Research Article

A Pressure-Sinkage Model for Deep-Sea Sediments Based on Variable-Order Fractional Derivatives

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1. Introduction

The deep sea contains rich mineral resources. Among them, polymetallic nodules have sizable reserves and are considered to be the most commercially promising resources [1–3]. At present, the deep-sea mining system is mainly composed of four parts: the surface mining vessel, the pump pipe conveying system, the subsea mining vehicle, and the system collaborative control simulation platform, which cooperate to lift polymetallic nodules with a depth of 5,000–6,000 meters to the surface [4–7]. Due to the complexity of the depositional environment, material source, and consolidation state, the mechanical properties of deep-sea sediments are quite different from those of onshore soft clay, which are characterized by low strength, high compressibility, and low permeability [8–10]. At the same time, subsea mining vehicles are the most critical equipment in pipeline hydraulic transportation systems. Subsea mining vehicles require adaptability and working reliability, and their stability directly determines the success or failure of mining. A schematic diagram of a subsea mining vehicle is shown in Figure 1. Subsea mining vehicles are in direct contact with the seafloor, relying on the shear resistance of the deep-sea sediments to pull the crawler. Therefore, it is important to analyze the mechanical properties of the deep-sea sediments and their interaction mechanism with the subsea mining vehicle, establish the pressure-sinkage model on deep-sea sediments, and reveal the influence of model parameters on the driving performance of the subsea mining vehicle. These factors provide a theoretical basis for the development and design of an undersea walking vehicle. However, due to the difficulty of carrying out in-situ experiments at the current level of technology, researchers often use mixtures of bentonite and water instead of deep-sea sediments to conduct experimental studies and obtain the mechanical properties of their interactions [11–15].

In recent years, domestic and foreign researchers have carried out a series of studies on the pressure-sinkage...
behaviours of deep-sea sediments [7, 16–18]. The Bekker bearing model has become the theoretical basis for analyzing the interaction characteristics between vehicles and the ground [19]. Schulte et al. [11] proposed the empirical pressure-sinkage model according to the stress-strain characteristics of deep-sea sediments using the classical Bekker theory and also briefly introduced the calculation method of static load and the variation trend of pressure sinkage with time. Wang et al. [17] obtained the pressure-sinkage curve of simulated sediments by carrying out pressure-sinkage tests and established a corresponding deep-sea sediment pressure-sinkage model based on the Bekker empirical model. According to the physicochemical properties of deep-sea sediments, Li [20] configured simulated soils with similar physical properties and established the shear stress-displacement and pressure-sinkage relationship equations based on the Bekker theory and the Reece theory. Compared with the abovementioned static prediction methods, related scholars have also proposed component models that consider the rheological properties of deep-sea sediments. The component model mainly characterizes the viscoelastic-plastic properties of rock through different combinations of standard elements [21, 22]. Innovative to deep-sea mining research, Qi et al. [14, 23] systematically studied the rheological properties of deep-sea sediments for the first time and established the rheological constitutive equations of deep-sea sediment-simulated soils under various loads. At the same time, the traction-creep characteristics of deep-sea sediment simulated soil were analyzed using a fractional derivative and the Burgers creep model. Xu et al. [24] used the Kelvin-Hooke and Burgers rheological models to analyze the compressive creep behavior and compressive-shear creep behavior of the deep-sea sediment simulated soil, employing TableCurve-3D to determine various parameters. The model-fitting results validate the experimental data, which can better reflect its rheological properties. According to the elastic-plastic theoretical model, Wang et al. [25] proposed a deep-sea sediment shear-stress-displacement relationship and verified the accuracy of their model through the deep-sea sediment shear test. The abovementioned constitutive models describe the mechanical properties of deep-sea sediments well, but due to differences between deep-sea sediments and terrestrial soils, whether the abovementioned empirical models are applicable to deep-sea sediments remains to be verified. At the same time, the deep-sea sediment pressure-sinkage process has obvious stage characteristics, and the fractional order model cannot describe the mechanical evolution of deep-sea sediments.

Historically, the variable-order fractional derivatives have attracted the attention of many researchers [26]. Valério and Sá da Costa discussed the definition of variable-order derivatives and their approximations under real complex numbers. Also, the variable-order approximations were obtained by using existing approximations for constant orders [27]. Moghaddam and Sá da Costa developed finite difference approach schemes for the variable-order and accuracy of the algorithm, which were verified by numerical examples [28]. Ortigueira et al. modified the previously proposed variable-order fractional derivatives and introduced a new approach based on the inverse Laplace transform [29]. Almeida et al. systematically introduced the theory of fractional calculus and the calculus of variations, and they presented a new numerical tool for the solution of differential equations involving Caputo derivatives of fractional variable order [30]. Variable-order fractional derivatives have powerful properties for building nonlinear constitutive models and are thus widely used in various fields of natural science and engineering applications [31–34], such as biome analysis [35], mathematical modelling [36], infectious disease transmission [37], medical image denoising [38], and tumor growth analysis [39]. Yet, no studies have established the pressure-sinkage model of deep-sea sediments based on the variable-order fractional differential theory.

