Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 5297688, 16 pages
https://doi.org/10.1155/2023/5297688

Research Article

Research on Optimization of Air Defense Firepower Configuration for Warship Cooperating with Strategic Location on the Sea

W. F. Zhao, H. J. Sun, M. Li, and K. N. Teng

Naval Aviation University, Yantai, 2H J Sun 64001, Shandong, China

Correspondence should be addressed to H. J. Sun; 14118420@bjtu.edu.cn

Received 5 August 2022; Revised 15 September 2022; Accepted 3 October 2022; Published 20 April 2023

Academic Editor: Dinesh Kumar Saini

Aiming at the problem of optimizing the firepower configuration for warships cooperating with strategic locations on the sea, the influence of air defense elements on firepower configuration is analyzed comprehensively, and the constraints of firepower configuration are put forward. Combined with factors such as the forward distance of the warship, the depth of the defense, the angle of the combat sector, and the direction of the enemy's attack, based on the coastal defense forces and the warship, the firepower configuration models of air defense at a strategic location on the sea for a single warship and two warships' coordinated air defense at important places are established, respectively. The forward range of a single warship and two warships' cooperative air defense at sea is determined, and the defense depth expectation calculation formula is given. And according to the configuration of air defense firepower of the warship cooperating with strategic locations on the sea, the warship cooperating with maritime key area defense effectiveness model is constructed by using the random service system theory. Finally, under the hypothetical conditions, the numerical simulation experiments show that the proposed model and calculation formula are reasonable and effective, which can provide a certain theoretical basis for the firepower configuration of warships cooperating with strategic locations on the sea.

1. Introduction

Strategic locations on the sea are often far away from the mainland, with small areas, scattered locations, harsh environments, and limited natural resources, but of significant potential military value [1]. The main combat modes of offensive and defensive operations at a strategic location on the sea are as follows: under the unified command of airborne early-warning aircraft; guided by enemy space-based satellites and long-range reconnaissance aircraft; covered by electromagnetic suppression of electronic warfare aircraft, the troops used various combat weapons (such as manned/unmanned attack aircraft and warships, submarine-launched long-range cruise missiles, precision-guided bombs, and shore-based medium- and short-range tactical missiles/long-range rockets) to carry out multidirectional, multistage, and multiform saturation attack. One of the bottlenecks of air defense/antimissile operations at a strategic location on the sea is the independent operation of a single strategic location on the sea. Because of the short defense depth, the troops cannot deploy a defense system with a large depth and cascade interception at a strategic location on the sea [2]. Therefore, to further expand the defense depth of important locations on the sea and build a defense system for multibatch interception of important locations on the sea, this paper studies the optimization of firepower configuration for coordinated air defense operations between warships and ground air defense forces at a strategic location on the sea.

The configuration of antiaircraft firepower units is not only the key link of air defense on the ground but also a hot issue in air defense/antimissile research [3–8]. At present, the research on the air defense firepower configuration of warships mainly focuses on the air defense formation configuration of aircraft carrier formations [9–11], ship formations [12–14], and air defense ships in maritime areas

[15–18]. Among these studies, Zhao and Liu [12] analyzed the basic principles of the formation of air defense area and established an optimization model and algorithm for the formation deployment of two ships in a maritime formation. Rao and Ravishankar [17] proposed a three-stage method based on game theory to study the deployment of air defense resources to maximize the coverage and performance of radar systems under various terrain conditions. Aiming at the problem of coordinated deployment of air defense weapon systems in multiple important bases to defend against UAV attacks, Xue et al. [18] constructed a multiweapon cooperative defense model according to the performance of each defense system, and further designed a genetic algorithm to solve it. However, few studies were conducted on the formation configuration of the naval formation and the cooperative air defense operations of the strategic locations on the sea. Limited literature [19] studied the issue of combat firepower distribution of island-ship integrated air defense dynamic weapons. With the support of warships, the decision-making model of island-ship cooperative dynamic weapon utilization was established. But there was no quantitative analysis of the firepower configuration of warships and islands and reefs for cooperative air defense operations. Ma et al. [20] studied the deployment of warships to support islands and reef air defense operations. Taking the maximum cover angle as the criterion, they established a configuration model of a single ship supporting air defense operations on islands and reefs and a two-ship configuration model. But they did not consider the firepower configuration of warships and air defense forces deployed at strategic locations on the sea for coordinated air defense operations.

Aiming at optimizing the configuration of warships in coordination with coastal air defense firepower units at strategic locations on the sea, this paper firstly builds a firepower unit configuration model of the ground air defense system to deal with the small area of strategic locations on the sea and analyzes the defense depth of the end defense line. Secondly, this paper builds a coordinated air defense configuration model between warships and air defense forces deployed at strategic locations on the sea and quantitatively analyzes the defense depth of the midrange defense line constructed in front of the warships to construct the maximum defense depth at strategic locations on the sea. Thirdly, according to the air defense firepower configuration of the ship's coordinated maritime strategic location; this paper constructs the ship's coordinated maritime strategic location defense effectiveness model according to the random service system theory. Finally, based on the midrange and terminal double-layer defense model constructed in this paper, an example simulation analysis is carried out, verifying that the model is reasonable and effective.

2. The Configuration Model of the Air Defense Firepower Unit at a Strategic Location on the Sea

The air attack weapons in air defense operations at strategic locations on the sea include various combat aircraft, tactical missiles, aerial bombs, and rocket bombs [21]. In this paper, aircraft are considered as incoming targets to study the configuration of antiaircraft fire units. The air defense operations of strategic locations on the sea are not only related to the cover capability of ground air defense weapon systems but also the performance of enemy aviation equipment, the distance to drop bombs, and the location of the ground antiaircraft weapon fire unit configuration.

2.1. Enemy Mission Line. During air defense operations at strategic locations on the sea, enemy aircraft carrying aviation munitions must be eliminated outside the enemy mission line. The enemy mission line [22] is a critical line for the defense of strategic locations on the sea. Once the enemy plane flies over this boundary line, it can attack the target at strategic locations on the sea and complete its air strike mission, as shown in Figure 1.

When the enemy plane launches air-to-ground missiles (or drops bombs) at a point T_1 (T_2), the distance between the enemy mission line and the center of the protected object is given as follows:

$$R_{rwx} = r_0 + r_s + r_k\left(r_f\right). \tag{1}$$

- r_0 is the radius of the defended object, and r_s is the effective radius of the air-to-ground missile (bomb)
- r_k is the horizontal distance of the air-to-ground missile or cruise missile after launch
- r_f is the horizontal flight distance of the bomb dropped by the aircraft

2.2. Defensive Depth and Route Shortcut. To protect strategic locations on the sea, antiaircraft fire units must destroy incoming targets outside the enemy mission line. To improve defense efficiency, when the air defense firepower unit is configured, it is necessary to configure a certain defense depth according to the incoming direction of the enemy. The size of the defense depth determines the number of shots fired by the antiaircraft fire units on incoming targets. Assuming that the time interval of the continuous firing of an antiaircraft fire unit is t_0, and the flight speed of the incoming target is v_0, the defense depth required by the antiaircraft fire unit to shoot n times can be estimated as follows:

$$L_{zs} = \Delta L + v_0 t_0 (n-1), \tag{2}$$

where ΔL represents the possible horizontal distance reduction of the kill zone under complex conditions. If the antiaircraft fire unit adopts the shooting-observing-shooting mode, the defense depth required for n shots can be estimated as follows:

$$L_{zs} = \Delta L + v_0 \left(t_{gc}(n-1) + \sum_{k=2}^{n} t_k \right). \tag{3}$$

- t_{gc} is the observation and evaluation time of the air defense weapon system

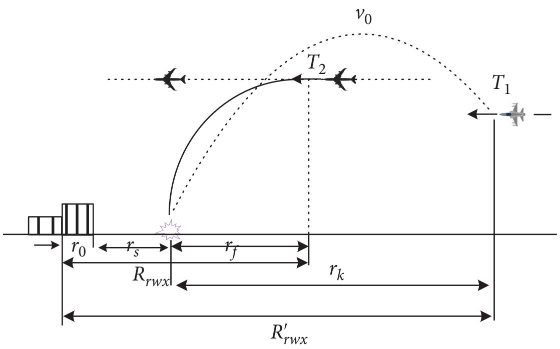


Figure 1: Enemy mission line.

t_k is the flight time between the k-th interceptor missile and the incoming target, which is related to factors such as the route shortcut and flight height of the incoming target

Route shortcut refers to the vertical distance from the center of the fire launch unit to the projection line or the tangent of the projection line of the air attack target track at sea level. As shown in Figure 2, the route shortcut $|O_1P|$, usually denoted by P, is an important parameter to determine whether the projectile can be launched or fired. In fact, as the route shortcut of the incoming target increases, the interception ballistic curvature will increase, and the required overload of the ship-to-air missile will also increase. But the available overload of the ship-to-air missile is unchanged. These will reduce the ratio of available overload to required overload, and increase the guidance error of the missile, thereby reducing the single-shot kill probability of the ship-to-air missile [23].

