Research Article

A Fusion Recognition Method Based on Temporal Evidence Reasoning

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In order to improve the effectiveness of system decision-making, the use of the evidence theory to identify target intentions has always been a research hotspot. In information fusion using the evidence theory, there are relatively few research studies on temporal domain evidence information fusion. Due to the obvious dynamic, sequential, and real-time characteristics of temporal domain information fusion, traditional spatial domain information fusion methods are not suitable. Therefore, it is very necessary to study new methods for the temporal evidence fusion problem. In this article, a temporal evidence fusion method under the framework of the evidence reasoning rule (the ER rule) is proposed. The method uses complementary reliability integration rules and the time-series evidence distance function to obtain the reliability of evidence at adjacent moments. According to the temporal domain evidence credibility decay model, the evidence weight of the temporal domain evidence is determined. Then, through the integration of the ER rule, the temporal domain evidence reliability and evidence weight are used to combine the evidence. The capability of this method is verified by numerical experiments and compared with other methods. The results show that the proposed method can effectively deal with the temporal domain evidence combination problem, has strong anti-interference ability, and can support target intent recognition.

1. Introduction

In order to improve the effectiveness of system decision-making, target intent recognition has always been a hotspot in decision making research [1–3]. Due to the increasing uncertainty and incompleteness of information, the use of information fusion technology for target intent recognition has become an important research direction in the field of pattern recognition [4–7]. Since time-series information is not acquired at the same time but sequentially along with the time series, it is impossible to fuse after all the information sequences are acquired, resulting in the obvious dynamic, sequential, and real-time characteristics of time-domain information fusion [8–11].

Since the emergence of the evidence theory, it has been widely used in many fields of information fusion [12, 13]. The evidence theory can provide a complete framework for the representation of uncertain information [14]. As an important tool for dealing with uncertain information, the evidence theory can also be applied to temporal information fusion [15–21]. According to the real-time state information of the target detected by the sensor in real time, the support calculation method for the target behavior intention is established, and then the evidence theory is used to combine each time series support to form the sequential identification of the target intention. The evidence theory was originally developed by Dempster in 1967 and extended by Shafer in 1976, so it is also known as Dempster-Shafer (DS) theory [22, 23]. Under the DS framework, the frame of discernment (FoD) consists of a set of mutually exclusive and integrally complete propositions. Its basic probability mass can be assigned not only to any single proposition set, but also to any subset of the proposition set [24]. Compared with the traditional probability theory, the DS theory can deal with the fusion of uncertain information. The core of the DS theory is the Dempster combination rule. Although it has...
a good ability to combine evidence, it has counterintuitive problems in dealing with conflicting evidence, which will lead to unreasonable results [25]. Many scholars have conducted research on the expression of DS theory input information and the improvement of combination rules, mainly focusing on the belief function and belief distribution. The belief function mainly expresses the probability characteristics of evidence from the perspective of the basic probability distribution [26], while the belief distribution mainly expresses the probability characteristics of evidence from the perspective of reliability and then integrates the weight and reliability of the evidence to achieve a hybrid expression. Xiao and Pedrycz [27] proposed a generalized negation method based on the quantum basic belief assignment. It provides a promising solution for knowledge representation, uncertainty measurement, and quantum information fusion. To solve the divergence measure between basic belief assignments (BBAs), Xiao et al. [28] proposed generalized evidential divergences to measure the differences between BBAs in the DS evidence theory. In terms of the confidence distribution, Yang and Singh [29] proposed the evidential reasoning (ER) method for the first time, which provided an idea for effectively solving the multiple-attribute decision making (MADM) problem. Wang et al. [30] applied the ER method to the environmental impact assessment, and gave an analytical algorithm for ER, which was helpful to solve the optimization problem of the ER algorithm. Under the framework of the evidence theory, Yang and Xu [31] proposed the ER rule, which is an extension of the traditional Dempster combination rule, which clearly distinguishes the concepts of evidence importance and reliability. ER rules have been applied in many fields such as safety assessment and fault diagnosis.

So far, there have been in-depth studies on information fusion in the spatial domain, but the information in the temporal domain has not been fully utilized. Wu et al. [32] studied the information fusion of the multisensor temporal domain and spatial domain, but failed to establish a specific model. Hong and Lynch [33] studied the spatiotemporal information fusion model based on the DS evidence theory earlier, and summarized the characteristics of the recursive centralized structure and recursive distributed structure. Different fusion models have a great impact on the fusion results, so an appropriate fusion model should be selected according to the respective fusion requirements and specific application characteristics. Although some scholars have studied the temporal domain evidence combination method, when it comes to specific combination methods, they cannot fully reflect the sequential characteristics of temporal domain information. Song et al. [34] noticed the sequentiality and dynamics in temporal information fusion and proposed a calculation method for the dynamic credibility of evidence, which is used as a dynamic discount factor to discount evidence. Liu and Xiao [35] believed that the time decay model was not suitable for the case where the time interval of sensor data was too long and proposed an ordered weight integration operator, which used the Q function to reduce the influence of the time interval on the final result. It can be seen that the current research on temporal information fusion has not made full use of temporal evidence sequences, and there are still many problems in the relationship between evidence combination rules and evidence weights.

This article focuses on the fusion of temporal domain evidence information and studies the reliability evaluation and combination rules of temporal domain evidence sequences. This article proposes a reliability evaluation method based on the complementary reliability combination rule and distance function, which can use adjacent evidence to obtain the reliability and then obtain the evidence importance through the time decay model and finally use the ER rule to fuse the temporal domain evidence. The temporal domain evidence combination method proposed in this article not only considers the influence of time factors on evidence combination, has strong dynamic evidence processing ability, but also considers the relationship between temporal domain evidence and has certain conflict evidence processing ability.

