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Research Article

Robust Control for Stochastic Nonlinear Delay Systems with Jumps

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The problem of infinite horizon H_∞ control for general delayed nonlinear stochastic Markov jump systems with the infinite jumping parameters is considered in this paper, in which the noise is dependent on the state, control, and external disturbance. The coupled Hamilton–Jacobi inequalities (HJIs)-based sufficient condition is given to ensure the existence of the H_∞ controller. As a corollary, infinite horizon H_∞ controllers are designed for nonlinear stochastic time-delay systems without jumps by solving a series of coupled HJIs. Besides, the effectiveness of the proposed method is verified by a numerical example.

1. Introduction

As we all know, Markov jump systems have been used widely both in theory and in engineering over the past decades [1]. In the practical life, the occurrence of parts failures and the change of the relationship between subsystems, as well as the sudden environmental disturbances, will cause the jump of systems structure or parameters. The Markov jump system may be a prefect model in describing these phenomena. As perturbations are unavoidable in practical systems [2], in recent years, many researchers have paid attention to the problem of stability and control for It $\hat{o}$-type Markov jumps stochastic systems; see [3–6] and the references therein.

It is noteworthy that most of the existing works on jump systems have been carried out in the finite state Markov process, that is, its state space is a finite set. In fact, some physical variables may be described more appropriately with infinite jump states. For instance, in the solar heat receiver model proposed in [7], the atmospheric parameters take the values in a Borel measurable infinite set. It is needed to emphasize that there are essential differences in performance between finite and infinite Markov jump systems (IMJSs), and the causal and anticausal Lyapunov operators of infinite Markov jump systems being no more adjoint is the reason. The authors [8] have pointed out that exponential stability and stochastic stability for IMJSs are no longer equivalent. In addition, IMJSs have attracted an increasing interest; see [9–16].

On the other hand, time delay and nonlinearity, which often occur in engineering, biological, and economic systems, are the important reasons for systems to be instable or performance being destroyed [17]. Notice that the robust stability and H_∞ control have been investigated a little for nonlinear stochastic systems with jumps and delay. The authors in [18] solved the infinite horizon H_∞ control for nonlinear IMJSs with disturbance-, control-, and state-dependent noise, but the effects of time delay is neglected. For the nonlinear delayed system with finite Markov jump, the authors in [19] designed its H_∞ controller. In conclusion, the research on stability and control of nonlinear stochastic systems with infinite Markov jumps and time delay has important theoretical meaning. However, as far as we know, these issues have not been fully investigated so far, which greatly inspires our research interest.

The problem of H_∞ control is mainly solved in this paper for general nonlinear delay stochastic systems with infinite Markov jumps and (x, u, v)-dependent noise. The main contributions are concluded as follows: First, we develop an infinite horizon asymptotically mean square stable H_∞ controller design method based on the complete square technique and Itô's formula. A numerical example shows the effectiveness of the proposed method. Second, a nonlinear stochastic bounded real lemma is derived as a byproduct. Compared with the previous work, our results have a wider range of applications. The work reported in [19], for example, is a special case of this paper.

The following notations are used in this study: R^n: the n-dimensional Euclidean space; $\|x\|$: the Euclidean norm of n-dimensional real vector x; $L^2_{\mathscr{F}}(R^+;R^l)$: the space of all nonanticipative stochastic processes $y(t)\in R^l$ with respect to an increasing σ-algebra $\mathscr{F}_t$ satisfying $\|y(t)\|_{L^2_{\mathscr{F}}(R^+_l;R^l)} = E(\int_0^\infty \|y(t)\|^2 \mathrm{d}t)^{1/2} < \infty$; I: the identity matrix; A': the transpose of matrix A; $A\geq 0\,(A>0)$: A is positive semi-definite (positive definite); $C^{2,1}(U;T)$: the class of functions $V(x,t)$ which are twice continuously differentiable with respect to $x\in U$ and once continuously differentiable with respect to $t\in T$ except possibly at the point $x=0$; $C([-\delta,0];R^n)$: the vector space of all continuous R^n-valued functions defined on $[-\delta,0]$; $col(x_1,x_2,\ldots,x_n) := [x_1,x_2,\ldots,x_n]'$; and S_n: the set of symmetric matrices.

2. Preliminaries

Consider the following nonlinear delay system with infinite jumps:

$$\begin{cases} \mathrm{d}x(t) = [f(x,x_\delta,t,\theta_t) + k(x,x_\delta,t,\theta_t)u + s(x,x_\delta,t,\theta_t)v]\mathrm{d}t, \\ +[g(x,x_\delta,t,\theta_t) + h(x,x_\delta,t,\theta_t)u + q(x,x_\delta,t,\theta_t)v]\mathrm{d}\omega(t), \\ z(t) = \mathrm{col}(m(x,x_\delta,t,\theta_t),u) := \begin{pmatrix} m(x,x_\delta,t,\theta_t) \\ u \end{pmatrix}, \\ x(t) = \Phi(t) \in \mathscr{C}^b_{\mathscr{F}_0}([-\delta,0];R^n), \end{cases} \tag{1}$$

where $x_\delta = x(t-\delta)$ is the time delay state, $x(t)\in R^n$ is the system state, $u(t)\in R^{n_u}$ is the control input, $v(t)\in R^{n_v}$ represents the multiplicative noise, and $z(t)\in R^{n_z}$ is the measured output. $\omega(t)$ is the standard one-dimensional Wiener process on a complete filtered space $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\in R^+},P)$, and the filtration $\{\mathscr{F}_t\}_{t\in R^+}$ satisfies usual conditions. $\mathscr{C}^b_{\mathscr{F}_0}([-\delta,0];R^+)$ defines all $\mathscr{F}$-measurable bounded $\mathscr{C}([-\delta,0];R^+)$-valued random variable $\varphi=\{\varphi(\xi): -\delta\leq\xi\leq 0\}$ with $E\|\varphi\|^2<\infty$, where $\|\varphi\| = \sup_{-\delta\leq\xi\leq 0}\|\varphi(\xi)\|$. The jumping process θ_t is a continuous-time discrete-state Markov process which takes values in an infinite set $D=\{1,2,\cdots\}$ with the generator $\Gamma=(\pi_{rh})_{r,h\in D}$, that is,