2.3. Configuration Model of Antiaircraft Firepower Units for Strategic Locations on the Sea. The configuration of air defense firepower units is a key link in air defense/antimissile defense operations at strategic locations on the sea. Due to the special geographical and natural environment conditions, only limited air defense forces can be deployed in strategic locations on the sea, and some small areas can only be equipped with portable air defense missiles. When establishing an antiaircraft fire unit, we must position the antiaircraft weapon platform at a certain distance from the target being defended to maximize the combat capability of the antiaircraft weapon system and ensure that it can be fired multiple times in front of the enemy mission line. For the air defense operations of strategic locations on the sea, considering its small area, we should try to deploy the fire units along the coastline according to the main combat direction of the enemy when configuring the air defense fire units, expanding the defense depth of air defense fire units (as shown in Figure 2).

Assuming that the center of the important sea area is O, and the horizontal main combat direction of the important sea area is BOD, where $\angle BOD = 2\varphi$. O_1 is the configuration point of the antiaircraft firepower unit, $d_{pz} = |OO_1|$ is the configuration distance from the center of the defended

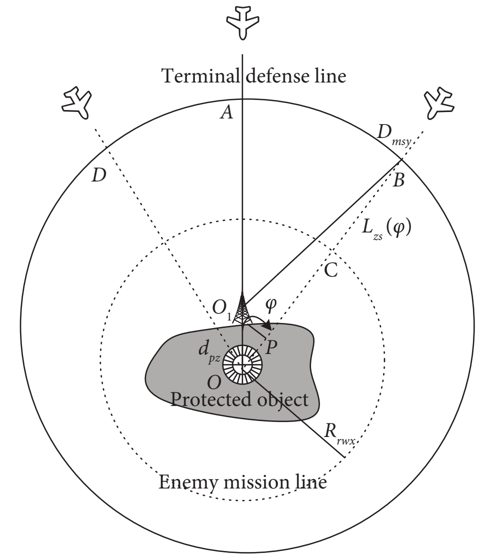


Figure 2: Schematic diagram of air defense firepower configuration in a strategic location on the sea.

object. The enemy mission line is R_{rwx}. The kill zone far boundary of the antiaircraft fire unit is $D_{msy} = |O_1B|$, and the antiaircraft fire unit has a border defense depth of $L_{zs}(\varphi) = |BC|$. In ΔO_1OB, using the triangular cosine theorem, we can obtain as follows:

$$D_{msy} = d_{pz}^2 + \left(R_{rwx} + L_{zs}(\varphi)\right)^2 - 2d_{pz}\left(R_{rwx} + L_{zs}(\varphi)\right). \quad (4)$$

From formula (4), it can be calculated that when air defense firepower is deployed on the coastline of important coastal areas, the minimum defense depth in the main combat direction is $L_{zs}(\varphi)$, and the corresponding route shortcut is $P_{\max} = (L_{zs}(\varphi) + R_{rwx})\sin\varphi$. In addition, it can be seen from Figure 2 that the maximum defense depth is $L_{zs} = D_{msy} - R_{rwx}$. when the route shortcut is zero.

3. The Air Defense Firepower Configuration Model of Warships Cooperating with the Strategic Location on the Sea

3.1. The Air Defense Firepower Configuration Model of a Single Warship Cooperating with the Strategic Location on the Sea. Figure 3 is a schematic diagram of the air defense firepower configuration of a single ship supporting a strategic position on the sea. The warship is preconfigured O', which forms a double-layer interception defense system with antiaircraft firepower at the strategic position on the sea. Taking into account the large forward distance of the warship and the range far boundary of the ship-to-air missile, and the relatively small area of the strategic position on the sea, the configuration distance $|OO_1|$ between the center and the antiaircraft firepower mentioned in Section 2.3, compared to ship forward distance and missile range boundaries, is negligible. For the convenience of modeling and analysis, it is assumed that the terminal defense line and the enemy mission line at the strategic position on the sea are concentric circles. The forward distance of the warship is $d_{pz} =$

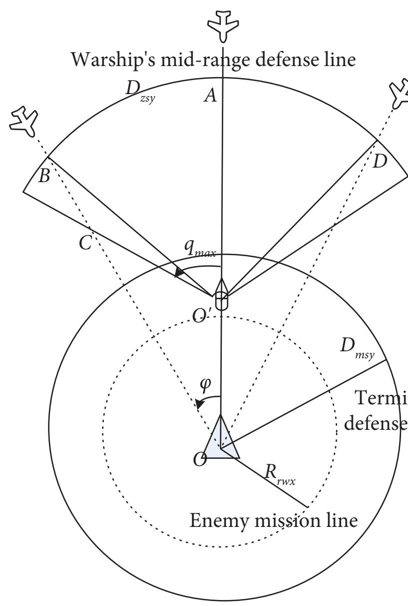


FIGURE 3: Schematic diagram 1 of the air defense configuration of a single warship cooperating with a strategic location on the sea.

$|OO'|$, and the maximum course angle is $q_{\max}$. The boundary of the ship-to-air missile kill zone is D_{zsy} and $L_{zs}(\theta)$ represents the defense depth of warships.

It is not difficult to see from Figure 3 that if the forward distance of the ship is small, its defense sector will be large and the defense depth will be small. As the forward distance of the ship increases, the defense sector of the ship becomes smaller and smaller, and the defense depth of the central area of the main combat direction gradually increases. However, the border defense depth ($|BC|$) showed a trend of increase in the beginning and then decrease gradually. What this paper considers is the forward distance of warships with the capability to defend the main combat direction and with a certain defense depth, to ensure that the key points at sea have the greatest defense depth.

3.1.1. Model of the Maximum Forward Distance of a Single Warship. Assuming that the main attack direction of the enemy is in the sector BOD, to ensure that the warships can effectively defend the enemy's incoming targets, it is required that the defense depth in the main combat direction is not less than the fixed value L, that is $L_{zs}(\varphi) = L$. Then, the maximum forward distance of the warships can be solved by the following equations:

$$\begin{cases} L + |OC| = |OB|, \\ D_{zsy}^2 = |OB|^2 + \left(d_{pz}^{\max}\right)^2 - 2|OB| d_{pz}^{\max} \cos\varphi, \\ |OC|\sin\varphi = \sqrt{|OC|^2 + \left(d_{pz}^{\max}\right)^2 - 2|OC| d_{pz}^{\max}\cos\varphi} \cdot \sin q_{\max}, \end{cases} \tag{5}$$

where $|OB|$ represents the far boundary of the midrange interception defense line constructed by the ship. Through the abovementioned equations, although the analytical solution of the ship configuration distance $d_{pz}^{\max}$ cannot be directly solved, when the parameter L, φ, $q_{\max}$, D_{zsy} is given, the maximum forward distance of the warship and the far boundary of the midrange defense line can be obtained.

3.2. Minimum Distance Model for Single Warship Forward Configuration. If the ship is too close to the key point at sea, it will lose the defensive depth formed by the front of the warship. Therefore, the minimum forward configuration distance of warships should be within the boundary area of the combat sector. The defensive depth formed by warships alone should not be less than the fixed value L, as shown in Figure 4.