In order to analyze the mechanical properties of deep-sea sediments and their interaction mechanism with subsea mining vehicle tracks, this article establishes the pressure-sinkage constitutive model for deep-sea sediments. Using the variable-order fractional order theory, the pressure-sinkage constitutive model for deep-sea sediments was proposed according to the pressure-sinkage characteristic curve, and the mechanical properties of deep-sea sediments were revealed. Then, by comparing with the experimental results from other literature, the new model is verified to predict the pressure-sinkage process of deep-sea sediments under static and dynamic loads. Finally, a sensitivity study of model parameters is carried out using the control variable method, and the effects of grounding pressure and time on the law and mechanism of deep-sea sediment pressure-sinkage are revealed.

2. Basic Theory of the Fractional Derivative

2.1. Fractional Order Calculus. Fractional order calculus is an extension of integer order calculus and represents an effective tool for studying derivatives of arbitrary order [40]. There are several definitions of fractional derivatives, such as Riemann–Liouville, Caputo, and Grunwald–Letnikov et al. The Riemann–Liouville (RL) fractional derivative first integral and then derivative operation sequence have certain advantages for model solving [41]. Therefore, the Riemann–Liouville (R-L) fractional order calculus is chosen to define and describe the pressure-sinkage properties of deep-sea sediments in this article. The Riemann–Liouville integral of β order is defined by

\[ D_t^{-\beta} f(t) = \frac{d^{-\beta} f(t)}{d\tau^{-\beta}} = \frac{1}{\Gamma(\beta)} \int_0^t (t - \tau)^{\beta - 1} f(\tau)d\tau, \]

where \( t \) is time, \( f(t) \) is the function in the integrable interval \([0, t]\), \( \Gamma(\beta) \) is the gamma function, \( \beta \) is the fractional order greater than 0, and \( \tau \) is the independent variable used for the Laplace transform.

Correspondingly, the \( \beta \) order differentiation of the function \( f(t) \) is defined as follows:
\[
d \frac{d \delta(t)}{dt^\beta} = \frac{d}{dt} \left[ \Gamma(n-\beta) f(t) \right]^{n-1} = \frac{d^n}{dt^n} \left[ \Gamma(n-\beta) f(t) \right],
\]

where \( n = 1 \) if \( 0 < \beta < 1 \), and \( \Gamma(\beta) \) is the gamma function, which is defined as follows:

\[
\Gamma(\beta) = \int_0^\infty t^{\beta-1} e^{-t} dt,
\]

when \( f(t) \) is integrable near \( t = 0 \) and \( 0 \leq \beta \leq 1 \). Let the Laplace transform of \( f(t) \) be \( \tilde{f}(s) \), and the Laplace transform formula for the above fractional calculus is as follows:

\[
\tilde{L} \left( \tilde{D}_0^\beta f(t) \right) = s^\beta \tilde{f}(s), \forall \beta > 0.
\]

The stress-strain relationship expressed in the fractional differential form is as follows:

\[
\delta(t) = \xi \frac{d^\beta \epsilon(t)}{dt^\beta} \quad (0 \leq \beta \leq 1),
\]

where \( \delta(t) \) is the stress, \( \epsilon(t) \) is the strain, \( \xi \) is the coefficient of viscosity in the element (MPa\( \cdot \)s\(^{-1}\)), and \( t \) is time. When \( \beta = 0 \), let \( \xi = E \). Equation (5) can be expressed as follows:

\[
\delta(t) = E \epsilon(t),
\]

where \( E \) is the modulus of elasticity of the spring element. In this case, the fractional order element represents the ideal elastic body (see Figure 2(a)). When \( \beta = 1 \), let \( \xi = \alpha \). Equation (5) can be expressed as follows:

\[
\delta(t) = \alpha \frac{d \epsilon(t)}{dt},
\]

where \( \alpha \) is the viscosity coefficient of the dashpot element and the fractional order element represents the ideal viscous body (see Figure 2(b)).

