

Research Article

Wave Propagations in Nonlinear Low-Pass Electrical Transmission Lines through Optical Fiber Medium

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Received 19 March 2021; Revised 2 April 2021; Accepted 16 December 2021; Published 5 January 2022

Academic Editor: Shahram Rezapour

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The present article discovers the new soliton wave solutions and their propagation in nonlinear low-pass electrical transmission lines (NLETs). Based on an innovative Exp-function method, multitype soliton solutions of nonlinear fractional evolution equations of NLETs are established. The equation is reformulated to a fractional-order derivative by using the Jumarie operator. Some new results are also presented graphically to understand the real physical importance of the studied model equation. The physical interpretation of waves is represented in the form of three-dimensional and contour graphs to visualize the underlying dynamic behavior of these solutions for particular values of the parameters. Moreover, the attained outcomes are generally new for the considered model equation, and the results show that the used method is efficient, direct, and concise which can be used in more complex phenomena.

1. Introduction

Considerable attention from scientists and researchers during the last two decades has highlighted that fractional differential simulations provide a better understanding than classical simulations to describe the complexities of physical scenarios in this real world. Various real life phenomena have been defined via the idea of fractional derivative such as porous medium, viscoelastic materials, optical fiber communication, fluid dynamics, signal processing, plasma physics, propagation of waves, ocean wave, photonic, electromagnetism, materials, chaotic systems, nanofiber technology, nuclear physics, and many others.

Currently, the most essential directions in the field of fractional calculus are to find the appropriate applications for describing the fractional operators. For this purpose, analytical and numerical techniques are used to produce the highly precise exact and approximate results. Recently, many authors have worked in this direction such as the efficient nonstandard finite difference scheme for a class of fractional chaotic systems [1]. A numerical technique has been used for resolving the fractional Hamilton equation with the help of

Caputo fractional operator [2], mathematical modelling of human liver with Caputo–Fabrizio fractional derivative [3], and mathematical model for Zika virus transmission [4]. Investigation of different types of soliton solutions of fractional differential equations (FDEs) can be identified through different techniques and has been examined by many authors [5–9]. Moreover, the application of fractional-order differential equations has been clearly expressed by studying the hyperchaotic behaviors of a nonautonomous cardiac conduction system in both frames of integer and fractional-order differential equations [10].

Consequently, several techniques and methods, for example, the method of Painleve analysis [11], the method of Riemann–Hilbert [12], the technique of $\exp(-\Phi(\xi))$ -expansion [13], the technique of generalized Kudryashov [14], the technique of $(G'/G, 1/G)$ -expansion [15], the trial solution approach [16], the Exp-function method [17], the modified simple equation method [18], and the technique of sine-Gordon expansion [19] are developed. He and Wu were the inventors of the Exp-function method [20] that was effectively extended to FDEs [21]. The Exp-function method has been applied to several nonlinear FDEs

[22–34]. Recently, many authors worked on the new results of the fractional calculus [35]. Many well-known problems have been resolved by using the fractional derivatives such as p -Laplacian nonperiodic boundary value problem [36] and conformable Laplace’s equation [37].

Transmission lines are utilized for the reason, such as high speed computer information buses, computer, arrange associations, trunk lines, steering calls between phone exchanging centers, dispersing cable tv signals, and interfacing radio transmitters and recipients with their receiving wires. The analytical solutions help in understanding the qualitative and quantity of many nonlinear phenomena and permit us to visualize the change mechanisms of various advanced nonlinear phenomena such as multiplicity or the absence of steady states below varied conditions, spatial localization of transfer processes, existence of peaking regimes, and lots of others. The examination of NLETs and its soliton solution have spectacular applications in electronic engineering, fluid dynamics, geophysics, optical fiber, laser optics, and communication systems such as the television cable lines that allocate signals and many others. Furthermore, for the transmission of the alternate current of radio frequency signal, NLETs are used as a special medium. The NLETs are used to distribute the higher manner to specific

the nonlinear excitation under nonlinear media and adjust the systems of recent structures. The NLETs play crucial role in the investigation of propagation phenomena of electrical solitons where they are traveling in nonlinear media of dispersion within the form of voltage waves.

The NLETs are derived with the help of Kirchoff’s laws that have been seen in the study by Abdoukary [38]. In preceding years, enormous research works have been done on NLETs. Many researchers have been obtaining numerous kind of analytic solution with the help of different methods like Jacobi elliptic method, auxiliary equation method, tanh function method, Ricatti equation technique, Kudryashov technique and modified version of Kudryashov technique, the sine-Gorden method, and the extended form of sine-Gorden equation [39–42].

In this paper, our main purpose is to establish, in the fractional context, important results for NLETs. For this purpose, the Exp-function method is used to resolve the nonlinear fractional evolution equation of NLETs by using the modified Riemann–Liouville (RL) derivative that was proposed by Jumarie [43] for nonlinear fractional evolution equation of NLETs. The modified RL derivative of order α is defined as follows:

$$D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(1-\beta)} \frac{d}{dt} \int_0^t (t-\xi)^{-\alpha} (f(\xi) - f(0)) d\xi, & 0 < \alpha < 1, \\ (f^n(t))^{(\alpha-n)}, & n \leq \alpha < n+1, n \geq 1. \end{cases} \tag{1}$$

The dynamical behavior for the presented model is studied in the form of graphs. To the best of our knowledge, the obtained results by using mathematical modelling and analytical technique presented in this paper for nonlinear fractional evolution equation of NLETs are new, more general, and comprise some valuable information.

This paper is organized as follows. Section 2 contains the methodology and general analysis of the Exp-function method. Section 3 includes the numerical implementation of the methods for solving the NLETs. In Section 4, results and discussion are given. Finally, conclusion is given in Section 5.

2. Analysis of Method

Consider the general nonlinear FDEs as follows:

$$Q(u, u_x, u_t, D_t^\alpha, \dots) = 0, \tag{2}$$

$$0 < \alpha \leq 1.$$

In equation (2), u is an unknown function, Q represents the polynomial, D_t^α represents the fractional derivative, and several linear and nonlinear terms with higher-order derivative are also included. The fractional traveling wave transformation is implemented for transforming equation (2) into ODEs as represented in the following equation:

$$u(x, t) = u(\eta), \tag{3}$$

$$\eta = k \frac{x^\alpha}{\alpha} + \omega \frac{t^\alpha}{\alpha},$$

where k is the width of soliton and ω represents the velocity. Applying the transformation equation (2) reduces to the following ODE:

$$R(u, u', u'', u''' \dots) = 0, \tag{4}$$

where the prime denotes the derivation with respect to η . If possible, we can integrate (4) one or more times. Suppose the soliton wave solution is

$$u(\eta) = \frac{\sum_{m=-r}^s a_m \exp[m\eta]}{\sum_{n=-g}^h b_n \exp[n\eta]}, \tag{5}$$

where $r, s, g,$ and h are positive integers which are unknown to be further determined and a_m and b_n are unknown constants. In equation (5), with the homogeneous balancing principle, the values of $r, s, g,$ and h will be determined.