It is not difficult to see from the figure when $|BC| = L$ the forward distance of the warship achieves the minimum value. Therefore, according to the geometric relationship of the figure, we can obtain

$$\begin{cases} \dfrac{|OB|}{\sin\angle OO'B} = \dfrac{|O'B|}{\sin\varphi} = \dfrac{d_{pz}^{\min}}{\sin\angle OBO'}, \\ |OB| = L + D_{msy}, \ |O'B| = D_{zsy}. \end{cases} \tag{6}$$

So, the minimum configuration distance of the ship is given as follows:

$$d_{pz}^{\min} = \frac{D_{zsy}\sin\left(\angle OO'B - \varphi\right)}{\sin\varphi}, \tag{7}$$

where $\angle OO'B = \pi - \arcsin(L + D_{msy})\sin\varphi / D_{zsy}$.

3.2.1. A Model of the Air Defense Depth of a Single Warship Cooperating with Strategic Positions on the Sea. After determining the ship's forward configuration distance d_{pz}, assuming that the enemy's incoming direction enters the ship's defense sector at the angle θ, and obeys a uniform distribution in the given threat sector, then the probability density function $f(\theta)$ of the enemy's incoming direction θ is defined as follows:

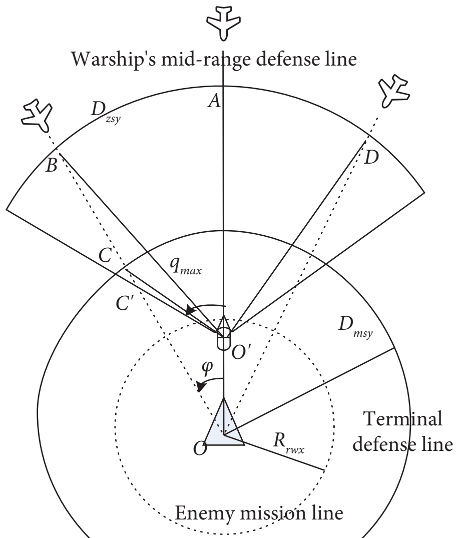


Figure 4: Schematic diagram 2 of the air defense configuration of a single warship cooperating with a strategic location on the sea.

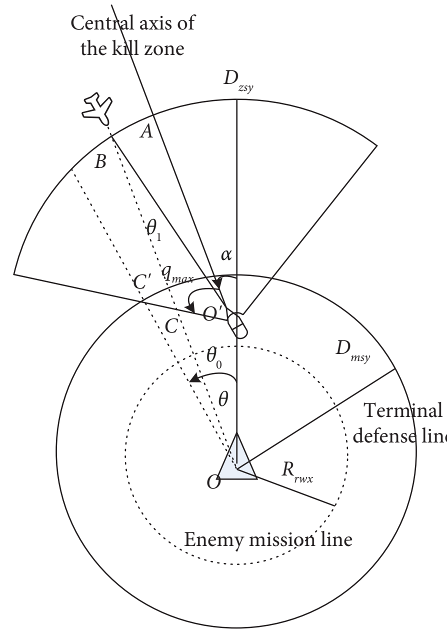


Figure 5: The defense depth of a single warship cooperating with a strategic location on the sea.

$$f(\theta) = \begin{cases} \dfrac{1}{b-a}, & a \le \theta \le b, \\ 0, & otherwise. \end{cases} \quad (8)$$

Assuming that the azimuth of the warship deployment is α. It can be seen from Figure 5 that the effective defense depth of the warship is defined as follows:

$$L_{zs}(\theta) = \frac{D_{zsy}\sin(q_{\max} + \alpha - \theta - \theta_1)}{\sin(q_{\max} + \alpha - \theta)}, \quad (9)$$

where $\theta_1 = \arcsin(d_{pz}\sin\theta / D_{zsy})$. Then, the expected defense depth of a warship in the main combat sector is defined as follows:

$$\begin{aligned} E(L_{zs}(a,b)) &= \int_a^b L_{zs}(\theta) f(\theta)\mathrm{d}\theta \\ &= \int_a^b \left(\frac{D_{zsy}\sin(q_{\max} + \alpha - \theta - \theta_1)}{\sin(q_{\max} + \alpha - \theta)}\right) f(\theta)\mathrm{d}\theta. \end{aligned} \quad (10)$$

The defense depth of the overlapping kill area of the warship's coordinated antiaircraft fire $L'_{zs}(\theta) = |CC'|$ is given as follows:

$$\begin{aligned} L'_{zs}(\theta) &= D_{msy} - (|OB| - |BC|) \\ &= D_{msy} - D_{zsy}\left(\frac{\sin(\theta + \theta_1)}{\sin\theta} - \frac{\sin(q_{\max} + \alpha - \theta - \theta_1)}{\sin(q_{\max} + \alpha - \theta)}\right). \end{aligned} \quad (11)$$

As can be seen from Figure 5, when there comes to a different target attack angle, there needs a different method to calculate the air defense firepower defense depth of the warship cooperating with strategic positions on the sea.

The specific calculation formula is defined as follows:

$$l_{zs}(\theta) = \begin{cases} L_{zs}(\theta) + \left(D_{msy} - R_{rwx} - L'_{zs}(\theta)\right), \theta \le \theta_0; \\ L_{zs}(\theta) + D_{msy} - R_{rwx}, \theta_0 < \theta < \varphi, \end{cases} \quad (12)$$

where $\theta_0 = q_{\max} + \alpha - \arcsin d_{pz}\sin(\alpha + q_{\max})/D_{msy}$ Then, the formula for calculating the expected defense depth of a single warship cooperating with strategic positions on the sea is given as follows:

$$E(l_{zs}(a,b)) = \int_a^b l_{zs}(\theta) f(\theta)\mathrm{d}\theta. \quad (13)$$

3.3. The Air Defense Firepower Configuration Model of the Two Warships Cooperating with Strategic Positions on the Sea. In the air defense operations of warship formations cooperating with strategic positions on the sea, the defense configuration includes linear configuration and circular configuration, etc. In the air defense of two warships cooperating with strategic positions on the sea, the central axis of the configuration of the two ships is configured as the central axis of the main combat direction of strategic positions on the sea, which can maximize the defense sector of the warship formation, as shown in Figure 6. The larger the configuration distance between the two ships, the larger the defense sector and the smaller the overlapping area of firepower. Conversely, the smaller the configuration distance between the two ships, the smaller the defensive sector and the larger the overlapping area of firepower. Meanwhile, the larger the forward configuration distance of the two ships, the smaller the defense sector and the greater the defense depth. Conversely, the smaller the forward configuration distance, the larger the defense sector and the smaller the defense depth. Therefore, when configuring the antiaircraft fire unit of the two warships cooperating

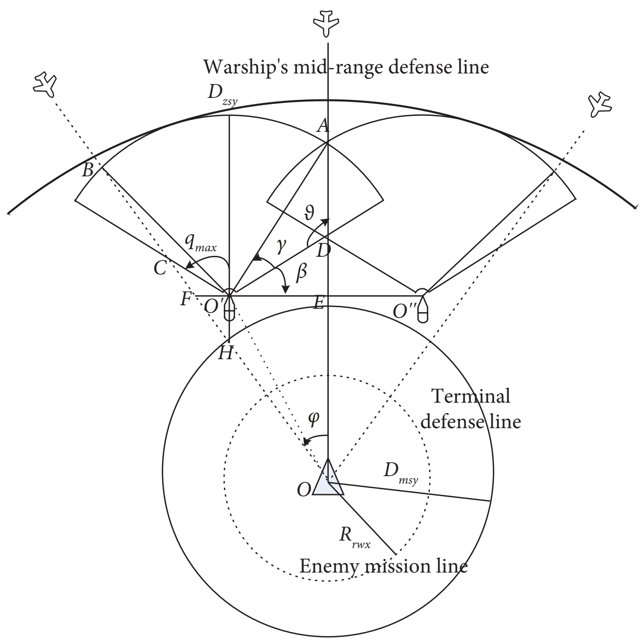


Figure 6: The configuration model of air defense firepower of two warships cooperating with a strategic location on the sea.

with strategic positions on the sea, one needs to consider the configuration distance between the two ships, as well as the forward configuration distance.

3.3.1. The Configuration Distance between the Two Warships. The limiting condition for the configuration distance between two warships is under the condition that the sum of the defense depths of the two warships' overlapping firepower area should not be less than the defense depth of a single ship's zero route shortcut, the configuration distance between the two warships should be as large as possible to expand the defense sector [24].