The rest of this article is arranged as follows: The next section briefly introduces the reliability function model, the Dempster combination rule, and the theoretical basis of the ER rule. In Section 3, the reliability evaluation of evidence sources is mainly studied from the aspects of the temporal decay factor, complementary information integration rules, and temporal evidence distance function, which provides the basis for temporal evidence fusion methods. In Section 4, a temporal evidence recursive fusion model is constructed, and a temporal evidence combination method based on ER rule is presented. In Section 5, numerical examples and experimental simulations are used to verify the temporal evidence fusion method proposed in this article and to compare and analyze with existing methods. Section 6 concludes this article and gives directions for future research.

2. Preliminaries

The evidence theory is also known as belief functions (BFs) or Dempster–Shafer theory (DST). Compared with the traditional probability theory, BFs use imprecise probability to model uncertainty and can deal with the fusion of uncertain information. It has become an important quantitative analysis method in the field of uncertainty information decision making.

2.1. Belief Function Model. BFs are viewed as a generalization of the probability theory that can address multiple possible propositions. Suppose \( \Theta = \{\theta_1, \theta_2, \ldots, \theta_N\} \) is a set of mutually exclusive and collectively exhaustive propositions, with \( \theta_i \cap \theta_j = \phi \) for any \( i, j \in \{1, 2, \ldots, N\} \) and \( i \neq j \) where \( \phi \) is an empty set. \( 2^\Theta \) or \( P(\Theta) \) is the power set of \( \Theta \), which consists of all subsets of \( \Theta \), i.e., \( 2^\Theta = P(\Theta) = \{\phi, \{\theta_1\}, \ldots, \{\theta_N\}, \{\theta_1, \theta_2\}, \ldots, \{\theta_1, \theta_2, \ldots, \theta_N\}, H\} \).

The core definitions in BFs are described as follows.

Definition 1 (see [22]). Let \( \Theta \) be the frame of discernment, then a BPA (also called a belief structure or a basic belief assignment) is a function \( m: 2^\Theta \rightarrow [0, 1] \), which is called a mass function, satisfying the two following conditions:
\[
\begin{align*}
\begin{cases}
  m(\phi) = 0, \\
  \sum_{\theta \subseteq \Theta} m(\theta) = 1,
\end{cases}
\end{align*}
\]

where \(m(\theta)\) is the BPA of \(\theta\). It measures the belief exactly assigned to \(\theta\) and represents how strongly the evidence supports \(\theta\). Each subset \(\theta \subseteq \Theta\) with \(m(\theta) > 0\) is called a focal element of \(m\). All related focal elements are collectively referred to as the body of evidence.

**Definition 2** (see [22]). Associated with each BPA is a belief measure, denoted by \(\text{Bel}(\theta)\), and a plausibility measure, denoted by \(\text{Pl}(\theta)\), which are defined as follows:

\[
\begin{align*}
\text{Bel}(\theta) &= \sum_{B \subseteq \theta} m(B), \\
\text{Pl}(\theta) &= \sum_{B \supseteq \theta} m(B),
\end{align*}
\]

where \(\theta\) and \(B\) are subsets of \(\Theta\). \(\text{Bel}(\theta)\) represents the sum of basic probability masses assigned to \(\theta\), whereas \(\text{Pl}(\theta)\) the sum of possible basic probability masses that could be assigned to \(\theta\). As such, \([\text{Bel}(\theta), \text{Pl}(\theta)]\) can be interpreted as the lower and upper bounds of probability to which \(\theta\) is supported. The two functions can be described as the following equation:

\[
\text{Pl}(\theta) = 1 - \text{Bel}(\overline{\theta}),
\]

where \(\overline{\theta}\) denotes the complement or negation of \(\theta\).

The core of BF's is Dempster’s rule, which can combine independent evidence. To combine evidence, the rule uses the orthogonal sum operation, which is based on computing the joint probability of independent events.

### 2.2. Dempster Combination Rule

**Definition 3** (see [22]). With two pieces of evidence represented by \(m_1\) and \(m_2\), respectively, Dempster’s rule is defined as follows:

\[
\begin{align*}
[m_1 \oplus m_2](\theta) &= \begin{cases} 
0, & \theta = \phi, \\
\frac{\sum_{B \subseteq \Theta} m_1(B) m_2(C)}{1 - \sum_{B \subseteq \Theta} m_1(B) m_2(C)}, & \theta \neq \phi,
\end{cases}
\end{align*}
\]

where \(\oplus\) is the operator of combination, \(B\) and \(C\) are both focal elements, \(1 - \sum_{B \subseteq \Theta} m_1(B) m_2(C)\) is called the normalization factor, \(\sum_{B \subseteq \Theta} m_1(B) m_2(C)\) is called the degree of conflict. Dempster’s rule satisfies commutativity and associativity, and forms a conjunctive probabilistic reasoning process.

### 2.3. ER Rule

The Shafer discount method is a widely recognized discount method, but in evidence reasoning, the weight of evidence and the reliability of evidence are not equivalent in many cases. The weight of evidence can be understood as the relative importance of the evidence compared to other evidence, and the reliability of evidence can be understood as the inherent characteristic of the evidence to make a correct judgment on the result. Due to the inconsistency of evidence reasoning application scenarios, there is no unified algorithm for solving evidence weight and evidence reliability. This article mainly studies the weight of evidence and the reliability of evidence according to the needs of temporal domain evidence reasoning.

