Research Article

Coupling of a Nonlinear Structure with Sloshing

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In this paper the role of solid nonlinearity on the fluid–structure interaction (FSI) is studied. First a simple mass-spring-damper system with a third-degree spring is coupled with a linear sloshing system to study the effect of solid nonlinearity. It shows that even both systems show similar behavior in free vibration tests, but nonlinearity caused 50% less on the amplitude of vibration.

Then a higher order solid system coupled with a sloshing tuned liquid damper which is modeled by boundary element method (BEM) is examined. Finally, the coupling of a nonlinear solid with the sloshing system calculated at high-resolution method of smoothed particle is analyzed. The results reveal that considering surface nonlinearity in BEM has not improved the prediction significantly and the full NavierStokes calculations is in another phase and amplitude to a step response of the system even after two periods of the system in comparison with linear estimation. As shown the nonlinearity in solid part can cause a big difference in response of coupled system that pointed the necessity of the use of accurate fluid models such as BEM or smoothed particle hydrodynamics method as well as solid system identification before the coupling.

1. Introduction

The design, construction, and maintenance of fluid storage tanks, which are widely used to store a variety of liquids in industrial plants and oil complexes, including oil depots, etc., is of great importance. The structure nonlinearity is a classic problem. Due to the complexity of seismic effects and the effect of various seismic parameters, including its frequency content on the vulnerability of liquid reservoirs, reservoir systems need more detailed study and analysis under near and far earthquakes. The simplest coupling of structure would be with a particle.

Considering solid nonlinearity in fluid–structure interactions (FSIs) can provide a more accurate and realistic representation of the behavior of the system. Solid nonlinearity refers to the fact that the deformation of a solid material is not always directly proportional to the applied force or stress. This means that the response of a solid material to an external force can be nonlinear, which can have important effects on the behavior of the fluid-solid system. For example, if a solid material is subjected to a large deformation, its stiffness may decrease, which can affect the way it interacts with the surrounding fluid. Similarly, if the fluid flow is strong enough to cause large deformations in the solid material, this can also affect the fluid flow itself. By considering solid nonlinearity in FSIs, it is possible to better understand and predict the behavior of the system under different conditions. This can be particularly important in engineering applications where accurate predictions of FSIs are necessary for the design and optimization of systems such as pipelines, dams, and offshore structures.

Vito [1] analyzed the stability of vibrations of a particle in a plane constrained by spring. Stability of vibrations of a particle in a plane constrained by identical nonlinear springs studied by Vito [1]. He found the region of stability of a particle in a 2D problem coupled by nonlinear structure. Further details of that work are out of scope of current study. The coupling of structure with a fluid where the fluid modeled by an inviscid model was done by Warnitchai and Pinkaew [2]. Tuned liquid dampers (TLD) used in engineering structures, civil and environmental engineering, require the modeling of liquid sloshing in a 2D rectangular tank with flow-dampening devices. Figure 1 presents the schematic of
an effective mass-spring-damper model. In Figure 1(a), a coupled system of sloshing tank over a rigid body with a mass of $M$ that connected with a spring-damper system ($C$ is damping coefficient and $K$ is the stiffness) to the moving ground (with known motion of $X_g$ time function). In Figure 1(b), the sloshing system simplified by a rigid body with a mass of $m$ that connected with a spring-damper system ($c$ is damping coefficient and $k$ is the stiffness) to the base structure. The theories and methods of finding those constants are presented in literature [2]. Warnitchai and Pinkaew [2] found the state of the fluid in a coupled motion with linear assumption. In linear theory of sloshing, the velocity potential governed with the Laplace equation with nonpenetrating boundaries at the bottom of the tank, right and left walls. Linearized dynamic and kinematic free surface boundary conditions approximated by the walls. Linearized dynamic and kinematic free surface boundary conditions are the first order are the specific features of this theory. Solution of those equations leads to perturbation based natural sloshing modes and natural sloshing frequencies, perturbation elevation, and perturbation velocity potential. Table 2 presents the equations and solution of nonlinear sloshing theory.

