Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 2181127, 9 pages
https://doi.org/10.1155/2023/2181127

Research Article

The Spectral Adomian Decomposition Method for the Solution of MHD Jeffery–Hamel Problem

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Received 1 May 2022; Revised 15 August 2022; Accepted 24 November 2022; Published 4 February 2023

Academic Editor: A. M. Bastos Pereira

In this study, the effects of magnetic field on the Jeffery–Hamel problem is studied using a powerful numerical method called the spectral Adomian decomposition method (SADM). The traditional Navier–Stokes equation of fluid mechanics and Maxwell's electromagnetism governing equations are reduced to nonlinear ordinary differential equations to model the problem. Comparisons with the numerical solutions are made to demonstrate the validity and high accuracy of the present approach. The velocity profile of the inner part of the divergent channel is studied for various values of magnetic field parameter and angle of channel. It was found that an increase in the magnetic field parameter leads to increase in the velocity profile. The results indicated that this technique is more efficient and converges faster than the standard Adomian decomposition method.

1. Introduction

Many problems of physical phenomena are formulated in several types of linear or nonlinear differential equations, and most natural models in physics, biology, or sciences are highly nonlinear in their nature. Finding solutions of differential equations is highly interesting for researchers and scientists, and there are available studies to find analytical or numerical solutions for linear or nonlinear differential equations. Finding an exact solution of differential equations is one of the most challenging problems. For this reason, the field of approximate techniques has been investigated extensively. There are many numerical methods for obtaining analytical and approximate solutions which include, among others, the Runge–Kutta methods, finite difference, cubic Hermite finite element, pseudospectral, Chebyshev-collocation, and finite element method. Some of these methods are facing a limitation in their accuracy, efficiency, and stability. To remove these limitations, several researchers used many perturbation or nonperturbation methods such as the Lyapunov artificial small parameter method [1], homotopy perturbation method [2, 3], homotopy analysis method [4], numerical methods of different type fuzzy equations [5–9], Volterra–Fredholm integral equations [10, 11], fractional differential equations (12) and (13), and Adomian decomposition method [14–16].

The Adomian decomposition method (ADM) was first reported in the 1980s, and it has been efficiently used to solve linear and nonlinear problems of differential and integral equations. An advantage of this method is that it can provide us the solution in terms of infinite series, which can be easily determined. The convergence of the series of solution has been discussed by [17]. There are many modifications of ADM which have been done by several authors in [18–24] in an attempt to improve the accuracy or expand the application of the original method.

However, the ADM was based on the assumptions that the main differential equation can be divided into linear and nonlinear parts, and the success of this technique depends mainly on the selection of the linear part. Some of the main limitations of the ADM are that it has limited choice of accepted linear operators (linear part) and initial approximations and it must be chosen to be simple in order to ensure that the higher order differential equations can be easily integrated. Complicated initial approximations and linear operators may result in higher order deformation equations that are difficult to integrate under the ADM. For some problems, the method of highest-order differential matching in which an auxiliary linear operator matching the highest derivative of the linear part of the governing nonlinear differential equation is prescribed.

In this paper, a modification the Adomian decomposition method has been obtained in order to address some of the limitations of the standard ADM. The modification suggests a standard way of choosing the linear operator and initial guess of the differential equation. The proposed method herein is referred to as the spectral Adomian decomposition method (SADM). The new technique was first studied by Yassir and Khidir [25], and they applied the new modification on the problem of boundary layer convective heat transfer over plate, and they showed that the new modification is more efficient than the standard ADM. This technique is based on the blending of the Chebyshev pseudospectral methods (see [26–35]) and the Adomian decomposition method. In this method, they expressed the linear operator in terms of the Chebyshev spectral differentiation matrix. Using this method, any selected initial guess can be used as long as it satisfies the boundary conditions. The application of the SADM leads the nonlinear differential equation to a system of algebraic linear equations that are easy to solve when compared to a system of ordinary differential equations obtained by ADM.

