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Research Article

Simultaneous Multiplicative and Additive Actuator Faults Estimation-Based Sliding Mode FTC for a Class of Uncertain Nonlinear System

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In this article, an adaptive sliding mode fault tolerant control (FTC) is improved in the case of uncertain nonlinear system which is affected by both multiplicative and additive faults in actuator. Especially, when the nonlinear system is modeled by Takagi–Sugeno (T-S) fuzzy system with local nonlinear model. The main contribution of this paper is developing a model of multiplicative faults, which offers a more realistic dynamic evolution of the actuator degradation. The degradation process is modeled by Wiener process and estimated by the maximum likelihood estimation (MLE). Sliding mode observer (SMO) is conceived to realize the additive actuator faults using convex multiobjective optimization. On these bases, the estimated multiplicative and additive actuator faults are used to design the adaptive sliding mode controller (SMC). Finally, the proposed fault-tolerant control scheme is demonstrated by the results of inverted pendulum system simulation.

1. Introduction

During the past few decades, the fields of fault estimation (FE) technique and fault tolerant control (FTC) have been the result drawing an intensive research interests due to the increasing demands for system's performance, safety, and reliability. Active FTC and FE to a category of nonlinear systems particularly T-S fuzzy models [1–3] have a significant position in recent control implementation, as well as in supervision and reliability of actuators. In recent decades, a variety of methods have been developed, using adaptive observer [4–6] or SMO [7–9]. The sliding mode scheme has an excellent application prospect in fault estimation and fault tolerant control due to its simple structure, strong applicability, and good robustness. Several publications have appeared in recent years regarding this issue. In [10], the authors studied Takagi–Sugeno fuzzy systems with uncertainties and multiplicative and additive actuator faults and then developed an adaptive sliding mode FTC design. However, in [11], a FTC design predicated using SMO for T-S fuzzy systems is developed. In [12], the authors used a nonquadratic Lyapunov function to estimate simultaneously actuator and sensor faults for T-S fuzzy systems. In a recent paper [13], the authors developed the FE and FTC for the T-S fuzzy systems subject to actuator and sensor faults. For nonlinear systems, popular fault detection methods have been elaborated in an effective and precise manner. In [14, 15], the authors modelled fault as an additive occurring in sensors or actuators. The main disadvantage of the previous techniques is that they regard sensor and actuator faults as additive. Nevertheless, some actuator and sensor errors, in addition to component faults, are frequently found in the multiplicative form. As a result, multiplicative faults and the system's inputs and outputs are mixed. Estimating the magnitude and characteristics of multiplicative faults has become an increasing attract in control theory due to the practical importance of decoupling their structure effects or parameter in the model or system and improving fault tolerant control concept for nonlinear systems. We can use stochastic process to model the actuator degradation [16–19]. The actuator environment can influence the deterioration process and strongly depends on several factors [20] (shock, temperature, and load variation) of the monitored system. Degradation process for industrial systems is influenced by both external and internal factors including operating conditions and dynamic environment [21]. Stochastic dynamics are the common characteristics involved in actuator increasing degradation process in actuator. This results in system uncertainties and measurements errors. For the past few decades, extensive research studies have been conducted in the area of stochastic degradation modelling [22–24]. Degradation models can be divided into shock-based degradation model [25], progressive degradation model [26, 27], and combined degradation model [28]. The degradation such as the wear out of engineering devices, the fatigue, and the corrosion of metals can be caused by multiple degradation processes and induces total failures of actuator.

In the following, the authors point out the main focus of the present study on developing effective and robust active FTC for a class of uncertain nonlinear system. This study offers a FE-based sliding mode FTC technique for nonlinear uncertain system affected by both multiplicative and additive faults. The major contributions of this article are as follows:

(i) In this study, for more simplicity and to find a model for the description of the interactions between the control system behavior and the actuator stochastic degradation process, the deterioration of the entire control system would be supposed to lie in the actuator loss of efficiency. In practice, when an actuator operates dynamically in a random environment, its capacity decreases overtime which is related to degradation process. Therefore, the multiplicative faults model was conceived based on not only the degradation process but also the capacity of actuator. Actually, it is straight forward to think multiplicative faults should be estimated in order to conceive a fault tolerant control design for dynamic systems. To describe the actuator degradation behavior, we use stochastic Wiener process model which offers a more realistic evolution of the deterioration. In order to estimate the degradation process, the maximum likelihood method is used.

(ii) We propose to conceive an adaptive SMO for T-S systems with the existence of uncertainties to estimate additive actuator faults. To study the stability of the proposed SMO, we use a linear matrix inequality (LMI) and the theories of Lyapunov. To improve the actuator faults estimation accuracy, we use an adaptive update term.

(iii) Using FE, we study an adaptive SMC for the T-S fuzzy system having models local nonlinear that complies with the condition of Lipschitz with additive and multiplicative faults. Particularly, it is demonstrated that the suggested sliding mode FTC is used for the parameters setting of the controller in order to obtain the desired performances of actuator even in the presence of both additive and multiplicative faults.

Compared to previous works, many studies have used only actuator or senor faults in system [29]. Then, most of the previous studies do not take into account multiplicative faults. Furthermore, we use adaptive law to design the SMO which gives more freedom in comparison with [30]; the model used incorporates output disturbance and Itô stochastic noise; and they introduced time delay in the state. However, in our case study, we suppose that the degradation of the total control system lies in the actuator loss of efficiency. Therefore, the multiplicative fault model was conceived based on not only the degradation process but the capacity of the actuator. To describe the actuator degradation behaviour, we use the stochastic Wiener process model (continuous models) which offers a more realistic deterioration. Compared to [31], the authors use only additive faults in both actuator and sensor in the system. They transform sensor faults into "pseudoactuator" faults by using an augmented T-S fuzzy system that causes many constraints in the application of the hypotheses; in fact, the total number of actuator faults must not exceed the number of outputs. This model may not be practical and conventional in all situations of stochastic degradation process in actuator. Moreover, it cannot adequately capture the dynamics of the actuator's degradation process. Indeed, the model of faults must describe the interaction between the actuator stochastic degradation process and the control system behaviour. This includes the dynamics of the actuator's performance, the control system's response to changes in the actuator's performance, and the uncertainty associated with the actuator's stochastic degradation process. The model must also take into account the physical characteristics of the actuator and the system environment, as well as the impact of external factors such as maintenance and other system parameters. In our study, we develop a model of multiplicative faults, which offers a more realistic dynamic evolution of the actuator degradation. The SMO is designed according to adaptive law which offers less conservative results and gives more liberty in comparison with [32]. Yang et al. in [33] have developed simultaneous multiplicative and additive faults in jump systems, which may be considered as a special class of stochastic systems. They use the adaptive backstepping technique to construct the fuzzy logic system based an online adaptive fault-tolerant compensation controller. However, in our study, we use continuous degradation models and we propose to conceive an adaptive SMO for Takagi–Sugeno fuzzy systems having local nonlinear models.

The outline of the article is organized as follows. Section 2 describes a nonlinear system with local nonlinear models, uncertainties, additive, and multiplicative (loss of efficiency) actuator faults. In Section 3, we use Wiener process to model the degradation process in the actuator and the maximum likelihood method to estimate the stochastic model for multiplicative fault. In Section 4, the proposed adaptive SMO is designed to estimate the additive faults of actuator. Section 5 presents the sliding mode FTC design in order to redress the impact of additive and multiplicative faults for the stabilization of the system. A simulation of the inverted pendulum and cart system is used in Section 6 in order to validate and illustrate the efficiency of the approach. Finally, Section 7 draws some conclusions.

2. Problem Formulation

In this study, we refer to a class of nonlinear uncertain systems affected both the additive and multiplicative actuator faults. Consider a nonlinear uncertain system represented by the following equations:

$$\begin{aligned}\dot{x}_t &= \varphi_{x_1}(x_t) + \varphi_{x_2}(x_t)u_t + \varphi_{x_3}(x_t)\xi(x,t) \\ &\quad + \varphi_{x_4}(x_t)\Gamma(x,t),\end{aligned} \tag{1}$$

$$y_t = \varphi(x_t), \tag{2}$$

$$y_{Lt} = \varphi_L(x_t). \tag{3}$$

$u_t \in R^m$ is the control input, $x_t \in R^n$ represents the state vector, $y_{Lt} \in R^{p1}$ stands for the controlled output, and $y_t \in R^p$ represents the measurement vector of output. The functions $\varphi_L(x_t), \varphi(x_t), \varphi_{x_i}(x_t)$, and $\Gamma(x,t)$ are always nonlinear for $i = 1,2,3,4$. $\xi(x,t) \in \mathrm{R}^l$ denotes the unknown uncertainties vector.

In feedback control system, the actuator is a significant part in the evaluation of the performance level. That is because, considering a degradation process in an electrical or mechanical element of the controlled system, the controlled action is affected as well, which eventually causes a poor performance of the control system.

Let us consider an actuator undergoing a progressive degradation process. It is subjected to both electrical and mechanical degradation that occurs stochastically over time, particularly in the case of the electrical actuator degradation where a rub impact between the rotor and the stator or a bend shaft can occur. It should be noted that rotor faults as well as stator faults are recognized electrical faults. Besides, there are other types of faults that depend on the failure mode. During its functioning, the actuator can be affected by several types of faults. Our study will focus on two types of faults: additive faults and multiplicative faults (loss of efficiency). In practice, the actuator operates dynamically in a random environment. Furthermore, its capacity decreases overtime which is related to degradation and depends on environmental factors as well as operational conditions of the feedback control system. Moreover, the wear or natural ageing of the electrical and/or mechanical components of the actuator due to the nondesired impacts of the working condition decreases the effectiveness of actuator in time. The actuator degradation process is a cause of the physical system performance deterioration. During the initial period of operation, the actuators function flawlessly.

