Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 9954702, 9 pages
https://doi.org/10.1155/2022/9954702

Research Article

Modeling Urban Exodus Dynamics Considering Settlers Adaptation Time and Local Authority Support

Li Xin [1] **and Fei Liu** [2]

[1]*School of Economics and Management, Yanshan University, Qinhuangdao 066004, China*
[2]*School of Public Administration, Hengshui University, Hengshui 05300, China*

Correspondence should be addressed to Li Xin; lx2319@126.com and Fei Liu; liufei937087578@126.com

Received 2 September 2022; Revised 1 November 2022; Accepted 4 December 2022; Published 21 December 2022

Academic Editor: Kavikumar Jacob

While there has been much work analyzing the effects of urban exodus on rural areas' development, particularly in improving these localities' access to better services and decent quality of life, models to date lacked important features such as adaptation time effect on ongoing agricultural projects of new settlers reflecting real difficulties related to individuals abilities. In this article, we show that newcomers individual abilities, educational backgrounds, motivation, and so forth are crucial to promote the development of rural areas and facilitate the relocation or return of a certain group of people in their region of interest. Using a systemic approach, we present a model of urban exodus based on constant delay differential equations considering the local authority and population support and the time needed before the successful settlement of newcomers in the region. Furthermore, we estimate that adaptation time was responsible both for successful settlement increase and failure decrease. To reflect this, we incorporate delay terms in both the successful settlement and failure differential equations. We performed a qualitative analysis of the proposed system and show in numerical simulation that newcomers should be selected in function of their skills and experiences to accelerate their successful settlement, achieve overall socioeconomic development and improve the quality of life and well-being of the inhabitants.

1. Introduction

In a fast-changing world, reinforcing rural areas' inhabitants and promoting the development of local economies are important for national economies and the overall well-being of the inhabitants of all regions. The disparity between economically well-developed urban and adjacent regions has been evoked to explain continuous rural exodus, the weakening of rural localities, and the lagging of their economies despite governments' efforts and support. Some research conducted around the world has pointed out that regional economies should have development strategies that take into account the characteristics of each region [1–4].

According to many reports and published research articles, promoting agricultural projects in rural areas may have many advantages such as improving health problems and sustaining food systems, which are critical to achieve in the world's development goals. Agricultural development is indeed one of the most powerful tools to end extreme poverty, boost shared prosperity, and feed projected millions of people for the next generation to come. It leads to social life growth, poverty reduction in the area, and reduced food security which is always at risk whenever any natural catastrophe occurs (Climate change, pandemic: COVID-19, pests, and conflicts: war), impacting food systems. The result might be higher food prices and growing hunger (inflation and famine) [5–7].

One of the reasons that are associated with the fact that many people leave rural areas is food insecurity in some regions of the world. Research has found that an estimated 3 billion people in the world cannot afford a healthy diet and poor diets are the leading cause of death everywhere in the world. Millions of people are either not eating enough or eating the wrong food due to poor agricultural systems or else, which results in a double burden of malnutrition that can lead to illnesses in many cases and health crises (FAO reports 2021 and 2022). More than 10% of the world's population is affected. To solve part of this problem, entrepreneurs should get involved in agricultural projects in rural areas for the benefit of not only the population livestock but also to decrease the various forms of disparities and economic laggings [8, 9]. Livestock is the most valuable household asset and an irreplaceable source of a nutritious diet for hundreds of millions of families across the globe, especially for children. The benefit of people moving or returning back to rural areas to invest in agriculture and other related industries would be the regional demand for livestock products that are rapidly growing in these areas [10, 11].

This might also bring to rural areas crop science programs that will work in close consultation with local farmers, and work hand in hand with the populations in order to pursue targeted innovation that can be adopted on the field. These innovations will provide local farmers with many options for adapting to the numerous challenges that climate change and many other natural crises can bring (flooding, high temperatures, an array of crop pests, and diseases). People going or returning back to rural areas for agricultural projects and other related activities usually focus on maintaining and expanding crop diversity in the area, developing related industries, and small businesses. They try to ensure that farmers have what they need to provide consumers with a wider menu of options for eating a healthy meal or a sustainable diet. Their bold new efforts in agriculture bring a dramatic improvement in the production of crops to the market place; build sustainable and resilient food systems. Their projects might also support local actors including farmers, processors, traders, food companies from big cities, and retailers [12, 13].