The far boundary of the warship defense is D_{zsy}, that is, $|A\ D| = 0.5D_{zsy}$ in Figure 6. In $\Delta\, AO'D$, by the law of sine, we obtain

$$\frac{|A\ D|}{\sin\gamma} = \frac{|AO'|}{\sin\vartheta}, \frac{|O'D|}{\sin(\gamma+\vartheta)} = \frac{|AO'|}{\sin\vartheta}, \tag{14}$$

where $\vartheta = \pi - q_{\max} = |AO'| = 2|A\ D| = D_{zsy}$, thus

$$|O'D| = D_{zsy}\left(\sqrt{1-\frac{1}{4}\sin^2 q_{\max}} - \frac{1}{2}\cos q\max\right). \tag{15}$$

Then, the configuration distance between the two warships is defined as follows:

$$\begin{aligned} d_{O'O''} &= |O'O''| \\ &= 2D_{zsy}\left(\sqrt{1-\frac{1}{4}\sin^2 q_{\max}} - \frac{1}{2}\cos q\max\right)\sin q_{\max}. \end{aligned} \tag{16}$$

3.3.2. The Model of the Forward Configuration Distance between the Two Warships. The principle of configuring the forward distance between two warships is the same as that of a single warship. When the configuration distance between the two warships and the main combat sector is determined, the distance between the two warships and the strategic positions on the sea gradually increases, and the air defense depth between the two warships and strategic positions on the sea becomes larger. When the forward configuration distance of the two warships reaches a certain level, the defense depth shows a decreasing trend with the greater forward configuration distance of the warships. As shown in Figure 6, the two warships are deployed at points O', O'' (outside the terminal defense line at strategic positions on the sea), and the main combat sector angle of the strategic positions on the sea is 2φ. Assuming that the minimum value of the horizontal defense depth of the warships is L_{zs}, then when the boundary of the combat sector is $|BC| = L_{zs}$, the configuration distance $|OO'| = |OO''|$ between the warship and the strategic positions on the sea reaches its maximum value.

At this moment, using the triangle sine theorem, we can obtain

$$\begin{cases} \dfrac{|BC|}{\sin\angle CO'B} = \dfrac{|O'B|}{\sin\angle BCO'}, \\[2ex] \dfrac{|O'C|}{\sin\angle CBO'} = \dfrac{|O'B|}{\sin\angle BCO'}, \\[2ex] \dfrac{|O'H|}{\sin\angle O'CH} = \dfrac{|O'C|}{\sin\varphi}, \end{cases} \tag{17}$$

where $\angle BCO' = \pi + \varphi - q_{\max} = |BC| = L_{zs}, |O'B| = D_{zsy}$, after simplification,

$$|O'H| = \frac{D_{zsy}\sin(q_{\max} - \varphi - \angle CO'B)}{\sin\varphi}, \tag{18}$$

where $\angle CO'B = \arcsin(L_{zs}\sin(q_{\max}-\varphi)/D_{zsy})$. From the proportional relationship between $|O'H|$ and $|OE|$, the distance from the strategic position on the sea to the configuration center E of the two warships can be obtained.

$$|OE| = \frac{D_{zsy}\sin(q_{\max} - \varphi - \angle CO'B) + 0.5d_{O'O''}\cot\varphi}{\sin\varphi}. \tag{19}$$

The maximum value of the distance from the strategic position on the sea to the two warships.

$$d_{oo'}^{\max} = \sqrt{\frac{1}{4}d_{O'O''}^2 + |OE|^2}. \tag{20}$$

When the deployment point O', O'' of the two warships is located within the terminal defense line at strategic positions on the sea, as shown in Figure 7, the overlapping airspace of antiaircraft firepower of warships cooperating with the strategic position on the sea is relatively large, as well as the defense sector angle. When the two warships are at the boundary distance $|BC| = L_{zs}$ of the main combat sector, the forward configuration distance of the warships takes the minimum value.

According to the geometric relationship in Figure 7, we can get

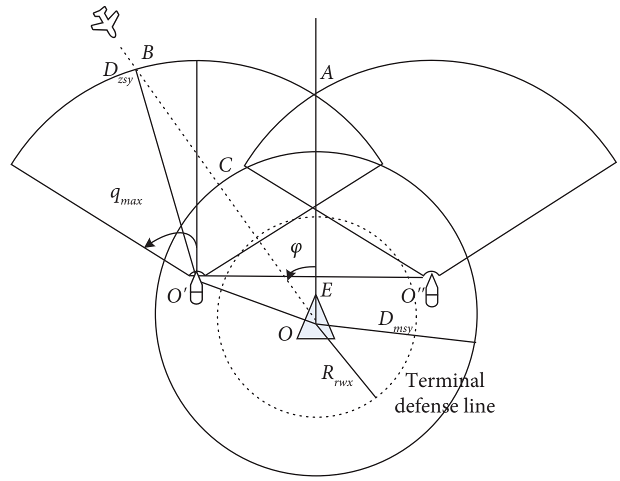


FIGURE 7: Schematic diagram of the minimum configuration of the distance between the warship and strategic location on the sea.

$$\begin{cases} \dfrac{|BC|}{\sin \angle CO'B} = \dfrac{|O'B|}{\sin \angle BCO'} = \dfrac{|O'C|}{\sin \angle CBO'}, \\ d_{OO'}^{\min} = \sqrt{\left(|OC|\cos\varphi - |O'C|\right)^2 + |O'E|^2}, \end{cases} \tag{21}$$

where $\angle BCO' = \pi - \varphi$, $|BC| = L, |O'B| = D_{zsy}, \infty|OC| = D_{msy}, |O'E| = 1/2 d_{O'O''}$. after simplification,

$$d_{OO'}^{\min} = \sqrt{\left(|OC|\cos\varphi + L\cos\varphi - \sqrt{D_{zsy}^2 - L^2\sin^2\varphi}\right)^2 + \frac{1}{4}d_{O'O''}^2}. \tag{22}$$

3.3.3. The Model of the Air Defense Depth of the Two Warships Cooperating with the Strategic Position on the Sea. It is assumed that the azimuth of the incoming target course is θ, and the course points to the strategic position on the sea O. At this time, as shown in Figure 8, the air defense ship adjusted the combat orientation according to the incoming direction, so that the central axis of the kill zone was parallel to the incoming direction, to ensure the best posture to defend the incoming target. The vertical line from point O'' to $|OA|$ intersects at point P, and the route shortcut is defined as follows:

$$p_{\max} = |O''P| = d_{OO''}\sin(\theta_0 - \theta). \tag{23}$$

In the right triangle APO'',

$$|AP| = \sqrt{|O''A|^2 - |O''P|^2} = \sqrt{D_{zsy}^2 - d_{OO''}^2\sin^2(\theta_0 - \theta)}. \tag{24}$$

The defense depth of the warship against the target is given as follows:

$$L_{zs}(\theta) = |AP| - |KP| = \sqrt{D_{zsy}^2 - d_{OO''}^2\sin^2(\theta_0 - \theta)} - d_{OO''}\sin(\theta_0 - \theta)\cot q_{\max}. \tag{25}$$

Therefore, the defense depth of the two warships is expected to be

$$E\left(l_{zs}(a,b)\right) = \int_b^b L_{zs}(\theta) f(\theta)\mathrm{d}\theta = \int_b^b \left(\sqrt{D_{zsy}^2 - d_{OO''}^2\sin^2(\theta_0 - \theta)} - d_{OO''}\sin(\theta_0 - \theta)\cot q_{\max}\right) f(\theta)\mathrm{d}\theta. \tag{26}$$

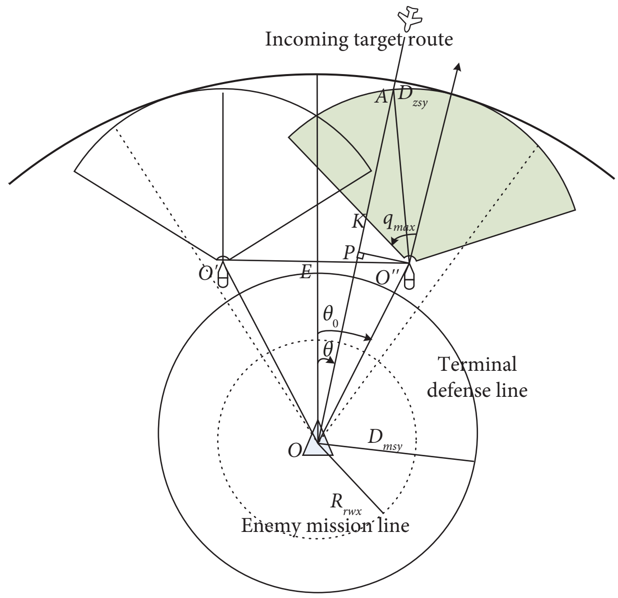


Figure 8: The defense depth of the two warships cooperating with a strategic location on the sea.