**Definition 4** (see [31]). Let \(w_i (0 \leq w_i \leq 1)\) be the importance of evidence \(e_i\), \(r_i (0 \leq r_i \leq 1)\) is the reliability of evidence \(e_i\), \(r_i = 1\) means that evidence \(e_i\) is completely reliable, and \(r_i = 0\) means that evidence \(e_i\) is completely unreliable. The basic probability masses \(m_{ij}\) represents the degree of support for \(\theta\) from \(e_i\) which is expressed as

\[
\begin{align*}
\mbar_{\theta,i} &= \begin{cases} 
0, & \theta = \emptyset, \\
c_{r_{ui}} m_{\theta,i}, & \theta \subseteq \Theta, \theta \neq \emptyset, \\
c_{r_{ui}} (1 - r_i), & \theta = P(\Theta),
\end{cases}
\end{align*}
\]

where \(m_{ij} = w_i p_{\theta,i}\), if \(\sum_{\theta \subseteq \Theta} p_{\theta,i} = 1\), then \(\sum_{\theta \subseteq \Theta} m_{\theta,i} + m_{P(\Theta),i} = 1, 1 - r_i\) is the unreliability of evidence \(e_i\), and \(c_{r_{ui}}\) is a normalization factor.

The mixed weight considering the reliability of the evidence and the weight of the evidence is

\[
\bar{w}_i = c_{r_{ui}} w_i,
\]

\[
= \frac{w_i}{1 + w_i - r_i},
\]

\(c_{r_{ui}} m_{\theta,i}\) is then equal to \(\bar{w}_i p_{\theta,i}\) and \(c_{r_{ui}} (1 - r_i)\) equal to \(1 - \bar{w}_i\). Using the mixed weight \(\bar{w}_i\) directly, Eq. (5) can be equivalently rewritten as follows:

\[
\begin{align*}
\mbar_{\theta,i} &= \begin{cases} 
0, & \theta = \emptyset, \\
\bar{w}_i p_{\theta,i}, & \theta \subseteq \Theta, \theta \neq \emptyset, \\
1 - \bar{w}_i, & \theta = P(\Theta),
\end{cases}
\end{align*}
\]

where \(\bar{w}_i\) can be interpreted as a hybrid importance and reliability coefficient for \(e_i\) to measure the degree of support from \(e_i\) with \(0 \leq \bar{w}_i \leq 1\).

Similarly, the reliability \(p_{\theta,i}\) of the original evidence can also be calculated in reverse, that is,

\[
p_{\theta,i} = \frac{\mbar_{\theta,i}}{1 - m_{P(\Theta),i}}, \theta \subseteq \Theta.
\]

**Definition 5** (see [31]). For two pieces of independent evidence \(e_1\) and \(e_2\), the ER rule can be used to combine them to obtain the degree of belief \(p_{\theta,e(2)}\) that \(e_1\) and \(e_2\) jointly support proposition \(\theta\).
3. Determination of Reliability of Temporal Evidence

In information fusion, in addition to spatial information fusion, it also includes temporal information fusion, which is mainly due to the influence of sensor performance and interference in a single measurement cycle, resulting in the acquired information not necessarily accurate. Therefore, it is necessary to use the information of multiple time nodes for fusion. Temporal evidence is sequential in time, and the processed information is not obtained at the same time, but is gradually obtained with the time series. Temporal evidence information fusion does not need to be fused after the information at all times is obtained but has high real-time requirements. Temporal evidence information fusion has obvious sequential, dynamic, and real-time characteristics. In order to weaken the influence of unreliable information on the fusion of temporal evidence, it is necessary to determine the reliability of evidence sources before combining evidence. This section focuses on the reliability evaluation of evidence sources and studies from the aspects of the time series decay factor, complementary information integration rules, and the time series evidence distance function. These are the basis for the fusion recognition method based on temporal evidence reasoning.

3.1. The Credibility Decay Model. In temporal evidence fusion, the information obtained by the sensor will become more and more accurate over time. The most recently obtained information has the greatest credibility, while the previously obtained information has a lower credibility. That is to say, after the sensor obtains information at a certain time, the credibility of the information will continue to decay with time and the impact on the fusion result will continue to decrease.

Definition 6 (see [34]). It is assumed that the time domain information fusion has the Markov property. When the fusion result of the system at time \( t_m \) is known, the fusion result of the system at time \( t_{m+1} \) (\( t_{m+1} > t_m \)) is only related to the state at time \( t_m \) and has nothing to do with the state before time \( t_m \).

Assume that the basic probability assignment (BPA) \( m_i \) defined on the frame of discernment \( \Theta \) represents the evidence generated by the sensor obtaining information at the time \( t_i, i = 1, 2, \cdots, n \), and the combination result of the evidence \( m_1, m_2, \cdots, m_n \) is recorded as \( f_n(m_1, m_2, \cdots, m_n) \).

Let \( g \) be a time-independent evidence combination rule for two BPAs, then the Markovian requirement of evidence fusion in time domain can be described as

\[
f_n(m_1, m_2, \cdots, m_n) = g(g(\cdots g(g(m_1, m_2), m_3), \cdots, m_{n-1}), m_n).
\]

In the process of temporal evidence fusion, the evidence obtained at a certain moment will continue to decay with time, and this process can be described by the credibility decay model [34].

Definition 7. Suppose \( m_j \) is the evidence obtained at time \( t_j \), then the dynamic credibility corresponding to time \( t_i (t_i > t_j) \) is defined as

\[
a_{ij} = e^{-\lambda(t_i - t_j)},
\]

where \( \lambda(\lambda > 0) \) is the credibility decay factor, in order to reduce the loss of information, usually let \( 0 < \lambda < \ln 2 \). In the temporal evidence fusion, the information collected by the sensor at different times can be converted into BPA, the dynamic credibility of each evidence at different times can be calculated according to the credibility decay model, and the evidence can be discounted. Then, select appropriate combination rules to combine evidence.

