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Research Article

Selective Maintenance Strategy for a Finite Planning Horizon Considering Imperfect Maintenance

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Most selective maintenance research strategies ignore the comprehensive evaluation of numerous following missions in favor of focusing just on the reliability of the next mission. In the same circumstance, however, assessing simply impending missions differs from considering the overall system's maintenance planning outcomes. In this study, a decision-making model for selective maintenance is developed by evaluating the total system reliability over a finite planning horizon. The purpose is to calculate the maintenance activities for each system component during each break and the best number of maintenance interruptions for the planning horizon to achieve maximum system reliability. Consequently, the selective maintenance problem is formulated as a max–min optimization model. Also, a hybrid imperfect maintenance model is used to formulate the component improvement after maintenance. Finally, simple case illustrations of maintaining the production system in coal transportation are given based on the assumed data.

1. Introduction

In recent years, the effective maintenance of complex systems has been regarded as a key factor in enhancing the core competitiveness of enterprises by academia and industry. The system's scale and structure have a substantial influence on reliability, and maintaining complex systems with high reliability have an increasing maintenance cost. In industrial environments, systems are intended to execute a sequence of missions with a finite break between two adjacent missions. Between each mission, there will be a break, which gives the system components time for maintenance. However, restricted to the limited maintenance resources, such as budget, time, manpower etc., it may not be possible to perform all desirable maintenance activities for the component in the system [1–4]. Therefore, it will be of great engineering significance to make scientific maintenance decisions for the complex systems with limited resources so that the system can operate stably and reliably and complete the expected missions. This maintenance strategy is known in the literature as selective maintenance.

The selective maintenance problem was first introduced by Rice et al. [5] in 1998. Since then, selective maintenance problems have been extensively investigated from various angles. Cassady et al. [6] extended the model proposed by Rice et al. [5] to a more general case, in which the component life follows the Weibull distribution and there are three optional maintenance actions, namely, minimal repair, corrective replacement, and preventive replacement. Subsequently, minimal repair and perfect maintenance were carried out by maintenance personnel as fundamental maintenance procedures in the several studies [7]. Recently, researchers have discovered that a maintenance action between minimal repair and perfect maintenance, known as imperfect maintenance, is better suited for engineering. Kijima et al. [8] type models Kijima [9], (p, q) models [10], age reduction models [11], improved factor models [12], hybrid models [13], geometric process models [14], and quasi renewal process model [15, 16] are examples of imperfect maintenance models. Liu and Huang [17] investigated imperfect maintenance of the system components in recent years. They thought that maintenance efforts had an impact on the system's age. Maaroufi et al. [18] used the Kijima type II model to reduce the costs in the selective maintenance of two-state systems while considering the economic importance of the components. Pandey et al. [19] constructed a selective maintenance decision-making model of a two-state system based on a hybrid model, in which imperfect maintenance actions can reduce the virtual life of components and change the change law of component failure intensity. Khatab and Aghezzaf [20] developed a selective maintenance decision model of two states' deteriorated systems using an imperfect maintenance model of randomly degraded components. Khatab et al. [21] extended Khatab and Aghezzaf's [20] study by considering the combined impact of periodic observations and imperfect maintenance. Gan et al. [22] proposed an innovative maintenance strategy for systems subject to normal degradation and impact. The shock process in the study is a nonhomogeneous Poisson process, and a changing factor is considered for the dependance between the normal degradation and the intensity of shocks. Gao and Xie [23] developed generalized reliability models and failure rate models of the mechanical systems, and the models consider strength degradation and imperfect maintenance.

The preceding studies demonstrated that selective maintenance activities have been expanding. However, in most research, imperfect maintenance is only considered as a single factor. Therefore, in this study, a more generalized hybrid imperfect maintenance model is employed to describe the component improvement following maintenance. It includes both age reduction and hazard adjustment factors. This assumption is more reasonable and general. We also discovered that many selective maintenance models were created without taking the overall system's reliability into account. Because the performance of the following missions may be affected by the maintenance approach stopped by the preceding work, several researchers have created novel selective maintenance models. Hou and Qian [24] develop a model where each subsystem operates on its own and has a specific sequence of missions with the different lengths. Yang et al. [25] consider a frequency-based selective maintenance model for a fleet-level multimission system. Pandey et al. [26] present a scheduling approach for determining the cost-optimal amount of periodic maintenance breaks over a finite horizon consisting of the nonidentical missions. Liu et al. [27] introduced a novel selected maintenance optimization approach for multistate systems capable of carrying out many continuous missions within a finite horizon. Yin et al. [28] considered the uncertainties associated with the durations of maintenance missions and breaks.

To summarize, we discovered that many selective maintenance strategies neglect the evaluation of overall system reliability. Simultaneously, they seldom consider such factors as a hybrid imperfect maintenance model, sufficient maintenance actions, and labor costs. Most selective maintenance models consider time and cost as resource constraints, with the implicit assumption of ample repair capacity (channels and repair crews) [29]. To address the foregoing problems, a novel selective maintenance model is devised and formulated as a max–min optimization. Compared to existing research, the unique contributions of this study are twofold:

(1) The proposed selective maintenance model can optimally allocate the limited maintenance resources for a repairable system executing multiple future missions.

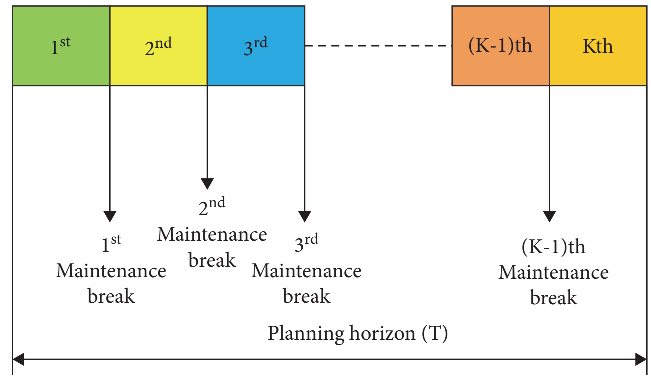

Figure 1: Maintenance break and execution missions within the finite planning horizon.