In order to succeed in agricultural projects in rural areas, the local authority's support is needed by entrepreneurs. The local authority should get involved and support the different efforts to develop good policies and practices that more efficiently connect local farmers to their markets, to ensure that consumers have consistent access to an assortment of healthy and affordable food. They have to work hand in hand to ensure that public and private-sector agricultural development efforts are well supported by different targeted resources and that are aligned with the goals of local and/or international institutions [14–16].

Local authority plays a crucial role in agricultural projects, they support efforts in key areas to develop diversified farming and food systems that can improve access to affordable prices and strengthen the market for smallholder producers. Furthermore, by doing so, the regional authority not only strengthens the central government capacity in different states or provinces but also helps in implementing agricultural development programs that serve the needs of smallholder producers. This might include helping government officials generate evidence to guide policy development, resource allocation, and strategies that expand opportunities for all involved actors. They should support partners in the public and private sectors across the countries that are committed to transforming smallholder agriculture into a sustainable and inclusive foundation of economic opportunities [17–26].

In this article, to give a systemic view of the development of rural areas based on the trending urban exodus and regional agricultural projects, we consider three species delayed differential equation system in which successfully settled people are interacting both with the authority and newcomers (particularly those who are failing to adapt to their new life conditions). In this approach, the local authority is playing a key role by setting a threshold representing the carrying capacity of the area of interest and is supporting newcomers to adapt and help in developing the locality. The inhabitants are also assisting newcomers in many ways, such as helping them in everyday life and immersing them into the local culture for better integration. We assume those who are talented and well prepared for their new life are more likely to adapt, settle and benefit the region after a certain time (delay) by starting new projects or bringing their contributions to ongoing agricultural and other projects. Those who are less talented and poorly prepared or who have difficulties adapting are somehow affecting both the inhabitants and the new settlers, and this will end up impacting the overall regional development. Based on individuals' motivation, ability to adapt, local authority, and populations help and support, the impact of those who are holding the regional development back could be minimized by organizing outdoor activities, special training, and financial support [27, 28].

The proposed dynamical system is proven to be stable locally around the coexistence equilibrium point at certain conditions, both for the delayed and autonomous version of the system. Numeral results pointed out that selecting newcomers in function of their adaptation ability and professional skills is a key to accelerating a successful settlement and creating business opportunities as well as strengthening the overall regional economic development.

2. The Mathematical Model

In this section, we formulate the proposed model differential equations and describe the model coefficients, mathematical implications, and assumptions.

Letting $x(t)$ be the total number of newcomers, supposed to arrive from big or middle size cities around the rural area of interest, $y(t)$ be the number of individuals who have succeeded to settle and who have benefited from the local authority and inhabitants' support, and $z(t)$ be the size of individuals, at time t, who failed to adapt and settle in the new locality. In this analysis, we assume $x(t)$, $y(t)$, and $z(t)$ could be increasing or decreasing size depending on regional policies and regulations, the social status of respective individuals, the effectiveness of social and financial support, and other interventions of all participating groups. This dynamic is submitted to each group of individuals' capacity of adaptation, professional skills, educational background, and willingness to experience and learn new things.

Furthermore, it is logical to assume that some of the unsuccessful newcomers could affect the harmony of the community and thus, have a negative impact on $x(t)$ and $y(t)$, depending on the situation at hand. In highly sensitive cases, for example, cases that could lead to the disturbance of the social tranquility of the locality, the local authority should take control of the ongoing crisis and find reasonable solutions such as monitoring respective individuals, providing them with assistance, and training. We design a control parameter to account for this effect of the authority actions on the social life of the inhabitants in the differential equations of the model. Let the model system be formulated as follows:

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= ax(t)\left(1-\frac{x(t)}{K}\right) - by(t)x(t-\tau) + nz(t)x(t-\tau),\\
\frac{\mathrm{d}y}{\mathrm{d}t} &= by(t)x(t-\tau) - my(t)z(t),\\
\frac{\mathrm{d}z}{\mathrm{d}t} &= my(t)z(t) - nz(t)x(t-\tau),\\
& t\in[t_0-\tau, t_0], \tau>0, a, b, m, n, K>0,\\
x(t) &= \Phi(t)>0; x(0)>0, y(0)>0, z(0)>0,
\end{aligned} \tag{1}$$

where a is the rate of the urban exodus in a given rural region representing the per capita $x(t)$ population growth per unit time, b is the rate at which newcomers are successfully settling after τ time of adaptation $by(t)x(t-\tau)$. n accounts for the effect of the authority intervention in assisting those who face difficulties to settle $nz(t)x(t-\tau)$. m represents the interaction effect of $z(t)$ or those who are having a hard time adapting and settling for various reasons. We consider that their bad experiences are affecting negatively the tranquility and harmony of the community in the short-term run as mentioned earlier, and could be reflected in the overall economic and social development of the region in the long-term run.