$$P\{\theta_{t+k}=h \mid \theta_t=r\} = \begin{cases} \pi_{rh}k + o(k), & \text{if } r\neq h, \\ 1+\pi_{rr}k+o(k), & \text{if } r=h, \end{cases} \tag{2}$$

where $k>0$, $\lim_{k\longrightarrow 0}(o(k)/k)=0$, $\pi_{rh}\geq 0\,(r,h\in D, r\neq h)$ is the switching rate from mode r at time t to mode h at time $t+k$ and $\pi_{rh} = -\sum_{h\in D, r\neq h}\pi_{rh} < \infty$ for all $r\in D$. The processes θ_t and $\omega(t)$ are supposed to be independent in this paper. For every $\theta_t = r\in D$, the local Lipschitz condition and the linear growth condition are satisfied for f,k,s,g,h,q, and m, which can ensure that system (1) has a unique strong solution [6].

Let $f(0,0,t,r)\equiv 0$, $g(0,0,t,r)\equiv 0$, and $\forall (t,r)\in R\times D$. For $V\in\mathscr{C}^{2,1}(R^n\times R\times D;R)$, the following infinitesimal generator $\mathscr{L}V$: $R^n\times R^n\times R\times D\longrightarrow R$ associated with (1) is denoted [6]:

$$\begin{aligned} \mathscr{L}V(x,y,t,r) = {} & \frac{\partial V(x,t,r)}{\partial t} + \frac{\partial V'(x,t,r)}{\partial x}[f(x,y,t,r)+k(x,y,t,r)u+l(x,y,t,r)v] \\ & + \frac{1}{2}[g(x,y,t,r)+h(x,y,t,r)u+q(x,y,t,r)v]'\frac{\partial V^2(x,t,r)}{\partial x^2} \\ & \times[g(x,y,t,r)+h(x,y,t,r)u+q(x,y,t,r)v] + \sum_{h=1}^{\infty}\pi_{rh}V(x,t,r). \end{aligned} \tag{3}$$

To design the infinite horizon H_∞ controller for system (1), the internal stability requirement is needed. Thus, the definition of stochastic stability is introduced as follows.

Definition 1 (see [6]). The nonlinear stochastic delayed system given by

$$\begin{cases} \mathrm{d}x(t) = f(x,x_\delta,t,\theta_t)dt + g(x,x_\delta,t,\theta_t)\mathrm{d}\omega, \\ x(t) = \Phi(t)\in\mathscr{C}^b_{\mathscr{F}_0}([-\delta,0];R^n), \end{cases} \tag{4}$$

is stable in probability (SIP) if

$$\lim_{x_0\longrightarrow 0} P\left(\sup_{t\geq 0}\|x(t)\|>\varepsilon\right) = 0, \quad \forall\varepsilon>0. \tag{5}$$

If system (4) is SIP and

$$P\left\{\lim_{t\longrightarrow\infty} x(t)=0\right\}=1, \tag{6}$$

then it is called to be globally asymptotically stable in probability (GASIP). System (4) is asymptotically stable in mean square (ASMS) if

$$\lim_{t\longrightarrow\infty} E\|x(t)\|^2=0. \tag{7}$$

Definition 2. For given $\gamma>0$, $u(t)=u^*(t)\in L^2_{\mathscr{F}}(R^+;R^{n_u})$ is called an infinite horizon H_∞ controller for system (1), if the following conditions are met:

(i) When $v=0$, system (1) with $u(t)=u^*(t)$ is internally stable, i.e., the system

$$\begin{aligned}\mathrm{d}x(t)&=[f(x,x_\delta,t,\theta_t)+h(x,x_\delta,t,\theta_t)u^*(t)]\mathrm{d}t\\&\quad+[g(x,x_\delta,t,\theta_t)+h(x,x_\delta,t,\theta_t)u^*(t)]\mathrm{d}\omega,\end{aligned} \tag{8}$$

is ASMS.

(ii) For $\forall v\in L^2_{\mathscr{F}}(R^+;R^{n_v})\neq 0$,

$$\|z\|_{L^2_{\mathscr{F}}(R^+;R^{n_z})}\leq\gamma\|v\|_{L^2_{\mathscr{F}}(R^+;R^{n_v})},\quad x(0)=0. \tag{9}$$

Remark 3. Let the perturbation operator $\|\mathscr{L}^{u^*}_\infty\|$ be denoted by $\mathscr{L}^{u^*}_\infty$: $L^2_{\mathscr{F}}(R^+;R^{n_v})\mapsto L^2_{\mathscr{F}}(R^+;R^{n_z})$ as

$$\mathscr{L}^{u^*}_\infty(v)=z(x(t,u^*,v,\theta_t)),\quad t\geq 0. \tag{10}$$

Its norm is

$$\left\|\mathscr{L}^{u^*}_\infty\right\|=\sup_{\substack{v\in L^2_{\mathscr{F}}(R^+;R^{n_z})\\ v\neq 0,x_0=0,\theta_0\in D}}\frac{\|z\|_{L^2_{\mathscr{F}}(R^+;R^{n_z})}}{\|v\|_{L^2_{\mathscr{F}}(R^+;R^{n_v})}}. \tag{11}$$

It is easy to verify that (9) is equivalent to $\|\mathscr{L}^{u^*}_\infty\|\leq\gamma$.

Lemma 4 (see [20]). *If there exists a positive Lyapunov function $V(x,t,r)\in C^{2,1}(R^n\times R\times D;R)$ satisfying $\mathscr{L}V(x,t,r)<0$ for $x\neq 0$ and $V(x,t,r)$ being radially unbounded, i.e.,*

$$\lim_{\|x\|\longrightarrow\infty}\inf_{t>0}V(x,t,r)=\infty, \tag{12}$$

then the point $x\equiv 0$ of (8) is GASIP.