(2) A hybrid imperfect maintenance model is used to formulate the component improvement after maintenance.

The remainder of this article is described below. The problem description and basic assumptions are given in Section 2. Maintenance options and resource consumption are explained in Section 3 to determine the maintenance cost and time for the system. The imperfect maintenance model and mission reliability model will be described in Section 4. The selective maintenance model and solution methodology are provided in Section 5. Results and discussion are given in Section 6. Conclusions are summarized in Section 7.

2. Problem Description and Basic Assumptions

2.1. Problem Description. In this paper, a series–parallel system is considered where i $(i = 1, 2, \ldots, m)$ independent subsystems are connected in series, and each subsystem i has n_i $(j = 1, 2, \ldots, n_i)$ components connected in parallel. The above m and n_i represent the number of components in series and subsystems in parallel. It is assumed that the lifetime of each component follows the Weibull distribution and that the shape parameters and scale parameters of each component are different. Assuming a planning implementation table is established for a finite planning horizon (0, T), it is necessary to interrupt the maintenance of the planning period and perform maintenance operations on the system's components by their states to consider the probability of successful mission completion within the finite horizon. The planning horizon (0, T) is divided into K discrete equal breaks denoted as T_k $(k = 1, \ldots, K)$. Each break includes one operation period and one maintenance opportunity at the end of each mission (except the last mission within the given planning horizon). The duration of the kth maintenance is denoted by M_k. The detailed schematic diagram is shown in Figure 1 below. The problem we want to solve in this paper is how to find an optimal number of interruptions over a finite planning horizon to ensure maximum system reliability under the resource constraints.

2.2. Basic Assumptions of This Paper. For the above problem description, the basic assumptions of the problems considered in this paper are given:

(1) All components are new at the beginning of the planning horizon.
(2) The components, as well as the system, are in a binary state, i.e., it is either functioning or failed.
(3) The system consists of several maintainable components.
(4) After replacement, the component is "as good as new" and if minimal repair is performed, it is "as bad as old." Maintenance is also possible such that the component's health may lie between as good as new and as bad as old, i.e., maintenance can be modeled by the imperfect repair.
(5) The resources available are limited (cost and time) and the amount of resources required for maintenance activities is known.
(6) The components and subsystems in the system are independent of each other.
(7) It should be noted that the maintenance break in this paper does not occupy the total planning horizon, it only consumes the total maintenance time.

3. Maintenance Options and Resource Consumption

3.1. Description of Maintenance Options. There are N_{ij} maintenance options available for the components. Based on the degree of maintenance, the available maintenance options for components can usually be divided into four categories: replacement (RE), minimal repair (MR), imperfect maintenance (IM), and doing nothing (DN). For all these maintenance options, let l_{ij} indicate the level of maintenance available for the component, where $l_{ij} \in [0, 1, 2, \ldots, N_{ij}]$ can be used for the component. Here, $l_{ij} = 0$ denotes the "Do nothing" case when no maintenance is performed on the component and $l_{ij} = N_{ij}$ denotes the replacement of the component. If the component fails before maintenance, $l_{ij} = 1$ and $2 \leq l_{ij} < N_{ij}$ correspond to MR and IM activities, respectively. If the components are in normal functioning condition, $2 \leq l_{ij} < N_{ij}$ denotes IM activity. For each component in the system, the available maintenance options may be different. Related to these alternatives, cost and time estimation are provided next.

3.2. Consumption of Maintenance Resources. Components can be selected for maintenance or not. When $\left(l_{ij} = 0\right)$ denotes that the corresponding maintenance cost is equal to zero. However, if you select a component maintenance $\left(l_{ij} > 0\right)$, it will consume some budget. The expression for maintenance cost for a component can be given as follows:

$$C_{ij}\left(l_{ij}\right) = c^{f}_{ij,l_{ij}} + c_{ij,l_{ij}}, \tag{1}$$

where $c^{f}_{ij,l_{ij}}$ denotes the fixed cost of disassembly and assembly and $c_{ij,l_{ij}}$ is the variable cost of component maintenance. For $l_{ij} = 0$, $c^{f}_{ij,l_{ij}} = 0$, $c_{ij,l_{ij}} = 0$, and $l_{ij} = N_{ij}$, $c_{ij,l_{ij}} = C^{R}_{ij}$, where C^{R}_{ij} is the replacement cost of the component. When the component is in a failed state and $l_{ij} = 1$ denotes the variable maintenance cost associated with the minimal repair, i.e., $c_{ij,l_{ij}} = C^{M}_{ij}$. The intermediate maintenance action of the functioning component is defined as the maintenance action between the no maintenance option $\left(l_{ij} = 0\right)$ and replacement option $\left(l_{ij} = N_{ij}\right)$. The intermediate maintenance operation of the failed component is defined as the maintenance operation between the minimal maintenance option $\left(l_{ij} = 1\right)$ and the replacement option $\left(l_{ij} = N_{ij}\right)$. Assuming that the cost of each inspection is the same, the total cost of the whole system can be determined as follows:

$$C_s = \sum_{i=1}^{m} \sum_{j=1}^{n_i} C_{ij}\left(l_{ij}\right) + k \cdot C_{\mathrm{w}}, \tag{2}$$

where C_{w} represents the salary paid for each failed inspection.