When \( 0 < \beta < 1 \), the fractional order element is called the constant phase element, which can describe the nonlinear strain process of a viscoelastic body between ideal elastic and viscous bodies [23], as shown in Figure 2(c). In equation (1), the fractional order \( \beta(t) \) is a function of time, which implies a strong memory of not only the past history but also the fractional order. Equation (5) can also be written in integral form which is as follows:

\[
\epsilon(t) = \frac{1}{\xi} \int_0^t \frac{d^\beta \delta(t)}{dt^\beta} (0 \leq \beta \leq 1),
\]

when \( \delta(t) = \delta_0 \) and the stress remains unchanged. Using the variable order operator in equation (1) and substituting the constant stress into equation (8), we can obtain the following equation:

\[
\epsilon(t) = \frac{1}{\xi} \int_0^t (t - \tau)^{\beta-1} \delta_0 d\tau \frac{1}{\Gamma(\beta)}.
\]

Using integration by parts on the right side, equation (9) can be expressed as follows:

\[
\epsilon(t) = \frac{\delta_0}{\xi} \frac{t^\beta}{\Gamma(1+\beta)}. \tag{10}
\]

2.2 Variable-Order Fractional Calculus. Variable-order fractional calculus evolves from fractional calculus, the difference being that for the former, the fractional order changes with the physical process and is a function of time \( (0 < \beta = \mu(t) < 1) \). The \( \mu(t) \) order integral for the function \( f(t) \) is defined as [42] follows:

\[
D_t^{-\mu(t)} f(t) = \frac{d}{dt} \frac{d^{\mu(t)} f(t)}{d^{\mu(t)} t} = \int_0^t (t - \tau)^{\mu(t) - 1} \frac{1}{\Gamma(\mu(t), t)} f(\tau) d\tau, \tag{11}
\]

where \( \mu(t, \tau) \) is the order of the fractional derivative, and its value varies with time \( t \). In equation (11), the fractional order \( \mu(t) \) is a function of time, which implies a strong memory of not only the past history but also the fractional order [42]. Thus, we change the fractional order in equation (5) into a variable order function which is as follows:

\[
\delta(t) = \xi \frac{d^{\mu(t)} \epsilon(t)}{d^{\mu(t)} t}. \tag{12}
\]
When the applied stress remains unchanged, by integrating both sides of equation (12) according to the variable-order fractional differential operator, the constitutive equation of the variable-order fractional element can be obtained, namely, which is as follows:

$$\varepsilon(t) = \sum_{k=1}^{n} \frac{\delta_{0}^k}{\zeta_k^k} (t - t_{k-1}) \Gamma(\mu_k + 1), \quad(13)$$

where $\mu_k = \mu(t)$ is the piecewise constant in the time domain, $k = 1, 2, 3, \ldots, n$, $t_{k-1} \leq t \leq t_k$, and $\zeta_k$ is the viscosity coefficient relative to $\mu_k$. Since the focus of this study is on the relationship between the ground pressure and pressure sinkage of deep-sea sediments, equation (13) is transformed into the following equation:

$$Z(t) = \sum_{k=1}^{n} P_0 \left( t - t_{k-1} \right) \Gamma(\lambda_k + 1), \quad(14)$$

where $Z(t)$ is the amount of pressure sinkage over time and $\zeta_k$ and $\lambda_k$ are the viscosity coefficients and segmentation constants of the components in the model, respectively; $P_0$ is the constant ground pressure.

### 3. Establishment of the Pressure-Sinkage Model

#### 3.1. Deep-Sea Sediment Pressure-Sinkage Process

Due to the extreme high pressure and the particularity of the sedimentary environment, the microstructure of deep-sea sediments is dominated by the lamellar linking structure and the honeycomb flocculation structure, which have superior water absorption. Therefore, its pressure-sinkage process has unique characteristics. Some researchers have investigated the deep-sea sediment pressure-sinkage process by simulating the crawler tooth plate of the subsea mining vehicle [23]. The deep-sea sediment pressure-sinkage process reveals obvious stages, as shown in Figure 3.

The result in Figure 3 illustrates that the sinkage depth of deep-sea sediments increases with time, with clear turning points. Details can be described as follows:

(i) Stage I: The sinkage depth occurs immediately when the load is applied to the deep-sea sediments, such that depth is directly related to the ground-specific pressure. This stage is called the instantaneous pressure sinkage.

(ii) Stage II: The amount of sinkage depth increases linearly with the loading time until it reaches the turning point A. It is called the initial pressure sinkage.

(iii) Stage III: The relationship between the amount of sinkage depth and the loading time increases nonlinearly and monotonically, and the sinkage rate is significantly lower than that in stage II. This stage is called the secondary pressure sinkage.

(iv) Stage IV: The amount of sinkage remains basically constant, and the sinkage rate approaches zero. It is called the stable creep. Therefore, the sinkage depth of deep-sea sediments can be expressed as follows:

$$Z = Z_d + Z_s + Z_c + Z_v, \quad(15)$$

where $Z$ is the total pressure sinkage; $Z_d$ is the instantaneous pressure sinkage; $Z_s$ is the initial pressure sinkage; $Z_c$ is the secondary pressure sinkage; and $Z_v$ is the stable creep.