For more complex geometry the numerical method presented by various researchers [4–10]. One of these methods is boundary element method (BEM) which used the green function solution. BEM is accurate in infinite domain problems due to the semianalytical nature and use of integrals used in method. As well that is more efficient in meshing due to the reduction of dimensions. Nakayama and Washizu [4] solved that nonlinear sloshing problem under the known external acceleration which was used as a benchmark solution of BEM analysis in many other works. BEM just focused on the boundaries of the problem. Another type of method is the finite element method (FEM) used extensively. FEM used a group of nodes over the domain and boundaries. For example, Ghattas and Li [5] developed a FEM enhanced by variational analysis for stationary nonlinear FSI. In the design of FSI system the fact that fluid motion affected by solid movement [2] or solid motion affected by fluid oscillation, the type of used method is difference. In their study, the effects of different frequency content of distant and near earthquakes by considering the horizontal components of earthquakes on the behavior of a rigid tank containing fluid with a constant height to diameter ratio have been studied. The general way is the two way (dual interaction of fluid motion and solid movement). Two-way FSI analysis is common in field of bioengineering in analysis of coronary artery bypass graft, pulsatile flow, and artery structure [6]. Another application of FSI is in transient analysis of sea–vessel interaction in underwater explosions. In order to simulating the responses of submerged structures to various underwater impacts, Zhang et al. [7] developed a smoothed particle hydrodynamics (SPH) shell combined with BEM method to discretize vessel surface structures, the second-order doubly asymptotic approximations of BEM. For this purpose, time history analysis has been used to dynamically analyze the tank. Reservoir and fluid modeling using SPH method has been done by considering the effects of structure and fluid interaction by the added mass method. They prove the compatibility of the used method with moving least square function. The dynamics of interaction between two quantum states in macroscopic quantum dynamics is like what happens in classical sloshing problem. Exact analytical study of Hamiltonian in a rescaled dimensionless time for resonance interactions of inviscid liquid sloshing waves is done by Pilipchuk [8]. He found analogies in nonlinear classical and macroscopic quantum self-trapping based on the Galerkin’s procedure in complete classification for nonlinear sloshing modes in a square-base tank.

The use of composite material in marine propeller is another application of nonlinear coupling of FSI which calculated with BEM, panel method, and FEM [9]. In the study of such hydroelastic problems the nonlinearity of the solid is important. Sometimes a simple fluid modeling is used to focus on nonlinearity of the solid. In such studies [10] the added-mass method is used to analyze the effect of presence
of fluid. But generally, in aeroelastic applications the evaluating of the local aerodynamic forces is required. Lee et al. [11] calculated distribution force on blade surface and wind pressure coefficients of various sections case for various wind speeds. For each velocity range they found another and classed them into the prestall (5–7 m/s), dynamic-stall (10–15 m/s), and deep-stall (20–25 m/s) regions. The results show that the elastic model used prevents the increase of fluctuations by changing the vibration frequency of the turbine and preventing the formation of rotational motion and reduces the base shear and the overturning anchor.

In the same manner in bioapplications usually both fluid (such as vitreous humor as a Burgers-type viscoelastic fluid) and solid (such as sclera and lens as hyperelastic solids) are modeled by nonlinear dynamics [12]. Their results indicate that there is a significant difference in the response of the reservoir-fluid system considering the simultaneous effects of the horizontal components of the applied. Arbitrary Lagrangian–Eulerian method usually used to solve that FSI problem [13]. Using a nonlinear model that simulates a soft spring oscillator where the soft spring differential equation is solved by the Poisson-Lindsted approximation method and a sequential approximation method, the researchers shows that there is a relation between the period and amplitude of the oscillator. Both analytical and experimental methods lead to similar results, which is that the structure period depends on the amplitude of vibration. Nevertheless, CFD method is more complex than BEM, but they can solve the transient fluid motion around the complex geometries [14].