3. Numerical Application

The nonlinear fractional evolution equation that describes the wave propagation in NLETs is defined as follows [44]:

$$D_t^{2\alpha} u + \beta D_t^{2\alpha} u^2 - d D_t^{2\alpha} u^3 + c^2 \beta D_x^{2\alpha} u^2 + \frac{c^4}{12} D_x^{4\alpha} u^2 = 0. \quad (6)$$

In equation (6), β , d , and c are the real constants and $u(x, t)$ characterizes to transmission line voltage. X signifies the distance, and t symbolizes the slow time. For acquiring the soliton solutions for NLETs, the Exp-function method will be applied to equation (6). For the conversion of equation (6) into an ordinary differential equation (ODE), we considered the fractional wave transformation as

$$u(x, t) = u(\eta)\eta = kx^\alpha/\alpha + \omega t^\alpha/\alpha \quad (7)$$

Substituting equation (3) in (6), we get

$$12(k^2 c^2 + \omega^2)u + 12\beta\omega^2 u^2 - 12 d\omega^2 u^3 + c^4 k^4 u'' = 0. \quad (8)$$

For simplicity, we set $r = s = g = h = 1$, and then equation (5) is reduced to

$$u(\eta) = \frac{a_{-1} \exp[-\eta] + a_0 + a_1 \exp[\eta]}{b_{-1} \exp[-\eta] + b_0 + b_1 \exp[\eta]}. \quad (9)$$

By substituting equation (9) in (8) and equating the coefficients of $\exp(n\eta)$ to zero, we find

$$\begin{aligned} A_1 &= (12b_{-1}^2 a_0 + 24a_{-1}(\beta a_0 + b_0)b_{-1} + 12a_{-1}^2(\beta b_0 - 3a_0))\omega^2 \\ &\quad + c^2 b_{-1} k^2 (a_0(k^2 c^2 + 12)b_{-1} - b_0 a_{-1}(k^2 c^2 - 24)) = 0, \\ A_2 &= \left(\begin{aligned} &12b_{-1}^2 a_1 + (24b_1 a_{-1} + 24a_0 b_0 + 12\beta(2a_{-1} a_1 + a_0^2))b_{-1} \\ &+ 12a_{-1}(\beta a_{-1} b_1 + 2\beta a_0 b_0 - 3a_{-1} a_1 - 3a_0^2 + b_0^2) \end{aligned} \right) \omega^2 \\ &\quad + 4c^2 k^2 \left(a_1(k^2 c^2 + 3)b_{-1}^2 + \left(\frac{(-k^2 c^2 a_{-1} + 6a_{-1})b_{-1}}{4} \right) b_{-1} + \frac{a_{-1} b_0^2 (k^2 c^2 + 12)}{4} \right) = 0, \\ A_3 &= \left(\begin{aligned} &(12\beta a_1^2 + 24b_1 a_1)b_{-1} + 12b_1^2 a_{-1} + (24a_0 b_0 + 12\beta(2a_{-1} a_1 + a_0^2))b_{1+} \\ &24a_1(\beta a_0 b_0 - \frac{3}{2}a_{-1} a_1 - \frac{3}{2}a_0^2 + \frac{1}{2}b_0^2) \end{aligned} \right) \omega^2 \\ &\quad - 4c^2 k^2 \left(\begin{aligned} &a_1 b_1 (k^2 c^2 - 6)b_{-1} + (-k^2 c^2 a_{-1} - 3a_{-1})b_1^2 - \frac{a_0 b_0 (k^2 c^2 - 24)b_1}{4} \\ &\frac{a_1 b_0^2 (k^2 c^2 + 12)}{4} \end{aligned} \right) = 0, \\ A_4 &= \left(\begin{aligned} &(24b_1 a_0 + 24a_1(\beta a_0 + b_0))b_{-1} + 24a_{-1}(\beta a_0 + b_0)b_1 + 12a_0 b_0^2 \\ &+ 12\beta(2a_{-1} a_1 + a_0^2)b_0 - 12a_0^3 - 72a_0 a_{-1} a_1 \end{aligned} \right) \omega^2 \\ &\quad - 6c^2 k^2 \left(\left((k^2 c^2 a_0 - 4a_0)b_1 - \frac{a_1 b_0 (k^2 c^2 + 8)}{2} \right) b_{-1} - \frac{b_0 (a_{-1} (k^2 c^2 + 8)b_1 + 4a_0 b_0)}{2} \right) = 0, \\ A_5 &= (12b_1^2 a_0 + 24a_1(\beta a_0 + b_0)b_{-1} + 12a_1^2(\beta b_0 - 3a_0))\omega^2 \\ &\quad + c^2 b_1 k^2 (a_0(k^2 c^2 + 12)b_{-1} - b_0 a_1(k^2 c^2 - 24)) = 0, \\ A_6 &= 12a_1((\beta a_1 b_1 - a_1^2 + b_1^2)\omega^2 + k^2 b_1^2 c^2) = 0, \\ A_7 &= 12a_{-1}((\beta a_{-1} b_{-1} - a_{-1}^2 + b_{-1}^2)\omega^2 + k^2 b_{-1}^2 c^2) = 0. \end{aligned} \quad (10)$$

By resolving the system of equations, we attain these results.

where a_{-1} , a_1 , and b_1 are free parameters.

Case 1

$$\begin{aligned}\omega &= \frac{cb_1k}{\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}}, \\ a_{-1} &= a_{-1}, \\ a_0 &= 0, \\ a_1 &= a_1, \\ b_{-1} &= \frac{b_1 a_{-1}}{a_1}, \\ b_0 &= 0, \\ b_1 &= b_1,\end{aligned}\tag{11}$$

$$u_1(x, t) = \frac{a_{-1} \exp\left[-Kx^\alpha/\alpha - cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + a_1 \exp\left[Kx^\alpha/\alpha + cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}{b_1 a_{-1}/a_1 \exp\left[-Kx^\alpha/\alpha - cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + b_1 \exp\left[Kx^\alpha/\alpha + cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}.\tag{12}$$

Case 2

where a_{-1} , a_0 , and b_0 are free parameters.

$$\begin{aligned}\omega &= \frac{cb_0k}{\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}}, \\ a_{-1} &= a_{-1}, \\ a_0 &= a_0, \\ a_1 &= 0, \\ b_{-1} &= \frac{b_0 a_{-1}}{a_0}, \\ b_0 &= b_0, \\ b_1 &= 0,\end{aligned}\tag{13}$$

$$u_2(x, t) = \frac{a_{-1} \exp\left[-Kx^\alpha/\alpha - cb_0kt^\alpha/\alpha\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}\right] + a_0}{b_0 a_{-1}/a_0 \exp\left[-Kx^\alpha/\alpha - cb_0kt^\alpha/\alpha\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}\right] + b_0}.\tag{14}$$