Comparing Figures 6 and 7, the forward distance of the two warships in Figure 6 is relatively large, and the defense depth of the two warships cooperating with the strategic position on the sea is large as well. In Figure 7, however, the overlapping area of the air defense firepower between the two warships and the strategic position on the sea is large, and the defense depth of the two warships cooperating with the strategic position on the sea is small. In fact, according to the forward distance $d_{OO''} \in (d_{OO''}^{\min}, d_{OO''}^{\max})$, the defense depth of the two warships cooperating with the strategic position on the sea can be obtained as follows:

$$l_{zs}(\theta) = \begin{cases} L_{zs}(\theta) + D_{msy} - R_{rwx}; \\ \quad if\, d_{OO''}(\cos(\theta_0 - \theta) + \sin(\theta_0 - \theta)\cot q_{\max}) \geq D_{msy} \\ \sqrt{D_{zsy}^2 - d_{OO''}^2 \sin^2(\theta_0 - \theta)} + d_{OO''}\cos(\theta_0 - \theta) - R_{rwx}; \text{other.} \end{cases} \tag{27}$$

The expected defense depth of the two warships cooperating with the strategic position on the sea is given as follows:

$$E(L_{zs}(a,b)) = \int_a^b l_{zs}(\theta) f(\theta) \mathrm{d}\theta. \tag{28}$$

4. Efficiency Analysis of the Air Defense of Warships Cooperating with the Strategic Position on the Sea

4.1. Efficiency Model of Air Defense of the Strategic Position on the Sea. The main air strikes faced by the strategic position on the sea are multidirectional, multistage, and multiform saturation attacks. In the event of a large-scale air attack, the defense system at the strategic position on the sea can be regarded as a random service system. The arrival process of the incoming target obeys the Poisson distribution with parameter λ, that is,

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}. \tag{29}$$

$k = 0, 1, 2, \ldots$ and $t > 0$, $P_k(t)$ demonstrates the probability that k targets will strike within time t. The antiair defense at the strategic position on the sea adopts the principle of "first to come, first to be served" and uses the shooting method of "shooting-observing-shooting." Each antiaircraft fire unit is regarded as a "service counter," and the "service" time for incoming targets obeys the negative exponential distribution of a parameter μ, that is,

$$P(\tau < t) = 1 - e^{-\mu t}, t \geq 0, \tag{30}$$

where $\mu = 1/t_{smean}$, t_{smean} represents the average "service" time to the target. It is assumed that the time of the incoming target flying over the kill zone obeys a negative exponential distribution with parameter υ, that is,

$$P(\tau < t) = 1 - e^{-\upsilon t}, \quad t \geq 0, \tag{31}$$

where $v = 1/wmean$ and t_{wmean} is the average time that the target stays in the kill zone, which depends on the incoming target's flight speed v_0 and the defense depth L_{zs} of the antiaircraft weapon fire units defending it. Incoming targets stay and fly in the launch area, which is equivalent to "customers" queuing for service in the service system. The incoming target flew away from the launch area and successfully penetrated the defense, which is equivalent to that of a "customer" who has been queuing for too long in the service system automatically left.

According to the above-mentioned analysis and assumptions, the air defense of the strategic position on the sea can be regarded as the M/M/n/c queuing model with a limited waiting time. Assuming that $N(t)$ represents the number of targets in the air defense weapon system at the t epoch, the possible states of the system are given as follows:

A_0: There are n firepower channels idle in the system, and there are 0 targets;

A_1: The system has 1 fire channel firing and 1 target;

......

A_n: All fire channels of the system are under firing, there are n targets;

A_{n+1}: All fire channels of the system are under firing, and there is 1 target waiting to be fired;

......

A_{n+s}: All fire channels of the system are under firing, and there are s targets waiting to be fired.

According to the principle of equilibrium state, the equation system can be obtained as follows:

$$\begin{cases} A_0:\ \mu p_1 - \lambda p_0 = 0; \\ A_1:\ 2\mu p_2 - (\lambda + \mu)p_1 + \lambda p_0 = 0; \\ \cdots \\ A_k:\ (k+1)\mu p_{k+1} - (\lambda + k\mu)p_k + \lambda p_{k-1} = 0; \\ \cdots \\ A_n:\ (n\mu + v)p_{n+1} - (\lambda + n\mu)p_n + \lambda p_{n-1} = 0; \\ A_{n+1}:\ (n\mu + 2v)p_{n+2} - (\lambda + n\mu + v)p_{n+1} + \lambda p_n = 0; \\ \cdots \\ A_{n+i}:\ (n\mu + (i+1)v)p_{n+i+1} - (\lambda + n\mu + v)p_{n+i} + \lambda p_{n+i-1} = 0; \\ \cdots \\ A_{n+s}:\ (n\mu + sv)p_{n+s} - \lambda p_{n+s-1} = 0. \end{cases} \tag{32}$$

According to $p_0 + p_1 + p_2 + \cdots + p_n + \cdots + p_{n+s} = 1$, the probability of stability can be obtained as follows:

$$\begin{cases} p_0 = \left[\sum_{k=0}^{n} \dfrac{\rho^k}{k!} + \dfrac{\rho^n}{n!} \cdot \sum_{s=1}^{\infty} \dfrac{\rho^s}{\prod_{m=1}^{s} (n + m\vartheta)}\right]^{-1}; \\ p_k = \dfrac{\rho^k}{k!} \cdot p_0, 0 \le k \le n; \\ p_{n+s} = \dfrac{\rho^n}{n!} \left(\rho^i \left(\prod_{k=1}^{i} (n + k\vartheta)\right)^{-1}\right) \cdot p_0, 1 \le i \le s, \end{cases} \tag{33}$$

where $\rho = \lambda P_f/\mu = \lambda P_f t_{smean}$, $\vartheta = v/\mu = v_0 t_{smean}/L_{zs}$, and P_f is the probability that the target is detected by the radar. The probability of successful penetration of the incoming target without being shot is denoted as P_{ref}. When there are on average m targets in the system waiting to be fired, the probability of the target not being fired is given as follows:

$$P_{ref} = \frac{v}{\lambda} \sum_{s=1}^{\infty} s p_{n+s} = \frac{v/\mu}{\lambda/\mu} \sum_{s=1}^{\infty} s p_{n+s} = \frac{\vartheta}{\rho} \sum_{s=1}^{\infty} s p_{n+s} = \frac{\vartheta}{\rho} \cdot \frac{\rho^n/n! \sum_{s=1}^{\infty} s\rho^s / \prod_{k=1}^{s} (n + k\vartheta)}{\sum_{k=0}^{n} \rho^k/k! + \rho^n/n! \cdot \sum_{s=1}^{\infty} \rho^s / \prod_{m=1}^{s} (n + m\vartheta)}. \tag{34}$$

Thus, the probability of the incoming target being shot is: $P_{shoot} = 1 - P_{ref}$. assuming the killing probability of the air defense firepower unit to the incoming target is P_{kill}, the defense efficiency of the strategic position on the sea is $DF = P_{shoot} \cdot P_{kill}$.