Complex geometries [15] and uncertainty quantification [16] can cause nonlinearities as well. Corner sloshing (bidi- directional effects) also could be effective in nonlinear sloshing behaviors specially in shallow tanks. The main purpose of uncertainty quantification is to numerically investigate the effect of variables uncertainty on passive control of earthquake-induced sloshing phenomenon in concrete fluid storage tanks [15]. Boundary layer modification of shallow water wave theory for the continuity and momentum equations are presented in Table 3 which affects the rigid part coupling. To solve such nonlinearities FSI solution based on shearing in pipes was developed [17]. The recent traumatic experiences have shown that liquid storage tanks have either been destroyed or severely damaged during earthquakes around the world. Damage or destruction of sloshing tanks can lead

### Table 1: Linear sloshing theory.

<table>
<thead>
<tr>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity potential with the Laplace equation</td>
<td>( \nabla^2 \phi = 0</td>
</tr>
<tr>
<td>Bottom of the tank</td>
<td>( \frac{\partial \phi}{\partial t} = 0</td>
</tr>
<tr>
<td>Right and left walls</td>
<td>( \frac{\partial \phi}{\partial t} = 0</td>
</tr>
<tr>
<td>Dynamic free surface</td>
<td>( \frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
<tr>
<td>Kinematic free surface</td>
<td>( \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
<tr>
<td>First order free surface</td>
<td>( \frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
<tr>
<td>Natural sloshing modes</td>
<td>( \cos \left( \frac{\pi}{4} x \right) \cosh \left( \pi (z + h) / l \right)</td>
</tr>
<tr>
<td>Natural sloshing frequencies</td>
<td>( \sqrt{g \frac{h}{2} \tanh \left( \frac{\pi \phi}{L} \right)}</td>
</tr>
</tbody>
</table>
| Elevation                                        | \( \eta = \sum_{i=1}^{\infty} a_i(t) \cos \left( \frac{\pi \phi}{L} \right) \)
| Velocity potential                               | \( \phi = \sum_{i=1}^{\infty} a_i(t) \cos \left( \frac{\pi \phi}{L} \right) \cosh \left( \pi (z + h) / l \right) \)

### Table 2: Nonlinear sloshing theory.

<table>
<thead>
<tr>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity potential with the Laplace equation</td>
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<td>Bottom of the tank</td>
<td>( \frac{\partial \phi}{\partial t} = 0</td>
</tr>
<tr>
<td>Right and left walls</td>
<td>( \frac{\partial \phi}{\partial t} = 0</td>
</tr>
<tr>
<td>Mass continuity of free surface</td>
<td>( \int \rho \delta z dx = 0.</td>
</tr>
<tr>
<td>Dynamic free surface</td>
<td>( \frac{\partial \phi}{\partial t} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial z} \frac{\partial \phi}{\partial x} + g z + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
<tr>
<td>Kinematic free surface</td>
<td>( \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial z} \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} \frac{\partial \phi}{\partial x} + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
<tr>
<td>First order free surface</td>
<td>( \frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} + \left( x - \frac{1}{2} \right) \frac{\partial^2 \phi}{\partial x^2} = 0</td>
</tr>
</tbody>
</table>
| Second order free surface                        | \( \frac{\partial^2 \phi}{\partial x^2} + g \frac{\partial \phi}{\partial z} = g \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} - g \frac{\partial \phi}{\partial z} \frac{\partial \phi}{\partial x} - \eta_1 \frac{\partial \phi}{\partial \phi} - \eta_2 \frac{\partial^2 \phi}{\partial \phi^2} \)
| Perturbation potential                           | \( \phi = \phi_1 + \varepsilon \phi_2 |
| Perturbation elevation                           | \( \eta = \eta_1 + \varepsilon \eta_2 |
| Natural sloshing modes                           | \( \cos \left( \frac{\pi}{4} x \right) \cosh \left( \pi (z + h) / l \right) \)
Table 3: Boundary layer modification of shallow-water wave theory.