#### 3.2. Construction of the Model

Deep-sea sediment rheology is a complex, time-related process in which elastic, viscous, plastic, and other deformations coexist. The pressure-sinkage process explains the interaction mechanism between the subsea mining vehicle and deep-sea sediments. Exploring the pressure-sinkage characteristics of deep-sea sediments represents an important prerequisite for constructing the relationship between load stress and sinkage. Therefore, we define the whole process of deep-sea sediment pressure sinkage and describe the changing characteristics of mechanical properties. According to the variable-order fractional derivative and the rheological model, a new four-element pressure-sinkage constitutive model that describes the mechanical properties of deep-sea sediments is established, drawing on the modelling idea of classical element combination. The pressure-sinkage model for deep-sea sediment is shown in Figure 4.

As shown in Figure 4, the pressure-sinkage model for deep-sea sediment is composed of basic mechanical elements in series. In the pressure-sinkage experiment, the deep-sea sediment will form an instantaneous deformation, independent of time, and the elastic element simulates elastic deformation in the rheological model, where $E$ is the elasticity modulus of the deep-sea sediments. The later stage is accompanied by viscous and plastic deformation and exhibits elastic recovery as well as some permanent deformation during the unloading process. The values $\lambda_1, \lambda_2, \lambda_3$ are the viscosity coefficients at different stages, and $\lambda_1$ and $\lambda_2$ are the orders of fractional order differentiation at different stages. According to the superposition principle, its constitutive model can be expressed as follows:

\[
\begin{align*}
  & \text{if } 0 \leq t \leq t_1, P_0 = P_1 = P_2, Z = Z_d + Z_s; \\
  & \text{if } t_1 \leq t \leq t_2, P_0 = P_1 = P_2 = P_3, Z = Z_d + Z_s + Z_c; \\
  & \text{if } t_2 \leq t \leq t_3, P_0 = P_1 = P_2 = P_3 + P_4, Z = Z_d + Z_s + Z_c + Z_v; \\
\end{align*}
\]

where $Z$ is the total pressure sinkage of the constitutive model for deep-sea sediments; $t_1, t_2$, and $t_3$ are the time of the different pressure-sinkage stages; $P_1, P_2, P_3$, and $P_4$ are the ground pressures; and $Z_d, Z_s, Z_c$, and $Z_v$ are the different pressure-sinkage periods.

One advantage of variable-order fractional calculus is that the different orders have a certain memory, because the variable order relates to time. We describe the pressure-sinkage process of deep-sea sediments accordingly. When $0 < t < t_1$, $D^{\frac{\lambda}{\alpha}}_t X = f(t)$ describes the instantaneous pressure sinkage. When $t_1 \leq t < t_2$, $D^{\frac{\lambda}{\alpha}}_t X = f(t)$ represents the initial pressure sinkage. When $t_2 \leq t < t_3$, $D^{\frac{\lambda}{\alpha}}_t X = f(t)$ exhibits the initial pressure sinkage. According to the structural features of the pressure-sinkage constitutive model for deep-sea sediments in Figure 4 and combined with equations
the pressure-sinkage constitutive equation of this study evolves as follows:

\[
Z(t) = \begin{cases} 
\frac{P}{E} + \frac{P}{\xi_1} t, & 0 \leq t < t_1, \\
\frac{P}{E} + \frac{P}{\xi_1} t_1 + \frac{P}{\zeta_2} \Gamma(1 + \lambda_1) t_1 \leq t < t_2, \\
\frac{P}{E} + \frac{P}{\xi_1} t_1 + \frac{P}{\zeta_2} \Gamma(1 + \lambda_1) + \frac{P}{\zeta_3} \Gamma(1 + \lambda_2) t_2 \leq t < t_3,
\end{cases}
\]  

(17)

3.3. Determination of Pressure-Sinkage Model Parameters.

From equation (17), the constitutive model of deep-sea sediments in the instantaneous pressure-sinkage and initial pressure-sinkage stages is described as follows:

\[
Z(t) = \frac{P}{E} + \frac{P}{\xi_1} t, \quad 0 \leq t < t_1,
\]

(18)

where \( E = p/Z(t) = p/Z(0) \), and equation (16) is written as a one-dimensional linear equation which is as follows:

\[
y = a_1 x + b_1.
\]

(19)

Using the inflection point data of the deep-sea sediment pressure-sinkage experiment and solving equation (18), the coefficients of equation (19) \( (a_1 \text{ and } b_1) \) can be calculated. Furthermore, \( E \) and \( \xi_1 \) are determined from \( a_1 \) and \( b_1 \). Similarly, according to equation (17), the constitutive model of deep-sea sediments in the secondary pressure-sinkage stage can be expressed as follows:

\[
Z(t) = \frac{P}{E} + \frac{P}{\xi_1} t_1 + \frac{P}{\zeta_2} \Gamma(1 + \lambda_1) t_1 \leq t < t_2.
\]

(20)

Taking the logarithm of both sides of equation (20), we obtain the following equation:

\[
\lg [Z(t) - Z(t_1)] = \lambda_2 \lg t - \frac{\zeta_2 \Gamma(1 + \lambda_2)}{P} t_1 < t < t_2,
\]

(21)

where \( Z(t_1) = P/E + P/\xi_1 t_1 \).