3.2. Combination Rule of Complementary Belief. Since the evidence in the time domain evidence combination has the Markov property, it is only necessary to combine the evidence at the previous moment and the evidence at the current moment. Since the sensor is affected by factors such as interference, the information at different times is uncertain. Therefore, it is necessary to correct the evidence at different times through dynamic reliability. Most of the existing methods for evaluating the dynamic reliability of evidence are based on the “majority principle” [36], that is to say, if most of the evidence supports a certain piece of evidence, the reliability of this piece of evidence is high. However, this method is not suitable for the reliability assessment of only two evidences. When there are only two evidences, the evidence reliability factor obtained by the evidence conflict measurement or the direct distance measurement method is all \( 1 \), that is, it is impossible to determine which evidence is reliable. Therefore, a new method must be explored to realize the reliability assessment of two adjacent evidences in the time domain evidence sequence. In this section, a complementary reliability combination rule is proposed. The time domain evidence is integrated through the complementary reliability combination rule to obtain the intermediate evidence value, and then the distance between the time domain evidence and the intermediate evidence value is calculated by the distance metric, and then the reliability of time domain evidence is determined.

Aiming at the shortcomings of the existing combination rules in the fusion of conflict beliefs, a reverse method is used to calculate the fusion result of the target beliefs, which is called the complementary belief integration rule.
Assuming a mutually exclusive and complete identification framework \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \), the BPA of single-class belief \( \theta_i \) is \( m(\theta_i) \), then \( \Theta_i = \{ \theta_1, \theta_2, \ldots, \theta_j, \ldots, \theta_n | j \neq i \} \) is called the complementary reliability of \( \theta_i \) on the identification frame \( \Theta \).

**Definition 8.** Assuming that on the identification frame \( \Theta \), the BPA of the single-class proposition in the power set space is \( m \), and \( m_2(\theta_i) \) is defined as the complementary reliability BPA of \( \theta_i \) on the identification frame \( \Theta \), which satisfies

\[
m_2(\theta_i) = 1 - m(\theta_i).
\]

It can be seen from the above definition that the complementary belief BPA describes the sum of the belief of all single-class propositions except the given proposition \( \theta_i \) on the identification frame \( \Theta \).

After defining the complementary beliefs, the complementary belief integration rules can be described by the following definitions.

**Definition 9.** Assuming that the BPAs of the two beliefs for the single-class proposition in the power set space on the identification frame \( \Theta \) are \( m_1 \) and \( m_2 \), respectively, \( A, B, C \subseteq \Theta \), let \( m_1 \oplus m_2 \) represent the complementary belief integration rule, then

\[
m(A) = m_1 \oplus m_2(A) = \frac{1 - \sum_{B \subseteq \Theta} m_1(B) \cdot m_2(C)}{1 - \sum_{B \subseteq \Theta} m_1(B) \cdot m_2(C)}.
\]

Through the above integration rules, the time domain evidence can be combined, and the complementary belief integration rules satisfy both the commutative law and the associative law. Complementary belief integration rules do not directly integrate the target belief, but first use the defined complementary belief to fuse and then convert it into a judgment of the target belief. This method works together by all the belief and is less affected by a single belief. It embodies an average integration idea, which can overcome the impact of conflicting reliability on the results.

**Example 1.** Let the identification frame be \( \Theta = \{ \theta_1, \theta_2, \theta_3 \} \), and the two independent belief values are

\[
m_1(\theta_1) = 1, \\
m_1(\theta_2) = 0, \\
m_1(\theta_3) = 0, \\
m_2(\theta_1) = 0.2, \\
m_2(\theta_2) = 0.6, \\
m_2(\theta_3) = 0.2.
\]

If the Dempster combination rule is used to fuse the above two independent beliefs, due to a certain conflict, the value of the conflict coefficient between \( m_1 \) and \( m_2 \) is \( k = 1 - (m_{1 \cap 2}(\theta_1) + m_{1 \cap 2}(\theta_2) + m_{1 \cap 2}(\theta_3)) = 0.8 \), and the combined result obtained is

\[
m_2(\theta_1) = 1, \\
m_2(\theta_2) = 0, \\
m_2(\theta_3) = 0.
\]

Obviously, the above fusion results using the Dempster combination rule are not intuitive. This is because the Dempster combination rule is a normalized multiplicative strategy integration operator, which makes this integration method easily affected by individual belief values. It is ultimately reflected in the impact of conflict belief.

If the complementary belief integration rule is adopted, since \( m_1(\theta_1) = 1 \), the integration numerator of \( \theta_1 \) is always 1 during the belief integration process, that is, \( 1 - (1 - 1) \times (1 - 0.2) = 1 \), which has nothing to do with the value of \( m_2(\theta_1) \). The main role of the respective belief of \( \theta_2 \) and \( \theta_3 \) is reflected in the respective belief integration process, and their numerators are \( 1 - (1 - 0) \times (1 - 0.6) = 0.6 \), \( 1 - (1 - 0) \times (1 - 0.2) = 0.2 \), and finally through the normalized integration, the calculation result that can be obtained as

\[
m_2(\theta_1) = 0.5556, \\
m_2(\theta_2) = 0.3333, \\
m_2(\theta_3) = 0.1111.
\]

It can be seen that the above calculation results are intuitive. The complementary belief integration rule is a normalized integration operator, similar to the additive averaging strategy. It reflects a group decision-making idea in the process of belief integration, which can weaken the influence of conflicting belief and make the belief integration more consistent with actual.

### 3.3 Distance Function of Temporal Evidence

The research on evidence conflict functions can be divided into two categories: (1) metric functions defined based on various distances; (2) metric functions using the concept of information entropy. Here we focus on the distance metric function. The fuzzy theory describes uncertain information through the membership function, and the distance between information can be defined by the membership function.

\[
d_F(m_1, m_2) = 1 - \frac{\sum_{\theta \in \Theta} \mu_1(\theta) \land \mu_2(\theta)}{\sum_{\theta \in \Theta} \mu_1(\theta) \lor \mu_2(\theta)},
\]

where \( \land \) and \( \lor \) represent disjunction and conjunction, respectively, and for the function, the plausibility function or the belief function can be taken.

