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Research Article

Multicriteria Ordered the Profile Clustering Algorithm Based on PROMETHEE and Fuzzy c-Means

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The purpose of multicriteria clustering is to locate groups of alternatives that have comparable qualities and have been examined across multiple criteria. An ordered profile clustering is a well-known problem, and the fuzzy c-means clustering (FCM) technique is one of the most broadly used in every field of life. At present, FCM is for the partitioning of information into numerous clusters which are still lacking priority relations. To address the problem of finding ranking in clusters based on multicriteria in the fuzzy environment, we propose a multicriteria ordered clustering algorithm based on the partial net outranking flow of the preference organization for enrichment evaluations method (PROMETHEE) and fuzzy c-means. Lastly, we apply the proposed algorithm to solve a real-world targeted clustering problem regarding the human development indexes. To test the efficacy of the proposed algorithm, a comparative analysis of ordered K-means clustering (OKM) and FCM is carried out with it.

1. Introduction

A classic challenge in multicriteria clustering is supervised categorization or allocating options to predefined classes. To be more specific, this subject has gotten a lot of interest from sectors like machine learning in every field [1], pattern recognition clustering [2], data retrieval [3], data mining [4], the clinical [5], sales [6], health [7], operational systems [8], and humanistic systems performance evaluation [9]. The primary concept of clustering, according to the clustering goal, is to divide a collection of data into a specified number of clusters (categories, groups, or subsets) with high similarity within each cluster.

Furthermore, several researchers have created innovative and outstanding methodologies in the framework of MCDA, including ELECTRE-SORT and TRI [10], Flowsort, a clustering algorithm [11], PROAFTN [12], UTADIS [13], PAIRCLASS [14], and so on. These approaches, in general, presuppose that the classifications are specified prior to a collection of options and their central or limit profiles.

The difficulty with supervised classification is that the groups are occasionally unknown a priori, i.e., no data structure information has been provided. Various solutions have recently been developed based on a multicriteria approach to deal with the difficulty of classifying related possibilities into undefined categories [15]. This area highlights three sorts of problems in clustering: relational, nonrelational Boujelben [16] and ordered Meyer and Olteanu [17]. Complete ordered clustering, in particular, may aid in the formation of targeting ordered clusters and the establishment of priority linkages among a subset of alternates for each cluster, making it an advantageous addition to the ranking process. Furthermore, multicriteria ordered clustering enables the display of created ranks in terms of their complexity, such as rankings of the foreign universities, economy, and the human development index (HDI), among other things.

Even though several statistical and mathematical methods, such as cluster analysis [18] and regression trees [19], may be used to find ordered clusters in a multicriteria environment, they have substantial drawbacks, such as the mischaracterization of independent/dependent variables and the removal of critical aspects [20]. To rank groupings of alternatives regarding a country's risk, De Smet et al.'s [21] initially created an enhanced and optimized PROMETHEE approach. Then, using the inconsistency matrix and the relations between pairwise preferences, they improved on their previous work and proposed an accurate technique for identifying a complete ordered partition [20]. On the other hand, De Smet et al.'s techniques partially used the structure of the data and measured the ordinal profile of the paired preference degrees [22].

Consequently, Chen et al. [22] have created an ordered K-means clustering algorithm (OKM) by combining the PROMETHEE method's partial net outranking flow with the K-means clustering algorithm. It has been observed that OKM produced a more robust and consistent outcome in HDI rankings. OKM has two primary advantages due to the partial net outranking flow: (1) it examines the weight of each criterion; and (2) it consists of the preferences in the same cluster of all the alternatives. Later, Liu et al. [23] presented a multicriterion ordered clustering algorithm based on PROMETHEE and K-Medoids clustering algorithms. The larger the cluster number, the better the results. However, this may not be appropriate for data in real-time with a large number of features and objects. As a result, a diagram method approach has been developed based on the net outranking of PROMETHEE and fuzzy c-means (OFCM) [24]. Consequently, we proposed a multi-criterion ordered profile clustering algorithm (MOPFCM) by utilizing the partial net outranking flow of PROMETHEE and fuzzy c-means, which not only provides the ordered clusters but also addresses the aforementioned problem through their centroids. In addition to that, earlier ordered clustering algorithms rewarded little attention to the fuzziness of each cluster's alternatives because they could resolve uncertainty in real-world situations.

The fuzzy c-means (FCM) algorithm is widely recognized as one of the most prominent fuzzy clustering algorithms 25]. Its ability to handle uncertainty in practical scenarios and ease of explanation has made it a popular choice. Over the years, researchers have extensively studied this innovative method, and its effectiveness has been further enhanced, as demonstrated by Wang et al. [26]. In addition, Zheng et al. [27] proposed a universal FCM and a hierarchical FCM to address image noise issues. Xu and Wu [28] present an extension of the fuzzy c-means approach to an intuitionistic fuzzy environment. Beg and Rashid [29] propose a modified dissimilarity measure for fuzzy data that is proposed and applied to real data with mixed feature variables. By employing relative entropy to optimize cluster dissimilarity, Zarinbal et al. [30] enhanced the goal. On the other hand, the current FCM algorithms are best suited to problems involving categorizing alternatives into predefined categories with no links between them. Decision-makers (DMs) in the field of MCDA may desire to receive "ordered clusters" with targeted ranking information between the clusters.

The goal of this article is to build on previous work and present a new multicriteria targeted rank clustering algorithm. Motivated by the partial net outranking flow/profile of the PROMETHEE [31] and the FCM, we propose a multicriteria-ordered profile clustering algorithm. MOPFCM first randomly assigns the membership values, then generates the cluster centroids, updates the membership values, and minimizes the partial net outranking flow of all rank clusters through the construction of an optimization model.

The rest of the paper is arranged as follows: Section 2 overviews the fuzzy c-means clustering algorithm and partial net outranking flow of the classical PROMETHEE methods. In Section 3, we propose an optimization model of MOPFCM based on the profile of PROMETHEE. In Section 4, to show the applicability and implementation procedure of the suggested method, we solved the human development index problem. This section also includes a comparison with other ordered clustering algorithms. Finally, Section 5 contains the paper's conclusion.

2. Preliminaries

This section focusing on the review of the FCM and the partial net outranking flow of the PROMETHEE methods is as follows.

2.1. Fuzzy c-Means Clustering Algorithm. To solve the clustering challenge, Bezdek [25] presented the classical fuzzy c-means clustering (FCM). The following objective function was determined by FCM to get the clustering result with fuzzy membership μ_{ij} and cluster centroid $\widehat{\mathbf{V}}_i$ cluster centroid:

$$J_{\mathrm{FCM}} = \sum_{\mathbf{i}=1}^{\mathbf{c}} \sum_{\mathbf{j}=\mathrm{sent}1}^{\mathbf{l}} \mu_{ij}^{\mathbf{m}} \left\| \widehat{\mathbf{x}}_{\mathbf{j}} - \widehat{\mathbf{V}}_{\mathbf{i}} \right\|, \tag{1}$$

where c is the cluster number of n objects, m is the weighting exponent ($m \in [1.5,\ 3.0]$ is determined to be analytically acceptable [32]), $\widehat{\mathbf{x}}_{\mathbf{j}}$ is an object of the set $X = \left\{ \widehat{\mathbf{x}}_{\mathbf{j}} \mid j = 1, 2, \ldots, l \right\} \subseteq \mathbb{R}^q$, q and l is the number of dimensions, and $\widehat{\mathbf{x}}_{\mathbf{j}}$ is the number of objects, respectively. Whereas, indicates the distance norm applied by FCM in Algorithm 1 [25]:

2.2. PROMETHEE Methods and Its Partial Net Outranking Flow. The following subsection introduces the PROMETHEE method's core ideas and the partial net outranking flow (see Brans and Mareschal [33] for more details). Let us consider the following multicriteria problem:

$$\max \left\{ g_1(a), g_2(a), \ldots, g_j(a), \ldots, g_k(a) \mid a \in A \right\}, \tag{2}$$

where A is a finite set of possible alternatives $\{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ and $G = \left\{ g_1(.), g_2(.), \ldots, g_j(.), \ldots, g_l(.) \right\}$ is a set of evaluation criteria, in which are may be consider maximized or to be minimized. The decision-maker expects to identify an alternative that optimizing all the criteria. Let $g_l(a_m)$ denotes the alternative a_m

Step 1: Randomly initialize memberships μ_{ij} of $\hat{\mathbf{x}}_\mathbf{j}$ belonging to cluster i, and $\sum_{i=1}^{l}\mu_{ij} = 1$.
Step 2: Centroid $\hat{\mathbf{V}}_\mathbf{i}$ of FCM is calculated as $\hat{\mathbf{V}}_\mathbf{i} = \sum_{j=1}^{l}(\mu_{ij})^m \hat{\mathbf{x}}_\mathbf{j} / \sum_{J=1}^{l}(\mu_{ij})^m$.
Step 3: Update μ_{ij} based on $\mu_{ij} = (1/\hat{\mathbf{x}}_\mathbf{j} - \hat{\mathbf{V}}_\mathbf{i})^{(1/m-1)} / \sum_{i=1}^{c}(1/\hat{\mathbf{x}}_\mathbf{j} - \hat{\mathbf{V}}_\mathbf{i})^{(1/m-1)}$.
Step 4: Steps 2 & 3 are repeated till the convergence of J_{FCM} i.e., J_{FCM} has negligible variations. Finally, we get the cluster results comprising of objects $\{\hat{\mathbf{x}}_\mathbf{j} \mid j = 1, 2, \ldots, l\}$ i.e., $C_k\,(k = 1, 2, \ldots, c)$.

Algorithm 1: Fuzzy c-mean clustering algorithm.

evaluation with respect to the criteria g_l The preference function $p_l(a_i, a_j)$, which is apply on the deviation $d_l(a_i, a_j)$. The preference function gives the degree of preference of the alternatives a_i to a_j with respect to the criterion g_l In PROMETHEE, six types of preference functions are can be utilized [33]. We can write preference function as:

$$P_l(a_i, a_j) = G_l(d_l(a_i, a_j)); \forall a_i, a_j \in A, \quad \text{Where } d_l(a_i, a_j) = g_l(a_i) - g_l(a_j), \tag{3}$$

and $G_l(.)$ Is a function which is monotonically non-decreasing and varying 0 to 1. $\forall a_i, a_j \in A$, we have

$$\pi(a_i, a_j) = \sum_{l=1}^{m} w_l P_l(a_i, a_j), \tag{4}$$

where $\pi(a_i, a_j)$ denotes the total preference of the alternative a_i over the a_j when all criteria are taken into consideration. Whereas, w_l is the relative weight of the criterion g_l from the set of all criteria.

Similarly,

$$\pi_{gl}(a_i, a_j) = \sum_{l=1}^{m} w_{gl} P_{gl}(a_i, a_j), \tag{5}$$

where $\pi_{gl}(a_i, a_j)$ indicates the profile (partially degree of preference) of the alternative a_i over the alternative a_j captivating into account of single criteria.

In order to acquire the ranking of all the alternatives, positive, negative and total net outranking flow were established by [33] as follows:

$$\begin{aligned} \varnothing^+(a_i) &= \frac{1}{m-1}\sum_{x\in A\setminus\{a_i\}} \pi(a_i, x), \\ \varnothing^-(a_i) &= \frac{1}{m-1}\sum_{x\in A\setminus\{a_i\}} \pi(x, a_i), \\ \varnothing(a_i) &= \varnothing^+(a_i) - \varnothing^-(a_i). \end{aligned} \tag{6}$$

Similarly, the partial positive outranking, negative outranking, and net outranking flows are given:

$$\begin{aligned} \varphi_{gl}^+(a_i) &= \frac{1}{m-1}\sum_{x\in A\setminus\{a_i\}} \pi_{gl}(a_i, x), \\ \varphi_{gl}^-(a_i) &= \frac{1}{m-1}\sum_{x\in A\setminus\{a_i\}} \pi_{gl}(x, a_i). \end{aligned} \tag{7}$$

Therefore,

$$\varnothing_{gl}(a_i) = \sum_{l=1}^{m}\varphi_{gl}^+(a_i) - \sum_{l=1}^{m}\varphi_{gl}^-(a_i). \tag{8}$$

Net outranking flow is

$$\varphi(a_i) = \sum_{l}^{m}\sum_{i}^{n}\varphi_{gl}(a_i), \tag{9}$$

where the partial positive flow $\varphi_{gl}^+(a_i)$ indicates how much the alternatives a_i prefers to the rest of all other alternatives in a single criterion. The larger $\varphi_{gl}^+(a_i)$ the better the alternatives a_i Similarly, the partial negative flow $\varphi_{gl}^-(a_i)$ indicates how much the alternatives are preferred by all other alternatives a_i in a single criterion. The smaller $\varphi_{gl}^-(a_i)$, the better the alternatives a_i. Usually, the larger the partial net outranking flow $\varnothing(a_i)$, the better the alternatives a_i.

3. Multicriteria Ordered Clustering Algorithm

We deal with a unique form of clustering problem called as ranks clustering in this study, which was first addressed by [21] for the nation risk rating problem. Identifying the ordered clusters can help the DM sort the possibilities, as previously noted. Unlike typical clustering problems, the ordered clustering problem separates the alternatives into a predetermined number of groups and also contains a complete ordering relationship between these clusters.

Let $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\} \subseteq \mathbb{R}^m$ be a sample set is assessing through a set of criteria $G = \{g_1, g_2, \ldots, g_i, \ldots, g_m\}$. We call a partition an ordered partition if it meets the following criteria:

(i) $A = \cup_{i=1,2,\ldots,K}\hat{C}_i$;
(ii) $\forall_{i\neq j}, \hat{C}_i \cap \hat{C}_j = \varphi$;
(iii) $\hat{C}_1 \succ \hat{C}_2 \succ \cdots \hat{C}_K$

where $\hat{C}_i$ signifies the i^{th} ordered cluster and $\succ$ indicating the priority relation among the clusters, such as if $\hat{C}_i \succ \hat{C}_j$, then the elements in $\hat{C}_i$ are better than $\hat{C}_j$.

By employing the Euclidean norm to assess similarity, FCM has been frequently utilized to cluster the alternatives with respect to their criteria. However, Euclidean norm cannot take into account the relative importance of the criteria being assessed. The PROMETHEE approach can determine the priority degree for each pair of alternatives based on their differences. As a result, we offer a novel supervised clustering technique based on partial net outranking, a multicriteria ordered clustering algorithm based on partial net outranking, and fuzzy c-means (MOPFCM). This algorithm will search for the best c-ordered partition of alternatives built on the FCM in composition with the partial net outranking of PROMETHEE.