Lemma 5 (see [21]). *For $z,b\in R^n$, $\mathscr{B}\in S_n$, and $\mathscr{B}^{-1}$ exists, we have*

$$z'\mathscr{B}z+b'z+z'b=\left(z+\mathscr{B}^{-1}b\right)'\mathscr{B}\left(z+\mathscr{B}^{-1}b\right)-b'\mathscr{B}^{-1}b. \tag{13}$$

3. Main Results

A sufficient condition is obtained for the infinite horizon H_∞ control of system (1) as follows.

Theorem 6. *For a given disturbance attenuation level $\gamma>0$, assume that there exist a set of positive functions $V(x,t,\theta_t)\in C^{2,1}(R^n\times R\times D;R)$ which have an infinitesimal upper limit (i.e., $\lim_{\|x(t)\|\longrightarrow\infty t>0}\inf V(x,t,\theta_t)=\infty$), $V(0,0,r)=0$, and $\partial^2V(x,t,\theta_t)/\partial x^2\geq 0$ for all nonzero $x\in R^n$, $r\in D$. Besides, one assumes that $V(x,t,\theta_t)>a\|x(t)\|^2$ for some $a>0$. If $V(x,t,\theta_t)$ solves the following HJIs:*

$$\begin{cases}\Pi_r:=\dfrac{\partial V_r}{\partial t}+\dfrac{\partial V_r'}{\partial x}f_r+\dfrac{1}{2}g_r'\dfrac{\partial^2V_r}{\partial x^2}g_r+m_r'm_r+\displaystyle\sum_{h=1}^{\infty}\pi_{rh}V_h\\[2ex]+\dfrac{1}{4}\left(g_r'\dfrac{\partial^2V_r}{\partial x^2}q_r+\dfrac{\partial V_r'}{\partial x}s_r\right)\left(\gamma^2I-q_r'\dfrac{\partial^2V_r}{\partial x^2}q_r\right)^{-1}\left(q_r'\dfrac{\partial^2V_r}{\partial x^2}g_r+s_r'\dfrac{\partial V_r}{\partial x}\right)\\[2ex]-\dfrac{1}{4}\left(g_r'\dfrac{\partial^2V_r}{\partial x^2}h_r+\dfrac{\partial V_r'}{\partial x}k_r\right)\left(I+h_r'\dfrac{\partial^2V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\dfrac{\partial^2V_r}{\partial x^2}g_r+k_r'\dfrac{\partial V_r}{\partial x}\right)<0,\\[2ex]\gamma^2I-q_r'\dfrac{\partial^2V_r}{\partial x^2}q_r>0,\quad r\in D,\end{cases} \tag{14}$$

where

$$\begin{aligned}&[V_r,f_r,k_r,s_r,g_r,h_r,q_r,m_r]=\\&\quad[V(x,t,\theta_t),f(x,y,t,\theta_t),k(x,y,t,\theta_t),s(x,y,t,\theta_t),g(x,y,t,\theta_t),h(x,y,t,\theta_t),q(x,y,t,\theta_t),m(x,y,t,\theta_t)],\end{aligned} \tag{15}$$

for all $x, y \in R^n$, $t \geq 0$, and $r \in D$, then

$$u_r^* = -\frac{1}{2}\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right), \quad (16)$$

is an asymptotically mean square H_∞ control for system (1).

Proof. First, we prove that system (8) is ASMS. For $r \in D$, we have the infinitesimal operator $\mathscr{L}_{u^*}$ of system (8).

$$\begin{aligned}
&\mathscr{L}_{u^*}V(x, y, t, \theta_t) \mid_{v=0} \\
&= \frac{\partial V_r}{\partial t} + \frac{\partial V_r'}{\partial x}(f_r + k_r u_r^*) + \sum_{h=1}^{\infty}\pi_{rh}V_h + \frac{1}{2}(g_r + h_r u_r^*)'\frac{\partial^2 V_r}{\partial x^2}(g_r + h_r u_r^*) \\
&= \frac{\partial V_r}{\partial t} + \frac{\partial V_r'}{\partial x}f_r + \frac{\partial V_r'}{\partial x}k_r u_r^* + \sum_{h=1}^{\infty}\pi_{rh}V_h + \frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}g_r \\
&\quad + \frac{1}{2}u_r^{*'}h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + \frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}h_r u_r^* + \frac{1}{2}u_r^{*'}h_r'\frac{\partial^2 V_r}{\partial x^2}h_r u_r^* \\
&= \frac{\partial V_r}{\partial t} + \frac{\partial V_r'}{\partial x}f_r + \frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}g_r + \sum_{h=1}^{\infty}\pi_{rh}V_h + \Gamma_{1r} + \Gamma_{2r},
\end{aligned} \quad (17)$$

where

$$\begin{aligned}
\Gamma_{1r} &= \frac{\partial V_r'}{\partial x}k_r u_r^* + \frac{1}{2}u_r^* h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + \frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}h_r u_r^* \\
&= -\frac{1}{2}\frac{\partial V_r'}{\partial x}k_r\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right) \\
&\quad - \frac{1}{4}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}h_r + \frac{\partial V_r'}{\partial x}k_r\right)\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}h_r'\frac{\partial^2 V_r}{\partial x^2}g_r \\
&\quad - \frac{1}{4}g_r'\frac{\partial^2 V_r}{\partial x^2}h_r\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right) \\
&= -\frac{1}{2}\left(\frac{\partial V_r'}{\partial x}k_r + g_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right),
\end{aligned} \quad (18)$$

and

$$\begin{aligned}
\Gamma_{2r} &= \frac{1}{2}u_r^{*'}h_r'\frac{\partial^2 V_r}{\partial x^2}h_r u_r^* \\
&= \frac{1}{8}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}h_r + \frac{\partial V_r'}{\partial x}k_r\right)\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}h_r'\frac{\partial^2 V_r}{\partial x^2}h_r \\
&\quad \times\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right) \\
&\leq \frac{1}{8}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}h_r + \frac{\partial V_r'}{\partial x}k_r\right)\left(I + h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r + k_r'\frac{\partial V_r}{\partial x}\right).
\end{aligned} \quad (19)$$

