = f_2 + g^* = c(\lambda T_1 + B_0) + n^* A - \frac{h_0}{2\lambda} q_0^2 + \frac{w}{2\lambda} B_0^2 + \frac{wh\lambda T_1^2}{2[n^* w + (n^* - 1)h]}. \quad (17)$$

Obviously, both f_1 and f_2 are constants and independent of decision variables; therefore, minimizing inventory cost during the period $[t_0, T]$ is equivalent to minimizing the buyer's inventory cost over the interval $[t_1, T]$.

Based on the discussion above, we can obtain the following two candidate optimal ordering policies in the horizon $[t_1, T]$ under strategy 1.

Policy π_1: $n^* = n_1$, $Q_1^* = w\lambda T_1/(n_1 w + (n_1 - 1)h) + B_0$, $Q^* = (w + h)\lambda T_1/(n_1 w + (n_1 - 1)h)$, $B^* = h\lambda T_1/(n_1 w + (n_1 - 1)h)$,

which applies to the case that $g_1 \le g_2$.

For this order policy, the minimum inventory cost in horizon $[t_1, T]$ is $g^* = g_1$.

Policy π_2: $n^* = n_1$, $Q_1^* = w\lambda T_1/(n_1 w + (n_1 - 1)h) + B_0$, $Q^* = (w + h)\lambda T_1/(n_1 w + (n_1 - 1)h)$, $B^* = h\lambda T_1/(n_1 w + (n_1 - 1)h)$,

which applies to the case that $g_1 < g_2$.

For this order policy, $g^* = g_2$.

4.2. Strategy 2: A Special Order Is Placed at t_0. By the inventory management theory, if we place a one-time special order at the current price before the price increases for both scenarios of $q_0 \ge 0$ and $q_0 < 0$, the inventory cost in the special replenishment cycle can be expressed as

$$f_s = c_0 Q_s + A + \frac{h_0}{2\lambda}(Q_s + q_0)^2 + \frac{w}{2\lambda} B_s^2. \quad (18)$$

Since the special order is exhausted at t_2 (see Figures 3 and 4), the next order will be placed at t_2; hence, the inventory system will run a new BEOQ inventory mechanism during the period $[t_2, T]$, and the shortage is B_s at t_2.

Suppose that the order sizes $Q_1', \ldots, Q_m'$ and the maximal shortage levels $B_1', \ldots, B_{m-1}'$ are made after the special order; again using a similar discussion in Section 3, we can obtain the inventory cost in horizon $[t_2, T]$

$$\begin{aligned} g_s(m, Q_1', \ldots, Q_m', B_1', \ldots, B_{m-1}') &= c(\lambda T_2 + B_s) + mA + \frac{h}{2\lambda}(Q_1' - B_s)^2 + \frac{h}{2\lambda}\sum_{i=2}^{m}(Q_i' - B_{i-1}')^2 + \frac{w}{2\lambda}\sum_{i=1}^{m-1} B_i'^2 \\ &= c(\lambda T_0 - q_0 - Q_s) + mA + \frac{h}{2\lambda}(Q_1' - B_s)^2 + \frac{h}{2\lambda}\sum_{i=2}^{m}(Q_i' - B_{i-1}')^2 + \frac{w}{2\lambda}\sum_{i=1}^{m-1} B_i'^2. \end{aligned} \quad (19)$$

Accordingly, the total inventory cost during the period $[t_0, T]$ is

$$\begin{aligned} F_s(m, Q_s, B_s, Q_1', \ldots, Q_m', B_1', \ldots, B_{m-1}') &= f_s + g_s(m, Q_1', \ldots, Q_m', B_1', \ldots, B_{m-1}') \\ &= c\lambda T_0 - cq_0 - (c - c_0)Q_s + (m+1)A + \frac{h_0}{2\lambda}(Q_s + q_0)^2 + \frac{w}{2\lambda}B_s^2 + \frac{h}{2\lambda}(Q_1' - B_s)^2 \\ &\quad + \frac{h}{2\lambda}\sum_{i=2}^{m}(Q_i' - B_{i-1}')^2 + \frac{w}{2\lambda}\sum_{i=1}^{m-1} B_i'^2. \end{aligned} \quad (20)$$

Thus, the problem of determining an optimal ordering policy of strategy 2 can be formulated as the following optimization problem:

$$\begin{aligned}&\min_{m,Q_s,B_s,Q_i',B_i'} F_s(m,Q_s,B_s,Q_1',\ldots,Q_m',B_1',\ldots,B_{m-1}')\\ &\text{s.t.}\, Q_s+Q_1'+Q_2'+\cdots+Q_m'=\lambda T_0-q_0\\ &Q_s\ge 0, B_s\ge 0,\\ &Q_i'\ge 0, i=1,\ldots,m,\\ &B_i'\ge 0, i=1,\ldots,m-1,\\ &m \text{ is a positive integer.}\end{aligned} \tag{21}$$

For problem (21), we have the following conclusion.