The following relationships are assumed:

\[
\begin{align*}
\lambda_1 &= a_2, \\
\zeta_2 &= 10^{-b_2} \frac{P}{\Gamma(\lambda_1 + 1)}.
\end{align*}
\]

(24)

Similarly, the constitutive model of deep-sea sediments in the stable creep stage can be expressed as follows:

\[
Z(t) = \frac{P}{E} + \frac{P}{\xi_1} t_1 + \frac{P}{\zeta_2} \Gamma(1 + \lambda_1) + \frac{P}{\zeta_3} \Gamma(1 + \lambda_2) t_2 \leq t < t_3.
\]

(25)

Taking the logarithms of both sides of equation (25), we obtain the following equation:

\[
\lg [Z(t) - Z(t_2)] = \lambda_2 \lg t - \frac{\zeta_2 \Gamma(1 + \lambda_2)}{P} t_2 < t < t_3,
\]

(26)

where \( Z(t_2) = P/E + P/\xi_1 t_1 + P/\zeta_2 (t_2 - t_1)^{\lambda_1}/\Gamma(1 + \lambda_1) \).

The following relationships are assumed:

\[
\begin{align*}
\lambda_2 &= a_3, \\
\zeta_3 &= 10^{-b_3} \frac{P}{\Gamma(\lambda_2 + 1)}.
\end{align*}
\]

(29)

4. Verification of the Deep-Sea Sediment Pressure-Sinkage Model

4.1. Experimental Research on Deep-Sea Sediment Pressure Sinkage Under Static Load. According to the geomechanical measurement test of deep-sea sediments, Xu et al. chose bentonite as the main raw material and prepared simulated soils with physicochemical properties similar to those of
deep-sea sediments [24]. A series of pressure-sinkage experiments with different ground pressures were carried out, and the pressure-sinkage curve of simulative soil for deep-sea sediments in the range of ground pressure \( P = 5 \sim 25 \text{kPa} \) was obtained. Because the design value of the ground pressure of the subsea mining vehicle is 5 kPa, the experimental axial pressure range should exceed the design value. As shown in Figure 5, the deep-sea sediments experience instantaneous strain at the moment of loading, and its magnitude increases directly with ground pressure. After the instantaneous elasticity, the test curve is attenuated and stable, and the sinkage depth gradually steadies to a constant value with the increase in action time. When the ground pressure continues to increase, the action time to enter the stable creep stage increases. Under the same ground pressure, the sinkage depth of the deep-sea sediment simulant gradually increased with increasing action time.

In this study, \( P = 5 \text{kPa} \) and \( P = 20 \text{kPa} \). Pressure-sinkage test data were selected; the parameter of the pressure-sinkage constitutive model was fitted and analysed, and the pressure-sinkage model for deep-sea sediments under the static load experiment was established. The sinkage rate curve of simulated soil for deep-sea sediments was acquired, and the sinkage rate appears in Figure 6.

Figure 6 demonstrates that when the ground pressure is between 5 kPa and 20 kPa, the sinkage rate of the deep-sea sediments gradually decreases with time, and the sinkage rate gradually stabilizes and eventually reaches zero. The pressure-sinkage curve for deep-sea sediments is nonlinear and exhibits obvious stage characteristics. The greater the ground pressure, the greater the variation range of the pressure-sinkage curve. The greater the ground pressure, the greater the deep-sea sediment sinkage depth. This occurs because the contact area of soil particles becomes larger, the pores of the soil expand, and fluidity increases under the action of the ground pressure. According to the trend of the pressure-sinkage and sinkage rate curves, the values of \( t_1 \) and \( t_2 \) at \( P = 5 \text{kPa} \) and \( P = 20 \text{kPa} \) can be determined, respectively. When \( P = 5 \text{kPa} \), \( t_1 = 3.2273 \) and \( t_2 = 32.8951 \); when \( P = 20 \text{kPa} \), \( t_1 = 8.9441 \) and \( t_2 = 67.076 \). With the increase in ground pressure, the values of \( t_1 \) and \( t_2 \) will gradually increase. Based on the pressure-sinkage experiment, the deep-sea sediments pressure-sinkage curves evolved from the proposed model, then the model parameters were fitted and analysed using equation (17). The fitting results of the pressure-sinkage experiment appear in Table 1. Table 1 exhibits that, during the sinkage of deep-sea sediments, the fractional order decreases gradually with time. The change of pores under external force represents an important manifestation of the pressure-sinkage of deep-sea sediments. Under external force, the pore water of deep-sea sediments discharges continuously, the soil viscosity increases, and the evolution of mechanical properties is roughly the same as the movement law of pore water.