Substituting (18) and (19) into (17), and considering (14), one gets

$$\begin{aligned}
&\mathscr{L}_{u^*}V(x,y,t,\theta_t)|_{v=0}\\
&=\frac{\partial V_r}{\partial t}+\frac{\partial V_r'}{\partial x}f_r+\frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}g_r+\sum_{h=1}^{\infty}\pi_{rh}V_h\\
&\quad-\frac{1}{2}\left(\frac{\partial V_r'}{\partial x}k_r+g_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)\left(I+h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r+k_r'\frac{\partial V_r}{\partial x}\right)\\
&\quad+\frac{1}{8}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}h_r+\frac{\partial V_r'}{\partial x}k_r\right)\left(I+h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r+k_r'\frac{\partial V_r}{\partial x}\right)\\
&\le\frac{\partial V_r}{\partial t}+\frac{\partial V_r'}{\partial x}f_r+\frac{1}{2}g_r'\frac{\partial^2 V_r}{\partial x^2}g_r+\sum_{h=1}^{\infty}\pi_{rh}V_h\\
&\quad-\frac{1}{4}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}h_r+\frac{\partial V_r'}{\partial x}k_r\right)\left(I+h_r'\frac{\partial^2 V_r}{\partial x^2}h_r\right)^{-1}\left(h_r'\frac{\partial^2 V_r}{\partial x^2}g_r+k_r'\frac{\partial V_r}{\partial x}\right)\\
&<-m_r'm_r-\frac{1}{4}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}g_r+\frac{\partial V_r'}{\partial x}s_r\right)\left(\gamma^2 I-q_r'\frac{\partial^2 V_r}{\partial x^2}q_r\right)^{-1}\left(g_r'\frac{\partial^2 V_r}{\partial x^2}q_r+\frac{\partial V_r'}{\partial x}s_r\right)\\
&\le 0.
\end{aligned}\tag{20}$$

Based on Lemma 4, (8) is GASIP. Besides, by using It$\hat{o}'s$ formula, for $t\ge s\ge 0$, one obtains

$$\begin{aligned}
EV(x(t),t,\theta_t)&=EV(x(s),s,\theta_s)+E\int_s^t\mathscr{L}_{u^*}V(x(\tau),\tau,\theta_\tau)\Big|_{v=0}\mathrm{d}\tau\\
&\quad+E\int_s^t[g(x(\tau),\tau,\theta_\tau)+h(x(\tau),\tau,\theta_\tau)u^*(\tau)]V(x(\tau),\tau,\theta_\tau)\mathrm{d}\omega(\tau)\\
&=EV(x(s),s,\theta_s)+E\int_s^t\mathscr{L}_{u^*}V(x(\tau),\tau,\theta_\tau)\Big|_{v=0}\mathrm{d}\tau\\
&\le EV(x(s),s,\theta_s).
\end{aligned}\tag{21}$$

It is easy to find that $E|V(x(t),t,\theta_t)|<\infty$. Setting $\tilde{\mathscr{F}}_t=\mathscr{F}_t\cup\sigma(y(s),0\le s\le t)$, then (21) yields

$$\begin{aligned}
E\left[V(x(t),t,\theta_t)|\tilde{\mathscr{F}}_s\right]&\le E\left[V(x(s),s,\theta_s)|\tilde{\mathscr{F}}_s\right]\\
&\le V(x(s),s,\theta_s)\ a.s.
\end{aligned}\tag{22}$$

Accordingly, considering $\{\tilde{\mathscr{F}}_t\}_{t\in R^+}$, $\{V(x(s),s,\theta_s),\tilde{\mathscr{F}}_t, 0\le s\le t\}$ is a non-negative supermartingale. By Doob's convergence theorem [22], it deduces that $V(x(\infty),\infty,\theta_\infty)=\lim_{t\longrightarrow\infty}V(x(t),t,\theta_t)=0$ *a.s.* Moreover, $\lim_{t\longrightarrow\infty}EV(x(t),t,\theta_t)=EV(x(\infty),\infty,\theta_\infty)=EV(0,\infty,\theta_\infty)=0.$

Because of $V(x(t),t,\theta_t)\geq a\|x(t)\|^2$ for some $a>0$, then $\lim_{t\longrightarrow\infty}E\|x(t)\|^2=0$.

Next, we will prove that (9) holds for system (1). For the initial state $x_0=0$, $\theta_0=r$, and any $T>0$, by using It $\hat{o}'s$ formula, we have

