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Research Article

Managing Emergency Procurement Using Option Contract under Supply Disruption and Demand Uncertainty

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Effective and flexible procurement and production strategies are capable of alleviating and mitigating supply disruption and demand risk. Considering the price fluctuation caused by environmental change, we investigate the optimal procurement and production strategies under supply disruption and demand uncertainty based on option contract in a two-stage supply chain consisting of a retailer who has two procurement opportunities and a supplier who has the emergency production chance. We explore the value of option contract by comparing it with the optimal decision making under no option contract. The result shows that option ordering and emergency procurement can coordinate the optimal strategies under uncertain environment, improving the economic performance of whole supply chain. When the disruption probability is high or the price of emergency procurement is lower, the higher option price can stimulate the supplier to produce more products to satisfy the retailer's emergency order at a low price, which is beneficial to both, and the value of option ordering is greater. Otherwise, the emergency procurement is worth more for the core enterprise. The moderate exercise price is conducive to the long-term cooperation of the supplier and the retailer.

1. Introduction

As supply chain becomes increasingly complex, supply disruption caused by unexpected situations (e.g., natural disaster, man-made destruction, machine failure, transport obstruction, economic crisis, and other factors) occurs frequently, thus causing catastrophic damage to relevant enterprises and even adversely affecting economic market and social stability [1, 2]. In 2018, ZTE blocked the chip supply from Qualcomm due to violating the U.S. export control policy on Iran, which led to a loss of more than 20 billion yuan and total chaos on market. Besides, the uncertainty of market demand is also one of the important factors affecting the profits of enterprises. The fast demand update and large market fluctuation of some industries (e.g., clothing and semiconductor industries) inevitably lead to surplus or shortage of products, which drops the profitability of upstream and downstream enterprises. However, one-time ordering and traditional bilateral procurement contracts, which are generally adopted in many industries, are difficult to cope with complex market demand change. Accordingly, supply uncertainty has gradually attracted the attention of enterprises and has also been widely concerned and studied by scholars [3]. Emergency replenishment and flexible supply have taken on a critical significance in dealing with supply uncertainty [4]. In fact, some enterprises have adopted more flexible contracts and more than one procurement opportunity. For instance, international enterprises such as IBM and Oracle and domestic enterprises such as HP have adopted the combination measure of option contract and two procurement opportunities in practice [5–7].

At the same time, the procurement price is not always fixed, which is usually ignored. Changes lead to fluctuation of price with purchasing relationship, social factors, policy changes, and the instability of the market environment. Japanese earthquake in 2011 led to supply disruption of many electronic products enterprises, causing the price of related products to increase significantly [8, 9]. The price of DRAM used by HP decreased by more than 90% in 2001 but increased by more than three times after the outbreak of SARS in 2002. Besides, the outbreak of COVID-19 in 2020 has led to the disruption of numerous global supply chains, resulting in the shortage of high-end components of Huawei and leading to a significant increase in price of related products. At the same time, the adverse impacts on the national economy have led to a decline in the purchasing power of consumers, so the prices of some medium- and low-end products have also been reduced to promote purchase. Therefore, it is necessary to consider the fluctuation of procurement price during the development of the two-stage model.

2. Literature Review

Our model is mainly related to two streams of established literature. One stream is about supply disruption management. The other stream is about response to demand uncertainty.

2.1. Supply Disruption Management. Supply disruption management has been widely concerned recently. Dong and Tomlin [10] pointed out that various measures should be made to prevent and respond to supply disruption including double sourcing, backup suppliers, and business disruption insurance. Snyder et al. [11] proposed multisource supply and backup supply to deal with the risk of raw material supply disruption. Wang et al. [12] considered that constructing a supply chain elastic cycle control framework, establishing standby coping strategies, and reserving standby personnel can deal with disruption risk. Li et al. [13] adopted strategic supplier and backup supplier to construct two option operation modes to cope with supply disruption. Yavari and Zake [14] built a resilient supply chain network to cope with disruption in the perishable food and power networks. Li et al. [15] developed dynamic response strategies to deal with the impacts of supply disruption on the supplier's inventory, including passive backup strategy without preventive measures and recovery backup strategy composed of proactive prevention and active strategy. Concerning the management of demand uncertainty, He and Li [16] showed how to manage the supply disruption and demand changes caused by the unexpected situation and established the recovery optimization model of the supply chain with uncertain disruption events.

The abovementioned literature investigates the methods of dealing with supply disruption from two aspects of prevention and response. However, there is less research on quantity decision under the risk of supply disruption, and most scholars ignore the price fluctuation under the change of market environment. Based on the above research, this study explores how each member of the supply chain decides the optimal quantity strategy under the background of supply disruption risk and uncertain market demand and investigates the impacts of emergency procurement price fluctuation caused by environmental change on the optimal decisions.

2.2. Response to Demand Uncertainty

2.2.1. Option Contract. There is a small but significant body of literature that concentrates on option contract under uncertain environment. Option contract is more flexible than other contracts, which is capable of increasing the flexibility of the supply chain effectively [17]. Accordingly, more and more scholars have studied the application of option contract in responding to changes in the external environment, especially in the semiconductor manufacturing and electronics industries [18]. Barnes-Schuster et al. [19] showed that option contract can make the retailer flexible to deal with the uncertainty of market demand and realize the coordination of the supply chain. Hu et al. [20] constructed an emergency material reservation supply model with option contract. As revealed by comparison, the option contract mechanism can achieve a win-win situation in two-level supply chain and the coordination of emergency material supply chain. Zhao et al. [21] explored the value of option contract in two-level supply with stochastic market demand and information update. Basu et al. [22] studied how to hedge demand uncertainty in a supply chain consisting of a risk-neutral supplier and a risk-averse retailer with a buyback option contract.

The above scholars have extended the value of option contract in the supply chain management. Nevertheless, most of them only consider demand uncertainty, not supply disruption at the same time. On the basis of the above research, we comprehensively consider the two, introduce the option contract in this context, and explore the value of it through comparison.

2.2.2. Two Procurement Opportunities. Our study is also related to the strategy of two procurement opportunities under uncertain environment. Many scholars assume that the retailer has one procurement opportunity: if the market demand cannot be satisfied, the retailer must bear the shortage loss caused by insufficient order. To solve the above problem, some scholars proposed that inventory can be replenished in sales season. Ma et al. [23] considered a risk-averse retailer with two procurement opportunities, which can order before and after the start of the sales season. Xue et al. [24] investigated the situation that reliable supplier can carry out emergency production to satisfy the demand in case of disruption and how core enterprises adopt the option contract and order commitment contract extensively employed in practice to deal with supply disruption and demand risk. On that basis, Xue et al. [25] considered the selection problem of enterprise between different ordering strategies with random emergency procurement price under disruption and demand risk.

Ma et al. [23] proposed the concept that enterprises have two procurement opportunities earlier but ignored the risk of supply disruption. The strategy of two procurement opportunities to deal with uncertainty was studied by Xue

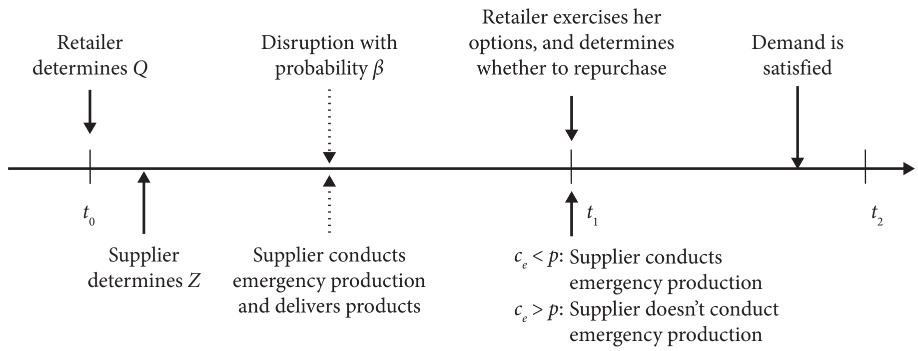

Figure 1: Sequence of events.

et al. [24, 25], but option contract was not considered. Under the background of disruption risk and uncertain market demand, we assume that the retailer has two procurement opportunities and the supplier has emergency production chance. Taking the price fluctuation of the emergency procurement caused by environmental changes as the node, we discuss the coordination between option orders and emergency procurement.

2.3. Innovation. As stated above, the supply disruption management and the procurement strategy have attracted extensive attention under demand uncertainty in academia and have been applied in different industries. The role of option contract has been fully understood with reliable supply, but it is rarely applied under unreliable supply and random demand. Although the strategy of two procurement opportunities is considered recently to deal with uncertain demand, most scholars ignore the impacts of emergency procurement price fluctuation on decision making. We employ the coordination of option ordering and emergency procurement to deal with the adverse effects caused by environmental change. Considering the risk of supply disruption, uncertainty of market demand, and the fluctuation of emergency procurement price comprehensively, we investigate how the retailer with two procurement opportunities and the supplier with emergency production chance make the optimal quantity decisions based on option contract. Furthermore, we analyze the value of option contract by comparison, expanding to demand uncertainty and supply disruption management.