| Continuity | \( \frac{\partial u}{\partial t} + (1 - \tanh(\frac{x+u}{a}))^2 \frac{\partial u}{\partial x} + 1.1025 \frac{\partial^2 u}{\partial x^2} + \frac{4}{3} \tanh(\frac{x+u}{l}) \frac{\partial^2 u}{\partial x^2} = 0 \) |
|---------------------------------------------------------------|
| Momentum | \( \frac{\partial^2 u}{\partial t^2} + 1.1025 \frac{\partial^2 u}{\partial x^2} + \frac{4}{3} \tanh(\frac{x+u}{l}) \frac{\partial^2 u}{\partial x^2} = -1.3685 \left[ \frac{\partial^2 u}{\partial x^2} \right] \left[ 1 + \frac{x}{a} \right] u - \dot{x}_s \) |
| Rigid part coupling | \( \ddot{x}_s + 2\omega_0^2 \dot{x}_s + \omega_0^2 x_s = -g_d - \frac{\omega_0^2}{2\Omega} (\eta_t + h)^2 - (\eta_t + h)^2 \) |

Table 4: TLD modeling as a solid mass damper.

<table>
<thead>
<tr>
<th>Rigid part coupling</th>
<th>( [m_d, 0, 0, m_t][\dot{x}_d; \ddot{x}<em>s] + [c_d - c</em>{ld}; c_d + c_d][\ddot{x}_d; \dot{x}<em>s] + [k_d - k</em>{ld}; -k_d + k_d][x_d; x_s] = {0; F_s } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>( \xi_d = 0.5 A^{0.35} ) (A = amplitude of excitation/length of the tank)</td>
</tr>
<tr>
<td>Stiffness hardening ratio ( (k_d/m_d \omega_s^2) )</td>
<td>( \kappa = 1.075 A^{0.007}, A \leq 0.03 )</td>
</tr>
<tr>
<td></td>
<td>( \kappa = 2.52 A^{0.35}, A &gt; 0.03 )</td>
</tr>
</tbody>
</table>

to adverse events such as shortages and cuts in drinking water and wastewater, uncontrolled fires, and dangerous fluid leaks. Nowadays accurate solutions of 3D problems are interested [18]. Also, it was found that the response of the reservoir-fluid system to external loads such as earthquakes with different frequency content has a significant difference, so the researchers recommended considering the simultaneous effect of different components and different frequency content in the analysis and design of this type of reservoirs. New methods of SPH modeling of challenging FSI [18], flexible structures based on the continuum-based shell element [19], etc., are developed [20–28]. For this reason, many studies have been performed on the dynamic behavior of fluid capacitors [20]. For this purpose, the Navier–Stokes equations are solved along with the techniques of fluid volume method as well as the general method of the moving body and the effect of the plane perpendicular to the water surface on reducing the seismic response of harmonic stimulated reservoirs with different frequencies is investigated. Most of these studies have been performed on cylindrical tanks and very few studies have been performed on the seismic response of rectangular tanks [21]. Some of them need one way chase of solid motion because of fast loading such as earthquake-induced motions [20, 22] and some of them need optimization of FSI coupling constraints [21]. Designed tanks are not ideally constructed and will have a geometric error value. After ensuring the accuracy of the coding results in the software, the sum of the percentages of the deformations of the five vibrating modules as the initial deformation has been assigned to all models [23]. TLD modeling as a solid mass damper is presented in Table 4. Rigid part coupling here is based on the damping ratio (amplitude of excitation/length of the tank) and stiffness hardening ratio.

