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Research Article

H_∞ Control for the Nonlinear Markov Networked Control System in the Presence of Data Packet Loss and DoS Attacks via Observer

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The H_∞ control issue for nonlinear Markov networked control systems in the presence of data packet loss and periodic denial-of-service (DoS) attacks is researched in this paper. First, two Bernoulli random variables are used to describe the packet loss between sensor to controller and controller to actuator. Considering the impact of DoS attacks, the probability of packet loss is set to be different during the attack sleeping interval and the attack active interval. Secondly, an observer is constructed on the controller side, and a comprehensive mathematical model including packet loss and DoS attacks is established. The sufficient conditions for the stochastic stability of the system are derived by the Lyapunov theory, and the design method of the controller and the minimum disturbance suppression performance index are also provided. Finally, a numerical example is utilized to reveal the applicability of the approach.

1. Introduction

Over the years, owing to the continuous development of network technology and the widespread application of computer networks, the structure of the control system is undergoing changes. The traditional point-to-point control method gradually loses its dominant position as the system becomes more complex and geographically dispersed. The network control system (NCS) formed by introducing a network into a control system and connecting sensors, controllers, and actuators has appeared [1–5]. NCS has far-going application backgrounds in different fields such as aerospace, national defense, transportation, wireless communications, and industrial automation, which makes networked control one of the research hot spots in academia [6–10].

NCS integrates various infrastructures through the network, thereby making human-computer interaction and physical processes more convenient. However, the introduction of communication networks impacts the stability of the system and also makes the system face the threat of network attacks. Generally speaking, typical network attacks are mainly divided into two types. The first type is the deception attack, which destroys the system by inserting incorrect data or processing raw data. The second type is the denial of service (DoS) attack, whose intent is to prevent intercourse between different system components, thereby degrading system functionality or destroying system stability [11–13]. DoS attacks are easy to implement. For this reason, they have been extensively researched [14–17]. For example, for a NCS under stochastic disturbances and DoS attacks, a distributed predictive controller was designed [18]. By using random perturbation information and explicitly considering packet loss due to DoS attacks, an observer was designed to reconstruct the state. For a class of NCS with DoS attacks, a resilient state feedback controller was designed [19]. The closed-loop system was modeled as an aperiodic sample data system that was related to the bound of the DoS attack duration. A state-error-dependent switched system model for NCS under resilient event-triggered communication schemes and periodic DoS attacks was proposed [20].

Data packet loss affects system performance as well as destabilizes the system, which has attracted a lot of attention and many research results have appeared. For instance, for the discrete NCSs subject to data packet loss in sensor-to-controller (S-C) and controller-to-actuator (C-A) channels, the range of successful packet transmission rates that made closed-loop NCSs stable was obtained with the asynchronous dynamical system theory [21]. For NCS suffering from S-C and C-A packet loss, an observer-based feedback control method was proposed. Considering the unmeasurable state of the controlled object, an observer was constructed to realize feedback control by describing the packet missing in S-C and C-A channels as stochastic Bernoulli variables [22]. The issue of predictive tracking control of NCSs subject to S-C data packet loss was investigated. The analysis of the packet miss impact on the performance of NCSs was carried out by taking input and output constraints into account [23]. Modeling the packet loss in the two channels as two independent Markov chains, the quantified dynamic output feedback control considering packet loss in the S-C and C-A channels was considered [24]. The above-mentioned results considered the problem of data packet loss without taking DoS attacks into account.

Currently, there are few results on the control problem for NCS in the presence of data packet loss and DoS attacks. The control issue for NCS with data packet dropout and DoS attacks was investigated [25]. However, the controlled plant in [25] was linear invariant system, and only the data packet loss in the S-C channel was considered. Moreover, resulted from the impacts such as component failures and environment changes, some control systems cannot be described by a definite model. The Markov jump system can be exploited to express the changes in the system parameters [26–29]. Therefore, the research for a networked Markov jump system subject to DoS attacks and data packet dropout has important theoretical and practical significance. The control problem of nonlinear Markov jump system subject to DoS attacks and data packet in both the S-C channel and C-A channel has not been researched, which motivates the current research. The contributions of this paper can be mainly exhibited as follows:

(1) Two independent Bernoulli variables are used to represent data packet loss in the S-C channel and C-A channel, respectively. Considering the fact that there is data packet transmission during the attack active interval, the probability of successful data packet transmission is set to be nonzero, which is much smaller than that during the attack sleeping interval.

(2) The closed-loop system is modeled as a class of control system with two variables. Under the designed controller, the closed-loop system is still stochastically stable and attains the H_∞ performance. And the connection between the disturbance suppression capability and DoS attacks is revealed.

The following content is divided into four sections. Section 2 establishes the mathematical model for nonlinear Markov NCS. Section 3 provides sufficient conditions on stochastic stability and the design method of the controller. In Section 4, numerical simulation example shows the effectiveness of the designed controller. Section 5 gives conclusions of this paper.

Notations: R^n represents the-dimensional Euclidean space. $*$ represents the transpose of the corresponding matrix block. A^{T} represents the transpose of the matrix A. If the matrix A is invertible, A^{-1} represents the inverse of A. The real positive definite matrix X is represented as $X > 0$. I represents the unit matrix of appropriate dimensions, and diag$\{a, b \ldots\}$ represents the diagonal matrix with a, b as the main diagonal. Pr $\{\cdot\}$ means mathematical probability. $E\,\{\cdot\}$ stands for mathematical expectation and Var $\{\cdot\}$ denotes variance.

2. Problem Formulation

The structure of nonlinear NCS subject to data packet loss and DoS attacks is exhibited in Figure 1, and the state equation of the plant is as follows:

$$\begin{cases} x(l+1) = A_{\delta(l)}x(l) + B_{\delta(l)}u(l) + D_{\delta(l)}\omega(l) \\ +f_{\delta(l)}(l, x(l)), \\ y(l) = C_{\delta(l)}x(l), \end{cases} \tag{1}$$

where $x(l) \in R^n$ represents the state, $u(l) \in R^m$ denotes the control input, $\omega(l) \in R^s$ stands for the disturbance, $y(l) \in R^r$ is the output, and $A_{\delta(l)}$, $B_{\delta(l)}$, $C_{\delta(l)}$, and $D_{\delta(l)}$ are known matrices with suitable dimensions. $\delta(l)$ takes value from $\ell = \{1, 2, \ldots, g\}$, and the transition possibility matrix of ℓ is $\Pi = [\pi_{ij}]$, $\pi_{ij} = \Pr\{\delta(l+1) = j \mid \delta(l) = i\}$, $\sum_{j=1}^{g}\pi_{ij} = 1$, and $\pi_{ij} \geq 0, i, j \in \ell$. $f_{\delta(l)}(l, x(l))$ is a nonlinear vector function and satisfies the following global Lipschitz condition [22]:

$$\left\| f_{\delta(l)}(l, x(l)) \right\| \leq \left\| G_{\delta(l)}x(l) \right\|, \tag{2}$$

$$\left\| f_{\delta(l)}(l, x(l)) - f_{\delta(l)}(l, y(l)) \right\| \leq \left\| G_{\delta(l)}(x(l) - y(l)) \right\|, \tag{3}$$

where $G_{\delta(l)}$ is a known real constant matrix.