Case 3

where a_1, a_0, b_{-1} , and b_1 are free parameters.

$$\begin{aligned} \omega &= \frac{cb_1k}{\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}}, \\ a_{-1} &= \frac{a_1 b_{-1}}{b_1}, \\ a_0 &= a_0, \\ a_1 &= a_1, \\ b_{-1} &= b_{-1}, \\ b_0 &= \frac{b_1 a_0}{a_1}, \\ b_1 &= b_1, \end{aligned} \tag{15}$$

$$u_1(x, t) = \frac{a_1 b_{-1}/b_1 \exp\left[-Kx^\alpha/\alpha - cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + a_0 + a_1 \exp\left[Kx^\alpha/\alpha + cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}{b_{-1} \exp\left[-Kx^\alpha/\alpha - cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + b_1 a_0/a_1 + b_1 \exp\left[Kx^\alpha/\alpha + cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]} \tag{16}$$

Case 4

where a_1, a_0 , and b_1 are free parameters.

$$\begin{aligned} \omega &= -\frac{cb_1k}{\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}}, \\ a_{-1} &= \frac{a_1 b_{-1}}{b_1}, \\ a_0 &= a_0, \\ a_1 &= a_1, \\ b_{-1} &= b_{-1}, \\ b_0 &= \frac{b_1 a_0}{a_1}, \\ b_1 &= b_1, \end{aligned} \tag{17}$$

$$u_4(x, t) = \frac{a_1 b_{-1}/b_1 \exp\left[-Kx^\alpha/\alpha + cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + a_0 + a_1 \exp\left[Kx^\alpha/\alpha - cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}{b_{-1} \exp\left[-Kx^\alpha/\alpha + cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + b_1 a_0/a_1 + b_1 \exp\left[Kx^\alpha/\alpha - cb_1 kt^\alpha/\alpha \sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]} \tag{18}$$

Case 5

where a_{-1} , a_1 , and b_1 are free parameters.

$$\omega = -\frac{cb_1k}{\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}},$$

$$a_{-1} = a_{-1},$$

$$a_0 = 0,$$

$$a_1 = a_1, \quad (19)$$

$$b_{-1} = \frac{b_1 a_{-1}}{a_1},$$

$$b_0 = 0,$$

$$b_1 = b_1,$$

$$u_5(x, t) = \frac{a_{-1} \exp\left[-Kx^\alpha/\alpha + cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + a_1 \exp\left[Kx^\alpha/\alpha - cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}{b_1 a_{-1}/a_1 \exp\left[-Kx^\alpha/\alpha + cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right] + b_1 \exp\left[Kx^\alpha/\alpha - cb_1kt^\alpha/\alpha\sqrt{-\beta a_1 b_1 + a_1^2 - b_1^2}\right]}. \quad (20)$$

Case 6

where a_{-1} , a_0 , and b_0 are free parameters.

$$\omega = -\frac{cb_0k}{\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}},$$

$$a_{-1} = a_{-1},$$

$$a_0 = a_0,$$

$$a_1 = 0, \quad (21)$$

$$b_{-1} = \frac{b_0 a_{-1}}{a_0},$$

$$b_0 = b_0,$$

$$b_1 = 0,$$

$$u_6(x, t) = \frac{a_{-1} \exp\left[-Kx^\alpha/\alpha + cb_0kt^\alpha/\alpha\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}\right] + a_0}{b_0 a_{-1}/a_0 \exp\left[-Kx^\alpha/\alpha + cb_0kt^\alpha/\alpha\sqrt{-\beta a_0 b_0 + a_0^2 - b_0^2}\right] + b_0}. \quad (22)$$