The advantages of this approach over the standard same ADM are as follows. (i) The current technique suggests a standard way of choosing the auxiliary linear operator of the differential equation which is the main motivation behind the selection of the current algorithm, whereas the other methods choose a linear operator to be simple in order to ensure that the differential equations can be easily integrated. (ii) It gives an excellent results in terms of convergence and accuracy of solutions. Some of the disadvantages of this method are that it cannot be applied for solving partial differential equations in the current form and that it cannot be applied directly on nonlinear differential equations; therefore, we need first to linearize the nonlinear differential equation using any available method.

We applied the new modification of ADM to find the approximate solution of nonlinear differential equation governing nonlinear MHD Jeffery–Hamel flow to show the efficiency of the SADM in comparison with the ADM. The mathematical model of Jeffery–Hamel flow was first introduced by Jeffery [36] and Hamel [37]. The Jeffery–Hamel model can be described by an exact similarity solution of the Navier–Stokes equations in the special case of two-dimensional flow. On the contrary, the term of MHD was first introduced by Bansal [38] in 1994. The classical Jeffery–Hamel problem was extended in [39] to include the effects of an external magnetic field on an electrically conducting fluid. The theory of MHD is inducing current in a moving conductive fluid in the presence of magnetic field, and such induced current results in force on ions of the conductive fluid. The theoretical study of MHD channel has been a subject of great interest due to its extensive applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps, and flow meters. The flow of various fluid types between converging/diverging channels have been extensively studied by many researchers. Adnan et al. [40, 41] studied the effects of cross diffusion for second grade fluids. Mohyud-Din et al. [42] investigated the MHD flow containing nano-sized metallic particles between nonparallel walls in the existence of stretching and shrinking. Noor et al. [43] explored the influence of thermophoretic parameters and presented graphical results for heat and mass transfer. The Jeffery and Hamel problem became very popular among the research community and authors focused on the study of Jeffery–Hamel flow from various aspects and studied the behavior of fluid flow characteristics. These applications included mechanical, environmental, and chemical engineering. The applications of Jeffery–Hamel flow were also applied on biomedical sciences.

In the present work, we applied SADM to solve MHD Jeffery–Hamel flow, and we made a comparison with the numerical solution. This paper is organized as follows. The mathematical formulation is given in Section 2. The descriptions of the standard and modified Adomian decomposition methods are given in Sections 3 and 4, respectively. The results are discussed and investigated in Section 5. Finally, the conclusions are given in Section 6.

2. Mathematical Formulation

We consider a system of cylindrical polar coordinates $u(r,\theta)$ in which steady two-dimensional flow of conducting viscous fluid from a source or sink at the intersection between two rigid plane walls that the angel between them is 2α is shown in Figure 1. The grid walls are considered to be divergent if $\alpha > 0$ and convergent if $\alpha < 0$. We assume that the velocity is only along radial direction and depends on r and θ so that $\mathbf{v} = (u(r,\theta), 0)$ only, and further, we assume that there is no magnetic field in the z-direction. The reduced forms of continuity, Navier–Stokes, and Maxwell's equations (44) are

$$\frac{\rho}{r}\frac{\partial}{\partial r}\left(ru(r,\theta)\right) = 0, \tag{1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + v\left[\frac{\partial^2 u(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2} - \frac{u(r,\theta)}{r^2}\right] - \frac{\sigma B_0^2}{\rho r^2}u(r,\theta), \tag{2}$$

$$\frac{1}{\rho r}\frac{\partial P}{\partial \theta} - \frac{2v}{r^2}\frac{\partial u(r,\theta)}{\partial \theta} = 0, \tag{3}$$

where P is the fluid pressure, B_0 is the electromagnetic induction, and σ is the conductivity of the fluid.

Continuity (1) implies that

$$u(r,\theta) = \frac{f(\theta)}{r}. \tag{4}$$

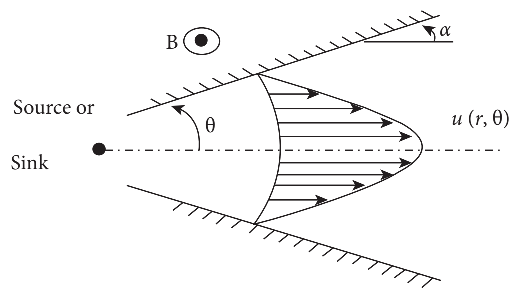

Figure 1: Geometry of the MHD Jeffery–Hamel flow.