$$\begin{aligned}
&E[V(x_T,T,\theta_T)-V(x(0),0,\theta_0)|\theta_0=r]\\
&=E\left\{\int_0^T \mathscr{L}V(x,x_\delta,t.\theta_t)\mathrm{d}t|\theta_0=r\right\}\\
&=E\left\{\int_0^T\left[\frac{\partial V_{\theta_t}}{\partial t}+\frac{\partial V_{\theta_t}'}{\partial x}\left(f_{\theta_t}+k_{\theta_t}u+s_{\theta_t}v\right)+\sum_{h=1}^{\infty}\pi_{rh}V_h\right.\right.\\
&\left.\left.+\frac{1}{2}\left(g_{\theta_t}+h_{\theta_t}u+q_{\theta_t}v\right)'\frac{\partial^2 V_{\theta_t}}{\partial x^2}\left(g_{\theta_t}+h_{\theta_t}u+q_{\theta_t}v\right)\right]dt|\theta_0=r\right\}\\
&=E\left\{\int_0^T\left[\frac{\partial V_{\theta_t}}{\partial t}+\frac{\partial V_{\theta_t}'}{\partial x}\left(f_{\theta_t}+k_{\theta_t}u+s_{\theta_t}v\right)+\sum_{h=1}^{\infty}\pi_{rh}V_h\left\|m_{\theta_t}\right\|^2+\|u\|^2-\gamma^2\|v\|^2\right.\right.\\
&\left.\left.+\frac{1}{2}\left(g_{\theta_t}+h_{\theta_t}u+q_{\theta_t}v\right)'\frac{\partial^2 V_{\theta_t}}{\partial x^2}\left(g_{\theta_t}+h_{\theta_t}u+q_{\theta_t}v\right)-\left\|z^2+\gamma^2\right\|v^2\right]dt|\theta_0=r\right\}\\
&=E\left\{\int_0^T\left[\frac{\partial V_{\theta_t}}{\partial t}+\frac{\partial V_{\theta_t}'}{\partial x}f_{\theta_t}+\frac{1}{2}g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}+m_{\theta_t}'m_{\theta_t}+\sum_{h=1}^{\infty}\pi_{rh}V_h\right.\right.\\
&+v'\left(-\gamma^2 I+\frac{1}{2}q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)v+\frac{1}{2}\left(g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}+\frac{\partial V_{\theta_t}'}{\partial x s_{\theta_t}}\right)v\\
&+\frac{1}{2}v'\left(s_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x}+q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}\right)+u'\left(I+\frac{1}{2}h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)u\\
&+\frac{1}{2}u'\left(h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}+k_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x}\right)+\frac{1}{2}\left(\frac{\partial V_{\theta_t}'}{\partial x}k_{\theta_t}+g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)u\\
&\left.\left.+\frac{1}{2}v' q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}u+\frac{1}{2}u' h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}v-\left\|z^2+\gamma^2\right\|v^2\right]dt|\theta_0=r\right\}\\
&=E\left\{\int_0^T\left[\Pi_1(x,x_\delta,t,\theta_t)+\Pi_2(v,x,x_\delta,t,\theta_t)+\Pi_3(u,x,x_\delta,\theta_t)\right.\right.\\
&\left.\left.+\frac{1}{2}v' q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}u+\frac{1}{2}u' h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}v-\left\|z^2+\gamma^2\right\|v^2\right]dt|\theta_0=r\right\},
\end{aligned}\tag{23}$$

where

$$\Pi_1(x, x_\delta, t, \theta_t) = \frac{\partial V_{\theta_t}}{\partial t} + \frac{\partial V_{\theta_t}'}{\partial x} f_{\theta_t} + \frac{1}{2} g_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} g_{\theta_t} + m_{\theta_t}' m_{\theta_t} + \sum_{h=1}^{\infty} \pi_{rh} V_h, \Pi_2$$

$$(v, x, x_\delta, t, \theta_t) = v' \left(-\gamma^2 I + \frac{1}{2} q_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} q_{\theta_t} \right) v + \frac{1}{2} \left(g_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} q_{\theta_t} + \frac{\partial V_{\theta_t}'}{\partial x s_{\theta_t}} \right) v$$

$$+ \frac{1}{2} v' \left(s_{\theta_t}' \frac{\partial V_{\theta_t}}{\partial x} + q_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} g_{\theta_t} \right), \quad (24)$$

$$\Pi_3(u, x, x_\delta, t, \theta_t) = u' \left(I + \frac{1}{2} h_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} h_{\theta_t} \right) u + \frac{1}{2} u' \left(h_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} g_{\theta_t} + k_{\theta_t}' \frac{\partial V_{\theta_t}}{\partial x} \right)$$

$$+ \frac{1}{2} \left(\frac{\partial V_{\theta_t}'}{\partial x} k_{\theta_t} + g_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} h_{\theta_t} \right) u.$$

Considering $\partial^2 V(x, t, \theta_t)/\partial x^2 \geq 0$ and $r \in D$, we assert

$$\frac{1}{2} \left(h_{\theta_t} u - q_{\theta_t} v \right)' \frac{\partial^2 V_{\theta_t}}{\partial x^2} \left(h_{\theta_t} u + q_{\theta_t} v \right) \geq 0, \quad (25)$$

which shows that

$$\frac{1}{2} v' q_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} h_{\theta_t} u + \frac{1}{2} u' h_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} q_{\theta_t} v$$
$$\leq \frac{1}{2} u' h_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} h_{\theta_t} u + \frac{1}{2} v' q_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} q_{\theta_t} v. \quad (26)$$

Therefore,

$$E[V(x_T, T, \theta_T) - V(x_0, 0, \theta_0) | \theta_0 = r]$$
$$\leq E\left\{ \int_0^T [\Pi_1(x, x_\delta, t, \theta_t) + \Pi_2(v, x, x_\delta, t, \theta_t) + \Pi_3(u, x, x_\delta, t, \theta_t) \right.$$
$$\left. + \frac{1}{2} u' h_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} h_{\theta_t} u + \frac{1}{2} v' q_{\theta_t}' \frac{\partial^2 V_{\theta_t}}{\partial x^2} q_{\theta_t} v - \|z\|^2 + \gamma^2 \|v\|^2 \right] \mathrm{d}t \,\Big|\, \theta_0 = r \Big\} \quad (27)$$
$$= E\left\{ \int_0^T [\Pi_1(x, x_\delta, t, \theta_t) + \widetilde{\Pi}_2(v, x, x_\delta, t, \theta_t) + \widetilde{\Pi}_3(u, x, x_\delta, t, \theta_t) \right.$$
$$\left. - \|z\|^2 + \gamma^2 \|v\|^2 \right] \mathrm{d}t \,\Big|\, \theta_0 = r \Big\},$$

where

$$\begin{aligned}\widetilde{\Pi}_2(v,x,x_\delta,t,\theta_t) &= v'\left(-\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)v + \frac{1}{2}\left(g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t} + \frac{\partial V_{\theta_t}'}{\partial x}s_{\theta_t}\right)v \\ &\quad + \frac{1}{2}v'\left(s_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x} + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}\right), \\ \widetilde{\Pi}_3(u,x,x_\delta,t,\theta_t) &= u'\left(I + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)u + \frac{1}{2}u'\left(h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t} + k_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x}\right) \\ &\quad + \frac{1}{2}\left(\frac{\partial V_{\theta_t}'}{\partial x}k_{\theta_t} + g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)u.\end{aligned} \tag{28}$$