4.2. Efficiency Model of the Defense of the Warship Cooperating with the Strategic Position on the Sea. The defense of the warships cooperating with the strategic position on the sea can be regarded as a double-layer air defense/antimissile defense system. It is the forward-configured warship that carries out the midrange interception defense at first, and it is the strategic position on the sea that performs terminal interception on targets that the warship has not effectively intercepted. Each layer of the defense system is treated as an M/M/n/c queuing model with a limited waiting time. Therefore, the efficiency model of the ship-ground double-layer air defense/antimissile defense is described as follows:

The efficiency model of the warship's midrange defense is given as follows:

$$JT: \begin{cases} \rho_1 = \lambda \cdot t_{\text{smean}}^{(jt)} \cdot P_{f_1}; \vartheta_1 = \dfrac{v_0 t_{smean}^{(jt)}}{L_{zs}^{(jt)}}; \\ P_{\text{shoot}}^{(jt)} = 1 - \dfrac{\vartheta_1/\rho_1 \sum_{m=1}^{\infty} m\rho_1^{n_1}/n_1!\left(\rho_1^m \left(\prod_{k=1}^{m}(n_1 + k\vartheta_1)\right)^{-1}\right)}{\left(\sum_{i=0}^{n_1} \rho_1^i/i! + \rho_1^{n_1}/n_1! \sum_{s=1}^{\infty}\left(\rho_1^s \left(\prod_{m=1}^{s}(n_1 + m\vartheta_1)\right)^{-1}\right)\right)}; \\ DF_1 = P_{\text{shoot}}^{(jt)} P_{\text{kill}}^{(jt)}; \end{cases} \tag{35}$$

Based on the warship interception, the efficiency model of the terminal interception defense of the warship cooperating with the strategic position on the sea is given as follows:

$$Y\ D: \begin{cases} \rho_2 = (1 - DF_1)\lambda \cdot t_{smean}^{(dj)} \cdot P_{f_2}; \vartheta_2 = \dfrac{v_0 t_{smean}^{(dj)}}{L_{zs}^{(dj)}}; \\ P_{\text{shoot}}^{(dj)} = 1 - \dfrac{\vartheta_2/\rho_2 \sum_{m=1}^{\infty} m\rho_2^{n_2}/n_2!\left(\rho_2^m \left(\prod_{k=1}^{m}(n_2 + k\vartheta_2)\right)^{-1}\right)}{\left(\sum_{i=0}^{n_1} \rho_2^i/i! + \rho_2^{n_2}/n_2! \sum_{s=1}^{\infty}\left(\rho_2^s \left(\prod_{m=1}^{s}(n_2 + m\vartheta_2)\right)^{-1}\right)\right)}; \\ DF_1 = P_{\text{shoot}}^{(dj)} P_{\text{kill}}^{(dj)}; \end{cases} \tag{36}$$

Overall, the efficiency model of the warship cooperating with the strategic position on the sea is given as follows:

$$DF = DF_1 + (1 - DF_1)DF_2. \tag{37}$$

$\rho_i\,(i = 1, 2)$ is the number of incoming targets found in the average firing time of warships and the strategic position on the sea. $\vartheta_i\,(i = 1, 2)$ represents the number of successful penetrations due to exceeding the waiting time during the average firing time. $n_i\,(i = 1, 2)$ represents the number of antiaircraft fire units on the warship and at the strategic position on the sea, respectively. The rest of the parameters are the same as those defined above.

5. Simulation Experiments

It is assumed that the coordinated air defense between a strategic position on the sea and a warship forms an effective double-layer interception defense system, with the warship's front-mounted ship-to-air missiles performing the midrange defense, and the air defense weapon systems deployed at the strategic position on the sea performing the terminal defense. Table 1 shows the performance of the relevant parameters of the firepower unit of each air defense weapon platform. The antiaircraft firepower units configured on warships and the strategic position on the sea are set to 4 and 6, respectively, and the probability of finding the target and the probability of killing are random numbers in the corresponding interval.

It is assumed that the maximum radius of the bombing circle of the enemy air attack weapon is 60 km, the flying height of the enemy air attack aircraft is 300 m at a flight speed $v_0 = 300 m/s$, and the attack intensity $\lambda = 8$ of planes per minute. The main combat sector angle at the strategic position on the sea $2\varphi = 120^{\circ}$, and the defensive combat shooting mode adopts the method of "shoot-look-shoot," and the observation and evaluation time $t_0 = 15\,s$. When configured with firepower, the warship is required to intercept the incoming targets in the main combat sector at least 2 times. Tables 2 and 3 show the relevant conclusions obtained from the air defense firepower configuration model of the warship cooperating with the strategic position on the sea constructed in Section 3.

As can be seen from Table 2, in the "shooting-observing-shooting" mode, the warship's defense depth in the main combat direction cannot be less than 30.9 km. To meet this requirement, when conducting the air defense of a single warship cooperating with the strategic position on the sea, the minimum forward distance is 27 km and the maximum

Table 1: Relevant parameters of firepower unit of air defense weapon system.

Parameter	Antiaircraft fire unit of the surface-to-air missile in the strategic position on the sea	Antiaircraft fire unit of warship surface-to-air missile
Kill zone far boundary (km)	80	100
Kill zone near the boundary (km)	5	3
Maximum course angle $q_{\max}$ (*)	75	80
Maximum route shortcut (km)	77	98
Observation evaluation time (s)	15	15
Antiaircraft fire transfer time (s)	10	10
Average flight speed of antiaircraft missiles (m/s)	900	900
Antiaircraft fire unit	8	6
Probability of finding the target P_f	Rand (0.8, 1)	Rand (0.8, 1)
Single-shot missile kill probability P_{kill}	Rand (0.5, 0.9)	Rand (0.5, 0.9)

Table 2: Conclusions about the distance of firepower configuration between the warship cooperating with a strategic location on the sea.

Parameter	Minimum value (km)	Maximum value (km)	Remarks (km)
The forward distance of air defense of single-warship cooperating with the strategic position on the sea	27	75	$L_{zs}^{\min} = 30.9$
The distance between the two warships	d_0	154	$d_{oo'} = 60$
The distance between the coordinated air defense of two warships and the strategic position on the sea	40	98	
The vertical distance between the configuration line of the two warships and the strategic position on the sea	33	75	

Table 3: Results of cooperative defense in depth.

Parameter	Single-warship	Two-warship	Remarks
Warship defense depth expectation (km)	90.5	98.4	Depth expectation of independent air defense of the strategic position on the sea: 51.2 km
Depth expectation in the overlapping area of coordinated antiaircraft fire (km)	39.8	22.7	
Coordinated air defense depth expectation (km)	100.7	125.7	

is 75 km. When conducting the air defense of two warships cooperating with the strategic position on the sea, without considering the communication distance and other factors, the maximum distance between the two warships is 154 km, and the minimum value is the minimum distance between the two warships under electromagnetic interference. When the distance between the two warships is 60 km, the configuration distance between the warships and the strategic position on the sea ranges from 40 km to 98 km, and the vertical distance between the two warships' configuration line and the strategic position on the sea ranges from 33 km to 55 km. Simulation parameters: when the single-warship coordinated air defense forward distance is 40 km, the two-warship coordinated air defense distance is 60 km, and the distance between the two warships is 60 km, to calculate the expected value of the defense depth within the range of the enemy's incoming direction. It is not difficult to see from Table 3 that the horizontal air defense depth of the two warships' coordinated air defense is better than that of the

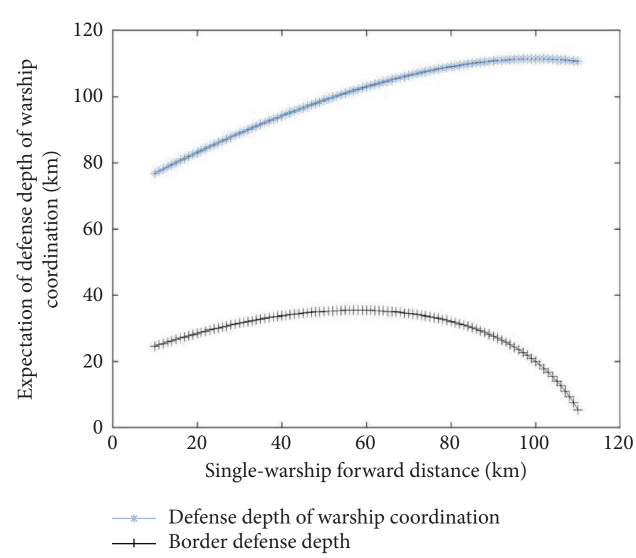


Figure 9: Single lead distance and defense depth.