3. The Steady State Equilibrium for $\tau = 0$

In this section, we solve the differential equations of the system when a delay is not applied or $\tau = 0$, and discuss the existence of the steady-state equilibrium.

If we suppose the adaptation time is negligent or zero for analysis purposes, then system (1) would become

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= ax(t)\left(1-\frac{x(t)}{K}\right) - by(t)x(t) + nz(t)x(t),\\
\frac{\mathrm{d}y}{\mathrm{d}t} &= by(t)x(t) - my(t)z(t),\\
\frac{\mathrm{d}z}{\mathrm{d}t} &= my(t)z(t) - nz(t)x(t).
\end{aligned} \tag{2}$$

Solving the equations using a standard method, we found that (2) admits a unique equilibrium point.

$$(x^*; y^*; z^*) = \left(K; \frac{nK}{m}; \frac{bK}{m}\right); \forall b, K, m, n > 0. \tag{3}$$

We can deduct that, to ensure continuous successful settlement of skilled migrant individuals and mitigate the impact of failure by maintaining lower the maximum to reach for z, $n < m < b$ must hold. This means there should be more newcomers succeeding to settle than failing for the local authority to reach its economic development goal. Furthermore, the growth rate of y must be maintained relatively higher, implying faster acculturation and/or assimilation (adaptation).

Setting

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= h_1(x, y, z),\\
\frac{\mathrm{d}y}{\mathrm{d}t} &= h_2(x, y, z),\\
\frac{\mathrm{d}z}{\mathrm{d}t} &= h_3(x, y, z),\\
& h_1, h_2, h_3 \in C^1 \forall (x, y, z) > 0, t \longrightarrow \infty.
\end{aligned} \tag{4}$$

The divergence and Laplacian of the system are computed as

$$\begin{aligned}
\nabla(h_1, h_2, h_3) &= a + nz(t) + bx(t) + my(t) - \left[\frac{2ax(t)}{K} + by(t) + nx(t) + mz(t)\right] \neq 0,\\
\nabla^2(h_1, h_2, h_3) &= -\frac{2a}{K} < 0,\\
\nabla(h_1, h_2, h_3)\big|^{(x^*;y^*;z^*)} &= -a.
\end{aligned} \tag{5}$$

The divergence is the measure of how much the vectors field diverge from each point of the phase space of a system, we have three possibilities. If $\nabla(h_1, h_2, h_3) > 0$, then the vectors are diverging from respective points of the phase space, and the system would be unstable. If $\nabla(h_1, h_2, h_3) < 0$, then the vectors will be converging to respective points of the phase space, indicating local stability at the vicinity of the given point of the trajectories. And for $\nabla(h_1, h_2, h_3) = 0$, we will have a zero divergence case, which could indicate that the system is conservative. The Laplacian $\nabla^2(h_1, h_2, h_3)$ being negative definite indicates that the gradient of the system is converging on average at each point of the phase space.

4. Stability Analysis When $\tau = 0$

Based on the determinant equation, the characteristic polynomial equation of (2) is computed as follows:

$$\begin{vmatrix} \lambda - a & -bK & nK \\ \dfrac{bnK}{m} & \lambda & -nK \\ -\dfrac{bnK}{m} & bK & \lambda \end{vmatrix} = 0. \tag{6}$$

It follows

$$\lambda^3 - T\lambda^2 + \Omega\lambda - D = 0, \tag{7}$$

where

$$\begin{aligned} T &= -a, \\ \Omega &= bnK^2\left(1 + \frac{b+n}{m}\right), \\ D &= -abnK^2. \end{aligned} \tag{8}$$

Routh–Hurwitz stability criteria says the system is locally asymptotically stable if

$$T < 0, \Omega > 0, D < 0, \Omega|T| > |D|. \tag{9}$$

This is satisfied as

$$abnK^2\left(1 + \frac{b+n}{m}\right) > abnK^2, \forall a, b, m, K > 0. \tag{10}$$

This will ensure that all traveling trajectories converge to $(x^*; y^*; z^*)$. The physical interpretation would be, as long as the urban exodus is going on in respective regions, a portion of newcomers will join the settlers group y and the rest of the people will remain at their current status or join z at relative speed, depending on parameters value and the effectiveness of the local authority support and control. The ideal scenario would be to have less and less people failing and a large portion of newcomers succeeding in integrating the local communities' social and economic life.