Since real-time is a factor that must be considered in the temporal domain evidence fusion process, complex algorithms are not suitable for fast calculation scenarios. In order to reduce the calculation amount of the distance metric function, the membership-pignistic probability distance function (MPPDF) is proposed. The specific definition is as follows:
Definition 10. Let the identification frame be \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \), \( \theta_i \in \Theta \), and \( m_1 \) and \( m_2 \) be the BPA on the identification frame \( \Theta \), then the MPPDF is defined as

\[
d_{FB}(m_1, m_2) = 1 - \frac{\sum_{\theta \in \Theta}(\text{Bet}_{P_1}(\theta) \land \text{Bet}_{P_2}(\theta))}{\sum_{\theta \in \Theta}(\text{Bet}_{P_1}(\theta) \lor \text{Bet}_{P_2}(\theta))},
\]

The MPPDF retains the disjunction and conjunction calculation of the membership degree function, and it can be proved that the MPPDF satisfies symmetry, consistency, non-negativity, and monotonicity. \( \text{Bet}_P(\cdot) \) is called the pignisitic probability function, which avoids solving the uncertain interval \( [\text{Bel}(\theta), P_l(\theta)] \), \( \forall \theta \in \Theta \), and the pignisitic probability function \( \text{Bet}_{P_m_\theta} : \Theta \rightarrow [0, 1] \)

\[
\text{Bet}_{P_m}(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, m(\emptyset) \neq 1,
\]

where \( |B| \) is the number of elements in the focus element set. \( \text{Bet}_P(\cdot) \) is mathematically the same as a general probability function. In particular, for one-element subsets, \( \forall \theta \in \Theta \), the pignistic probability of \( \theta \) is

\[
\text{Bet}_{P_m}(\theta) = \sum_{B \subseteq \Theta} \frac{1}{|B|} \frac{m(B)}{1 - m(\emptyset)}.
\]

\( \text{Bet}_{P_m}(\theta) \) evenly distributes the reliability of the composite focal element to the single elements it contains and realizes the conversion from BPA to the probability distribution.

4. Temporal Evidence Reasoning Fusion Recognition Method

In order to realize the fusion and identification of temporal evidence, it is necessary to first evaluate the importance and reliability of temporal evidence and then use the ER rule combination rule for evidence reasoning. This process is called the temporal evidence fusion based on the ER rule (the TEF-ER rule).

4.1. Recursive Fusion Model of Temporal Evidence. For the time domain information fusion problem, the corresponding fusion models can be divided into three categories: the recursive centralized fusion model, recursive distributed nonfeedback fusion model, and recursive distributed feedback fusion model. According to the recursive centralized fusion model, a recursive fusion method of spatiotemporal evidence based on the ER rule is obtained, as shown in Figure 1.

The target identification information accumulated at time \( t_{k-1} \) is called \( m(k-1) \). In the recursive centralized spatiotemporal information model, \( m(k-1) \) is combined with the target identification information measured by \( N \) sensors at time \( t_k \), then the total target recognition information fusion at time \( t_k \) is obtained. It can be seen that the accumulated information in the temporal domain at time \( t_k \) is the result of the spatial fusion of the information of each sensor at this time. The spatiotemporal information fusion model is equivalent to first fuse the spatial information of each time point and then perform the temporal domain information fusion.

4.2. Temporal Evidence Combination Method. In the recursive fusion of spatiotemporal evidence, the multisensor spatial information is first fused to obtain the fusion result \( m(k) \) at time \( t_k \), and then the temporal evidence is fused with the accumulated fusion information \( m(k-1) \) at time \( t_{k-1} \). The combination rule adopted here is the ER rule, which needs to evaluate the importance and reliability of the temporal evidence \( m(k-1) \) and \( m(k) \). In the case of only two evidences, the traditional evaluation method based on evidence distance cannot be used to evaluate the reliability of \( m(k-1) \) and \( m(k) \), which makes it impossible to use the ER rule to realize the fusion of temporal evidence. Therefore, a new temporal evidence fusion method is proposed, which first evaluates the importance and reliability of the two evidences and then performs evidence fusion based on the ER rule.

Assuming that the identification framework is \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \), according to the dynamic reliability evaluation method of temporal evidence, the fusion of temporal evidence can be completed according to the following process.

1. According to the temporal evidence decay model, calculate the decay factor \( \alpha \) of the accumulated fusion information \( m(k-1) \) at the current moment.

\[
\alpha = e^{-\lambda(t_k-t_{k-1})}.
\]

The decay factor \( \alpha \) is assigned as the evidence weight of the temporal evidence at the current moment, then the evidence weights of \( m(k-1) \) and \( m(k) \) are \( w_{k-1} = \alpha \) and \( w_k = 1 \), respectively.
(2) According to the complementary reliability integration rule, the two temporal evidences \( m(k-1) \) and \( m(k) \) are integrated to obtain the temporary evidence combination result \( m_{\text{Tem}}(k) \).

\[
m_{\text{Tem}}(A) = \left[ m_{k-1} \oplus m_k \right](A) = \frac{1 - \sum_{B \subseteq \Theta \cap A} m_{k-1}(B) \cdot m_k(C)}{\sum_{B \subseteq \Theta} m_{k-1}(B) \cdot m_k(C)}.
\]

(22)

(3) Using the MPPDF, the distances between \( m_{\text{Tem}}(k) \) and \( m(k-1) \) and \( m(k) \) are calculated, respectively, and \( d_{\text{FB}}(m_{k-1}, m_{\text{Tem}}) \) and \( d_{\text{FB}}(m_k, m_{\text{Tem}}) \) are obtained, which are converted into the reliability of time domain evidence \( r_{k-1} \) and \( r_k \).

\[
\begin{cases}
  r_{k-1} = 1 - d_{\text{FB}}(m_{k-1}, m_{\text{Tem}}), \\
  r_k = 1 - d_{\text{FB}}(m_k, m_{\text{Tem}}).
\end{cases}
\]

(23)

(4) Using the mixed factor \( \tilde{u}_i \) composed of the weight of evidence \( w_i \) and the reliability \( r_i \) to discount the belief of the evidence, the weighted belief distribution \( m_{\tilde{B}} \) with reliability is obtained.