Theorem 2. *For inventory model (21) with $q_0 \ge 0$ or $q_0 < 0$, assume m is fixed; after the special order, all optimal maximum shortage levels are the same, and the optimal ordering sizes are the same. Furthermore, the optimal special ordering size is*

$$Q_s=\begin{cases}\lambda T_0-q_0-\dfrac{m\lambda(w+h)\alpha}{mh_0(w+h)+wh}, & \text{if } \alpha>0 \text{ and } \beta\le 0,\\ & \text{or } \alpha>0, \beta>0 \text{ and } m<\delta,\\ \lambda T_0-q_0, & \\ & \text{otherwise,}\end{cases} \tag{22}$$

the ordering size after special order is

$$Q'=\begin{cases}\lambda T_0-q_0-\dfrac{\lambda(w+h)\alpha}{mh_0(w+h)+wh}, & \text{if } \alpha>0 \text{ and } \beta\le 0,\\ & \text{or } \alpha>0, \beta>0 \text{ and } m<\delta,\\ 0, & \\ & \text{otherwise,}\end{cases} \tag{23}$$

and the maximal shortage level is

$$B_s=B'=\begin{cases}\dfrac{\lambda h\alpha}{mh_0(w+h)+wh}, & \text{if } \alpha>0 \text{ and } \beta\le 0,\\ & \text{or } \alpha>0, \beta>0 \text{ and } m<\delta,\\ 0, & \\ & \text{otherwise,}\end{cases} \tag{24}$$

where $\alpha = h_0T_0 + c_0 - c$, $\beta = q_0h_0 + \lambda(c_0 - c)$, $\delta = (\lambda T_0 - q_0)wh/(w+h)\beta$.

Proof. See Appendix B.

From the proof of Theorem 2, if $\alpha \ge 0$ and $\beta \le 0$, or $\alpha \ge 0, \beta > 0$ and $m < \delta$ hold, then

$$\begin{aligned}F_s &= c\lambda T_0 + cq_0 + (c_0-c)Q_s + (m+1)A + \frac{h_0}{2\lambda}(Q_s+q_0)^2\\ &\quad + \frac{mh}{2\lambda}(Q'-B')^2 + \frac{mw}{2\lambda}B'^2\\ &= c_0\lambda T_0 - c_0q_0 + (m+1)A + \frac{h_0\lambda T_0^2}{2} - \frac{m\lambda(w+h)\alpha^2}{2[mh_0(w+h)+wh]}.\end{aligned} \tag{25}$$

Otherwise,

$$m=0, Q_s=\lambda T_0-q, Q'=0, B'=0, \tag{26}$$

$$F_s = c_0\lambda T_0 - c_0q_0 + A + \frac{h_0\lambda T_0^2}{2}. \tag{27}$$

Now, we consider the optimal ordering time under condition (58) or (59).

For the function F_s given by (25),

$$F_s''(m) = \frac{w^2h\lambda(w+h)\alpha^2}{[mh_0(w+h)+wh]^3} > 0, \tag{28}$$

so that $F_s(m)$ is a convex function in m, and its minimum can be obtained at the stationary point of the function F_s, which yields

$$m = \frac{1}{h_0}\sqrt{\frac{w\lambda h}{2A(w+h)}}\left(\alpha - \sqrt{\frac{2Awh}{\lambda(w+h)}}\right). \tag{29}$$

Denote $\gamma = \sqrt{(2Awh/\lambda(w+h))}$ and $m_1 = \lfloor m \rfloor$, $m_2 = \lceil m \rceil$. Taking the fact m is a positive integer into consideration, we know that the optimal ordering time is $m = m_1$ when $F_s(m_1) \ge F_s(m_2)$; otherwise it is $m = m_2$.

Based on the above discussion, and if $\alpha \le \gamma$, the optimal ordering time is $m^* = 0$. We obtain the following three candidate optimal order policies in horizon $[t_0, T]$ under strategy 2.