The actual capacity of actuator $K_a(t) = K_{a_{\text{int}}}$ where $K_{a_{\text{int}}}$ is the initial nominal capacity. If $d(t)$ outlined the actuator degradation, the capacity of actuator (see Figure 1) can be represented by the following equation:

$$K_a(t) = K_{a_{\text{int}}} - d(t). \tag{4}$$

The efficiency factor (see Figure 2) can be written as follows:

$$\beta = \frac{K_a(t)}{K_{a_{\text{int}}}}. \tag{5}$$

The minimum efficiency is reached when $K_a(t) = K_{\min}$, and we define the minimum efficiency factor $\varepsilon = (K_{\min}/K_{a_{\text{int}}}) > 0$ such as $0 < \varepsilon \le \beta \le 1, \forall t \ge t_f$ and t_f is the time occurrence of multiplicative defect.

The loss of actuator effectiveness is examined to consequence from the dynamic progression of the degradation process $\beta = (K_{a_{\text{int}}} - d(t))/K_{a_{\text{int}}}$.

In this way, the following three cases are defined:

(i) $K_a(t) = K_{a_{\text{int}}}$: the actuator operates without degradation $(d(t) = 0)$, and the efficiency factor is $\beta = 1$

(ii) $K_a(t) = K_{a_{\text{int}}} - d(t)$: the actuator deteriorates, its capacity $K_{a_{\min}} < K_a(t) < K_{a_{\text{int}}}$, and the efficiency factor is $\beta = (K_{a_{\text{int}}} - d(t))/K_{a_{\text{int}}}$

(iii) $K_a(t) = K_{a_{\min}}$: the actuator operates with its minimum capacity, and then the efficiency factor is $\beta = \varepsilon = (K_{a_{\min}}/K_{a_{\text{int}}})$

The nonlinear system (1)–(3) affected by additive and multiplicative faults at the same time can be described by the following uncertain structure and local nonlinearities as follows:

$$\begin{aligned}\dot{x}_t &= \varphi_{x_1}(x_t) + \varphi_{x_2}(x_t)\beta u_t + \varphi_{x_3}(x_t)f_a(t) \\ &\quad + \varphi_{x_4}(x_t)\Gamma(x,t) + \varphi_{x_5}(x_t)\xi(x,t),\end{aligned} \tag{6}$$

$$y_t = \varphi(x_t), \tag{7}$$

$$y_{Lt} = \varphi_L(x_t), \tag{8}$$

where $f_a(t) \in \mathrm{R}^q$ is the additive actuator faults.

Multiplicative fault may be rearranged as follows:

$$\begin{aligned}\varphi_{x_2}(x_t)\beta u_t &= \varphi_{x_2}(x_t)\beta u_t + \varphi_{x_2}(x_t)u_t - \varphi_{x_2}(x_t)u_t \\ &= \varphi_{x_2}(x_t)u_t + \varphi_{x_2}(x_t)(\beta - 1)u_t \\ &= \varphi_{x_2}(x_t)u_t + F(u_t,t),\end{aligned} \tag{9}$$

where $F(u_t,t)$ are the multiplicative faults.

Then, the nonlinear system (1)–(3) affected by additive and multiplicative faults at the same time can be expressed in terms of uncertain structures and local nonlinearities as follows:

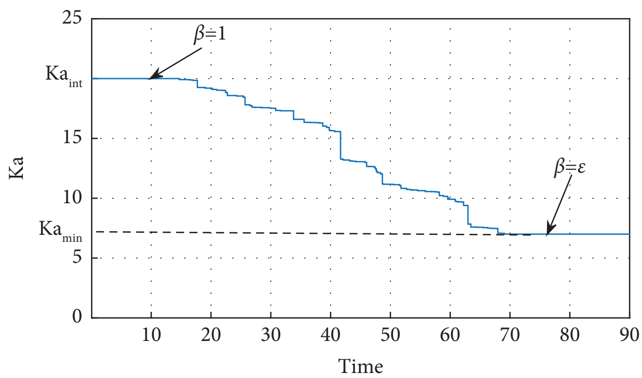

FIGURE 1: The actuator capacity $K_a(t)$.

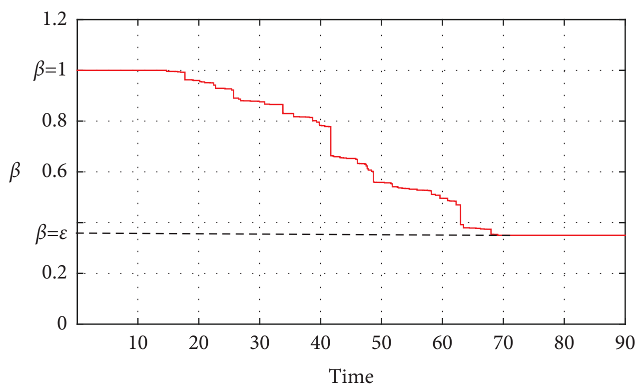

FIGURE 2: The efficiency factor β.

$$\dot{x}_t = \varphi_{x_1}(x_t) + \varphi_{x_2}(x_t)u_t + F(u,t) + \varphi_{x_3}(x_t)f_a(t) + \varphi_{x_4}(x_t)\Gamma(x,t) + \varphi_{x_5}(x_t)\xi(x,t), \quad (10)$$

$$y_t = \varphi(x_t), \quad (11)$$

$$y_{Lt} = \varphi_L(x_t). \quad (12)$$

The multiplicative faults are $F(u,t) = \varphi_{x_2}(x_t)(\beta - 1)u_t = \varphi_{x_2}(x_t)((K_{a_{\text{int}}} - d(t)/K_{a_{\text{int}}}) - 1)u_t$, and the following three case are defined:

(i) $d(t) = 0$: the actuator operates without degradation, the multiplicative faults are neglected $F(u,t) = 0$, so we have only the additive faults

(ii) $0 < d(t) < L$ (L is a maximum degradation process): the actuator operates with degradation process, and the multiplicative faults are

$$F(u,t) = \varphi_{x_2}(x_t)\left(\frac{K_{a_{\text{int}}} - d(t)}{K_{a_{\text{int}}}} - 1\right)u_t. \quad (13)$$

(iii) $d(t)$ reached L: the multiplicative faults are $F(u,t) = \varphi_{x_2}(x_t)((K_{a_{\min}}/K_{a_{\text{int}}}) - 1)u_t$

Given the nonlinear system (10)–(12) affected by both the additive and multiplicative actuator faults, respectively, $f_a(t)$, $F(u,t)$, and the uncertainties $\xi(x,t)$, our objective to achieve an adaptive sliding mode FTC resides principally on solving the following three problems:

(1) First problem: develop and estimate the degradation process to conceive a multiplicative faults model which offers a more realistic evolution of the deterioration in the actuator. It is very important to be able to estimate the faults before the performance systems degradation.

(2) Second problem: estimate T-S fuzzy system states and additive actuator faults, with the adaptive SMO.

(3) Third problem: we need to stabilize the closed loop of nonlinear systems, with the simultaneous occurrence of additive and multiplicative faults, using the robust adaptive sliding mode controller (10)–(12).

3. Actuator Degradation Models Estimation

In this study, we suppose that the system (10)–(12) is subject to Wiener process. The actuator degradation is denoted by a random variable d_t at time t. In this paper, we suppose that degradation is an increasing Lévy process [34] supported by the following assumptions:

(i) The initial degradation is denoted $d_0 = 0$

(ii) The degradation process is described with one-dimensional stochastic process $\{d\}_{t\geq 0}$

(iii) The increments $\{d\}_{t\geq 0}$ are independent and stationary

In this way, Wiener process has been frequently used to conceive a degradation model, particularly, when it was successfully applied to describe the increasing degradation in an actuator. The Wiener process is one of the most classic processes used in many progressive degradation modeling area; the basic idea is to model the cumulative increasing degradation $d_{W,t}$ by the stochastic Wiener process such that

$$d_{W,t} = d_{W,t_0} + W_t(\mu,\sigma), \forall t \geq 0, \quad (14)$$

where μ is a linear drift parameter and σ is a diffusion coefficient parameter and $W_t(\mu,\sigma) = \mu t + \sigma B_t$.

We consider that the Wiener process [24] is used such that the increment $W_{t_2} - W_{t_1}$ follows a Gaussian distribution with mean $E(W_{t_2} - W_{t_1}) = \mu(t_2 - t_1)$ and variance $\text{Var}(W_{t_2} - W_{t_1}) = \sigma^2(t_2 - t_1)$. If we consider the initial condition d_{W,t_0}, we can approximate the degradation measure $d_{W,t}$ with the following equation:

$$d_{W,t} = d_{W,t_0} + W(t - t_0), \forall t \geq 0. \quad (15)$$

Here in, we can deduce that

$$d_{W,t} = d_{W,t_0} + \mu(t - t_0) + \sigma B(t - t_0). \quad (16)$$

In particular, if $d_{W,t_0} = 0$, the degradation process $d_{W,t} = W_t(\mu,\sigma)$

The objective is to estimate the linear drift μ and diffusion parameter σ. We apply the maximum likelihood estimation (MLE) method.

Considering the degradation increment $\Delta d_{i,j} = (d_{i,j+1} - d_{i,j})$ of ith items at time j where $\rho = (\mu, \sigma)$, $j = 1, 2, \ldots m$ and $i = 1, 2, \ldots n$. The degradation measurements for item i, $\Delta d_i = (\Delta d_{i,1}, \Delta d_{i,2}, \ldots, \Delta d_{i,m})$. The density function is given by the following equation:

$$f_{(\mu\Delta t_{i,j}, \sigma^2 \Delta t_{i,j})}(\Delta d_{i,j}) = \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{i,j}}} * e^{a}, \tag{17}$$

where $a = -((\Delta d_{i,j} - \mu\Delta t_{i,j})^2 / 2\sigma^2 \Delta t_{i,j})$.

