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Research Article

Triple Homogeneously Weighted Moving Average Charts for Monitoring Process Dispersion

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Homogeneously weighted moving average (HWMA) charts have recently achieved popularity for monitoring small changes in process parameters (location and/or dispersion). Furthermore, the DHWMA (double HWMA) and THWMA (triple HWMA) are the advanced versions of the HWMA charts. The HWMA chart for the process dispersion is designed to detect only the upward (one-sided) shift (i.e., process deterioration). Employing a two-sided chart for concurrently detecting both process improvement and process deterioration is an important aspect of statistical process monitoring. By taking this point as motivation, one and two-sided THWMA charts (symbolized as the THWMAV) are proposed for monitoring the process dispersion. The Monte Carlo simulations are performed to investigate the performance behavior of the THWMAV charts in terms of certain performance indicators, including ARL, SDRL, EQL, RARL, and PCI. The comparison among the THWMAV versus existing charts (DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV) indicates that the THWMAV charts outperform the existing charts. Finally, a dataset is also analyzed to illustrate the implementation of the THWMAV charts.

1. Introduction

Every manufacturing and production process involves two kinds of variations: one common cause and other special cause variations. The common cause variations are natural and essential parts of any stable process. A process that operates with the common cause variations is known to be statistically IC (in-control). On the other hand, the special cause variations are unnatural and deteriorate the process stability. A process that functions with the special cause variations is called statistically OC (out-of-control). The special cause variations demand special attention for quick detection and removal to get the process back on track (i.e., statistically IC).

Control charts are the most well-known SPM (statistical process monitoring) tools for detecting special cause variations that trigger shifts in the process parameters (location and/or dispersion). Shewhart [1] introduced the basic chart, which is used to monitor the process parameters and decide whether the process is IC or OC. The Shewhart $\overline{X}$ chart is used to monitor the mean level of the process, whereas the Shewhart R, S, and S^2 charts are implemented to detect the changes in the process dispersion. The Shewhart charts are also referred to as memoryless-type control as they only use the current process information; consequently, the Shewhart charts are less sensitive to small changes in the process parameters. The memory-type charts, on the other hand, incorporate both current and previous process information and are more sensitive than the Shewhart charts. The memory-type charts include the CUSUM (cumulative sum) proposed by Page [2], EWMA (exponentially weighted moving average) designed by Roberts [3], MA (moving average) introduced by Roberts [3], and DMA (double moving average) suggested by Khoo and Wong [4] and Alevizakos et al. [5].

The classical EWMA chart has become the most powerful tool for researchers to detect the small shifts that ruin the manufacturing and production process stability. Numerous authors have improved the performance of the classical EWMA chart by integrating different techniques, emphasizing the process mean monitoring (see [6–10]). It is important to note that, in general, many special cause variations can potentially affect the process dispersion (variance or standard deviation). Furthermore, dispersion monitoring is of utmost importance because the mean structure is based on it [11]. Moreover, an increase in process dispersion leads to process deterioration, while a decrease in process dispersion improves process performance [12]. Various authors have investigated the classical EWMA-type charts in the context of process dispersion monitoring. For example, Crowder and Hamilton [13] developed the EWMA chart, which uses the log transformation to the sampling variance S^2 and detects changes in process standard deviation. Following Crowder and Hamilton [13], Shu and Jiang [14] introduced the EWMA chart to monitor the process dispersion, which has a better detection ability than the Crowder and Hamilton [13] chart. In the same way, Huwang et al. [15] designed the EWMA charts for process dispersion, and they demonstrated that their charts uniformly outperformed the Crowder and Hamilton [13] and Shu and Jiang [14] charts. Similarly, Castagliola [16] proposed the bilateral EWMA chart using a three-parameter log transformation to S^2, which efficiently monitors the process dispersion. Likewise, Ali and Haq [17] proposed the GWMA (generally weighted moving) and HEWMA (hybrid EWMA) charts to detect the process dispersion shifts. In addition, Chatterjee et al. [18] used the transformation of Castagliola [16] and introduced the TEWMA (triple EWMA) chart for monitoring the process dispersion.

Abbas [19] introduced the HWMA (homogeneously weighted moving average) chart and showed that the HWMA chart outperformed the classical EWMA chart. Later, Abid et al. [20] proposed the DHWMA (double HWMA) chart for the process mean that outperformed the HWMA chart. Similarly, Riaz et al. [21] proposed the THWMA (triple HWMA) chart for the process mean, which uniformly outperformed the HWMA and DHWMA charts. Other HWMA-based studies for mean process monitoring are offered by Rasheed et al. [22], Rasheed et al. [23], and Zhang et al. [24]. After that, Knoth et al. [25] highlighted a few concerns about the HWMA chart when compared to the EWMA chart in the case of the steady-state comparison scenario. However, Riaz et al. [26] reinvestigated the zero-state as well as the steady-state performances of the HWMA chart for various shift sizes and smoothing parameters. They performed a comprehensive comparative analysis of the run-length profiles of the HWMA and EWMA charts for several values of the design parameters. The results indicate that the concerns of Knoth et al. [25] are not always true, i.e., the HWMA chart outperforms the EWMA chart under a zero state for numerous regions of shifts, and that it can maintain its dominance over the EWMA chart despite varied delays in process shifts.