3. Model Description and Symbol Description

3.1. Model Description. The Stackelberg model is constructed in a two-level supply chain system with a supplier (he) as a follower and a retailer (she) as a leader. The retailer purchases products from the supplier and sells them to the end market to satisfy the need of customers. Figure 1 presents the complete decision process of supply chain members.

Before sales season t_0, the retailer and the supplier sign option contract (o, w). After that, the retailer decides the number of option ordering Q, and the supplier decides the quantity of production Z (the minimum promised production quantity meets $Z_{\min} \geq Q$). After sales season of t_1, the retailer exercises her options not to exceed Q at unit price of w and decides whether to repurchase according to the update of demand information. When option ordering quantity and market demand meet $Q < \zeta$, the retailer repurchases products from the supplier, which is called emergency procurement, and the price in this stage is also adjusted with environmental change. When the quantity of demand exceeds supply, the procurement price rises; otherwise, the price falls. At the same time, the production cost of products is not constant in different periods. When $Z < \zeta < Q$, the supplier's first production quantity is enough to satisfy all of the retailer's orders. When $\zeta > Z$, the supplier decides whether to carry out emergency production. If the emergency order leads to positive returns for the supplier, he uses surplus warehouse $[Z - Q]^+$ produced at first stage and then puts into emergency production $[\zeta - Z]^+$ to satisfy all of the retailer's orders. Otherwise, the maximum emergency order that he can deliver to the retailer is $[Z - Q]_+$, which is his surplus warehouse.

Throughout the decision-making process, the supplier is subject to a random disruption (exogenous) whose probability of occurrence is β. The disruption is a type disruption of "all-or-nothing," which is widely used in emergency management [24, 26, 27]. If supply disruption occurs, $Z = 0$, the supplier has to implement emergency production under the option commitment so that the retailer can exercise her rights normally. The emergency production refers to the overtime work, the use of spare production lines, and high-end or similar products to replace each other, which leads to a higher cost [25, 28]. For example, in 2000, a fire broke out in a semiconductor factory owned by Philips Electronics NV, which disrupted the supply chain of ASCI chips to Nokia and Ericsson. Philips replaced the original chips with similar ones and enabled the backup production line [6]. The remaining products are treated after the end of the sales season according to the salvage value.

3.2. Parameters in the Model. The notations and definitions are shown in Table 1.

(1) $v < c < c_e$: prevent the supplier from over production, profiting from salvage value.

Table 1: Notions and definitions.

	Illustration
c	Supplier's production cost of per unit
w	Retailer's first ordering price per unit without option contract
o	Option price per unit
e	Option exercise price per unit
r	Retail price per unit
c_e	Supplier's emergency production cost per unit
p	Retailer's emergency procurement price per unit
v	Unit salvage value after sales season
β	Disruption probability
Z	Supplier's production quantity (decision variable)
Q	Retailer's option ordering quantity (decision variable)
ζ	The market demand is stochastic, which is a continuous stochastic variable, $\zeta \sim [0, \mathrm{A}]$, whose probability density function and cumulative distribution function are $f(\cdot)$ and $F(\cdot)$, with mean value $\mu = a/2$
π_i	$i = s, r$, denotes the profit function of the supplier and the retailer
$*$	Denotes optimum

(2) $c < w < o + e < p < r$: the price of terminal sales is high to ensure that both the supplier and the retailer can gain profits from the transaction.

(3) In order to simplify the model, the shortage cost is not considered temporarily, and the price fluctuation is only considered when the supplier purchases products from the manufacturer after the sales season starts.

(4) The supplier and the retailer are independent risk-neutral individuals.

(5) The information among supply chain members is completely symmetrical.

(6) The option contract involved is American call option.

4. Model Analysis

Based on the relative size between the emergency production cost c_e and procurement price p, i.e., $c_e < p$ and $c_e > p$, this section discusses the optimal production strategy of the supplier and the optimal ordering strategy of the retailer in two cases, respectively.

4.1. Scenario I ($c_e < p$). Under the condition of no disruption occurring, when $c_e < p$, the supplier profits from the emergency order regardless of using surplus stock or carrying out emergency production. Thus, he satisfies all of the retailer's orders on the premise of unlimited production capacity.

4.1.1. Supplier's Production Decision. When $Q < \zeta$, the retailer repurchases from the supplier. Under the situation of $Q < \zeta$, when $\zeta \leq Z$, the surplus inventory of the supplier at the first production is enough to meet the retailer's emergency order; when $\zeta > Z$, he makes an emergency production to achieve the part that exceeds the initial production quantity. If supply disruption occurs, the supplier has to carry out emergency production to promise the normal exercise of options due to option commitment, i.e., min $\{Q, \zeta\}$. Accordingly, the supplier's profit function can be expressed as

$$\begin{aligned}\pi_s(Z|c_e < p) = {}& -cZ + oQ + eE[\min\ \{Q, \zeta\}] + pE[\{\zeta - Q\}^+] \\ & - \beta c_e E[\zeta] - (1-\beta)c_e E[\{\zeta - Z\}^+] \\ & + (1-\beta)vE[\{Z - \zeta\}^+],\end{aligned} \tag{1}$$

where the first three items denote the supplier's initial production cost and the revenues from option ordering and exercise and the last four items denote the income from satisfying the retailer's emergency order, the investments of emergency production when disruption occurs and does not occur, and the salvage value. Combined with the random demand distribution of products, (1) can be expressed as

$$\begin{aligned}\pi_s(Z|c_e < p) - cZ + oQ + e\left(\int_0^Q xf(x)\mathrm{d}x + \int_Q^A Qf(x)\mathrm{d}x\right) + p\int_Q^A (x - Q)f(x)\mathrm{d}x \\ -\beta c_e \int_0^A xf(x)\mathrm{d}x - (1-\beta)c_e \int_Z^A (x - Z)f(x)\mathrm{d}x + (1-\beta)v\int_0^Z (Z - x)f(x)\mathrm{d}x.\end{aligned} \tag{2}$$

4.1.2. Retailer's Purchase Decision. All of the orders will be met, and the profit function of the retailer can be expressed as

$$\pi_r(Q|c_e < p) = -oQ - eE[\min\{Q, \zeta\}] - pE[\{\zeta - Q\}^+] + rE[\zeta], \tag{3}$$

where the first two items denote the retailer's cost of option purchase and exercise and the last two items are investments in emergency procurement and sales revenue, respectively. Combined with the random demand distribution of products, (3) can be expressed as

$$\begin{aligned}\pi_r(Q|c_e < p) = -oQ - e\left(\int_0^Q xf(x)\mathrm{d}x + \int_Q^A Qf(x)\mathrm{d}x\right) \\ - p\int_Q^A (x - Q)f(x)\mathrm{d}x + r\int_0^A xf(x)\mathrm{d}x.\end{aligned} \tag{4}$$

Proposition 1. *When $c_e < p$, with a threshold $\hat{\beta}_1 = 1 - c(p - e)/o(c_e - v) + v(p - e)$, the optimal production quantity of the supplier Z^* and the optimal ordering quantity of the retailer Q^* meet the following conditions:*

(1) If $0 \le \beta < \hat{\beta}_1, Z^* > Q^*, F(Z^*) = (1 - \beta)c_e - c/(1 - \beta)(c_e - v), F(Q^*) = 1 - o/p - e$.

(2) If $\hat{\beta}_1 \le \beta \le 1 - c/c_e, Z^* = Q^*, F(Z^*) = F(Q^*) = 1 - o/p - e$.

Appendix presents the Proof of Proposition 1 and all proofs of the following propositions and corollaries. By Proposition 1 and Figure 2, when $c_e < p$, the retailer's optimal ordering quantity is independent of the disruption probability, and the supplier's optimal production quantity decreases with the increase of the disruption probability. When the disruption probability reaches the threshold $\hat{\beta}_1$, both optimal quantities are consistent.

No matter whether the supply chain is disrupted or not, all of the retailer's orders are met. Thus, her optimal ordering quantity has nothing to do with supply disruption probability. When the disruption probability is relatively low, the supplier produces more products as backup inventory and satisfies the retailer's emergency order at a low cost if the option order is not enough to meet uncertain demand. The higher the probability of disruption is, the greater risk of loss the supplier needs to bear. When supply disruption probability exceeds $\hat{\beta}_1$, the optimal production quantity reaches the lowest, which is consistent with the optimal ordering quantity. If the market demand cannot be met, he carries out emergency production to continue supplying.

Corollary 1. *When $c_e < p$, there is*

(1) If $0 \le \beta < \hat{\beta}_1, \partial Z^*/\partial\beta < 0, \partial Z^*/\partial c < 0, \partial Z^*/\partial c_e > 0, \partial Z^*/\partial v > 0, \partial Q^*/\partial o < 0, \partial Q^*/\partial e < 0, \partial Q^*/\partial p > 0$.