5. Stability Analysis When $\tau > 0$

In this section, we perform a stability analysis of the model when the adaptation time before settling is taken into account using the exponential solutions method. The transcendental characteristic equation is determined and given by

$$\begin{aligned} &\lambda^3 + s_0\lambda^2 + \left[(a-b)\lambda^2 + s_1\lambda - s_3 - \rho_1\right]e^{-\lambda\tau} + s_2\lambda + s_3K \\ &+\left[\left(bn + \rho_0 + n^2\right)\lambda + n\left(ab + \rho_0\right) + \rho_1\right]e^{-2\lambda\tau} - bn^2e^{-3\lambda\tau} = 0, \end{aligned} \tag{11}$$

where

$$\begin{aligned} s_0 &= (b-n)K + a, \\ s_1 &= a(1-b) - 2nbK, \\ s_2 &= aK(b-n) + 2nbK^2, \\ s_3 &= 2abnK, \\ \rho_0 &= b^2n\frac{K}{m}, \\ \rho_1 &= b^2n^2\frac{K^2}{m}. \end{aligned} \tag{12}$$

If we take $\lambda = i\mu, \mu \in \mathbb{R}^*$ to have periodic curves orbiting around the steady state equilibrium, then substituting in (11) we will get

$$\begin{aligned} P(\lambda):\ &-i\mu^3 - s_0\mu^2 + \left[-(a-b)\mu^2 + is_1\mu - s_3 - \rho_1\right]e^{-i\mu\tau} - is_2\mu\tau + s_3K \\ &+\left[\left(bn + \rho_0 + n^2\right)i\mu\tau + n\left(ab + \rho_0\right) + \rho_1\right]e^{-2i\mu\tau} - bn^2e^{-3i\mu\tau} = 0, \end{aligned} \tag{13}$$

After transformation and simplification using trigonometric formulas, and after separating real and imaginary parts, we obtain

$$\begin{aligned} \mathfrak{Re}(\lambda) &= -s_0\mu^2 - \left[(1-b)\mu^2 + s_3 + \rho_1\right]\cos\mu\tau + s_3K + \left(abn + \rho_0 n + \rho_1\right)\cos 2\mu\tau - \left(bn\mu + \mu\rho_0 + \mu n^2\right)\sin 2\mu\tau - bn^2\cos 3\mu\tau = 0, \\ \mathfrak{Im}(\lambda) &= -\mu^3 - \left[(1-b)\mu^2 + s_3 + \rho_1\right]\sin\mu\tau + s_2\mu - \left(abn + \rho_0 n + \rho_1\right)\sin 2\mu\tau - \left(bn\mu + \mu\rho_0 + \mu n^2\right)\cos 2\mu\tau - bn^2\sin 3\mu\tau = 0. \end{aligned} \tag{14}$$

Setting $X = \cos\mu\tau, Y = \sin\mu\tau$ gives

$$\begin{cases} -4bn^2X^3 + \varepsilon_0 X^2 + \left(3bn^2 - \varepsilon_1\right)X - \varepsilon_0 Y^2 + s_1\mu Y - 2\varepsilon_2 XY + s_3 K - s_0\mu^2 = 0, \\ kY^2 + \varepsilon_1 Y - 2\varepsilon_2 XY - \varepsilon_2 X^2 + s_1\mu X + \mu\left(s_2 - \mu^2\right) = 0, \end{cases} \tag{15}$$

where

$$\begin{aligned} \varepsilon_0 &= abn + \rho_0 n + \rho_1, \\ \varepsilon_1 &= (1-b)\mu^2 + s_3 + \rho_1, \\ \varepsilon_2 &= \mu\left(bn + \rho_0 + n^2\right), \\ k &= 3bn^2 - \varepsilon_2. \end{aligned} \tag{16}$$

Applying the Newton–Raphson iterative method with a globally convergent strategy to bound the progress toward the solution at each iteration