Applying Lemma 5 to $\widetilde{\Pi}_2(v,x,x_\delta,t,\theta_t)$ and $\widetilde{\Pi}_3(u,x,x_\delta,t,\theta_t)$, we arrive at

$$\begin{aligned}&\widetilde{\Pi}_2(v,x,x_\delta,t,\theta_t) \\ &= (v+\Lambda_1)'\left(-\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)(v+\Lambda_1) \\ &\quad - \frac{1}{4}\left(\frac{\partial V_{\theta_t}'}{\partial x}s_{\theta_t} + g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)\left(-\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)^{-1}\left(s_{\theta t}'\frac{\partial V_{\theta_t}}{\partial x} + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}\right),\end{aligned} \tag{29}$$

and

$$\begin{aligned}\widetilde{\Pi}_3(u,x,x_\delta,t,\theta_t) &= (u+\Lambda_2)'\left(I + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)(u+\Lambda_2) \\ &\quad - \frac{1}{4}\left(g_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t} + \frac{\partial V_{\theta_t}'}{\partial x}k_{\theta_t}\right)\left(I + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)^{-1}\left(k_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x} + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t}\right),\end{aligned} \tag{30}$$

where

$$\Lambda_1 = \frac{1}{2}\left(\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)^{-1}\left(q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t} + s_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x}\right),$$

$$\Lambda_2 = \frac{1}{2}\left(I + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}\right)^{-1}\left(h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}g_{\theta_t} + k_{\theta_t}'\frac{\partial V_{\theta_t}}{\partial x}\right). \tag{31}$$

Substituting (29) and (30) into (27), and considering (14), one infers that

$$\begin{aligned} &E\left[V(x_T, T, \theta_T) - V(x_0, 0, \theta_0) | \theta_0 = r\right] \\ &\leq E\left\{\int_0^T \left[(v+\Lambda_1)'\left(-\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)(v+\Lambda_1)\right.\right. \\ &\left.\left.+(u+\Lambda_2)'\left(I + h_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}h_{\theta_t}h_{\theta_t}\right)(u+\Lambda_2) - \|z\|^2 + \gamma^2\|v\|^2\right]\mathrm{d}t \,\middle|\, \theta_0 = r\right\}. \end{aligned} \tag{32}$$

In view of (14), if one takes $u = u^* = -\Lambda_2$, then (32) becomes

$$\begin{aligned} &E\left(\int_0^T \|z\|^2 \mathrm{d}t \,\middle|\, \theta_0 = r\right) \\ &\leq -E\left[V(x_T, T, \theta_T) | \theta_0 = r\right] + \gamma^2 E\left[\int_0^T \|v\|^2 \mathrm{d}t \,\middle|\, \theta_0 = r\right] \\ &\quad - E\left[\int_0^T (v+\Lambda_1)'\left(\gamma^2 I + q_{\theta_t}'\frac{\partial^2 V_{\theta_t}}{\partial x^2}q_{\theta_t}\right)(v+\Lambda_1)\mathrm{d}t \,\middle|\, \theta_0 = r\right] \\ &< \gamma^2 E\left(\int_0^T \|v\|^2 \mathrm{d}t \,\middle|\, \theta_0 = r\right). \end{aligned} \tag{33}$$

Letting $T \longrightarrow \infty$, it can be seen that (9) is established, which achieves the desired result. □

Setting $u(t) \equiv 0$, then (1) becomes the following unforced nonlinear system:

$$\begin{cases} dx(t) = \left[f(x, x_\delta, t, \theta_t) + s(x, x_\delta, t, \theta_t)v\right]\mathrm{d}t + \left[g(x, x_\delta, t, \theta_t) + q(x, x_\delta, t, \theta_t)v\right]\mathrm{d}\omega(t), \\ z(t) = m(x, x_\delta, t, \theta_t), \\ x(t) = \Phi(t) \in \mathscr{C}^b_{\mathscr{F}_0}([-\delta, 0]; R^n). \end{cases} \tag{34}$$

Remark 7. It is generally HJIs (14) that are not easy to be solved. Maybe we can try getting the approximate solution by a fuzzy method.

The nonlinear stochastic bounded real lemma for system (34) will be obtained by Theorem 6.

Corollary 8. *For $\gamma > 0$, assume that there exist a set of positive functions $V(x, t, \theta_t) \in C^{2,1}(R^n \times R \times D; R)$ which satisfy $\lim_{\|x(t)\| \longrightarrow \infty} \inf_{t>0} V(x, t, \theta_t) = \infty$, $V(0, 0, r) = 0$, and $\partial^2 V(x, t, \theta_t)/\partial x^2 \geq 0$ for all nonzero $x \in R^n$, $r \in D$, as* *well as $V(x,t,\theta_t) > a\|x(t)\|^2$ for some $a > 0$. If $V(x,t,\theta_t)$ satisfies the following HJIs:*

$$\begin{cases} \dfrac{\partial V_r}{\partial t} + \dfrac{\partial V_r'}{\partial x} f_r + \dfrac{1}{2} g_r' \dfrac{\partial^2 V_r}{\partial x^2} g_r + m_r' m_r + \sum_{h=1}^{\infty} \pi_{rh} V_h \\ + \dfrac{1}{4}\left(g_r' \dfrac{\partial^2 V_r}{\partial x^2} q_r + \dfrac{\partial V_r'}{\partial x} s_r\right)\left(\gamma^2 I - q_r' \dfrac{\partial^2 V_r}{\partial x^2} q_r\right)^{-1}\left(q_r' \dfrac{\partial^2 V_r}{\partial x^2} g_r + s_r' \dfrac{\partial V_r}{\partial x}\right) < 0, \\ \gamma^2 I - q_r' \dfrac{\partial^2 V_r}{\partial x^2} q_r > 0, r \in D, t \geq 0, \end{cases} \tag{35}$$

then system (34) is internally stable and $\|\mathscr{L}_{\infty}^{u^*}\| \leq \gamma$.