(2) If $\hat{\beta}_1 \le \beta \le 1 - c/c_e, \partial Z^*/\partial o = \partial Q^*/\partial o < 0, \partial Z^*/\partial e = \partial Q^*/\partial e < 0, \partial Z^*/\partial p = \partial Q^*/\partial p > 0$.

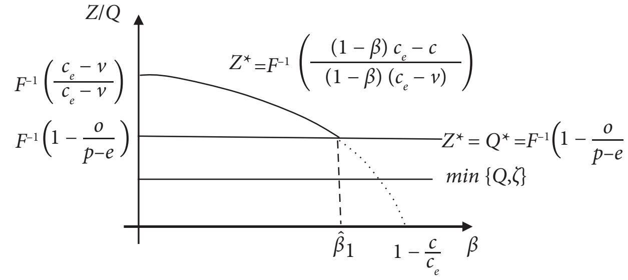

FIGURE 2: Correlation between optimal decisions and disruption probability ($c_e < p$).

Based on Corollary 1 and under the condition of $c_e < p$, the higher unit price of option ordering and exercise means that the larger the option ordering is, the greater risk of loss the retailer has to bear. Accordingly, she reduces option ordering quantity. With the increase of emergency procurement price, the retailer's profits from the order become less. Therefore, she increased option ordering quantity to minimize her emergency investment. If the supply disruption probability is relatively low ($0 \le \beta < \hat{\beta}_1$), the supplier reduces his initial production quantity with increase of the first production cost in order to decrease his risks under uncertain demand. But when the cost of the emergency production is higher, he produces more before sales season. Supposing that the demand cannot be met by the initial order, the supplier tries to use surplus products in the initial production to supply, avoiding the huge cost caused by large emergency production. If the supply disruption probability is relatively high ($\hat{\beta}_1 \le \beta \le 1 - c/c_e$), the supplier's optimal production quantity is equal to the retailer's optimal ordering quantity.

4.2. Scenario II ($c_e > p$). When $c_e > p$, the supplier gets negative income in case that he makes emergency production. Therefore, he only uses the surplus inventory in the initial production to satisfy the retailer's emergency order.

4.2.1. Supplier's Production Decision. When $\zeta > Q$, the supplier does not produce urgently under no disruption occurring. If the disruption occurs, he has to produce urgently $\min\{Q, \zeta\}$ with option commitment. Accordingly, supplier's profit function can be expressed as

$$\begin{aligned}\pi_s(Z|c_e > p) = -cZ + oQ + eE[\min\{Q, \zeta\}] \\ - \beta c_e E[\min\{Q, \zeta\}] \\ + (1 - \beta)\{pE[\{\min\{Z, \zeta\} - Q\}^+] + vE[\{Z - \zeta^+\}]\},\end{aligned} \tag{5}$$

where the first four items denote the supplier's initial production cost, the income of option ordering and exercise, and the investments in emergency production when the disruption occurs and the last two items are the revenues for satisfying the retailer's emergency order and the salvage value. Combined with the random demand distribution of products, (5) can be expressed as

$$\pi_s(Z|c_e > p) = -cZ - \beta c_e\left(\int_0^Q xf(x)\mathrm{d}x + \int_Q^U Qf(x)\mathrm{d}x\right) + oQ + e\left(\int_0^Q xf(x)\mathrm{d}x + \int_Q^U Qf(x)\mathrm{d}x\right)$$
$$+(1-\beta)\left\{p\left[\int_Q^Z (x-Q)f(x)\mathrm{d}x + \int_Z^U (Z-Q)f(x)\mathrm{d}x\right] + v\int_0^Z (Z-x)f(x)\mathrm{d}x\right\}. \quad (6)$$

4.2.2. Retailer's Purchase Decision. Option order of the retailer is supplied, while the emergency order achieves the maximum supply of $[Z-Q]^+$. Therefore, the profit function of the retailer can be expressed as

$$\pi_r(Q_2|c_e > p) = -oQ - eE[\min\{Q,\zeta\}] + rE[\min\{Q,\zeta\}]$$
$$+(1-\beta)(r-p)E[\{\min\{Z,\zeta\} - Q^+], \quad (7)$$

where the first two items denote the retailer's investments in option ordering and exercise and the last two items are the sales revenues and the emergency procurement cost. Combined with the random demand distribution of products, (7) can be expressed as

$$\pi_r(Q|c_e > p) = -oQ + (r-e)\left(\int_0^Q xf(x)\mathrm{d}x + \int_Q^A Q(x)dx\right)$$
$$+(1-\beta)(r-p)\left[\int_Q^Z (x-Q)f(x)\mathrm{d}x + \int_Z^A (Z-Q)f(x)\mathrm{d}x\right]. \quad (8)$$

Proposition 2. *When $c_e > p$, with a threshold $\widehat{\beta}_2$, which is the solution of β in the equation $(1-\beta)p - c/(1-\beta)(p-v) = 1 - o/r - e - (1-\beta)(r-p)$. $0 < \beta < 1 - c/p$, $\widehat{\beta}_2 < 1 - c/p$. The optimal production quantity of the supplier Z^* and the optimal ordering quantity of the retailer Q^* meet the following conditions:*

(1) *If $0 \le \beta < \widehat{\beta}_2$, $Z^* > Q^*$, $F(Z^*) = (1-\beta)p - c/(1-\beta)(p-v)$, $F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$.*

(2) *If $\widehat{\beta}_2 \le \beta < 1 - \frac{c}{p}$, $Z^* = Q^*$, $F(Z^*) = F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$.*

As revealed by Proposition 2 and Figure 3, when $c_e > p$, the retailer's optimal ordering quantity is positively related to the disruption probability. The supplier's optimal production quantity is negatively related to the disruption probability before $\widehat{\beta}_2$ and positively related to it after $\widehat{\beta}_2$, which is consistent with that of the retailer.

Since $c_e > p$, negative income is generated when the supplier takes the initiative to make an emergency production. Considering that supply disruption leads to a large loss, the number of products put into production for the initial time by the supplier decreases with the increase of disruption probability, until it just meets the retailer's initial order. Accordingly, the maximum number of products meeting the emergency order becomes less. In order to promote the supplier to produce more products for responding to uncertain demand, the retailer increases his optimal ordering quantity instead.

Corollary 2. *Under $c_e > p$, the conclusions are as follows:*

(1) *If $0 \le \beta < \widehat{\beta}_2$, $\partial F(Z^*)/\partial\beta < 0$, $\partial F(Z^*)/\partial p > 0$, $\partial F(Z^*)/\partial v > 0$, $\partial F(Q^*)/\partial\beta > 0$, $\partial F(Q^*)/\partial o < 0$, $\partial F(Q^*)/\partial e < 0$, $\partial F(Q^*)/\partial p > 0$, $\partial F(Q^*)/\partial r > 0$.*

(2) *If $\widehat{\beta}_2 \le \beta \le 1 - c/p$, $\partial F(Z^*)/\partial\beta = \partial F(Q^*)/\partial\beta > 0$, $\partial F(Z^*)/\partial o = \partial F(Q^*)/\partial o < 0$, $\partial F(Z^*)/\partial e = \partial F(Q^*)/\partial e < 0$, $\partial F(Z^*)/\partial p = \partial F(Q^*)/\partial p > 0$, $\partial F(Z^*)/\partial r = \partial F(Q^*)/\partial r > 0$.*

Based on Corollary 2 and under the condition of $c_e > p$, the impacts of the cost of emergency procurement, option ordering, and exercise are the same as those of $c_e < p$. The difference is that the optimal ordering quantity is related to the retail price and the disruption probability. If the probability of supply disruption is relatively low $(0 \le \beta < \widehat{\beta}_2)$, the retailer purchases more options with increase of the emergency procurement cost in order to reduce large expenses incurred by emergency order. At the same time, the supplier increases production quantity as inventory in order to satisfy the emergency order at a low investment. If the probability of supply disruption is relatively high $(\widehat{\beta}_2 \le \beta \le 1 - c/c_e)$, the supplier's optimal production quantity is minimized, which is consistent with the optimal ordering quantity.

4.3. Special Case of No Option Contract. When the option ordering price or exercise price is large enough, the retailer does not accept the option contract anymore. The initial procurement price of unit w_0 is determined by consultation. Whether the supplier produces urgently to satisfy the retailer's emergency order depends on the price relationship. The production is no longer limited by the promise of option, so the risks are jointly borne by the two.