Communication between NCS components usually suffers damage from malicious attackers. We suppose that the DoS attacks occur in the S-C channel. For the intent to describe the DoS attacks more conveniently, a power-constrained periodic jamming signal is illustrated as follows:

$$\lambda = \begin{cases} 1, l \in \left[(d-1)T, (d-1)T + T_{\mathrm{off}} \right), \\ 2, l \in \left[(d-1)T + T_{\mathrm{off}}, \mathrm{d}T \right), \end{cases} \tag{4}$$

where d indicates the period number, T indicates the attack period, T_{off} indicates attack sleeping interval, $\lambda = 1$ indicates it is in the sleeping interval of attack in the dth attack cycle, and $\lambda = 2$ indicates it is in the active interval of attack in the dth attack cycle.

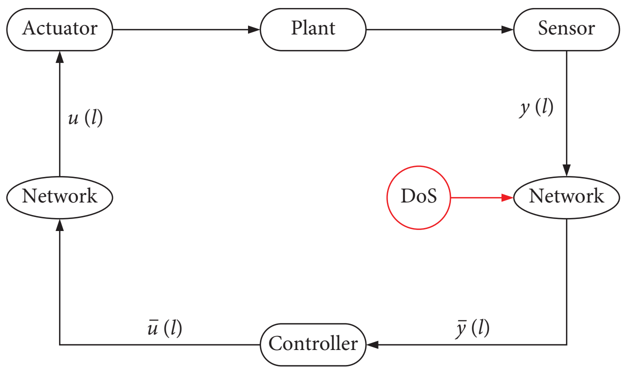

Figure 1: Nonlinear networked control system with data packet loss and DoS attacks.

Owing to the nature of the network, such as bandwidth limitations and network overcrowding, packets are unavoidably lost during transference. We suppose that the packet loss scenarios also exist during the sleeping interval. A random variable $\alpha(l) \in \{0,1\}$ is defined to describe the data packet transmission in the S-C channel. If $\alpha(l) = 0$, the data packet in the S-C channel is lost; if $\alpha(l) = 1$, the packet is successfully transmitted.

The random variable $\alpha(l)$ follows a Bernoulli distribution. For the sleeping interval, we suppose that the probability of successful data packet transference is $\Pr\{\alpha(l) = 1\} = \alpha_1$, and the probability of data packet transference failure is $\Pr\{\alpha(l) = 0\} = 1 - \alpha_1$. For the active period, the probability of successful data packet transmission is $\Pr\{\alpha(l) = 1\} = \alpha_2$, and the probability of data packet transference failure is $\Pr\{\alpha(l) = 0\} = 1 - \alpha_2$.

Therefore, the following equation can be obtained:

$$\begin{cases} \Pr\{\alpha(l) = 1\} = \alpha_\lambda, \\ \Pr\{\alpha(l) = 0\} = 1 - \alpha_\lambda, \end{cases} \tag{5}$$

where $\lambda = \begin{cases} 1, l \in [(d-1)T, (d-1)T + T_{\text{off}}) \\ 2, l \in [(d-1)T + T_{\text{off}}, dT) \end{cases}$ and $\text{Var}\{\alpha(l)\} = E\{(\alpha_\lambda(l) - \alpha_\lambda)^2\} = (1-\alpha_\lambda)\alpha_\lambda = \overline{\alpha}_\lambda^2$.

Remark 1. In the attack active interval, the data packet loss phenomenon is more severe than that in the sleeping period, therefore, $\alpha_2 < \alpha_1$.

Similarly, we define a random variable $\beta(l) \in \{0,1\}$ to describe the data packet transmission in the C-A channel. When $\beta(l) = 0$, the data packets transmitted in the C-A channel are lost; when $\beta(l) = 1$, the data packets are successfully transmitted, and the Bernoulli distribution white sequence of random variable $\beta(l)$ is as follows:

$$\begin{aligned} &\Pr\{\beta(l) = 1\} = \beta, \\ &\Pr\{\beta(l) = 0\} = 1 - \beta, \\ &\quad \text{Var}\{\beta(l)\} = E\{(\beta(l) - \beta)^2\} = (1-\beta)\beta = \overline{\beta}^2. \end{aligned} \tag{6}$$

The state equation of the observer is as follows:

$$\begin{cases} \hat{x}(l+1) = A_{\delta(l)}\hat{x}(l) + B_{\delta(l)}\overline{u}(l) + f_{\delta(l)}(l, \hat{x}(l)) \\ \quad + L(\overline{y}(l) - \alpha_\lambda C_{\delta(l)}\hat{x}(l)), \\ u(l) = \beta(l)\overline{u}(l), \end{cases} \tag{7}$$

where $\overline{y}(l) = \alpha_\lambda(l)y(l)$, $\hat{x}(l) \in R^n$ means the state of the observer, and $\overline{u}(l) \in R^m$ means the control input of the observer.

Due to the possible data packet loss in C-A channel, the control input of the controlled plant is as follows:

$$\overline{u}(l) = K\hat{x}(l). \tag{8}$$

Define the state estimation error $e(l) = x(l) - \hat{x}(l)$. Substituting (7) into (1) and (8), the closed-loop system model can be written as follows:

$$\begin{cases} x(l+1) = (A_{\delta(l)} + \beta B_{\delta(l)}K)x(l) - \beta B_{\delta(l)}Ke(l) \\ \quad + (\beta(l) - \beta)B_{\delta(l)}Kx(l) + D_{\delta(l)}\omega(l) \\ \quad - (\beta(l) - \beta)B_{\delta(l)}Ke(l) + f_{\delta(l)}(l, x(l)), \\ e(l+1) = (\beta - 1)B_{\delta(l)}Kx(l) - (\beta(l) - \beta)B_{\delta(l)}Ke(l) \\ \quad + (A_{\delta(l)} - (\beta - 1)B_{\delta(l)}K - \alpha_\lambda LC_{\delta(l)})e(l) \\ \quad + (\beta(l) - \beta)B_{\delta(l)}Kx(l) + D_{\delta(l)}\omega(l) \\ \quad - (\alpha_\lambda(l) - \alpha_\lambda)LC_{\delta(l)}e(l) + F_{\delta(l)}(l), \end{cases} \tag{9}$$

where $F_{\delta(l)}(l) = f_{\delta(l)}(l, x(l)) - f_{\delta(l)}(l, \hat{x}(l))$.