More particularly, consider the following nonlinear time-delay system with (x,u,v)-dependent noise but without Markov jumps:

$$\begin{cases} dx(t) = [f_1(x,x_\delta,t) + k_1(x,x_\delta,t)u + s_1(x,x_\delta,t)v]dt \\ +[g_1(x,x_\delta,t) + h_1(x,x_\delta,t)u + q_1(x,x_\delta,t)v]d\omega(t), \\ z(t) = col(m_1(x,x_\delta,t),u) := \begin{pmatrix} m_1(x,x_\delta,t), \\ u \end{pmatrix}, \\ x(t) = \Phi(t) \in \mathscr{C}_{\mathscr{F}_0}^b([-\delta,0];R^n). \end{cases} \tag{36}$$

For $\tilde{V} \in \mathscr{C}^{2,1}(R^n \times R; R)$, an infinitesimal generator $\mathscr{L}\tilde{V}$: $R^n \times R^n \times R \longrightarrow R$ associated with (36) is defined as follows [23]:

$$\begin{aligned} \mathscr{L}\tilde{V}(x,y,t) = {} & \frac{\partial \tilde{V}(x,t)}{\partial t} + \frac{\partial \tilde{V}'(x,t)}{\partial x}[f_1(x,y,t) + k_1(x,y,t)u + l_1(x,y,t)v] \\ & + \frac{1}{2}[g_1(x,y,t) + h_1(x,y,t)u + q_1(x,y,t)v]' \frac{\partial \tilde{V}^2(x,t)}{\partial x^2} \\ & \times [g_1(x,y,t) + h_1(x,y,t)u + q_1(x,y,t)v]. \end{aligned} \tag{37}$$

For system (36), according to Theorem 6, we can directly obtain the following corollary.

Corollary 9. *For a given $\gamma > 0$, assume that there exist a set of positive functions $\tilde{V}(x,t) \in C^{2,1}(R^n \times R; R)$ which satisfy $\lim_{\|x(t)\| \longrightarrow \infty} \inf_{t>0} \tilde{V}(x,t) = \infty$, $V(0,0) = 0$, and $\partial^2 \tilde{V}(x,t)/\partial x^2 \geq 0$ for all nonzero $x \in R^n$, as well as $\tilde{V}(x,t) > a\|x(t)\|^2$ for some $a > 0$. If $\tilde{V}(x,t)$ solves the following HJIs:*

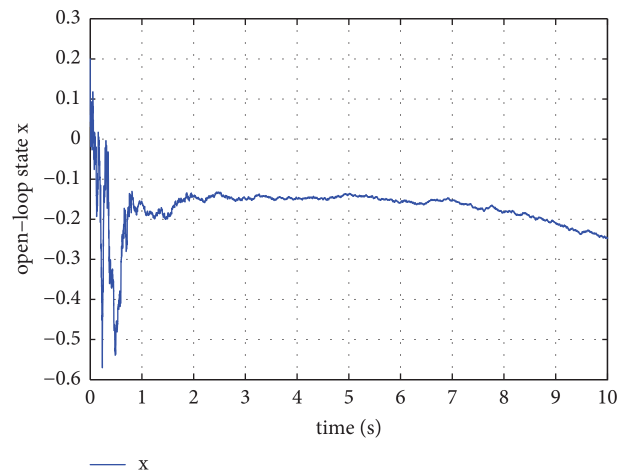

FIGURE 1: State trajectories of the unforced system.

$$\begin{cases} \Pi_r := \dfrac{\partial \tilde{V}}{\partial t} + \dfrac{\partial \tilde{V}'}{\partial x} f_1 + \dfrac{1}{2} g_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} g_1 + m_1' m_1 \\ + \dfrac{1}{4} \left(g_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} q_1 + \dfrac{\partial \tilde{V}'}{\partial x} s_1 \right) \left(\gamma^2 I - q_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} q_1 \right)^{-1} \left(g_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} q_1 + \dfrac{\partial \tilde{V}'}{\partial x} s_1 \right) \\ - \dfrac{1}{4} \left(g_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} h_1 + \dfrac{\partial \tilde{V}'}{\partial x} k_1 \right) \left(I + h_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} h_1 \right)^{-1} \left(h_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} g_1 + k_1' \dfrac{\partial \tilde{V}}{\partial x} \right) < 0, \\ \gamma^2 I - q_1' \dfrac{\partial^2 \tilde{V}}{\partial x^2} q_1 > 0, t \geq 0, \end{cases} \quad (38)$$

then

$$u^* = -\frac{1}{2} \left(I + h_1' \frac{\partial^2 \tilde{V}}{\partial x^2} h_1 \right)^{-1} \left(h_1' \frac{\partial^2 \tilde{V}}{\partial x^2} g_1 + k_1' \frac{\partial \tilde{V}}{\partial x} \right), \quad (39)$$

is an asymptotically mean square H_∞ control for system (36).

4. A Simulation Example

A simulation example is presented to indicate the correctness of the results obtained in this paper.