Recently, Riaz et al. [27] developed the HWMA chart for effective process dispersion monitoring. They showed that the HWMA dispersion chart outperformed the EWMA dispersion charts proposed by Crowder and Hamilton [13], Shu and Jiang [14], and Huwang et al. [15], respectively. It should be noted, the HWMA chart has a significant disadvantage, i.e., it is a one-sided chart and detects only the upward shifts in the process, i.e., process deterioration. However, in practice, both the process deterioration and the process improvement are important, which necessitates the use of a single two-sided chart that monitors the process deterioration and the process improvement. Also, as mentioned earlier, Riaz et al. [21] improved the performance of the HWMA-type charts by introducing the THWMA chart for efficient mean process monitoring. So, there is a clear research gap to further enhance the performance of existing charts by extending the idea of Riaz et al. [21] and designing a two-sided chart for monitoring the upward and downward process dispersion shits at the same time. Capitalizing the aforementioned research gap, this paper aims to present the design of one and two-sided THWMA charts with time-varying control limits, which effectively monitor the process dispersion. These charts are labeled as THWMAV charts. The upper-sided THWMAV chart is formulated that identifies the upward changes (process deterioration), while the two-sided THWMAV chart is developed to monitor both upward and downward changes (process deterioration and process improvement) in the dispersion parameter. The Monte Carlo simulations are performed to compute the approximate run-length indicators, such as ARL (average run length), SDRL (standard deviation run length), EQL (extra quadratic loss), RARL (relative average run length), and PCI (performance comparison index). Based on these run-length indicators, the one and two-sided THWMAV charts are compared to the one and two-sided competing charts denoted as DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts. The comparison demonstrates that the THWMAV charts achieve better performance against the competing charts in monitoring small and large shifts in the process dispersion. Finally, to illustrate the practical implementation of the THWMAV charts, the wind farm data analysis is also provided.

The structure for the rest of the paper is given as follows. Section 2 offers the theory and background of some competing charts for dispersion monitoring. Similarly, the methodologies of the THWMAV charts are specified in Section 3. Likewise, Section 4 presents the design of the THWMAV charts. In addition, Section 5 includes a comprehensive comparative study of the THWMAV charts. Furthermore, the application of the THWMAV chart to the wind farm dataset is given in Section 6. The final section addresses the summary, conclusions, and recommendations of the study.

2. Competing Methods

This section provides the concepts and background of some charts that monitor the process dispersion shifts. Section 2.1 introduces the process variable and transformation. Similarly, Sections 2.2–2.5 present the methodologies and formulation of the one and two-sided EWMAV, DEWMAV, TEWMAV, HWMAV, and DHWMAV charts.

2.1. Process Variable and Transformation. Suppose that $X_{1t}, X_{2t}, \ldots, X_{it}, \ldots, X_{nt}$ are n independent identically normal process random variables, i.e., $X_{it} \sim N(\mu_0, \delta\sigma_0)$, where μ_0 and σ_0 are the IC process mean and dispersion, respectively, $t = 1, 2, \ldots$ is the subgroup number, and δ refers to the amount of shift in process dispersion. As the only concern is to monitor the shifts in process dispersion, the process mean is considered to be zero, i.e., $\mu_0 = 0$. Here, the shifts δ can be defined as $\delta = \sigma/\sigma_0$, where σ is the OC process dispersion. So, for the IC process, $\delta = 1$, and for the OC process with decreasing shifts, $\delta < 1$, while for increasing shifts, $\delta > 1$.

Assuming the sample variance $S_t^2 = 1/n - 1 \sum_{i=1}^{n} (X_{it} - \overline{X}_t)^2$ computed from the t^{th} subgroup having size n, where $\overline{X}_t = 1/n \sum_{i=1}^{n} X_{it}$ is the sample mean, then it is well known that for the IC process, S_t^2 has a chi-square distribution with degrees of freedom equal to $n-1$, i.e., $S_t^2 \sim \sigma_0^2/n - 1\chi^2_{(n-1)}$. As the sample variance S_t^2 has a skewed distribution, it is not an appropriate statistic for the design of certain memory charts. To overcome this limitation, Quesenberry [28] suggested the transformation for the process dispersion, defined as follows:

$$V_t = \Phi^{-1}\left(G\left(\frac{(n-1)S_t^2}{\sigma_0^2}; n-1\right)\right), \tag{1}$$

where $G(\bullet; n-1)$ denotes the CDF (cumulative distribution function) of chi-square distribution at $n-1$ degrees of freedom and $\Phi^{-1}(\bullet)$ denotes the inverse standard normal CDF. If the underlying process is IC, then the statistic V_t follows the standard normal distribution, i.e., $V_t \sim N(0,1)$.