The likelihood function of the ith path $L_i(\rho) = f_i(\Delta d_i) = f_i(\Delta d_{i,1}, \ldots, \Delta d_{i,m}/\mu, \sigma)$ is given by the following equation:

$$\mathrm{Li}(\rho) = \prod_{j=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{i,j}}} \mathrm{e}^{(a)}. \tag{18}$$

The log-likelihood function for the ith item can be expressed by the following equation:

$$\mathrm{li}(\rho) = \ln\left[\prod_{j=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2 \Delta \mathrm{t}_{\mathrm{i,j}}}} e^{(a)}\right]. \tag{19}$$

Since the measurements $d_{i,j}$ are independents, we can express $\mathrm{l}(\rho) = \ln(\Delta d_1, \Delta d_2, \Delta d_3, \ldots \Delta d_n)$ as

$$\mathrm{l}(\rho) \sum_{i=1}^{n} \ln\left(\prod_{j=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2 \Delta \mathrm{t}_{\mathrm{i,j}}}} e^{\left(-\left(\Delta d_{i,j} - \mu\Delta t_{i,j}\right)^2 / 2\sigma^2 \Delta t_{i,j}\right)}\right). \tag{20}$$

We find the MLE $\hat{\rho} = [\hat{\sigma}, \hat{\mu}]$ by maximizing with respect to σ and μ, the partial derivatives of the log-likelihood function.

As a result, we write the partial derivative of log-likelihood function compared to μ as

$$\frac{\partial \mathrm{l}(\rho)}{\partial \mu} = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\Delta d_{i,j} - \mu\Delta t_{i,j}}{\sigma^2} = 0, \tag{21}$$

and compared to σ as

$$\frac{\partial \mathrm{l}(\rho)}{\partial \sigma} = \frac{-\mathrm{mn}}{\sigma^2} + \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\Delta \mathrm{d}_{\mathrm{i,j}} - \mu\Delta \mathrm{t}_{i,j}}{\sigma^3 \Delta \mathrm{t}_{\mathrm{i,j}}} = 0. \tag{22}$$

Then, we obtain the expression as follows:

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \Delta d_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{m} \Delta t_{i,j}}, \tag{23}$$

$$\hat{\sigma}^2 = \frac{1}{mn} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\Delta d_{i,j} - \hat{\mu}\Delta t_{i,j}}{\sigma^3 \Delta t_{i,j}}. \tag{24}$$

The measurements data $\Delta_{i,j}$ are generated by MATLAB with the parameters $\mu = 0.4$, $\sigma = 0.2$. We compute equations (23) and (24), and the estimated parameters are obtained as follows: $\mu = 0.398$ and $\sigma = 0.213$.

A design procedure for multiplicative fault development and estimation is described as follows:

(i) Step 1: we define the expression of the efficiency factor using the degradation process and the actuator capacity to conceive the multiplicative faults model.

(ii) Step 2: we use the Wiener process to model the stochastic degradation in actuator.

(iii) Step 3: the maximum likelihood method is used to estimate the linear drift and diffusion parameter.

4. Additive Actuator Faults Estimation

It was shown that using T-S fuzzy system with local nonlinear models concept was well suited to the study of a several class of systems. The nonlinear system (10)–(12) affected by additive and multiplicative faults at the same time is written by the T-S fuzzy system with uncertainty and models local nonlinear

$$\dot{x}(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{A_i x(t) + B_i u(t) + F(u,t) + M_i f_a(t) + D_i \xi(x,t) + \Gamma(x,t)\}, \tag{25}$$

$$y(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_i x(t)\}, \tag{26}$$

$$y_L(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_{(L,i)} x(t)\}, \tag{27}$$

where M_i, D_i, C_i, B_i, and A_i are matrices with known real values. $F(u,t)$ is the multiplicative fault. We suppose that (A_i, B_i) and (A_i, C_i) are, respectively, controllable and observable. $f_a(t)$ and $F(u,t)$ represent the additive and multiplicative fault in control channel.

The functions $\mu_i(\zeta_t)$ (fuzzy normalized membership) must satisfy the properties of sum convex

$$\forall i \in [1, 2, \ldots, k], \sum_{i=1}^{k} \mu_i(\zeta_t) = 1, 1 \geq \mu_i(\zeta_t) \geq 0. \tag{28}$$

We will use the following assumptions in this paper.

Assumption 1. We assume that the uncertainties and faults are unknown and bounded. For the faults $f_a(t), F(u,t)$ and the uncertainties $\xi(x,t)$, there exist known positive constants ξ_0, ρ_F and ρ_a such that $\|f_a(t)\| \leq \rho_a$, $\|F(u,t)\| \leq \rho_F$ and $\|\xi(x,t)\| \leq \xi_0$.

Assumption 2. (A_i, M_i, C_i) is minimum phase and relative degree one, and we verify that for all complex numbers s when $\mathrm{Re}(s) \geq 0$ that

$$\mathrm{rank}\begin{bmatrix} sI_n - A_i & -M_i \\ C_i & 0 \end{bmatrix} = n + q, \tag{29}$$

Ensures that the nonasymptotically stable modes are observable which means it is detectable.

Assumption 3. The distribution matrix M_i of the additive fault in equation (24) satisfies

$$\mathrm{rank}(C_i M_i) = q. \tag{30}$$

Assumption 4. $\Gamma(x,t)$ the known nonlinear function satisfies the local condition of Lipschitz on $\mathbb{M} \subset \mathrm{R}^n$ with

$$\left\|\left(\Gamma\left(x_{a_1}, t\right) - \Gamma\left(x_{a_2}, t\right)\right)\right\| \leq \left\|\left(x_{a_1} - x_{a_2}\right)\right\| \gamma, \forall\left(x_{a_2}, x_{a_1}\right) \in \mathbb{M}. \tag{31}$$

The constant of Lipschitz $\gamma > 0$ is unknown and $\Gamma(x,t)$ is globally Lipschitz if $\mathbb{M} = \mathrm{R}^n$. Edwards and Spurgeon [35] have studied an SMO to estimate faults and stats taking into account the following required assumptions.

The following lemma and definition are utilized to achieving the principal results.

4.1. Definition and Notation. Let $Z \in \mathrm{R}^{n\times m}$ a random matrix; if $Z^+ \in \mathrm{R}^{m\times n}$ satisfies $Z^+ Z = I_m$, then $Z^+ = (Z^T Z)^{-1} Z^T$ is a left-inverse of Z.

Lemma 5. *For the two matrix Y and Z, the next condition carries*

$$Z^T Y + Y^T Z \leq \varepsilon^{-1} Z^T Z + \varepsilon Y^T Y, \tag{32}$$

$\varepsilon > 0$.

We develop a novel adaptive SMO defined as

$$\dot{\hat{x}}(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\left\{A_i \hat{x}(t) + B_i u(t) + \hat{F}(u,t) + \Gamma(\hat{x},t) + G_{l,i} e_y(t) + G_{n,i} v(t)\right\}, \tag{33}$$

$$\hat{y}(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_i \hat{x}(t)\}, \tag{34}$$

where $\hat{y}(t)$ and $e_y(t)$ denote, respectively, the output of the observer and estimation error, $\hat{x}(t)$ represents the observer state. $G_{n,i}$ and $G_{l,i}$ are suitable gain matrices. In this way, the signal $v(t)$ of the robust adaptive sliding mode is written as follows:

$$v(t) := \begin{cases} \eta(t)\dfrac{P_e}{\|P_e\|}, & \text{if } e_y(t) \neq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{35}$$

where $P_e = P_2 e_y(t)$, $\eta(t) = \hat{\rho} + \varrho$ with ϱ is a scalar positive, $P_2 \in \mathbb{R}^{p\times p}$ is positive definite and symmetric and $\hat{\rho}$ is the adaptive term can be updated given by the following equation:

$$\dot{\hat{\rho}} = \alpha\|P_e\|, \hat{\rho}(0) \geq 0, \tag{36}$$

$\alpha > 0$ is a gain.

Under condition in equation (30), we can use a change of coordinates as follows:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = T_i x(t). \tag{37}$$

The matrices $A_i, B_i, D_i, M_i, C_i, G_{l,i}$, and $G_{n,i}$ become

$$\begin{aligned} A_i &= \begin{bmatrix} A_{11,i} & A_{12,i} \\ A_{21,i} & A_{22,i} \end{bmatrix}, \\ B_i &= \begin{bmatrix} B_{1,i} \\ B_{2,i} \end{bmatrix}, \\ D_i &= \begin{bmatrix} D_{1,i} \\ D_{2,i} \end{bmatrix}, \\ M_i &= \begin{bmatrix} 0 \\ M_{2,i} \end{bmatrix}, \\ C_i &= \begin{bmatrix} 0 & C_{2,i} \end{bmatrix}, \\ G_{l,i} &= \begin{bmatrix} G_{l1,i} \\ G_{l2,i} \end{bmatrix}, \\ G_{n,i} &= \begin{bmatrix} G_{n1,i} \\ G_{n2,i} \end{bmatrix}, \end{aligned} \tag{38}$$

where $A_{11,i} \in \mathbb{R}^{n_p\times n_p}$, $n_p = (n-p)$, $D_{1,i} \in \mathbb{R}^{n_p\times l}$, $M_{2,i} \in \mathbb{R}^{p\times(q)}$ and $C_{2,i} \in \mathbb{R}^{p\times p}$ is nonsingular.