Like maintenance cost, the time model for component maintenance is as follows:

$$T_{ij}\left(l_{ij}\right) = t^{f}_{ij,l_{ij}} + t_{ij,l_{ij}}, \tag{3}$$

where $t^{f}_{ij,l_{ij}}$ is fixed time, $t_{ij,l_{ij}}$ is the variable time associated with component maintenance, depending on the maintenance option $l_{ij} \in [0, 1, 2, \ldots, N_{ij}]$. For $l_{ij} = 0$, $t^{f}_{ij,l_{ij}} = 0$, $t_{i,j,l_{ij}} = 0$, and $l_{ij} = N_{ij}$, $t_{ij,l_{ij}} = T^{R}_{ij}$, where T^{R}_{ij} is the maintenance time of replacement. If a component fails, $t_{ij,l_{ij}} = \mathrm{T}^{M}_{ij}$ and $l_{i,j} = 1$, where T^{M}_{ij} is the time to perform minimal repair on the failed component. For $2 \leq l_{ij} < N_{ij}$, intermediate maintenance operations are performed on the failed components. If a component is functioning, $t_{ij,l_{ij}} = \mathrm{T}^{R}_{ij}$ and $l_{ij} = N_{ij}$, where T^{R}_{ij} is the replacement time of the functioning component. For $2 \leq l_{ij} < N_{ij}$, intermediate maintenance operations are performed on the functioning components. Therefore, the total maintenance time for the whole system can be determined as follows:

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n_i} T_{ij}\left(l_{ij}\right). \tag{4}$$

4. Imperfect Maintenance Model and Mission Reliability Model

4.1. Imperfect Maintenance Model. When replacement maintenance is performed, it is in the state of "as good as new" imperfect maintenance puts it somewhere between "as bad as old," and "as good as new." Nakagawa [10] proposed two preventive maintenance models where adjustment/improvement factors were considered in the hazard rate and the effective age for a PM policy. The combined mixed model including hazard adjustment and reduced age can be expressed as follows:

$$g\left(t_{ij} + x\right) = ah\left(bB_{ij} + x\right), \tag{5}$$

where $a\,(a \geq 1)$ and $b\,(0 \leq b \leq 1)$ represent the hazard rate adjustment factor and the age reduction factor, respectively. $h_i(\cdot)$ is the hazard rate function before maintenance, and $g(\cdot)$

is the hazard rate function after maintenance. B_{ij} and t_{ij} represent the component lifetime before and after maintenance, respectively. For selective maintenance, different improvement factor values are obtained according to different maintenance levels.

Using the hybrid model, the hazard rate function for a component i for $x \geq 0$ after the kth PM, in the $(k+1)$th mission, can be determined as follows by Liu et al. [27]:

$$g_{ij,k+1}(t_{k+1}+x) = A_{ij,k}h_o(b_{ij,k}B_{ij,k}+x), \tag{6}$$

where t_{k+1} denotes the start time of the $(k+1)$th mission; $A_{ij,k} = \prod_{q=1}^{k} a_{ij,q}$ represents the cumulative effect of the hazard adjustment on the hazard rate, and $B_{ij,k}$ is the effective age just before the kth maintenance. The corresponding hazard adjustment factor and age reduction factor are $(a_{ij,1}, a_{ij,2}, \ldots, a_{ij,k}) \geq 1$ and $(b_{ij,1}, b_{ij,2}, \ldots, b_{ij,k}) \leq 1$ for a component from the first to the kth maintenance. The effective age of the component after the kth maintenance becomes $b_{ij,k}B_{ij,k}$.

4.1.1. Cost-Based Age Reduction Factor. Generally, the quality of maintenance in component improvement depends on the number of resources used and the relative lifetime of components. For a specific imperfect maintenance action, younger components will result in the better maintenance efficiency. The hybrid imperfect maintenance model used in this study takes component age and maintenance cost into account. The age reduction factor can be defined as follows by Pandey et al. [19]:

$$b(B_{ij}, l_{ij}) = \begin{cases} 1 - \left(\dfrac{c_{ij}(l_{ij}) - c_{ij}^{M}}{c_{ij}^{R}}\right)^{m(B_{ij})}, & \text{for } Y_{i,j} = 0,\ 1 \leq l_{ij} > N_{ij} \\ 1 - \left(\dfrac{c_{ij}(l_{ij})}{c_{ij}^{R}}\right)^{m(B_{ij})}, & \text{otherwise.} \end{cases} \tag{7}$$

In Equation (7), $Y_{i,j} = 0$ denotes that the component is in the failed state and $m(B_{ij})$ is the characteristic index reflecting the relative age of the component. It is defined as the ratio of the effective age of the component, and the index (m) value for a component as given by Pandey et al. [19]:

$$m(B_{ij}) = \frac{B_{ij}}{\text{MRL}} = \frac{B_{ij} \times R(B_{ij})}{\int_{B_{ij}}^{\infty} R_{ij}(x)dx}. \tag{8}$$

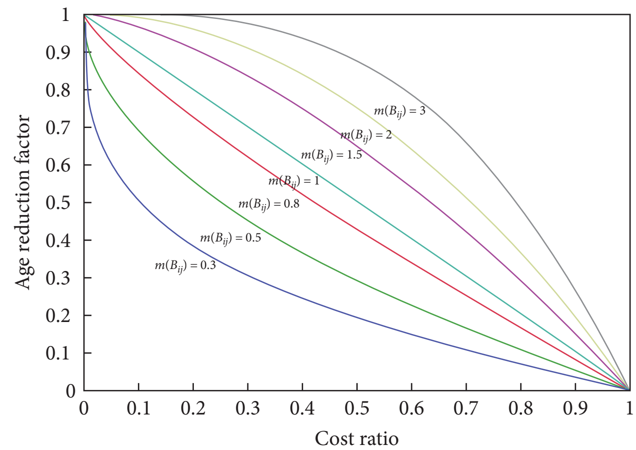

Figure 2: Age reduction factor versus cost ratio for different values of 'm'.

