Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 2691439, 16 pages
https://doi.org/10.1155/2023/2691439

Research Article

Intelligent System for Optimal Geometric Design Using Fuzzy Soft PDE

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Received 17 October 2022; Revised 22 January 2023; Accepted 6 April 2023; Published 26 April 2023

Academic Editor: Arunava Majumder

In this research, a methodology is discussed to develop an intelligent system for optimal geometric design for the culinary product. An optimum geometric design meets certain specified criteria in the most efficient or cost-effective way possible. The criterion for an optimum design will depend on the specific goals and constraints of the specific problem. In this methodology, partial differential equation and weighted Bonferroni mean are consolidated to develop the intelligent system for optimal geometric designs for the required product with the blend of fuzzy soft sets. Fuzzy soft sets allow capturing the incorporation of subjective or personal opinions into decision-making processes, as well as the consideration of multiple conflicting criteria. A parameter known as the smoothness parameter is used to control the shape of the optimal geometric model. The smoothness parameter, used as a fuzzy number, is important in this developed system as it fulfills the requirements for the desired intelligent system for product design according to the industries' demands. To verify the credibility of this system, an illustrated example is presented to design a culinary product, which is profitable for the hotel industry.

1. Introduction

Computer-aided design (CAD) is a technique discussed by Sarcar et al. in [1]. This technique is used in the creation, modification, analysis, and optimization of the geometric modeling of products with the help of computers. As mentioned in [2], the fewer number of design variables and the intuitive control of the design for the user are the key aspects of efficiency in systems used for the mathematical modeling of three-dimensional surfaces. Surface modeling, solid modeling, and particle system modeling are the different types of three-dimensional modeling discussed in [3].

Authors discussed in [4], when one surface patch of a set of patches is varied, it creates difficult situations such as maintaining the smooth continuity of the shape. According to Frey and Borouchaki in [5], a surface can be defined geometrically by moving a line in space in a direction other than the line itself. The surface under consideration can be defined as the area swept by the movement. If either the line or the movement is curved, the resulting surface is curved. The direction of the line is known as the g direction in mathematics, and the direction of movement of the line is known as the h direction. The h parameter notation is crucial in CAD concepts and plays an important role in generating surfaces as discussed in [6]. Patches are classified based on the shape of their originating curves. As mentioned in [7], Bezier splines produce a Bezier patch, and two B-spline curves produce a B-spline patch, as discussed in [8]. A patch will inherit features such as control points from its original spline curves. As a result, the number and position of a patch's control points are determined by the control points associated with each of the spline curves. These control points can be used to modify the patch in the same way that control points can be used to manipulate a curve.

The smoothness and extendibility of the surface's topology are the two most important issues in the computer modeling of surfaces. As mentioned in [9], patches, which are mathematically truly curved, are the solution for smooth surfaces. They must, however, be constructed with grid-like rows and columns. According to the author's explanation in

[10], surfaces are subdivided to make them smooth. To achieve the smooth curvature of the surface, the simple polygonal model is subdivided into smaller polygons in this technique. The extent to which the original polygons are subdivided is determined by the curvature of the end surface. When a curve or surface is infinitely subdivided, the result is truly curved and is known as a limit curve or limit surface as discussed in [11]. The most important aspect of the subdivision technique is that it allows to manipulate the shape of the model with both the original polygonal model and the subdivision model. When the original polygonal model is used, the subdivision result is also modified. Moreover, if the subdivision model is manipulated, the changes will have no effect on the original polygon model.

The surface generation method based on partial differential equations, mentioned in [12], is another efficient mathematical model for representing real-world objects that addresses some of the drawbacks discussed in the preceding methods. It is particularly appealing due to the fact that it allows for intuitive manipulation of the shape of the object with minimal user interaction. With a slight change in its design parameters, it generates a wide range of shapes from a single equation. This technique, also known as the PDE method discussed in [13, 14], entails solving a suitable partial differential equation over a set of boundary conditions. A design parameter is also included in the equation to give the user more flexibility in changing the shape of the object with less effort. This method's rendering process is also simple, as it entails solving a mathematical equation to generate points and plot them to create the desired surfaces and shapes. Patches of the surfaces generated by this method can be used to create objects with complex shapes.

According to Ugail and Wilson in [14, 15], geometric modeling, using the spline approach for design optimization, has the disadvantage of making it difficult to maintain a smooth transition between adjacent patches during the design process. It happens because they typically represent an object shape as a mesh of rectangular curvilinear parts. The parameterizations of a product's shape are considered relatively simple if it is made up of standard geometric constructs such as circles, squares, cubes, and cylinders. However, in general, most products cannot be completely constructed using these standard shapes alone but must also include free form surfaces in their design to achieve the desired shape. Regardless of how minor these surfaces are in the overall shape of the object, they may be critical if they are part of the object's functionality. It should be noted that the PDE method is similar in some ways to previously established surface design methods such as Bezier patches and rational B-splines discussed in [15–18]. However, PDE's global smoothing approach, in conjunction with its elliptic boundary-value formulation, clearly distinguishes it from those traditional spline-based techniques. When compared to the hundreds of control points required by existing spline-based techniques, PDE requires only a small set of design variables. As mentioned in [19], Ugail et al. research in the area of interactive design demonstrated that PDE surfaces could be defined and manipulated efficiently in real time. Authors also demonstrated that surface manipulation could be performed in an interactive environment with an interactively defined set of parameters, discussed in [19–22]. These parameters, by definition, make their effect on the geometry of the surface obvious.

Regarding interactive design using the PDE method, Bloor and Wilson showed in [23] that, this method can produce simple surfaces in an interactive environment. This work was then extended by Ugail et al., mentioned in [19, 20]. They used four boundary conditions in the PDE method to generate surface. Later, their work was extended by Ugail et al., discussed in [24], in which they used six boundary conditions to generate the geometric surface. It was easier to work with six boundary conditions than with four boundary conditions, as a more curvier geometric model can be generated and a complex geometric model can be generated more easily. In this article, their work is further extended by using eight boundary conditions to develop the complex geometric model with the help of a set of interactively defined parameters. The boundary conditions are responsible to control the overall form of the surface of the designed model, while a set of parameters control all the coordinates of the boundary conditions. One of the parameters involved in the designing of the desired model is the modified smoothness parameter, responsible for the smoothness and quality of the geometric model, which is developed with the help of the fuzzy soft matrix (FSM) along with weighted Bonferroni mean (WBM).

The fuzzy soft matrix (FSM) is a very useful tool to gather requirements of the industry for designing their products, while weighted Bonferroni mean (WBM) blend the requirements of industry into a single entity. According to Zadeh discussed in [25], fuzzy sets theory is useful for modeling uncertain circumstances better than standard theories. In 1999, Molodtsov introduced the concept of soft set, discussed in [26], and successfully solved some uncertainties. Later, Maji et al. [27] modified the concept of soft set theory and successfully applied it to decision-making problems. In 2001, Maji et al. in [28] presented the concept of the fuzzy soft set by combining the concept of the fuzzy set with the soft set. One of the main advantages of fuzzy soft sets is that they allow for the incorporation of subjective or personal opinions into decision-making processes, as well as the consideration of multiple conflicting criteria. This can be particularly useful in situations where there is a lack of objective data or where multiple stakeholders have different viewpoints or priorities.

Fuzzy soft sets have been applied in a wide range of fields, including decision analysis, artificial intelligence, information retrieval, and control systems. They are often used in conjunction with other techniques, such as fuzzy logic or artificial neural networks, to improve the accuracy and robustness of decision-making systems. This theory opened the path to many new concepts. Neog and Sut mentioned in [29] used fuzzy soft complement and fuzzy soft matrix operation to solve decision-making problems. Sut in [30] utilizes fuzzy soft relations in a decision-making problem.

Cagman et al. in [31] proposed a decision-making method that makes use of a fuzzy soft aggregation operator and a cardinal set. Celik and Yamak in [32] used fuzzy soft set theory with a fuzzy aggregation operator to diagnose patients' diseases. In 2018, Beg et al. [33] developed a time-dependent model to analyse human attributes by fuzzy soft matrix (FSM) and Bonferroni mean (BM). BM was developed by Bonferroni in [34], which was helpful in the multiple comparison test. Yager in [35, 36] and Beliakov et al. in [37] discussed useful properties of BM to capture the interrelationships among arguments. Using the concept of BM with fuzzy soft matrix, Beg et al. explained in [33] and defined Bonferroni fuzzy soft matrix (BFSM) and weighted Bonferroni fuzzy soft matrix (WBFSM) for data representation. They also used WBFSM for design making.

An optimum geometric design is a design that meets certain specified criteria in the most efficient or cost-effective way possible. The criteria for an optimum design will depend on the specific goals and constraints of the problem at hand and may include factors such as strength, stiffness, weight, cost, and manufacturability.

In this article, the concept of WBFSM is used to calculate the modified smoothness parameter used in the partial differential equation. Since the requirements for designing an intelligent system to generate a geometric model for an industry can only be provided in the form of fuzzy soft set and single value is required for smoothness parameter, so this intelligent system was needed to convert fuzzy soft set in a single number. For this purpose, WBFSM is used in this article as it is an ideal tool to blend multiple values in a single value.