We have

$$\begin{aligned} F(X,Y) &= -4bn^2X^3 + \varepsilon_0 X^2 + \left(3bn^2 - \varepsilon_1\right)X - \varepsilon_0 Y^2 + s_1\mu Y - 2\varepsilon_2 XY + s_3 K - s_0\mu^2, \\ G(X,Y) &= kY^2 + \varepsilon_1 Y - 2\varepsilon_2 XY - \varepsilon_2 X^2 + s_1\mu X + \mu\left(s_2 - \mu^2\right), \\ J &= \begin{pmatrix} -12bn^2X^2 + 2\varepsilon_0 X + 3bn^2 - \varepsilon_1 - 2\varepsilon_2 Y & -2\varepsilon_0 Y + s_1\mu - 2\varepsilon_2 X \\ -2\varepsilon_2 Y - 2\varepsilon_2 X + s_1\mu & 2kY + \varepsilon_1 - 2\varepsilon_2 X \end{pmatrix} \\ X^{(0)} &= X(0) > 0, Y^{(0)} = Y(0) > 0, j = 1, 2, 3, \ldots \\ \delta X &= -J^{-1}.F, \\ X^{(j+1)} &= X^{(j)} + \delta X, \\ \delta Y &= -J^{-1}.G, \\ Y^{(j+1)} &= Y^{(j)} + \delta Y, \end{aligned} \tag{17}$$

where $X^{(0)}$ and $Y^{(0)}$ are initial guesses of the iteration.

Case 1. For $X^* = \cos\mu\tau > 0, Y^* = \sin\mu\tau > 0$,

$$\tau^* < \frac{\pi}{2\mu} + 2k\pi, k = 1, 2, 3. \tag{18}$$

Case 2. For $X^* = \cos\mu\tau > 0, Y^* = \sin\mu\tau < 0$,

$$\frac{3\pi}{2\mu} < \tau^* < \frac{2\pi}{\mu} + 2k\pi, k = 1, 2, 3. \tag{19}$$

Case 3. For $X^* = \cos\mu\tau < 0, Y^* = \sin\mu\tau > 0$,

$$\frac{\pi}{2\mu} < \tau^* < \frac{\pi}{\mu} + 2k\pi, k = 1, 2, 3. \tag{20}$$

Case 4. For $X^* = \cos\mu\tau < 0, Y^* = \sin\mu\tau < 0$,

$$\frac{\pi}{\mu} < \tau^* < \frac{3\pi}{2\mu} + 2k\pi, k = 1, 2, 3. \tag{21}$$

We can conclude that the population growth of the rural region submitted to continuous urban exodus and the development goal of respective local authority could be sustained and reached as long as the carrying capacity of the region is not saturated and the local authority, with the help of the inhabitants are assisting newcomers to settle down and to start new activities as fast and effective as possible. Their success would have a positive impact on the locality's economic and social development plan. This outcome is highly sensitive to the adaptation time of the settlers as the equations (18)–(21) suggest it.

6. Numerical Results and Discussion

In this section, we present the result of a computer simulation carried out to test the proposed system behavior for different parameters value representing different urban exodus determinants and regional development scenarios. As described earlier, all model parameters are positive constant numbers to reflect a continuous exodus and constant effort of both the local authority and the inhabitants to assist and facilitate the settlement of those who are motivated enough to adapt to their new life. The outcome of the interaction occurring is submitted to initial conditions or the size of each group at the start of the experimentation, to a, formalizing how many newcomers are joining the community in terms of ratio or proportion per unit of time. The dynamic of the interaction between $x(t)$, $y(t)$, and $z(t)$ is also submitted to b, formalizing the rate of $x(t)$ individuals turning into $y(t)$, to n, representing the fraction of $x(t)$ individuals joining $z(t)$ after τ time (here nondimensional unit of time for analysis purposes). $y(t)$ and $z(t)$ evolution over time is submitted to m,

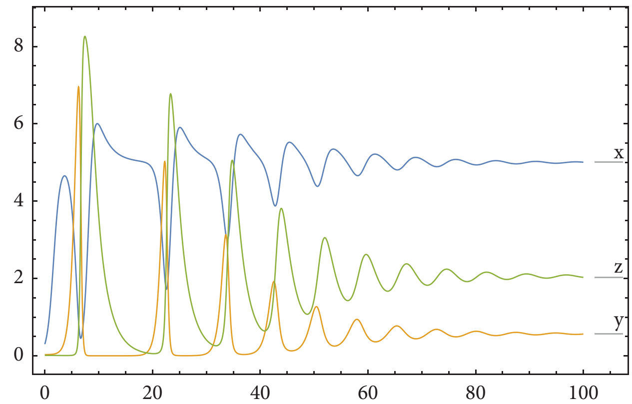

Figure 1: Dynamic of interaction when more people are failing to settle for $x(0) = 0.3$; $y(0) = 0.03$; $z(0) = 0.01$; $a = 1.681$; $b = 0.3$; $K = 5$; $n = 0.083$; $m = 0.73$; $\tau = 0.3$.