$e(t) = x(t) - \hat{x}(t)$ is the estimated error of the state.

$$\dot{e}_1(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\left\{A_{11,i}e_1(t) + \left(A_{12,i} - G_{l1,i}\right)e_y(t) - \Gamma_1(\hat{x},t) + \Gamma_1(x,t) + D_{1,i}\xi(x,t) - G_{n1,i}v(t) + e_{F_{a1}}(t)\right\} \quad (39)$$

$$\begin{aligned}\dot{e}_2(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\Big\{&A_{21,i}e_1(t) + \left(A_{22,i} - G_{l2,i}\right)e_y(t) - \Gamma_2(\hat{x},t) + \Gamma_2(x,t) \\ &+ D_{2,i}\xi(x,t) + M_{2,i}f_a(t) - G_{n2,i}v(t) + e_{F_{a2}}(t)\Big\}\end{aligned}, \quad (40)$$

where $T_i\Gamma(x,t) = \left[\Gamma_1^T(x,t)\;\; \Gamma_2^T(x,t)\right]^T$ and $\left[e_{F_{a1}}(t)\;\; e_{F_{a2}}(t)\right]^T = T_i e_{F_a}(t) = T_i[F_a(t) - \widehat{F}_a(t)]$

According to equation (37), the nonlinear gain is described by the following equation:

$$\begin{bmatrix} G_{n1,i} \\ G_{n2,i} \end{bmatrix} = \begin{bmatrix} -L_i\overline{C}_{2,i}^{-1} \\ C_{2,i}^{-1} \end{bmatrix}, \quad L_i = \left[L_{1,i}\;\; 0\right]. \quad (41)$$

4.2. Sliding Motion Stability. In addition, another change of coordinates is expressed by the following equation:

$$T_{L,i} = \begin{bmatrix} I_{n-p} & L_i \\ 0_{p\times(n-p)} & C_{2,i} \end{bmatrix}, \quad (42)$$

where L_i is discussed later. In the new coordinates system, we obtain

$$\begin{aligned}\widetilde{A}_i &= \begin{bmatrix} \widetilde{A}_{11,i} & \widetilde{A}_{12,i} \\ \widetilde{A}_{21,i} & \widetilde{A}_{22,i} \end{bmatrix}, \\ \widetilde{D}_i &= \begin{bmatrix} \widetilde{D}_{1,i} \\ \widetilde{D}_{2,i} \end{bmatrix}, \\ \widetilde{M}_i &= \begin{bmatrix} 0 \\ \widetilde{M}_{2,i} \end{bmatrix}, \\ \widetilde{C}_i &= \left[0\;\; I_p\right].\end{aligned} \quad (43)$$

If $\widetilde{A}_{11,i} = A_{11,i} + L_iA_{21,i}$ should be stable, $\widetilde{D}_{1,i} = D_{1,i} + L_iD_{2,i}$, $\widetilde{D}_{2,i} = C_{2,i}D_{2,i}$ and $\widetilde{M}_{2,i} = C_{2,i}M_{2,i}$.

The observer gain matrices are

$$\begin{aligned}\begin{bmatrix} \widetilde{G}_{n1,i} \\ \widetilde{G}_{n2,i} \end{bmatrix} &= T_{L,i}\begin{bmatrix} G_{n1,i} \\ G_{n2,i} \end{bmatrix} = \begin{bmatrix} 0 \\ I_p \end{bmatrix}, \\ \begin{bmatrix} \widetilde{G}_{l1,i} \\ \widetilde{G}_{l2,i} \end{bmatrix} &= T_{L,i}\begin{bmatrix} G_{l1,i} \\ G_{l2,i} \end{bmatrix} = \begin{bmatrix} \widetilde{A}_{12,i} \\ \widetilde{A}_{22,i} - \widetilde{A}_{22}^s \end{bmatrix},\end{aligned} \quad (44)$$

where $\widetilde{A}_{22}^s$ is the stable design matrix.

By referring to equation (42), the error system from equations (39)–(40) can be handled as follows:

$$\dot{\widetilde{e}}_1(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\left\{\widetilde{A}_{11,i}\widetilde{e}_1(t) + \widetilde{D}_{1,i}\xi(x,u,t) + T_{L_{1,i}}\widetilde{\Gamma}_1(x,t) + T_{L_{1,i}}e_{F_{a1}}(t)\right\}, \quad (45)$$

$$\dot{e}_y(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\left\{\widetilde{A}_{21,i}\widetilde{e}_1(t) + \widetilde{A}_{22}^s e_y(t) + \widetilde{D}_{2,i}\xi(x,t) + T_{L_{2,i}}\widetilde{\Gamma}_2(x,t) + \widetilde{M}_{2,i}f_a(t) - v(t) + T_{L_{2,i}}e_{F_{a2}}(t)\right\}, \quad (46)$$

where $\widetilde{\Gamma}_1(x,t) = \Gamma_1(x,t) - \Gamma_1(\hat{x},t)$ and $\widetilde{\Gamma}_2(x,t) = \Gamma_2(x,t) - \Gamma_2(\hat{x},t)$.

Our aim is to estimate the actuator faults and the states variables in the presence of multiplicative faults. Define

$$\begin{aligned} r(t) &= H\begin{bmatrix} \widetilde{e}_1(t) \\ e_y(t) \end{bmatrix} \\ &= H\widetilde{e}(t), \end{aligned} \quad (47)$$

where H is the weight matrix and it is supposed that $H = \mathrm{diag}(H_1, H_2)$.

We define the measure of performance in worst case as follows:

$$\|H\|_\infty := \sup_{\|\xi\|_2 \neq 0} \frac{\|r(t)\|_2^2}{\|\xi(x,u,t)\|_2^2}. \quad (48)$$

Theorem 6. *Consider the T-S fuzzy system (25)–(27) and suppose that Assumptions 3 and 2 are checked. The asymptotically stability is archived for the state estimation errors (45)–(46) with both minimisation gain ς^* for $\xi(x,t)$ and maximisation admissible of γ^* for $\Gamma(x,t)$, if $\exists \varsigma, \alpha, \varepsilon$, $1 \geq \lambda \geq 0$ and matrices $P_1 > 0, P_2 > 0, W_i$ as*

$$\begin{bmatrix} \Xi_{1,i} + H_1^T H_1 & C_{2,i}^T A_{3,i}^T P_2 & P_1 D_{1,i} + W_i D_{2,i} & P_1 & 0 & I_{n-p} & 0 \\ (*) & \Xi_{2,i} + H_2^T H_2 & P_2 D_{2,i} & 0 & P_2 & 0 & I_p \\ (*) & (*) & -\varsigma I & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & -\varepsilon I & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & -\varepsilon I & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & -\alpha I_{n-p} & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & -\alpha I_p \end{bmatrix} < 0, \tag{49}$$

where

$$\begin{aligned} \Xi_{1,i} &= A_{11,i}^T P_1 + P_1 \overline{A}_{11,i} + W_i \overline{A}_{21,i} + A_{21,i}^T W_i^T + P_{F_1}, \\ \Xi_{2,i} &= \widetilde{A}_{22}^{sT} P_2 + P_2 \widetilde{A}_{22}^{s} + P_{F_2}. \end{aligned} \tag{50}$$

Proof. (see Appendix A)

Our aim is estimate the additive actuator faults in nonlinear system (25)–(27). The adaptive SMO donated by equations (33)–(34) has been developed and satisfies the condition of reachability, and then $e_y(t) = 0$ and $\dot{e}_y(t) = 0$. Then, equation (47) is then expressed as follows:

$$0 = \sum_{i=1}^{k} \mu_i(\zeta_t) \left\{ \widetilde{A}_{21,i} \widetilde{e}_1(t) + T_{L2,i} \widetilde{\Gamma}_2(N,t) + \widetilde{D}_{2,i} \xi(N,t) + \widetilde{M}_{2,i} f_a(t) + T_{L2,i} e_{F_{a2}}(t) - v(t) \right\}, \tag{51}$$

where $N = T_{L,i}^{-1} x$.

The approximate equivalent of the output error injection signal $v_{eq}(t)$ is

$$v_{eq}(t) = \eta(t) \frac{P_e}{\|P_e\| + \sigma}, \tag{52}$$

where $\sigma > 0$ is a scalar to decrease the impact of chattering.

We define the next relation

$$\phi(\widetilde{e}_1, x, t) = \sum_{i=1}^{k} \mu_i(\zeta_t) \left\{ T_{L2,i} \widetilde{\Gamma}_2(N,t) + \widetilde{A}_{21,i} \widetilde{e}_1(t) + \widetilde{D}_{2,i} \xi(N,t) + T_{L2,i} e_{F_{a2}}(t) \right\}. \tag{53}$$

It is clear that

$$\|\phi(\widetilde{e}_1, x, u)\|_2 \leq \sum_{i=1}^{k} \mu_i(\zeta_t) \varsigma_{\max,i}, \tag{54}$$

where $\varsigma_{\max,i} = \|\widetilde{D}_{2,i}\|_2 \xi_0 + (\|\widetilde{A}_{21,i}\|_2 + \gamma)\varpi$.

It remains to conclude from equation (54) that

$$\|\phi(\widetilde{e}_1, x, u)\| \leq \varsigma_{\max}. \tag{55}$$

Then, approximately, for a small $\varsigma_{\max}$, it seems that

$$0 = \sum_{i=1}^{k} \mu_i(\zeta_t) \left\{ -v(t) + \widetilde{M}_{2,i} f_a(t) \right\}. \tag{56}$$

Therefore, the additive actuator faults estimation is given by

$$f_a(t) \approx \left(\sum_{i=1}^{k} \mu_i(\zeta_t) \widetilde{M}_{2,i} \right)^{+} v(t). \tag{57}$$

The method for FE with an adaptive SMO is outlined as

(i) Step 1: pick out the weight matrix in equation (47).

(ii) Step 2: pick out the suitable scalar "$0 \leq \lambda \leq 1$" and resolve the equation (49) (LMI optimization problem). So, we can obtain P_1; W; P_2 and ε, ς and γ.

(iii) Step 3: design the adaptive SMO (32)-(33); then, according to equations (51) and (57), the estimation of additive actuator faults can be accomplished. □

5. Sliding Mode FTC Design

5.1. Structure of Adaptive Sliding Mode Controller. This part of the article is devoted to explore an adaptive sliding mode FTC design founded on estimated state variables as well as actuator faults information (magnitude, type, and occurrence time). To stabilize the nonlinear system and compensate additive and multiplicative actuator faults effects, the proposed SMC with adaptive law was used to envisage a corrective action. Therefore, we assume that the nonlinear system with uncertainties (6)–(8) can be modeled by T-S fuzzy representation with both local nonlinearities and uncertainties as follows:

$$\dot{x}(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{A_i x(t) + F(u,t) + B_i u(t) + D_i \xi(x,t) + \Gamma(x,t) + M_i f_a(t)\}, \tag{58}$$

$$y(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_i x(t)\}, \tag{59}$$

$$y_L(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_{(L,i)} x(t)\}. \tag{60}$$

First, the sliding motion takes place on a sliding surface denoted as $\mathbb{S}$ which is defined as follows:

$$\mathbb{S} = \{S_c(t) = 0\colon\ y_c(t) \in \mathbb{R}^p\}. \tag{61}$$

We describe the linear switching function $S_c(t) \in \mathbb{R}^m$ using the feedback information of the output

$$S_c(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{N_{c,i} y_c(t)\}, \tag{62}$$

$N_{c,i} = -h(-C_i B_i (C_i B_i)^+ (+I_p)) + (C_i B_i)^+$ and $((C_i B_i)^+ C_i B_i)^{-1} (C_i B_i)^T = (C_i B_i)^+$ where $h \in \mathbb{R}^{m\times p}$ is an arbitrary matrix.