Policy π_3:

$$m^* = m_1, \quad Q_s^* = \lambda T_0 - q_0 - \frac{m_1\lambda(w+h)\alpha}{m_1h_0(w+h)+wh}, \tag{30}$$

$$Q'* = \frac{\lambda(w+h)\alpha}{m_1h_0(w+h)+wh}, B'^* = \frac{\lambda h\alpha}{m_1h_0(w+h)+wh}. \tag{31}$$

It applies to

$$\begin{aligned}&\alpha>\gamma, \beta\le 0,\\ &F_s(m_1)\le F_s(m_2),\end{aligned} \tag{32}$$

$$\text{or } \alpha>\gamma, \beta>0, m_2<\delta \text{ and } F_s(m_1)\le F_s(m_2), \tag{33}$$

$$\text{or } \alpha>\gamma, \beta>0, m_1<\delta\le m_2. \tag{34}$$

For policy π_3, the minimum inventory cost over the interval $[t_0, T]$ is $F_s^* = F_s(m_1)$.

Policy π_4:

$$m^* = m_2, Q_s^* = \lambda T_0 - q_0 - \frac{m_2\lambda(w+h)\alpha}{m_2h_0(w+h)+wh}, \tag{35}$$

$$Q'^* = \frac{\lambda(w+h)\alpha}{m_2h_0(w+h)+wh}, B'^* = \frac{\lambda h\alpha}{m_2h_0(w+h)+wh}, \tag{36}$$

which applies to

$$\alpha>\gamma, \beta\le 0, \text{ and } F_s(m_1)>F_s(m_2), \tag{37}$$

or

$$\text{or } \alpha > \gamma, \beta > 0, m_2 < \delta \text{ and } F_s(m_1) > F_s(m_2). \tag{38}$$

For this policy, the minimum inventory cost over the interval $[t_0, T]$ is $F_s^* = F_s(m_2)$.

Policy π_5:

$$m^* = 0, Q_s^* = \lambda T_1 - q_0, Q'^* = B'^* = 0, \tag{39}$$

i.e., no order is made after the special order; the minimum inventory cost over the interval horizon $[t_0, T]$ is

$$F_s^* = c_0 \lambda T_0 - c_0 q_0 + A + \frac{h_0 \lambda T_0^2}{2}. \tag{40}$$

According to the analysis above, the difference of the minimal total costs under two strategies is $G^* = F^* - F_s^*$. Certainly, placing a special order or not is determined by the sign of G^*. If $G^* \geq 0$, then the buyer placing a special order can save costs than not placing special orders, and the candidate optimal policy is π_3, π_4, or π_5 after the special order.

On the contrary, if $G^* < 0$, the buyer will not place a special order, and the candidate optimal policy is π_1 or π_2 after the regular order replenishment cycle.

Considering all the ordering policies discussed above, we can present our algorithm for our problem in this paper.

5. Algorithm

The algorithm flowchart for problem is shown in Figure 5. See Appendix C.

To compute the remaining stock q_0 at t_0, we first give the optimal order policy under the current price. For an inventory mechanism, if there is not price change, this is an inventory model with backordering in the finite horizon of length T under the current price c_0, and the stock level is zero at the beginning and the end of the inventory system; based on the conclusions (5) and (6) in Theorem 1, the optimal ordering size Q_0, maximum shortage level B_0, and the optimal order times k under the current price are, respectively,

$$Q_0 = \frac{(w + h_0)\lambda T}{kw + (k-1)h_0}, \qquad B_0 = \frac{h_0 \lambda T}{kw + (k-1)h_0}, \tag{41}$$

$$k = \left\lfloor T\sqrt{\frac{w\lambda h_0}{2A(w+h_0)}} + \frac{h}{w+h_0} \right\rfloor \triangleq k_1 \quad \text{or} \quad \left\lceil T\sqrt{\frac{w\lambda h_0}{2A(w+h_0)}} + \frac{h_0}{w+h_0} \right\rceil \triangleq k_2. \tag{42}$$

According to these, we can compute that

$$q_0 = \left\lceil \frac{\lambda t_0}{Q_0} \right\rceil Q_0 - B_0 - \lambda t_0. \tag{43}$$

6. Computational Experiments and Sensitivity Analysis

In this section, in order to show the applicability of the presented models, we will perform some numerical examples and sensitivity analysis on major parameters. Based on the algorithm above, if the basic parameters of the models are given, we can obtain an optimal ordering policy. The results of computational experiments and sensitivity analysis can be obtained by the Matlab procedures of algorithm above, so we mainly list the important results and list the simple solution process.