Using the following dimensionless parameters,

$$F(\eta) = \frac{f(\theta)}{f_{\max}}, \qquad \eta = \frac{\theta}{\alpha}, \tag{5}$$

and with eliminating P from (2) and (3), we obtain the following third order nonlinear ordinary differential equation for the normalized function profile $F(\eta)$:

$$F'''(\eta) + 2\alpha R_e F'(\eta) F(\eta) + \alpha^2 (4 - H) F'(\eta) = 0. \tag{6}$$

Subject to the boundary conditions,

$$F(0) = 1, \quad F'(0) = 0, \quad F(1) = 0, \tag{7}$$

where R_e is the Reynolds number given by

$$R_e = \frac{U_{\max} r \alpha}{v} = \begin{pmatrix} \text{divergent chanel: } \alpha > 0, f_{\max} > 0 \\ \text{convergent chanel: } \alpha < 0, f_{\max} < 0 \end{pmatrix}, \tag{8}$$

where $U_{\max}$ is the velocity at the center of the channel $(r = 0)$ and $H = \sqrt{\sigma B_0^2 / \rho v}$ is the Hartmann number.

In this study, we use the spectral Adomian decomposition method (SADM) to find an approximate solution of MHD Jeffery–Hamel (6) together with the boundary conditions (6). For the convenience of the reader, we first present a brief review of the standard Adomian decomposition method; this is then followed by a description of the algorithm of the proposed spectral Adomian decomposition method differential equations.

3. Standard Adomian Decomposition Method (ADM)

In this section, the review of the standard Adomian decomposition method [45–48] is presented. We start by considering the following differential equation:

$$Lu(x) + Ru(x) + N(u(x)) = g(x), \tag{9}$$

where L is the highest-order derivative which is assumed to be invertible, R is a linear differential operator of less order than L, Nu represents the nonlinear terms, and $g(x)$ is known analytic function. The method is based on applying the inverse operator L^{-1} formally to the expression:

$$Lu(x) = g(x) - Ru(x) - Nu(x). \tag{10}$$

So, by using the given conditions, we obtain

$$u(x) = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \tag{11}$$

where the function $f(x)$ represents the terms arising from integrating the source term $g(x)$, and from using the given conditions, all are assumed to be prescribed. The standard Adomian decomposition method defines the solution $u(x)$ by the series as follows:

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{12}$$

where the components $u_0, u_1, u_2, \ldots$, are usually determined recursively by using the relation:

$$u_{k+1}(x) = -L^{-1}(Ru_k) - L^{-1}(Nu_k), k \geq 0. \tag{13}$$

It is important to note that the decomposition method suggests that the zeroth component u_0 is usually identified by the function f described above. For nonlinear equations, the nonlinear operator $Nu = F(u)$ is usually represented by an infinite series of the so-called Adomian polynomials:

$$F(u) = \sum_{k=0}^{\infty} A_k, \tag{14}$$

where Adomian s polynomials A_n may be computed by the formula:

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N \left(\sum_{i=0}^{n} \lambda^i u_i \right) \right]_{\lambda=0}. \tag{15}$$

4. Spectral Adomian Decomposition Method (SADM)

In this section, we present the modified Adomian decomposition method [25], this technique is based on the blending of the Chebyshev pseudospectral methods and the standard ADM. We start by transforming the domain of the problem from $[0, 1]$ to the domain $[-1, 1]$ on which the Chebyshev spectral method can be implemented, using the algebraic mapping as follows:

$$x = 2\eta - 1, x \in [-1, 1]. \tag{16}$$

It is also convenient to make the boundary conditions homogeneous by making use of the transformation:

$$F(\eta) = f(x) + f_m(\eta), \tag{17}$$

where $f_0(\eta)$ is chosen to satisfy boundary conditions (6). Substituting (10) and (11) in (5) gives

$$a_1 f'''(x) + a_2 f'(x) + a_3 f(x) + 4\alpha R_e f(x) f'(x) = R. \tag{18}$$

Subject to conditions, we obtain

$$f(-1) = f(1) = f'(-1) = 0, \tag{19}$$

where

$$a_1 = 8a_2(\eta) = 4\alpha R_e f_m + 8\alpha^2 - 2\alpha^2 H a_3(\eta) = 2\alpha R_e f'_m$$
$$R = -\left(f'''_m + 2\alpha R_e f_m f'_m + 4\alpha^2 f'_m - \alpha^2 H f'_m\right). \tag{20}$$