To verify this intelligent system, an example is taken from the hotel industry to design a drinking glass for fizzy drinks. According to a case study discussed in [38], drinking behavior of a person highly depends on the design of a glass. If the glass has an effective design and is easy to drink from, then the person will drink more than usual. The design of a glass also depends on the type of drinks. It is also discussed in the same case study, how design of glass affects the drinking behavior of a person. Considering the drinking behavior and the type of drink, it can be very helpful in designing a glass. In the making of a drinking glass, many parameters are considered (such as quality, material used in glass, and price) according to the requirement of the customer. But the problem is that a customer is not able to see it until the manufacturing of the glass is complete. Sometimes, the final result is not according to the customer's requirement. For this purpose, a three-dimensional design of a drinking glass according to the required parameters is generated. The intelligent system developed in this article is used to generate multiple designs of drinking glass according to customer's requirements. After modeling these designs, multicriteria decision-making (MCDM) is used to select the glass which efficiently fulfills the customer requirements.

The structure of this article is as follows: Section 2 is devoted, the basic definitions used in this research, which will help the reader in understanding this research. Section 3 provides the detail on the PDE used in this research along with some geometric models generated using the involved PDE method. In Section 4, forming of the smoothness parameter with the help of WBFSM is discussed in detail. In this section, we also developed an intelligent system and with the help of illustrated examples, we verified the credibility of the developed system. Finally, the last section of this article is concluded with the discussion on the benefits and utilization of the developed system in different designing industries. A path for further research is also mentioned in this section.

2. Preliminaries

Partial differential equation (PDE) is a tool which is used in every field of mathematics. In this article, a hybrid methodology is developed by using fuzzy sets and soft sets theory with blend of eighth order PDE with some boundary conditions and a set of parameters. In this chapter, some basic definitions are discussed which will be helpful in further discussion.

Definition 1 [25]. Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $\mu_A\colon X \longrightarrow [0,1]$, and $\mu_A(X)$ is interpreted as the degree of membership of element x in the fuzzy set A for each $x \in X$.

Definition 2 [26]. Let Y be a universal set, E be a set of parameters, and A be a subset of E. Power set of Y is denoted by $P(Y)$, then the pair $(F; A)$ is said to be a soft set over the universal set Y, where F is mapping given by $F\colon A \longrightarrow P(Y)$. In fact, a soft set is a parameterized family of subsets over the universal set Y . Every set $(F(e); e)$ represents the set of elements of the soft set $(F; A)$.

Definition 3 [39]. If Y be the universal set, E be the set of parameters, and $B \subseteq E$ and P^Y be the set of all fuzzy subsets of Y, then (G, B) is said to be the fuzzy soft set over Y, where G is a mapping such that $G\colon B \longrightarrow P^Y$. Tabular representation of a fuzzy soft set is represented in Table 1.

Definition 4 [37]. Let there be two natural numbers s, t and $r_i \geq 0$ for $(i = 1, 2, \cdots, n)$. Then, Bonferroni mean $F^{s,t}$ is as follows:

$$F^{s,t} = \left(\frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^{n} r_i^s r_j^t \right)^{(1/s+t)}. \quad (1)$$

The Bonferroni mean (BM) is a useful aggregative operator that is an extension of the arithmetic mean. The Bonferroni mean (BM) has the property of capturing the interrelationships between factors that are significant for multicriteria decision-making, according to Yager [35]. The following scenario for BM was studied by Dyckhoff and Pedrycz [40] for various s and t values.

(i) If $s = 1$ and $t = 0$, then

$$F^{1,0} = \left(\frac{1}{n} \sum_{i=1}^{n} r_i \right). \quad (2)$$

(ii) If $s = 2$ and $t = 0$, then

Table 1: Tabular representation of a fuzzy soft set (G, B).

	y_1	y_2	y_3	$\cdots$	y_n
e_1	a_{11}	a_{12}	a_{13}	$\cdots$	a_{1n}
e_2	a_{21}	a_{22}	a_{23}	$\cdots$	a_{2n}
e_3	a_{31}	a_{32}	a_{33}	$\cdots$	a_{3n}
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
e_m	a_{m1}	a_{m2}	a_{m3}	$\cdots$	a_{mn}

$$F^{2,0} = \left(\frac{1}{n}\sum_{i=1}^{n} r_i\right)^{(1/2)}. \quad (3)$$

(iii) If $s \longrightarrow +\infty$ and $t = 0$, then

$$\lim_{s \longrightarrow +\infty} F^{s,0} = \max_i \{r_i\}. \quad (4)$$

(iv) If $s \longrightarrow 0$ and $t = 0$, then

$$\lim_{s \longrightarrow 0} F^{s,0} = \left(\prod_{i=1}^{n} r_i\right)^{(1/n)}. \quad (5)$$

(v) If $s = 1$ and $t = 1$, then

$$F^{1,1} = \left(\frac{1}{n(n-1)}\sum_{i,j=1, i\neq j}^{n} r_i r_j\right)^{(1/2)}. \quad (6)$$

In equation (6), the operator's interpretation is a combination of the relational operators "and" and "average." Product activities, in particular, can be leveraged to implement two satisfaction requirements. As a result, r_i and r_j denote the degree to which the two requirements are satisfied. As a result, $F^{1,1}(r_1, r_2, \cdots r_n)$ estimates the reference pair's average satisfaction.

Definition 5 [35]. If s and t are two natural numbers and $r_i \geq 0$ for all $(i = 1, 2, 3, \cdots, n)$ and for any weighted vector $W = (w^i \geq 0)^T$ of r_i, which has condition $\sum_{i=1}^{n} w_i = 1$, then weighted Bonferroni mean is

$$WF^{s,t} = \left(\frac{1}{n(n-1)}\sum_{i,j=1, i\neq j}^{n} (w_i r_i)^s (w_j r_j)^t\right)^{(1/s+t)}. \quad (7)$$

Definition 6 [33]. Let there be a fuzzy soft matrix $F_{mn}^h = [f_{ij}^h]$ where $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ and $h = 1, 2, \cdots, q$. Then, Bonferroni fuzzy soft matrix (BFSM) is defined as $F_{mn} = [f_{mn}]$, where $f_{mn} = F^{s,t}(f_{mn}^1, f_{mn}^2, \ldots, f_{mn}^q)$. Here, s and t are natural numbers.

Definition 7 [33]. Let a fuzzy soft matrix be $F_{m\times n} = [f_{i,j}]$, where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. Then, weighted Bonferroni fuzzy soft matrix (WBFSM) is defined as $F_{m\times 1} = [d_{i,1}]$ for weight vector $W = w_j$, which is the weight of $f_{i,j}$. The weight vector has the condition $\sum_j^n w_j = 1$. Here, $d_{i,1} = WF^{s,t}(f_{i,1}, f_{i,2}, \ldots, f_{i,n})$.

Definition 8 [33]. Optimum fuzzy soft constant (OFSC) is Optimum $= \max_i (c_{i,1})$, where $c_{i,1}$ is from the last definition.

2.1. Partial Differential Equation Surfaces. Wood ward introduced in [41] that the PDE method mainly aims to provide a skillful function accepting some specific boundary and continuity conditions, and this function works as a connecting surface for primary neighboring surfaces. Follow that a second order elliptic PDE method was used by Bloor and Wilson [42] to generate blend surfaces in the field of CAGD. They then extended their work and generated PDE surfaces using fourth order PDE [19, 20], which used four boundary conditions to generate surfaces. In this article using their work as a reference, eight order PDE surfaces are generated using eight boundary conditions.

$$\left(\frac{\partial^2}{\partial\alpha^2} + \eta^2\frac{\partial^2}{\partial\beta^2}\right)^4 \Omega(\alpha, \beta) = 0, \quad (8)$$

where $\Omega(\alpha, \beta) = (x(\alpha, \beta), y(\alpha, \beta), z(\alpha, \beta))$ is a parametric function.

To preserve the shape of the surface, the selection of boundary conditions and parametric domain are essential for equation (8). Here, Ω is normally taken as a rectangle, that is, Ω: $\alpha_0 \leq \alpha \leq \alpha_1, \beta_0 \leq \beta \leq \beta_1$. The parameter η is known as a smoothness parameter which plays an important role for the shape reconstruction of a model [41].

To solve the eighth order PDE, equation (8) required eight boundary conditions so that a unique solution in x, y, and z coordinates can be obtained. Due to this, in this research, the required PDE surface is generated by the solution of equation (8) with eight positional curves taken by the given geometric shape with the help of CAGD and used as boundary conditions. An illustrated solution of equation (8) with 8 boundary conditions is drawn in Figure 1 as representing the 3D model of an apple.