3.1. Minimum Partial Net Outranking Flow Objective Function. Similar to the FCM clustering algorithm, we propose an objective function based on partial net outranking flow of PROMETHEE to minimize:

$$\widehat{J}_{\mathrm{MOPFCM}}(U,V)=\sum_{i=1}^{K}\sum_{a_j\in C_i}\left(\mu_{ij}\right)^m\left|\varphi_{gl}^{C_i}(a_i)\right|^2, \tag{10}$$

where C_i is the group of alternatives in i^{th} cluster of the ordered clustering and

$$\varphi_{gl}^{C_k}(a_i)=\frac{\left(\mu_{ij}\right)^m}{\left|C_k\right|}\left(\sum_{a_i\in C_k}\pi_{gl}\left(a_i,a_j\right)-\sum_{a_i\in C_k}\pi_{gl}\left(a_j,a_i\right)\right). \tag{11}$$

With the fuzzy centroids θ_{gl}^i is

$$\theta_{gl}^i=\frac{\sum_{i=1}^n\left(\mu_{ij}\right)^m\varphi_{gl}(a_i)}{\sum_{i=1}^n\left(\mu_{ij}\right)^m}. \tag{12}$$

To capture the similarity of alternatives with respect to criteria and reconstruct the optimization model (10) for clustering of the alternatives, we propose the partial net outranking flow. The suggested model offers the following important benefits over the traditional FCM clustering algorithm: (1) $\widehat{J}_{\mathrm{MOFCM}}$ takes the relative importance of each criterion into account. (2) each criterion partial net outranking flow considers the preferences of all the alternatives in the same cluster; (3) centroids of partial net outranking clustering are used to classify the clusters and criteria; and (4) the net outranking will be used to rank the alternatives within and between clusters.

Theorem 1. *Clustering based on partial net outranking converges to local minima of $\widehat{J}_{MOFCM}$ in the finite iterations.*

Proof. In the PROMETHEE method, there are six kinds of preference functions [33]. These functions are monotonically increasing or decreasing and the partial net outranking flow $\varnothing_{gl}(a_i)$ for the alternatives a_i is one-to-one and converges in MOPFCM. Bezdek et al. [32] has previously mathematically validate the convergence of FCM and consequently, convergence of local optimal solution of $\widehat{J}_{\mathrm{MOFCM}}$. □

Property 1. Suppose that, $A=\{a_1,a_2,\ldots,a_i,\ldots,a_n\}$ be the set of alternatives having fuzzy centroids $\theta_{gl}^i\in\mathbb{R}^m$ for $i=1,2,3,\ldots,C$ of each cluster C_i, the membership μ_{ij} of the alternative a_j belonging to C_i, and $\widehat{J}_{\mathrm{MOPFCM}}(U,V)$ is the objective function of MOPFCM. If $\widehat{J}_{\mathrm{MOPFCM}}(U,V)$is <1, then there will be a distinct partitions of A.

Proof. According to Algorithm 1 and (12), we have

$$\begin{aligned}\theta_{gl}^i&=\frac{\sum_{i=1}^n\left(\mu_{ij}\right)^m\varphi_{gl}^{C_i}(a_i)}{\sum_{i=1}^n\left(\mu_{ij}\right)^m}\geq\frac{\sum_{i=1}^n\left(\mu_{ij}\right)^m\varphi_{\min(gl)}^{C_i}(A)}{\sum_{i=1}^n\left(\mu_{ij}\right)^m}=\varphi_{\min(gl)}^{C_i}(A),\\ \theta_{gl}^i&=\frac{\sum_{i=1}^n\left(\mu_{ij}\right)^m\varphi_{gl}^{C_i}(a_i)}{\sum_{i=1}^n\left(\mu_{ij}\right)^m}\geq\frac{\sum_{i=1}^n\left(\mu_{ij}\right)^m\varphi_{\max(gl)}^{C_i}(A)}{\sum_{i=1}^n\left(\mu_{ij}\right)^m}=\varphi_{\max(gl)}^{C_i}(A),\end{aligned} \tag{13}$$

where, $\varphi_{\min(gl)}^{C_i}(A)$ and $\varphi_{\max(gl)}^{C_i}(A)$ represents the maximal and minimal of partial net outranking flows $\varphi_{gl}^{C_i}(a_i)$ respectively. □

3.2. Update the Cluster. The shortest Euclidean distance is being used for updating the membership of clusters in the conventional FCM techniques. Only the actual distance between alternatives is taken into account by the Euclidean distance. But it is not appropriate for MCDM's targeting ordered clustering of criterion. The partial net outranking flow can identify the relative relevance of each choice for the cluster. We attempt to figure out the relationship between the partial net outranking and the cluster center that corresponds to it, and then we get the result. From (11), we have

$$\begin{aligned}
\varphi_{gl}^{C_k}(a_i) &= \frac{(\mu_{ij})^m}{|C_k|}\left(\sum_{a_i\in C_k}\pi_{gl}(a_i,a_j)-\sum_{a_i\in C_k}\pi_{gl}(a_j,a_i)\right),\\
\varphi_{gl}^{C_k}(a_i) &= \frac{(\mu_{ij})^m}{|C_k|}\left(\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}P_{gl}(a_i,a_j)-\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}P_{gl}(a_j,a_i)\right),\\
\varphi_{gl}^{C_k}(a_i) &= \frac{(\mu_{ij})^m}{|C_k|}\left(\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}G_{gl}(d_{gl}(a_i)-d_{gl}(a_j))-\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}G_{gl}(d_{gl}(a_j)-d_{gl}(a_i))\right),\\
\varphi_{gl}^{C_k}(a_i) &= \frac{(\mu_{ij})^m}{|C_k|}\left(\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}G_{gl}(d_{gl}(a_i)-d_{gl}(\theta_{gl}^i)+d_{gl}(\theta_{gl}^i)-d_{gl}(a_j))-\sum_{a_i\in C_k}\sum_{l=1}^{m}w_{gl}G_{gl}(d_{gl}(a_j)-d_{gl}(\theta_{gl}^i)+d_{gl}(\theta_{gl}^i)-d_{gl}(a_i))\right),
\end{aligned}\tag{14}$$

where $d_{gl}(a_i)$ denotes the calculation of the alternatives a_i with respect to the criteria gl, and $\theta_{gl}^{i}\varepsilon\mathbb{R}^m$ represents the center of the k^{th} cluster. If the function $G_{gl}(.)$ is a linear, then

$$\varphi_{gl}^{C_k}(a_i) = \varphi_{gl}^{\theta_{gl}^i}(a_i) + \varphi_{gl}^{C_k}(\theta_{gl}^i), \tag{15}$$

where θ_{gl}^i represents the center of the k^{th} cluster C_k and

$$\varphi_{gl}^{\theta_{gl}^i}(a_i) = \frac{(\mu_{ij})^m}{|C_k|}\left(\sum_{a_i\in C_k}\pi_{gl}(a_i,\theta_{gl}^i)-\sum_{a_i\in C_k}\pi_{gl}(\theta_{gl}^i,a_i)\right). \tag{16}$$

Brans and Vincke [34] presented six types of specific preference functions which are linear (functions except the 6^{th} (Gaussian criterion). Consequently, the conversion can be established in transformation. If the linear preference functions are applied on a big data set, we can see $\varphi_{gl}^{C_k}(a_i) \approx \varphi_{gl}^{\theta_{gl}^i}(a_i) + \varphi_{gl}^{C_k}(\theta_{gl}^i)$. Where, θ_{gl}^i is the cluster center to denote the relevant cluster and calculate the distance between cluster's center and the alternatives.