The initial approximation $f_0(x)$ for the solution of (18) is obtained from the solution to the linear part of (18):

$$a_1 f'''_0(x) + a_2 f'_0(x) + a_3 f_0(x) = R. \tag{21}$$

Subject to the boundary conditions, we have

$$f_0(-1) = f_0(1) = f'_0(-1) = 0. \tag{22}$$

Equation (21) solved using the Chebyshev pseudo-spectral method where the unknown function $f_0(x)$ is approximated as truncated series of Chebyshev polynomials is of the form as follows:

$$f_i(x) \approx \sum_{k=0}^{N} f_i(x_k) T_k(x_j),\ j = 0, 1, \ldots, N, \tag{23}$$

where T_k is the kth Chebyshev polynomial defined as

$$T_k(x) = \cos\left[k \cos^{-1}(x)\right]. \tag{24}$$

The derivatives of the variables at the collocation points are represented as

$$\frac{d^r f_i}{dx^r} = \sum_{k=0}^{N} \mathscr{D}^r_{kj} f_i(x_k),\ j = 0, 1, \ldots, N, \tag{25}$$

where r is the order of differentiation and $\mathscr{D}$ being the Chebyshev spectral differentiation matrix whose entries are defined as (see, for example, [49])

$$\left.\begin{aligned} \mathscr{D}_{jk} &= \frac{c_j}{c_k} \frac{(-1)^{j+k}}{\xi_j - \xi_k}\ j \neq k;\ j, k = 0, 1, \ldots, N, \\ \mathscr{D}_{kk} &= -\frac{\xi_k}{2\left(1 - \xi_k^2\right)} k = 1, 2, \ldots, N-1, \\ \mathscr{D}_{00} &= \frac{2N^2 + 1}{6} = -\mathscr{D}_{NN}. \end{aligned}\right\}. \tag{26}$$

Here, $c_0 = c_N = 2$ and $c_j = 1$ with $1 \leq j \leq$ N−1, x_j are the Chebyshev-collocation points defined by

$$x_j = \cos\frac{j\pi}{N},\ j = 0, 1, 2 \ldots, N. \tag{27}$$

Substituting equations (15)–(19) in (21) yields an equation of the form,

$$A f_0 = R_0, \tag{28}$$

where

$$\begin{aligned} A &= a_1 \mathscr{D}^3 + \operatorname{diag}[a_2]\mathscr{D} + \operatorname{diag}[a_3], \\ R_0 &= -\left[\left(f_m'''(\eta_i) + 2\alpha R_e f_m(\eta_i) f'_m(\eta_i) + 4\alpha^2 f'_m(\eta_i) - \alpha^2 H f'_m(\eta_i)\right)\right]^T, \end{aligned} \tag{29}$$

where T denotes transpose and diag[ ] is a diagonal matrix of size $(N+1) \times (N+1)$. The matrix A has dimension $(N+1) \times (N+1)$, while matrices f_0 and R_0 have dimensions $(N+1) \times 1$. To implement the boundary conditions (14) into systems (20), we delete the first and the last rows and columns of A and delete the first and last elements of f_0 and R_0; also, we replace the resulting of last row of the modified matrix A by the last row of the matrix $\mathscr{D}$ and set the resulting last row of the modified matrix R_0 to be zero.

The solution of the linear algebraic (21) is obtained by

$$f_0 = A^{-1} R_0, \tag{30}$$

which is the first approximation. To find highest approximations of (18), we begin by defining the following linear operator:

$$\mathscr{L} = a_1 \frac{d^3}{dx^3} + a_2 \frac{d}{dx} + a_3. \tag{31}$$

According to (7), equation (12) must be written as follows:

$$\mathscr{L} f_{k+1}(x) = -(N f_k) = \sum_{i=0}^{k} B_k,\ k \geq 0, \tag{32}$$

where B_k are the Adomian polynomials. In matrices form, (32) can be written as

$$A f_{k+1} = R_{k+1}. \tag{33}$$

With the boundary conditions, we have

$$f_{k+1}(-1) = f_{k+1}(1) = f_{k+1}(-1) = 0, \tag{34}$$

where

$$\begin{aligned} A &= a_1 \mathscr{D}^3 + \operatorname{diag}[a_2]\mathscr{D} + \operatorname{diag}[a_3], \\ R_{k+1} &= -4\alpha R_e \left[\sum_{i=0}^{k} f_i\left(\mathscr{D} f_{k-i}\right)\right]^T. \end{aligned} \tag{35}$$