(5) The ER rule is used to fuse two independent pieces of evidence, and the degree of support for hypothesis \( \theta \subseteq \Theta \) in the fusion result can be calculated by the following formula:

\[
\tilde{m}_{\theta_{(2)}} = \left[ (1 - r_2) m_{\theta_{(1)}} + (1 - r_1) m_{\theta_{(1)}} \right] + \sum_{B \subseteq \Theta} m_{\theta_{(1)}} \cdot m_{\theta_{(1)}}.
\]

(24)

where \( m_{\theta_{(i)}} = c_{\text{mix}} \cdot m_{\theta_{(i)}}, \forall \theta \subseteq \Theta, m_p(\theta) \).

(6) By normalizing the fusion result \( \tilde{m}_{\theta_{(2)}} \), the final result \( m_{\theta_{(2)}} \) of temporal evidence fusion can be obtained.

5. The Illustrative Example

In this section, the performance of the TEF-ER rule is analyzed through numerical examples and experimental simulations in decision-level fusion.

Example 2. Let the identification framework be \( \Theta = \{ \theta_1, \theta_2, \theta_3 \} \), the BPA corresponding to the accumulated identification results at time \( t_1 = 10s \) is \( m_1 \), and the BPA corresponding to the latest identification information obtained at the current time \( t_2 = 12s \) is \( m_2 \). The belief values of each category of \( m_1 \) and \( m_2 \) are given below.

\[
m_1(\theta_1) = 0.6, \quad m_1(\theta_2) = 0.1, \quad m_1(\theta_3) = 0.3, \\
m_2(\theta_1) = 0.0, \quad m_2(\theta_2) = 0.8, \quad m_2(\theta_3) = 0.2.
\]

(25)

According to the temporal evidence decay model, the weights of the two evidences are

\[
w_1 = e^{-\lambda(t_2 - t_1)}, \\
w_2 = e^{-\lambda(t_2 - t_1)}.
\]

(26)

Using complementary evidence integration rules, combine \( m_1 \) and \( m_2 \) to get the temporary evidence combination result at \( t_2 \) time.

\[
m_{\text{Tem}}(\theta_1) = 0.3226, \\
m_{\text{Tem}}(\theta_2) = 0.4409, \\
m_{\text{Tem}}(\theta_3) = 0.2366.
\]

(27)

Using the MPPDF, the distances between \( m_{\text{Tem}} \) and \( m_1 \) and \( m_{\text{Tem}} \) and \( m_2 \) are calculated, respectively, and \( d_{\text{FB}}(m_1, m_{\text{Tem}}) \) and \( d_{\text{FB}}(m_2, m_{\text{Tem}}) \) are obtained, and the reliability of the temporal evidence is \( r_1 = 0.4355 \) and \( r_2 = 0.5645 \).

Using the ER rule to fuse the temporal evidence, the final result is

\[
m_{12}(\theta_1) = 0.2066, \\
m_{12}(\theta_2) = 0.5337, \\
m_{12}(\theta_3) = 0.2597.
\]

(28)

It can be seen that the final fusion result has the greatest support for proposition \( \theta_2 \), which is consistent with the proposition supported by \( m_2 \). If the Dempster combination rule is used to directly combine the temporal evidence, due to the “one-vote veto” problem, it will lead to \( m_{12}(\theta_2) = 0 \), that is, the support of the proposition \( \theta_1 \) will always be 0. It can be seen that the temporal evidence combination method proposed in this article can better deal with the conflict between adjacent evidences and help make more reasonable decisions.
Example 3. On the basis of Example 2, set the corresponding BPA at time \( t_2 = 15s \) to be \( m_3 \), and divide it into two cases, the first supports proposition \( \theta_1 \) and the second supports proposition \( \theta_2 \).

\[
\begin{align*}
(1) & \quad m_3 (\theta_1) = 0.5, m_3 (\theta_2) = 0.3, m_3 (\theta_3) = 0.2 m_{13} (\theta_1) = 0.3646, m_{13} (\theta_2) = 0.4227, m_{13} (\theta_3) = 0.2126 \\
(2) & \quad m_3 (\theta_1) = 0.1, m_3 (\theta_2) = 0.75, m_3 (\theta_3) = 0.15 m_{13} (\theta_1) = 0.1141, m_{13} (\theta_2) = 0.7237, m_{13} (\theta_3) = 0.1621
\end{align*}
\]

Evidential reasoning has been widely used in air target fusion recognition. In order to further illustrate the effectiveness of time sequence evidence fusion method based on evidential reasoning rules, the following is a numerical example of evidential reasoning in the field of air target recognition to verify and analyze it.

Example 4. It is assumed that radar sensors are used for continuous observation of air targets, and then fusion identification of targets is carried out after the observation information is obtained. The main observation targets are the fighter jet, bomber, and helicopter, and the BPA of the same target at different times is obtained. Let \( \theta_1 \) stand for fighter jet, \( \theta_2 \) for bomber, and \( \theta_3 \) for helicopter, and the identification framework of the target to be identified is \( \Theta = \{\theta_1, \theta_2, \theta_3\} \). The observation information of multiple sensors at the same time can be fused in the spatial domain. There are many methods that can be used for spatial information fusion, such as the classic Dempster combination method, which can get ideal combination results. Since this article mainly studies the combination of temporal evidence, the spatial fusion results of 5 time points are directly given here.