3.3. Update the Centroids. In classical fuzzy c-means clustering algorithm we used Algorithm 1 to calculate the fuzzy centroids and update the membership value. To apprehend the ordered centroids of the clustering, the cluster centers can be obtained by

$$\theta_{gl}^i = \arg\min\left|\varphi_{gl}^{C_k}(\theta_i)\right|^2, i = 1,2,\ldots,n, \tag{17}$$

where $\varphi_{gl}^{C_k}(\theta_i)$ represents the partial net outranking flow of data and $\theta_{gl}^i\varepsilon\,\mathbb{R}^m$ can be computed using (10).

3.4. Classification of Ordered Clusters and Criteria. Centroids of clusters $\theta_{gl}^{C_i}\in\mathbb{R}^m$ are representing the middle value of all the alternatives lies within the clusters. These centroids are being used for classification of different levels of clusters and their corresponding criteria.

3.5. Ranking of Alternatives within and between the Clusters. Assign ranks to each clustered alternatives based on net outranking within and between the clusters i.e., (Cluster #, Rank of $\varphi_{gl}^{C_k}(a_i)$, Rank of $\varnothing(a_i)$).

3.6. The Proposed Algorithm. (Algorithm 2)

4. Case Study and Comparative Analysis

In this section, we apply the MOPFCM on real-life situation problem associated with the Human Development Index (HDI) adapted from [20, 22] to validate the efficiency of MOPFCM. The United Nations Development Program (UNDP) ranks the 179 countries in the HDI ranking based on three criteria i.e., $G = \{g_1, g_2, g_3\}$ = {life expectancy, education, income index}. We are not concerned with the precise ranking problem of nations in this section, and instead divide the countries according to the three criteria. Our goal is to use a targeted rank-based regrouping method that takes partial net outranking of all three criteria into consideration, then compare the results to the OKM to verify the MOPFCM. Moreover, our framework not only ranks the clusters but also ranks the alternative within-cluster and between the clusters. In the end, cluster centroids of partial net outranking are being used for classification of ordered clusters and their criteria.

4.1. MOPFCM Algorithm for Regrouping on the Basis of Partial Net Outranking Flow. This subsection demonstrates how it will implement for clustering the countries (see Appendix of [20]) based on their performance in aforementioned three criteria for the year 2008. Let $A = \{A = \{a_i \mid \text{i} = 1, 2, \ldots, \text{n}\}$ represented as alternatives of all countries against the three criteria $G = \{g_1, g_2, g_3\}$. The a_i links to the g^{th} place of country in ranking of HDI. Step-by-step implementation process for ordered clustering built on Algorithm 2 is stated as follows.

The process of the MOPFCM is as follows:
Step 1: Compute the partial net outranking flow $\varphi_{gl}(a_i)$ and $\varphi(a_i)$ net outranking.
Step 2: Randomly initialize membership of $\varphi_{gl}^{C_k}(a_i)$ belonging to the cluster i.
Step 3: Compute the fuzzy centroids θ_{gl}^{i} $\theta_{gl}^{i} = \sum_{i=1}^{n}(\mu_{ij})^m \varphi_{gl}(a_i)/\sum_{i=1}^{n}(\mu_{ij})^m$.
Step 4: Targeted rank the clusters based on the fuzzy centroids θ_{gl}^{i} of clusters. For example, if $\theta_{gl}^{i} \succ \theta_{gl}^{j}$ then $\varphi_{gl}^{C_i}(a_i) \succ \varphi_{gl}^{C_j}(a_i)$, where$\succ$ is determined by net outranking flow.
Step 5: update the μ_{ij} based on $\mu_{ij} = (1/\|\varphi_{gl}^{C_k}(a_i) - \theta_{gl}^{i}\|)^{(1/m-1)}/(1/\sum_{i=1}^{c}\|\varphi_{gl}^{C_k}(a_i) - \theta_{gl}^{i}\|)^{(1/m-1)}$.
Step 6: Step 3 and 4 are repeating till the convergence of $\widehat{J}_{\text{MOPFCM}}$ in (10) i.e., $\widehat{J}_{\text{MOPFCM}}$ has negligible change.
Step 7: Centroids of clusters $\theta_{gl}^{C_i} \in \mathcal{R}^m$ are being used for classification of ordered clusters and their profiles.
Step 8: Assign ranks to each clustered alternatives based on net outranking within and between the clusters i.e., (Cluster #, Rank of $\varphi_{gl}^{C_k}(a_i)$, Rank of $\varnothing(a_i)$).

Algorithm 2: Multicriteria ordered profile fuzzy c-means clustering algorithm.

Step 1: Calculate preference degree of each criterion $\pi_{gl}(a_i, a_j)$ between each pair of alternatives and compute the partial net outranking flow $\varnothing_{gl}(a_i)$ and net outranking flow $\varnothing(a_i)$ of each country using equations (8) and (9). We select the same linear preference function for each criterion as mentioned in [20]:

$$f_k(v) = \begin{cases} 0, & v \le 0, \\ \dfrac{v}{p_l}, & 0 \le v \ge p_l, \quad l = 1, 2, 3, \\ 1, & v \ge p_l, \end{cases} \tag{18}$$

where threshold value p_l and the corresponding weights of each criterion has been determined in Table 1.

Then, for each criterion we can find the preferences between two alternatives are shown in Figures 1–3 and their partial net outranking $\varnothing_{gl}(a_i)$ for i $= 1, 2, \ldots, 179$ is obtained as shown in Figure 4.

Step 2: Randomly initialize membership of $\varphi_{gl}^{C_k}(a_i)$ that belongs to the cluster i.

Step 3: By using equation (12) compute cluster centroids θ_{gl}^{i} of each cluster and let $m = 2$.

Step 4: Based on partial net outranking flow, compute the targeted ranked clusters based on the fuzzy centroids θ_{gl}^{i} of each cluster. Such as, if $\theta_{gl}^{i} \succ \theta_{gl}^{j}$ then $\varphi_{gl}^{C_i}(a_i) \succ \varphi_{gl}^{C_j}(a_i)$, where $\succ$ is determined by the net outranking flow.

Step 5: Update the memberships μ_{ij} of $\varphi_{gl}^{C_k}(a_i)$ using the Step 5 of Algorithm 2.

Step 6: Steps 3 and 4 are repeated till the convergence of $\widehat{J}_{\text{MOPFCM}}$ i.e., $|\widehat{J}_{\text{MOPFCM}}^{t} - \widehat{J}_{\text{MOPFCM}}^{t-1}| \le \varepsilon$, where t indicates the iteration and $\varepsilon = 0.001$.

Step 7: Classification of ordered clusters with respect to centroids of clusters $\theta_{gl}^{C_i} \in \mathcal{R}^m$.

Step 8: Assign ranks to each clustered alternatives based on net outranking within and between the clusters i.e., (Cluster # , Rank of $\varphi_{gl}^{C_k}(a_i)$, Rank of $\varnothing(a_i)$) see Table 2.