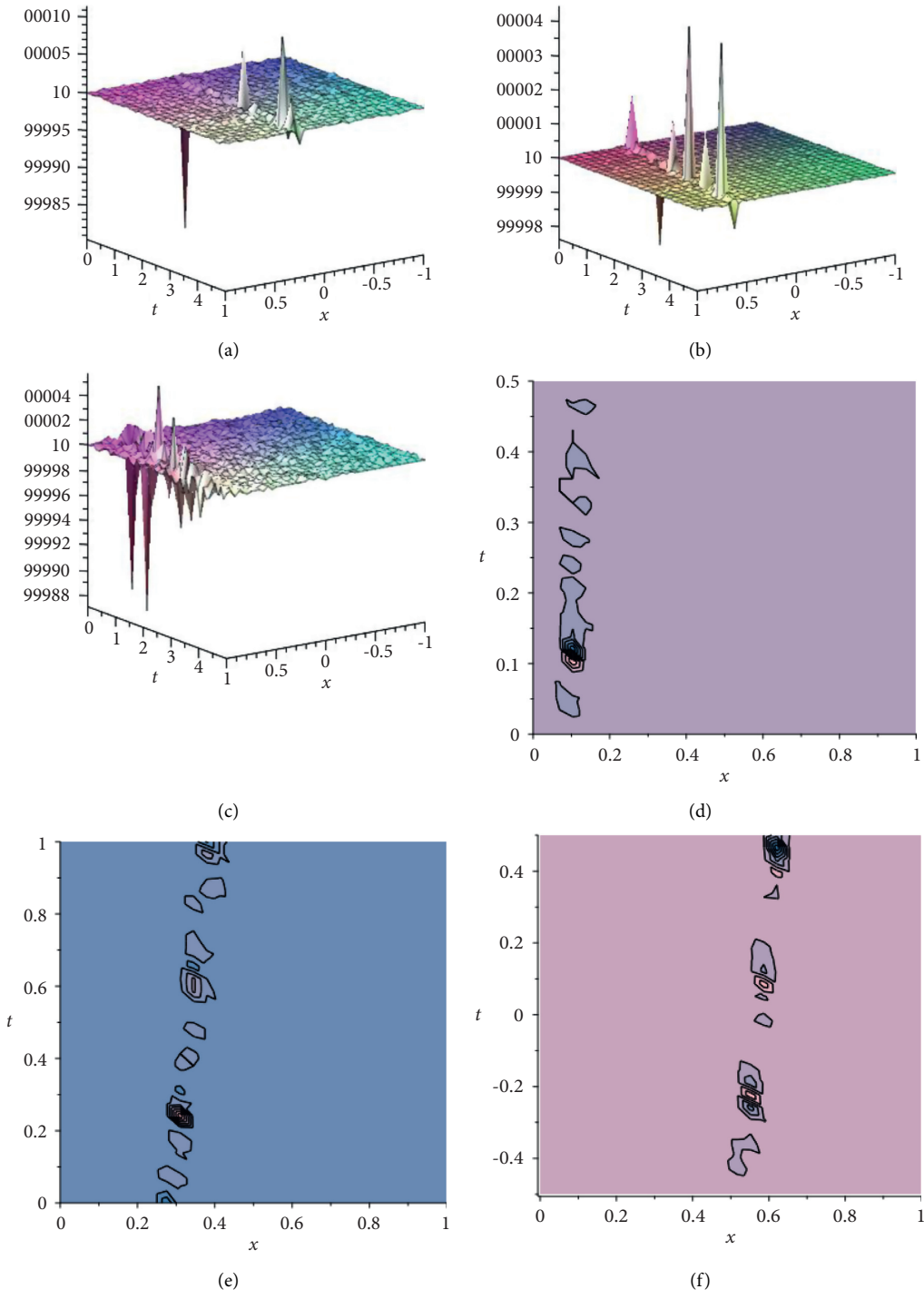


FIGURE 1: For $u_1(x,t)$, (a)–(c) with $-0.5 \leq x \leq 1, 0 \leq t \leq 2$ signifying the 3D plots (d),(e), and (f) with $0 \leq x \leq 1, 0 \leq t \leq 0.5$; $0 \leq x \leq 1, 0 \leq t \leq 1$; and $0 \leq x \leq 1, -0.5 \leq t \leq 0.5$, respectively, show the contour graph for different values of α : (a) $\alpha=0.5$; (b) $\alpha=0.7$; (c) $\alpha=1$; (d) $\alpha=0.5$; (e) $\alpha=0.7$; (f) $\alpha=1$.

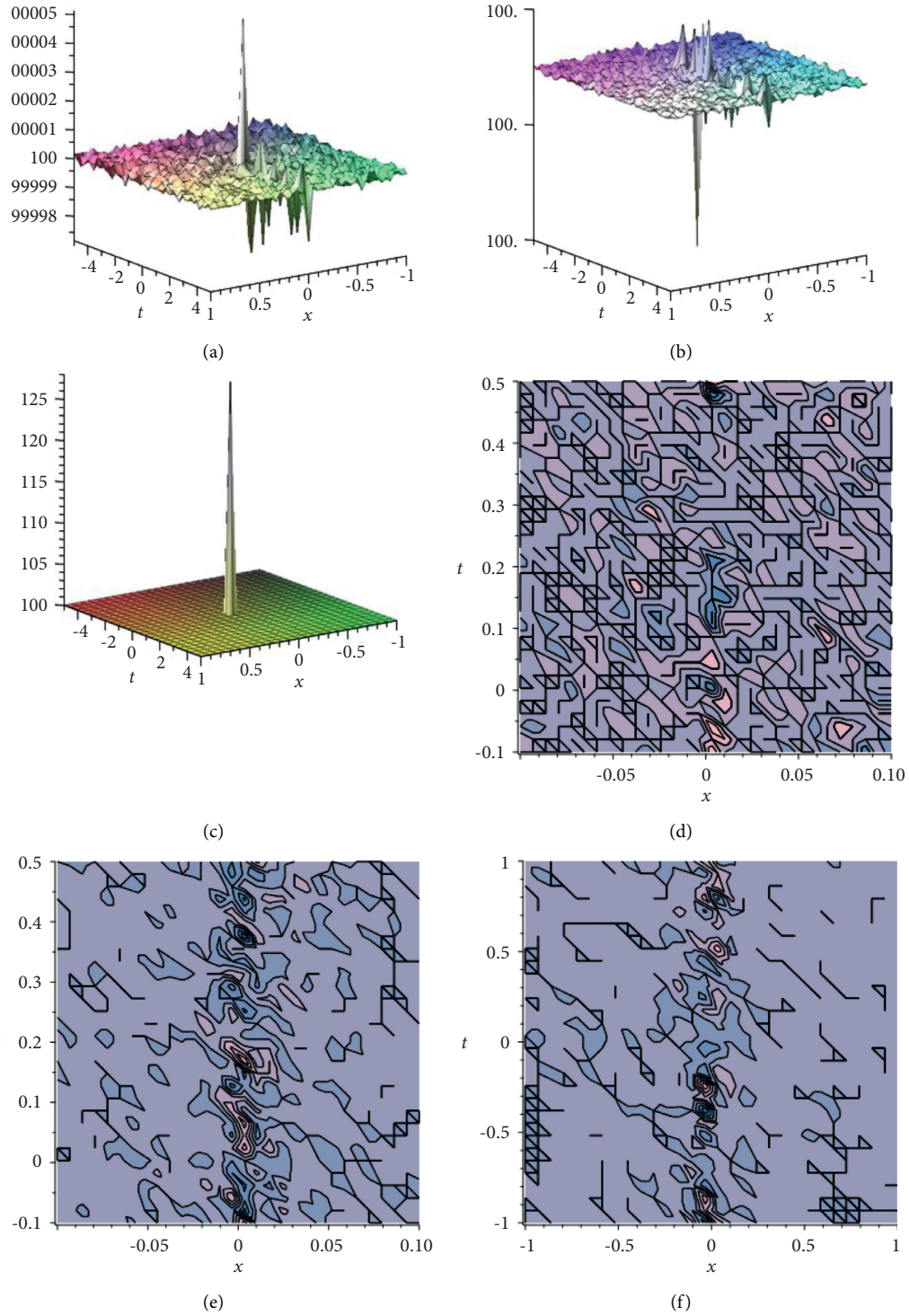


FIGURE 2: For $u_2(x,t)$, (a)–(c) with $-1 \leq x \leq 1, -5 \leq t \leq 5$ signifying the 3D plots (d), (e), and (f) with $-0.1 \leq x \leq 0.1, -0.1 \leq t \leq 0.5$; $-0.1 \leq x \leq 0.1, -0.1 \leq t \leq 0.5$; and $-1 \leq x \leq 1, -1 \leq t \leq 1$, respectively, show the contour graph for different values of α : (a) $\alpha=0.5$; (b) $\alpha=0.7$; (c) $\alpha=1$; (d) $\alpha=0.5$; (e) $\alpha=0.7$; (f) $\alpha=1$.