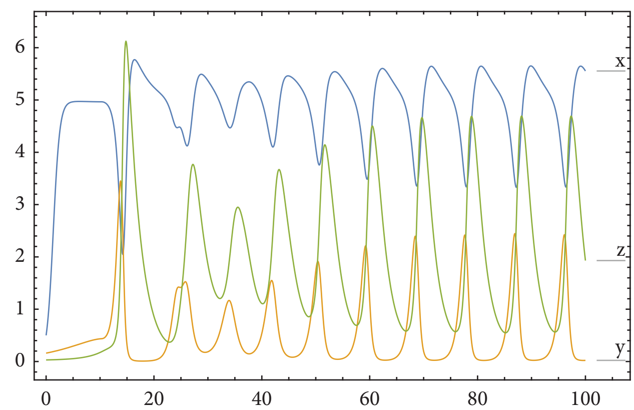

Figure 2: Dynamic of interaction when the adaptation time is relatively longer for $x(0) = 0.5$; $y(0) = 0.16$; $z(0) = 0.03$; $a = 1.681$; $K = 5$; $b = 0.3$; $n = 0.083$; $m = 0.73$; $\tau = 10.28$.

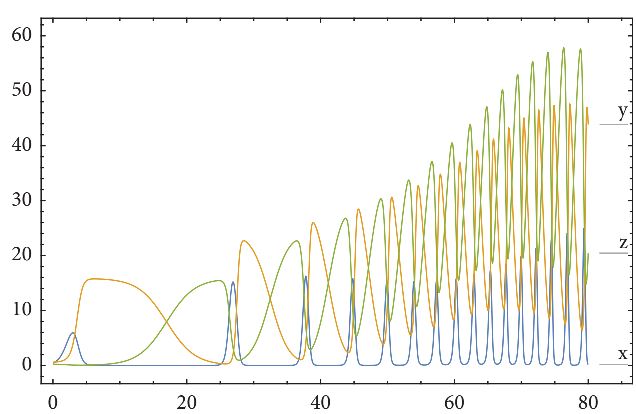

Figure 3: Dynamic of interaction when the settler individuals reach saturation earlier for $x(0) = 0.42$; $y(0) = 0.6$; $z(0) = 0.21$; $a = 1.41$; $K = 15$; $b = 0.23$; $n = 0.15$; $m = 0.03$; $\tau = 0.04$.

related to the percentage of failure and negative impact on the local community social and economic development plan. In each simulation scenario, the initial condition $x(0)$ will be considered as a historic function.

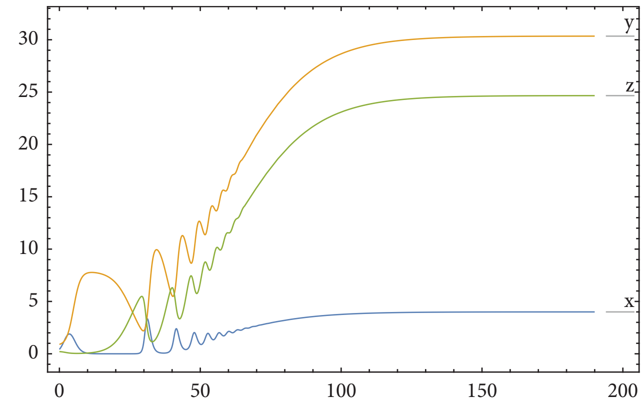

Figure 4: Time series of interactions showing a desirable outcome where the settlers adapt very rapidly for $x(0) = 0.42$; $y(0) = 0.9$; $z(0) = 0.21$; $a = 1.01$; $K = 4$; $b = 0.23$; $n = 0.283$; $m = 0.0373$; $\tau = 0.01$.