The corresponding BPA after spatial domain fusion at each time is shown in Table 1. It can be seen that at time \( t_2 \), the spatial fusion result has greater support for the target category \( \theta_1 \) but less support for category \( \theta_3 \). At time \( t_1 \) and \( t_3 \), the spatial domain fusion results support category \( \theta_1 \) to a greater extent. In order to analyze the information fusion ability of the TEF_ERrule temporal evidence combination method proposed in this article, the Dempster combination method, TEC-CFR [36], and TEF_ER rule were used to perform fusion comparison experiments on temporal evidence at five time points. Table 2 presents the fusion results of the TEF_ERrule temporal evidence combination method at each time point.

As can be seen from Table 2, it can be considered that the system is disturbed at time \( t_3 \), which leads to an increase in the support for category \( \theta_1 \) and a decrease in support for category \( \theta_3 \) in the temporal evidence fusion result at this time. However, with the addition of subsequent time information, the TEF_ERrule method can gradually decrease the support for category \( \theta_1 \) and gradually increase the support for category \( \theta_3 \). For the convenience of intuitive analysis, Figure 2 shows the change trend of BPA over time when temporal evidence fusion is performed based on the TEF_ERrule method. It can be seen that at \( t_4 = 23s \), the recognition result of the target can be judged as \( \theta_1 \) based on the TEF_ERrule method. At time \( t_5 = 26s \), the support for the target category \( \theta_3 \) is further improved based on the TEF_ERrule method.

Figure 2 shows the temporal evidence fusion results based on TEF_ERrule. Figures 3–5 show the changing trends of the BPA of the three evidence combination methods for different categories of targets \( \theta_1, \theta_2, \) and \( \theta_3 \), respectively. In Figure 3, because the system is affected by interference at \( t_2 = 8s \), the BPA of the target category \( \theta_1 \) corresponding to the three methods is improved, but the TEF_ERrule method in this article is least affected by the interference. That is, the increase of BPA in the TEF_ERrule method is the smallest at time \( t_2 \). After time \( t_2 \), the BPA obtained by the Dempster combination method decreases slowly, while the BPA obtained by the TEF-ER rule and TEC_CFR method in this article decreases faster. In Figure 4, since all the temporal evidences have low support for the target category \( \theta_2 \), the corresponding BPA of the three methods are not very different. In Figure 5, for the target category \( \theta_3 \), it can be seen that the TEF_ERrule method in this article is interfered at \( t_2 \), and the decrease of BPA is the smallest. It can be seen that it is precisely because the TEF_ERrule algorithm in this article can combine the reliability of evidence and the weight of evidence, so the fusion recognition system has better anti-interference ability after the time domain interference.

When the Dempster combination method is used for temporal evidence fusion, the temporal evidence sequences are simply combined in sequence, and the influence of time factors is not considered. The TEF_ERrule method in this article can effectively reflect the influence of time factors and the relationship between adjacent evidences. In order to further verify the influence of time factor on the result of time domain evidence fusion, the evidence \( m_{13} \) at time \( t_1 \) and the evidence \( m_{13} \) at time \( t_2 \) in this example are exchanged. The corresponding BPA after spatial domain fusion at each time is shown in Table 3. Except for the BPA changes at \( t_1 \) and \( t_3 \), the spatial fusion recognition results at other times remain unchanged. The TEF_ERrule method is used to accumulate the spatial fusion results in Table 3 in the time domain, and then the cumulative fusion results in the time domain at different time points are shown in Table 4. In Table 3, the spatial domain fusion results at time \( t_1 \) have a larger conflict than the spatial domain fusion results at other times, while the conflicts between the fusion results at subsequent times are small. Therefore, it can be considered that the system is disturbed at time \( t_1 \), which leads to a large deviation of the spatial fusion results. Without the continuous addition of airspace fusion results at subsequent times, the support of the cumulative results in the time domain for the target category \( \theta_1 \) continues to decline, while the support for the target category \( \theta_3 \) continues to rise. At time \( t_4 \), a reasonable identification can be made according to the accumulated results in the time domain.

Comparing the fusion results in Tables 4 and 2, it can be seen that the cumulative fusion results obtained by using the TEF_ERrule method at different times are different. It shows that the change of any two time points and the change of the acquisition sequence of identification information may bring about changes in the results of temporal evidence fusion.
fusion. This shows that the TEF_ERrule method is sensitive to time and is an effective temporal evidence fusion method.

In order to intuitively analyze the influence of exchanging evidence $m_1$ and $m_2$ on the time-domain cumulative fusion results, Figure 6 shows the change trend of BPA over time when time-domain fusion is performed based on the TEF_ERrule method. Figures 2–5 show the changing trends of the BPA of the three evidence combination methods for different categories of targets $\theta_1$, $\theta_2$, and $\theta_3$, respectively. Comparing Figure 7 and Figure 3, it can be seen that when the Dempster combination method is used for time domain fusion, except that the BPA of the category target $\theta_1$ is different at the initial moment of $t_1$, the exchange of information at the first two moments does not affect the fusion results at subsequent moments. It shows that the Dempster combination method does not consider the influence of time factors, and does not meet the requirement of sequential evidence fusion in the time domain. As can be seen in Figure 7, the time domain cumulative result obtained by the TEF_ERrule method is different from that in Figure 3, which indicates that the algorithm is sensitive to time. Meanwhile, the fusion result shows an overall downward trend, consistent with the actual input BPA variation. Although the input evidence BPA changed at time $t_2$, the fusion result of the TEF_ERrule changed less than that of TEC_CFR and more than that of Dempster. At time $t_3$, the support degree of input evidence for $\theta_1$ is still not high, and the Dempster method basically has no change, while the fusion result obtained by the TEF_ERrule method is between the results of TEC_CFR and Dempster. As the evidence gradually tends to be consistent at subsequent times, the TEF_ERrule method also achieves a time-domain accumulation result that is basically consistent with the TEC_CFR method at $t_4$ and $t_5$. In Figure 8, since all temporal evidences have low support for the target category $\theta_2$, the corresponding BPAs of the three methods are not much