When the cost of emergency production c_e meets $c_e < p$, the profit functions of the supplier and the retailer can be expressed as

$$
\begin{aligned}
\pi_s(Z|c_e<p) &= -cZ + wQ + (1-\beta)pE[\{\zeta - Q\}^+] - (1-\beta)c_eE[\{\zeta - Z\}^+] + (1-\beta)vE[\{Z-\zeta\}^+] + \beta pE[\zeta] - \beta c_eE[\zeta] \\
&= -cZ + wQ + (1-\beta)\left[p\int_Q^A (x-Q)f(x)dx - c_e\int_Z^A (Z-x)f(x)dx + v\int_0^Z (Z-x)f(x)dx\right] + \beta(p-c_e)\int_0^A xf(x)dx \\
&= -cZ + wQ + (1-\beta)\left[p\int_Q^A (x-Q)f(x)dx - c_e\int_Z^A (Z-x)f(x)dx + v\int_0^Z (Z-x)f(x)dx\right] + \beta(p-c_e)\int_0^A xf(x)dx, \\
\pi_r(Q|c_e<p) &= -wQ - (1-\beta)pE[\{\zeta - Q\}^+] - \beta pE[\zeta] + rE[\zeta] = -wQ - (1-\beta)p\int_Q^A (x-Q)f(x)dx + (r-\beta p)\int_0^A xf(x)dx.
\end{aligned}
\tag{9}
$$

When the cost of emergency production c_e meets $c_e > p$, the profit functions of the supplier and the retailer can be expressed as

$$
\begin{aligned}
\pi_s(Z|c_e<p) &= -cZ + wQ + (1-\beta)[pE[\{\min\{Z,\zeta\} - Q]^+) + vE[\{Z-\zeta^+\}]] \\
&= -cZ + wQ + (1-\beta)\left[p\left(\int_Q^Z (x-Q)f(x)dx + \int_Z^A (Z-Q)f(x)dx\right) + v\int_0^Z (Z-x)f(x)dx\right] \\
\pi_r(Q|c_e>p) &= -wQ + (1-\beta)rE[\min\{Q,\zeta\}] + (1-\beta)(r-p)E[\{\min\{Z,\zeta\} - Q^+] \\
&= -wQ + (1-\beta)r\left(\int_0^Q xf(x)dx + \int_Q^A Qf(x)dx\right) + (1-\beta)(r-p)\left[\int_Q^Z (x-Q)f(x)dx + \int_Z^A (Z-Q)f(x)dx\right].
\end{aligned}
\tag{10}
$$

Proposition 3. *Under the condition of no option contract, with a threshold $\widehat{\beta}_3 = 1 - cp - w(c_e - v)/vp$, the optimal production quantity of the supplier Z^* and the optimal initial ordering quantity Q^* of the retailer meet the following conditions:*

$$
\begin{aligned}
&0 \le \beta < \min\left\{1 - \frac{c}{c_e}, 1 - \frac{w}{p}\right\}, \\
&F(Z^*) = \frac{(1-\beta)c_e - c}{(1-\beta)(c_e - v)}, \\
&F(Q^*) = 1 - \frac{w}{(1-\beta)p}.
\end{aligned}
\tag{11}
$$

As revealed by Proposition 3 and Figures 4 and 5, the retailer's optimal initial ordering quantity and the supplier's optimal production quantity are both negatively correlated with the supply disruption probability. Under the condition of $c_e < p$, the supplier's first production quantity is larger when $0 < \beta < \widehat{\beta}_3$ and smaller when $\widehat{\beta}_3 < \beta < 1 - w/p$. Under the condition of $c_e > p$, the supplier's first production quantity is always larger than the initial ordering quantity.

If option contract is invalid, the risk of supply disruption is shared by the retailer and the supplier who bears more. Therefore, the optimal ordering and production decisions of both decrease with supply disruption probability, and the latter decreases faster. When $c_e < p$, the supplier is willing to put into emergency production actively. If the emergency production exceeds the certain threshold $\widehat{\beta}_3$, the supplier would rather bear more cost of emergency ordering. When $c_e > p$, the emergency production will not be considered under no disruption, so the supplier's first production quantity is always larger than the retailer's ordering quantity in order to have more surplus inventory to deal with demand uncertainty. At the same time, the rational supplier does not produce too many products, which will inevitably lead to risks and losses.

Corollary 3. *Under the condition of no option contract, the conclusion is as follows:*

$$
0 \le \beta < \min\left(1 - \frac{c}{c_e}, 1 - \frac{w}{p}\right), \frac{\partial Z^*}{\partial \beta} < 0, \frac{\partial Z^*}{\partial c} < 0, \frac{\partial Z^*}{\partial c_e} > 0, \frac{\partial Z^*}{\partial v} > 0, \frac{\partial Q^*}{\partial \beta} < 0, \frac{\partial Q^*}{\partial w} < 0, \frac{\partial Q^*}{\partial p} > 0. \tag{12}
$$

Based on Corollary 3, with the increase of the first procurement price, the retailer decreases the quantity of initial ordering to reduce the waste caused by the excessive ordering. As the emergency procurement price increases, the

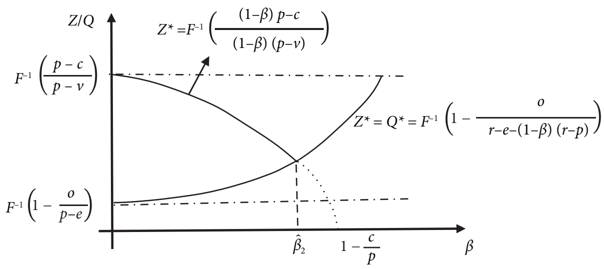

FIGURE 3: Correlation between optimal decision and disruption probability $(c_e < p)$.

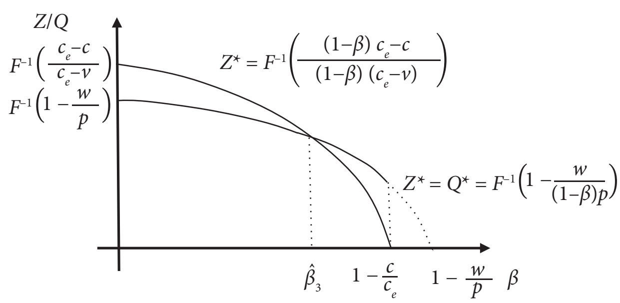

FIGURE 4: The relationship between optimal decision and disruption probability under no option contract $(c_e < p)$.

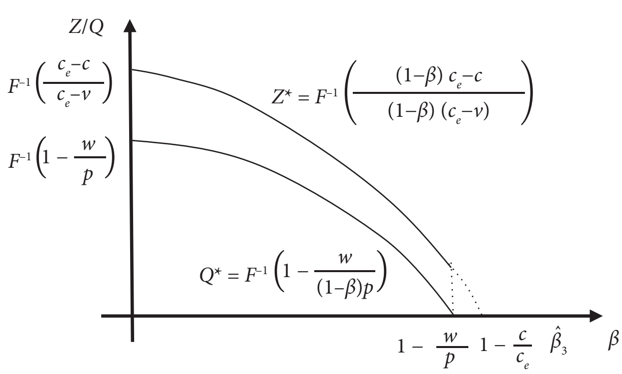

FIGURE 5: The relationship between optimal decision and disruption probability under no option contract $(c_e > p)$.

retailer's profits decrease. Accordingly, she increases initial ordering quantity to decrease large expense caused by emergency procurement.

4.4. Value of Option Contract. Compared the supplier's and the retailer's optimal quantity decisions with and without option contract, the value of option contract is explored under uncertain demand and supply disruption risk.

Proposition 4. *Compared with low probability $(\beta_4 \le \beta < 1 - c/c_e)$, the value of option contract is more obvious when the disruption probability is relatively higher $(\beta_4 \le \beta < 1 - c/c_e)$. Define $\widehat{\beta}_4$ satisfying $\widehat{\beta}_4 = 1 - w(p-e)/op$.*

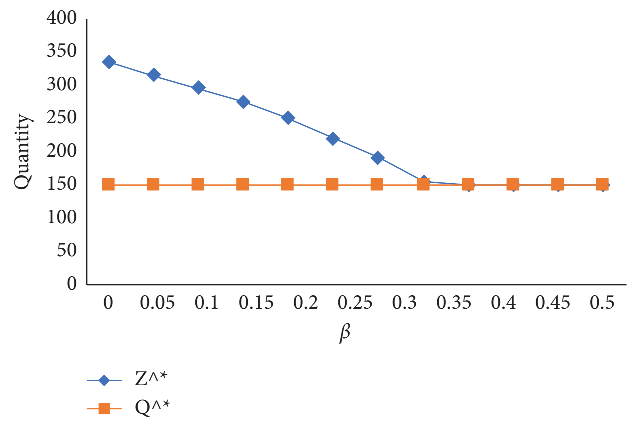

FIGURE 6: Q^* and Z^* with different $\beta (c_e < p)$.

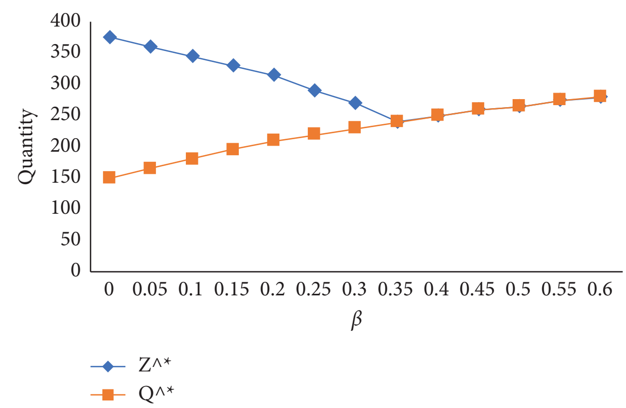

FIGURE 7: Q^* and Z^* with different $\beta (c_e > p)$.