Example 10. Consider a one-dimensional nonlinear stochastic delayed system with infinite Markov jumps, and the following parameters are listed:

$$\begin{aligned} f_\rho &= -\frac{\rho x x_\delta}{\xi} - \frac{x}{2(\xi)}, k_\rho = \frac{7}{\xi}, s_\rho = \frac{1}{\xi}, \\ g_\rho &= \frac{x}{\xi}, h_\rho = 1, q_\rho = 1, m_\rho = \frac{\rho x x_\delta}{\xi}, \end{aligned} \quad (40)$$

where $\xi = \rho + 1$. Let $\gamma = \sqrt{2}$. The transition rate of $\{\theta_t\}_{t \geq 0}$ is given by $-\pi_{\rho\rho} = \pi_{\rho,\xi} = 1$ and $\pi_{\rho h} = 0, \rho \in D, h \in D/\{\rho, \xi\}$. Setting $V(x, t, \rho) = \rho x^2 / 2\xi$, the coupled HJIs (14) become

$$\begin{aligned} \Pi = & \frac{\rho x}{\xi} \cdot \left(-\frac{\rho x x_\delta}{\xi} - \frac{x}{2\xi} \right) + \frac{1}{2} \frac{x}{\xi} \cdot \frac{\rho}{\xi} \cdot \frac{x}{\xi} + \frac{\rho x x_\delta}{\xi} \cdot \frac{\rho x x_\delta}{\xi} - \frac{\rho x^2}{2\xi} \cdot \frac{\xi x^2}{2\xi} \\ & + \frac{1}{4} \left(\frac{x}{\xi} \cdot \frac{\rho}{\xi} + \frac{\rho x}{\xi} \cdot \frac{1}{\xi} \right) \left(2 - \frac{\rho}{\xi} \right)^{-1} \left(\frac{\rho}{\xi} \cdot \frac{x}{\xi} + \frac{1}{\xi} \cdot \frac{\rho x}{\xi} \right) \end{aligned}$$

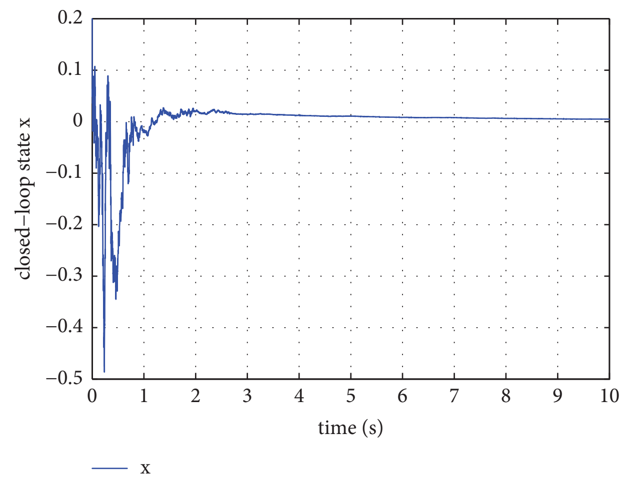

Figure 2: State trajectories of the controlled system.

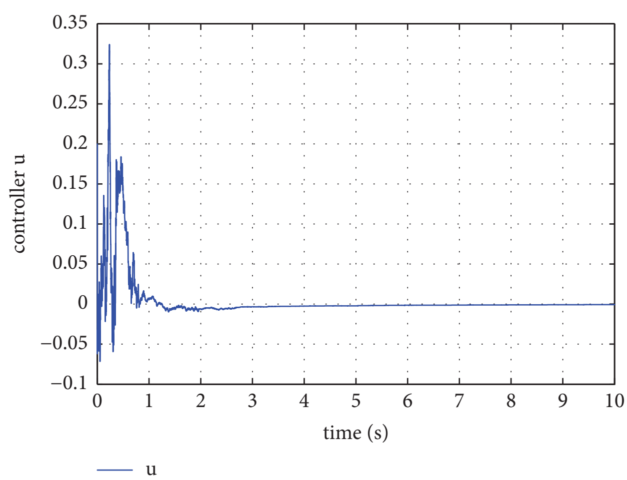

Figure 3: State trajectories of the controller.

$$-\frac{1}{4}\left(\frac{x}{\xi}\cdot\frac{\rho}{\xi}+\frac{\rho x}{\xi}\cdot\frac{7}{\xi}\right)\left(1+\frac{\rho}{\xi}\right)^{-1}\left(\frac{\rho}{\xi}\cdot\frac{x}{\xi}+\frac{7}{\xi}\cdot\frac{\rho x}{\xi}\right)$$
$$=\frac{\left(-24\rho^3-50\rho^2+9\rho+2\right)x^2}{2(\xi)^3(\rho+2)(2\xi-1)}<0, \quad (41)$$
$$\gamma^2 I - q_\rho'\frac{\partial^{'} V_\rho}{\partial x^2}q_\rho = 2I-\frac{\rho}{\xi}=\frac{2+\rho}{\xi}>0.$$

According to Theorem 6, the H_∞ controller of system (1) is

$$u_\rho^* = -\frac{4\rho}{\xi(2\xi-1)}x. \quad (42)$$

Select the initial condition $\Phi(t)=0.15$ for any $t\in[-\delta,0]$ with $\delta=0.15$ and $v(t)=e^{-(\rho-1)}\sin(0.1\rho\pi)$. Figures 1–3 show the trajectories of the states of the unforced system $(u(t)=0)$, the states of the controlled system $(u(t)=u_\rho^*)$, and the control input $u(t)$, respectively. It can be seen from the simulation results that the controlled system can not only achieve stability but also satisfy the attenuation performance by using the H_∞ controller.

5. Conclusion

This paper has solved the problem of infinite horizon H_∞ control for general nonlinear stochastic jump systems with (x,u,v)-dependent noise and delay. And the asymptotic mean square H_∞ controller has been designed by solving a series of coupled HJIs. Finally, the validity of the obtained results has been demonstrated by a numerical example. Some more difficult and meaningful topics need to be studied in the future, including infinite horizon H_2/H_∞ control and filter problems for nonlinear stochastic systems with infinite Markov jumps and time-varying delays.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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