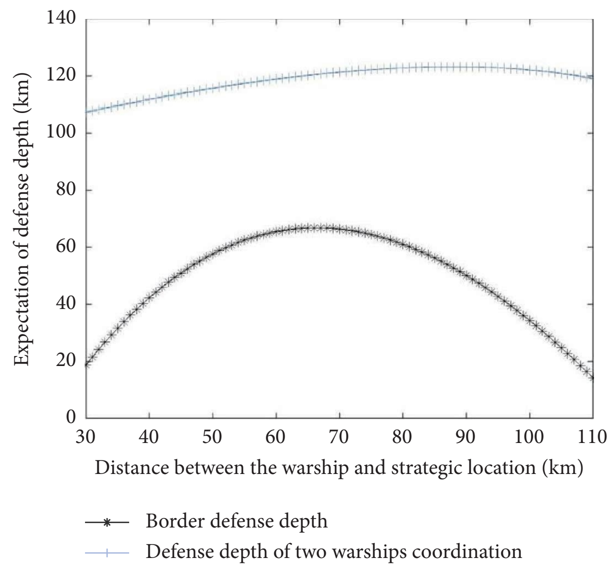


FIGURE 10: The forward distance and defense depth of the two warships.

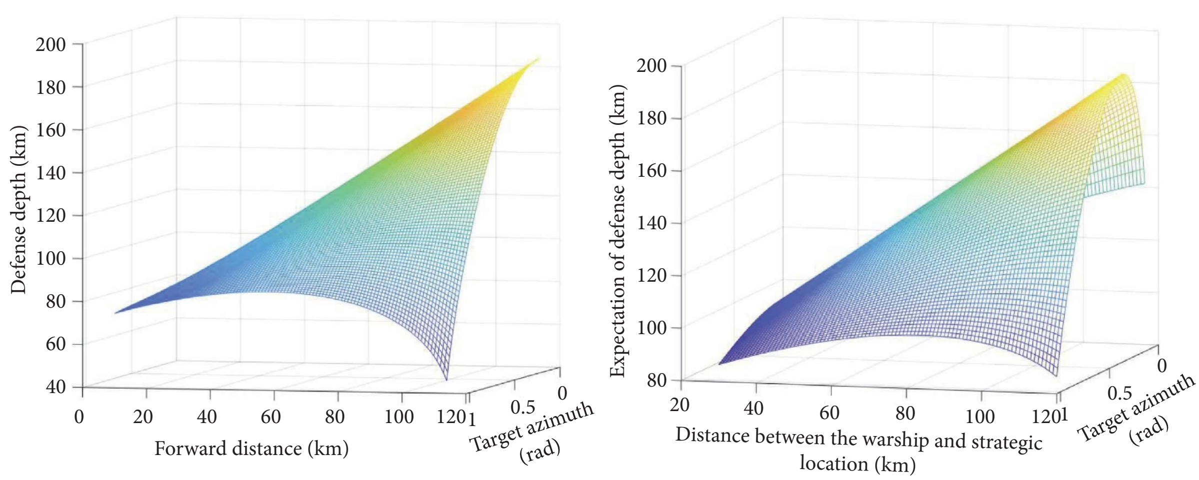
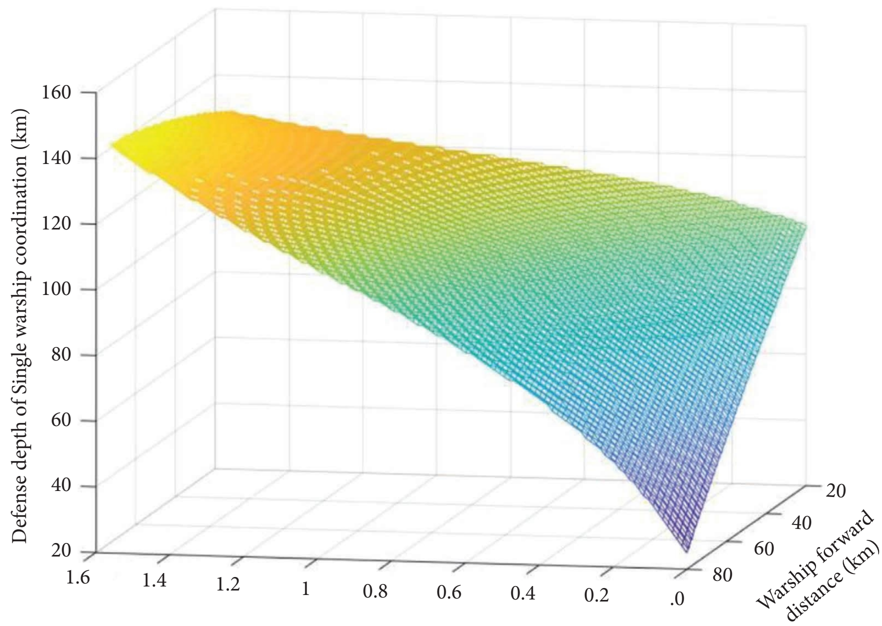


FIGURE 11: Schematic diagram of the depth, forward distance, and target azimuth of the warship cooperating with a strategic location on the sea.

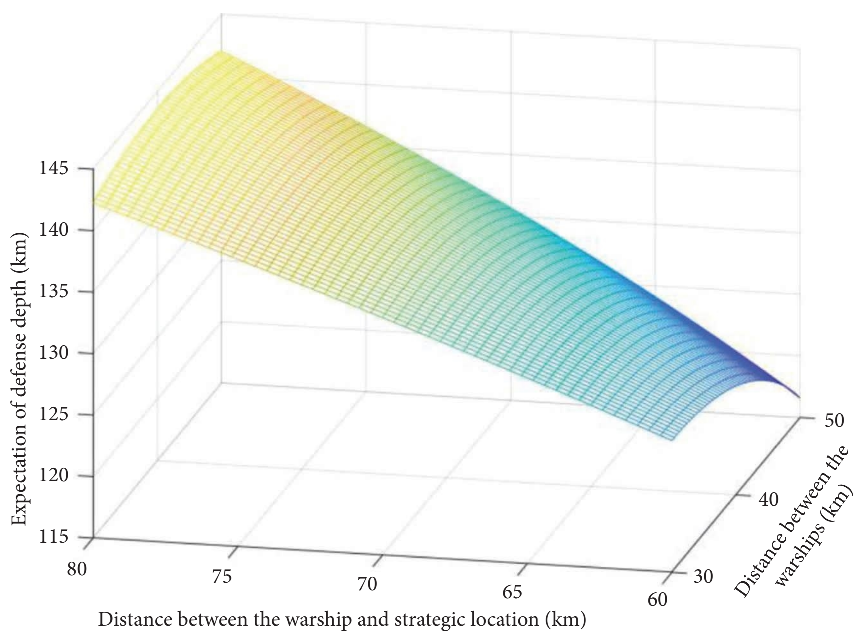
single warship's coordinated air defense. The expected value of the horizontal defense depth of a single warship cooperating with the strategic position on the sea is 100.7, while that of two warships cooperating with the strategic position on the sea is 125.7. In the independent air defense operation of the strategic position on the sea, the expected defense depth is 51.2 km. The fact is that the conclusions of Table 3 are obtained when the configuration distance is determined. To further analyze the relationship between the defense depth and the configuration distance of air defense firepower units, this paper uses the configuration distance as a variable in the simulation, and obtains a schematic diagram of the horizontal defense depth, the border defense depth, and the configuration distance, as shown in Figures 9 and 10.

It can be seen from the figure that whether the coordinated air defense is conducted by a single warship or two warships, the expected value of the horizontal defense depth increases with the increase of the warship configuration distance. When the configuration distance reaches a certain level, the defense depth tends to be stable. However, if the forward distance is too large or too small, the border defense depth in the main combat direction cannot meet the requirements of warships for performing two interceptions. Combining Table 2 and Figures 9 and 10, it can be seen that under a single-warship coordinated defense, the configuration distance is less than 27 km or greater than 75 km, or under a two-warship coordinated defense, the configuration distance is less than 40 km or greater than 97 km. In this case, the border defense depth is less than 30.9 km, which does not meet the defense requirements.