As shown in Figure 7, the pressure-sinkage curves calculated by the pressure-sinkage constitutive model, presented in the article at different ground pressures, are consistent with the experimental data of deep-sea sediments. The proposed pressure-sinkage model based on the variable-order fractional derivative can accurately describe the sinkage process of deep-sea sediments under static loads and can fully reflect the evolution law and mechanism of deep-sea sediments under the different pressure-sinkage stages. In order to further verify the accuracy of the proposed model, we used \( R^2 \), MAPE, and MFE to evaluate the model fitting curve and the fitting accuracy. The fitting correlation coefficient \( R^2 \) is greater than 0.98. From the calculated MFE values in Table 1, when \( P = 5 \text{kPa} \), the predicted value of the model is generally lower than the experimental data (negative deviation). When \( P = 20 \text{kPa} \), the predicted value of the model is generally higher than the experimental data (positive deviation). The MAPE values of the pressure-sinkage prediction results at \( P = 5 \text{kPa} \) and \( P = 20 \text{kPa} \) are 1.51% and 2.47%, respectively. In addition, the comparisons between the variable order model and the constant fractional order model indicate that the pressure-sinkage constitutive model presented in the article is reliable.

For the subsea mining vehicle in China, the length, width, and height are 9.2 m, 5.2 m, and 3 m, respectively, and the driving speed is 1 m/s. Therefore, it is necessary to focus on the pressure-sinkage process and change the law of the subsea mining vehicle in the first ten seconds. Combining equation (17) and the measured values in the above article, the sinkage depth of simulated soil for deep-sea sediments in the first ten seconds is calculated (presented in Figure 8 with measured values). The error between the calculated and measured values is small, and the results illustrate that the proposed model achieves accurate predictions for the first ten seconds of subsea mining vehicle operation.

4.2. Experimental Research on Deep-Sea Sediment Pressure-Sinkage under Dynamic Load. A pressure-sinkage experiment of simulated soil for deep-sea sediments under different dynamic load conditions was carried out by Ma et al. [23]. The dynamic load experiment of simulated soil for deep-sea sediments includes weight drops on the measuring plate from a certain height. Then, according to the law of conservation of mass, the relationship between the impact height of the object and its kinetic energy is established. The ground pressure is controlled at 5 kPa, 10 kPa, 15 kPa, 20 kPa, and 25 kPa, and the design velocity varies between 0.84 m/s, 1.19 m/s, 1.46 m/s, 1.68 m/s, and 1.88 m/s. When the ground pressure \( P = 5 \text{kPa} \), the dynamic load pressure-sinkage curve of the simulated soil for deep-sea sediments at different walking velocities is shown in Figure 9.

In this study, the experimental data of \( v = 0.84 \text{m/s} \) and \( v = 1.68 \text{m/s} \) were selected, the parameters of the pressure-sinkage prediction model were fitted and analysed, and the deep-sea sediment pressure-sinkage model under the dynamic load experiment was established. The sinkage rate curve of simulated soil for deep-sea sediments was acquired, and the sinkage rate is plotted in Figure 10.

As shown in Figures 9 and 10, when \( v = 0.84 \text{m/s} \) and \( v = 1.68 \text{m/s} \), the dynamic load pressure-sinkage curve of the
simulated soil for deep-sea sediments has the same trend as the static load pressure-sinkage curve. Four stages occur in total:

1. The instantaneous pressure-sinkage stage has nothing to do with time, and deep-sea sediments will form an instantaneous sinkage at the moment of loading;
2. The initial pressure-sinkage stage, in which the sinkage rate exhibits a decaying characteristic and the rate changes greatly;
3. The secondary pressure-sinkage stage, where the strain rate exhibits a decaying and stable characteristic;
4. The stable creep stage, in which the sinkage rate is smaller and remains constant.