In order to assess the ordering in the HDI problem, 4 clusters are pre requests which is justified later by De Smet et al. [20, 22]. Figure 5 validate the aforementioned claim by drifting of the total sum of all country's' partial net outranking flow in respect of the cluster numbers. As a result, we select 4 clusters, which are equal to very high, high, medium, and low human-developed countries. Then, we extant the ordered clustering results got from MOPFCM in Figure 6. The x-axis indicates the HDI aggregate scores for the 179 nations, whereas the y-axis indicates the number of clusters are 4. MOPFCM results are very similar with HDI ranks and OKM and De Semet et al. [20]. First 52 countries are belonging to cluster very high human development index, 74 country belongs to high human development index, 30 countries are belonging to medium level human development index, whereas 23 are belonging to low level human development index as shown in Figure 6. All alternatives are assigned ordered ranking i.e., cluster number, ranking within cluster and ranking among the clusters. For example, the status of QATAR is (1, 34, 36) which it belongs to very high human development countries having 34$^{\text{th}}$ position among 52 and 36$^{\text{th}}$ position among 179 countries. In summary, Table 2 shows the overall rankings of nations as well as their grouping using various clustering approaches.

4.2. Comparison of MPOFCM with Other Clustering Algorithms with FCM and OKM. We compare the outcomes of MOPFCM with the traditional FCM and the OKM for addressing the identical HDI issue as indicated above in order to further verify our suggested clustering technique. We apply a Python "FCM" module to group the alternatives that use the conventional FCM clustering technique based on three different HDI criteria (see Table 2), showing that the partition result and HDI ranking are incompatible. The fundamental reason for this might be that the classic FCM uses the Euclidean distance to evaluate the degree of similarity between any two alternatives. In other words, the typical FCM is unable to offer preference correlations between alternatives and clusters owing to the symmetry of the Euclidean distance.

Table 1: Normalized data based, indifference, weight and preference thresholds of each criterion.

Parameters	Life expectancy	Adult literacy index	GDP
Strict preference threshold: p_l	0.704	0.719	0.828
Indifference threshold: q_l	0	0	0
Weight of criteria: w_l	0.333	0.333	0.333

Table 2: Ranking of alternatives based on net outranking within and between the clusters, i.e., (Cluster #, Rank of $\varphi_{gl}^{C_k}(a_i)$, Rank of $\varnothing(a_i)$) is given.

Countries	FCM	MOPFCM	OKM
Iceland	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
Norway	(1, 2, 2)	(1, 2, 2)	(1, 2, 2)
Canada	(1, 3, 3)	(1, 3, 3)	(1, 3, 3)
Australia	(1, 4, 4)	(1, 4, 4)	(1, 4, 4)
Ireland	(1, 5, 5)	(1, 6, 6)	(1, 6, 6)
Netherland	(1, 6, 6)	(1, 7, 7)	(1, 7, 7)
Sweden	(1, 7, 7)	(1, 5, 5)	(1, 5, 5)
Japan	(1, 8, 8)	(1, 8, 8)	(1, 8, 8)
Luxembourg	(1, 8, 8)	(1, 11, 11)	(1, 11, 11)
Switzerland	(1, 10, 10)	(1, 10, 10)	(1, 10, 10)
France	(1, 11, 11)	(1, 9, 9)	(1, 9, 9)
Finland	(1, 12, 12)	(1, 12, 12)	(1, 12, 12)
Denmark	(1, 13, 13)	(1, 13, 13)	(1, 13, 13)
Austria	(1, 14, 14)	(1, 14, 14)	(1, 14, 14)
United States	(1, 14, 14)	(1, 16, 16)	(1, 16, 16)
Spain	(1, 16, 16)	(1, 15, 15)	(1, 15, 15)
Belgium	(1, 17, 17)	(1, 17, 17)	(1, 17, 17)
Greece	(1, 18, 18)	(1, 18, 18)	(1, 18, 18)
Italy	(1, 19, 19)	(1, 20, 20)	(1, 20, 20)
New Zealand	(1, 20, 20)	(1, 19, 19)	(1, 19, 19)
United Kingdom	(1, 22, 22)	(1, 21, 21)	(1, 21, 21)
Hong Kong, China (SAR)	(1, 21, 21)	(1, 22, 22)	(1, 22, 22)
Germany	(1, 23, 23)	(1, 23, 23)	(1, 23, 23)
Israel	(1, 24, 24)	(1, 24, 24)	(1, 24, 24)
Korea (Republic of)	(1, 25, 25)	(1, 25, 25)	(1, 25, 25)
Slovenia	(1, 26, 26)	(1, 26, 26)	(1, 26, 26)
Brunei Darussalam	(1, 27, 27)	(1, 27, 27)	(1, 27, 27)
Singapore	(1, 28, 28)	(1, 28, 28)	(1, 28, 28)
Kuwait	(1, 29, 29)	(1, 30, 30)	(1, 30, 30)
Cyprus	(1, 30, 30)	(1, 29, 29)	(1, 29, 29)
United Arab Emirates	(1, 31, 31)	(1, 32, 32)	(1, 32, 32)
Bahrain	(1, 32, 32)	(1, 33, 33)	(1, 33, 33)
Portugal	(1, 33, 33)	(1, 31, 31)	(1, 31, 31)
Qatar	(1, 34, 34)	(1, 36, 36)	(1, 36, 36)
Czech Republic	(1, 35, 35)	(1, 34, 34)	(1, 34, 34)
Malta	(1, 36, 36)	(1, 35, 35)	(1, 35, 35)
Barbados	(1, 37, 37)	(1, 37, 37)	(1, 37, 37)
Hungary	(1, 39, 39)	(1, 39, 39)	(1, 39, 39)
Poland	(1, 40, 40)	(1, 40, 40)	(1, 40, 40)
Chile	(1, 41, 41)	(1, 38, 38)	(1, 38, 38)
Slovakia	(1, 38, 38)	(1, 46, 47)	(1, 48, 47)
Estonia	(1, 42, 42)	(1, 41, 41)	(1, 41, 41)
Lithuania	(1, 43, 43)	(1, 42, 42)	(1, 42, 42)
Latvia	(1, 44, 44)	(1, 43, 66)	(1, 43, 66)
Croatia	(1, 45, 45)	(1, 44, 44)	(1, 45, 44)
Argentina	(1, 46, 46)	(1, 45, 45)	(1, 46, 45)
Uruguay	(2, 1, 47)	(1, 1, 43)	(1, 44, 43)
Cuba	(2, 2, 48)	(1, 2, 46)	(1, 47, 46)
Bahamas	(1, 47, 49)	(1, 47, 48)	(1, 49, 48)
Costa Rica	(2, 3, 50)	(1, 3, 49)	(1, 50, 49)
Mexico	(2, 4, 51)	(1, 4, 50)	(1, 51, 50)
Libyan Arab Jamahiriya	(1, 48, 52)	(1, 48, 51)	(1, 52, 51)

Table 2: Continued.