2.2. EWMAV *Chart.* The EWMA chart for the process dispersion, based on the statistic V_t, is proposed by Huwang et al. [15] and denoted by the HHW2 chart. Hereafter this chart is labeled by EWMAV chart. If E_t denotes the plotting statistic of the EWMAV chart, then it can be defined by the relation given as follows:

$$E_t = \lambda V_t + (1-\lambda)E_{t-1}, \tag{2}$$

where λ is the smoothing parameter, satisfying $0 < \lambda \leq 1$. The initial value of the statistic E_t is set to zero, i.e., $E_0 = 0$. The expected value and variance of the statistic E_t are, respectively, given as follows:

$$E(E_t) = 0, \ \operatorname{Var}(E_t) = \frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2t}\right\}. \tag{3}$$

If UCL (upper control limit) and LCL (lower control limit) denote the upper and lower control limits that monitor the two-sided shifts in process dispersion, then its time-varying form for the EWMAV chart is given as follows:

$$\left.\begin{aligned} \mathrm{UCL}_{t(\mathrm{EWMAV})} &= L_{\mathrm{EWMAV}}\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2t}\right\}}, \\ \mathrm{LCL}_{t(\mathrm{EWMAV})} &= -L_{\mathrm{EWMAV}}\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2t}\right\}}, \end{aligned}\right\} \tag{4}$$

where $L_{\mathrm{EWMAV}} > 0$ is the width coefficient of the EWMAV chart, and it is determined so that the ARL_0 (in-control ARL) is equal to its desired value. The two-sided EWMAV chart is constructed by plotting the statistic E_t against the subgroup number t and the process is declared to be OC if $E_t \geq \mathrm{UCL}_{t(\mathrm{EWMAV})}$ or $E_t \leq \mathrm{LCL}_{t(\mathrm{EWMAV})}$; otherwise, the process is considered to be IC.

Similarly, to detect the upward changes in the process dispersion with the EWMAV chart, only the upper limit $\mathrm{UCL}_{t(\mathrm{EWMAV})}$ is considered, i.e.,

$$\mathrm{UCL}_{t(\mathrm{EWMAV})} = L_{\mathrm{EWMAV}}\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2t}\right\}}. \tag{5}$$

The statistic E_t is plotted against the subgroup number t and the upper-sided EWMAV chart will detect the process to be OC if $E_t \geq \mathrm{UCL}_{t(\mathrm{EWMAV})}$; otherwise, the process remains IC.

2.3. DEWMAV *Chart.* The DEWMA chart for process dispersion (hereafter labeled by DEWMAV chart) is based on the statistic V_t and can be designed by combining the features of the two EWMA charts. If the plotting statistic for the DEWMAV chart is denoted by DE_t, then it can be defined as follows:

$$\mathrm{DE}_t = \lambda E_t + (1-\lambda)\mathrm{DE}_{t-1}. \tag{6}$$

The starting value of DE_t is equal to zero, i.e., $\mathrm{DE}_0 = 0$. The mean and variance of DE_t for the IC process are, respectively, given as follows:

$$E(\mathrm{DE}_t) = 0, \ \operatorname{Var}(\mathrm{DE}_t) = \frac{\lambda^4\left\{1+(1-\lambda)^2-(t+1)^2(1-\lambda)^{2t}+\left(2t^2+t-1\right)(1-\lambda)^{2t+2}-t^2(1-\lambda)^{2t+4}\right\}}{\left\{1-(1-\lambda)^2\right\}^3}. \tag{7}$$