Define $\eta(l) = [x^T(l) \; e^T(l)]^T$; then the closed-loop system (9) can be expressed as follows:

$$\begin{aligned} \eta(l+1) = {} & \overline{A}\eta(l) + (\beta(l) - \beta)\overline{B}\eta(l) + \overline{D}\omega(l) \\ & + (\alpha_\lambda(l) - \alpha_\lambda)\overline{C}\eta(l) + \overline{F}(l), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \overline{A} &= \begin{bmatrix} A_{\delta(l)} + \beta B_{\delta(l)}K & -\beta B_{\delta(l)}K \\ (\beta - 1)B_{\delta(l)}K & A_{\delta(l)} - (\beta - 1)B_{\delta(l)}K - \alpha_\lambda LC_{\delta(l)} \end{bmatrix}, \\ \overline{B} &= \begin{bmatrix} B_{\delta(l)}K & -B_{\delta(l)}K \\ B_{\delta(l)}K & -B_{\delta(l)}K \end{bmatrix}, \overline{C} = \begin{bmatrix} 0 & 0 \\ 0 & -LC_{\delta(l)} \end{bmatrix}, \\ \overline{D} &= \begin{bmatrix} D_{\delta(l)} \\ D_{\delta(l)} \end{bmatrix}, \overline{F}(l) = \begin{bmatrix} f_{\delta(l)}(l, x(l)) \\ F_{\delta(l)}(l) \end{bmatrix}. \end{aligned} \tag{11}$$

Definition 2. (see [30]) When $\omega(l) = 0$, if for whichever initial mode $\delta(0)$ and state $\eta(0)$, such that

$$E\left\{\sum_{l=0}^{\infty} \|\eta(l)\|^2 \mid \eta(0), \delta(0)\right\} < \infty, \tag{12}$$

then system (10) is stochastically stable.

System (10) under random data packet loss and DoS attacks is stochastically stable and attains the H_∞ performance requirement, if the following two requirements are satisfied:

(1) System (10) under consideration is stochastically stable

(2) Under the zero initial condition, for all $\omega(l) \neq 0$, the controlled output $y(l)$ satisfies

$$E\left\{\sum_{l=0}^{\infty} y^{\mathrm{T}}(l)y(l)\right\} < \gamma^2 E\left\{\sum_{l=0}^{\infty} \omega^{\mathrm{T}}(l)\omega(l)\right\}, \tag{13}$$

where $\gamma > 0$ is a prescribed scalar.

Remark 3. Only the DoS attacks in the S-C channel is considered in this paper. The obtained results can be extended to the case where the DoS attacks exist in both the S-C and C-A channel, and the probability of packet loss in the C-A channel can be considered for both attack active interval and attack sleeping interval.

Lemma 4. *(see [31]) (S-procedure) Letting $T_i \in R^{n\times n}$ $(i = 0, 1, \ldots, p)$ be symmetric matrices, T_i $(i = 0, 1, \ldots, p)$, $\varsigma^{\mathrm{T}} T_0 \varsigma > 0$, $\forall \varsigma \neq 0$, s.t. $\varsigma^{\mathrm{T}} T_i \varsigma \geq 0$ $(i = 0, 1, \ldots, p)$ holds if there exist $\tau_i \geq 0$ $(i = 1, 2, \ldots, p)$ such that $T_0 - \sum_{i=1}^{p} \tau_i T_i > 0$.*

3. Main Results

The following theorem presents a sufficient condition on the stochastic stability of system (10).

Theorem 5. *For the communication channel parameters $0 \leq \alpha_\lambda \leq 1$ and $0 \leq \beta \leq 1$, if there exist positive definite matrices $P_i > 0$, $Y_i > 0$, controller gain matrix K, observer gain matrix L, and nonnegative scalars $\tau_1 \geq 0$, $\tau_2 \geq 0$, such that*

$$\begin{bmatrix} \Gamma_{11} & * \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} < 0, \quad P_i Y_i = I, \tag{14}$$

where

$$\Gamma_{11} = \begin{bmatrix} -P_i + \tau_1 G_i^{\mathrm{T}} G_i & * & * & * \\ 0 & -P_i + \tau_2 G_i^{\mathrm{T}} G_i & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Gamma_{21} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 \\ \Xi_{21} & \Xi_{22} & 0 & \Xi_{24} \\ \Xi_{31} & \Xi_{32} & 0 & 0 \\ \Xi_{31} & \Xi_{32} & 0 & 0 \\ \Xi_{41} & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} \Xi & * & * & * & * \\ 0 & \Xi & * & * & * \\ 0 & 0 & \Xi & * & * \\ 0 & 0 & 0 & \Xi & * \\ 0 & 0 & 0 & 0 & \Xi \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} &= \left[\sqrt{\pi_{i1}}\left(A_i + \beta B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(A_i + \beta B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{12} &= \left[\sqrt{\pi_{i1}}\left(-\beta B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(-\beta B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{13} &= \left[\sqrt{\pi_{i1}}\, I, \ldots, \sqrt{\pi_{ig}}\, I\right]^{\mathrm{T}}, \\
\Xi_{21} &= \left[\sqrt{\pi_{i1}}\left((\beta - 1)B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left((\beta - 1)B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{22} &= \left[\sqrt{\pi_{i1}}\left((A_i - (\beta - 1)B_i K - \alpha_\lambda L C_i)\right)^{\mathrm{T}}, \ldots, \right. \\
&\qquad \left. \sqrt{\pi_{ig}}\left((A_i - (\beta - 1)B_i K - \alpha_\lambda L C_i)\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{24} &= \left[\sqrt{\pi_{i1}}\, I, \ldots, \sqrt{\pi_{ig}}\, I\right]^{\mathrm{T}}, \\
\Xi_{31} &= \left[\sqrt{\pi_{i1}}\left(\overline{\beta} B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(\overline{\beta} B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{32} &= \left[\sqrt{\pi_{i1}}\left(-\overline{\beta} B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(-\overline{\beta} B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi_{41} &= \left[\sqrt{\pi_{i1}}\left(\overline{\alpha}_\lambda \mathrm{L} C_i\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(\overline{\alpha}_\lambda \mathrm{L} C_i\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Xi &= \mathrm{Diag}\left\{-Y_1, \ldots, -Y_g\right\}, \\
\overline{\alpha} &= \left[(1 - \alpha_\lambda)\alpha_\lambda\right]^{\frac{1}{2}}, \overline{\beta} = \left[(1 - \beta)\beta\right]^{\frac{1}{2}}.
\end{aligned} \tag{15}$$

then system (10) is stochastically stable.