Countries	FCM	MOPFCM	OKM
Oman	(1, 49, 53)	(1, 49, 52)	(2, 1, 52)
Seychelles	(1, 51, 55)	(1, 50, 55)	(2, 4, 55)
Saudi Arabia	(1, 50, 54)	(1, 51, 56)	(2, 5, 56)
Bulgaria	(2, 5, 56)	(2, 5, 53)	(2, 2, 53)
Trinidad and Tobago	(1, 52, 57)	(1, 52, 58)	(2, 7, 58)
Panama	(2, 6, 58)	(2, 6, 54)	(2, 3, 54)
Antigua and Barbuda	(1, 53, 59)	(1, 53, 71)	(2, 19, 71)
Saint Kitts and Nevis	(2, 7, 60)	(2, 7, 57)	(2, 6, 57)
Venezuela (Bolivarian Republic of)	(2, 8, 61)	(2, 8, 59)	(2, 8, 59)
Romania	(2, 9, 62)	(2, 9, 60)	(2, 9, 60)
Malaysia	(2, 10, 63)	(2, 12, 63)	(2, 12, 63)
Montenegro	(2, 11, 64)	(2, 10, 61)	(2, 10, 61)
Serbia	(2, 12, 65)	(2, 11, 62)	(2, 11, 62)
Saint Lucia	(2, 13, 66)	(2, 13, 64)	(2, 13, 64)
Belarus	(2, 14, 67)	(2, 14, 65)	(2, 14, 65)
Macedonia (TFYR)	(2, 15, 68)	(2, 16, 68)	(2, 16, 68)
Albania	(2, 16, 69)	(2, 15, 67)	(2, 15, 67)
Brazil	(2, 17, 70)	(2, 18, 70)	(2, 18, 70)
Kazakhstan	(2, 17, 70)	(2, 19, 72)	(2, 20, 72)
Ecuador	(2, 19, 72)	(2, 17, 69)	(2, 17, 69)
Russian Federation	(2, 20, 73)	(2, 21, 74)	(2, 22, 74)
Mauritius	(2, 21, 74)	(2, 22, 75)	(2, 23, 75)
Bosnia and Herzegovina	(2, 22, 75)	(2, 20, 73)	(2, 21, 73)
Turkey	(2, 23, 76)	(2, 24, 76)	(2, 25, 76)
Dominican Republic	(2, 24, 77)	(2, 23, 88)	(2, 24, 88)
Lebanon	(2, 25, 78)	(2, 25, 77)	(2, 26, 77)
Peru	(2, 26, 79)	(2, 27, 79)	(2, 28, 79)
Colombia	(2, 27, 80)	(2, 26, 78)	(2, 27, 78)
Thailand	(2, 28, 81)	(2, 29, 81)	(2, 30, 81)
Ukraine	(2, 29, 82)	(2, 28, 80)	(2, 29, 80)
Armenia	(2, 31, 84)	(2, 30, 82)	(2, 31, 82)
Iran (Islamic Republic of)	(2, 30, 83)	(2, 32, 84)	(2, 33, 84)
Tonga	(2, 32, 85)	(2, 31, 83)	(2, 32, 83)
Grenada	(2, 32, 85)	(2, 45, 98)	(2, 46, 98)
Jamaica	(2, 34, 87)	(2, 33, 85)	(2, 34, 85)
Belize	(2, 43, 96)	(2, 42, 95)	(2, 43, 95)
Suriname	(2, 35, 88)	(2, 35, 87)	(2, 36, 87)
Jordan	(2, 36, 89)	(2, 34, 86)	(2, 35, 86)
Dominican Republic	(2, 37, 90)	(2, 36, 88)	(2, 37, 88)
Saint Vincent and the Grenadines	(2, 38, 91)	(2, 38, 90)	(2, 39, 90)
Georgia	(2, 39, 92)	(2, 37, 91)	(2, 38, 91)
China	(2, 40, 93)	(2, 39, 92)	(2, 40, 92)
Tunisia	(2, 41, 94)	(2, 41, 94)	(2, 42, 94)
Samoa	(2, 42, 95)	(2, 40, 93)	(2, 41, 93)
Azerbaijan	(2, 44, 97)	(2, 43, 96)	(2, 44, 96)
Paraguay	(2, 45, 98)	(2, 44, 97)	(2, 45, 97)
Maldives	(2, 46, 99)	(2, 46, 99)	(2, 47, 99)
Algeria	(2, 47, 100)	(2, 49, 102)	(2, 50, 102)
El Salvador	(2, 48, 101)	(2, 48, 101)	(2, 49, 101)
Philippines	(2, 49, 102)	(2, 47, 100)	(2, 48, 100)
Fiji	(2, 50, 103)	(2, 51, 104)	(2, 52, 104)
Sri Lanka	(2, 51, 104)	(2, 50, 103)	(2, 51, 103)
Syrian Arab Republic	(2, 52, 105)	(2, 52, 105)	(2, 53, 105)
Occupied Palestinian Territories	(2, 53, 106)	(2, 53, 106)	(2, 54, 106)
Gabon	(2, 54, 107)	(2, 61, 114)	(2, 62, 114)
Turkmenistan	(2, 55, 108)	(2, 56, 109)	(2, 57, 109)
Indonesia	(2, 56, 109)	(2, 54, 107)	(2, 55, 107)
Guyana	(2, 57, 110)	(2, 55, 108)	(2, 56, 108)
Bolivia	(2, 58, 111)	(2, 60, 113)	(2, 61, 113)
Mongolia	(2, 59, 112)	(2, 58, 112)	(2, 59, 112)

Table 2: Continued.