Let the domain of Ω be finite defined as Ω: $0 \leq \alpha \leq 1, 0 \leq \beta \leq 2\pi$ such that

$$\begin{aligned}
\Omega(0, \beta) &= \hbar_0(\beta),\\
\Omega(k, \beta) &= \hbar_k(\beta),\\
\Omega(l, \beta) &= \hbar_l(\beta),\\
\Omega(m, \beta) &= \hbar_m(\beta),\\
\Omega(n, \beta) &= \hbar_n(\beta),\\
\Omega(o, \beta) &= \hbar_o(\beta),\\
\Omega(p, \beta) &= \hbar_p(\beta),\\
\Omega(1, \beta) &= \hbar_1(\beta),
\end{aligned} \quad (9)$$

$\hbar_0(\beta), \hbar_k(\beta), \hbar_l(\beta), \hbar_m(\beta), \hbar_n(\beta), \hbar_0(\beta), \hbar_p(\beta), \hbar_1(\beta)$ are the boundary conditions. The relative position of the boundary curve depends on the parameters $0 < k < l < m < n < o < p < 1$.

Figure 1: 3D model of an apple.

In this research, equation (8) is solved by using the method of separation of variables as follows:

$$\Omega(\alpha, \beta) = \overline{\varrho}_0(\alpha) + \sum_{i=1}^{F} \left[\overline{\sigma}_i(\alpha)\cos(i\beta) + \overline{\tau}_i(\alpha)\sin(i\beta)\right], \tag{10}$$

where

$$\overline{\varrho}_0 = \overline{\mu}_{00} + \overline{\mu}_{01}\alpha + \overline{\mu}_{02}\alpha^2 + \overline{\mu}_{03}\alpha^3 + \overline{\mu}_{04}\alpha^4 + \overline{\mu}_{05}\alpha^5 + \overline{\mu}_{06}\alpha^6 + \overline{\mu}_{07}\alpha^7, \tag{11}$$

$$\overline{\sigma}_i = \overline{\nu}_{i1}e^{\eta i\alpha} + \overline{\nu}_{i2}e^{-\eta i\alpha} + \overline{\nu}_{i3}\alpha e^{\eta i\alpha} + \overline{\nu}_{i4}\alpha e^{-\eta i\alpha} + \overline{\nu}_{i5}\alpha^2 e^{\eta i\alpha} + \overline{\nu}_{i6}\alpha^2 e^{-\eta i\alpha} + \overline{\nu}_{i7}\alpha^3 e^{\eta i\alpha} + \overline{\nu}_{i8}\alpha^3 e^{-\eta i\alpha}, \tag{12}$$

$$\overline{\tau}_i = \xi_{i1}e^{\eta i\alpha} + \overline{\xi}_{i2}e^{-\eta i\alpha} + \xi_{i3}\alpha e^{\eta i\alpha} + \xi_{i4}\alpha e^{-\eta i\alpha} + \overline{\xi}_{i5}\alpha^2 e^{\eta i\alpha} + \overline{\xi}_{i6}\alpha^2 e^{-\eta i\alpha} + \xi_{i7}\alpha^3 e^{\eta i\alpha} + \xi_{i8}\alpha^3 e^{-\eta i\alpha}. \tag{13}$$

Here, all the values of $\mu, \nu,$ and ξ are unknown.

The effect in the final shape of a model due to these boundary conditions is shown in Figure 1. A slight change in the position of the curves can change the shape of the entire model. The unknown parameters, μ, ν, and ξ, also have a huge role in obtaining the final shape.

2.2. Error Analysis. The unknown constants are Fourier coefficients. They can be calculated using suitable numerical techniques on the given boundary conditions. The shape of the final model involves some error. The following equation shows that error:

$$\Omega(\alpha, \beta) = \overline{\varrho}_0(\alpha) + \sum_{i=1}^{F} \left[\overline{\sigma}_i(\alpha)\cos(i\beta) + \overline{\tau}_i(\alpha)\sin(i\beta)\right] + \overline{R}(\alpha, \beta), \tag{14}$$

where the error is shown by $\overline{R}$ and F is a finite natural number.

In the boundary conditions, the coefficients $\rho(\alpha)$ and $\tau(\alpha)$ can be calculated using the amplitude of the i^{th} mode. The role of high frequency modes to the final model is shown by the term $\overline{R}(\alpha, \beta)$. Because of the finite value of F, this term has an effect on the overall shape of the model. The following equation is used to find this term:

$$\overline{R}(\alpha, \beta) = \overline{\omega}_1(\beta)e^{l\alpha} + \overline{\omega}_2(\beta)e^{-l\alpha} + \overline{\omega}_3(\beta)\alpha e^{l\alpha} + \overline{\omega}_4(\beta)\alpha e^{-l\alpha} + \overline{\omega}_5(\beta)\alpha^2 e^{l\alpha} + \overline{\omega}_6(\beta)\alpha^2 e^{-l\alpha} + \overline{\omega}_7(\beta)\alpha^3 e^{l\alpha} + \overline{\omega}_8(\beta)\alpha^3 e^{-l\alpha}. \tag{15}$$

Here, $\overline{\omega}_1, \overline{\omega}_2, \overline{\omega}_3, \overline{\omega}_4, \overline{\omega}_5, \overline{\omega}_6, \overline{\omega}_7, \overline{\omega}_8$, and l are obtained, by taking the difference of the original boundary conditions with the conditions fulfilled by the function in equation (10).

2.3. Different Geometric Models Using PDE Surfaces. The various geometrical figures can be generated from a variety of curve options. The examples mentioned demonstrate the efficacy and comprehensibility of the PDE approach for the generation of complex geometry shapes, which supports the main goal of this study. The figures from 2 to 9 are generated using eight boundary conditions with Ω: $0 \leq \alpha \leq 1, 0 \leq \beta \leq 2\pi$, where $x = \alpha \cos(\beta), y = \alpha \sin(\beta)$ and $0 \leq z \leq 1$. In Figure 2, only a single point is taken for the first curve and the surface is generated by joining that point to each point of the next curve. Figure 3 is also generated using a point for top and bottom curves. A circular curve is used for the first curve of Figure 4 and a point for its bottom curve which gave it the shape of a pot. To generate the shape of a peach in Figure 5, the position of the first curve is taken between the second and third curves. (Figures 6–9) are also drawn in the same manner according to the related boundaries conditions.

3. Intelligent Systems for Designing of the Culinary Product Based on the Fuzzy Soft Partial Differential Equation

In optimum designing of the culinary product, the Bonferroni mean has the ability to capture desirable properties among arguments' interrelationship. While the fuzzy soft set deals with the situation where uncertainty occurs, differential equation is a useful tool to analyse the rate of change of dependent variables with respect to the independent variable. Therefore, the characteristic of BM with blend of fuzzy soft sets and differential equation becomes a very useful instrument to design an optimum culinary product according to the requirement of the industry. In this section, fuzzy soft matrix is used with weighted Bonferroni mean to calculate the value of the smoothness parameter, η, in the PDE defined in equation (8). An example is also developed to demonstrate the reliability of the developed technique.

3.1. An Intelligent System for Optimum Geometric Design. In the previous section, equation (8) was used to generate a PDE surface using eight curves. This equation has a parameter η, known as the smoothness parameter, which plays an important role in the shape and smoothness of a surface. Usually, η is considered 1 by default. But when this PDE method is applied to design a product for some customers, a specific value of η is needed as it plays an important role in fulfilling the requirements of the customer. The parameter, η, consists of all the specifications for designing a product, for example, price, quality, design, volume, and shape. Due to the uncertainty involved during product designing, it is not possible to take $\eta = 1$. To develop an intelligent system for designing the required product using η in the PDE given in equation (8) for different requirements of the industry, the following steps should be considered.

Step 1: there are r decision makers, who provide r number of decision matrices, in the form of fuzzy soft matrices $A^r_{mn} = [a_{mn}]$, where $m = 1, 2, \ldots, i$ and $n = 1, 2, \ldots, j$, according to industrial requirements to design a specific product. A weight vector $W = \{w_1, w_2, w_3, \ldots, w_n\}$ is also given according to the industry requirements. W must satisfy the conditions $w_p > 0$ for $p = 1, 2, 3, \cdots, j$ and $\sum_{p=1}^{j} \omega_p = 1$, then using Definition 7, we calculate WBFSM for each $A_{mn} = [a_{mn}]$. For each decision matrix Amn, a unique resultant matrix of the form WAs,t $= [a_{m1}]$, for m = 1, 2, 3, . . ., i and s, t are natural numbers, are developed. Each entry of $WA^{s,t}$ (say η_{mn}) provides a unique product design by using Equation (8) as specified by the industry.

Step 2: a matrix $E = [\eta_{mn}]$, where $m = 1, 2, \cdots, i$ and $n = 1, 2, \cdots j$, will be generated by the calculated values of η_{mn} in step 1.

Step 3: to select the best product design between the generated designs by each η_{mn}, we use

$$\text{Optimum Smoothing Parameter}\left(\eta^{-}\right) = \max\left(\max\left(\eta_{mn}\right)\right), \tag{16}$$

where $m = 1, 2, \ldots, i$ and $n = 1, 2, \ldots, j$. This intelligent system helps obtain an optimal geometric design with the help of $\overline{\eta}$. This is the best design of the desired product which completely fulfills the requirements of the industry.