Proof. Define Lyapunov function for system (12) as follows:

$$V(l) = x^{\mathrm{T}}(l) P_{\delta(l)} x(l) + e^{\mathrm{T}}(l) P_{\delta(l)} e(l), \tag{16}$$

where $P_{\delta(l)} > 0$. From (9) with $\omega(l) = 0$, we can get

$$\begin{aligned}
&\Delta V(l) \\
&\quad = E\{V(l+1) \mid x(l), \ldots, x(0), e(l), \ldots, e(0), \delta(l) = i\} \\
&\qquad - V(l), \\
&\quad = E\left\{x^{\mathrm{T}}(l+1) \sum_{j\in\ell} \pi_{ij} P_j x(l+1) - x^{\mathrm{T}}(l) P_i x(l)\right. \\
&\qquad \left. + e^{\mathrm{T}}(l+1) \sum_{j\in\ell} \pi_{ij} P_j e(l+1)\right\} - e^{\mathrm{T}}(l) P_i e(l), \\
&\quad = E\{[(A_i + \beta B_i K)x(l) - \beta B_i K e(l) + f_i(l, x(l)) \\
&\qquad + (\beta(l) - \beta) B_i K x(l) - (\beta(l) - \beta) B_i K e(l)]^{\mathrm{T}}
\end{aligned}$$

$$
\begin{aligned}
&\sum_{j\in\ell}\pi_{ij}P_j\left[\left(A_i+\beta B_iK\right)x(l)-\beta B_iKe(l)+f_i(l,x(l))\right.\\
&\left.+(\beta(l)-\beta)B_iKx(l)-(\beta(l)-\beta)B_iKe(l)\right]\\
&+\left[(\beta-1)B_iKx(l)+F_i(l)+(\beta(l)-\beta)B_iKx(l)\right.\\
&+\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC\right)e(l)-(\beta(l)-\beta)B_iKe(l)\\
&\left.-\left(\alpha_\lambda(l)-\alpha_\lambda\right)LC_ix(l)\right]^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\left[(\beta-1)B_iKx(l)\right.\\
&+F_i(l)+\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC\right)e(l)\\
&+(\beta(l)-\beta)B_iKx(l)-(\beta(l)-\beta)B_iKe(l)\\
&\left.-\left(\alpha_\lambda(l)-\alpha_\lambda\right)LC_ix(l)\right]\}-x^{\mathrm{T}}(l)P_ix(l)-e^{\mathrm{T}}(l)P_ie(l).
\end{aligned}
\tag{17}
$$

Owing to $E\left\{(\beta(l)-\beta)^2\right\}=\overline{\beta}^2$, $E\left\{(\alpha_\lambda(l)-\alpha_\lambda)^2\right\}=\overline{\alpha}_\lambda^2$, thus we can get

$$
\begin{aligned}
&\Delta V(l)\\
&=\left[\left(A_i+\beta B_iK\right)x(l)-\beta B_iKe(l)+f_i(l,x(l))\right]^{\mathrm{T}}\\
&\quad\sum_{j\in\ell}\pi_{ij}P_j\left[\left(A_i+\beta B_iK\right)x(l)-\beta B_iKe(l)+f_i(l,x(l))\right]\\
&\quad+\left[\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC\right)e(l)\right.\\
&\quad\left.+(\beta-1)B_iKx(l)+F_i(l)\right]^{\mathrm{T}}\\
&\quad\sum_{j\in\ell}\pi_{ij}P_j\left[\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC\right)e(l)\right.\\
&\quad\left.+(\beta-1)B_iKx(l)+F_i(l)\right]\\
&\quad+\overline{\beta}^2\left[B_iKx(l)-B_iKe(l)\right]^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\\
&\quad\left[B_iKx(l)-B_iKe(l)\right]\\
&\quad+\overline{\beta}^2\left[B_iKx(l)-B_iKe(l)\right]^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\\
&\quad\left[B_iKx(l)-B_iKe(l)\right]\\
&\quad+\overline{\alpha}_\lambda^2x^{\mathrm{T}}(l)C_i^{\mathrm{T}}L^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jLC_ix(l)\\
&\quad-x^{\mathrm{T}}(l)P_ix(l)-e^{\mathrm{T}}(l)P_ie(l)\\
&=\begin{bmatrix}x(l)\\e(l)\\f_i(l,x(l))\\F_i(l)\end{bmatrix}^{\mathrm{T}}\Lambda\begin{bmatrix}x(l)\\e(l)\\f_i(l,x(l))\\F_i(l)\end{bmatrix}\\
&\triangleq=\xi^{\mathrm{T}}(l)\Lambda\xi(l),
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
\Lambda&=\begin{bmatrix}\Lambda_{11}&*&*&*\\\Lambda_{21}&\Lambda_{22}&*&*\\\Lambda_{31}&\Lambda_{32}&\sum_{j\in\ell}\pi_{ij}P_j&*\\\Lambda_{41}&\Lambda_{42}&0&\sum_{j\in\ell}\pi_{ij}P_j\end{bmatrix},\\
\Lambda_{11}&=\left(A_i+\beta B_iK\right)^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\left(A_i+\beta B_iK\right)\\
&\quad+\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad+\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad+\overline{\alpha}_\lambda^2C_i^{\mathrm{T}}L^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jLC_i-P_i\\
&\quad+\left((\beta-1)B_iK\right)^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\left((\beta-1)B_iK\right),\\
\Lambda_{21}&=-\beta K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_j\left(A_i+\beta B_iK\right)\\
&\quad-\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad-\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad+\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC_i\right)^{\mathrm{T}}\\
&\quad\sum_{j\in\ell}\pi_{ij}P_j(\beta-1)B_iK,\\
\Lambda_{22}&=\beta^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK-P_i\\
&\quad+\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad+\overline{\beta}^2K^{\mathrm{T}}B_i^{\mathrm{T}}\sum_{j\in\ell}\pi_{ij}P_jB_iK\\
&\quad+\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC_i\right)^{\mathrm{T}}\\
&\quad\sum_{j\in\ell}\pi_{ij}P_j\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC_i\right),\\
\Lambda_{31}&=\sum_{j\in\ell}\pi_{ij}P_j\left(A_i+\beta B_iK\right),\\
\Lambda_{32}&=\sum_{j\in\ell}\pi_{ij}P_j\left(-\beta B_iK\right),\\
\Lambda_{41}&=(\beta-1)\sum_{j\in\ell}\pi_{ij}P_jB_iK,\\
\Lambda_{42}&=\sum_{j\in\ell}\pi_{ij}P_j\left(A_i-(\beta-1)B_iK-\alpha_\lambda LC_i\right).
\end{aligned}
\tag{19}
$$

It follows from (2)-(3) that

$$
\begin{aligned}
f_i^{\mathrm{T}}(l, x(l)) f_i(l, x(l)) & \\
&= \left\| f_i(l, x(l)) \right\|^2 \\
&\leq \left\| G_i x(l) \right\|^2 \\
&= x^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i x(l),
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
F_i^{\mathrm{T}}(l) F_i(l) & \\
&= \left\| F_i(l) \right\|^2 \\
&\leq \left\| G_i e(l) \right\|^2 \\
&= e^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i e(l),
\end{aligned} \tag{21}
$$

which indicates that

$$
\begin{aligned}
& f_i^{\mathrm{T}}(l, x(l)) f_i(l, x(l)) - x^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i x(l) \\
&= \xi^{\mathrm{T}}(l) \begin{bmatrix} -G_i^{\mathrm{T}} G_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi(l) \\
&\triangleq = \xi^{\mathrm{T}}(l) \Lambda_1 \xi(l) \\
&\leq 0,
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
& F_i^{\mathrm{T}}(l) F_i(l) - e^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i e(l) \\
&= \xi^{\mathrm{T}}(l) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -G_i^{\mathrm{T}} G_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \xi(l) \\
&\triangleq = \xi^{\mathrm{T}}(l) \Lambda_2 \xi(l) \\
&\leq 0.
\end{aligned} \tag{23}
$$