6.1. Computational Experiments

Example 1. Consider the inventory system with the following parameters: $\lambda = 500, A = 50, c_0 = 10, c = 10.5, h_0 = 4, h = 4.2, w = 3, T = 12, t_0 = 2.04$.

Solution: For this inventory system, first, we calculate the optimal regular order times, order size, the maximum shortage level before t_0, and the remaining stock at t_0 before the price increase does not take place according to formulas (41)–(43) as follows:

$$k = 36, Q_0 = 169.35, B_0 = 96.77, q_0 = 68.71. \tag{44}$$

If we adopt strategy 1, then the candidate optimal policy set is $\{\pi_1, \pi_2\}$.

However, if we adopt strategy 2, since $\alpha = 39.34 > \gamma = 0.5916$, and $\beta = 24.84 > 0, m_2 = 29 < (\lambda T_0 - q_0)wh/(w+h)\beta = 346.02$; based on Theorem 2 in Section 4, the candidate optimal policy set is $\{\pi_3, \pi_4\}$. The numerical results by algorithm are listed in Table 2.

From Table 2, we can see that $F^* = 54493$, $F_s^* = 54525$, and thus $G^* = -32$; hence the optimal ordering policy of Example 1 is π_1, and the optimal order times, optimal ordering sizes, and the optimal maximum shortage level over the interval [t1,T] are, respectively,

$$m^* = 29, Q_1^* = 167.37, Q^* = 169.43, B^* = 98.83. \tag{45}$$

In this way, the buyer's inventory cost savings are 32.

Example 2. For the inventory system considered in Example 1, set $t_0 = 2.5$ and other parameters remain unchanged.

For this inventory system, the optimal regular order times, order size, and the maximum shortage level are the same as in Example 1, and the remaining stock at t_0 is $q_0 = 8.06$.

Similar to Example 1, since $\alpha = 39.34 > \gamma = 0.5916$, and $\beta = -375.16 < 0$, if we apply strategy 2, the candidate optimal policy set is $\{\pi_3, \pi_4\}$. The numerical results are listed in Table 3, from which we can see that the cost of applying π_3 is minimal, so the optimal ordering policy of Example 2 is π_3, and the buyer's inventory cost saving of placing a special order compared to not placing special order is 16.