According to the trend of the pressure sinkage and the sinkage rate curves, the \( t_1 \) and \( t_2 \) values when \( v = 0.84 \text{ m/s} \) and \( v = 1.68 \text{ m/s} \) can be determined. When \( v = 0.84 \text{ m/s} \), \( t_1 = 39.81182 \) and \( t_2 = 55.1132 \); when \( v = 1.68 \text{ m/s} \), \( t_1 = 13.5833 \) and \( t_2 = 45.2296 \). As the walking velocity increases, the values of \( t_1 \) and \( t_2 \) gradually decrease, and the deep-sea sediment pressure-sinkage process enters the stable creep faster with time. After fitting and analyzing the model parameters under dynamic loading with the above formula, the calculated model parameters and prediction effect evaluation results are shown in Table 2. Deep-sea sediments belong to saturated soils, with the increase in sinkage time, pore water gradual discharge, and viscous response. The fractional order is consistent with the change in viscosity and represents the evolution of mechanical properties. The results of this study are the same as those of other researchers [43].

Figure 11 exemplifies that the instantaneous pressure-sinkage stage after loading is described correctly by the proposed constitutive model. Meanwhile, the secondary pressure-sinkage stage and the stable creep stage are also expressed correctly. The correlation coefficients (\( R^2 \)) of the fitted curves are all above 0.97. In addition, the proposed constitutive model has fewer parameters, and the physical meaning of the model parameters is clear, which reflects the

<table>
<thead>
<tr>
<th>( P ) (kPa)</th>
<th>( E_0 )</th>
<th>( \zeta_1 )</th>
<th>( \lambda_1 )</th>
<th>( \zeta_2 )</th>
<th>( \lambda_2 )</th>
<th>( \zeta_3 )</th>
<th>( \lambda_3 )</th>
<th>( R^2 )</th>
<th>MAPE (%)</th>
<th>MFE</th>
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<td>3.572</td>
<td>30.826</td>
<td>0.677</td>
<td>50.912</td>
<td>0.437</td>
<td>189.12</td>
<td>0.9823</td>
<td>1.51</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>3.669</td>
<td>91.869</td>
<td>0.9002</td>
<td>371.27</td>
<td>0.6829</td>
<td>1030.3</td>
<td>0.9807</td>
<td>2.47</td>
<td>0.036</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Typical segments of pressure-sinkage curves for deep-sea sediments [23].
change characteristics of the initial and secondary pressure-sinkage stages during sinkage. When \( v = 0.84 \text{ m/s} \), the predicted value of the model has a negative deviation from the measured value; when \( v = 1.68 \text{ m/s} \), the predicted value of the model has a positive deviation from the measured value. In addition, the MAPE values of the pressure-sinkage prediction results at \( v = 0.84 \text{ m/s} \) and \( v = 1.68 \text{ m/s} \) are 0.79% and 1.42%, respectively. Based on this analysis, the proposed constitutive model based on variable-order fractional derivatives is better than the constant fractional order model in predicting the pressure-sinkage process of deep-sea sediments.

### 5. Parameter Sensitivity Analysis of the Deep-Sea Sediment Pressure-Sinkage Model

Consequently, the ground pressure and action time are the main factors of deep-sea sediment pressure sinkage and exhibit crucial influence on the construction of the pressure-sinkage constitutive model for deep-sea sediments. Therefore, this article will use the control variable method to conduct sensitivity analysis on the above model parameters, study the influence of ground pressure \( P \) and secondary pressure-sinkage time \( t_2 \) on deep-sea sediments, and analyse the evolution of model parameters on the dynamic characteristics of pressure sinkage. The model parameters are selected from the experimental data under static load, and the specific parameters are as follows: \( P = 5 \text{ kPa} \), \( E_0 = 3.572 \text{ MPa} \), \( \zeta_1 = 30.826 \text{ MPa·S} \), \( \lambda_1 = 0.677 \), \( \zeta_2 = 50.912 \text{ MPa·S}^{0.51} \), \( \lambda_2 = 0.437 \), and \( \zeta_3 = 189.12 \text{ MPa·S}^{1.2} \).

#### 5.1. Effect of the Ground Pressure \( P \)

Based on the experimental data, the above fitted parameters \( E_0 = 3.572 \text{ MPa} \), \( \zeta_1 = 30.826 \text{ MPa·S} \), \( \lambda_1 = 0.677 \), \( \zeta_2 = 50.912 \text{ MPa·S}^{0.51} \), \( \lambda_2 = 0.437 \), and \( \zeta_3 = 189.12 \text{ MPa·S}^{1.2} \) are input to equation (15) to quantitatively analyse the influence of the ground pressure \( P \) on the proposed constitutive model. The ground pressure level varies between 2 kPa, 5 kPa, 8 kPa, and 11 kPa. The results of the pressure-sinkage tests at different ground pressure levels are shown in Figure 13. The sinkage depth of deep-sea sediments directly increases with ground pressure levels. However, ground pressure has no significant effect on the sinkage rate. Higher ground pressure levels result in greater sinkage depth in the instantaneous, secondary, and initial pressure-sinkage stages, increasing the delay in entering the stable creep stage. In addition, with the increase in the ground pressure level, the form of the change curve of the secondary pressure-sinkage stage of the deep-sea sediments gradually changes to the exponential type.