3.2. Illustrated Example. Mr. Khan is an owner of a juice bar, who wants to buy cold drink glass for serving fizzy drinks. He wants the design of the glass to be attractive but simple. Also, the glass should be of fine quality with price as low as possible, and the volume of the glass should be appropriate for one serving. He went to a glass design company. He selected five different basic designs for glass according to his given parameters which are $P = \{p(\text{price}), v(\text{volume}), \text{and } q(\text{quality})\}$. Mr. Khan also gave some weight $W = (0.4\ 0.3\ 0.3)$ to each of these parameters. Then, these designs were given to five experts de singers $= \{d_1, d_2, d_3, d_4, d_5\}$. All the designers gave their suggestions for each glass according to Mr. Khan's requirements. Following is the fuzzy soft matrix for the basic design of the first glass according to the suggestions of the experts.

$$G_1 = \begin{bmatrix} 0.30 & 0.70 & 0.20 \\ 0.90 & 0.60 & 0.70 \\ 0.60 & 0.75 & 0.40 \\ 0.80 & 0.60 & 0.50 \\ 0.40 & 0.50 & 0.35 \end{bmatrix}. \tag{17}$$

Then, using the definition (2.0.9) of weighted Bonferroni fuzzy soft matrix (WBFSM), the WBFSM for the first glass design G_1 is according to the given weights for the parameters, which is as follows:

$$B_1 = \begin{bmatrix} 0.1225 \\ 0.2437 \\ 0.1913 \\ 0.2102 \\ 0.1373 \end{bmatrix}. \tag{18}$$

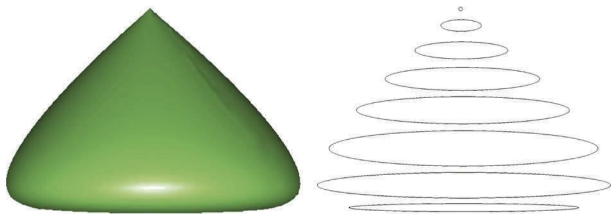


Figure 2: Tomb of a mosque with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 1.0)$, $C_2 = (0.1\cos(\beta), 0.1\sin(\beta), 0.8)$, $C_3 = (0.3\cos(\beta), 0.3\sin(\beta), 0.7)$, $C_4 = (0.5\cos(\beta), 0.5\sin(\beta), 0.6)$, $C_5 = (0.6\cos(\beta), 0.6\sin(\beta), 0.5)$, $C_6 = (0.8\cos(\beta), 0.8\sin(\beta), 0.3)$, $C_7 = (0.9\cos(\beta), 0.9\sin(\beta), 0.2)$, and $C_8 = (0.7\cos(\beta), 0.7\sin(\beta), 0.0)$.

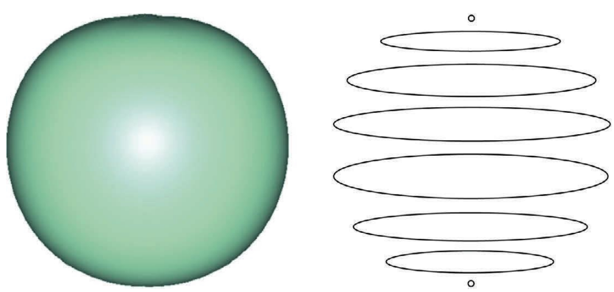


Figure 3: Sphere with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 1.0)$, $C_2 = (0.2\cos(\beta), 0.2\sin(\beta), 0.8)$, $C_3(0.6\cos(\beta), 0.6\sin(\beta), 0.7)$, $C_4 = (1.0\cos(\beta), 1.0\sin(\beta), 0.5)$, $C_5 = (0.6\cos(\beta), 0.6\sin(\beta), 0.3)$, $C_6 = (0.2\cos(\beta), 0.2\sin(\beta), 0.2)$, $C_7 = (0.1\cos(\beta), 0.1\sin(\beta), 0.1)$, and $C_8 = (0.0\cos(\beta), 0.0\sin(\beta), 0.0)$.

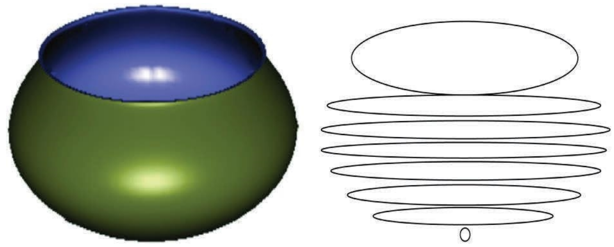


Figure 4: A cooking pot with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (0.7\cos(\beta), 0.7\sin(\beta), 1.0)$, $C_2 = (0.5\cos(\beta), 0.5\sin(\beta), 0.95)$, $C_3 = (0.6\cos(\beta), 0.6\sin(\beta), 0.7)$, $C_4 = (0.7\cos(\beta), 0.7\sin(\beta), 0.6)$, $C_5 = (0.8\cos(\beta), 0.8\sin(\beta), 0.5)$, $C_6 = (1.0\cos(\beta), 1.0\sin(\beta), 0.4)$, $C_7 = (0.4\cos(\beta), 0.4\sin(\beta), 0.2)$, and $C_8 = (0.1\cos(\beta), 0.1\sin(\beta), 0.0)$.

Now, for matrix G_1, there are five different values. Each value defines a smoothness parameter η_{mn} for a separate glass. Each value will be used in equation (8) as η_{mn} to get a slightly different glass design. Here, the values are $\eta_{11} = 0.1225$, $\eta_{21} = 0.2437$, $\eta_{31} = 0.1913$, $\eta_{41} = 0.2102$, and $\eta_{51} = 0.1373$. Putting these values in equation (8), the following designs are generated:

As it can be seen from the data $\eta_{21} > \eta_{41} > \eta_{31} > \eta_{51} > \eta_{11}$, so the glass designed in Figure 10(b) with the value η_{21} is the best between these five glass designs described in Figures 10(a)–10(e). The difference between these glasses is

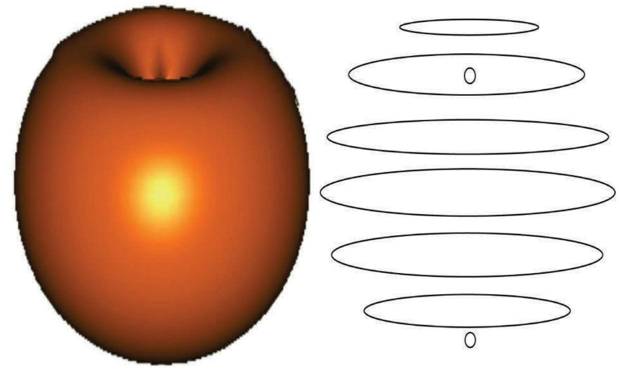


Figure 5: A peach model with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (0.0\cos(\beta), 0.0\sin(\beta), 0.8)$, $C_2 = (0.3\cos(\beta), 0.3\sin(\beta), 1.0)$, $C_3 = (0.5\cos(\beta), 0.5\sin(\beta), 0.7)$, $C_4 = (0.8\cos(\beta), 0.8\sin(\beta), 0.6)$, $C_5 = (1.0\cos(\beta), 1.0\sin(\beta), 0.5)$, $C_6 = (0.8\cos(\beta), 0.8\sin(\beta), 0.3)$, $C_7 = (0.3\cos(\beta), 0.3\sin(\beta), 0.2)$, and $C_8 = (0.0\cos(\beta), 0.0\sin(\beta), 0.0)$.

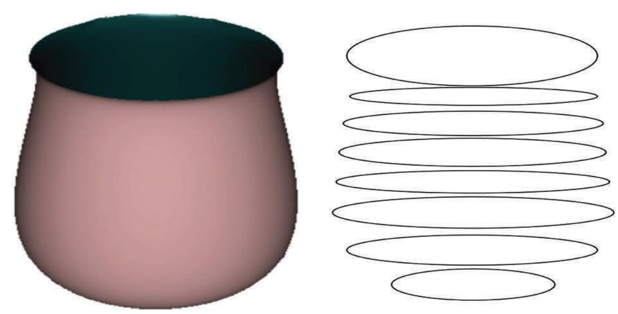


Figure 6: A glass bowl with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (0.8\cos(\beta), 0.8\sin(\beta), 1.0)$ $C_2 = (0.7\cos(\beta), 0.7\sin(\beta), 0.9)$, $C_3 = (0.8\cos(\beta), 0.8\sin(\beta), 0.7)$, $C_4 = (0.85\cos(\beta), 0.85\sin(\beta), 0.6)$, $C_5 = (0.9\cos(\beta), 0.9\sin(\beta), 0.5)$, $C_6 = (1.0\cos(\beta), 1.0\sin(\beta), 0.3)$, $C_7 = (0.9\cos(\beta), 0.9\sin(\beta), 0.15)$, and $C_8 = (0.5\cos(\beta), 0.5\sin(\beta), 0.0)$.