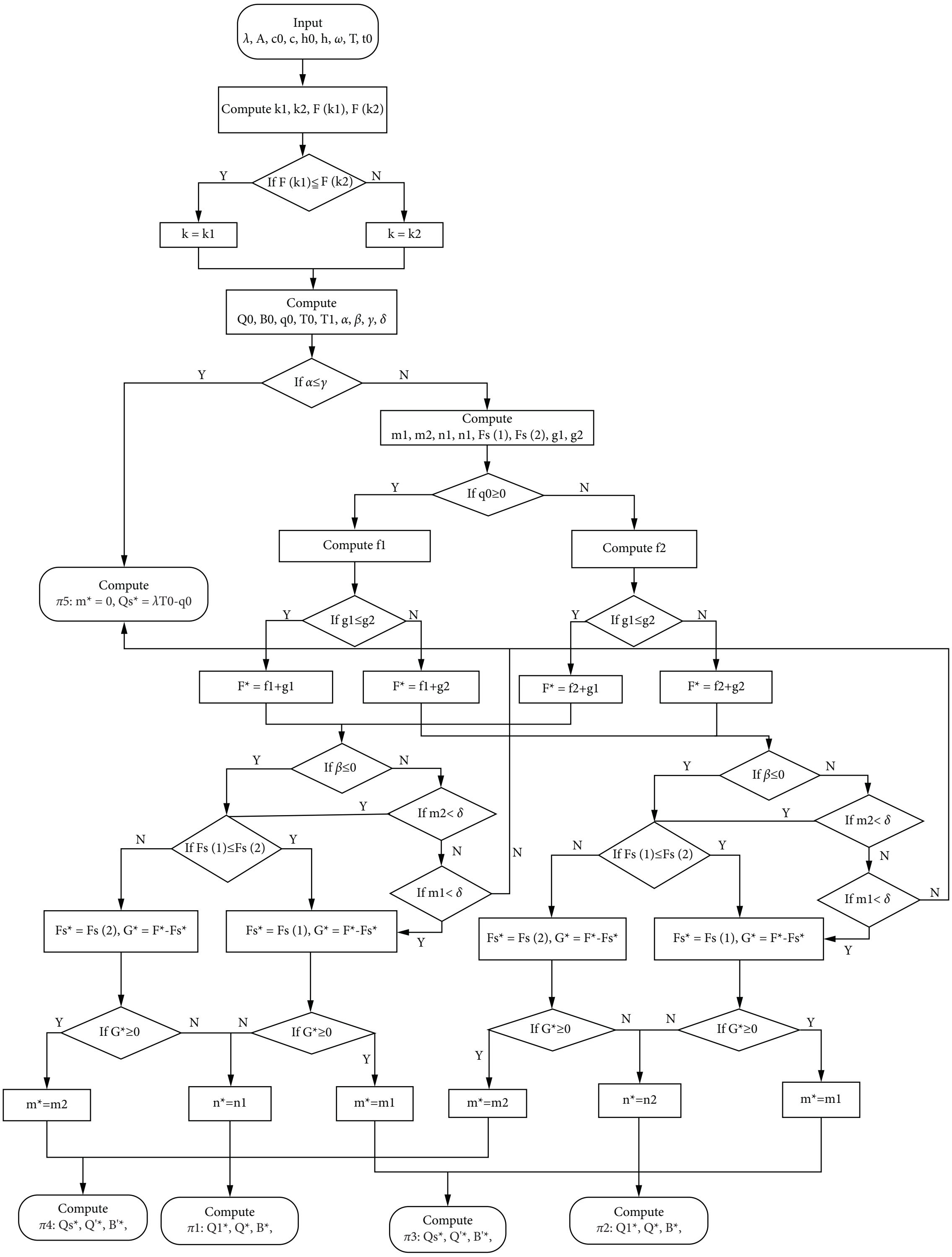

Figure 5: Algorithm flowchart.

Table 2: Numerical results on Example 1.

Policy	Q_s	$m\backslash n$	$Q\backslash Q'$	$B\backslash B'$	$F\backslash F_s$
π_1	—	29	169.43	98.83	54493
π_2	—	30	163.67	95.47	54494
π_3	69.44	28	172.92	100.87	54525
π_4	66.87	29	167.05	97.45	54525

Table 3: Numerical result on Example 2.

Policy	Q_s	$m\backslash n$	$Q\backslash Q'$	$B\backslash B'$	$F\backslash F_s$
π_1	—	28	169.43	98.83	52596
π_2	—	29	163.67	95.47	52597
π_3	129.18	27	170.84	99.656	52580
π_4	126.55	28	164.84	96.15	52581

Table 4: Numerical result on Example 3.

Policy	Q_s	$m\backslash n$	$Q\backslash Q'$	$B\backslash B'$	$F\backslash F_s$
π_1	—	29	169.43	98.83	54457
π_2	—	30	163.67	95.47	54458
π_3	214.89	27	173.94	101.47	54368
π_4	212.21	28	167.82	97.90	54366

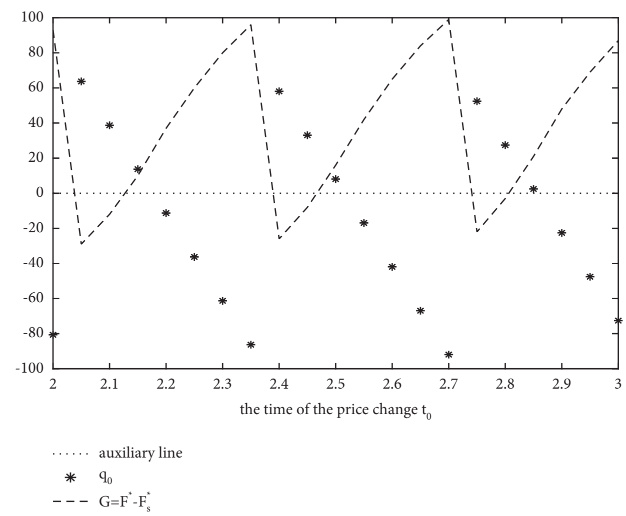

Figure 6: The changes of the q_0 and G with the price growth rate.