The control input may be described as follows:

$$u(t) = u_l(t) + u_n(t). \tag{63}$$

The linear part denoted as $u_l(t)$ and depending on system states as well as both additive and multiplicative actuator faults estimation is expressed as follows:

$$u_l(t) = \sum_{j=1}^{k} \mu_j(\zeta_t)\{-K_j \hat{x}(t) - q_a \hat{f}_a(t) - q_F \hat{F}(u,t)\}, \tag{64}$$

where $-q_a(\hat{f}_a(t) + \hat{F}(u,t))$ is created to compensate the additive and multiplicative faults influence. Assume that $q_a = B_i^+ M_i, q_F = B_i^+$ and $K_j \in \mathbb{R}^{m\times n}$.

The adaptive nonlinear control input part $u_n(t)$ is proposed as follows:

$$u_n(t) := \begin{cases} \eta_c(t)\dfrac{S_c(t)}{\|S_c(t)\|}, & \text{if } S_c(t) \neq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{65}$$

$\eta_c(t) = \hat{\rho}_c + \varepsilon_c + \varrho_c$ where ε_c and ϱ_c are small and positive constants. $\eta_c(t)$ is determined by using $\hat{\rho}_c$.

$$\dot{\hat{\rho}}_{S_c} = \|S_c(t)\|\epsilon_c, \hat{\rho}_c(0) \geq 0, \epsilon_c > 0. \tag{66}$$

5.2. FTC Design. Using the part $u_n(t)$ of control input, we need to prove the sliding and the reaching of $\mathbb{S}$ in a finite time. Construct the function of Lyapunov as follows:

$$V_c(t) = \frac{1}{2\epsilon_c}\tilde{\rho}_c^2 + \frac{1}{2}S_c^T(t)S_c(t), \tag{67}$$

$\tilde{\rho}_c = \rho_c - \hat{\rho}_c$.

Referring to equation (58), the derivative of (67) is given by the following equation:

$$\begin{aligned}\dot{V}_c(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j(\zeta_t)\{-\rho_c - K_j\|x(t)\| - \eta_c(t)\}\|S_c(t)\| - \|S_c(t)\|\tilde{\rho}_c \\ &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j(\zeta_t)\{(-K_j)\|x(t)\| - \rho_c - \varepsilon_c\}\|S_c(t)\|.\end{aligned} \tag{68}$$

Define Ω_c as

$$\Omega_c := \{x\colon \|x(t)\| \leq \kappa_c\}. \tag{69}$$

If the sliding surface S is reached, the condition of reachability is satisfied. Then, ϱ_c is select to fulfill $\varrho_c > \kappa_c(C_i A_i - K_j)$ as

$$-\|S_c(t)\|\varepsilon_c \geq S_c^T(t)S_c(t). \tag{70}$$

An perfect sliding motion is guaranteed to occur in finite time by the suggested SMC with adaptive law, $\forall t \geq t_c\, S_c(t) = \dot{S}_c(t) = 0$. When the SM is required, we examine the stability. We suppose that $u_{eq}(t)$ such that $\dot{S}_c(t) = 0$, as

$$u_{eq}(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{-[\Gamma(x,t) + A_i x(t)] + u_l(t) - D_i \xi(x,t)\}. \tag{71}$$

The closed-loop dynamic system with equation (71) is

$$\dot{x}(t) = \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i \mu_j(\zeta_t)\left\{\left((-B_i K_j) + (\theta_i A_i)\right)x(t) + \overline{B}_{i,j}\phi(t) + \theta_i \Gamma(x,t)\right\}, \tag{72}$$

$$y_c(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\{y(t)\}, \tag{73}$$

$$y_L(t) = \sum_{i=1}^{k} \mu_i(\zeta_t)\left\{C_{(L,i)} x(t)\right\}, \tag{74}$$

where $\theta_i = I_n, \overline{B}_{i,j} = [B_i K_j M_i I_n \theta_i D_i]$ and $\phi(t) = \begin{bmatrix} e^T(t) \\ e_{f_a}^T(t) \\ e_F^T(t) \\ \xi^T(x,t) \end{bmatrix}$.

Assumption 8. The condition of Lipschitz is satisfied by

$$\|\Gamma(x,t)\| \le \|x(t)\|, \tag{75}$$

γ denotes the constant of Lipschitz.

Theorem 9. *The T-S fuzzy system with local nonlinear models (72)–(74) is stable robustly with a maximization of the Lipschitz constant γ, positive scalars $\lambda_c, \alpha_c, \delta_c$, and the minimization of attenuation level ς_c, if there exist the matrices $Y = Y^T, Q_j$ and $\overline{P}_x = \overline{P}_x^T > 0$ where the optimization problem of multiobjective linear matrix inequality has a resolution.*

Minimize $[(1-\lambda_c)\varsigma_c + \lambda_c(\delta_c + \alpha_c)]$, subject to

$$\Delta_{\text{control},ij} := \begin{bmatrix} \Upsilon i,j & B_i Q_j & M_i & \theta_i D_i & \overline{P}_x C_{L,i}^T & \overline{P}_x & I_n \\ (*) & -2Y + \varsigma_c I_n & 0 & 0 & 0 & 0 & 0 \\ (*) & (*) & -\varsigma_c I_q & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & -\varsigma_c I_l & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & -I_{p1} & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & -\delta_c I_n & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & -\alpha_c I_n \end{bmatrix} < 0, \tag{76}$$

where

$$\begin{aligned} \Upsilon i,j &= \theta_i A_i \overline{P}_x + \overline{P}_x A_i^T \theta_i^T - Q_j^T B_i^T - B_i Q_j, \\ Y &= \mu_c \overline{P}_x. \end{aligned} \tag{77}$$

According to the results, it is possible to obtain the adaptive SMC from $K_j = Q_j \overline{P}_x^{-1}$.

Proof. (see Appendix B)

A design method for adaptive FTC with an adaptive SMO is outlined as follows:

(i) Step 1: select "$0 \le \lambda_c \le 1$", resolve the LMI optimization problem (76); so we get the matrices Q_j and the scalars δ_c, ς_c and γ_c

(ii) Step 2: calculate $K_j = Q_j \overline{P}_x^{-1}$

(ii) Step 3: concept the Adaptive SMC (65), then the stability of (51), (57) is accomplished. □

6. Case Study

The sliding mode FTC design according to the sliding mode observer is accomplished by considering the inverted pendulum and cart system. The objective is to conceive an adaptive stabilization controller that the considered inverted pendulum benchmark system [36, 37] consists of a moveable carriage having one degree (see Figure 3). The carriage is freely rotatable in driving direction on which a pendulum is mounted and actuated by a motor.

6.1. The Nonlinear System Modelling. We start by examining the model inverted pendulum and cart system

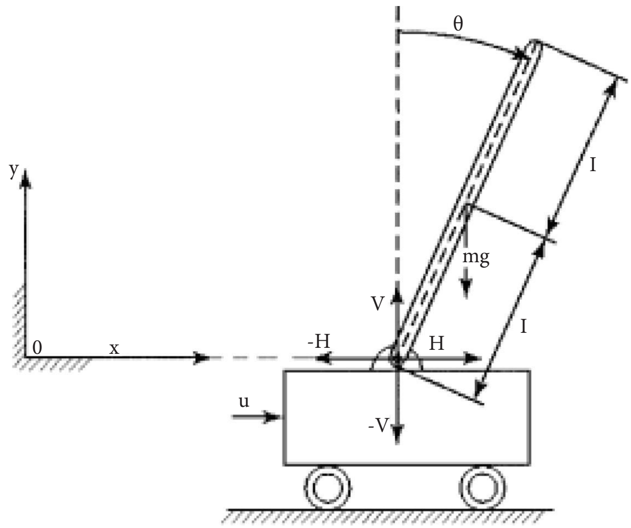

FIGURE 3: Inverted pendulum.

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \dfrac{g\sin(x_1(t)) - m\,l\,a\,x_2^2(t)\,(\sin(2x_1(t))/2) - ba\cos(x_1(t))\,x_4(t)}{(4l/3) - m\,l\,a\cos(x_1(t))^2}, \\ -\dfrac{a\cos(x_1(t))(\beta u(t) - f_c)}{(4l/3) - m\,l\,a\cos(x_1(t))^2}, \\ \dot{x}_3(t) = x_4(t), \\ \dot{x}_4(t) = \dfrac{-m\,g\,a(\sin(2x_1(t))/2) + (4\,m\,l\,a/3)\,x_2^2(t)\sin(x_1(t)) - b\,a\,x_4(t)}{(4/3) - m\,a\cos(x_1(t))^2}, \\ -\dfrac{(4a/3)(u(t) - f_c)}{(4/3) - m\,a\cos(x_1(t))^2}, \end{cases} \tag{78}$$

where β is the efficiency factor. $x_4(t), x_3(t), x_2(t)$, and $x_1(t)$ represent, respectively, speed of cart, angular velocity of pendulum, cart position, and angle position of pendulum. Let us consider $f_c = \text{csign}(x_4(t))$ and $a = 1/(M+m)$.

The parameters of the system are shown in Table 1.

The approximation of the nonlinear faulty system may be obtained by T-S fuzzy system. We use in this study the models with local nonlinearity as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{k} \mu_i(\zeta_t)\{A_i x(t) + B_i u(t) + F(u,t) + M_i f_a(t) + D_i \xi(x,t) + \Gamma(x,t)\}, \\ y(t) &= \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_i x(t)\}, \\ y_L(t) &= \sum_{i=1}^{k} \mu_i(\zeta_t)\{C_{(L,i)} x(t)\}. \end{aligned} \tag{79}$$

Table 1: System parameters.