In Equation (8), MRL represents the mean residual life of the component. Based on MRL and effective age B_{ij}, the characteristic index of the components is determined. $R(B_{ij})$ is the reliability of component (i,j) at an effective age B_{ij}, and $R_{ij}(x)$ is the reliability function of component (i,j) at $t > B_{ij}$. For $B_{ij} < \text{MRL}$, $m(B_{ij}) < 1$, and $m(B_{ij}) > 1$, $B_{ij} > \text{MRL}$ [19]. In Figure 2, we can see the change of the age reduction factor relative to the cost ratio under the different $m(B_{ij})$.

Figure 2 above shows that, for the same set of circumstances, the factors corresponding to various m values are also different. When using fewer budgets for component maintenance or when the ratio of maintenance cost to maintenance replacement cost is low, the degree of component renewal is also low. In contrast, when using more budgets for component maintenance, the degree of component renewal is likewise larger. It also shows that for a fixed age reduction factor, the required budget increases as the component ages (i.e., m increases).

When multiple missions need to be considered within a planning horizon, a component may undergo multiple maintenance operations. In this case, the hazard rate and reliability function $R_{ij}(x)$ of the component will change. It is then required to derive a formulation for the characteristic index m that could be used in the subsequent break in a maintenance scheduling problem. Assuming the age of the component follows a Weibull distribution with scale and shape parameters α_i and β_i, respectively, and the formulation is redefined as follows by Liu et al. [27]:

$$m(B_{ij,k}) = \frac{B_{ij,k} \times R(B_{ij,k})}{\exp\left(\frac{A_{ij,k-1}}{\alpha_{ij}^{\beta_{ij}}}(b_{ij,k-1}B_{ij,k-1})^{\beta_{ij}}\right) \times \int_{B_{ij,k}}^{\infty} \exp\left(-\frac{A_{ij,k-1}}{\alpha_{ij}^{\beta_{ij}}}(b_{ij,k-1}B_{ij,k-1}+x)^{\beta_{ij}}\right)dx}. \tag{9}$$

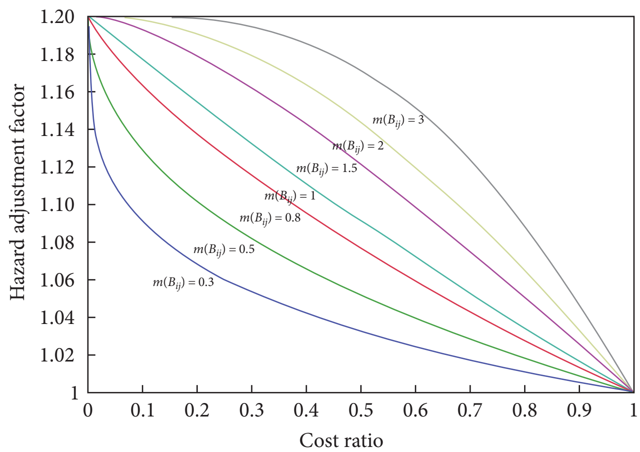

FIGURE 3: Hazard adjustment factor versus cost ratio for different values of ‘m’.

4.1.2. Cost-Based Hazard Adjustment Factor. The cost-based hazard adjustment factor model, like the cost-based age reduction model, may be stated as follows:

$$a\left(B_{ij}, l_{ij}\right) = \begin{cases} \dfrac{h}{\left((h-1) + \left(\dfrac{\left(c_{ij}\left(l_{ij}\right) - c_{ij}^{M}\right)}{c_{ij}^{R}}\right)^{m\left(B_{ij}\right)}\right)}, & \text{for } Y_{i,j} = 0,\ 2 \leq l_{ij} > N_{ij} \\ 1, & \text{for } l_{ij} = 0\text{ , and } Y_{ij} = 0,\ l_{ij} = 1, h > 1 \\ \dfrac{h}{\left((h-1) + \left(\dfrac{c_{ij}\left(l_{ij}\right)}{c_{ij}^{R}}\right)^{m\left(B_{ij}\right)}\right)}, & \text{otherwise.} \end{cases} \tag{10}$$

In Equation (10), h determines the maximum allowable hazard increment for a component, i.e., it defines the upper limit of the hazard adjustment factor that a component can achieve after a maintenance break. For the same cost ratio, with the aging of components, the value of m increases, and the hazard adjustment factor of components also increases. It also shows that under the fixed value of the characteristic constant (i.e., the fixed effective age), the hazard adjustment factor varies with the usage of the maintenance budget. At the time of selective maintenance, the component’s effective age is known, and the only decision variable is the level of imperfect maintenance corresponding to which the PM cost $C_{ij}\left(l_{ij}\right)$ for a component is determined. When the cost ratio is at a specific imperfect maintenance level l_{ij}, the corresponding hazard adjustment factor can be found using the Equation (10). Figure 3 below shows the change of the hazard adjustment factor relative to the cost ratio when $h = 6$ under different m. The maximum allowable value of the hazard adjustment factor calculated in Figure 3 is 1.2.

It can be seen from Figure 3 above that under the same conditions, the hazard adjustment factors corresponding to the different m values are also different. When the cost ratio between maintenance cost and maintenance replacement is small, that is, when less maintenance budget is used to carry out maintenance actions on the components, the hazard rate after maintenance will increase faster (the value of the hazard adjustment factor is higher). On the contrary, when more maintenance budgets are used to carry out maintenance actions on the components, the hazard rate after maintenance will increase more slowly (the value of the hazard adjustment factor is lower). It also shows that for a fixed hazard adjustment factor, the budget required will increase as the component ages (i.e., m increases).

4.2. Mission Reliability Model. The maintenance effect of various components in the system is related to their maintenance activities, and the effect can be defined as follows:

$$g_{ij,k}\left(t_{ij,k} + x\right) = \begin{cases} h_{ij,k-1}\left(B_{ij,k-1} + x\right), & l_{ij,k-1} \in \{0, 1\} \\ h_{ij,k-1}(x), & l_{ij,k-1} = N_{ij} \\ A_{ij,k-1} h_{ij,k-1}\left(b_{ij,k-1} B_{ij,k-1} + x\right), & \text{otherwise,} \end{cases} \tag{11}$$

where $l_{ij,\,k-1}$ represents the maintenance level at the end of the $(k-1)$th mission, when $l_{ij,\,k-1} \in \{0, 1\}$, the age of the component does not change, that is, it has no impact on the reliability of the component; when $l_{ij,k-1} = N_{ij}$, it means that the components are replaced and the age is cleared; in other cases, it means imperfect maintenance, which can be obtained through the above formulation.