Table 5: Impact of parameter q_0 on the retailer's ordering policy, $c = 10.5$.

t_0	q_0	π_1	π_2	π_3	π_4	G^*	Q_s	Optimal policy
2.00	−80.65	56333	56334	56241	56240	93	216.53	π_4
2.05	63.71	54490	54492	54520	54519	−29	—	π_1
2.10	38.71	54480	54482	54492	54492	−12	—	π_1
2.15	13.71	54476	54476	54465	54465	10	123.60 or 121.06	π_3 or π_4
2.20	−11.29	54474	54475	54437	54438	36	148.21	π_3
2.25	−36.29	54470	54472	54410	54412	60	172.83	π_3
2.30	−61.29	54463	54464	54384	54383	80	197.44	π_4
2.35	−86.29	54452	54453	54356	54356	96	224.73 or 222.06	π_3 or π_4
2.40	58.06	52609	52611	52635	52635	−26	—	π_1
2.45	33.06	52600	52602	52608	52608	−8	—	π_1
2.50	8.06	52596	52597	52580	52581	16	129.18	π_3
2.55	−16.94	52595	52596	52553	52554	42	153.78	π_3
2.60	−41.94	52590	52592	52527	52525	65	178.38	π_4
2.65	−66.94	52582	52584	52500	52498	84	202.98	π_4
2.70	−91.94	52570	52572	52472	52471	99	227.58	π_4
2.75	52.42	50728	50730	50751	50750	−22	—	π_1
2.80	27.42	50720	50722	50723	50723	−3	—	π_1
2.85	2.42	50717	50719	50696	50696	11	134.76 or 132.03	π_3 or π_4
2.90	−22.58	50716	50717	50668	50670	48	159.34	π_3
2.95	−47.58	50710	50712	50643	50641	69	183.93	π_4
3.00	−72.58	50701	50703	50615	50614	87	208.51	π_4

The underlined values indicate the ordering cost under the optimal ordering strategy.

Table 6: Impact of price increase ratio parameter on the retailer's ordering policy, $t_0 = 2.04$.

$\Delta c/c_0$ (%)	h	π_1	π_2	π_3	π_4	G^*	Q_s	Optimal policy
1	4.04	52506	52505	52555	52554	−49	—	π_2
2	4.08	53003	53002	53049	53049	−47	—	π_2
3	4.12	53501	53499	53543	53542	−43	—	π_2
4	4.16	53996	53998	54035	53034	−38	—	π_1
5	4.20	54493	54494	54525	54525	−32	—	π_1
6	4.24	54990	54991	55015	55014	−24	—	π_1
7	4.28	55487	55488	55503	55502	−15	—	π_1
8	4.32	55983	55984	55989	55989	−6	—	
9	4.36	56480	56482	56475	56474	6	117.24	π_4
10	4.4	56977	56977	56958	56958	18	132.40 or 129.82	π_3 or π_4
11	4.44	57473	57474	57441	57441	32	144.98 or 142.39	π_3 or π_4
12	4.48	57970	57970	57922	57922	48	157.55 or 154.96	π_3 or π_4
13	4.52	58466	58466	58402	58402	64	170.12 or 167.53	π_3 or π_4
14	4.56	58963	58963	58881	58881	82	182.69 or 180.09	π_3 or π_4
15	4.60	59459	59459	59358	59358	101	195.25 or 192.65	π_3 or π_4
16	4.64	59995	59995	59834	59834	121	207.81 or 205.21	π_3 or π_4
17	4.68	60451	60451	60308	60309	143	220.36	π_3
18	4.72	60948	60947	60782	60782	165	232.91 or 230.30	π_3 or π_4
19	4.76	61444	61443	61254	61254	189	245.452 or 242.85	π_3 or π_4
20	4.80	61940	61939	61724	61725	215	258.99	π_3

The underlined values indicate the ordering cost under the optimal ordering strategy.

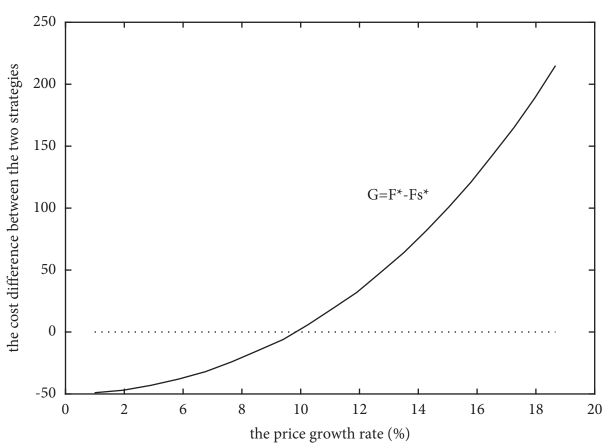

Figure 7: The cost difference between the two strategies with the price growth rate.