The lattice Boltzmann method (LBM) [23], SPH methods [24], ALE formulation [27], and high-order partitioned FSI framework [28] also shown their efficiencies on FSI calculations. Therefore, most of study is mainly related to linear elastic solids [29]. Therefore, the dynamic behavior of a solid capacitor under the influence of motion caused by an environment in three-dimensional space is studied based on linear analysis in the time domain [30]. Although the results of numerical studies were compared with the results of the laboratory model and validated but they have limited applications [31]. In some applications nonlinearity of the solid comes from its internal mechanisms [25] or over coupling by fluid motion [26, 29, 30]. A particle–spring approach is usually used for structural dynamics [1, 32]. The elastic constants of springs are related with the material mechanical properties [31]. Such spring has the ability of adding viscoelastic structural damping coefficient and the geometrical parameters such as cross-section area and rotary inertia of the flexible cylinder [33]. For example, the thin-walled shell structures have light weight and high strength. Shell structures have many applications in various fields [34]. Due to the small wall thickness of the shell structures, it is possible to create any deformation and disturbance on wall surfaces due to various errors during construction. The initial geometric imperfection on the amount of sloshing has been negligible [35]. Yu et al. [36] and Sun et al. [37] performed classical nonlinear works on sloshing while the nonlinearity refers to the fluid part. Jamalabadi [38, 39] solved the FSI problem in analytical sloshing, sloshing with the improved boundary layer approximation [40], sloshing with finite element [41, 42], LBM [43], and sloshing with SPH method [44]. In many of studies a linear elastic model is adopted for solid part and focus of accuracy is on fluid section [36–51].

The weakness of linear elasticity in structure displacement calculation is that it assumes that the deformation of a structure is proportional to the applied load, and that the material properties remain constant throughout the deformation. This assumption may not hold true for structures that undergo large deformations or are made of materials that exhibit nonlinear behavior, such as plastic deformation or creep. As a result, linear elasticity may underestimate the displacement and stress in such structures, leading to inaccurate predictions of their behavior and potentially compromising their safety. More literature survey for reviewing the literature on the recent papers about vibration control systems (e.g., TLD) and soil–structure interaction [52–56] could show the advantages of the considering solid nonlinearity in FSI estimation.

As stated above in many of research works a linear model for the solid part is assumed specially for the TLD devices. That assumption can cause the errors in coupled FSI systems.
large response to a small initial disturbance would be detected. The equations and analytical solution of common method in sloshing analysis are introduced in tables as linear sloshing equations (Table 1), nonlinear sloshing equations (Table 2), boundary layer modification of shallow water wave theory (Table 3), and TLD modeling as a solid mass damper (Table 4).

The BEM is a numerical computational method of the solving linear partial differential equations. For example, here Laplace’s equation represents the continuity of the velocity potential of the flow of an inviscid incompressible as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad (2)$$

with free surface boundary conditions of velocity potential obtained from Bernoulli’s equation for irrotational flow

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left( \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right) + (a_y + g) \eta + a_x x = 0, \quad (3)$$

and

$$\frac{\partial \phi}{\partial n} = n_y \frac{\partial \eta}{\partial t}. \quad (4)$$

The harmonic velocity potential is also subject to the non-penetration condition in other walls. To use the BEM, the velocity potential should be formulated as integral equations (i.e., in boundary integral form).

Green’s first identity is obtained by multiplying the Laplace equation by a twice-differentiable function. Green’s second identity is obtained by subtracting Green’s first identity with its transposed form (interchanging the roles of functions). As well the Green’s function of the Laplace equation satisfies Laplace’s equation everywhere except at an arbitrarily chosen point. At that point, the Green’s function is singular, that is, it takes an infinite. To solve Laplace equation using BEM, the Green’s theorem was applied resulting in

$$\frac{\alpha(x)}{2\pi} \phi + \oint_{\text{domain}} \frac{\partial \psi}{\partial n} \, ds = \int_{\text{domain}} \psi(x, y) \frac{\partial \phi}{\partial n} \, ds, \quad (5)$$

where \( r = (x, y) \) is an arbitrary point on the boundary, \( \alpha(x) \) is the angle between two tangent lines to \( x \) boundary node (for straight element is \( \pi/2\pi \)), and fundamental solution (\( \psi \)) for 2D problems is

$$\psi(x, y) = \frac{1}{2\pi} \ln \frac{1}{r}, \quad (6)$$

and the Green’s function for potential problem in 3D is \( 1/4\pi \). The above equation is exact due to the use of the Green’s function except the singularity of the Green’s
function. Green’s theorem is used to replace domain integrals to boundary integrals as follows:

\[
\frac{\partial(I)}{2\pi} \varphi(I) + \int_{\text{boundary}} \varphi \frac{\partial}{\partial n} \left( \frac{1}{2\pi \ln \frac{1}{r}} \right) ds
= \int_{\text{free surface}} n_r \frac{\partial}{\partial t} \left( \frac{1}{2\pi} \ln \frac{1}{r} \right) ds.
\]  