Based on $E(\mathrm{DE}_t)$ and $\mathrm{Var}(\mathrm{DE}_t)$ in equation (7), the control limits, $\mathrm{UCL}_{t(\mathrm{DEWMAV})}$ and $\mathrm{LCL}_{t(\mathrm{DEWMAV})}$, of the DEWMAV chart are defined as follows:

$$\left.\begin{aligned}\mathrm{UCL}_{t(\mathrm{DEWMAV})} &= L_{\mathrm{DEWMAV}}\sqrt{\frac{\lambda^4\left\{1+(1-\lambda)^2-(t+1)^2(1-\lambda)^{2t}+\left(2t^2+t-1\right)(1-\lambda)^{2t+2}-t^2(1-\lambda)^{2t+4}\right\}}{\left\{1-(1-\lambda)^2\right\}^3}},\\ \mathrm{LCL}_{t(\mathrm{DEWMAV})} &= -L_{\mathrm{DEWMAV}}\sqrt{\frac{\lambda^4\left\{1+(1-\lambda)^2-(t+1)^2(1-\lambda)^{2t}+\left(2t^2+t-1\right)(1-\lambda)^{2t+2}-t^2(1-\lambda)^{2t+4}\right\}}{\left\{1-(1-\lambda)^2\right\}^3}},\end{aligned}\right\} \quad (8)$$

where L_{DEWMAV} is the DEWMAV chart width coefficient. To diagnose the upward or downward changes in the process dispersion, DE_t is plotted against the subgroup number t and if $\mathrm{DE}_t \geq \mathrm{UCL}_{t(\mathrm{DEWMAV})}$ or $\mathrm{DE}_t \leq \mathrm{LCL}_{t(\mathrm{DEWMAV})}$, then the underlying process is considered to be OC; otherwise, it is considered to be IC.

Similarly, the control limit $\mathrm{UCL}_{t(\mathrm{DEWMAV})}$ for the upper-sided DEWMAV chart is given as follows:

$$\mathrm{UCL}_{t(\mathrm{DEWMAV})} = L_{\mathrm{DEWMAV}}\sqrt{\frac{\lambda^4\left\{1+(1-\lambda)^2-(t+1)^2(1-\lambda)^{2t}+\left(2t^2+t-1\right)(1-\lambda)^{2t+2}-t^2(1-\lambda)^{2t+4}\right\}}{\left\{1-(1-\lambda)^2\right\}^3}}. \quad (9)$$

To detect the increasing shifts in process dispersion, DE_t is plotted against the subgroup number t and the process is said to be OC when $\mathrm{DE}_t \geq \mathrm{UCL}_{t(\mathrm{DEWMAV})}$; otherwise, the process is said to be IC.

2.4. TEWMAV Chart. The TEWMA chart based on the three-parameter log transformation is constructed by Chatterjee et al. [18], which effectively monitors the process dispersion. Similarly, the TEWMA chart based on transformation V_t, i.e., TEWMAV chart, can also be formulated to monitor the process dispersion changes. The plotting statistic of the TEWMAV chart, i.e., TE_t, can be defined as follows:

$$\mathrm{TE}_t = \lambda\mathrm{DE}_t + (1-\lambda)\mathrm{TE}_{t-1}. \quad (10)$$

The initial values of TE_t are set as zero, i.e., $\mathrm{TE}_0 = 0$. The mean of TE_t is given as $E(\mathrm{TE}_t) = 0$, whereas its variance is defined as follows:

$$\begin{aligned}\mathrm{Var}(\mathrm{TE}_t) = \Bigg[&\frac{\eta^3\lambda^6}{4}\left\{-\frac{t\left(t^2-1\right)(t-2)\eta^{t-3}}{1-\eta}-\frac{4t\left(t^2-1\right)\eta^{t-2}}{(1-\eta)^2}-\frac{12t(t+1)\eta^{t-1}}{(1-\eta)^3}-\frac{24(t+1)\eta^t}{(1-\eta)^4}+\frac{24\left(1-\eta^{t+1}\right)}{(1-\eta)^5}\right\}\\ &+2\eta^2\lambda^6\left\{-\frac{t\left(t^2-1\right)\eta^{t-2}}{1-\eta}-\frac{3t(t+1)\eta^{t-1}}{(1-\eta)^2}-\frac{6(t+1)\eta^t}{(1-\eta)^3}+\frac{6\left(1-\eta^{t+1}\right)}{(1-\eta)^4}\right\}\\ &+\frac{7\eta\lambda^6}{2}\left\{-\frac{t(t+1)\eta^{t-1}}{1-\eta}-\frac{2(t+1)\eta^t}{(1-\eta)^2}+\frac{2\left(1-\eta^{t+1}\right)}{(1-\eta)^3}\right\}+\lambda^6\left\{\frac{1-\eta^{t+1}}{(1-\eta)^2}-\frac{(t+1)\eta^t}{1-\eta}\right\}\Bigg],\end{aligned} \quad (11)$$

where $\eta = (1-\lambda)^2$. The limits $\mathrm{UCL}_{t(\mathrm{TEWMAV})}$ and $\mathrm{LCL}_{t(\mathrm{TEWMAV})}$ of the two-sided TEWMAV chart can be defined as follows:

$$\left.\begin{aligned}\mathrm{UCL}_{t(\mathrm{TEWMAV})} &= L_{\mathrm{TEWMAV}}\sqrt{\mathrm{Var}(\mathrm{TE}_t)},\\ \mathrm{LCL}_{t(\mathrm{TEWMAV})} &= -L_{\mathrm{TEWMAV}}\sqrt{\mathrm{Var}(\mathrm{TE}_t)},\end{aligned}\right\} \quad (12)$$

where L_{TEWMAV} denotes the width of the TEWMAV chart. To detect the increasing or decreasing changes in the dispersion parameter, the two-sided TEWMAV chart is constructed, and the statistic TE_t is plotted against the subgroup number t. If the statistic TE_t falls outsides the control limits, i.e., $\mathrm{TE}_t \geq \mathrm{UCL}_{t(\mathrm{TEWMAV})}$ or $\mathrm{TE}_t \leq \mathrm{LCL}_{t(\mathrm{TEWMAV})}$, then the process is deemed to be OC; otherwise, it is IC. On the other hand, to detect the upward changes in the process, however, only $\mathrm{UCL}_{t\,(\mathrm{TEWMAV})}$ in equation (12) is used. In this case, the process is said to be OC when $\mathrm{TE}_t \geq \mathrm{UCL}_{t\,(\mathrm{TEWMAV})}$; otherwise, the process is said to be IC.

2.5. HWMAV *Chart.* The HWMA chart, based on the statistic V_t, can be used to detect the process dispersion shifts. This chart is referred to as the HWMAV chart. If H_t denotes the plotting statistic of the HWMAV chart, then it can be given as follows:

$$H_t = \lambda V_t + (1-\lambda)\overline{V}_{t-1}, \tag{13}$$

where $\overline{V}_{t-1} = \sum_{i=1}^{t-1} V_i/t-1$ is the mean of previous $t-1$ observations and λ $(0 < \lambda \leq 1)$ is the smoothing constant. The initial values H_0 and $\overline{V}_0$ are zero, i.e., $H_0 = \overline{V}_0 = 0$. The IC mean and variance of H_t, from Appendix A.1, are given as follows:

$$E(H_t) = 0, \mathrm{Var}(H_t) = \lambda^2, \text{ for } t = 1, \mathrm{Var}(H_t) = \lambda^2 + \frac{(1-\lambda)^2}{t-1}, \text{ for } t > 1. \tag{14}$$

The two-sided time-varying control limits, $\mathrm{UCL}_{t\,(\mathrm{HWMAV})}$ and $\mathrm{LCL}_{t\,(\mathrm{HWMAV})}$, for the HWMAV chart are given as follows:

$$\left.\begin{array}{l} \mathrm{UCL}_{t\,(\mathrm{HWMAV})} = \begin{cases} L_{\mathrm{HWMAV}}\,\lambda, & \text{for } t = 1 \\ L_{\mathrm{HWMAV}}\sqrt{\lambda^2 + \dfrac{(1-\lambda)^2}{t-1}}, & \text{for } t > 1 \end{cases} \\ \mathrm{LCL}_{t\,(\mathrm{HWMAV})} = \begin{cases} -L_{\mathrm{HWMAV}}\,\lambda, & \text{for } t = 1 \\ -L_{\mathrm{HWMAV}}\sqrt{\lambda^2 + \dfrac{(1-\lambda)^2}{t-1}}, & \text{for } t > 1 \end{cases} \end{array}\right\}, \tag{15}$$

where L_{HWMAV} is called the width coefficient and it is chosen to obtain ARL_0 close to prespecified value. To monitor an increase or decrease in process dispersion, the two-sided HWMAV chart is constructed, and H_t is plotted versus the subgroup number t. The underlying process is considered to be OC when $H_t \geq \mathrm{UCL}_{t\,(\mathrm{HWMAV})}$ or $H_t \leq \mathrm{LCL}_{t\,(\mathrm{HWMAV})}$; otherwise, it is considered to be IC. Likewise, based on equation (14), the time-varying upper control limit, $\mathrm{UCL}_{t\,(\mathrm{HWMAV})}$, of the HWMAV chart is given as follows:

$$\mathrm{UCL}_{t\,(\mathrm{HWMAV})} = \begin{cases} L_{\mathrm{HWMAV}}\,\lambda, & \text{if } t = 1, \\ L_{\mathrm{HWMAV}}\sqrt{\lambda^2 + \dfrac{(1-\lambda)^2}{t-1}}, & \text{if } t > 1. \end{cases} \tag{16}$$

To identify the upward changes in the process dispersion, the HWMAV statistic H_t and if $H_t \geq \mathrm{UCL}_{t\,(\mathrm{HWMAV})}$, then the process is considered to be OC; otherwise, the process is regarded to be IC.