To implement the boundary conditions (25) to system (24), we delete the first and the last rows and columns of A and delete the first and last elements of f_{k+1} and R_{k+1}; also, we replace the resulting of last row of the modified matrix A by the last row of the matrix $\mathscr{D}$ and set the resulting last

Table 1: Comparison of absolute errors between the ADM [50] and SADM results with the numerical solution of $F(\eta)$ for different values of H when $\alpha = 5^0$ and $R_e = 25$.

	$H = 0$		$H = 100$		$H = 250$		$H = 500$	
η	ADM	SADM	ADM	SADM	ADM	SADM	ADM	SADM
0.1	$3.2e^{-5}$	$7.9e^{-10}$	$7.2e^{-4}$	$2.8e^{-10}$	$1.6e^{-3}$	$7.3e^{-11}$	$2.5e^{-3}$	$2.9e^{-11}$
0.2	$1.2e^{-4}$	$3.1e^{-9}$	$2.7e^{-3}$	$1.1e^{-9}$	$5.9e^{-3}$	$2.9e^{-10}$	$9.6e^{-3}$	$1.2e^{-10}$
0.3	$2.3e^{-4}$	$6.7e^{-9}$	$5.3e^{-3}$	$2.4e^{-9}$	$1.2e^{-2}$	$6.4e^{-10}$	$1.4e^{-5}$	$2.6e^{-10}$
0.4	$3.3e^{-4}$	$1.1e^{-8}$	$7.7e^{-3}$	$3.9e^{-9}$	$9.6e^{-3}$	$1.1e^{-9}$	$1.2e^{-2}$	$4.4e^{-10}$
0.5	$3.8e^{-4}$	$1.5e^{-8}$	$9.2e^{-3}$	$5.3e^{-9}$	$1.0e^{-2}$	$1.5e^{-9}$	$1.4e^{-2}$	$6.2e^{-10}$
0.6	$3.7e^{-4}$	$1.6e^{-8}$	$9.2e^{-3}$	$5.9e^{-9}$	$6.1e^{-3}$	$1.7e^{-9}$	$6.9e^{-3}$	$7.5e^{-10}$
0.7	$2.9e^{-4}$	$1.4e^{-8}$	$7.6e^{-3}$	$5.4e^{-9}$	$4.2e^{-3}$	$1.7e^{-9}$	$8.6e^{-3}$	$7.7e^{-10}$
0.8	$1.7e^{-4}$	$9.8e^{-9}$	$4.7e^{-3}$	$3.8e^{-9}$	$4.8e^{-3}$	$1.2e^{-9}$	$1.4e^{-2}$	$6.3e^{-10}$
0.9	$4.5e^{-5}$	$4.5e^{-9}$	$1.6e^{-3}$	$1.8e^{-9}$	$1.3e^{-3}$	$6.2e^{-10}$	$7.4e^{-3}$	$3.6e^{-10}$
Time/Sec	—	1.19	—	0.86	—	0.81	—	0.82

element of the modified matrix R_{k+1} to be zero. The final solution of (24) is given by

$$f_{k+1} = A^{-1} R_{k+1}. \quad (36)$$

Thus, starting from the first approximation f_0, which is obtained by (30), higher order approximations $f_{k+1}, k = 0,, 1, 2 \ldots$ can be obtained through recursive formula (36).