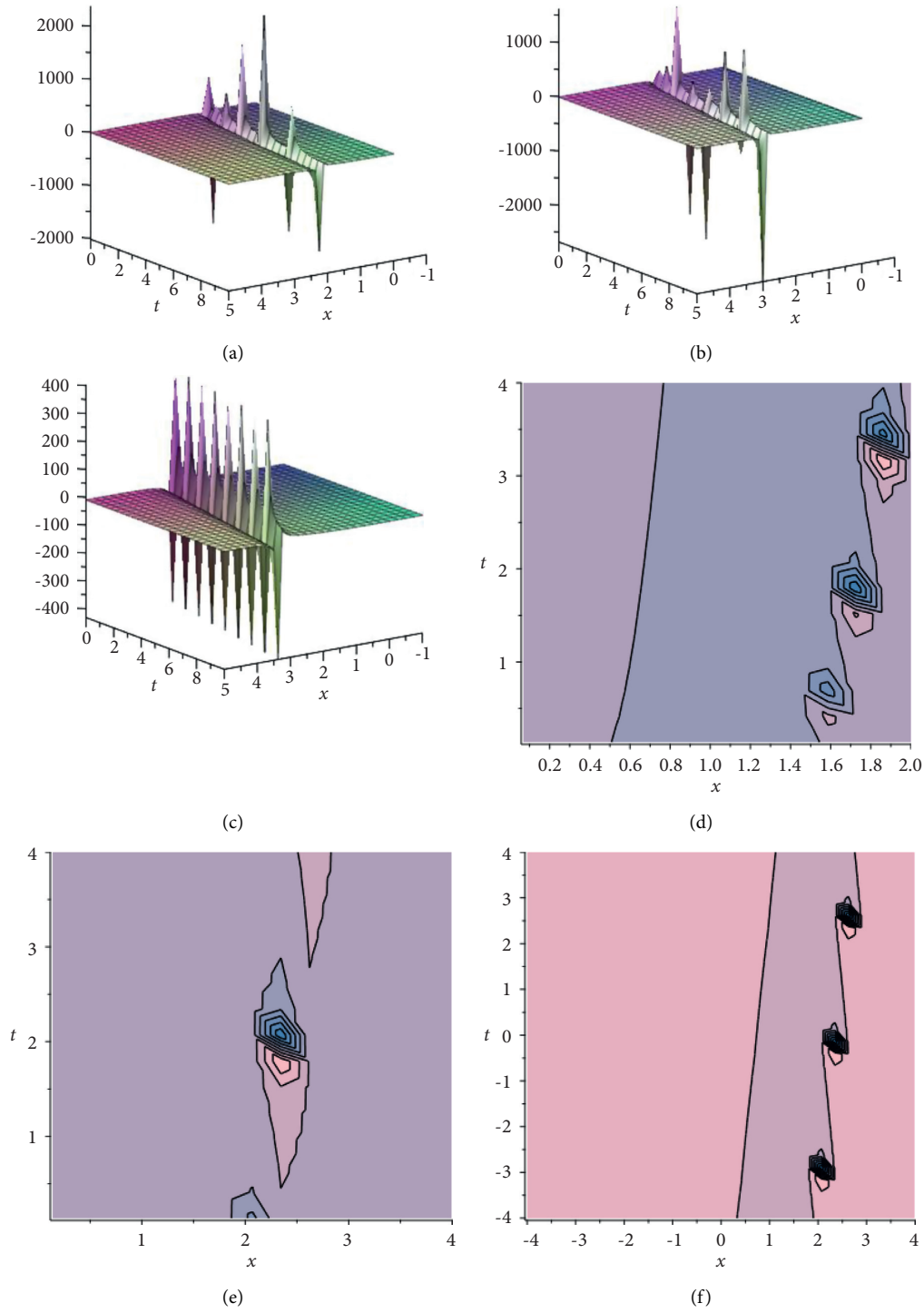


FIGURE 3: For $u_3(x,t)$, (a)-(c) with $-1 \leq x \leq 5, 0 \leq t \leq 5$ signifying the 3D plots (d), (e), and (f) with $-2 \leq x \leq 2, -4 \leq t \leq 4$; $-4 \leq x \leq 4, -4 \leq t \leq 4$; and $-4 \leq x \leq 4, -4 \leq t \leq 4$, respectively, characterize the contour graph for different values of α : (a) $\alpha = 0.5$; (b) $\alpha = 0.7$; (c) $\alpha = 1$; (d) $\alpha = 0.5$; (e) $\alpha = 0.7$; (f) $\alpha = 1$.