2.6. DHWMAV *Chart.* The DHWMA chart is based on V_t, symbolized by DHWMAV chart, can be used to monitor the process dispersion, and detects both the downward and upward shifts. The DHWMAV statistic is DH_t and it can be given as follows:

$$\mathrm{DH}_t = \lambda H_t + (1-\lambda)\overline{V}_{t-1}. \tag{17}$$

It is assumed that $\mathrm{DH}_0 = H_0 = \overline{V}_0 = 0$. The statistic DH_t in equation (17) can be written as follows:

$$DH_t = \lambda^2 V_t + \left(1-\lambda^2\right)\overline{V}_{t-1}. \tag{18}$$

The IC mean and variance of DH_t, from Appendix A.2, are provided as follows:

$$E\left(\mathrm{DH}_t\right)=0, \mathrm{Var}\left(\mathrm{DH}_t\right)=\lambda^4, \text{ for } t=1, \mathrm{Var}\left(\mathrm{DH}_t\right)=\lambda^4+\frac{\left(1-\lambda^2\right)^2}{t-1}, \text{ for } t>1. \tag{19}$$

The control limits, i.e., $\mathrm{UCL}_{t(\mathrm{DHWMAV})}$ and $\mathrm{LCL}_{t(\mathrm{DHWMAV})}$, for the two-sided DHWMAV chart are given as follows:

$$\left.\begin{aligned}\mathrm{UCL}_{t(\mathrm{DHWMAV})}&=\begin{cases}L_{\mathrm{DHWMAV}}\lambda^2, & \text{for } t=1\\ L_{\mathrm{DHWMAV}}\sqrt{\lambda^4+\dfrac{\left(1-\lambda^2\right)^2}{t-1}}, & \text{for } t>1\end{cases}\\ \mathrm{LCL}_{t(\mathrm{DHWMAV})}&=\begin{cases}-L_{\mathrm{DHWMAV}}\lambda^2, & \text{for } t=1\\ -L_{\mathrm{DHWMAV}}\sqrt{\lambda^4+\dfrac{\left(1-\lambda^2\right)^2}{t-1}}, & \text{for } t>1\end{cases}\end{aligned}\right\}, \tag{20}$$

where L_{DHWMAV} denotes the width coefficient of the DHWMAV chart, and its value is chosen so that the ARL_0 is approximately equal to a specified value. To detect an increase or decrease in the process dispersion, the two-sided DHWMAV chart is constructed, and DH_t is plotted against t. The process is stated to be OC when $\mathrm{DH}_t \geq \mathrm{UCL}_{t(\mathrm{DHWMAV})}$ or $\mathrm{DH}_t \leq \mathrm{LCL}_{t(\mathrm{DHWMAV})}$; otherwise, the process is stated to be IC.

Similarly, the control limit, $\mathrm{UCL}_{t(\mathrm{DHWMAV})}$, of the DHWMAV chart is given as follows:

$$\mathrm{UCL}_{t(\mathrm{DHWMAV})}=\begin{cases}L_{\mathrm{DHWMAV}}\lambda^2, & \text{for } t=1,\\ L_{\mathrm{DHWMAV}}\sqrt{\lambda^4+\dfrac{\left(1-\lambda^2\right)^2}{t-1}}, & \text{for } t>1.\end{cases} \tag{21}$$

To diagnose the upward changes, the DHWMAV statistic DH_t is plotted against the subgroup number t and if $\mathrm{DH}_t \geq \mathrm{UCL}_{t(\mathrm{DHWMAV})}$, the process is stated as OC; otherwise, the process is considered IC.

3. THWMAV Chart

This section describes the methodology of the THWMA chart, referred to as the THWMAV chart, that efficiently monitors the process dispersion shifts. To construct the THWMAV chart, the charting statistic for the THWMAV chart is given as follows:

$$\mathrm{TH}_t=\lambda \mathrm{DH}_t+(1-\lambda)\overline{V}_{t-1}, \tag{22}$$

where DH_t is the charting statistic of the DHWMAV chart, specified by equation (17), and λ is a design parameter. Here, $\mathrm{TH}_0=\mathrm{DH}_0=\overline{V}_0=0$. Equation (22), alternatively, can be written as follows:

$$\mathrm{TH}_t=\lambda^3 V_t+\left(1-\lambda^3\right)\overline{V}_{t-1}. \tag{23}$$

The mean and variance of the statistic TH_t, from Appendix A.3, are given as follows:

$$E\left(\mathrm{TH}_t\right)=0, \mathrm{Var}\left(\mathrm{TH}_t\right)=\lambda^6, \text{ for } t=1, \ \mathrm{Var}\left(\mathrm{TH}_t\right)=\lambda^6+\frac{\left(1-\lambda^3\right)^2}{t-1}, \text{ for } t>1. \tag{24}$$