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$m(\theta_1)$</th>
<th>$m(\theta_2)$</th>
<th>$m(\theta_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 5$</td>
<td>0.23</td>
<td>0.13</td>
<td>0.64</td>
</tr>
<tr>
<td>$t_2 = 8$</td>
<td>0.95</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>$t_3 = 16$</td>
<td>0.44</td>
<td>0.10</td>
<td>0.55</td>
</tr>
<tr>
<td>$t_4 = 23$</td>
<td>0.20</td>
<td>0.10</td>
<td>0.70</td>
</tr>
<tr>
<td>$t_5 = 26$</td>
<td>0.25</td>
<td>0.20</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$m(\theta_1)$</th>
<th>$m(\theta_2)$</th>
<th>$m(\theta_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 5$</td>
<td>0.2300</td>
<td>0.1300</td>
<td>0.6400</td>
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<tr>
<td>$t_2 = 8$</td>
<td>0.7203</td>
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<td>0.2274</td>
</tr>
<tr>
<td>$t_3 = 16$</td>
<td>0.5753</td>
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<tr>
<td>$t_4 = 23$</td>
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<td>0.0676</td>
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</tr>
<tr>
<td>$t_5 = 26$</td>
<td>0.2672</td>
<td>0.1069</td>
<td>0.6258</td>
</tr>
</tbody>
</table>
different. In Figure 9, since the BPA of the target category $\theta_3$ is only low at the initial time of $t_1$ and the BPA of the rest of the time is maintained at a large value, the time-domain cumulative result obtained by the TEF_ERrule method shows a gradual upward trend after the time of $t_1$. In the TEC_CFR method, the BPA of the target category $\theta_3$ decreases at time $t_3$, which does not conform to the intuitive analysis. Since the fusion result obtained by the Dempster combination method is not affected by the information exchange of the previous two moments, it does not meet the requirement of sequential evidence fusion in the time domain. Through the above analysis, it can be seen that the TEF_ERrule method is sensitive to time and can effectively reflect the trend of evidence change in time.

Figure 3: BPA comparison of the category $\theta_1$ for different methods.

Figure 4: BPA comparison of the category $\theta_2$ for different methods.
Table 3: Spatial domain fusion results of different time nodes.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$m(\theta_1)$</th>
<th>$m(\theta_2)$</th>
<th>$m(\theta_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 5$</td>
<td>0.95</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>$t_2 = 8$</td>
<td>0.23</td>
<td>0.13</td>
<td>0.64</td>
</tr>
<tr>
<td>$t_3 = 16$</td>
<td>0.44</td>
<td>0.10</td>
<td>0.55</td>
</tr>
<tr>
<td>$t_4 = 23$</td>
<td>0.20</td>
<td>0.10</td>
<td>0.70</td>
</tr>
<tr>
<td>$t_5 = 26$</td>
<td>0.25</td>
<td>0.20</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 4: Cumulative fusion results in temporal domain of different time nodes using the TEF_ERrule method.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$m(\theta_1)$</th>
<th>$m(\theta_2)$</th>
<th>$m(\theta_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1 = 5$</td>
<td>0.9500</td>
<td>0.0200</td>
<td>0.0300</td>
</tr>
<tr>
<td>$t_2 = 8$</td>
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<td>0.0586</td>
<td>0.2628</td>
</tr>
<tr>
<td>$t_3 = 16$</td>
<td>0.5504</td>
<td>0.0620</td>
<td>0.3876</td>
</tr>
<tr>
<td>$t_4 = 23$</td>
<td>0.3268</td>
<td>0.0681</td>
<td>0.6051</td>
</tr>
<tr>
<td>$t_5 = 26$</td>
<td>0.2596</td>
<td>0.1067</td>
<td>0.6336</td>
</tr>
</tbody>
</table>

Figure 5: BPA comparison of the category $\theta_3$ for different methods.

Figure 6: Temporal domain fusion results based on the TEF_ERrule.
Figure 7: BPA comparison of the category $\theta_1$ for different methods.

Figure 8: BPA comparison of the category $\theta_2$ for different methods.
6. Conclusions

As an important part of information fusion, temporal evidence fusion shows obvious sequentiality and dynamics. The traditional spatial evidence fusion methods are not all suitable for temporal evidence fusion. How to fuse temporal evidence is a problem worth studying. This article establishes the evidence reliability and weight model and proposes the complementary reliability integration rule and the time-series evidence distance function. The intermediate evidence is obtained by combining the time domain evidence of adjacent time nodes, and the distance between the adjacent time node evidence and the intermediate evidence is calculated and converted into evidence reliability. The evidence weight in the time domain evidence sequence is determined by using the evidence time domain credibility decay model. Combined with evidence reliability and weight, a temporal evidence combination method based on the ER rule (the TEF-ER rule) is proposed, and a sequential fusion model of spatiotemporal information is constructed based on the TEF-ER rule. The performance characteristics of this method are verified by numerical examples and experimental simulations, and a comparative analysis with other methods is carried out. The results show that the TEF-ER rule method can effectively deal with the conflicts between time domain evidences, has strong anti-interference ability, and can reflect the influence of time factors on the temporal evidence combination. It should be pointed out that temporal information fusion is a complex engineering problem, especially for the fusion of different levels, which has both real-time and computational requirements. Therefore, it is necessary to combine the method proposed in this article with other related theories, which will be our next research direction.

Data Availability

The evaluation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors’ Contributions

Haibin Wang contributed to the conceptualization, research ideas, modeling methods, and programming of the article. Xin Guan contributed to the research ideas, modeling, and verification methods of the article. Xiao Yi contributed to the result analysis and experimental design of the article. Ying Liu contributed to the verification of the article and the visualization of experimental results. Guidong Sun contributed to the revision opinions and the improvement of the grammar of the article.

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