Let $X_{ij,k}$ and $Y_{ij,k}$ denote the state of the component at the beginning of mission k $(k = 1, 2, \ldots)$ and at the end of mission k $(k = 1, 2, \ldots)$, respectively. At the beginning of mission k, the state of the component can be expressed as follows:

$$X_{ij,k} = \begin{cases} 1, & \text{if component} j \text{ of subsystem } i \\ & \text{is functioning at the start of mission } k. \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

Similarly, the state of the subsystem and the whole system at the beginning of mission k is also represented by (0, 1), where 0 represents the failure state and 1 represents the functioning state.

Likewise, at the end of mission k, the state of the component can be written as follows:

$$Y_{ij,k} = \begin{cases} 1, & \text{if component of subsystem } i \\ & \text{is functioning at the start of mission } k. \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

The state of the subsystem and the whole system at the end of mission k is also represented by (0, 1), where 0 represents the failure state and 1 represents the functioning state. Then, the reliability of the component in the kth mission is expressed as follows:

$$R_{ij,k} = \exp\left(-\int_0^{T_k} h_{ij,k}\left(t_{ij,k} + x\right)dx\right) \cdot X_{ij,k}. \quad (14)$$

The reliability of subsystem in the kth mission is given as follows:

$$R_{i,k} = 1 - \prod_{j=1}^{n_i}\left(1 - R_{ij,k}\right). \quad (15)$$

The state of the entire system at the end of a mission k can be determined as follows:

$$R_k = \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n_i}\left(1 - R_{ij,k}\right)\right). \quad (16)$$

The probability to finish the next mission can be recursively determined for each component using its initial state, age at the beginning of the next mission, and the mission duration. Thus, the reliability of the whole system can be determined using Equation (16).

The system's reliability is composed of multiple missions. Therefore, combined with the above analysis, the system reliability within the finite planning horizon is given as follows:

$$R_s = \prod_{k=1}^{K} R_k = \prod_{k=1}^{K}\left(\prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n_i}\left(1 - R_{ij,k}\right)\right)\right), \quad (17)$$

where R_s represents the system reliability within the finite planning horizon. It is found in Equation (17) that we should not only consider the reliability of a single mission but also comprehensively consider the reliability of multiple subsystems.

5. Selective Maintenance Model and Solution Methodology

5.1. Selective Maintenance Model. Designing the number of mission interruptions within the finite planning horizon has a critical impact on the reliability of the whole system. Different numbers of mission interruptions affect the reliability of each subsystem mission and then affect the whole system. Suppose that the length of a planning horizon is 300. According to the above analysis, if the mission is interrupted three times, the length of each subsystem mission is 75. If the mission is interrupted four times, the length of each subsystem mission is 60. It can be found that as the number of interruptions increases, the length of each mission decreases, and it may be possible to ensure the high reliability of the next mission without taking on the maintenance activities. If the number of mission interruptions decreases, the mission length of the subsystem will become longer, and coupled with the lack of reasonable maintenance activities, the reliability of the whole system will become extremely poor. In addition, if the planning horizon is interrupted too much, the cost of fault detection will increase, so it is necessary to find the optimal interruption number.

This study adopts a max–min optimization model [30]. Assuming that the total maintenance cost and total maintenance time in the planning horizon are $C_{\max}$ and $T_{\max}$, respectively. When the system enters the first mission break, all components are in a new state. The decision model for this objective can be obtained from the max–min criteria as follows:

Objective:

$$\max\text{–}\min\left(R_{s,k=2}, R_{s,k=3}, R_{s,k=4}, \ldots, R_{s,k=K}\right). \quad (18)$$

Subject to:

$$\sum_{\mathrm{k}=1}^{K} M_{\mathrm{k}} \leq T_{\max}, 1 \leq k \leq K - 1, \quad (19)$$

$$C_s \leq C_{\max}, \quad (20)$$

$$0 \leq l_{ij} \leq N_{ij}. \quad (21)$$

Equation (18) represents the max–min criterion, and the optimal schedule is selected by comparing the results of the worst case. Constraints in the Equations (19) and (20) exhibit the limited available resources to perform maintenance, and Equation (21) denotes the scope of maintenance levels for a component.

5.2. Solution Methodology. The selective maintenance decision model in this paper is a complex nonlinear programing problem with continuous decision variables, and the problem will become more complex with the increase in the number of units and the number of mission stages. At present, a variety of intelligent optimization algorithms can find the global optimal solution (or global approximate optimal solution) within a reasonable time. Such as genetic algorithm (GA), Stochastic Fractal Search Approach, Tabu Search, and Ant Colony Optimization (ACO). In this study, a modified GA, namely the simulated annealing-based genetic algorithm [30], is customized for our specific problem.

5.2.1. Construct Population Individuals. The key to using the simulated annealing-based GA to solve optimization problems is to express feasible solutions as population individuals in the GA. In this paper, the maintenance cost is divided into several maintenance levels, so the feasible solution corresponding to the individual population is expressed as follows:

$$L = \left\{ \underbrace{l_{1,1}, \cdots, l_{q,1}}_{\substack{\text{Maintenance}\\ \text{break}}} 1^{\text{st}}, \cdots \underbrace{l_{1,k}, \cdots, l_{q,k}}_{\substack{\text{Maintenance}\\ \text{break}}} k\text{th}, \underbrace{l_{1,k+1}, \cdots, l_{q,k+1}}_{\substack{\text{Maintenance}\\ \text{break}}} (k+1)\text{th} \cdots, \underbrace{l_{1,K}, \cdots, l_{q,K}}_{\substack{\text{Maintenance}\\ \text{break}}} K\text{th} \right\}, \tag{22}$$

where $l_{q,k}$ is a decimal integer denoting the discretized maintenance level for a component in the kth break.