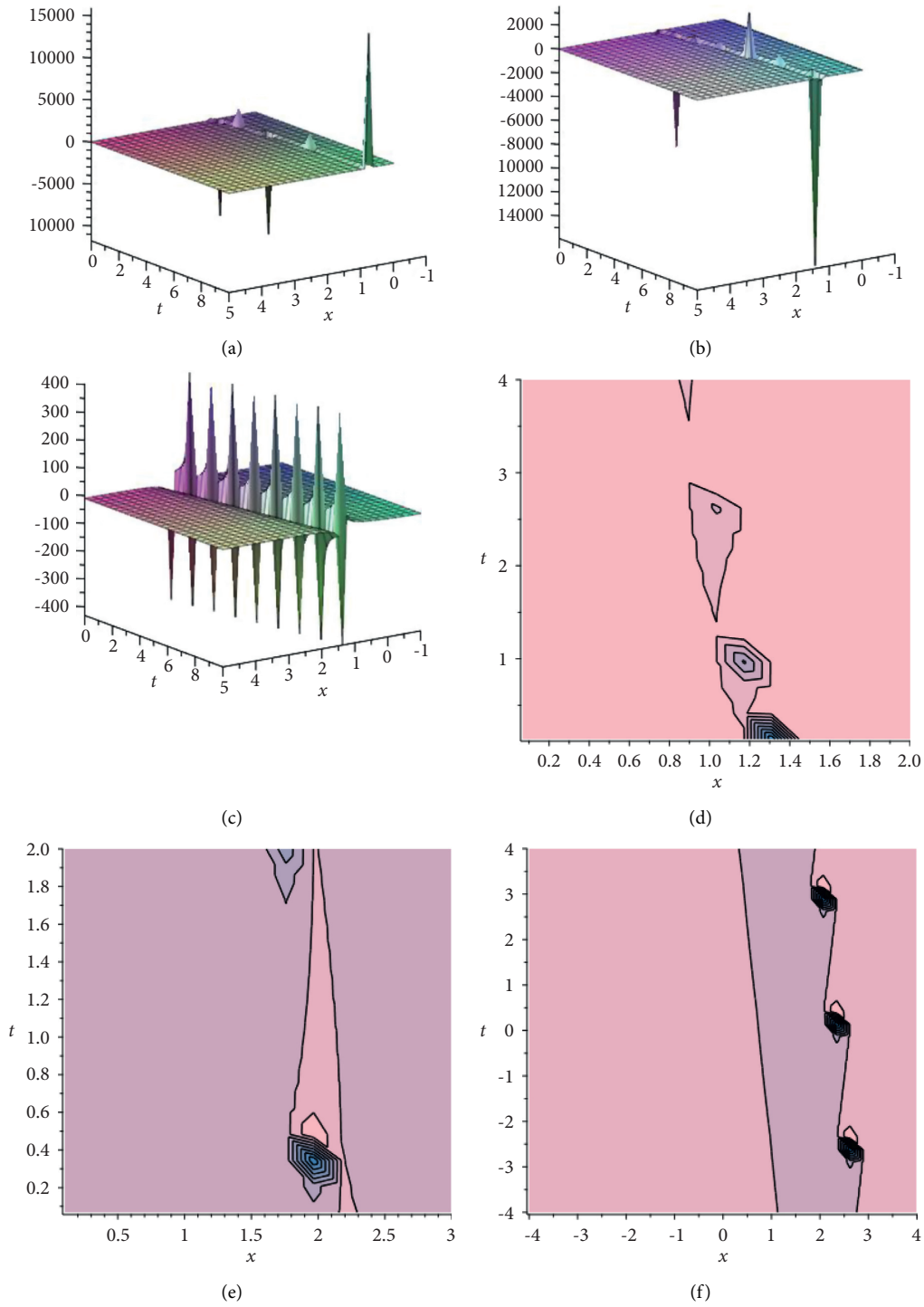


FIGURE 4: For $u_4(x, t)$, (a)–(c) with $-1 \leq x \leq 5, 0 \leq t \leq 5$ characterizing the 3D plots, (d), (e), and (f) with $-2 \leq x \leq 2, -4 \leq t \leq 4$; $-3 \leq x \leq 3, -2 \leq t \leq 2$; and $-4 \leq x \leq 4, -4 \leq t \leq 4$, respectively, signify the contour graph for different values of α : (a) $\alpha = 0.5$; (b) $\alpha = 0.7$; (c) $\alpha = 1$; (d) $\alpha = 0.5$; (e) $\alpha = 0.7$; (f) $\alpha = 1$.

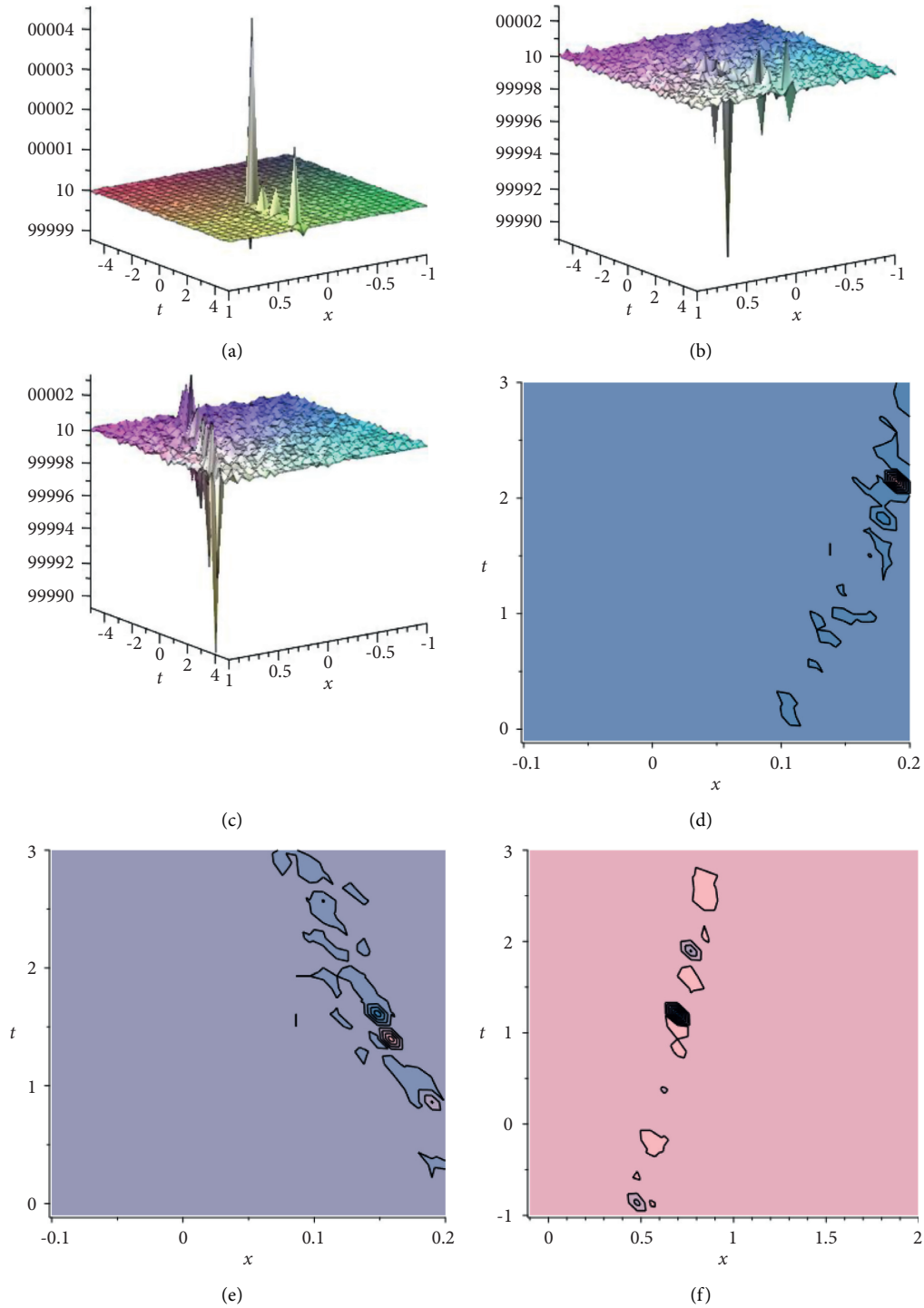


FIGURE 5: For $u_5(x,t)$, (a)–(c) with $-1 \leq x \leq 1, -5 \leq t \leq 5$ signifying the 3D plots (d), (e), and (f) with $-0.1 \leq x \leq 0.2, -0.1 \leq t \leq 3$; $-0.1 \leq x \leq 0.2, -0.1 \leq t \leq 3$; and $-0.1 \leq x \leq 2, -1 \leq t \leq 3$, respectively, indicate the contour graph for different values of α : (a) $\alpha=0.5$; (b) $\alpha=0.7$; (c) $\alpha=1$; (d) $\alpha=0.5$; (e) $\alpha=0.7$; (f) $\alpha=1$.