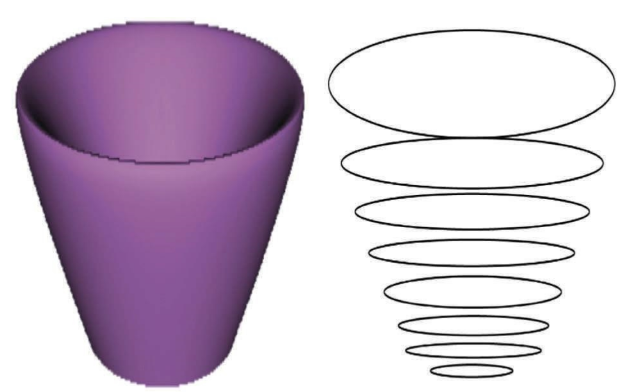


Figure 7: Drinking glass with the following boundary condition in which $0 \leq \beta \leq 2\pi$: $C_1 = (1.0\cos(\beta), 1.0\sin(\beta), 1.0)$, $C_2 = (0.9\cos(\beta), 0.9\sin(\beta), 0.8)$, $C_3 = (0.7\cos(\beta), 0.7\sin(\beta), 0.7)$, $C_4 = (0.6\cos(\beta), 0.6\sin(\beta), 0.6)$, $C_5 = (0.5\cos(\beta), 0.5\sin(\beta), 0.5)$, $C_6 = (0.4\cos(\beta), 0.4\sin(\beta), 0.3)$, $C_7 = (0.3\cos(\beta), 0.3\sin(\beta), 0.2)$, and $C_8 = (0.2\cos(\beta), 0.2\sin(\beta), 0.0)$.

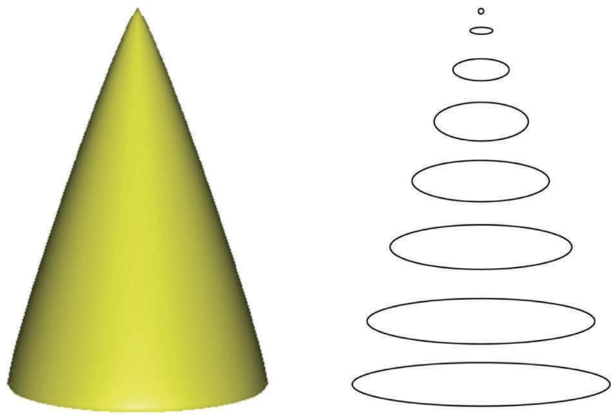


Figure 8: A cone model with the following boundary condition in which $0 \le \beta \le 2\pi$: $C_1 = (0.0\cos(\beta),\ 0.0\sin(\beta),\ 1.0)$, $C_2 = (0.1\cos(\beta),\ 0.1\sin(\beta),\ 0.9)$, $C_3 = (0.2\cos(\beta),\ 0.2\sin(\beta),\ 0.8)$, $C_4 = (0.4\cos(\beta),\ 0.4\sin(\beta),\ 0.6)$, $C_5 = (0.6\cos(\beta),\ 0.6\sin(\beta),\ 0.5)$, $C_6 = (0.8\cos(\beta),\ 0.8\sin(\beta),\ 0.3)$, $C_7 = (0.9\cos(\beta),\ 0.9\sin(\beta),\ 0.2)$, and $C_8 = (1.0\cos(\beta),\ 1.0\sin(\beta),\ 0.0)$.

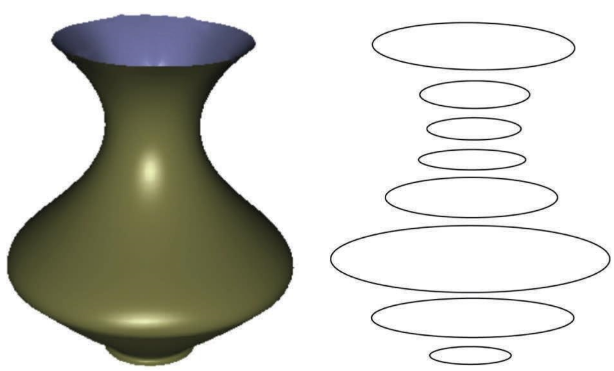


Figure 9: Vase with narrow mouth with the following boundary condition in which $0 \le \beta \le 2\pi$: $C_1 = (0.5\cos(\beta),\ 0.5\sin(\beta),\ 1.0)$, $C_2 = (0.3\cos(\beta),\ 0.3\sin(\beta),\ 0.8)$, $C_3 = (0.25\cos(\beta),\ 0.25\sin(\beta),\ 0.7)$, $C_4 = (0.3\cos(\beta),\ 0.3\sin(\beta),\ 0.6)$, $C_5 = (0.4\cos(\beta),\ 0.4\sin(\beta),\ 0.5)$, $C_6 = (0.75\cos(\beta),\ 0.75\sin(\beta),\ 0.3)$, $C_7 = (0.5\cos(\beta),\ 0.5\sin(\beta),\ 0.2)$, and $C_8 = (0.15\cos(\beta),\ 0.15\sin(\beta),\ 0.0)$.

so small that one cannot detect it just by looking at their pictures, but when the data are observed, it can be seen that the glass with the best quality and moderate volume is selected. Here, volume of 0.5 is approximately equal to 250 ml that is also considered as one serving of a drink. If the given data of Figure 10(e) are observed, it has the exact volume of one serving, but the quality of the material of glass is very low, so it is not a good choice for the customer.

The fuzzy soft matrix for the second design of the glass according to the suggestions of the experts is as follows:

$$G_2 = \begin{bmatrix} 0.10 & 0.65 & 0.30 \\ 0.60 & 0.53 & 0.70 \\ 0.40 & 0.60 & 0.50 \\ 0.50 & 0.57 & 0.60 \\ 0.80 & 0.48 & 1.00 \end{bmatrix}. \tag{19}$$

Similarly, WBFSM for the matrix G_2 is as follows:

$$B_2 = \begin{bmatrix} 0.0982 \\ 0.2016 \\ 0.1631 \\ 0.1835 \\ 0.2485 \end{bmatrix}. \tag{20}$$

Here, $\eta_{12} = 0.0982$, $\eta_{22} = 0.2016$, $\eta_{32} = 0.1631$, $\eta_{42} = 0.1835$, and $\eta_{52} = 0.2485$. Putting these values of η_{mn} in equation (8), the generated designs are as follows:

Here, $\eta_{52} > \eta_{22} > \eta_{42} > \eta_{32} > \eta_{12}$. So, η_{52} gives the best glass from the abovementioned five glasses shown in Figures 11(a)–11(e). It can be noticed from the data that η_{52} has the highest quality and moderate volume for one serving.

The fuzzy soft matrix for the third glass design is as follows:

$$G_3 = \begin{bmatrix} 0.20 & 0.25 & 0.35 \\ 0.70 & 0.58 & 0.50 \\ 1.00 & 0.48 & 0.65 \\ 0.40 & 0.25 & 0.50 \\ 0.10 & 0.10 & 0.30 \end{bmatrix}. \tag{21}$$

For matrix G_3, the WBFSM is as follows:

$$B_3 = \begin{bmatrix} 0.0862 \\ 0.1973 \\ 0.2336 \\ 0.1255 \\ 0.0500 \end{bmatrix}. \tag{22}$$

Here, $\eta_{13} = 0.0862$, $\eta_{23} = 0.1973$, $\eta_{33} = 0.2336$, $\eta_{43} = 0.1255$, and $\eta_{53} = 0.0500$. Figures 12(a)–12(e) designs are generated by the values of η_{mn} after putting them in equation (8):

As $\eta_{33} > \eta_{23} > \eta_{43} > \eta_{13} > \eta_{53}$, so the selected design for G_3 seen in Figure 12(c) is the design made by η_{33}. This design has the highest quality, and the volume is close to 250 ml.

Now, for the fourth glass design, the fuzzy soft matrix is as follows:

$$G_4 = \begin{bmatrix} 0.20 & 0.10 & 0.30 \\ 0.80 & 0.60 & 0.50 \\ 0.40 & 0.37 & 0.60 \\ 0.10 & 0.20 & 0.10 \\ 1.00 & 0.40 & 0.80 \end{bmatrix}. \tag{23}$$

The weighted Bonferroni fuzzy soft matrix (WBFSM) for fuzzy soft matrix G_4 is as follows:

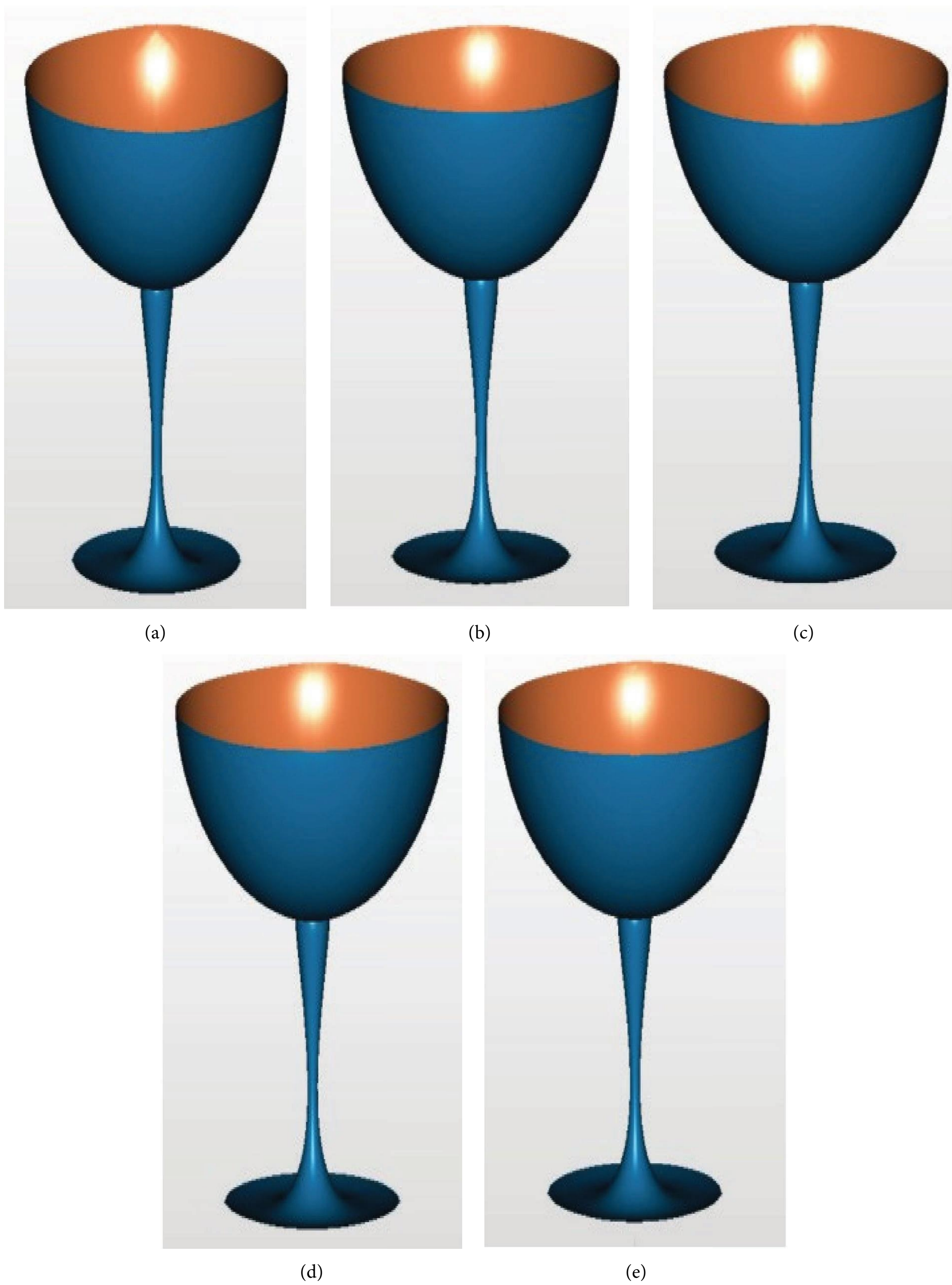


Figure 10: (a) First glass model when the smoothness parameter is $\eta_{11} = 0.1225$. (b) First glass model when the smoothness parameter is $\eta_{21} = 0.2437$. (c) First glass model when the smoothness parameter is $\eta_{31} = 0.1913$. (d) First glass model when the smoothness parameter is $\eta_{41} = 0.2102$. (e) First glass model when the smoothness parameter is $\eta_{51} = 0.1373$.

$$B_4 = \begin{bmatrix} 0.0640 \\ 0.2102 \\ 0.1489 \\ 0.0424 \\ 0.2400 \end{bmatrix}. \quad (24)$$

$\eta_{14} = 0.0640$, $\eta_{24} = 0.2102$, $\eta_{34} = 0.1489$, $\eta_{44} = 0.0424$, and $\eta_{54} = 0.2400$. Putting these values in equation (8) for η_{mn}gives slightly different designs.

$\eta_{54} > \eta_{24} > \eta_{34} > \eta_{14} > \eta_{44}$. Similarly, by observing the data given for this design, it is noticed that η_{54} has the finest quality with volume closer to 250 ml. So, the glass generated

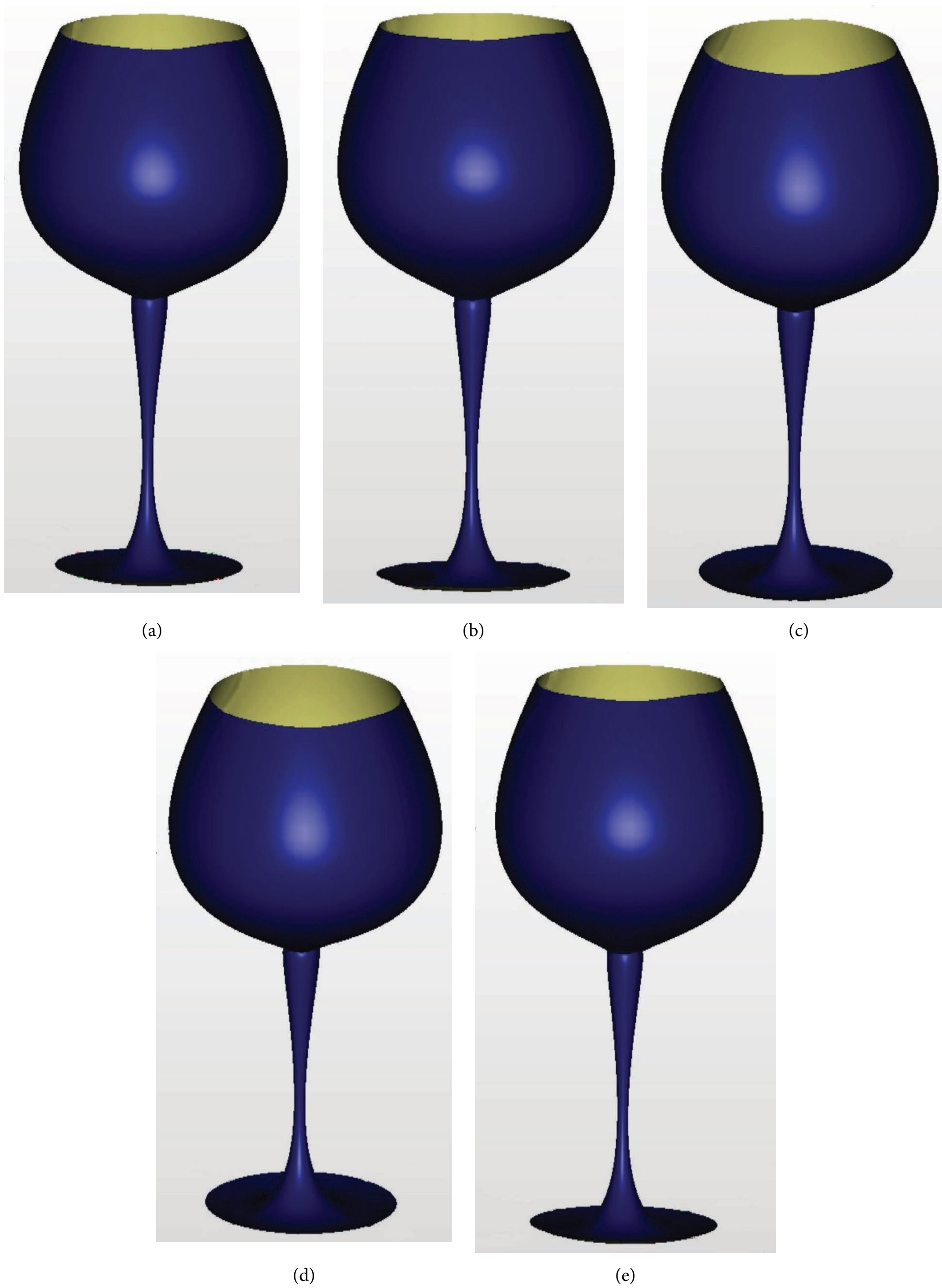


Figure 11: (a) Second glass model when the smoothness parameter is $\eta_{12} = 0.0982$. (b) Second glass model when the smoothness parameter is $\eta_{22} = 0.2016$. (c) Second glass model when the smoothness parameter is $\eta_{32} = 0.1631$. (d) Second glass model when the smoothness parameter is $\eta_{42} = 0.1835$. (e) Second glass model when the smoothness parameter is $\eta_{52} = 0.2485$.