The feasible solution must not be the ideal solution if the total maintenance cost associated with solution L is less than the maintenance budget; if the overall maintenance cost exceeds the maintenance budget, the solution is not feasible. In the case that the total maintenance cost is insufficient or exceeds the maintenance budget, this paper adopts the methods of cyclic increase and cyclic decrease to make the total maintenance cost just reach the maintenance budget.

5.2.2. Simulated Annealing-Based Genetic Algorithm. The GA imitates the principle of "survival of the fittest" in the process of biological evolution, regards the set of feasible solutions in the optimization problem as a population, selects, crossover, and mutation individuals from the population, obtains a new generation of the population with better fitness, and continuously iterates to obtain the optimal solution of the problem. This algorithm has the advantages of high efficiency and global optimization in solving complex nonlinear optimization problems and has been widely used in the field of reliability engineering [31–33]. Simulated annealing algorithms and GAs can complement each other, which can effectively overcome the premature phenomenon of the traditional GAs. Therefore, this paper will use a simulated annealing GA to solve the optimization problem of selective maintenance decisions under a finite planning horizon.

(1) Algorithm parameter setting

The parameters of the simulated annealing-based GA include maximum iteration number N_{iter}, population individual number N_p, crossover probability P_c, mutation probability P_m, exchange probability P_s, annealing temperature T_0, annealing coefficient η, and error function limit τ.

(2) Coding and individual fitness

The coding methods for decision variables include binary and real-valued coding [30]. Since real-value coding is more efficient for solving combinatorial optimization problems, real-value coding will be used in this study. Real-valued coding means that each gene value in an individual is represented by a real number, and the length of individual coding is equal to the number of its decision variables.

(3) Error function

The simulated annealing-based GA uses the error function to represent the individual differences in the population, and its expression is:

$$E_r = \frac{\|Y_{\text{best}} - Y_{\text{worst}}\|_2}{\|Y_{\text{best}}\|_2}, \tag{23}$$

where Y_{best} and Y_{worst} represent the optimal individual and the worst individual in the population, respectively.

(4) Termination condition

If the number of algorithm iterations reaches the maximum number of iterations or the error function is less than the tolerance of the error function, the algorithm terminates.

(5) Selection, crossover, mutation, and swap

Table 1: Component parameters (unit of cost: US \$1,000; unit of time: d).

ID	β_i	α_i	l_{ij}	$c_{ij,l_{ij}}$	$t_{ij,l_{ij}}$	ID	β_i	α_i	l_{ij}	$c_{ij,l_{ij}}$	$t_{ij,l_{ij}}$
1	1.5	300	1	8	0.35	8	2.0	375	1	12	0.3
			2	9	0.4				2	15	0.35
			3	21	0.75				3	24	0.55
			4	40	1.25				4	42	1
2	2.4	300	1	5	0.36	9	1.2	400	1	8	0.55
			2	7	0.4				2	10	0.6
			3	20	0.65				3	25	0.8
			4	36	1				4	45	1.4
3	1.6	250	1	6	0.38	10	1.4	400	1	8	0.4
			2	8	0.4				2	9	0.45
			3	18	0.75				3	20	0.7
			4	40	1.25				4	42	1.45
4	2.6	400	1	9	0.5	11	2.8	450	1	9	0.38
			2	10	0.55				2	10	0.4
			3	25	0.8				3	20	0.65
			4	45	1.3				4	35	1.15
5	1.8	400	1	9	0.5	12	1.5	450	1	8	0.38
			2	10	0.55				2	9	0.4
			3	25	0.8				3	20	0.65
			4	45	1.3				4	38	1.25
6	2.4	375	1	9	0.38	13	2.4	425	1	9	0.5
			2	10	0.4				2	10	0.55
			3	25	0.65				3	25	0.75
			4	38	1.15				4	40	1.15
7	2.5	400	1	9	0.4	14	2.2	400	1	10	0.5
			2	10	0.45				2	10	0.55
			3	30	0.7				3	20	0.8
			4	40	1.05				4	40	1.45

The selection, crossover, and mutation operations in simulated annealing-based GAs are the same as those in traditional GAs.

The roulette wheel selection method is used in the selection operation in this study. The crossover operation generates new individuals Ξ_{c1} and Ξ_{c2} with probability P_c, and the mutation operation generates new individuals Ξ_m with probability P_m. To order to make the population more diverse, the simulated annealing-based GA in this study increases the swap operation compared with the traditional GA. The swap operation refers to the swap of genes at both sides (left and right) of the same individual with probability P_s to generate a new individual Ξ_s. For example, an individual is coded as $x = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, and a new individual $\Xi_s = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ is obtained after the swap operation.

(6) Individual acceptance criteria

For the individual $x_i\,(i = 1, 2, \cdots, N_p)$ in the population, a new individual set $\{\Xi_{c1}, \Xi_{c2}, \Xi_m, \Xi_s\}$ is obtained through crossover, mutation, and swap operations, and the optimal individual Ξ_{new} is selected from it. The individual acceptance criterion is as follows: (1) If Ξ_{new} is better than x_i, update x_i to Ξ_{new}; (2) if Ξ_{new} is worse than x_i, Ξ_{new} shall be accepted according to metropolis criteria. The Metropolis criterion accepts the generated new solution with probability, which is described as randomly generating a random number h between 0 and 1. If the condition $h < \min\{1, \exp\{-\| \Xi_{\mathrm{new}} - x_i\|_2/T_0\}\}$ is satisfied, x_i is updated to Ξ_{new}.