By the well-known S-procedure, that is, Lemma 4, we can get $\Delta V(l) = \xi^{\mathrm{T}}(l) \Lambda \xi(l) < 0$ with constrains (22) and (23) holding, if there exist non-negative real scalars $\tau_1 \geq 0$, $\tau_2 \geq 0$ such that

$$
\Lambda - \tau_1 \Lambda_1 - \tau_2 \Lambda_2 < 0. \tag{24}
$$

From (24), we have

$$
\begin{aligned}
\Delta V(l) & \\
&= \xi^{\mathrm{T}}(l) \Lambda \xi(l) \\
&\leq -\lambda_{\min}(-\Lambda) \xi^{\mathrm{T}}(l) \xi(l) \\
&= -\lambda_{\min}(-\Lambda) \left(\eta^{\mathrm{T}}(l) \eta(l) + F_i^{\mathrm{T}}(l) F_i(l) \right. \\
&\quad \left. + f_i^{\mathrm{T}}(l, x(l)) f_i(l, x(l)) \right) \\
&= -\lambda_{\min}(-\Lambda) \left(\|\eta(l)\|^2 + \left\| F_i(l) \right\|^2 + \left\| f_i(l, x(l)) \right\|^2 \right) \\
&\leq -\lambda_{\min}(-\Lambda) \|\eta(l)\|^2.
\end{aligned} \tag{25}
$$

From (25), for any $N \geq 0$, we have

$$
\begin{aligned}
& E\left\{ \sum_{l=0}^{N} \|\eta(l)\|^2 \right\} \\
&\leq \frac{E\{V(0)\} - E\{V(N+1)\}}{\lambda_{\min}(-\Lambda)} \\
&\leq \frac{E\{V(0)\}}{\lambda_{\min}(-\Lambda)} \\
&\leq \infty.
\end{aligned} \tag{26}
$$

Therefore, in accordance with Definition 2, that system (10) under consideration is stochastically stable. □

Remark 7. Theorem 5 provides sufficient conditions, which makes sure system (12) is stochastically stable. The following Theorem 8 will provide the proof that system (12) under consideration is stochastically stable and attain the H_∞ performance requirement (13).

Theorem 8. *When $\omega(l) \neq 0$, take as given the communication channel parameters $0 \leq \alpha_\lambda \leq 1$, $0 \leq \beta \leq 1$ and a scalar $\gamma > 0$, if there exist positive definite matrices $P_i > 0$, $Y_i > 0$, matrices K, L, and scalars $\tau_1 \geq 0$, $\tau_2 \geq 0$, such that*

$$
\begin{bmatrix} \Upsilon_{11} & 0 \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} < 0, \tag{27}
$$

$$
P_i Y_i = I, \tag{28}
$$

where

$$
\Upsilon_{11} = \begin{bmatrix} \chi_{11} & * & * & * & * \\ 0 & \chi_{22} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},
$$

$$
\Upsilon_{21} = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & 0 \\ \Theta_{21} & \Theta_{22} & \Theta_{13} & 0 & \Theta_{14} \\ C_i & 0 & 0 & 0 & 0 \\ \Theta_{31} & \Theta_{32} & 0 & 0 & 0 \\ \Theta_{31} & \Theta_{32} & 0 & 0 & 0 \\ \Theta_{41} & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\Upsilon_{22} = \begin{bmatrix} \Theta & * & * & * & * & * \\ 0 & \Theta & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & \Theta & * & * \\ 0 & 0 & 0 & 0 & \Theta & * \\ 0 & 0 & 0 & 0 & 0 & \Theta \end{bmatrix},
$$

$$
\begin{aligned}
\chi_{11} &= -P_i + \tau_1 G_i^{\mathrm{T}} G_i, \chi_{22} \\
\Theta_{11} &= \left[\sqrt{\pi_{i1}}\,(A_i + \beta B_i K)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\,(A_i + \beta B_i K)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{12} &= \left[\sqrt{\pi_{i1}}\,(-\beta B_i K)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\,(-\beta B_i K)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{13} &= \left[\sqrt{\pi_{i1}}\,D_i^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\,D_i^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{14} &= \left[\sqrt{\pi_{i1}}\,I, \ldots, \sqrt{\pi_{ig}}\,I\right]^{\mathrm{T}}, \\
\Theta_{21} &= \left[\sqrt{\pi_{i1}}\,((\beta-1)B_i K)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\,((\beta-1)B_i K)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{31} &= \left[\sqrt{\pi_{i1}}\left(\overline{\beta} B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(\overline{\beta} B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{22} &= \left[\sqrt{\pi_{i1}}\,((A_i - (\beta-1)B_i K - \alpha_\lambda L C_i))^{\mathrm{T}}, \ldots,\right. \\
&\qquad \left.\sqrt{\pi_{ig}}\,((A_i - (\beta-1)B_i K - \alpha_\lambda L C_i))^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{32} &= \left[\sqrt{\pi_{i1}}\left(-\overline{\beta} B_i K\right)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\left(-\overline{\beta} B_i K\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta_{41} &= \left[\sqrt{\pi_{i1}}\,(\overline{\alpha}_\lambda L C_i)^{\mathrm{T}}, \ldots, \sqrt{\pi_{ig}}\,(\overline{\alpha}_\lambda L C_i)^{\mathrm{T}}\right]^{\mathrm{T}}, \\
\Theta &= \mathrm{Diag}\left\{-Y_1, \ldots, -Y_g\right\}, \\
\overline{\alpha} &= [(1-\alpha_\lambda)\alpha_\lambda]^{\frac{1}{2}}, \overline{\beta}
\end{aligned} \tag{29}
$$

then system (10) attains the H_∞ performance requirement (13).