Now, the final solution of the MHD Jeffery–Hamel problem is obtained as

$$F(\eta) = f_m + \sum_{i=0}^{\infty} f_i. \quad (37)$$

5. Results and Discussion

In this study, the objective was to apply the spectral Adomian decomposition method to obtain a numerical solution of the MHD Jeffery–Hamel problem in order to test the applicability, accuracy, and efficiency of SADM. Here, we use a personal computer of 2.5 GHz CPU speed including MATLAB 2007 – 7.5 package to perform the simulation results and also used the inbuilt MATLAB boundary value problem solver bvp4c for the numerical solution approach. In generating the presented results, it was determined through numerical experimentation that $N = 40$ gave sufficient accuracy for the method. We also generated the computational times of the results to show the computational efficiency of the solution.

A comparison between absolute errors of ADM [50] and SADM results for velocity $F(\eta)$ at different values of H when $R_e = 25$ and $\alpha = 5$ is shown in Table 1 to illustrate the accuracy of SADM. The table shows a high accuracy of SADM, and it is more accurate than the standard ADM which confirms the validity and convergence of this method. We also observed that inTable 1, the presented algorithm computationally is very fast which needs a fraction of a second to be generated, and the results are shown in the bottom of the table.

Errors of ADM and SADM for $F(\eta)$ at different iterations can be seen in Figure 2. It can be noticed that the errors becomes minimized after 12 iterations for ADM solution, while the error gets minimized after only 4 iterations for SADM. As we can see, the SADM has a high accuracy, and it has faster convergence than the standard ADM. In Table 2, we give a comparison of the SADM varied at different orders of approximation against the numerical results for different values of $H, R_e,$ and α. It can be seen from the table that the results of the SADM are comparable and converge to the numerical solution. Again, we note that the SADM results seem to have converged at the 4th order of approximation for seven decimal places.

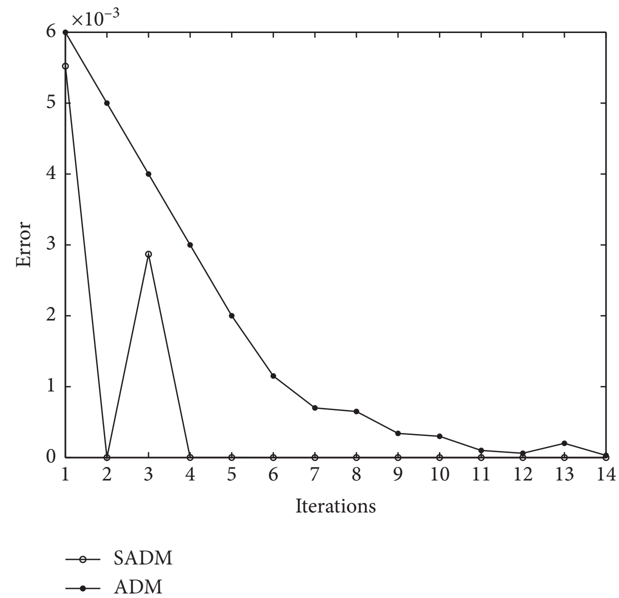

Figure 2: Comparison between the ADM (filled circles) and SADM (white circles) error at different steps of $F(\eta)$ when $H = 100$, $\alpha = 5^o$ and $R_e = 50$.

Tables 3 and 4 gave a comparison of the SADM results at different orders of the solution series against the numerical results for convergent and divergent channels. The SADM give the same level of accuracy as the numerical results at the 3rd order of the solution series approximation for only six decimal places.

Figure 3 shows, firstly, a comparison between the numerical results at the 3rd order of SADM approximations and, secondly, the effect of magnetic field on

Table 2: Comparison of the values of the SADM for $F''(0)$ with the numerical solution for various values of H, R_e, and α.