6. Results and Discussion

To further evaluate the proposed method and explain the advantages of the proposed model, we have considered the example of a coal transportation system [17]. It includes five basic subsystems, as shown in Figure 3. Feeder 1 (Subsystem 1) transfers coal from the bin to Conveyor 1. Conveyor 1 (Subsystem 2) transports the coal from Feeder 1 to the stacker reclaimer (Subsystem 3), which lifts the coal to the burner level. Feeder 2 (Subsystem 4) then loads Conveyer 2 (Subsystem 5), which transfers the coal to the boiler's burner feeding system. The parameter values of each component, such as Weibull life distribution parameters, maintenance actions, related costs, and time, are shown in Table 1, in which the maintenance fixed cost and maintenance fixed time are assumed to be 0.

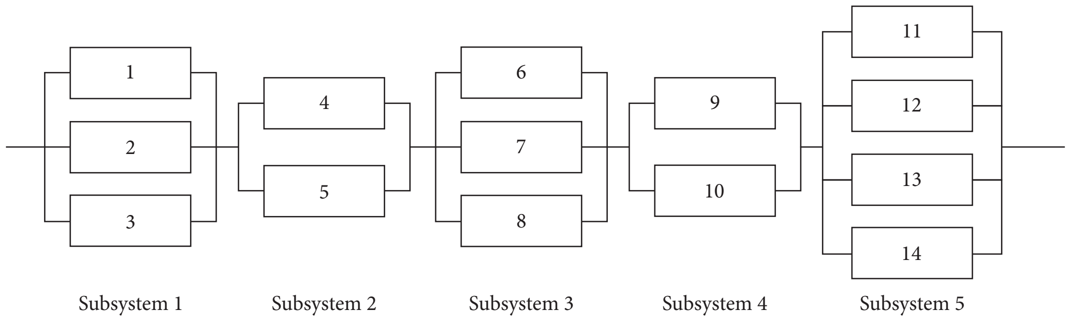

FIGURE 4: Coal transmission system.

TABLE 2: Optimal selective maintenance decision.

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
Component			2, 4, 9, 10	3, 4, 6	2, 5, 7	
Action	DN	DN	RE, RE, RE, IM	IM, IM, RE	IM, RE, IM	
Level			4, 4, 4, 3	3, 3, 4	3, 4, 3	
Time			4.4	2.7	2.65	
Reliability	0.9930	0.9537	0.9369	0.9489	0.9637	0.9758
Total cost	472					

We assume the coal transmission system needs to complete the mission with a total length of 360, and the cost of each fault detection is 30. The maximum maintenance time $T_{\max}$ and the maximum maintenance cost $C_{\max}$ are 10 units and 500 units, respectively. First, the ages and states of various components in the system under different interruption times are obtained through the Monte Carlo simulation, then the optimal maintenance schedule is obtained according to the selective maintenance decision model proposed in this paper (Figure 4), and finally, the system reliability value of the system after the optimal maintenance decision schedule can be obtained, as shown in Table 2. It can be seen from Table 2 that after the first mission, the reliability during the second mission is high even if no maintenance action is taken due to the short length of the completed mission.

Our goal is to maximize the reliability of the overall system. It is assumed that operational errors after the first mission may lead to a lack of resources for subsequent maintenance. Therefore, low-reliability components should be maintained first, and the reliability of weak links in the system should be improved to ensure that each link can operate normally. In addition, it can be found that due to the limitation of maintenance time, the remaining 28 maintenance cost units can no longer be used.

To better understand the relationship between different numbers of interruptions and system reliability, we have made the relationship between the number of interruptions and system reliability, as shown in Figure 5.

It can be seen from Figure 5 that when the number of interruptions is less than or equal to 6, the system reliability increases with the increase in the number of divided interruptions. When the number of interruptions exceeds 6, the system's reliability decreases. The important reason for this is that with the increase in the number of interruptions, the

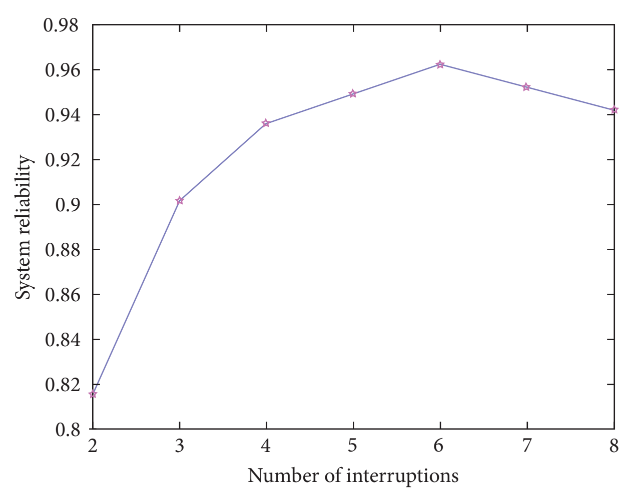

FIGURE 5: Relationship between the number of interruptions and system reliability.

detection cost also increases. The decrease in the cost ratio of limited resources to component maintenance will inevitably lead to a decrease in the system reliability. Table 2 also shows that with the increase in the number of divided breaks, the first few missions have good reliability without maintenance, but there will be a reliability trough after multiple missions. If the limited maintenance time is used when the previous reliability is high, the reliability of the later missions may be lower. Therefore, it is the optimal decision-making schedule to divide the finite planning horizon six times under the limited resources, and the average reliability of the system reaches 0.962.