The two-sided time-varying control limits, $\mathrm{UCL}_{t(\mathrm{THWMAV})}$ and $\mathrm{LCL}_{t(\mathrm{THWMAV})}$, of the THWMAV chart, using the defined mean and variance in equation (24), are given as follows:

$$\left.\begin{aligned}\mathrm{UCL}_{t(\mathrm{THWMAV})} &= \begin{cases} L_{\mathrm{THWMAV}}\lambda^3, & \text{for } t=1\\ L_{\mathrm{THWMAV}}\sqrt{\lambda^6+\dfrac{\left(1-\lambda^3\right)^2}{t-1}}, & \text{for } t>1\end{cases}\\ \mathrm{LCL}_{t(\mathrm{THWMAV})} &= \begin{cases} -L_{\mathrm{THWMAV}}\lambda^3, & \text{for } t=1\\ -L_{\mathrm{THWMAV}}\sqrt{\lambda^6+\dfrac{\left(1-\lambda^3\right)^2}{t-1}}, & \text{for } t>1\end{cases}\end{aligned}\right\}, \tag{25}$$

where L_{THWMAV} denotes the width coefficient of the THWMAV chart, and its value is chosen so that ARL_0 is approximately equal to the desired value. For monitoring an increase or a decrease in the process, the two-sided THWMAV chart is constructed, and TH_t is plotted against t. The two-sided THWMAV chart declares the process to be OC when $\mathrm{TH}_t \geq \mathrm{UCL}_{t(\mathrm{THWMAV})}$ or $\mathrm{TH}_t \leq \mathrm{LCL}_{t(\mathrm{THWMAV})}$; otherwise, it declares that the process is IC.

Similarly, the upper-sided time-varying limit, $\mathrm{UCL}_{t(\mathrm{THWMAV})}$, of the THWMAV chart is given as follows:

$$\mathrm{UCL}_{t(\mathrm{THWMAV})} = \begin{cases} L_{\mathrm{THWMAV}}\lambda^3, & \text{for } t=1,\\ L_{\mathrm{THWMAV}}\sqrt{\lambda^6+\dfrac{\left(1-\lambda^3\right)^2}{t-1}}, & \text{for } t>1.\end{cases} \tag{26}$$

To detect the upward shifts in the process dispersion, TH_t is plotted against t. If $\mathrm{DH}_t \geq \mathrm{UCL}_{t(\mathrm{THWMAV})}$, then the process is considered OC; otherwise, it is considered IC.

4. Design and Implementation of THWMAV Charts

This section provides the performance evaluation measures in Section 4.1. Similarly, Section 4.2 offers the overall performance measures. Likewise, Section 4.3 provides the run-length distribution of the THWMAV chart. Also, Section 4.4 explains the IC design of the THWMAV chart. Moreover, the OC performance of the THWMAV chart is also provided in Section 4.4.

4.1. Performance Measures. The ARL and SDRL are the most popular and frequently used measures to evaluate the performance of the chart at a specified shift. ARL is defined as the expected number of sample points plotted on the chart before the chart indicates an OC signal [29]. ARLs are of two types, i.e., ARL_0 (IC ARL) and ARL_1 (OC ARL). The ARL_0 should be high enough to prevent many false alarms if the process operates in an IC state, whereas the ARL_1 should be lower to rapidly identify process shifts [30]. To evaluate the effectiveness of two or more charts, it is necessary to set the common ARL_0 for all of them. For a certain shift, the chart with the lower ARL_1 value is considered sensitive and may identify a shift quicker than the other charts.

4.2. Overall Performance Measures. Other performance indicators that assess the overall performance of a control chart over the entire range of shifts are known as EQL (extra quadratic loss), RARL (relative average run length), and PCI (performance comparison index). The EQL is the weighted ARL over the entire range of shifts, i.e., $\delta_{\min} \leq \delta \leq \delta_{\max}$, under the square of the shift as a weight. Symbolically, EQL can be defined as follows:

$$\mathrm{EQL} = \frac{1}{\delta_{\max}-\delta_{\min}}\int_{\delta_{\min}}^{\delta_{\max}} \delta^2 \mathrm{ARL}(\delta)\mathrm{d}\delta, \tag{27}$$

where $\mathrm{ARL}(\delta)$ is the ARL of a specific chart at a shift δ. The RARL presents the ratio between the ARL of a particular chart (i.e., $\mathrm{ARL}(\delta)$) and the benchmark chart (i.e., $\mathrm{ARL}_{\mathrm{BM}}(\delta)$). Symbolically, the RARL can be defined as follows:

$$\mathrm{RARL} = \frac{1}{\delta_{\max}-\delta_{\min}}\int_{\delta_{\min}}^{\delta_{\max}} \frac{\mathrm{ARL}(\delta)}{\mathrm{ARL}_{\mathrm{BM}}(\delta)}\mathrm{d}\delta. \tag{28}$$