Example 3. For the inventory system considered in Example 1, set $t_0 = 2.33$ and other parameters remain unchanged.

For this example, the remaining stock at t_0 is $q_0 = -76.29$.

Similar to Example 1, since $\alpha = 38.18 > \gamma = 0.5916$, and $\beta = -555.16 < 0$, if a special order is placed, the candidate optimal policy set is $\{\pi_3, \pi_4\}$. The numerical results are listed in Table 4, from which we can see that the optimal ordering policy is π_4, and the buyer's inventory cost saving of placing a special order compared to not placing special order is 91.

In the numerical experiments, we see that the remaining stock q_0 at t_0 significantly affects the buyer's ordering policy. Using the same data as those in Example 1, we study the sensitivity analysis.

6.2. Sensitivity Analysis. To better understand the effect of q_0 on the ordering policy, we conduct a sensitivity analysis of the models by varying the parameter q_0 while keeping the other parameters fixed as same as in Example 1. The numerical results are presented in Table 5 and Figure 6.

Figure 6 exhibits that $G \geq 0$ when $q_0 \leq 24$, the buyer should place a special order at t_0 for this scenario, and the optimal special order quantity decreases with the increase of q_0. However, when $q_0 > 24$, the buyer makes a special order more cost-saving than not making a special order, and his optimal replenishment policy is one of the candidate optimal policy sets π_1, π_2.

In general, whether the buyer places a special order not only relates to q_0, but also relates to the extent of price increase. In order to discuss this issue, we conduct a sensitivity analysis of the model by varying the parameters c and h while keeping the other parameters fixed as same as in Example 1. The numerical results are presented in Table 6 and Figure 7.

From Table 6 and Figure 7, it is not difficult to see that when q_0 is fixed, a special order is not required when the price increase rate is small. When the price increases to a certain level, placing a special replenishment is the buyer's optimal replenishment strategy. The buyer placing a special replenishment can save costs when the price increase rate is more than 8.5%.

7. Conclusion and Extensions

This study investigated the finite horizon inventory system with backordering and a known price increase, which is observed in common items like grain crops, wheat, corn, and soybeans, etc. The supplier allows the buyer to place a special order when the price increases. The objective of this study is to determine whether or not to purchase additional stock and optimal number of orders so that the inventory cost is minimum. For this mechanism, based on the retailers cost analysis, the problem is formulated as an optimization model and is solved by optimal techniques. A global optimal solution for the model was obtained through the algorithm given in the paper. From numerical studies, we show that the results of the numerical analysis are consistent with managerial implications and economical common sense. It has been shown that if those basic parameters of the models are given, we can get an optimal ordering policy, and under those basic parameters such as in Example 1 being fixed, the optimal order policy changes with the change of parameters q_0 and Δc, and the special replenishment size increases as q_0 decreases and c increases.

These results in the paper are important to the real world. The prices of goods, for example, wheat or soybeans, will increase after two or three months after harvest. Also when and how much the price increases are generally due to the interval between two normal orders. When buyers have known the price changes, they need to adjust their ordering strategy. Our model provides the decision-maker useful for this mechanism, and the algorithm can provide an optimal replenishment and stocking strategy to the buyers quickly. Numerical experiments were also carried out to illustrate the validity of the models. Sensitivity analysis has been performed in order to determine the robustness of the models presented above.

To make the concerned model more practical, we may introduce the shortages and partial backordering into the model. Further, the time of price change may be stochastic. This will be considered in the future research.

Appendix

A. Proof of Theorem 1

The proof of Theorem 1 is divided into two steps.

First, we discuss the optimal solution of problem (3) when k is fixed. Since the constraints are linear, any optimal solution is a KKT point which satisfies one of the following systems. See [11, 25].

$$\begin{cases} \dfrac{\partial f}{\partial Q_1} = \dfrac{h}{\lambda}(Q_1 + q) - \mu = 0, \\ \dfrac{\partial f}{\partial Q_i} = \dfrac{h}{\lambda}(Q_i - B_{i-1}) - \mu = 0, i = 2, \ldots, k, \\ \dfrac{\partial f}{\partial B_i} = -\dfrac{h}{\lambda}(Q_{i+1} - B_i) + \dfrac{w}{\lambda}B_i = 0, i = 1, \ldots, k-1, \end{cases} \tag{46}$$

where

$$f = F(k, Q_1, \ldots, Q_k, B_1, \ldots, B_{k-1}) - \mu\left(\sum_{i=1}^{k} Q_i - \lambda T + q\right) \tag{47}$$

is the Lagrange function, and $\mu \geq 0$ is the Lagrange multiplier corresponding to the equality constraint.