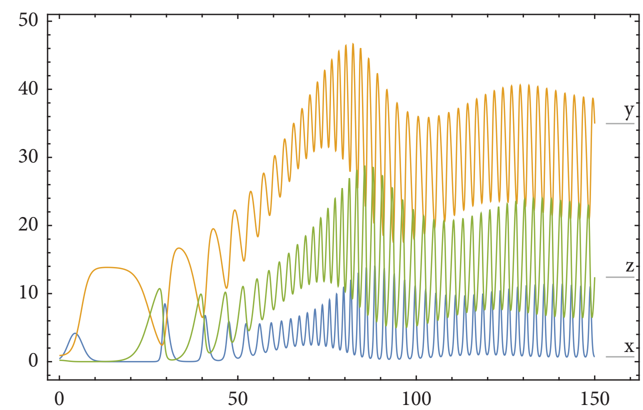

Figure 5: Time series of interactions showing a rapid adaptation for the settlers and closed curves around the steady state equilibrium for $x(0) = 0.42$; $y(0) = 0.9$; $z(0) = 0.21$; $a = 1.01$; $K = 7$; $b = 0.13$; $n = 0.283$; $m = 0.0373$; $\tau = 0.08$.

Figures 1 and 2 illustrate two urban exodus scenarios where more newcomers are having a hard time adapting and settling. The cumulative effort of the authority and the inhabitants in assisting newcomers is not helping very much as $n < m < b$ condition is not fulfilled with $m > b$.

In Figure 2, it is shown the effect of adaptation time on the system behavior when trajectories approach the steady state equilibrium for $\tau = 10.28$. The strong presence of newcomers does not have expected effects as $x(t)$ and $z(t)$ dominate the debate and $y(t)$ remains lower in the long run. In case of longer adaptation time, when an important fraction of the population is failing to settle, this group of individuals will influence significantly the dynamic of those who have the potential to settle and to accelerate the overall development of respective rural regions.

More effort should be dedicated to assist, accompany, train, and help $z(t)$ individuals in achieving their immersion into the local community society. This would be reflected directly in the adaptation time τ by boosting the adaptation

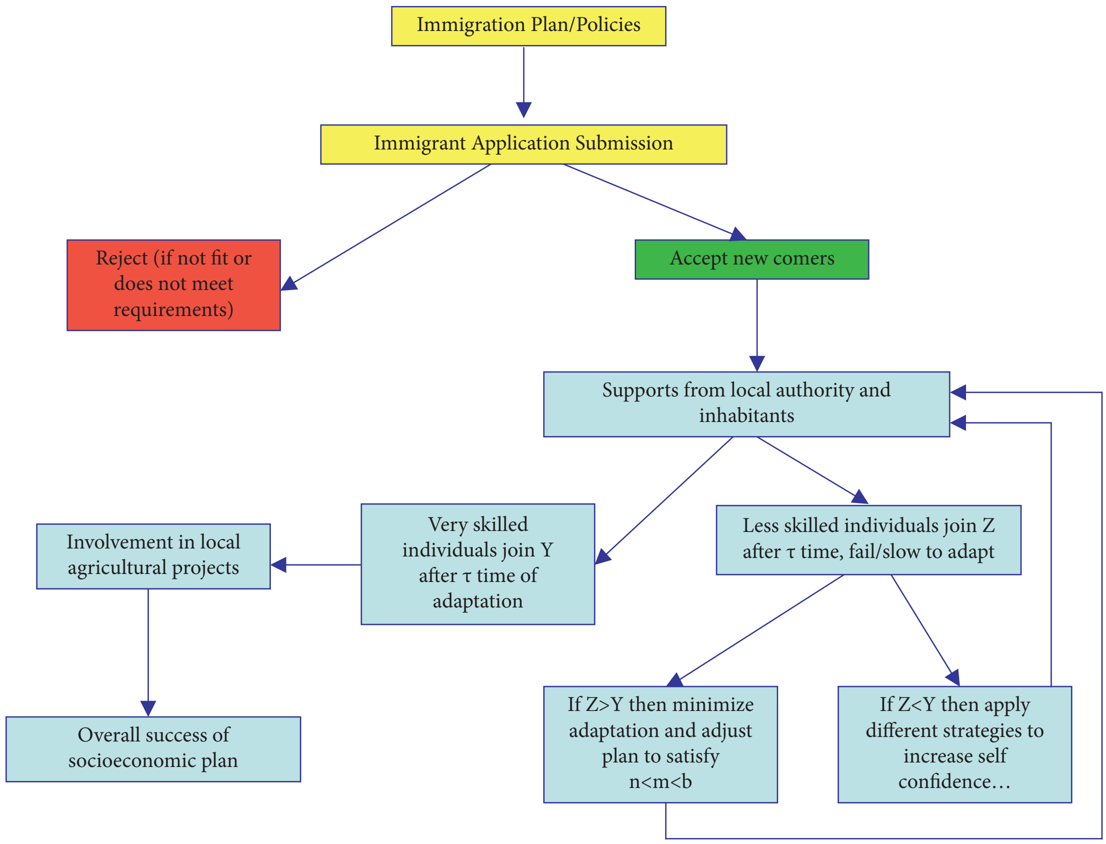

Figure 6: Urban exodus diagram for reaching the overall socioeconomic goal.