#### 5.2. Effect of the Secondary Pressure-Sinkage Time \( t_2 \)

To analyse the influence of secondary pressure-sinkage time \( t_2 \) on the mechanical properties of deep-sea sediments, the control variable method helps obtain the pressure-sinkage curves under different secondary pressure-sinkage times \( t_2 \), as shown in Figure 14.
Figure 6: The sinkage rate of the deep-sea sediments under static load for (a) $P = 5\, \text{kPa}$ and (b) $P = 20\, \text{kPa}$.

Figure 7: Experimental data and the fitting curves by the variable order and constant fractional order models for (a) $P = 5\, \text{kPa}$ and (b) $P = 20\, \text{kPa}$.

Figure 8: Comparison of calculated and measured values under static load.

Figure 9: The curve of the deep-sea sediment pressure-sinkage tests under dynamic load [23].
Figure 10: The sinkage rate of the deep-sea sediments under dynamic load. (a) $v = 0.84\, \text{m/s}$; (b) $v = 1.68\, \text{m/s}$.

Figure 11: Experimental data and the fitting curves by the variable order model and the constant fractional order model for (a) $v = 0.84\, \text{m/s}$ and (b) $v = 1.68\, \text{m/s}$.

Figure 12: Comparison of calculated and measured values under dynamic load.

Figure 13: Sensitivity analysis under different ground pressures $P$. 
As shown in Figure 14, the secondary pressure-sinkage time $t_2$ gradually increases from 27.5 to 42.5. The first two stages are unrelated to secondary pressure-sinkage time $t_2$; they do not change with time, but the stable creep stage will gradually increase in duration with increasing secondary pressure-sinkage time $t_2$. In addition, the sinkage depth under different secondary pressure-sinkage times $t_2$ will change. As the secondary pressure-sinkage time $t_2$ increases, the sinkage depth of the secondary pressure-sinkage stage continues to deepen, but the sinkage rate gradually decreases.

6. Discussion

The fractional order theory has always been a research hotspot. The variable-order fractional derivative is an extension of the fractional derivative, and the fractional order is a function of time. It can be closer to the mechanical properties of viscoelastic materials and can characterize their time-dependent mechanical properties.

In this study, a new four-element pressure-sinkage model was established by introducing variable-order fractional derivatives into the modelling idea of classic element combination to describe the full pressure-sinkage regions of deep-sea sediments. Through fitting a curve to experimental results, the proposed model’s accuracy reached 97%. We found that the fractional order gradually decreased with time during the sinkage of deep-sea sediments, implying that the mechanical properties of deep-sea sediments have changed. The fractional order is consistent with the process of pore water and deep-sea sediment skeleton, which preliminarily reveals the gradual transformation process of sediments from an elastic state to a viscous state during sinkage.

7. Conclusion

The mechanical properties of deep-sea sediments and their interaction mechanism with subsea mining vehicle tracks are analyzed better, and the pressure-sinkage constitutive model is established for deep-sea sediments. In this study, a four-element pressure-sinkage model is established by introducing variable-order fractional order into the modelling idea of classic element combination, and the constitutive equations of different pressure-sinkage stages are constructed. Then, by comparing with experimental results from other literature, the proposed model is verified to predict the pressure-sinkage process of deep-sea sediments under static and dynamic loads. Finally, through sensitivity analysis of the model parameters, the influence of the ground pressure and time on the pressure-sinkage law and mechanism for deep-sea sediments is revealed. The following conclusions may be drawn:

1. According to the curve of deep-sea sediment sinkage rate, the pressure-sinkage process is divided into four stages, and the time-dependent mechanical properties of deep-sea sediments during sinkage emerge. The Riemann–Liouville theory, variable-order fractional derivatives, and rheological mechanics theory support the establishment of a new pressure-sinkage constitutive model for deep-sea sediments.

2. The pressure-sinkage constitutive model in this article is validated under static and dynamic loads. Through experimental data verification, the results demonstrate that the proposed model is optimal for simulating the pressure-sinkage process of deep-sea sediments. The model’s accuracy reached 95%, which is satisfactory for engineering applications.

3. The sensitivity analysis of the main influencing parameters in the proposed model, including ground pressure $P$ and the secondary pressure-sinkage time $t_2$, illustrates that ground pressure and time are directly related to the sinkage depth and rate.

Data Availability

The data supporting the findings of the current study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References


[36] X. Zhang and H. Yan, “Medical image fusion and noise suppression with fractional-order total variation and multi...