A straightforward computing of the system of equations gives that

$$\begin{cases} Q_1 = \dfrac{w\lambda T}{kw + (k-1)h} - q, \\ Q_2 = \cdots = Q_k = \dfrac{(w+h)\lambda T}{kw + (k-1)h} \triangleq Q, \\ B_1 = \cdots = B_{k-1} = \dfrac{h\lambda T}{kw + (k-1)h} \triangleq B, \end{cases} \tag{48}$$

and thus, $Q_1 = Q - B - q$.

Substitute the above solutions into formula (2); we obtain the inventory cost over the planning horizon of length T is

$$\begin{aligned} F &= c(\lambda T - q) + kA + \frac{kh}{2\lambda}(Q - B)^2 + \frac{(k-1)h}{2\lambda}B^2 \\ &= c(\lambda T - q) + kA + \frac{wh\lambda T^2}{2[kw + (k-1)h]}. \end{aligned} \tag{49}$$

Now, we consider the optimal ordering times k.

Since

$$F(k) = c(\lambda T - q) + kA + \frac{wh\lambda T^2}{2[kw + (k-1)h]}, \tag{50}$$

by

$$F''(k) = -\frac{wh\lambda T^2 (w+h)^2}{[kw + (k-1)h]} < 0, \tag{51}$$

we know that $F(k)$ is concave in k, and its minimum value is reached at the stationary point of the function $F(k)$ which gives

$$k = T\sqrt{\frac{w\lambda h}{2A(w+h)}} + \frac{h}{w+h}. \tag{52}$$

Considering k is an integer, we conclude that the optimal ordering time is $k_1 = \lfloor k \rfloor$ or $k_2 = \lceil k \rceil$. In detail, if $F(k_1) \leq F(k_2)$, then optimal ordering time is k_1; otherwise, it is k_2. □

B. Proof of Theorem 2

Similarly to the proof of Theorem 1, under the condition that m is fixed, we have

$$\begin{aligned} &Q_1' = \cdots = Q_m' \triangleq Q', \\ &B_s = B_1' = \cdots = B_{m-1}' \triangleq B', \end{aligned} \tag{53}$$

$$Q_s = \lambda T_0 - q_0 - \frac{m\lambda(w+h)(h_0 T_0 + c_0 - c)}{mh_0(w+h) + wh}, \tag{54}$$

$$Q' = \frac{\lambda(w+h)(h_0 T_0 + c_0 - c)}{mh_0(w+h) + wh}, \quad B' = \frac{\lambda h(h_0 T_0 + c_0 - c)}{mh_0(w+h) + wh}. \tag{55}$$

Considering the requirements that $Q_s, Q', B' \geq 0$, we conclude that Q_s, Q', B' given in (54)-(55) is a solution to model (21) provided that $h_0 T_0 + c_0 - c \geq 0$ and

$$(\lambda T_0 - q_0)\left[mh_0 (w + h) + wh\right] > m\lambda (w + h)\left(h_0 T_0 + c_0 - c\right), \tag{56}$$

i.e., $m(w+h)\left[q_0 h_0 + \lambda (c_0 - c)\right] < (\lambda T_0 - q_0) wh$. Obviously, if $q_0 h_0 + \lambda (c_0 - c) \leq 0$, then (56) holds; otherwise,

$$m < \frac{(\lambda T_0 - q_0) wh}{(w + h)\left[q_0 h_0 + \lambda (c_0 - c)\right]}. \tag{57}$$

For simplicity, we denote $\alpha = h_0 T_0 + c_0 - c$, $\beta = q_0 h_0 + \lambda (c_0 - c)$, $\delta = (\lambda T_0 - q_0) wh / (w + h)\left[q_0 h_0 + \lambda (c_0 - c)\right]$. Then Q_s, Q', B' given by (54), (55) is a solution to problem (21) provided that

$$\alpha \geq 0 \text{ and } \beta \leq 0, \tag{58}$$

$$\text{or } \alpha \geq 0, \beta > 0 \text{ and } m < \delta. \tag{59}$$

Otherwise, the solution of problem (21) is

$$m = 0, Q_s = \lambda T_0 - q_0, Q' = B' = 0. \tag{60}$$

The desired result follows by combining the discussion above.

Data Availability

The data used in our numerical experiments are taken randomly, and all used data released in this paper can be used directly.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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