Symbol	Description	Value	Unit
m	Pendulum point mass	0.2	kg
M	Cart mass	0.8	kg
l	From mass point to joint distance	0.5	m
g	Gravitational constant	9.81	ms^{-2}
L	Rail length	2	m

In this case, $\Gamma(x,t) = x_2^2(t)\mathrm{lmsin}(x_1(t)) - f_c$, the fuzzy weights, are defined by the following equations:

$$\mu_1(t) = \frac{1 - (1/1 + \exp(-14(x_1(t) - (\Pi/8))))}{1 + \exp(-14(x_1(t) - (\Pi/8)))}, \quad \mu_2(t) = 1 - \mu_1(t). \tag{80}$$

We assume that the actuator faults as well as the control input are in the same direction $B_i = M_i$. The matrices of the local nonlinear models are

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 17.3118 & 0 & 0 & 0.0106 \\ 0 & 0 & 0 & 1 \\ -1.7312 & 0 & 0 & -0.0053 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -0.1765 \\ 0 \\ 0.1176 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 14.3223 & 0 & 0 & 0.0069 \\ 0 & 0 & 0 & 1 \\ -1.0127 & 0 & 0 & -0.049 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -0.1147 \\ 0 \\ 0.1081 \end{bmatrix},$$
$$D_1 = D_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad C_{L1} = C_{L2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{81}$$

6.2. Adaptive Sliding Mode Observer and Controller Design. We will design an adaptive SMC based on the requirements of the observer information to stabilize the system.

6.3. SMO Design. The parameters are $\widetilde{A}_{22}^s = \mathrm{diag}(-3, -5, -7), \lambda = 0.5, H_1 = I_{1\times1}, H_2 = 10 * I_{3\times3}$.

We obtain using Theorem 6

$$\begin{bmatrix} \varepsilon^* & \alpha^* & \widetilde{\gamma}^* \end{bmatrix} = \begin{bmatrix} 1.4913 & 09233 & 0.8522 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 1.8794 & 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 2.0548 & 0 & 0 \end{bmatrix}, \quad P_1 = 3.6952, \quad P_2 = \begin{bmatrix} 1.4048 & 0 & -0.23 \\ 0 & 5.3765 & 0 \\ -0.2300 & 0 & 2.6683 \end{bmatrix}, \tag{82}$$

The adaptive sliding mode observer (33) and (34) design is as follows:

$$G_{l,1} = \begin{bmatrix} 31.8794 & 0 & -1.5 \\ 73.6923 & -0.0106 & -104.9894 \\ 0 & 49 & 1 \\ -1.7318 & -49.9947 & 69.9947 \end{bmatrix}, \quad G_{l,2} = \begin{bmatrix} 3.5548 & 0 & -1.0607 \\ 17.4045 & -0.0069 & -3.7054 \\ 0 & 1.5 & 1 \\ -1.0127 & -2.4951 & 3.4951 \end{bmatrix}, \quad G_{n,1} = \begin{bmatrix} 1 & 0 & 0 \\ 1.8794 & 0 & -1.5 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \quad G_{n,2} = \begin{bmatrix} 1 & 0 & 0 \\ 2.548 & 0 & -1.0607 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}. \tag{83}$$

6.4. SMC Design. By using Theorem 9 for $\lambda_c = 0.95$, we find that

$$\varsigma_c^* = 0.3465, \quad \delta_c^* = 6.0749, \quad \alpha^* = 17.5312, \quad \gamma^* = 0.0969, \quad P_x = \begin{bmatrix} 0.4532 & -1.3951 & 0.2459 & -0.6225 \\ -1.3951 & 7.1217 & -0.0516 & -1.6025 \\ 0.2459 & -0.0516 & 5.6132 & -4.8928 \\ -0.6225 & -1.6025 & -4.8928 & 8.0061 \end{bmatrix}. \tag{84}$$

The sliding mode controller gains are expressed by the following equation:

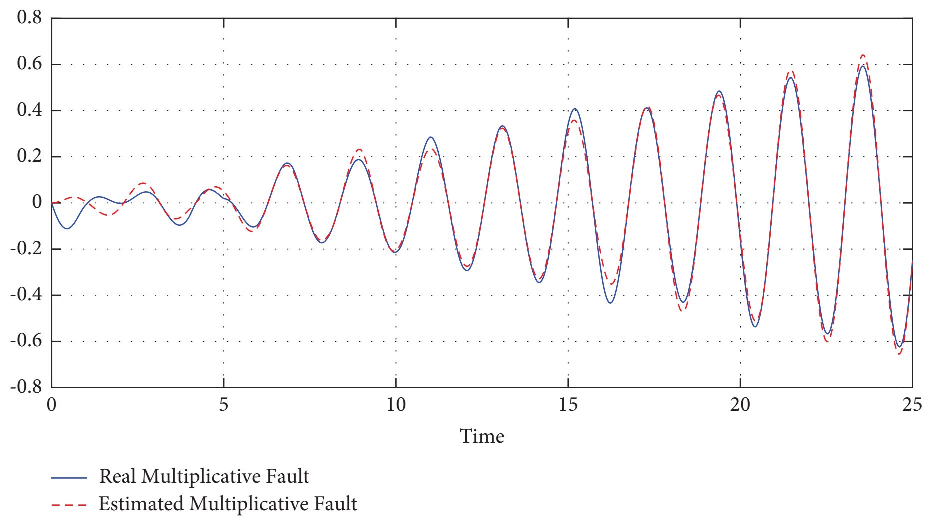

FIGURE 4: Multiplicative actuator FE.

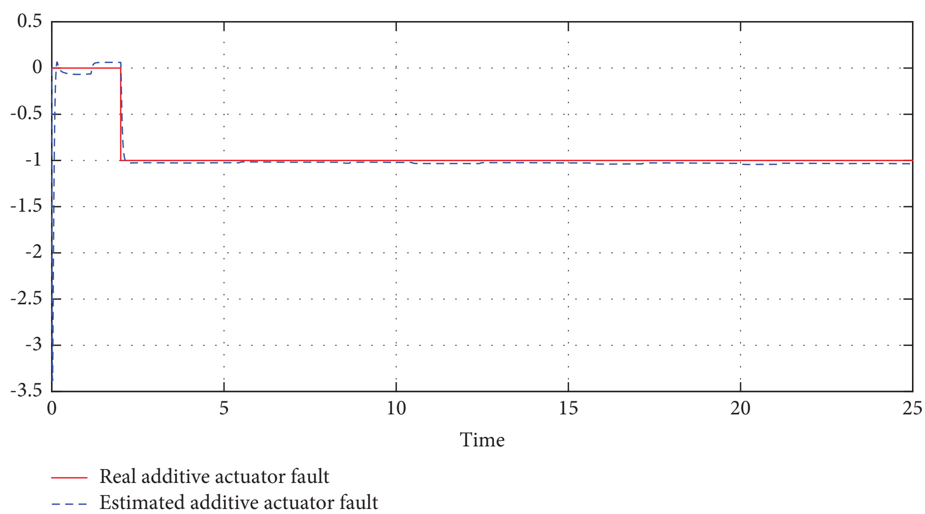

FIGURE 5: Additive actuator FE.

$$K_1 = K_2 = [\,-1898 \quad -486.4 \quad -238.3 \quad -370\,]. \tag{85}$$

6.5. Simulation Results. It is worth mentioning that the results are obtained using online simultaneous additive and multiplicative faults estimation.

Figure 4 illustrates multiplicative actuator fault estimation. In our example, we consider an additive linear time varying actuator fault. The developed adaptive sliding mode observer (33) and (34) can reject the effects of system uncertainties and make an estimation of actuator fault with satisfactory accuracy. Meanwhile, at $t = 2$, the additive actuator fault (Figure 5) has been introduced, in order to demonstrate the capacity of the developed estimation method to additive faults.

As shown in Figure 5, it is worth noting that in spite of the existence of uncertainties, the SMO can still track the additive faults $f_a(t)$. Hence, the simulation results outline that the suggested fault estimation with the adaptive law for the inverted pendulum and cart system described by T-S fuzzy representation with local nonlinear models accomplishes the first objective of this article (actuator faults estimation) with an excellent performance in terms of robustness and precision despite the presence of the uncertainties.

Figures 6–8 compare nonlinear output referring to three cases: output without faults, output without sliding mode FTC, and output responses with the conceived FTC design.

As can be seen from the comparison with the output responses without faults, the conceived adaptive sliding mode controller (63) is capable of the stabilization of the system.

Zoomed versions of the nonlinear inverted pendulum and cart system output responses illustrated in Figures 6–8 highlight a good satisfactory precision of the proposed adaptive FTC, which ensures the stability of the system subject to both additive and multiplicative faults. More precisely, it can be seen clearly that the integrated adaptive law is an effective way to improve both faults estimation and compensation.

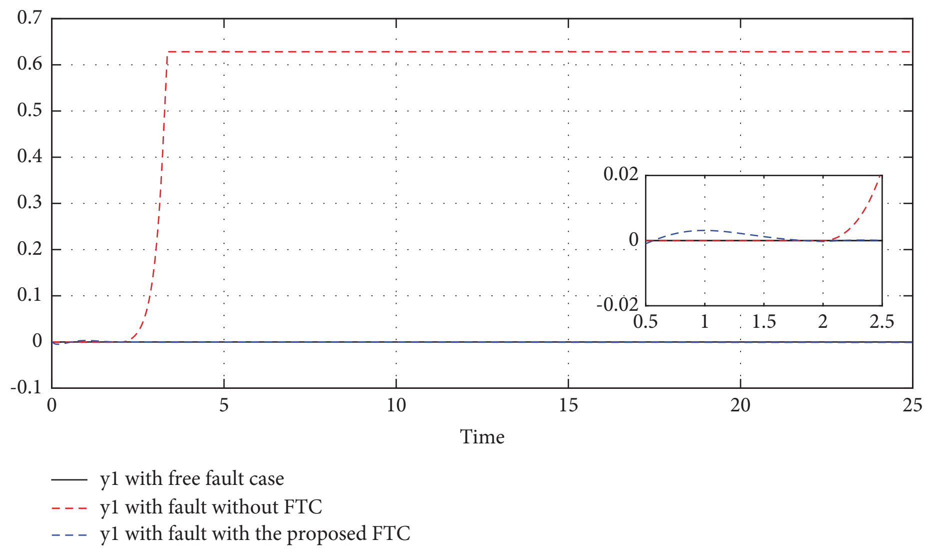

Figure 6: y_1 (black line) with free faults, y_1 (red line) without FTC, and y_1 (blue line) with FTC.