Proof. When $\omega(l) \neq 0$, from (11) we can get

$$
\begin{aligned}
&E\left\{\Delta V(l) + y^{\mathrm{T}}(l) y(l) - \gamma^2 \omega^{\mathrm{T}}(l)\omega(l)\right\} \\
&\quad = E\{[(A_i + \beta B_i K)x(l) - \beta B_i K e(l) + D_i \omega(l) \\
&\qquad + (\beta(l) - \beta) B_i K x(l) - (\beta(l) - \beta) B_i K e(l) \\
&\qquad + f_i(l, x(l))]^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j [(A_i + \beta B_i K)x(l) \\
&\qquad - \beta B_i K e(l) + D_i \omega(l) + f_i(l, x(l)) \\
&\qquad + (\beta(l) - \beta) B_i K x(l) - (\beta(l) - \beta) B_i K e(l)] \\
&\qquad + [(\beta - 1) B_i K x(l) + (\beta(l) - \beta) B_i K x(l) + F_i(l) \\
&\qquad + D_i \omega(l) + (A_i - (\beta - 1) B_i K - \alpha_\lambda L C) e(l) \\
&\qquad - (\beta(l) - \beta) B_i K e(l) - (\alpha_\lambda(l) - \alpha_\lambda) L C_i x(l)]^{\mathrm{T}} \\
&\qquad \sum_{j\in\ell} \pi_{ij} P_j [(\beta - 1) B_i K x(l) + D_i \omega(l) \\
&\qquad + F_i(l) + (A_i - (\beta - 1) B_i K - \alpha_\lambda L C) e(l) \\
&\qquad + (\beta(l) - \beta) B_i K x(l) - (\beta(l) - \beta) B_i K e(l) \\
&\qquad - (\alpha_\lambda(l) - \alpha_\lambda) L C_i x(l)]\} - x^{\mathrm{T}}(l) P_i x(l) \\
&\qquad - e^{\mathrm{T}}(l) P_i e(l) + x^{\mathrm{T}}(l) C_i^{\mathrm{T}} C_i x(l) - \gamma^2 \omega^{\mathrm{T}}(l)\omega(l) \\
&\quad = \zeta^{\mathrm{T}}(l) \Omega \zeta(l),
\end{aligned} \tag{30}
$$

where

$$
\begin{aligned}
\zeta^{\mathrm{T}} &= \left[x^{\mathrm{T}}(l)\;\; e^{\mathrm{T}}(l)\;\; \omega^{\mathrm{T}}(l)\;\; f_i^{\mathrm{T}}(l, x(l))\;\; F_i^{\mathrm{T}}(l)\right], \\
\Omega &= \begin{bmatrix}
\varphi_{11} & * & * & * & * \\
\varphi_{21} & \varphi_{22} & * & * & * \\
\varphi_{31} & \varphi_{32} & \varphi_{33} & * & * \\
\varphi_{41} & -\beta P_i B_i K & P D_i & P_i & * \\
\varphi_{51} & \varphi_{52} & P D_i & 0 & P_i
\end{bmatrix}, \\
\varphi_{11} &= (A_i + \beta B_i K)^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (A_i + \beta B_i K) \\
&\quad + \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K + \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K \\
&\quad + \overline{\alpha}_\lambda^2 C_i^{\mathrm{T}} L^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j L C_i - P_i \\
&\quad + ((\beta - 1) B_i K)^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (\beta - 1) B_i K + C^{\mathrm{T}} C, \\
\varphi_{21} &= -\beta K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (A_i + \beta B_i K) \\
&\quad - \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K - \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K \\
&\quad + (A_i - (\beta - 1) B_i K - \alpha_\lambda L C_i)^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (\beta - 1) B_i K, \\
\varphi_{22} &= \beta^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K + \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K \\
&\quad + \overline{\beta}^2 K^{\mathrm{T}} B_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} Q_j B_i K - P_i \\
&\quad + (A_i - (\beta - 1) B_i K - \alpha_\lambda L C_i)^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j \\
&\quad (A_i - (\beta - 1) B_i K - \alpha_\lambda L C_i), \\
\varphi_{31} &= D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (A_i + \beta B_i K) \\
&\quad + D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (\beta - 1) B_i K, \\
\varphi_{32} &= -\beta D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j B_i K \\
&\quad + D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j (A_i - (\beta - 1) B_i K - \alpha_\lambda L C_i), \\
\varphi_{33} &= D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j D_i + D_i^{\mathrm{T}} \sum_{j\in\ell} \pi_{ij} P_j D_i - \gamma^2 I, \\
\varphi_{41} &= P_i A_i + \beta P_i B_i K, \\
\varphi_{51} &= (\beta - 1) P_i B_i K, \\
\varphi_{52} &= P_i (A_i - (\beta - 1) B_i K - \alpha_\lambda L C_i).
\end{aligned} \tag{31}
$$

From (18)-(19), we can obtain

$$
\begin{aligned}
& f_i^{\mathrm{T}}(l, x(l)) f_i(l, x(l)) - x^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i x(l) \\
& = \zeta^{\mathrm{T}}(l) \begin{bmatrix} -G_i^{\mathrm{T}} G_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \zeta(l) \\
& \triangleq = \zeta^{\mathrm{T}}(l) \Omega_1 \zeta(l) \\
& \leq 0,
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
& F_i^{\mathrm{T}}(l) F_i(l) - e^{\mathrm{T}}(l) G_i^{\mathrm{T}} G_i e(l) \\
& = \zeta^{\mathrm{T}}(l) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -G_i^{\mathrm{T}} G_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \zeta(l) \\
& \triangleq = \zeta^{\mathrm{T}}(l) \Omega_2 \zeta(l) \\
& \leq 0.
\end{aligned} \tag{33}
$$

By the well-known S-procedure, that is, Lemma 4, we can get

$$
\zeta^{\mathrm{T}}(l) \Omega \zeta(l) < 0, \tag{34}
$$

with constrains (32) and (33) holding. If there exist non-negative real scalars $\tau_1 \geq 0$, $\tau_2 \geq 0$ such that

$$
\Omega - \tau_1 \Omega_1 - \tau_2 \Omega_2 < 0. \tag{35}
$$

From (26)–(30), we can conclude that

$$
E\left\{ \Delta V(l) + y^{\mathrm{T}}(l) y(l) - \gamma^2 \omega^{\mathrm{T}}(l) \omega(l) \right\} < 0. \tag{36}
$$

Adding up (39) from $l = 0$ to $l = \infty$:

$$
\begin{aligned}
& \sum_{l=0}^{\infty} E\left\{ y^{\mathrm{T}}(l) y(l) \right\} \\
& < \gamma^2 \sum_{l=0}^{\infty} E\left\{ \omega^{\mathrm{T}}(l) \omega(l) \right\} + E\{V(0)\} - E\{V(\infty)\}.
\end{aligned} \tag{37}
$$

Duo to the condition that system (10) is stochastically stable, we can get

$$
\sum_{l=0}^{\infty} E\left\{ y^{\mathrm{T}}(l) y(l) \right\} < \gamma^2 \sum_{l=0}^{\infty} E\left\{ \omega^{\mathrm{T}}(l) \omega(l) \right\}. \tag{38}
$$

which implies the H_∞ performance index (12) is achieved. This ends the proof. □

Remark 10. The prerequisites in Theorem 8 are a series of matrix inequalities under matrix inverse constraints, which can be settled by the cone complementary linearization method as follows:

$$
\operatorname{Mintr}\left(\sum_{i=1}^{g} P_i Y_i \right), \text{s.t.}\ (24) \text{ and } (32), \tag{39}
$$

$$
\begin{bmatrix} P_i & I \\ I & Y_i \end{bmatrix} > 0, i \in \ell. \tag{40}
$$

The computing steps are illustrated in Algorithm 1, where μ is a suitable scalar.