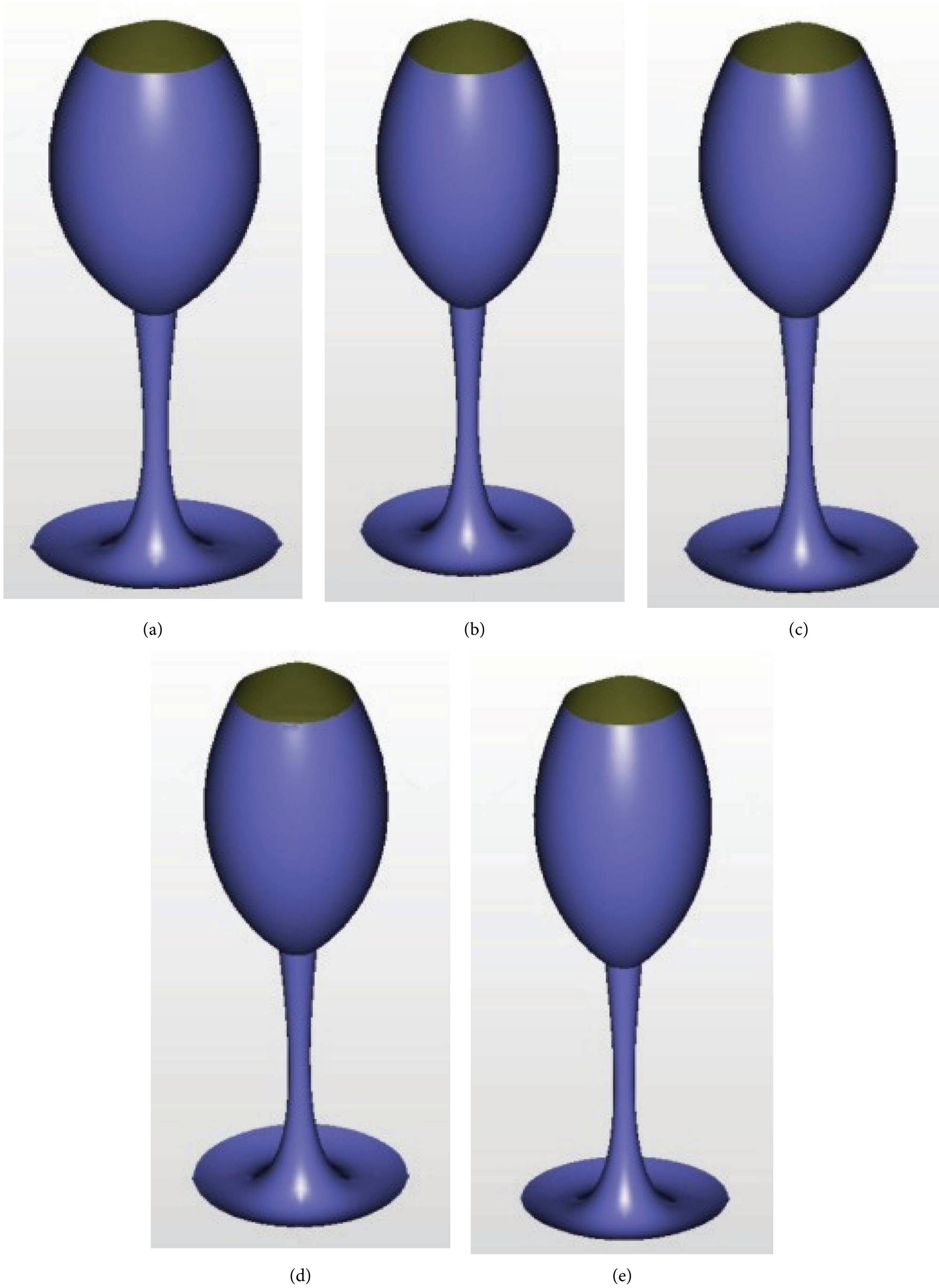
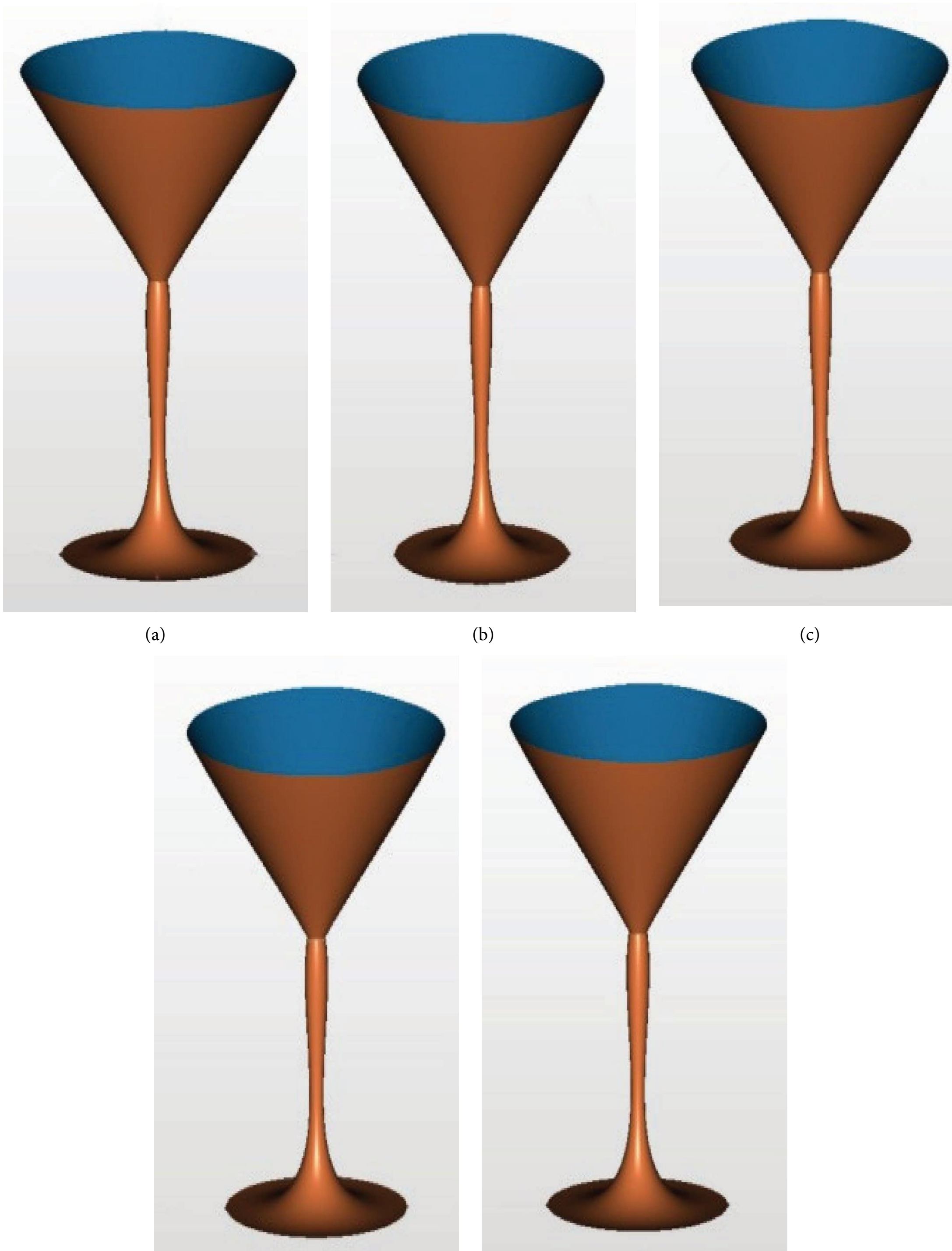


Figure 12: (a) Third glass model when the smoothness parameter is $\eta_{13} = 0.0862$. (b) Third glass model when the smoothness parameter is $\eta_{23} = 0.1973$. (c) Third glass model when the smoothness parameter is $\eta_{33} = 0.2336$. (d) Third glass model when the smoothness parameter is $\eta_{43} = 0.1255$. (e) Third glass model when the smoothness parameter is $\eta_{53} = 0.0500$.



(a) (b) (c)

(d) (e)

Figure 13: (a) Forth glass model when the smoothness parameter is $\eta_{14} = 0.0640$. (b) Forth glass model when the smoothness parameter is $\eta_{24} = 0.2102$. (c) Forth glass model when the smoothness parameter is $\eta_{34} = 0.1489$. (d) Forth glass model when the smoothness parameter is $\eta_{44} = 0.0424$. (e) Forth glass model when the smoothness parameter is $\eta_{54} = 0.2400$.