of those who are struggling to settle. $y(t)$ reaches saturation level faster than $z(t)$ in the early stages in Figure 1 compared to Figure 2. This indicates that the faster people adapt and settle, the more likely the region will develop for the benefit of the inhabitants' social well-being.

In Figure 3, $n < m < b$ being satisfied, with a strong presence of successful settlers $y(0) = 0.6$; $z(0) = 0.21$, and benefiting from more people joining $y(t)$, the settlers growth is sustained at the early stages. In the long run, however, due to the negative effect of $z(t)$ individuals (having a relatively large population size) on the local community's overall social and economic development, more and more people are affected as illustrated in the late stages in Figure 3. To reduce the negative effect of people who are failing to adapt, the local authority and inhabitants should collaborate with $y(t)$ individuals in training, hiring or helping $z(t)$ individuals to solve their problem and adapt to the local culture and conditions.

By reducing the speed of the exodus with $a = 1.01$ vs. $a = 1.41$, and accelerating the adaptation process with $\tau = 0.01$ vs. $\tau = 0.04$, clearly, the effects of failure have been relatively mitigated, and $y(t)$ dominates the competition and participates, for example, more actively in promoting local businesses in Figures 4 and 5. The activities of early settlers on the economic performance of respective rural regions are highly important for achieving higher development goals in rural regions. Their positive experiences could be emulated by those who are willing to adapt and settle. The outcome of the interaction will be submitted to initial conditions, the adaptation time, and the nature and sign of the eigenvalues of the Jacobian matrix; whether they are real or imaginary numbers as in Figure 5 at late stages where we have periodic motion around the steady state equilibrium point.

This result confirms the complexity of adopting new ideas and a new way of life for most people. The process of adaptation could be extremely complex for some individuals, especially those coming from different cultural backgrounds, those who have not experienced that type of challenges before, or those who are slow in assimilating gradual changes in social class relations. For these individuals, local authorities of hosting regions could adopt different acculturation or integration strategies to optimize their adaptation process as stated in much earlier and recent related literature [29–33].

For example, in many cases, those who fail to adapt are coming from strongly stressed urban life. Most of them wish to drop their burden and start a new life in remote areas, but they are not very well prepared for that challenge. The regional authority could apply an acculturation strategy based on J. Berry model by concentrating efforts towards reinforcing the integration of y individuals or assimilation of newcomers. Careful attention should be given to individuals who have the ability to integrate new cultures or values by maintaining their original culture or habits [34].

Another possible solution would be to apply TPB (Theory of Planned Behavior) with the ultimate goal of increasing the perceived confidence of newcomers such that they would have enough mental resources to engage in personal sacrifice, as they would gradually feel that they are needed and are at their place in the respective community. Figure 6 summarizes the process of successfully controlled continuous exodus for a region of interest.

7. Conclusion

In this article, an interactive delay differential equations system is proposed to analyze the underlying urban exodus dynamics in rural areas, considering the effects of adaptation and constant support of the local authority with the ultimate goal of socioeconomic long-term development goal. The model positive solutions exhibit complex dynamical behavior as traveling curves could be periodic and the steady state equilibrium point could be the center of some nearby orbiting trajectories, depending on control parameters and initial conditions. We showed that optimizing the integration of newcomers by accelerating their adaptation time through training, support, and implication of the inhabitants would reduce the rate of failure, enhance the economic, social, and environmental competitiveness of the region and benefit the locality in achieving decent quality of life and global economy goal. The results suggest adjusting the exodus determinants by selecting newcomers in function of their motivation, skills, and other related factors such as adaptation to change influencing respective individuals' integration in their new community.

Data Availability

All data generated or analyzed during the current study are available from the corresponding author Fei Liu upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this research article.

Authors' Contributions

Xin Li designed and carried out the experiments; Xin Li provided mathematical contents, and Xin Li and F. Liu wrote the manuscript.

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