(7)

Velocity potential expanded over each element as a function of dimensionless distance and end point values \((k\) and \(k+1\)) as

\[
\varphi(\zeta) = \left[ 1 - \zeta \right] \begin{bmatrix} \varphi(k) \\ \varphi(k+1) \end{bmatrix},
\]

and free surface elevation as:

\[
\eta(\zeta) = \left[ 1 - \zeta \right] \begin{bmatrix} \eta(k) \\ \eta(k+1) \end{bmatrix}.
\]

(8)

(9)

For numerical integration of the above formula the Gauss–Legendre interpolation are used. As well the governing equation changed to the form of the single-layer and double-layer potentials at the elements midpoints integrated over the segments. That set up a linear system for the normal velocity.

\[
\frac{Dv_j}{Dt} = g - m_j \sum_j \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla W_{ij} + 0.03 h \sum_j \frac{(c_i + c_j)(v_i - v_j) \cdot r_{ij}}{(\rho_i + \rho_j)(r_{ij}^2 + 0.01 h^2)} \nabla W_{ij},
\]

(10)

(11)

where \(W\) is the kernel function (cubic spline kernel), \(P\) denote pressure, \(m\) is mass, \(\rho\) is density, \(c\) is speed of sound, \(v\) is velocity, \(r\) coordinates \((r_{ij} = r_i - r_j)\), and \(g\) acceleration of gravity. As well the pressure evaluated from [49]

\[
P_i = \rho_0 c_0^2 \frac{1}{\gamma} \left( \left( \frac{\rho_i}{\rho_0} \right)^\gamma - 1 \right),
\]

(12)

where \(\gamma = 7\), \(\rho_0 = 1.000 \text{ kg/m}^3\); \(c_0\) represents the speed of sound which is set as more than 10 times of the maximum speed of the flow field.

3. Results and Discussion

The limitations of nonlinear FSI are that it is a complex and computationally intensive process, which requires advanced mathematical models and numerical methods to accurately simulate. Additionally, the accuracy of the simulations is highly dependent on the quality of the input data and assumptions made during the modeling process. The hypothesis of nonlinear FSI is that the interaction between a fluid and a solid object can be accurately modeled using the nonlinear equations that account for the dynamic behavior of both the fluid and the solid. The construction of nonlinear FSI involves developing mathematical models that describe the behavior of both the fluid and the solid, as well as numerical methods for solving these equations. These models typically include a range of nonlinear effects, such as turbulence, viscosity, and non-Newtonian behavior, and may also account for factors such as fluid–solid contact and deformation. The resulting simulations can be used to analyze a wide range of phenomena, from the behavior of ocean waves to the performance of aircraft wings, and TLD devices for seismic dampers.

Figure 3 presents a comparison of the free vibration of linear and nonlinear solid.

As well for the SPH the by assumption of kernel and particle approximation, the density equation for any pair of interacting particles \((i \text{ and } j)\) is [49]

\[
\frac{D \rho_i}{Dt} = -\rho_i \sum_j (v_i - v_j) \cdot \nabla W_{ij} v_j,
\]

and the momentum equation is expressed as follows [50]:

\[
\frac{Dv_i}{Dt} = g - m_i \sum_j \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla W_{ij} + 0.03 h \sum_j \frac{(c_i + c_j)(v_i - v_j) \cdot r_{ij}}{(\rho_i + \rho_j)(r_{ij}^2 + 0.01 h^2)} \nabla W_{ij},
\]
(natural logarithm of the ratio of any two successive amplitudes) are both systems are same the test could be failed in analysis of those systems. The application of just frequency and damping factor in detection made a similar result.