	H	2nd order	3rd order	4th order	5th order	Numerical	Time/sec
$R_e = 50$ $\alpha = 5$	0	−3.4952323	−3.5393850	−3.5394156	−3.5394156	−3.5394156	0.00046
	50	−3.4283287	−3.4285241	−3.4285241	−3.4285241	−3.4285241	0.00084
	100	−3.3213169	−3.3214993	−3.3214993	−3.3214993	−3.3214993	0.00119
	250	−3.0208444	−3.0222270	−3.0222270	−3.0222270	−3.0222270	0.00156
	500	−2.5856085	−2.5884479	−2.5884481	−2.5884481	−2.5884481	0.00191
$R_e = 50$ $\alpha = -5$	0	−1.1337278	−1.1219917	−1.1219891	−1.1219891	−1.1219891	0.00052
	50	−1.0891656	−1.0891429	−1.0891429	−1.0891429	−1.0891429	0.00093
	100	−1.0574854	−1.0574643	−1.0574643	−1.0574643	−1.0574642	0.00130
	250	−0.9691038	−0.9689408	−0.9689408	−0.9689408	−0.9689408	0.00166
	500	−0.8408516	−0.8405132	−0.8405132	−0.8405132	−0.8405132	0.00201
$R_e = 100$ $\alpha = 5$	0	−5.0430609	−5.8509930	−5.8691587	−5.8691651	−5.8691651	0.00047
	50	−5.6994848	−5.7001821	−5.7001821	−5.7001821	−5.7001821	0.00085
	100	−5.5352801	−5.5359433	−5.5359433	−5.5359433	−5.5359433	0.00121
	250	−5.0655255	−5.0706222	−5.0706229	−5.0706229	−5.0706229	0.00157
	500	−4.3691447	−4.3800940	−4.3800972	−4.3800972	−4.3800972	0.00193
$R_e = 100$ $\alpha = -5$	0	−0.6962841	−0.6402980	−0.6401781	−0.6401781	−0.6401781	0.00046
	50	−0.6229102	−0.6228964	−0.6228964	−0.6228964	−0.6228964	0.00084
	100	−0.6061917	−0.6061788	−0.6061788	−0.6061788	−0.6061788	0.00119
	250	−0.5592927	−0.5591913	−0.5591913	−0.5591913	−0.5591913	0.00154
	500	−0.4904798	−0.4902632	−0.4902632	−0.4902632	−0.4902632	0.00189

Table 3: Comparison of the numerical results against the SADM approximate solutions for $F(\eta)$ when $\alpha = 5, R_e = 25$, and $H = 500$.

η	1st order	2nd order	3rd order	4th order	Numerical
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.990221	0.990222	0.990221	0.990221	0.990221
0.2	0.960937	0.960941	0.960939	0.960939	0.960939
0.3	0.912283	0.912290	0.912287	0.912287	0.912287
0.4	0.844398	0.844412	0.844405	0.844405	0.844405
0.5	0.757307	0.757327	0.757317	0.757317	0.757317
0.6	0.650745	0.650771	0.650758	0.650758	0.650758
0.7	0.523938	0.523967	0.523953	0.523953	0.523953
0.8	0.375318	0.375344	0.375331	0.375331	0.375331
0.9	0.202146	0.202161	0.202153	0.202153	0.202153
1.0	0.000000	0.000000	0.000000	0.000000	0.000000
Time/Sec	0.000688	0.001305	0.001867	0.002480	—

Table 4: Comparison of the numerical results against the SADM approximate solutions for $F(\eta)$ when $\alpha = -5, R_e = 25$, and $H = 500$.

η	1st order	2nd order	3rd order	4th order	Numerical
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.994438	0.994378	0.994409	0.994409	0.994409
0.2	0.977299	0.977052	0.977177	0.977177	0.977177
0.3	0.947182	0.946624	0.946907	0.946907	0.946907
0.4	0.901649	0.900681	0.901172	0.901172	0.901172
0.5	0.837077	0.835677	0.836387	0.836387	0.836387
0.6	0.748480	0.746761	0.747633	0.747633	0.747633
0.7	0.629347	0.627585	0.628478	0.628478	0.628478
0.8	0.471546	0.470122	0.470844	0.470844	0.470844
0.9	0.265394	0.264636	0.265020	0.265020	0.265020
1.0	0	0	0	0	0

the velocity profiles for convergent and divergent channels for fixed Reynolds numbers. We note that the SADM technique is able to match the accuracy of the numerical results at the third order showing the efficiency and reliability of this technique. Figure 3 also shows moderate increases in the velocity with increasing with Hartmann numbers for both convergent and divergent channels.