Under the risk of supply disruption and demand uncertainty, the optimal production quantity of the supplier with option contract is always not larger than that without option contract, which indicates that option contract can reduce investments and inventory risks of the supplier. When $0 \le \beta < \beta_4$, the supplier takes an active role in producing more products as inventory in order to obtain higher income by satisfying more demand. Therefore, the retailer who has an emergency procurement opportunity reduces her initial ordering quantity under option contract. When $\beta_4 \le \beta < 1 - c/c_e$, the supplier bears more risks so that he reduces investments in production subjectively. The retailer uses the characteristics of option contract to increase the initial ordering quantity to stimulate production. However, the number of option ordering should not exceed the predicted demand for a rational retailer.

Proposition 5. *The value of option contract is more significant under $c_e < p$ than that under $c_e > p$.*

When $c_e < p$, the value of option contract is mainly reflected in securing the first phase of the supply process in case of supply disruption occurring because the supplier takes the initiative to fulfill the retailer's emergency order. When $c_e > p$, the supplier has to satisfy the retailer's option order. The retailer increases his inventory by purchasing more options to stimulate the supplier to produce more products, and the option contract is worth more.

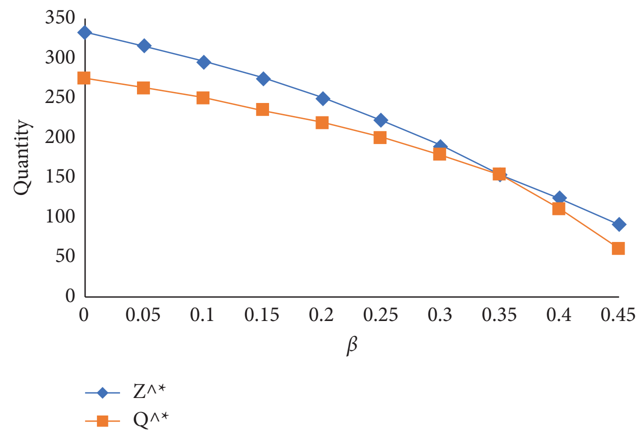

FIGURE 8: Q^* and Z^* with different $\beta (c_e < p)$.

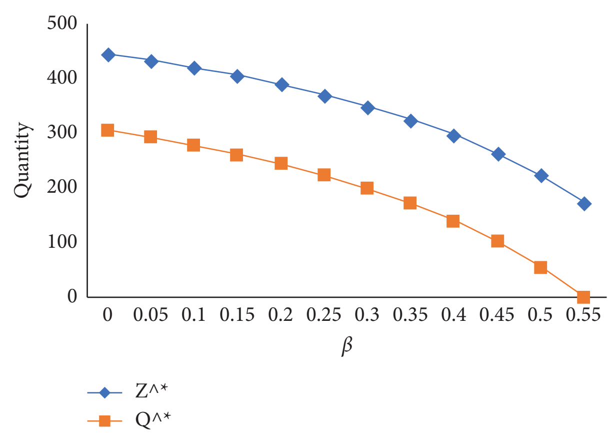

FIGURE 9: Q^* and Z^* with different $\beta (c_e < p)$.

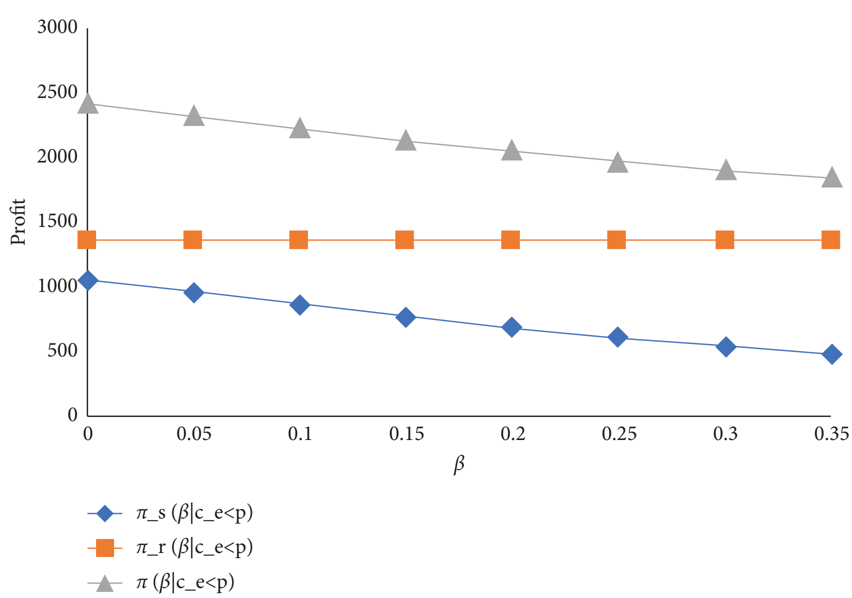

FIGURE 10: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $\beta (c_e < p)$.

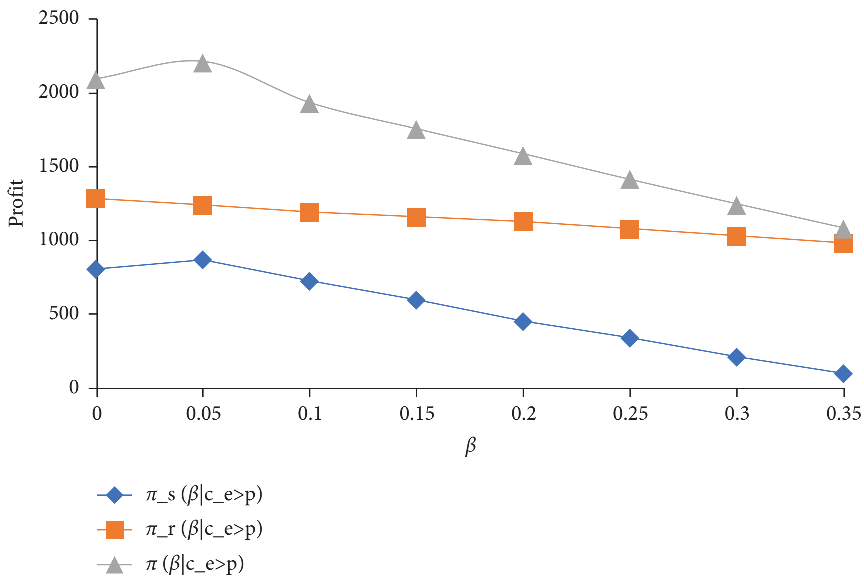

Figure 11: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $\beta\,(c_e > p)$.

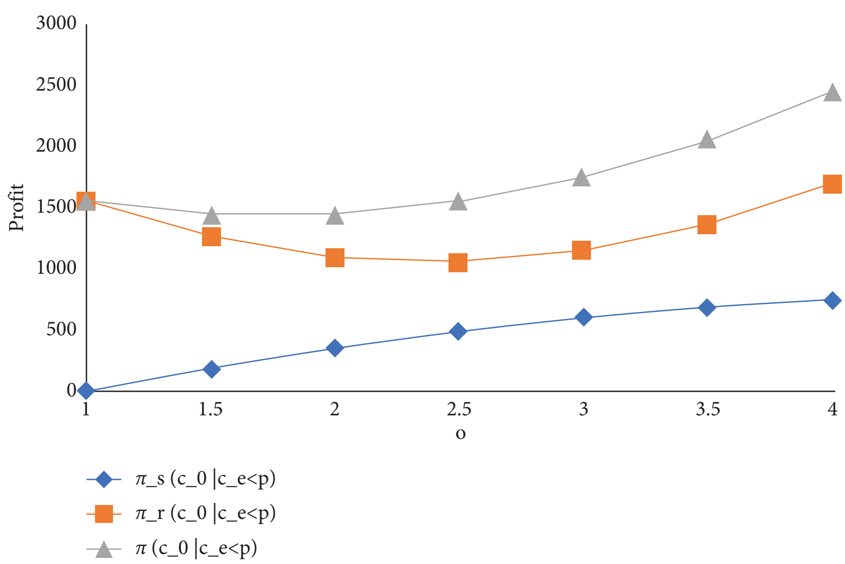

Figure 12: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $o\,(c_e < p)$.

5. Numerical Examples

We apply an example to study the impacts of disruption probability β, option price c_0, and exercise price w on the quantity strategies and profit changes of each party. The demand for the product follows $\zeta \sim U(0.500)$. Other parameters are as follows: $c = 4$, $c_0 = 3.5$, $w = 5$, $p = 10$, $r = 15$, $c_e = 8$ or $c_e = 12$, and $v = 2$.

5.1. Impacts of Disruption Probability on Quantity. Option contract can affect the quantity decision making of the supplier and the retailer, effectively dealing with supply disruption and demand uncertainty. In the following, the value of option contract in this model is further verified by analyzing the trends of optimal quantity decisions with disruption probability β in different situations, respectively.