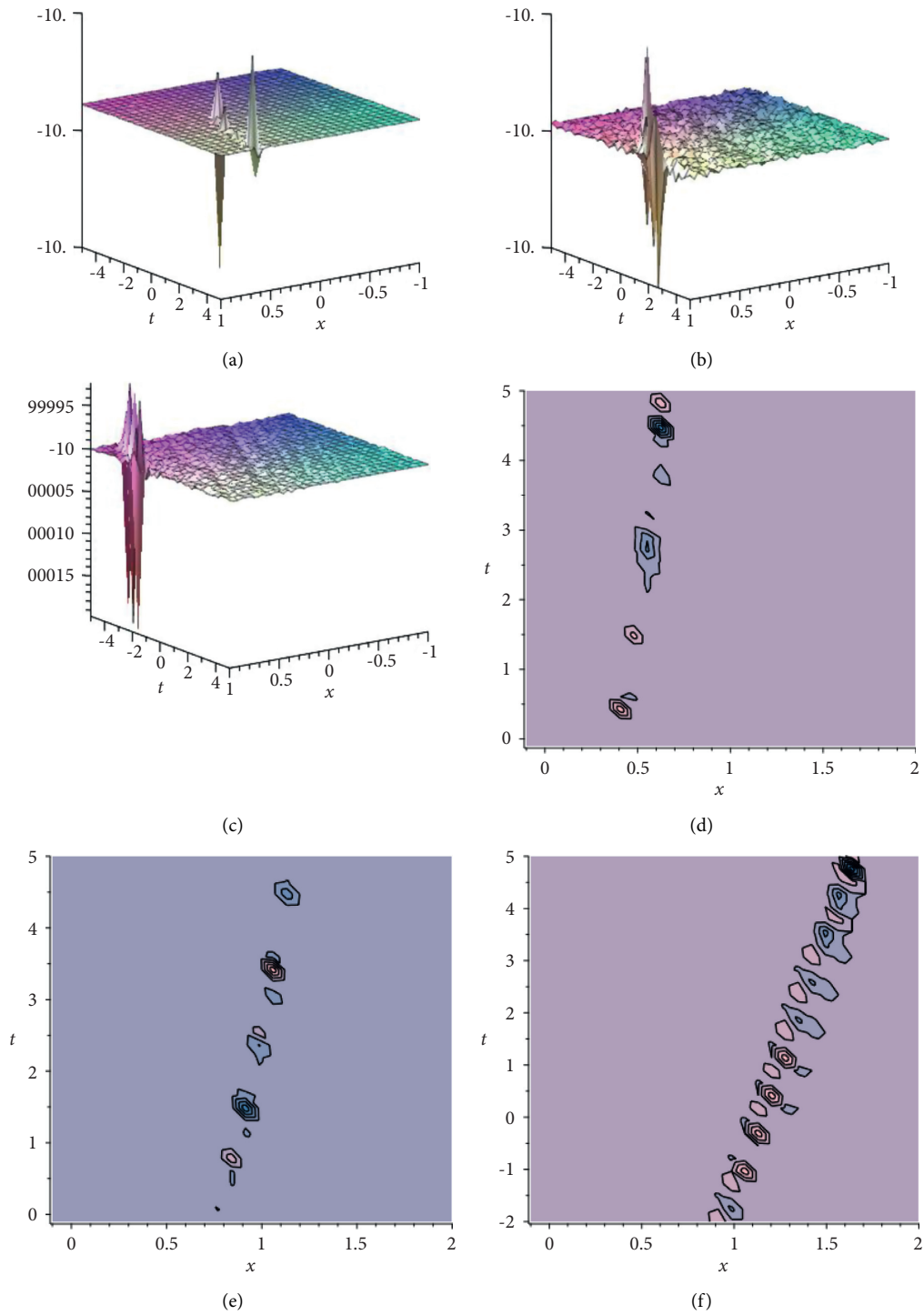


FIGURE 6: For $u_6(x,t)$, (a)–(c) with $-1 \leq x \leq 1, -5 \leq t \leq 5$ signifying the 3D plots (d), (e), and (f) with $-0.1 \leq x \leq 2, -0.1 \leq t \leq 5$; $-0.1 \leq x \leq 2, 0.10 \leq t \leq 5$; and $-0.1 \leq x \leq 2, -2 \leq t \leq 5$, respectively, show the contour graph for different values of α : (a) $\alpha = 0.5$; (b) $\alpha = 0.7$; (c) $\alpha = 1$; (d) $\alpha = 0.5$; (e) $\alpha = 0.7$; (f) $\alpha = 1$.