The horizontal defense depth of air defense of warships cooperating with the strategic position on the sea is not only related to the configuration distance but also related to parameters such as the azimuth angle of the incoming target, the azimuth angle of warship deployment, and the configuration distance between the two warships. Figure 11 reflects



(a)



(b)

FIGURE 12: The relationship between the depth of defense for the azimuth of the warship, and the configuration distance of warship-ground.

the relationship between the defense depth, the configuration distance, and the direction of the incoming target under air defense of the single warship and the two warships cooperating with the strategic position on the sea. As can be seen from the figure, when the incoming direction is facing the center of the defense kill zone, the defense depth is relatively large, and the configuration distance gradually increases. When the attack angle is relatively large and close to the boundary of the combat direction, the defense depth first increases and then decreases with the configuration distance. In addition, the depth expectation of two-warship coordinated defense is significantly higher than the data of single-warship coordinated defense.

Figure 12(a) takes the direction angle of the incoming target as an example and shows the relationship between the air defense depth of the warship cooperating with the strategic position on the sea, the warship's deployment azimuth, and the forward distance. The smaller the warship's deployment azimuth angle and the incoming direction angle, the greater the defense depth. As the deployment azimuth increases, the greater the deployment distance, the greater the defense depth.

Table 4: Simulation results of cooperative defense effectiveness between the warship and strategic location on the sea.

	Coordinated defense type	ρ	ϑ	P_{ref}	P_{shoot}	DF
Single-warship coordinated defense	Single-warship medium-range defense effectiveness	10.08	0.2913	0.6296	0.3704	0.3148
	Terminal defense effectiveness at the strategic position on the sea	6.4050	0.2388	0.1337	0.8663	0.7797
	Total	—	—	—	—	0.8491
Two-warship coordinated defense	Two-warship medium-range defense effectiveness	9.3600	0.2453	0.1709	0.8291	0.7047
	Terminal defense effectiveness at the strategic position on the sea	2.7603	0.2388	0.0043	0.9957	0.8962
	Total	—	—	—	—	0.9693

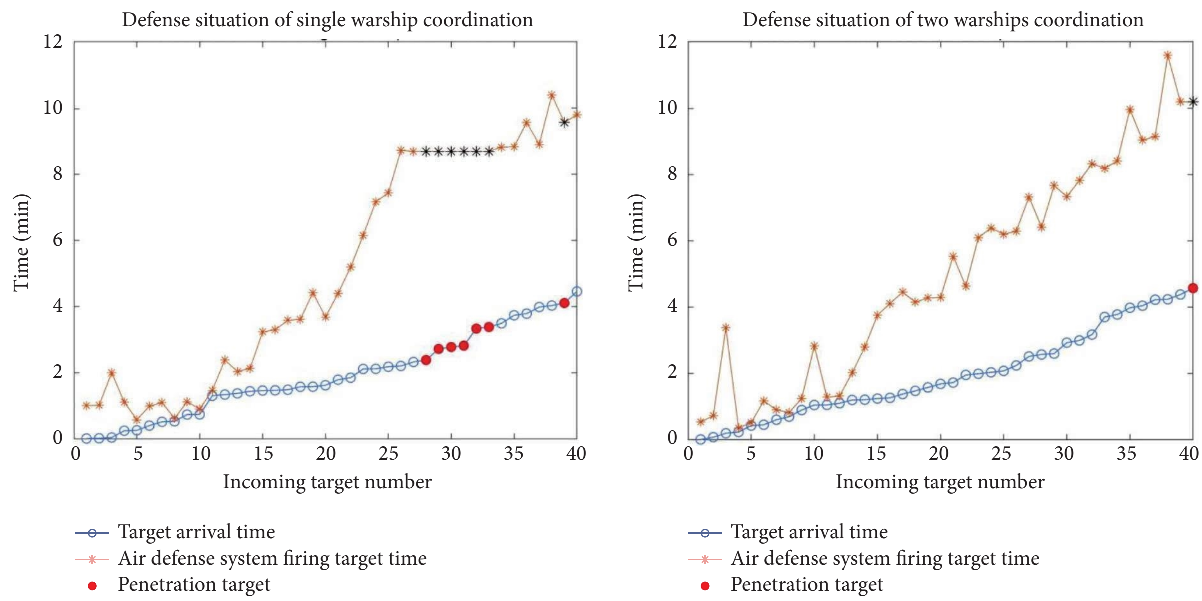


Figure 13: Target arrival time and penetration.

Figure 12(b) reflects the relationship between the defense depth, the configuration distance between the two warships, and the distance between the warship and the ground under the air defense of two warships cooperating with the strategic position on the sea. As can be seen from the figure, as the distance between the two warships increases, the defense depth decreases. But in fact, the larger the defense sector of the two warships, the greater the distance between the two warships and the strategic position on the sea, and the defense depth also increases.

Therefore, ignoring other factors, when optimizing the configuration of air defense firepower units of a single warship cooperating with the strategic position on the sea, the azimuth angle of the warship should maintain a small angle with the direction of the incoming target. Premised by meeting the requirements of the warship's defense depth in the main combat direction, the forward distance of the warship should be as forward as possible. Under the defense of two warships cooperating with the strategic position on the sea, the configuration distance between the two warships is determined according to the enemy's main attack sector and the antiaircraft far boundary of the antiaircraft fire unit. On the premise of meeting the defense depth requirements of the main combat sector, the small distance between the two warships is conducive to the coordinated defense of the two warships. It effectively increases the overlapping area of air defense firepower in the center of the combat sector and increases the probability of intercepting incoming targets. In addition, under the condition of meeting the defense depth requirements of the main combat sector, the greater the distance between the two warships and the strategic position on the sea, the greater the expected value of the defense depth.

According to the abovementioned analysis of the coordinated defense depth of warships cooperating with the strategic position on the sea, the efficiency model of coordinated defense, and the parameters of the combat scenario, this paper uses MATLAB to conduct random simulations. After 100 simulations, the conclusions related to cooperative defense effectiveness between the warship and strategic location on the sea are shown in Table 4 and Figure 13.

It can be seen from the simulation results that the cooperative defense effectiveness between two warships and strategic locations on the sea is 0.9693, which is superior to that between a single warship and strategic location on the sea at 0.8491. This is because the defense depth in the defense of two warships cooperating with the strategic position on the sea is larger than that in a single-warship coordinated defense. Moreover, the two-warship coordinated defense usually has a large number of air defense firepower units, which can effectively form alternate echelon interceptions, thereby improving interception efficiency. It can be seen from Figure 13 that when the number of attacking warships reaches 40, in the single-warship coordinated defense system, 7 attacking targets penetrated the defense without being shot, while only 1 penetrated the defense in the two-warship cooperated defense system. In fact, among the plans on warships supporting the air defense of the strategic location on the sea, the independent air defense of the strategic location on the sea shows an efficiency of 0.6218, which is far lower than the efficiency of the air defense of warships cooperating with the strategic position on the sea. It can be seen that under the simulation background, the defense efficiency of the two-warship coordinated defense is 96.93%, which is much higher than that of the independent air defense at the strategic location on the sea. Furthermore, when the strength of the incoming target is A (frames/min), the defense efficiency of two-warship coordinated defense remains above 80%, while that of the independent defense at the strategic location on the sea is less than 50%. Overall, it is fair to say that the two-warship coordinated air defense can greatly improve defense efficiency.

6. Conclusion

Aiming at the optimization of the firepower configuration for air defense operations at a strategic location on the sea, this paper comprehensively analyzes the influence of various elements of air defense on the firepower configuration according to the actual situation of the air defense combat battlefield environment. Based on the coastal defense forces and warship antiaircraft firepower units deployed at the strategic location on the sea, this paper constructs a firepower configuration model for coordinated air defense operations between warships and the strategic location on the sea. Accordingly, the calculation formulas of the forward distance and the expected value of the defense depth of the single warship and two warship air defenses cooperating with the strategic position on the sea are proposed. In addition, through simulation examples, this paper also analyzes the influence of parameters such as the azimuth angle, forward distance, and incoming target azimuth angle of warship deployment on the configuration optimization of air defense firepower units. The optimization model and calculation formulas constructed in this paper can provide a certain theoretical basis for the configuration of coordinated air defense firepower units between warships and the strategic location on the sea. In the follow-up research, we will further discuss the configuration optimization of the integrated air defense firepower unit of reefs, seas, and air under the coordination of long-range aviation, early-warning aircraft, and other forces.

Data Availability

The data that support the findings of this study are available within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by grant JY2020A016 from the Military Postgraduate Funding Projects.

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