Figure 4(a) presents the BEM discretization used here. To validate the BEM code used here, a horizontal motion with a sinusoidal function with amplitude of 2 mm and time argument of 5.5 is used \( (a_x = -0.0605 \sin (5.5t)) \). Figure 4(b) presents the BEM validation. As shown the current method is in agreement with the previous published benchmark for the displacement of the contact node of free surface and right wall.

### Table 5: Parameters of the linear and nonlinear model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>0.464</td>
<td>Tank length</td>
</tr>
<tr>
<td>( m_f )</td>
<td>5.64</td>
<td>Fluid mass</td>
</tr>
<tr>
<td>( k )</td>
<td>2,424.7</td>
<td>Stiffness</td>
</tr>
<tr>
<td>( m_s )</td>
<td>564</td>
<td>Structure mass</td>
</tr>
<tr>
<td>( c_s )</td>
<td>9.45</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>242.47</td>
<td>Stiffness</td>
</tr>
<tr>
<td>( h )</td>
<td>0.304</td>
<td>Fluid height</td>
</tr>
<tr>
<td>( w )</td>
<td>0.08</td>
<td>Tank width</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>1,000</td>
<td>Fluid density</td>
</tr>
</tbody>
</table>

![Figure 4: (a) BEM discretization and (b) BEM validation.](image)

![Figure 5: Comparison of the coupled vibration of linear and nonlinear solid.](image)
Figure 5 shows the comparison of the coupled vibration of linear and nonlinear solid with sloshing. As shown the coupled solid system with linear assumptions cause higher estimation of displacement of ground motion (TLD tank position). It shows that even both systems show similar behavior in free vibration tests, but nonlinearity caused 50% less on the amplitude of vibration. The higher the amplitude, the higher the energy. To sum up, if we neglect the drift of frequency because of nonlinearity, the lower the amplitude means the more energy absorbed in structure. The energy transported by an oscillator is directly proportional to the square of the amplitude. So whatever change occurs in the amplitude, the square of that effect impacts the energy. This means that two-thirds of the amplitude results in a four-ninth of the energy.
For a better understanding of such behavior the higher accuracy modeling of sloshing system is done by SPH. The details of system parameters are presented in Table 5. Figure 6 illustrates that the use of SPH method made exact free surface profiles. As shown system because of limited height of the tank liquid experiences the wave breaking which cannot be modeled with simple mass-spring models. As well some of surface particles experience the splashing from free surface to the air, which also cannot be modeled by BEM models.

Finally, the comparison of all results is compared in Figure 7 that reveals the comparison of the various methods. As shown BEM results have a better accuracy than Yu et al. [36]. The results reveal that considering surface nonlinearity in BEM has not improved the prediction significantly. As well the real system is in another phase and half amplitude after two seconds in comparison with linear estimation. As shown the nonlinearity in solid part can cause a big difference in the response of coupled system that pointed the necessity of the use of accurate fluid models such as BEM or SPH method as well as solid system identification before the coupling.

4. Conclusion

In this paper the role of solid nonlinearity on the FSI is studied. First a simple mass-spring-damper system with a third-degree spring is coupled with a linear sloshing system to study the effect of solid nonlinearity. It shows that even both systems show similar behavior in free vibration tests, but nonlinearity caused 50% less on the amplitude of vibration in nonlinear case which equals to more than half of energy absorbed in linear vibration. Then a higher order solid system coupled with a sloshing TLD which is modeled by BEM is examined. Finally, the coupling of a nonlinear solid with the sloshing system calculated at high-resolution method of smoothed particle is analyzed. The results reveal that considering surface nonlinearity in BEM has not improved the prediction significantly. As well the real system is in another phase and half amplitude after 2 s in comparison with linear estimation. As shown the nonlinearity in solid part can cause a big difference in response of coupled system that pointed the necessity of the use of accurate fluid models such as BEM or SPH method as well as solid system identification before the coupling.

Data Availability

This study contains no extra or additional data.

Conflicts of Interest

The author declares that there is no conflicts of interest.

References