4. Numerical Example

In this section, an example is presented to illustrate the effectiveness of the obtained method.

The parameter of the controlled plant is as follows:

$$
\begin{cases} x(l+1) = A_{\delta(l)} x(l) + B_{\delta(l)} u(l) + D_{\delta(l)} \omega(l) \\ + f_{\delta(l)}(l, x(l)), \\ y(l) = C_{\delta(l)} x(l), \end{cases} \tag{41}
$$

where

$$
\begin{aligned}
& A_1 = \begin{bmatrix} 0.8266 & -0.6330 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1.0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, D_1 = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}, \\
& A_2 = \begin{bmatrix} 0.9226 & -0.6330 & 0 \\ 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}, D_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\
& C_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \delta(l) \in \{1, 2\}, \\
& f_1(l, x(l)) = \begin{bmatrix} 0.01 \sin x_1(l) \\ 0.01 \sin x_2(l) \\ 0.01 \sin x_3(l) \end{bmatrix}, x(l) = \begin{bmatrix} x_1(l) \\ x_2(l) \\ x_3(l) \end{bmatrix}, \\
& f_2(l, x(l)) = \begin{bmatrix} 0.02 \sin x_1(l) \\ 0.02 \sin x_2(l) \\ 0.02 \sin x_3(l) \end{bmatrix}, \tau_1 = 0.5, \tau_2 = 0.3, \\
& G_1 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, G_2 = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix}.
\end{aligned} \tag{42}
$$

Suppose the attack period is $T = 10$, the sleeping period is set as $T_{\text{off}} = 6$, and the total operating time is $l = 60$, which is $d = \{1, 2, \ldots, 6\}$. The transition possibility matrices of $\delta(l)$ is $Q = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$. To analyze the impact caused by the DoS attacks, the probabilities of success to transmission data packet in the sleeping period are fixed, that is $\alpha_1 = 0.9$ and $\beta = 0.8$. Table 1 lists the allowable minimum value of γ. It is clear that when α_2 gets bigger, $\gamma_{\min}$ gets smaller. The impact of DoS attacks is clear, demonstrating the importance of investigating security issues.

According to Theorem 8, when $\alpha_1 = 0.9$, $\alpha_2 = 0.3$, and $\beta = 0.8$, K, L, and $\gamma_{\min}$ can be obtained as follows:

(1) Set $R_{\max}$ as the maximal iteration number, and let $\gamma = \gamma_0$
(2) Obtain a feasible solution (P_i^0, Y_i^0, K^0, L^0) satisfying (23) and (31), and let $l = 0$
(3) Settle the optimization issue bellow: Min $\operatorname{tr}(\sum_{i=1}^{g} P_i^l Y_i + Y_i^l P_i)$ such that (27) and (40)
(4) Set $P_i^l = P_i, Y_i^l = Y_i, K^l = K, L^l = L$
(5) **while** iterations number $< R_{\max}$ **do**
(6) **if** (27) and (28) hold, **then**
(7) $\gamma = \gamma - \mu$, $l = l + 1$, go to step 3
(8) **else**
(9) $l = l + 1$, go to step 3
(10) **end if**
(11) **end while**
(12) **if** $\gamma < \gamma_0$, **then**
(13) $\gamma_{\min} = \gamma + \mu$
(14) **else**
(15) No solution can be obtained within $R_{\max}$
(16) **end if**

ALGORITHM 1: Computing steps of (24) and (25).

TABLE 1: The minimum value of γ for different α_2.

α_2	$\gamma_{\min}$
0.3	0.4164
0.4	0.4157
0.5	0.4150
0.6	0.4141
0.7	0.4133
0.8	0.4124
0.9	0.4111

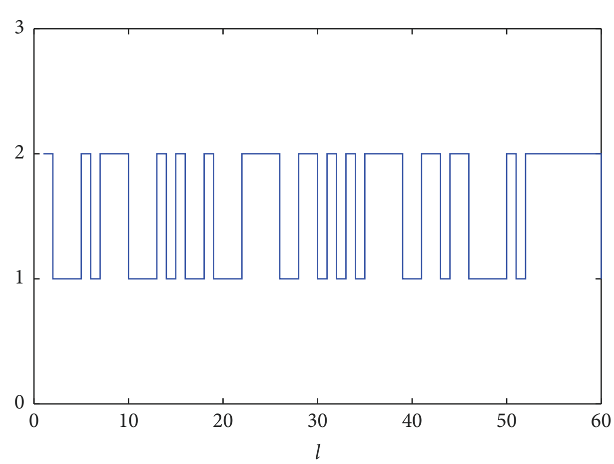

FIGURE 2: The mode of the closed-loop system $\delta(l)$.

$$K = \begin{bmatrix} -0.1339 & -0.0274 & -0.0026 \end{bmatrix}, L = \begin{bmatrix} 1.0868 \\ 0.1465 \\ -0.7119 \end{bmatrix},$$
$$\gamma_{\min} = 0.417. \tag{43}$$

The initial conditions of the nonlinear NCS are supposed as $x_0 = \begin{bmatrix} 0.2 & 0.3 & 0.1 \end{bmatrix}^{\mathrm{T}}$, $\hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$, and the

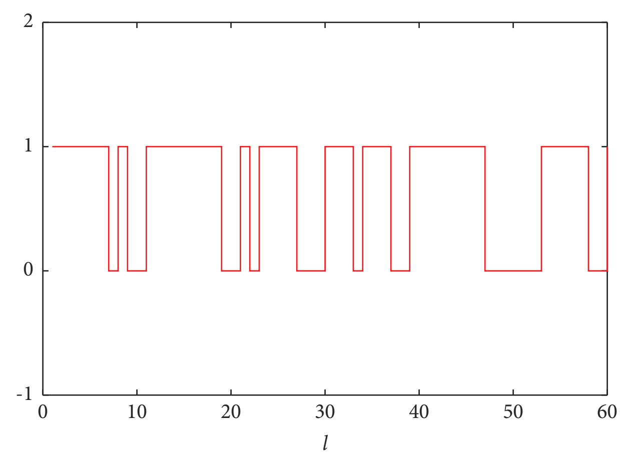

FIGURE 3: The data packet dropout in the S-C channel $\alpha(l)$.