As the cost of fault detection needs to be consumed, if the number of interruptions increases, the remaining maintenance

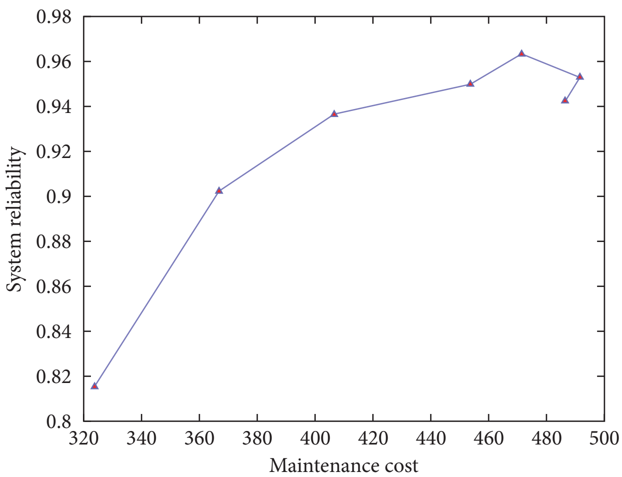

Figure 6: Relationship between maintenance cost and average system reliability.

cost will be reduced, resulting in a decline in the system reliability. To understand the relationship between the cost required for each interruption number and the average system reliability, we have made a diagram of the relationship between maintenance cost and system reliability under the different interruption numbers, as shown in Figure 6.

It can be seen from Figure 6 that the system reliability increases with the increase in maintenance costs at the beginning. When the average maintenance cost exceeds 472 units, the system's reliability begins to decline. Therefore, we should learn to make full use of maintenance resources. For example, when the number of interruptions is 7, the average maintenance is 492 units, but the reliability value obtained is not the highest. The main reason is that too many interruptions consume more maintenance costs for the fault detection. Therefore, it is a major challenge for decision-makers to spend limited resources on the critical points. In addition, we can also observe that the number of interruptions is not directly proportional to the maintenance costs, and the specific maintenance costs are related to the selection of maintenance schedules. The relationship between interruption times, maintenance costs, and system reliability is shown in Figure 7.

In many studies, they believe that maintenance is immediate, ignoring maintenance time. However, maintenance does take some time, and it is important to include it in the modeling and find out its impact on the maintenance operations and system reliability. To reflect the impact of maintenance time on the system reliability, we analyzed the total maintenance time in Table 2 with three different time limits, as shown in Figure 8 below.

Figure 8 shows that system reliability depends on the total maintenance time. When the required system reliability increases from 95% to more than 96%, the total maintenance time increases by 1 unit, from 9 units to 10 units, which is enough to reach the required reliability limit. However, when the expected reliability increases from 96% to more than 97%, the required maintenance time increases from 10 units to 13 units (an increase of 3 units). Therefore, whether the

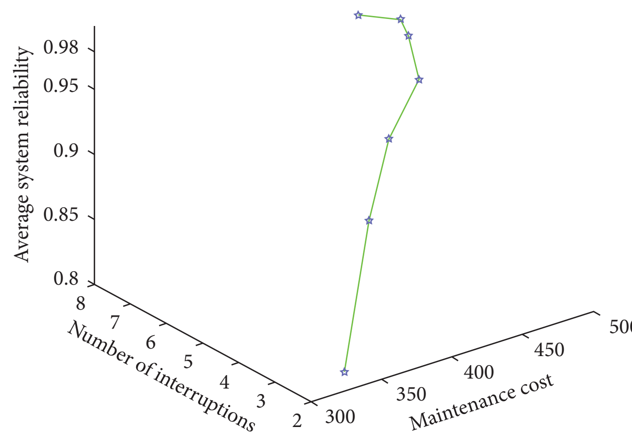

Figure 7: Relationship among average system reliability, interruption, and maintenance cost.

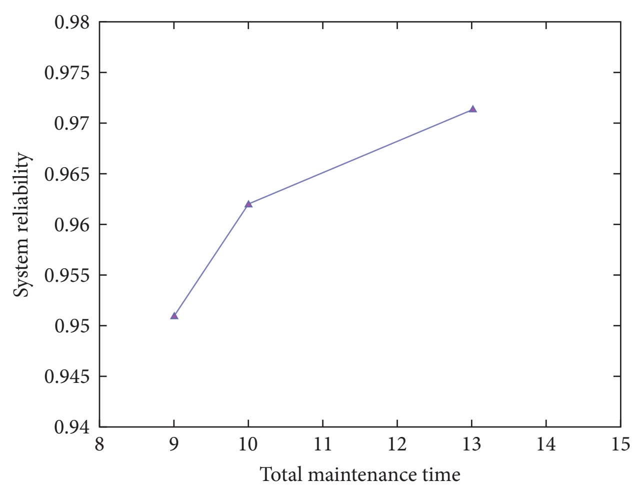

Figure 8: Relationship between different total maintenance times and average reliability.

expected reliability limit can be reached depends on the available maintenance time. This shows that it is inappropriate to ignore the limited time in the research work, especially in the scheduling within a finite planning horizon, because the system reliability that can be achieved after maintenance is sensitive to the available maintenance duration. Therefore, for maintenance managers, it is important to check the changes between maintenance time and system reliability so that they can determine whether the expected reliability limit can be reached within a given maintenance time and accurately set maintenance time to avoid resource waste.

7. Conclusions

In this research, a maintenance scheduling model under imperfect maintenance is developed for a given finite planning horizon. A hybrid imperfect maintenance model is used to formulate the component improvement after maintenance. It includes both age reduction and hazard adjustment factors. The model comprehensively evaluates the impact of maintenance actions on the subsequent missions. Through the max–min criterion, the optimal maintenance schedule can be calculated. The case analysis shows that the decision model proposed in this paper is feasible and effective for the selective maintenance problem in the finite planning horizon. It can help decision-makers to find the optimal number of mission interruptions to determine a plan of action for each component in the system while maximizing overall system reliability over the planning horizon. We also found that the optimal number of interruptions can save resources, maintenance time has a great impact on reliability, and the cost of failure inspection also affects maintenance. In addition, it is assumed in this paper that the length of missions divided within the finite planning horizon is equal and that there is no correlation between the system components involved. The mission length may change with the demand, and there may be a variety of correlations between components. In the future, we can further study and explore this problem and expand the model.

Data Availability

The relevant data of calculation used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

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