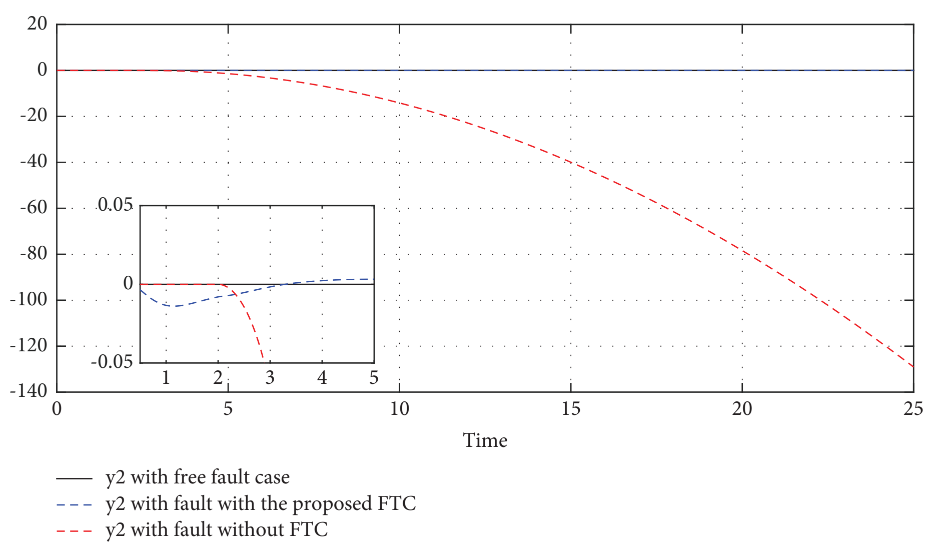

Figure 7: y_2 (black line) with free faults, y_2 (red line) without FTC, and y_2 (blue line) with FTC.

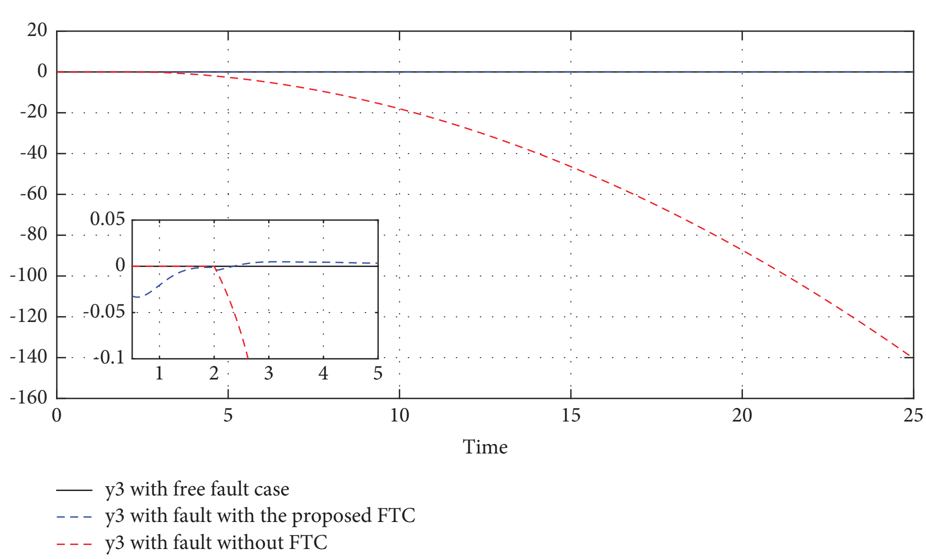

Figure 8: y_3 (black line) with free faults, y_3 (red line) without FTC, and y_3 (blue line) with FTC.

7. Conclusions

This paper addresses two powerful problems: the SMO scheme and the FE-based sliding mode FTC, concerning system described by T-S fuzzy representation and local nonlinear models. We start by the study of the multiplicative faults, and the actuator degradation process is considered as a source of performance deterioration. The degradation process in the actuator is modeled by Wiener process. The additive actuator faults are estimated using robust adaptive SMO in the existence of uncertainties so that the proposed observer's stability has been derived using H_∞ performances in order to minimize the uncertainties' effect on the dynamics of estimation error and solved within the linear matrix inequalities (LMI's) optimization design. To compensate the effects of both additive and multiplicative actuator faults and guarantee the system stability, an adaptive FTC with sliding mode control is studied. The existence conditions are expressed via Lyapunov approach in terms of LMI. Convex multiobjective optimization is employed to simultaneously maximize the nonlinear term Lipschitz constant in the T-S fuzzy modeling in addition to the uncertainties attenuation level, in order to obtain satisfactory gain of the adaptive SMO and controller. The results of the simulation on the application clearly display the efficiency of the proposed additive actuator FE and the adaptive FTC design with uncertainties in system. In our future work, we can use shock models (Poisson process) or combined models to characterize the stochastic deterioration process in an actuator. Indeed, the actuator is a device that operates dynamically in a random environment; its capacity may decrease in an accelerated way. So, we can integrate covariates in the degradation model with accelerated mode deterioration. The derivative of the output in the designer of observers could be taken into account to increase the performance of the estimation.

Appendix

Construct Lyapunov function as follows:

$$V(t) = V_{e_1}(t) + V_{e_y}(t), \tag{A.1}$$

$V_{e_y}(t) = (1/\beta)\tilde{\rho}^2 + e_y^T(t)P_e, V_{e_1}(t) = \tilde{e}_1^T(t)P_1\tilde{e}_1(t), P_1 \in \mathbb{R}^{(n_p)\times(n_p)}$. Taking $\dot{V}(t)$ along a trajectory of the taken system error,

$$\begin{aligned}\dot{V}_{e_1}(t) = \sum_{i=1}^{k}\mu_i(\zeta_t)\Big\{&\tilde{e}_1^T(t)\Big(\tilde{A}_{11,i}^T P_1 + P_1\tilde{A}_{11,i}\Big)\tilde{e}_1(t) + 2\tilde{e}_1^T(t)P_1\tilde{D}_{(1,i)}\xi(x,t)\\ &+ 2\tilde{e}_1^T(t)P_1 T_{L_1,i}\tilde{\Gamma}_1(x,t) + 2\tilde{e}_1^T(t)P_1 T_{L_1,i}e_{F_{a1}}(t)\Big\},\end{aligned} \tag{A.2}$$

$$\begin{aligned}\dot{V}_{e_y}(t) = \sum_{i=1}^{k}\mu_i(\zeta_t)\Big\{&e_y^T(t)\Big(\tilde{A}_{22}^{sT}P_2 + P_2\tilde{A}_{22}^{s}\Big)e_y(t) + 2e_y^T(t)P_2\tilde{A}_{21,i}\tilde{e}_1(t)\\ &+ 2e_y^T(t)P_2\tilde{D}_{2,i}\xi(x,t) + 2e_y^T(t)P_2 T_{L_2,i}\tilde{\Gamma}_2(x,t) + 2e_y^T(t)P_2 T_{L_2,i}e_{F_{a2}}(t)\\ &+ 2e_y^T(t)P_2\big[\tilde{M}_{2,i}f_a(t) - v(t)\big] - \frac{1}{\beta}\tilde{\rho}\dot{\hat{\rho}}\Big\},\end{aligned} \tag{A.3}$$

where $\tilde{\rho} = \rho - \hat{\rho}$.

By applying Lemma 5 and Assumption 4, it could be proving that

$$\begin{aligned}&2\tilde{e}_1^T(t)P_1 T_{L1,i}\tilde{\Gamma}_1(x,t) \le \varepsilon\tilde{\gamma}^2\|\tilde{e}_1(t)\|^2 + \frac{1}{\varepsilon}\tilde{e}_1^T(t)P_1^2\tilde{e}_1(t),\\ &2\tilde{e}_y^T(t)P_2 T_{L_2,i}\tilde{\Gamma}_2(x,t) \le \varepsilon\tilde{\gamma}^2\|e_y(t)\|^2 + \frac{1}{\varepsilon}e_y^T(t)P_2^2\tilde{e}_y(t),\end{aligned} \tag{A.4}$$

$\tilde{\gamma} = \gamma\|T_{L1,i}\|$, where γ is the constant of Lipschitz of $\Gamma(x,t)$

From the expression of $v(t)$ in equation (34), we deduce that

$$\begin{aligned} e_y^T(t)P_2M_{2,i}(-v(t)+f_a(t)) - \frac{1}{\beta}\tilde{\rho}\dot{\tilde{\rho}} &\le e_y^T(t)M_{2,i}^TP_2\|f_a(t)\| - \eta(t)e_y^T(t)P_2\frac{P_e}{\|P_e\|} - \tilde{\rho}\|P_e\| \\ &\le e_y^T(t)M_{2,i}^TP_2\|f_a(t)\| - (\hat{\rho}+\varrho)\|P_e\| - \tilde{\rho}\|P_e\| \\ &\le \kappa_{i,\max}\|P_e\| - \varrho\|P_e\| \\ &< 0, \end{aligned} \tag{A.5}$$

where $\rho\|M_{2,i}\|_{\max} = \kappa_{i,\max}, \rho \ge \kappa_{i,\max}$.

For the multiplicative fault, we can write the inequality as follows:

$$\begin{aligned} 2\tilde{e}_1^T(t)\left[P_1T_{L1,i}\right]e_{F_{a1}}(t) &\le \tilde{e}_1^T(t)P_1\tilde{e}_1(t) + \delta_{F_1}, \\ 2e_y^T(t)\left[P_2T_{L2,i}\right]e_{F_{a2}}(t) &\le e_y^T(t)P_2e_y(t) + \delta_{F_2}, \end{aligned} \tag{A.6}$$

where $\delta_{F_2} = \|e_{F_{a_2}}(t)\|^2\lambda_{\max}(X_2^TP_2^{-1}X_2)$ and $\delta_{F_1} = \|e_{F_{a_1}}(t)\|^2\lambda_{\max}(X_1^TP_1^{-1}X_1)$.