5.1.1. Under Option Contract. Under option contract, the retailer's order is promised. We get $\widehat{\beta}_1 = 0.355$, $\widehat{\beta}_2 = 0.35$, and $1 - c/c_e = 0.5$ based on assumptions above. When $c_e = 8$, it yields that $0 < \beta \leq 0.35$; when $c_e = 12$, it yields that $\beta \leq 0.35$. The influence of β on the maximum quantity of each member and the whole supply chain is verified by calculation. There is $Z^* \geq Q^*$ in the β range. The results of numerical analysis are shown in Figures 6 and 7.

5.1.2. Under No Option Contract. When $c_e = 8$, it yields that $\beta_3 = 0.35$ and $0 \leq \beta \leq 0.45$; when $c_e = 12$, it yields that $0 \leq \beta \leq 0.55$. The influence of β on the maximum quantity of the retailer, the supplier, and the whole supply chain is verified by calculation. $Z^* > Q^*$ in $0 \leq \beta \leq 0.35$ and $Z^* < Q^*$ in $0.35 \leq \beta \leq 0.45$. The results of numerical analysis are shown in Figures 8 and 9.

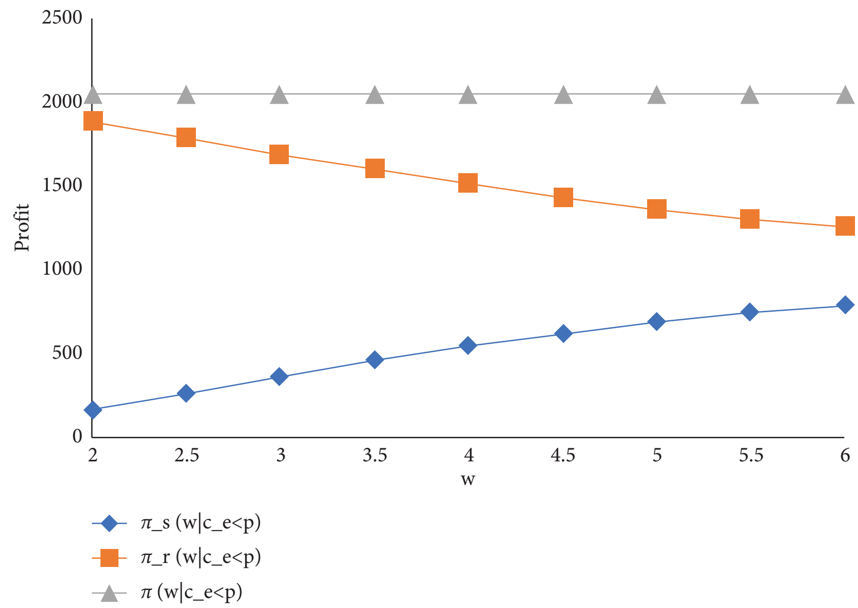

FIGURE 13: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $w\,(c_e < p)$.

Based on Figures 6–9, we once again verify the correctness and effectiveness of the above analysis results and further get the conclusion that the option contract transfers the risks of inventory and supply disruption to the upstream supplier. At the same time, it ensures the retailer's supply stability and increases his product reserve.

5.2. Impacts of Disruption Probability on Profits

5.2.1. Scenario I $(c_e < p)$. When $c_e = 8$, $\beta \leq 0.35$, the influence of β on the maximum profits of the retailer, the supplier, and the whole supply chain is explored by calculation. There is $Z^* > Q^*$ in the β range. The results of numerical analysis are shown in Figure 10. The retailer's profits are almost unchanged with the increase of supply disruption probability, and the supplier's and the whole supply chain's profits decrease instead. When the probability of supply disruption is zero, the profits of the supplier and the supply chain reach the maximum.

Since the option commitment indicates that the risk of disruption is solely borne by the supplier, it will not affect the retailer's profits. With the probability of disruption increase, the supplier has to make an emergency production to achieve his option commitment in the event of disruption, which will incur a large loss. Therefore, he reduces production and his profits also decrease.

5.2.2. Scenario II $(c_e > p)$. When $c_e = 12$, $\beta \leq 0.35$, the relationship between the optimal quantities is $Z^* > Q^*$. The impacts of disruption probability β on the maximum profits of the retailer, supplier, and whole supply chain are explored by calculation. The results of numerical analysis are shown in Figure 11. The profits of the supplier and the whole supply chain are greatly affected by disruption probability. They increase slightly at first and then decrease with disruption probability increase, reaching the maximum at $\beta = 0.05$. The retailer's profits are negatively correlated with the disruption probability.

When $\beta = 0.05$, the retailer partly decreases her production in order to reduce the loss risk caused by supply disruption. At the same time, her risk of inventory also reduces. In fact, the loss risk of disruption is very small because of the low disruption probability. When $\beta = 0$, the retailer bears more inventory risk instead. Accordingly, the profits of the retailer are higher when $\beta = 0.05$. Besides, the retailer bears more loss risk and his profits also decrease with the increase of disruption probability when β is larger than the threshold (0.05).

5.3. Impacts of Option Price and Exercise Price on Profits

5.3.1. Scenario I $(c_e < p)$. When $c_e < p$, assuming that the disruption probability $\beta = 0.2$, $Q^* = 150$, and $Z^* = 250$ with other parameters unchanged, different option ordering price o and exercise price w are set, respectively. Because $o + w < p$, when $w = 5$, 1 is the minimum value of o and 4.5 is the maximum value of o. When $o = 3.5$, 2 is the minimum value of w and 6 is the maximum value of w. The following will continue to study the influence of o and w on the maximum profits of the supplier, the retailer, and the overall supply chain. The results of numerical analysis are shown in Figures 12 and 13. With the increase of option ordering price, the supplier's profits increase, while the retailer's profits decrease first and then increase. With the increase of option exercise price, the supplier's profits increase, but the retailer's profits decrease. The profits of the whole supply chain unchanged. The influence of option ordering price is greater than that of exercise price for the whole supply chain.

Under the condition of $c_e < p$, the closer the option ordering price is to the critical maximum, the closer the profits of the supplier, the retailer, and the whole supply chain are to the maximum. The higher the option exercise

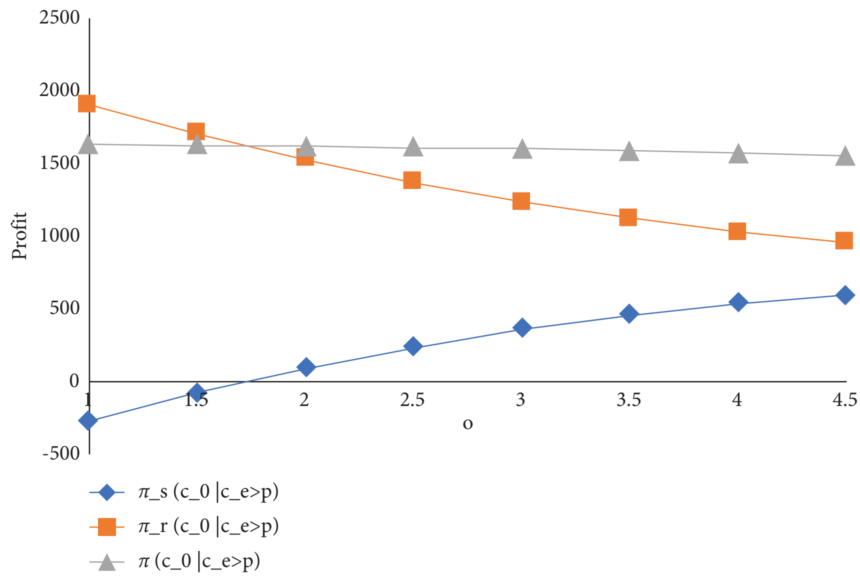

FIGURE 14: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $o\,(c_e > p)$.

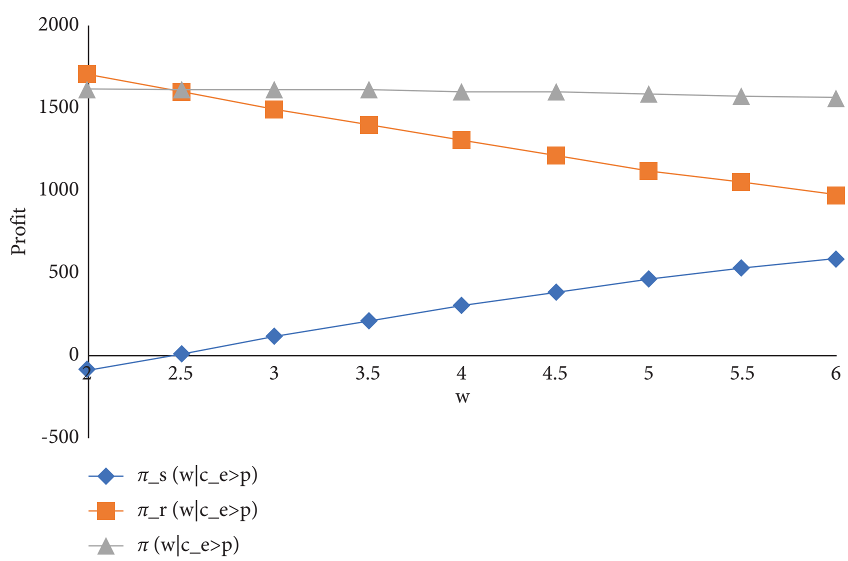

FIGURE 15: $\pi_s{}^*$, $\pi_r{}^*$, and π^* with different $w\,(c_e > p)$.

price is, the better it is for the retailer and the worse it is for the supplier. The impacts on both offset each other, resulting in no impacts on the profits of the whole supply chain. When the option ordering price exceeds a certain threshold, the higher option price motivates the supplier to produce more products and makes the retailer reduce his option ordering quantity, leading to the higher profits of the supplier and the retailer. The above analysis results show that the effects of option contract are more obvious when the option ordering price is higher under the condition of $c_e < p$.