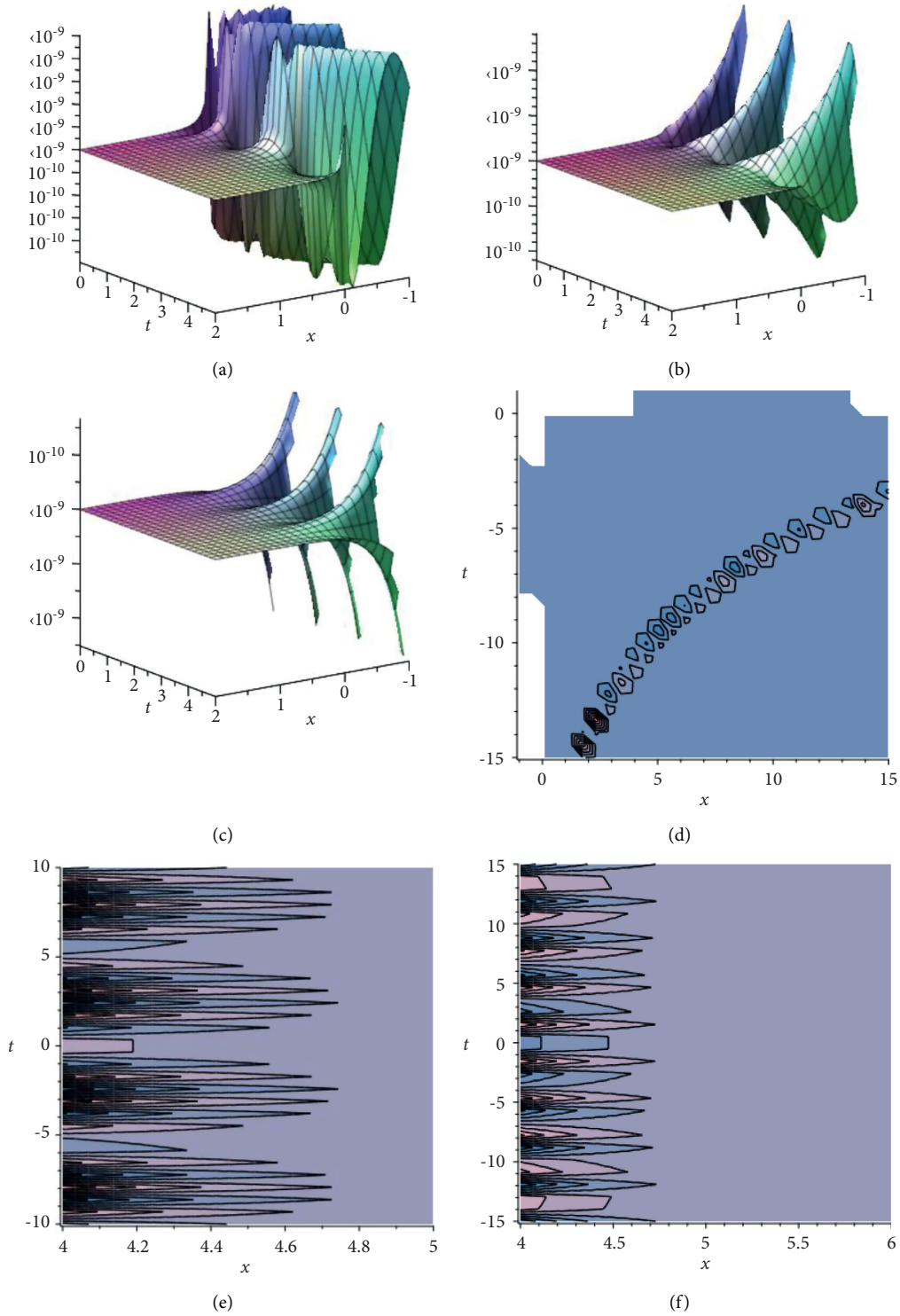


FIGURE 7: For $u_7(x,t)$, (a)-(c) with $-1 \leq x \leq 2, 0 \leq t \leq 5$ signifying the 3D plots (d), (e), and (f) indicate the contour graph for $10 \leq x \leq 15, -15 \leq t \leq 15$; $4 \leq x \leq 5, -10 \leq t \leq 10$; and $4 \leq x \leq 6, -15 \leq t \leq 15$, respectively, for different values of α : (a) $\alpha = 0.5$; (b) $\alpha = 0.7$; (c) $\alpha = 1$; (d) $\alpha = 0.5$; (e) $\alpha = 0.7$; (f) $\alpha = 1$.

Case 7.

$$\omega = -\frac{\sqrt{-3c^2k^2 - 36ck}}{6},$$

$$a_{-1} = 0,$$

$$a_0 = a_0,$$

$$a_1 = 0,$$

$$b_{-1} = b_{-1},$$

$$b_0 = -\frac{\beta a_0(k^2c^2 + 12)}{3k^2c^2},$$

$$b_1 = \frac{a_0^2(2\beta^2k^4c^4 + 9k^4c^4 + 48\beta^2k^2c^2 + 108k^2c^2 + 288\beta^2)}{72b_{-1}k^4c^4},$$
(23)

where a_0 and b_{-1} are free parameters.

$$u_7(x, t) = \frac{a_0}{b_{-1} \exp\left[-Kx^\alpha/\alpha + \sqrt{-3c^2k^2 - 36ckt^\alpha}/6\alpha\right] - \beta a_0(k^2c^2 + 12)/3k^2c^2 + a_0^2(2\beta^2k^4c^4 + 9k^4c^4 + 48\beta^2k^2c^2 + 108k^2c^2 + 288\beta^2)/72b_{-1}k^4c^4 \exp\left[Kx^\alpha/\alpha - \sqrt{-3c^2k^2 - 36ckt^\alpha}/6\alpha\right]}.$$
(24)

4. Results and Discussion

The erection of the soliton wave by determining nonlinear fractional evolution equation has been studied using the competent Exp-function method. We compare our acquired results with the previously existing solutions in the literature obtained by using different techniques. The main concentration of this work is to obtain new and more general solutions of fractional order at different values of parameters. In the literature, many different kinds of solutions have been obtained by using different techniques like dark, bright, trigonometric, hyperbolic, and rational type of solitary wave solutions as given in [38, 42, 44].

Graphical data illustrate the physical behavior of solitons at different values of parameters. Figures 1–7 show the soliton solutions of NLETL equation of fractional order in the form of 3D and contour plots at $\alpha = 0.4$, $\alpha = 0.7$, and $\alpha = 1$. Figure 1 signifies the physical behavior of $u_1(x, t)$ for $a_{-1} = 0.1$, $a_1 = -0.01$, $b_1 = -0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 2 signifies the plot of $u_2(x, t)$ at $a_{-1} = 0.1$, $a_0 = -0.1$, $b_0 = -0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 3 demonstrates the plot for $u_3(x, t)$ for $a_{-1} = 0.1$, $a_0 = 0.1$, $a_1 = 0.01$, $b_{-1} = 1.5$, $b_1 = -0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 4 characterizes the physical behavior of $u_4(x, t)$ for $a_{-1} = 0.1$, $a_0 = 0.1$, $a_1 = 0.01$, $b_{-1} = 1.5$, $b_1 = -0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 5 denotes the plot of $u_5(x, t)$ with the parameter $a_{-1} = 0.1$, $a_1 = -0.01$, $b_1 = -0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 6 signifies the plot of $u_6(x, t)$ for

$a_{-1} = 0.1$, $a_0 = -0.01$, $b_0 = 0.001$, $c = 1$, $\beta = 0.1$, and $k = 2$. Figure 7 indicates the plot of $u_7(x, t)$ at $a_0 = 1 \times 10^{-12}$, $b_0 = -0.00001$, $c = 1$, $\beta = 0.1$, and $k = 3$.

We represented seven cases for different values of parameter using fractional derivative in this research work. It is clear from this work that fractional derivative has very important role in understanding the structure of the presented nonlinear evolution equation and also describes the continuous behavior of the solution wave through out the process. By comparing our results with the existing results, we conclude that our obtained results using the Exp-function method are generally new which have not been presented in the previous existing work. The complete examination proves that the presented technique is more consistent, competent, and dominant to examine different kinds of nonlinear fractional evolution equations.

5. Conclusion

In conclusion, the main attempt is to find and analyse the new soliton solutions of the fractional nonlinear fractional evolution equation that explains the wave propagation in nonlinear low-pass electrical transmission lines. The innovative Exp-function method with fractional traveling wave transform was utilized to obtain the well-known soliton solutions. Furthermore, from the graphical depiction, we have observed that different values of the parameters provide the different types of solutions for the low-pass electrical

transmission lines. The solutions obtained from this model equation are applicable to various fields of science and engineering. Accordingly, we come to the conclusion that the presented model in this paper using the properties of fractional calculus is more flexible and genuinely analyzes real-world dynamical systems via the fractional-order differential operators from which the associated complicated dynamical behaviors of the problem can be revealed more precisely than the integer-order ones. Thus, it is well known that solitons have a great importance in the telecommunication system where the results obtained will probably help to carry information and increase the bit-rate of data. Also, we can extend our work for resolving the more complex biological and engineering problems using the properties and suitable operator of fractional calculus.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest or personal relationships that could have appeared to influence the work reported in this paper.

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