Countries	FCM	MOPFCM	OKM
Moldova	(2, 60, 113)	(2, 59, 111)	(2, 60, 111)
Viet Nam	(2, 61, 114)	(2, 57, 110)	(2, 58, 110)
Equatorial Guinea	(2, 62, 115)	(2, 67, 120)	(2, 68, 120)
Egypt	(2, 63, 116)	(2, 62, 115)	(2, 63, 115)
Honduras	(2, 64, 117)	(2, 63, 116)	(2, 64, 116)
Cape Verde	(2, 65, 118)	(2, 64, 117)	(2, 65, 117)
Uzbekistan	(2, 66, 119)	(2, 65, 118)	(2, 66, 118)
Nicaragua	(2, 67, 120)	(2, 66, 119)	(2, 67, 119)
Guatemala	(2, 68, 121)	(2, 69, 122)	(2, 70, 122)
Kyrgyzstan	(2, 69, 122)	(2, 68, 121)	(2, 69, 121)
Vanuatu	(2, 70, 123)	(2, 71, 124)	(2, 72, 124)
Tajikistan	(2, 71, 124)	(2, 70, 123)	(2, 71, 123)
South Africa	(2, 72, 125)	(2, 72, 125)	(2, 73, 125)
Botswana	(2, 73, 126)	(2, 73, 126)	(2, 74, 126)
Morocco	(3, 1, 127)	(3, 2, 128)	(3, 2, 128)
Sao Tome and Principe	(3, 2, 128)	(3, 1, 127)	(3, 1, 127)
Namibia	(3, 3, 129)	(3, 3, 129)	(3, 3, 129)
Congo	(3, 4, 130)	(3, 4, 130)	(3, 4, 130)
Bhutan	(3, 5, 131)	(3, 6, 132)	(3, 6, 132)
India	(3, 9, 135)	(3, 9, 135)	(3, 9, 135)
Lao People's Democratic Republic	(3, 6, 132)	(3, 5, 131)	(3, 5, 131)
Solomon Islands	(3, 7, 133)	(3, 7, 133)	(3, 7, 133)
Myanmar	(3, 8, 134)	(3, 8, 134)	(3, 8, 134)
Cambodia	(3, 10, 136)	(3, 10, 136)	(3, 10, 136)
Comoros	(3, 11, 137)	(3, 11, 137)	(3, 11, 137)
Yemen	(3, 12, 138)	(3, 12, 138)	(3, 12, 138)
Pakistan	(3, 13, 139)	(3, 13, 139)	(3, 13, 139)
Mauritania	(3, 14, 140)	(3, 14, 140)	(3, 14, 140)
Swaziland	(3, 15, 141)	(3, 27, 153)	(3, 27, 153)
Ghana	(3, 16, 142)	(3, 16, 142)	(3, 16, 142)
Madagascar	(3, 17, 143)	(3, 15, 141)	(3, 15, 141)
Kenya	(3, 18, 144)	(3, 18, 144)	(3, 18, 144)
Nepal	(3, 19, 145)	(3, 17, 143)	(3, 17, 143)
Sudan	(3, 20, 146)	(3, 21, 147)	(3, 21, 147)
Bangladesh	(3, 21, 147)	(3, 19, 145)	(3, 19, 145)
Haiti	(3, 22, 148)	(3, 20, 146)	(3, 20, 146)
Papua New Guinea	(3, 23, 149)	(3, 22, 148)	(3, 22, 148)
Cameroon	(3, 24, 150)	(3, 23, 149)	(3, 23, 149)
Djibouti	(3, 25, 151)	(3, 24, 150)	(3, 24, 150)
Tanzania (United Republic of)	(3, 26, 152)	(3, 25, 151)	(3, 25, 151)
Senegal	(3, 27, 153)	(3, 26, 152)	(3, 26, 152)
Nigeria	(4, 1, 154)	(4, 2, 155)	(4, 2, 155)
Lesotho	(4, 2, 155)	(4, 3, 156)	(4, 3, 156)
Uganda	(4, 3, 156)	(4, 1, 154)	(4, 1, 154)
Angola	(4, 4, 157)	(4, 4, 159)	(4, 4, 159)
Timor-Leste	(3, 28, 158)	(3, 28, 157)	(3, 28, 157)
Togo	(3, 29, 159)	(3, 29, 158)	(3, 29, 158)
Gambia	(4, 5, 160)	(4, 5, 160)	(3, 30, 160)
Benin	(4, 6, 161)	(4, 7, 162)	(4, 6, 162)
Malawi	(4, 7, 162)	(4, 6, 161)	(4, 5, 161)
Zambia	(4, 8, 163)	(4, 8, 163)	(4, 7, 163)
Eritrea	(4, 9, 164)	(4, 9, 164)	(4, 8, 164)
Rwanda	(4, 10, 165)	(4, 10, 165)	(4, 9, 165)
Cote d'Ivoire	(4, 11, 166)	(4, 11, 166)	(4, 10, 166)
Guinea	(4, 12, 167)	(4, 12, 167)	(4, 11, 167)
Mali	(4, 13, 168)	(4, 14, 169)	(4, 13, 169)
Ethiopia	(4, 14, 169)	(4, 13, 168)	(4, 12, 168)
Chad	(4, 15, 170)	(4, 17, 172)	(4, 16, 172)
Guinea-Bissau	(4, 16, 171)	(4, 16, 171)	(4, 15, 171)
Burundi	(4, 17, 172)	(4, 15, 170)	(4, 14, 170)

Table 2: Continued.

Countries	FCM	MOPFCM	OKM
Burkina Faso	(4, 18, 173)	(4, 19, 174)	(4, 18, 174)
Niger	(4, 19, 174)	(4, 18, 173)	(4, 17, 173)
Mozambique	(4, 20, 175)	(4, 22, 177)	(4, 21, 177)
Liberia	(4, 21, 176)	(4, 20, 175)	(4, 19, 175)
Congo (Democratic Republic of the)	(4, 22, 177)	(4, 21, 176)	(4, 20, 176)
Central African Republic	(4, 23, 178)	(4, 23, 178)	(4, 22, 178)
Sierra Leone	(4, 24, 179)	(4, 24, 179)	(4, 23, 179)

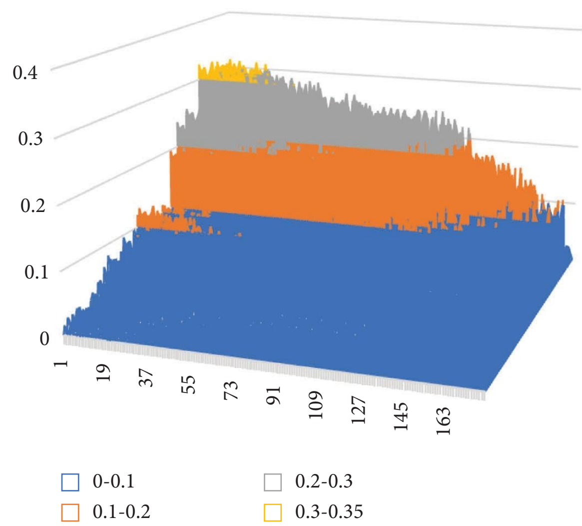

Figure 1: The pairwise preference degree of life expectancy for each pair among 179 countries. The yellow color shows highest preference degree and the blue color indicates the low preference degree in targeting number of intervals.

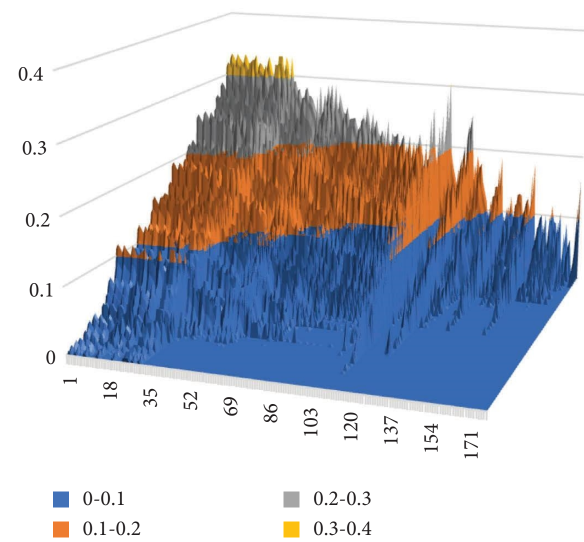

Figure 3: The pairwise preference degree of GDP for each pair among 179 countries. The Yellow color shows a high preference degree and blue color indicates a low preference degree in targeting the number of intervals.

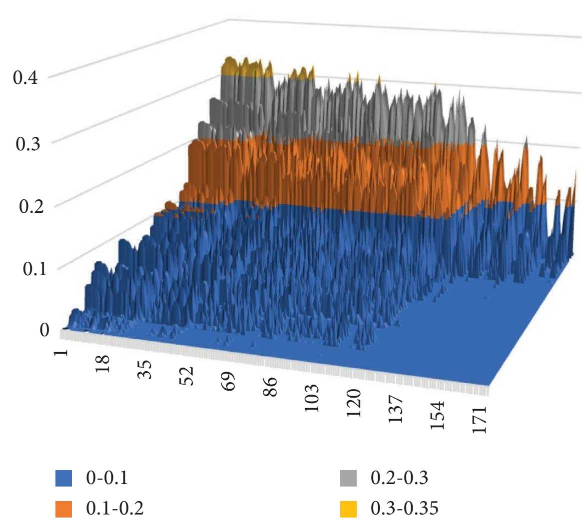

Figure 2: The pairwise preference degree of adult literacy index for each pair among 179 countries. The yellow color shows a high preference degree and the blue color indicates the low preference degree in targeting the number of intervals.