5.3.2. Scenario II $(c_e > p)$. Assume $c_e = 12$ and $\beta = 0.2$, if other parameters unchanged, then we can get $Q^* = 210$, $Z^* = 315$. Set different option price o and option exercise price w, respectively. We study how the profits of the retailer, the supplier, and the whole supply chain change with the changes of option price o and exercise price w. The results of numerical analysis are shown in Figures 14 and 15. Option price has a great impact on the profits of the supplier and the retailer but almost has no impact on the whole supply chain. With increase of option ordering price and exercise price, the supplier's profits tend to rise, and the retailer's profits show a declining trend which is the same as the trend under the situation of $c_e < p$. The effects of option ordering and exercise price on both are almost offset, resulting in the fact that the overall profits of the supply chain are almost unaffected by the rise in option price.

When $c_e > p$, the impacts of option ordering price on the retailer are different from those when $c_e < p$. Even though higher option ordering price will bring higher initial input cost, the retailer only encourages the supplier to produce more products to meet uncertain demand by increasing her option ordering quantity under the condition of $c_e > p$. Higher option ordering price or exercise price is beneficial to one part but harms the interests of the other. When the exercise price and option price are moderate, it is conducive to the long-term cooperation and development.

6. Conclusions

In order to reduce the adverse impacts of supply disruption and demand uncertainty on enterprises, we investigate the optimal quantity strategies based on option contract in the two-stage supply chain, which consists of a supplier with emergency production chance and a retailer with two procurement opportunities, and explore the role of coordination between option ordering and emergency procurement in dealing with uncertain risks. In addition, the influence of disruption probability and option parameters on profits is further discussed through vertical simulation. Through the above analysis, the following conclusions can be drawn.

Firstly, if the retailer needs to pay a higher cost of emergency procurement ($c_e < p$), she just makes her decision of option ordering quantity based on the predictable demand without considering the disruption risk under option commitment and mainly deals with uncertainty demand through the emergency ordering which will be met. Otherwise ($c_e > p$), she increases option ordering which is worth more so that she can get more product inventory.

Secondly, when the emergency procurement cost is higher ($c_e < p$), it is profitable for the supplier to satisfy the retailer's emergency order by emergency production. Otherwise ($c_e > p$), the supplier just uses surplus inventory to fulfill the retailer's emergency order. At the same time, the lower the probability of disruption occurring, the more output the supplier will put into production for the first time. But for a rational supplier, his production should not exceed the predictable market demand. With the probability of supply disruption increase, the supplier reduces the production volume until it is equal to the option ordering of the supplier to ensure exercise of option contract.

Thirdly, when the price of emergency production is lower ($c_e < p$), setting a higher option ordering price and a lower exercise price is beneficial to the retailer because the adverse impacts caused by supply disruption are not obvious on her under option contract. On the contrary ($c_e > p$), it is profitable for the retailer to set a lower option ordering and exercise price. However, in both cases, the supplier's profits will be damaged. Therefore, a moderate option related price is conducive to the long-term cooperation of the supply chain.

Some possible extensions of this research are as follows. First, the fluctuation of the emergency procurement price is expressed by parameter in the article, and it can be set as a variable in further study. Second, this article only analyzes the supply chain composed of a supplier and a retailer, which can be extended to the competition between two suppliers. Third, this study is conducted on the premise of information symmetry, so we can continue to suppose information asymmetry on this basis. In addition, the two-way option contract can be studied in the future. Last, we can also study the optimal decisions of companies with multiple contract combinations.

Appendix

A. Proofs of Propositions 1–5

Proof of Proposition 1. The first derivative and second derivative of supplier's profit function $\pi_s(Z|c_e < p)$ with respect to Z are

$$\frac{\partial \pi_s(Z|c_e < p)}{\partial Z}$$
$$\frac{\partial^2 \pi_s(Z|c_e < p)}{\partial Z^2} = (1-\beta)(v-c_e) f(Z) < 0. \tag{A.1}$$

Since the second derivative of Z of $\pi_s(Z|c_e < p)$ is less than zero, it yields that $\pi_s(Z|c_e < p)$ is a concave function about Z. Thus, if the first derivative is 0, it yields that $F(Z^*) = (1-\beta)c_e - c/(1-\beta)(c_e - v)$. Since $F(Z^*) > 0$, it yields $\beta < 1 - c/c_e$, so the optimal production quantity of the supplier meets $F(Z^*) = \max\{F(Q), (1-\beta)c_e - c/(1-\beta)(c_e - v)\}$, as demonstrated by $Z^* \geq Q$.

The first derivative and second derivative of the supplier's profit function $\pi_r(Q|c_e < p)$ with respect to Q are

$$\frac{\partial \pi_r(Q|c_e < p)}{\partial Q}$$
$$\frac{\partial^2 \pi_r(Q|c_e < p)}{\partial Q^2} = -(p-e) f(Q) < 0. \tag{A.2}$$

Since the second derivative of Q of $\pi_r(Q|c_e < p)$ is less than zero, it yields that $\pi_r(Q|c_e < p)$ is a concave function about Q. Thus, if the first derivative is 0, it yields that $F(Q^*) = 1 - o/p - e$.Thus, the optimal production quantity of the supplier meets $F(Z^*) = \max\{F(Q^*), (1-\beta)c_e - c/(1-\beta)(c_e - v)\}$.

When $(1-\beta)c_e - c/(1-\beta)(c_e - v) \leq 1 - o/p - e$, $F(Z^*) \leq F(Q^*)$, it yields $1 - c(p-e)/o(c_e - v) + v(p-e) \leq \beta < 1 - c/c_e$. Since $p > o + w$, $1 - c(p-e)/o(c_e - v) + v(p-e) < 1 - c/p$. At this time, F $(Z^*) = F(Q^*) = 1 - o/p - e$.

When $(1-\beta)c_e - c/(1-\beta)(c_e - v) > 1 - o/p - w, 0 \leq \beta < 1 - c(p-w)/c_0(c_e - v) + v(p-w), F(Z^*) > F(Q^*)$, i.e., $Z^* > Q^*, F(Z^*) = (1-\beta)c_e - c/(1-\beta)(c_e - v), F(Q^*) = 1 - o/p - e$. Since $\widehat{\beta}_1 = 1 - c(p-e)/o(c_e - v) + v(p-e)$, Proposition 1 is achieved. □

Proof of Proposition 2. The first derivative and second derivative of supplier's profit function $\pi_s(Z|c_e > p)$ with respect to Z are

$$\frac{\partial \pi_s (Z|c_e > p)}{\partial Z}$$

$$\frac{\partial^2 \pi_s (Z|c_e > p)}{\partial Z^2} = (1-\beta)(-p+v) f(Z) < 0. \quad (A.3)$$

Since the second derivative of Z of $\pi_s (Z|c_e > p)$ is less than zero, it yields that $\pi_s (Z|c_e > p)$ is a concave function about Z. Thus, if the first derivative is 0, it yields that $F(Z^*) = (1-\beta)p - c/(1-\beta)(p-v)$. Since $F(Z^*) > 0$, it yields $\beta < 1 - \frac{c}{p}$, so the optimal production quantity of the supplier meets $F(Z^*) = \max\ \{F(Q), (1-\beta)p - c/(1-\beta)(p-v)\}$, as demonstrated by $Z^* \geq Q$.

The first derivative and second derivative of the supplier's profit function $\pi_r (Q|c_e > p)$ with respect to Q are

$$\frac{\partial \pi_r (Q|c_e > p)}{\partial Q}$$

$$+ (1-\beta)(r-p)(F(Q)-1), \quad (A.4)$$

$$\frac{\partial^2 \pi_r (Q|c_e > p)}{\partial Q^2} = [w - p - \beta (r-p)] f(Q).$$

Since the second derivative of Q of $\pi_r (Q|c_e > p)$ is less than zero, it yields that $\pi_r (Q|c_e > p)$ is a concave function about ?. Thus, if the first derivative is 0, it yields that $F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$. Since $o + w < p$, it yields that $1 - o/r - e - (1-\beta)(r-p) > 0$ is constantly valid, so the optimal production quantity of the supplier meets $F(Z^*) = \max\ \{F(Q^*), (1-\beta)p - c/(1-\beta)(p-v)\}$.