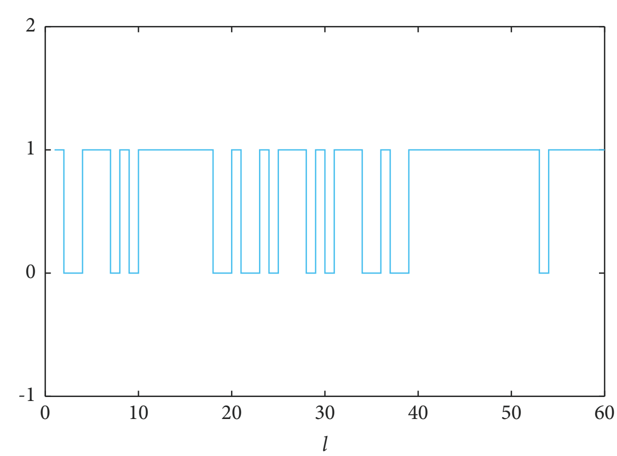

FIGURE 4: The data packet dropout in the C-A channel $\beta(l)$.

disturbance input is assumed to be $\omega(l) = 1/l^2$. The mode of the system under consideration is shown in Figure 2. The data packet dropout in the S-C channel is shown in Figure 3, and the data packet dropout in the C-A channel is exhibited in

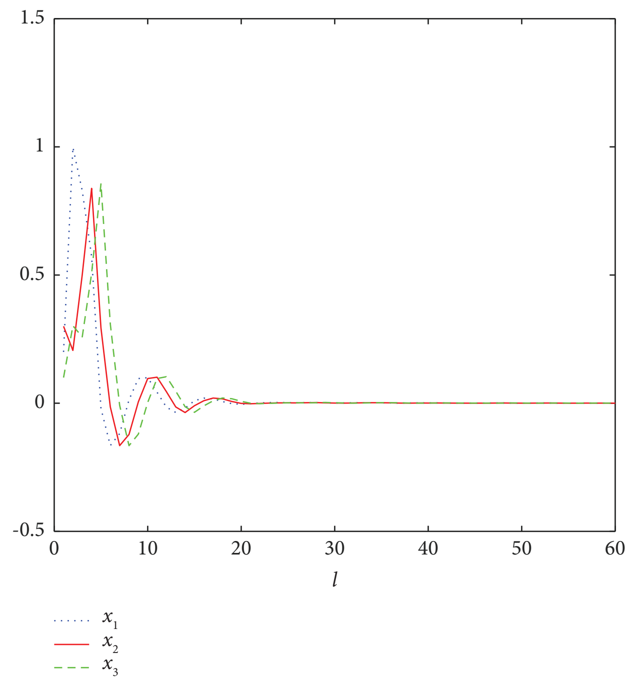

Figure 5: The state responses of the open-loop system.

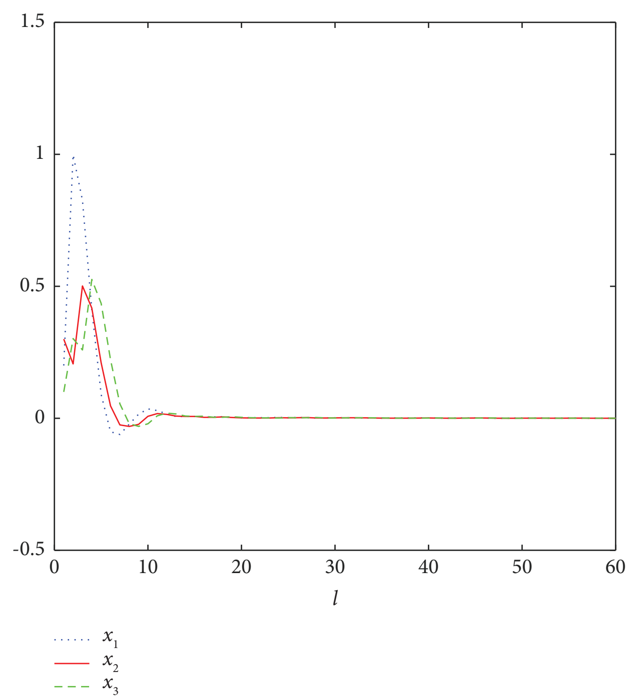

Figure 6: The state responses of the closed-loop system.

Figure 4. As can be seen from Figure 3, the probability of data packet loss during the active attack period is significantly greater than that during the attack sleep period. In the total running time, the number of successful packet transfers is 34. The number of packet transmission failures is 26, which is much higher than the average packet transmission failure probability of 0.1 when there is no network attack. In Figure 4, the number of successful data packet transmission in the total running time is 51. The number of packet transmission failures is 6, which is very close to the average data packet loss failure probability of 0.2 when there is no network attack.

We give the response curve of the open-loop system, as shown in Figure 5. Since both subsystems of the controlled object are stable, the system state converges to zero in the

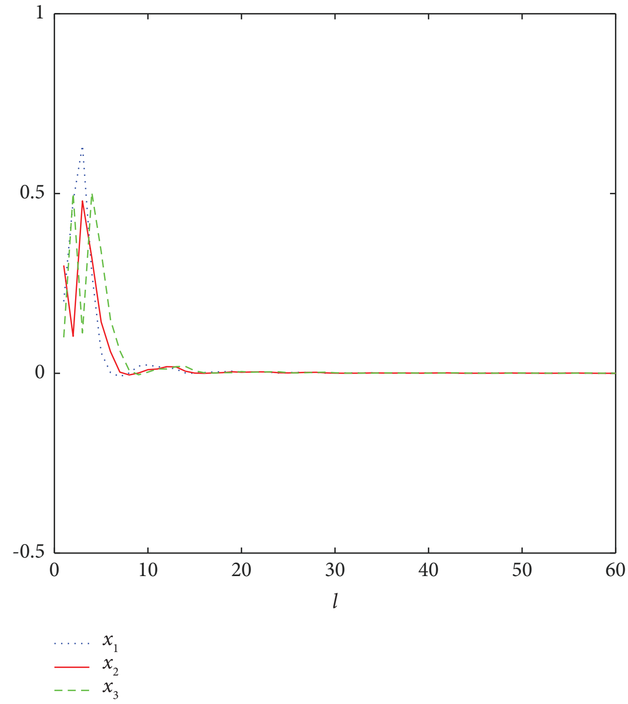

Figure 7: The state responses of the closed-loop system without data packet dropout.

open-loop case. As can be seen from Figure 6, under the action of the designed controller, the system performance such as convergence time and overshoot is better than that of the open-loop system.

If there is no packet loss in S-C and C-A channels, the system state curve is shown in Figure 7. Compared with Figure 6, the system performance is better than that in the case of packet loss. Hence, data packet loss will reduce system performance.

5. Conclusions

This paper researches the H_∞ control problem for a nonlinear Markov NCS with random data packet loss and periodic DoS attacks via observer. Premeditating the data packet loss in the S-C channel and C-A channel, sufficient conditions for the stochastic stability of the system are derived, and the controller design method is also proposed. Simulation results demonstrate the effectiveness of the proposed method. Under the event-triggered mechanism, the controller design for NCS subject to data packet dropout and DoS attacks in both the S-C channel and C-A channel will be researched in the future.

Data Availability

The data used in this study are available upon reasonable request to the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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