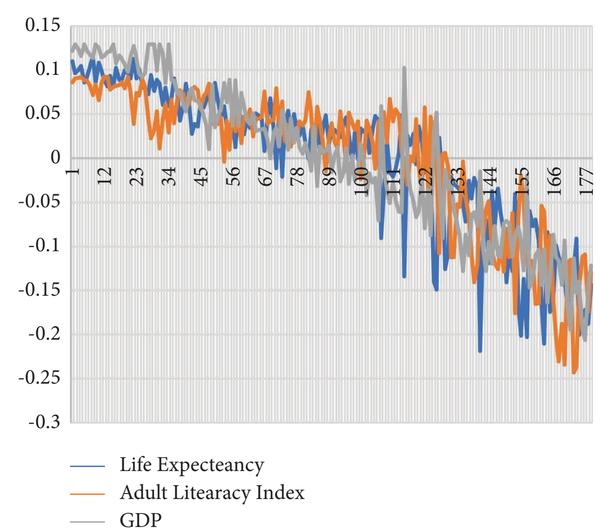

Figure 4: Partial net outranking's of life expectancy, adult literacy & GDP of 179 countries are showing very close moments with each other.

By using the Python "K-Means" package, we use the OKM clustering technique to regroup the nations-based partial net out rankings of three unique HDI criteria see Table 2, which show that the results are similar to the suggested MOPFCM and the partitioning results of HDI-ranking. The fundamental reason for this might be

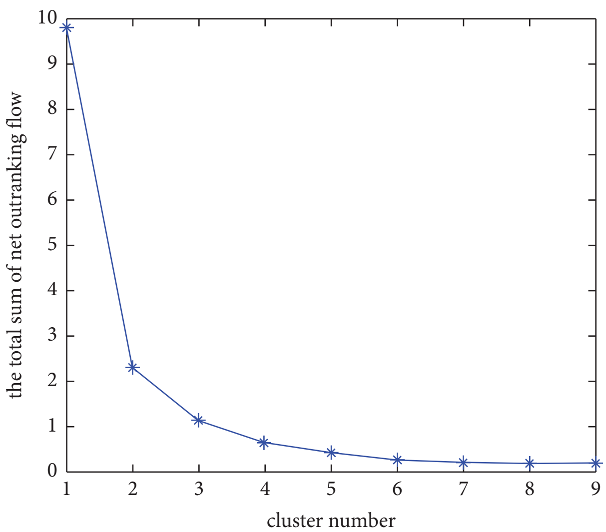

Figure 5: The sum of partial net outranking flows is proportional to the number of clusters.

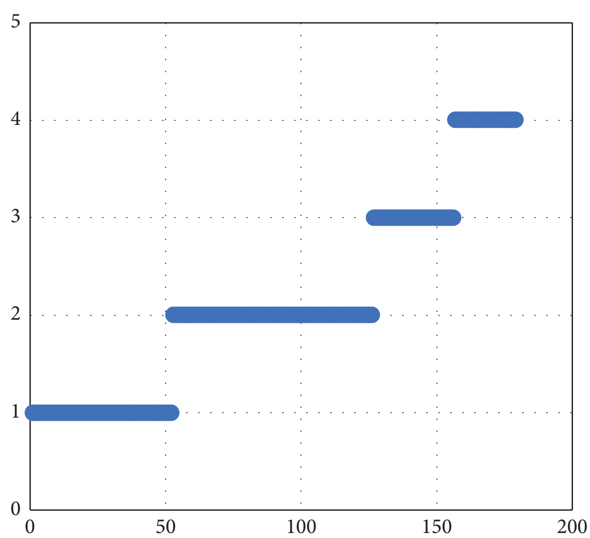

Figure 6: The x-axis represents the HDI ranking and the y-axis represents the its number of clusters.

because we estimate the similarity of any two nations based on their preference profile. In other words, preference correlations between items and clusters are provided by partial net outranking the symmetry of the Euclidean distance.

4.3. Analysis of MOPFCM Based on Centroids. The Ward minimum variance clustering technique is being used to create a double dendrogram heatmap based on four MOPFCM centroids. Rows and columns are converted into similar groups in this heat map based on their ordered centroids. Where, c_1 is the first cluster which is "Very High Human Development Index", c_2 "High Human Development Index", c_3 is "Medium Human Development Index" and c_4 is "Low Human Development Index"

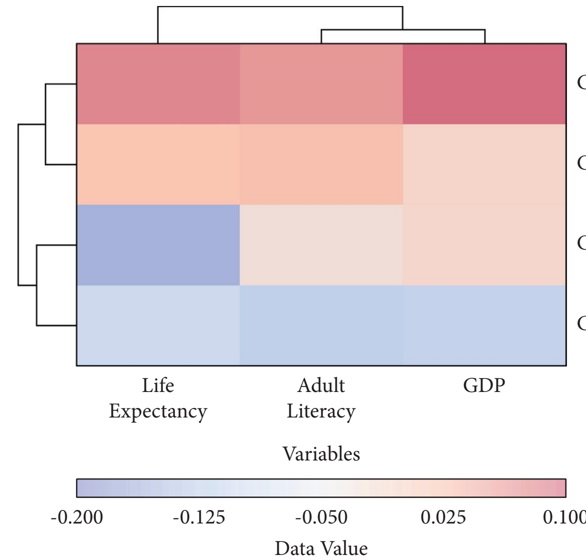

Figure 7: Double dendrogram heatmap have been applied on centroids of 4 clusters.

countries. It has been observed that, very high human development countries have low results in adult literacy. Whereas, a group of countries belongs to high human development have low GDP and group of countries belongs to medium level index have very low results in life expectancy as shown in Figure 7.

In summary, we present the multi-criteria ordered clustering algorithm for rankings of countries based on the partial and total net outranking in fuzzy environment. Rank the clustering results between the clusters and among the 179 based on total net outranking (cluster number, ranking within the cluster, overall ranking) as shown in Appendix A. The MOPFCM segmentation is compatible with OKM's ranks. The boundaries between the various clusters are adequate because they split developing, developed, and undeveloped countries in proportions that are appropriate.

5. Conclusion

In this research, we present MOPFCM, a targeted ranked clustering model based on partial net outranking and fuzzy c-means (FCM) clustering algorithm, to handle multi-criteria ordered clustering. The partial net outranking flow\ profile in the PROMETHEE approach differs from the classical FCM utilizing Euclidean norms. Several significant MOPFCM features are also theoretically supported. The human development index (HDI) issue has proved the usefulness of MOPFCM. Meanwhile, the ordered K-means (OKM) clustering technique and the traditional FCM clustering algorithm are given for comparison. The findings of the targeted rank clustering show that MOPFCM not only assists decision-makers in ranking clusters, but also in obtaining alternate rankings inside clusters and among all based on total net outranking. Based on the clustering and validation results, the advantages of MOPFCM can be summarized as follows:

(i) MOPFCM takes the weight of each criterion into account.
(ii) Each criterion profile reflects the alternatives preferences in the same cluster.
(iii) Centroids of the ordered profile clustering clarify the reasons that are being included in the cluster i.e., low, medium, high, and very high level of that particular group of countries.
(iv) Based on clustering results and total net outranking, complete ranking results are presented for clarification of the position of alternatives within the cluster and between the clusters.

As a result, we will use MOPFCM to execute big data clustering in a much quicker iterative manner in the future. More study is being investigated to see how the nonlinear preference function in PROMETHEE might be used to improve performance.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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