When $(1-\beta)p - c/(1-\beta)(p-v) \leq 1 - o/r - e - (1-\beta)(r-p), F(Z^*) \leq F(Q^*)$, it yields $\widehat{\beta}_2 \leq \beta < 1 - c/p$. At this time, $F(Z^*) = F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$; when $(1-\beta)p - c/(1-\beta)(p-v) > 1 - o/r - e - (1-\beta)(r-p)$, i.e., $0 \leq \beta < \widehat{\beta}_2, F(Z^*) > F(Q^*), Z^* > Q^*, F(Z^*) = (1-\beta)p - c/(1-\beta)(p-v), F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$. To be specific, $\widehat{\beta}_2$ satisfies $c - (1-\beta)v/(1-\beta)(p-v) = o/p - r - w - (1-\beta)(r-p)$, and Proposition 2 is achieved. □

Proof of Proposition 3. Based on the case of $c_e < p$, the same is true for $c_e > p$.

Since the second derivative of Q of $\pi_r (Q|c_e < p)$ is less than 0, it yields that $\pi_r (Q|c_e < p)$ is a concave function about Q. Thus, if the first derivative is 0, it yields that $F(Q^*) = 1 - w/(1-\beta)p$. Since $F(Q^*) > 0$, it yields $\beta < 1 - w/p$, so the optimal production quantity of the supplier meets $F(Z^*) = \max\ \{F(Q^*), (1-\beta)c_e - c/(1-\beta)(c_e - v)\}$.

When $(1-\beta)c_e - c/(1-\beta)(c_e - v) > 1 - w/(1-\beta)p, 0 \leq \beta < 1 - cp - w(c_e - v)/vp$, $F(Z^*) > F(Q^*)$, i.e., $Z^* > Q^*, F(Z^*) = (1-\beta)p - c/(1-\beta)(p-v)$, $F(Z^*) = 1 - w/(1-\beta)p$.

When $c - (1-\beta)c_e/(1-\beta)(c_e - v) \leq 1 - w/(1-\beta)p$, $F(Z^*) \leq F(Q^*)$, it yields $1 - cp + w(v - c_e)/vp < \beta \leq 1 - w/p$. Set $\widehat{\beta}_3 = 1 - cp - w(c_e - v)/vp, 0 < \widehat{\beta}_3 < 1$, and it is proven that $1 - w/p > 1 - c/c_e$ is established, so Proposition 3 is achieved. □

Proof of Proposition 4. When the option contract is valid in the model and $c_e < p$, the retailer's optimal order quantity is $1 - o/p - e$. When $c_e > p$, her optimal ordering quantity is $1 - o/r - e - (1-\beta)(r-p)$. When the option contract is invalid, her optimal initial ordering quantity is $1 - w/(1-\beta)p$. $1 - o/r - e - (1-\beta)(r-p) \geq 1 - o/p - e$ is constantly correct. When $1 - w/(1-\beta)p > 1 - o/p - e$, $\beta < 1 - w(p-e)/op$. At this point, the retailer's optimal order quantity with option contract is less than that without option contract. When $\beta > 1 - w(p-e)/op$, the optimal order quantity of the retailer with option contract is more than that without option contract, which is recorded as $\beta_4 = 1 - w(p-e)/op$.

$(1-\beta)p - c/(1-\beta)(p-v) < (1-\beta)c_e - c/\ (1-\beta)(c_e - v)$ is always valid, so the optimal production of the supplier with option contract is always less than that without option contract. □

Proof of Proposition 5. When $c_e < p$, $F(Q^*) = 1 - o/p - e$, $F(Q^*)$ is independent of disruption probability and emergency procurement price. The retailer has an opportunity of emergency procurement that the supplier can profit from. All of the retailer's orders can be fulfilled under no disruption occurring. Therefore, the role of the option ordering is less than that of emergency procurement.

When $c_e > p$, $F(Q^*) = 1 - o/r - e - (1-\beta)(r-p)$. Since $\partial F(Q^*)/\partial \beta > 0$, $\partial F(Q^*)/\partial p > 0$, $F(Q^*)$ is positively correlated with disruption probability and the emergency procurement price. At this point, the retailer relies more on option ordering to promote production. Accordingly, the role of option ordering is greater than that of emergency procurement. □

B. Proofs of Corollaries 1–3

Proof of Corollary 1. When $0 \leq \beta < \widehat{\beta}_1, \alpha F(Z^*)/\alpha\beta = -c/(1-\beta^2 (v - c_e) < 0, \alpha F(Z^*)/\alpha c = -1/(1-\beta)(c_e - v) < 0, \alpha F(Z^*)/\alpha c_e = c - (1-\beta)v/(1-\beta)(c_e - v)^2 > 0, \alpha F(Z^*)/\alpha v = (1-\beta)c_e - c/(1-\beta)(c_e - v)^2 > 0, \alpha F(Q^*)/\alpha o = -o/p - e < 0, \alpha F(Q^*)/\alpha w = -o/(p-w)^2 < 0, \alpha F(Q^*)/\alpha p = o/(p-e)^2 > 0, \alpha F(Q^*)/\alpha o = -o/p - e < 0, \alpha F(Q^*)/\alpha w = -o/(p-e)^2 < 0, \alpha F(Q^*)/\alpha p = o/(p-e)^2 > 0$.

When $\widehat{\beta}_1 \leq \beta < 1 - c/c_e$, $\alpha F(Z^*)/\alpha o = \alpha F(Q^*)/\alpha o = -o/p - e < 0, \alpha F(Z^*)/\alpha e = \alpha F(Q^*)/\alpha e = -o/(p-e)^2 < 0, \alpha F(Z^*)/\alpha p = \alpha F(Q^*)/\alpha p = o/(p-e)^2 > 0$. □

Proof of Corollary 2. When $0 \leq \beta < \widehat{\beta}_2, \partial F(Z^*)/\partial \beta = -c/(1-\beta^2 (p-v) < 0, \partial F(Z^*)/\partial p = c/(1-\beta^2 (p-v) > 0, \partial F(Z^*)/\partial v = (1-\beta)[(1-\beta)p - c]/(1-\beta)^2 (p-v)^2 > 0, \partial F(Z^*)/\partial c = -1/(1-\beta)(p-v) < 0$; $\partial F(Q^*)/\partial \beta = -(r-p)o/[r - w - (1-\beta)(r-p)]^2 < 0, \partial F(Q^*)/\partial o = -1/r - e - (1-\beta)(r-p) < 0, \partial F(Q^*)/\partial w = -o/[r - e - (1-\beta)(r-p)]^2 < 0, \partial F(Q^*)/\partial r =$

$\beta o/[r-e-(1-\beta)(r-p)]^2>0, \partial F(Q^*)/\partial p = (1-\beta)o/[r-e-(1-\beta)(r-p)]^2>0$.

When $\widehat{\beta}_2 \leq \beta < 1-c/p$,

$$\begin{aligned}
&\frac{\partial F(Z^*)}{\partial \beta}\\
&\frac{\partial F(Z^*)}{\partial o} = \frac{\partial F(Q^*)}{\partial o} = -\frac{1}{r-e-(1-\beta)(r-p)} < 0,\\
&\frac{\partial F(Z^*)}{\partial w} = \frac{\partial F(Q^*)}{\partial w} = -\frac{o}{[r-e-(1-\beta)(r-p)]^2} < 0,\\
&\frac{\partial F(Z^*)}{\partial p} = \frac{\partial F(Q^*)}{\partial p},\\
&= \frac{(1-\beta)o}{[r-e-(1-\beta)(r-p)]^2} > 0.
\end{aligned} \tag{B.1}$$

□

Proof of Corollary 3. When $0 \leq \beta < \min(1-c/c_e, 1-w/p)$, $F(Z^*) = (1-\beta)c_e - c/(1-\beta)(c_e-v), F(Q^*) = 1 - w/(1-\beta)p$.

$$\begin{aligned}
&\frac{\alpha F(Z^*)}{\alpha \beta}\\
&\frac{\alpha F(Z^*)}{\alpha c} = -\frac{1}{(1-\beta)(c_e-v)} < 0,\\
&\frac{\alpha F(Z^*)}{\alpha c_e} = \frac{c-(1-\beta)v}{(1-\beta)(c_e-v)^2} > 0,\\
&\frac{\alpha F(Z^*)}{\alpha v} = \frac{(1-\beta)c_e-c}{(1-\beta)(c_e-v)^2} > 0,\\
&\frac{\alpha F(Q^*)}{\alpha \beta} = -\frac{w}{(1-\beta)^2 p} < 0,\\
&\frac{\alpha F(Q^*)}{\alpha w} = -\frac{1}{(1-\beta)p} < 0,\\
&\frac{\alpha F(Q^*)}{\alpha p} = \frac{w}{(1-\beta)p^2} > 0.
\end{aligned} \tag{B.2}$$

□

Data Availability

The proofs of Propositions 1–5 and Corollaries 1–3 used to support the findings of this study are included within Appendix A and Appendix B of the article, respectively.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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