The time derivative of $V_e(t)$ and $V_{e_y}(t)$

$$\begin{aligned} \dot{V}_e(t) \le \sum_{i=1}^{k}\mu_i(\zeta_t)\Big\{&\tilde{e}_1^T(t)\Big[\tilde{A}_{11,i}^TP_1 + P_1\tilde{A}_{11,i} + \frac{1}{\varepsilon}P_1^2 + \varepsilon\tilde{\gamma}^2I_{n-p} + P_1\Big]\tilde{e}_1(t) \\ &+ 2\tilde{e}_1^T(t)P_1\tilde{D}_{1,i}\xi(x,t)\Big\}, \end{aligned} \tag{A.7}$$

and

$$\begin{aligned} \dot{V}_{e_y}(t) \le \sum_{i=1}^{k}\mu_i(\zeta_t)e_y^T(t)&\Big[P_2\tilde{A}_{22}^s + \tilde{A}_{22}^{sT}P_2 + \frac{1}{\varepsilon}P_2^2 + \varepsilon\tilde{\gamma}^2I_p + P_2\Big]e_y(t) \\ &+ 2e_y^T(t)P_2\tilde{D}_{2,i}\xi(x,t) + 2e_y^T(t)P_2\tilde{A}_{21,i}\tilde{e}_1(t)\Big\}. \end{aligned} \tag{A.8}$$

To obtain the robustness of the proposed adaptive SMO (33)-(34), let $V_0(t)$ be defined as

$$V_0(t) := \dot{V}(t) - \varsigma\xi^T(x,t)\xi(x,t) + r^T(t)r(t). \tag{A.9}$$

We define the following variable α as

$$\alpha := \frac{1}{\varepsilon\tilde{\gamma}^2} \longrightarrow \tilde{\gamma} = \frac{1}{\sqrt{\alpha\varepsilon}}. \tag{A.10}$$

To maximize $\tilde{\gamma}$ for $\Gamma(x,t)$, we simultaneously minimize both ε and α. Referring to equation (A.10), we shall write the abovementioned expression (A.9) as follows:

$$V_0(t) \le \sum_{i=1}^{k}\mu_i(\zeta_t)\left\{\begin{bmatrix}\tilde{e}_1(t)\\ e_y(t)\\ \xi(x,t)\end{bmatrix}\Delta_{\mathrm{est},i}\begin{bmatrix}\tilde{e}_1(t)\\ e_y(t)\\ \xi(x,t)\end{bmatrix}\right\}, \tag{A.11}$$

where

$$\begin{aligned} \Delta_{est,i} &= \begin{bmatrix}\Pi_{1,i} & \tilde{A}_{21,i}^TP_2 & P_1\tilde{D}_{1,i}\\ P_2\tilde{A}_{21,i} & \Pi_{2,i} & P_2\tilde{D}_{2,i}\\ \tilde{D}_{1,i}^TP_1 & \tilde{D}_{2,i}P_2 & -\varsigma I_l\end{bmatrix}, \\ \Pi_{1,i} &= \tilde{A}_{11,i}^TP_1 + P_1\tilde{A}_{11,i} + \frac{1}{\varepsilon}P_1^2 + \alpha^{-1}I_{n-p} + P_{F_1}, \\ \Pi_{2,i} &= \tilde{A}_{22}^{sT}P_2 + P_2\tilde{A}_{22}^s + \frac{1}{\varepsilon}P_2^2 + \alpha^{-1}I_p + P_{F_2}. \end{aligned} \tag{A.12}$$

Thus, we conclude that $V_0(t)$ is negative, if

$$\sum_{i=1}^{k} \mu_i(\zeta_t)\Delta_{\text{est},i} < 0. \tag{A.13}$$

From the Schur complement, we deduce that the relation (A.13) and the relation (49) are equivalent. If equation (49) is satisfied, $V_0(t)$ is negative.

For proving that the error system of equation (45)–(47) is stable asymptotically with ς such that

$$\|r(t)\|_2^2 \le \|\xi(x,t)\|_2^2 \varsigma, \tag{A.14}$$

where ς is the attenuation level.

This completes the proof.

B

To guarantee the stability of system, we should investigate the next Lyapunov function

$$V_x(t) = x^T(t)P_x x(t), \tag{B.1}$$

$P_x \in \mathbb{R}^{n\times n}$ is definite positive symmetric.

According to (72)–(74),

$$\begin{aligned}\dot{V}_x(t) = \sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j(\zeta_t)\Big\{&x^T(t)\Big(\big(-B_iK_j+\theta_iA_i\big)^T P_x + P_x\big(-B_iK_j+\theta_iA_i\big)\Big)x(t)\\ &+2x^T(t)P_x\big[\overline{B}_{i,j}\phi(t)+\theta_i\Gamma(x,t)\big]\Big\}.\end{aligned} \tag{B.2}$$

Using Assumption 8 and Lemma 5, we obtain

$$\begin{aligned}2x^T(t)P_x\theta_i\Gamma(x,t) &\le \delta_c\Gamma^T(x,t)\theta_i^T\theta_i\Gamma(x,t)+\frac{1}{\delta_c}x^T(t)P_xP_xx(t)\\ &\le x^T(t)\left[\delta_c\big(\gamma\|\theta_i\|\big)^2+\frac{1}{\delta_c}P_x^2\right]x(t).\end{aligned} \tag{B.3}$$

Let us define $J(t)$

$$J(t) = y_L^T(t)y_L(t)+\dot{V}_x(t)-\varsigma_c\phi^T(t)\phi(t). \tag{B.4}$$

We write equation (B.4) as follows:

$$\begin{aligned}J(t) = \sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j(\zeta_t)\Big\{&x^T(t)\Big[+P_x\big(-B_iK_j+\theta_iA_i\big)+\big(-B_iK_j+\theta_iA_i\big)^T P_x + C_L^T C_L\\ &+\delta_c\tilde{\gamma}_c^2+\frac{1}{\delta_c}P_x^2\Big]x(t)-\varsigma_c\phi^T(t)\phi(t)+2x^T(t)P_x\overline{B}_{i,j}\phi(t)\Big\},\end{aligned} \tag{B.5}$$

where $(\|\theta_i\|\gamma) = \tilde{\gamma}_c$.

A novel variable is defined as

$$\alpha_c := \frac{1}{\delta_c\tilde{\gamma}_c{}^2} \Longleftrightarrow \tilde{\gamma}_c = \frac{1}{\sqrt{\alpha_c\delta_c}}. \tag{B.6}$$

The simultaneous minimization of α_c and δ_c, i.e., minimization of $\alpha_c+\delta_c$, can maximize $\tilde{\gamma}_c$ according to the nonlinear function of Lipschitz

Therefore, it must be proved that $J(t)$ are negative, if

$$\begin{gathered}\begin{bmatrix}\psi_{i,j} & P_x\overline{B}_{i,j}\\ \overline{B}_{i,j}^T P_x & -I\varsigma_c\end{bmatrix} < 0,\\ \psi_{i,j} = \big(-B_iK_j+\theta_iA_i\big)^T P_x + P_x\big(-B_iK_j+\theta_iA_i\big)+C_{L,i}^T C_{L,i}+\frac{1}{\delta_c}P_x^2+\alpha_c^{-1}I_n.\end{gathered} \tag{B.7}$$

Using the Shur complement, the following relation is easily obtained as follows:

$$\begin{bmatrix} \Theta_{i,j} & P_xB_iK_j & P_xM_i & P_x\theta_iD_i & C_{L,i}^T & P_x & I_n \\ (*) & -\varsigma_cI_n & 0 & 0 & 0 & 0 & 0 \\ (*) & (*) & -\varsigma_cI_q & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & -\varsigma_cI_l & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & -I_{p1} & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & -\delta_cI_n & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & -\alpha_cI_n \end{bmatrix} < 0, \tag{B.8}$$

where $\Theta_{i,j} = P_x\theta_iA_i + \theta_i^TA_i^TP_x - P_xB_iK_j - K_j^TB_i^TP_x$.

The following matrix X will be made as $X = \mathrm{di\,ag}\{(P_x)^{-1}, (P_x)^{-1}, I_q, I_l, I_{p_1}, I_n, I_n\}$ post, pre-multiplying by X^T, X in (B.8), we obtain

$$\begin{bmatrix} \Upsilon i,j & B_iQ_j & M_i & \theta_iD_i & \overline{P}_xC_{L,i}^T & \overline{P}_x & I_n \\ (*) & -\varsigma_c\overline{P}_x\overline{P}_x & 0 & 0 & 0 & 0 & 0 \\ (*) & (*) & -\varsigma_cI_q & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & -\varsigma_cI_l & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & -I_{p1} & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & -\delta_cI_n & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & -\alpha_cI_n \end{bmatrix} < 0 \tag{B.9}$$

where

$$\Upsilon_{i,j} = \theta_iA_i\overline{P}_x + \overline{P}_xA_i^T\theta_i^T - Q_i^TB_i^T - B_iQ_j, P_x^{-1} = \overline{P}_x$$
$$K_jP_x^{-1} = Q_j. \tag{B.10}$$

Using Lemma 5, we obtain

$$-\overline{P}_x\overline{P}_x\varsigma_c \le -2\overline{P}_x\varsigma_c + I_n\varsigma_c. \tag{B.11}$$

We can deduce that (B.11) holds, if

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j(\zeta_t)\Delta_{\mathrm{control},ij} < 0 \tag{B.12}$$

where $\Delta_{\mathrm{control},ij}$ has a similar structure as (76). Moreover, whether (B.12) is fulfilled, then $J(t)$ are negative, the T-S fuzzy system (72)–(74) is robustly stable in regard to γ and ς_c.

The proof is completed.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Additional Points

The authors use Matlab simulink for simulation results, and they will improve it for other publication.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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