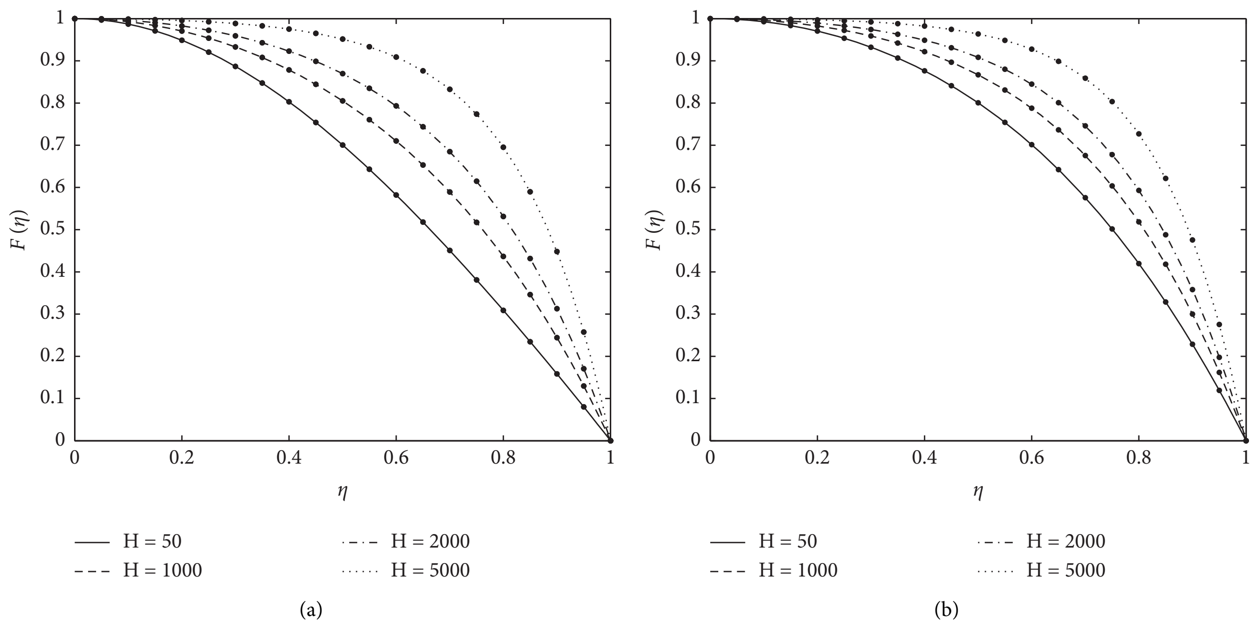

Figure 3: Comparing the numerical solution (filled circles) with the 3rd order SADM approximation for the velocity profile $F(\eta)$ varying H for (a) $\alpha = 5^o$, $R_e = 25$ and (b) $\alpha = -5^o$, $R_e = 25$.

6. Conclusions

In this study, we applied the spectral Adomian decomposition method to solve the 3rd order nonlinear differential equation that governs the MHD Jeffery–Hamel equation. Also, this problem is solved by a numerical method of the inbuilt MATLAB boundary value problem solver bvp4c. We made comparisons between the SADM, andADM, and numerical results show the efficiency of SADM. We summarized our results as follows:

(i) The SADM has been shown to have certain advantages over the ADM; for example, the SADM has a standard way of choosing the auxiliary linear operators and initial approximations, but in the ADM, we have limited choices of acceptable linear operators which is a limitation of the ADM

(ii) The comparisons between ADM, SADM, and numerical results show that SADM is highly accurate, efficient, and converges rapidly with only three or four iterations required to achieve the accuracy of the numerical results

(iii) The results indicate that an increase in the Hartmann number leads to an increase in the velocity profile.

Generally, the results show that the SADM is an effective tool for solving nonlinear differential equation that arises in nonlinear sciences. In future, we intend to show that the SADM can be extended to coupled nonlinear partial differential equations in place of the traditional methods such as Runge–Kutta, finite differences, finite element, or Keller-Box methods.

Nomenclature

R_e : Reynolds number
B_0 : Electromagnetic induction
H : Hartmann number
P : Pressure
r : Cylindrical coordinates
f : Nondimensional velocity
u : Velocity component in radial direction
r : Cylindrical coordinates
η : Nondimensional angle

Greek symbols

ρ : Density of the fluid
σ : Conductivity of the fluid
α : Angle between two plates
θ : Cylindrical coordinates.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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