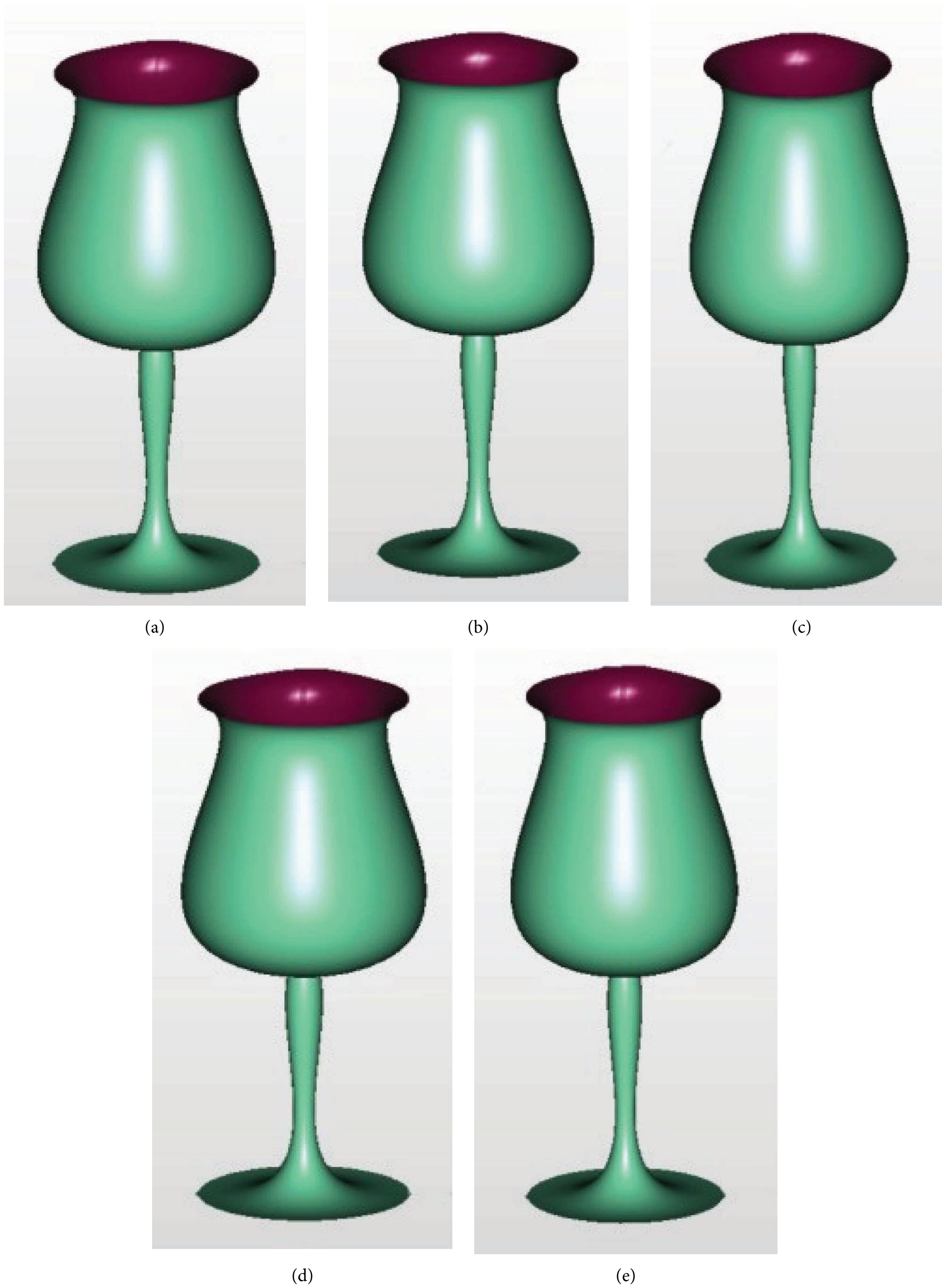


FIGURE 14: (a) Fifth glass model when the smoothness parameter is $\eta_{15} = 0.2163$. (b) Fifth glass model when the smoothness parameter is $\eta_{25} = 0.2365$. (c) Fifth glass model when the smoothness parameter is $\eta_{35} = 0.1741$. (d) Fifth glass model when the smoothness parameter is $\eta_{45} = 0.1494$. (e) Fifth glass model when the smoothness parameter is $\eta_{55} = 0.1192$.

in Figure 13(d), with η_{54}, is the selected glass from the glass design for G_4 shown in Figures 13(a)–13(e).

Fuzzy soft matrix for the last glass design according to the suggestions of the designers is as follows:

$$G_5 = \begin{bmatrix} 0.90 & 0.30 & 0.80 \\ 1.00 & 0.35 & 0.83 \\ 0.60 & 0.70 & 0.30 \\ 0.50 & 0.63 & 0.25 \\ 0.40 & 0.20 & 0.50 \end{bmatrix}. \quad (25)$$

Now, for this last fuzzy soft matrix G_5, WBFSM is as follows:

$$B_5 = \begin{bmatrix} 0.2163 \\ 0.2365 \\ 0.1741 \\ 0.1494 \\ 0.1192 \end{bmatrix}. \quad (26)$$

Here, $\eta_{15} = 0.2163$, $\eta_{25} = 0.2365$, $\eta_{35} = 0.1741$, $\eta_{45} = 0.1494$, and $\eta_{55} = 0.1192$. Putting these values for η_{mn} in equation (8) gives slightly different designs shown in Figures 14(a)–14(e):

$\eta_{25} > \eta_{15} > \eta_{35} > \eta_{45} > \eta_{55}$. Here, η_{25} has the greatest value, so it is our selected glass for G_5. The selected glass shown in Figure 14(b) has volume closer to 250 ml and finest quality.

Now, all the values of η_{mn} will generate a matrix such that

$$\begin{bmatrix} 0.1225 & 0.0982 & 0.0862 & 0.0640 & 0.2163 \\ 0.2437 & 0.2016 & 0.1973 & 0.2102 & 0.2365 \\ 0.1913 & 0.1631 & 0.2336 & 0.1489 & 0.1741 \\ 0.2102 & 0.1835 & 0.1255 & 0.0424 & 0.1494 \\ 0.1373 & 0.2485 & 0.0500 & 0.2400 & 0.1192 \end{bmatrix}. \quad (27)$$

A maximum value has been selected from each column of the abovementioned matrix, and by the definition of the optimum smoothness parameter, $\overline{\eta}_{mn}$ defined in Step 3 will be used, which is as follows:

$$\begin{aligned} \overline{\eta} &= \max(0.2437, 0.2485, 0.2336, 0.2400, 0.2365), \\ \overline{\eta} &= 0.2485. \end{aligned} \quad (28)$$

As $\overline{\eta} = 0.2485$, so the glass design generated from this value is the best design according to the customer's requirement. If the data of the glass designs mentioned previously are observed, it can be seen that the final selected design has the lowest price and finest quality among all the five designs. Also, the volume of the selected glass design is slightly less than 250 ml, which is in the favor of our customer. The shape of the designed glass is also favorable for fizzy drinks. According to the case study about glass design on drinking behavior discussed in [38], it can be concluded that the selected design has the best effect on the drinking behavior of a person since the selected glass design has the curve that makes it easy for a person to take a sip of his drink. Also, its shape does not allow the fizz in the drink to die down immediately, so a person can enjoy his drink for a long time. Hence, the selected glass design has the best result in the favor of the customer, and it also fully satisfies his requirements.

From the influence of each glass design on the drinking behavior of a person, it can be proved that the selected glass is the best choice for Mr. Khan. Looking at the design of G_1 glass, it can be seen that it is shaped like a parabola. It also has a wide mouth, which makes it easier for a person to drink from it. Although it is a good shape of glass to serve juices, but it is not an ideal shape for fizzy drinks since it does not hold the fizz in the drink for a long time. Due to its wide mouth and parabolic shape, the fizz in the drink dies down fast. The design of glass G_3 is like a cylinder. Although it has a curve in its shape, which makes it a good choice for fizzy drinks, but its mouth is so narrow which makes it difficult for a person to drink from it. Its narrow mouth and low volume does not make it a good choice for fizzy drinks, but it is a good choice for alcoholic drinks because these types of glasses reduce the consumption of alcoholic drinks which is also mentioned in a case study on drinking behaviors [38]. The next glass design G_4 is a V-shaped glass. As it can be seen in the figure mentioned previously, this glass has a wide mouth with no curve in its shape, which makes it a bad choice for fizzy drinks. The reason for this glass not being a good choice for fizzy drinks is the same as the design of glass G_1, but unlike glass design G_1, it has a small volume, which makes it a good choice for nonfizzy drinks taken in small quantity. They are also a good choice for small serving of juice for kids. The glass design G_5 has a design like a vase or a tulip flower. Although it has an attractive shape, its deep curve and narrow mouth does not make it a good choice for fizzy drinks. This type of glasses is best for serving milk drinks such as milk shakes or coconut milk. Now, on observing the selected design of the glass which is G_2 glass design, it is clear that its shape is the ideal shape for fizzy drinks. It has a right amount of curve to hold the fizz in the drink for a long time and its mouth is not narrow, which allow a person to drink from it easily and enjoy his drink. On observing all the designs of the glasses, it is proven that the selected glass not only has the best shape for fizzy drinks but also fulfills the requirements of the customer, Mr. Khan.

4. Conclusion

In this article, an intelligent system is developed for geometric designing of products according to industrial requirements. This intelligent system is developed using the fuzzy soft partial differential equation with weighted Bonferroni mean to design the geometric models of the required product. For this purpose, a parameter known as the optimum smoothness parameter is defined, using WBFSM, in the PDE. This parameter fulfills industrial requirements and generates the optimal design of the desired product. The overall shape of the surface is determined by the boundary conditions (which are effectively defined by curves in 3-space) at the edge of the surface patch. As a result, it is relatively simple for a designer to use this intelligent system to create objects of practical significance using standard computer devices. As this intelligent system uses close curves, rather than using standard shapes, such as ellipse, square, and cylinder, it is easier to shape it according to the need. Finally, to show the effectiveness of the developed technique, an illustrated example is developed for the hotel industry to design a drinking glass according to their requirements. From these geometric designs of the glasses, the design developed using the optimum smoothness parameter was selected, as it fulfilled the customer's requirements in the best way possible. All of this process was performed prior to the development of the product, which reduced the time and money spent on the development of the glass.

The developed intelligent system of PDE with weighted Bonferroni fuzzy soft matrix can be useful in many industries. In the future, this developed system can further be extended in the development of many products. It can be used in the development of capsule shells in pharmaceutical companies because temperature has a major effect on these capsule shells. Extreme temperature can melt them, so this PDE method can be used by taking temperature as the main parameter. Similarly, it can also be used in the manufacturing of robots, aircrafts, and home decor products.

Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Additional Points

Human and Animal Rights. We would like to mention that this article does not contain any studies with animals and does not involve any studies on human beings.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors equally conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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