A benchmark chart is often recognized as the chart with the lowest EQL or as one of the existing standard charts. The PCI is the ratio of the EQL of a chart and the EQL of the benchmark chart. Mathematically, it can be defined as follows:

$$\mathrm{PCI} = \frac{\mathrm{EQL}}{\mathrm{EQL}_{\mathrm{BM}}}. \tag{29}$$

The benchmark chart has $\mathrm{PCI} = 1$, and for the remaining charts, $\mathrm{PCI} > 1$ [31].

4.3. Run-Length Distribution of THWMAV *Chart.* To compute the approximate run-length distribution of the THWMAV chart, the Monte Carlo simulation technique is used. A simulation algorithm is developed using the statistical software *R* package, which can be explained in the steps given as follows:

(i) Choose sample size n, smoothing parameter λ, and parametric values, i.e., in the case of IC process $N(\mu_0, \delta\sigma_0)$.

(ii) Generate random observations $(X_{1t}, X_{2t}, \ldots, X_{nt})$, for $t = 1, 2, \ldots 50,000$, from $N(\mu_0, \delta\sigma_0)$.

(iii) Compute the statistic V_t in equation (1).

(iv) Using the statistic V_t, calculate the HWMAV statistic H_t in equation (13).

(v) Using H_t as input, compute the DHWMAV statistic, DH_t, in equation (17).

(vi) Using the statistic DH_t as input, compute the THWMAV statistic, TH_t, in equation (22).

(vii) Select L_{THWMAV} for desired ARL_0 and compute $\mathrm{UCL}_{t(\mathrm{THWMAV})}$ and $\mathrm{LCL}_{t(\mathrm{THWMAV})}$ in equation (25).

(viii) Plot the statistic TH_t versus the subgroup number t. If $\mathrm{TH}_t \geq \mathrm{UCL}_{t(\mathrm{THWMAV})}$ or $\mathrm{TH}_t \leq \mathrm{LCL}_{t(\mathrm{THWMAV})}$, record sequence order, known as run length (RL).

(ix) Repeat steps (ii)–(viii) for $m = 10^6$ times and record RLs and hence compute the approximate ARL by

$$\mathrm{ARL} = \frac{\sum_{t=1}^{m} \mathrm{RL}_t}{m}, \tag{30}$$

and approximate SDRL by

$$\mathrm{SDRL} = \sqrt{\frac{\sum_{t=1}^{m} (\mathrm{RL}_t - \mathrm{ARL})^2}{m-1}}. \tag{31}$$

The above simulation algorithm is used to construct the two-sided (i.e., $\mathrm{TH}_t \geq \mathrm{UCL}_{t(\mathrm{THWMAV})}$ and $\mathrm{TH}_t \leq \mathrm{LCL}_{t(\mathrm{THWMAV})}$) THWMAV chart; however, only one of the two control limits in equation (25) is required to design the one-sided (i.e., $\mathrm{TH}_t \geq \mathrm{UCL}_{t(\mathrm{THWMAV})}$ or $\mathrm{TH}_t \leq \mathrm{LCL}_{t(\mathrm{THWMAV})}$) THWMAV chart. In addition, the ARL and SDRL values of the competing DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts are computed similarly.

4.4. IC Design and OC Performance of THWMAV *Charts.* The IC designs of the THWMAV one and two-sided THWMAV charts are based on smoothing parameters λ and the width coefficient, L_{THWMAV}. The combinations $(\lambda, L_{\mathrm{THWMAV}})$ are chosen such that the ARL_0 is close to the desired value. Also, time-varying control limits are considered for the one and two-sided THWMAV charts. The smoothing parameter values are set as follows: $\lambda = 0.05$, 0.1, 0.2, and 0.3, for each of the n values of 5, 10, and 20, respectively, to execute the simulation study. Each combination of n and λ is then used to compute the values of the THWMAV chart coefficient, i.e., L_{THWMAV}, to make sure that ARL_0 is roughly equal to the desired value. Table 1 lists the L_{THWMAV} values computed for the one and two-sided THWMAV charts to get ARL_0 close to 200, 370, and 500. Furthermore, Table 1 also presents the SDRL_0 (IC SDRL) for one and two-sided THWMAV charts.

To address the OC performance of the one and two-sided THWMAV charts, different combinations $(\lambda, L_{\mathrm{THWMAV}})$ are used for each $n = 5$, 10, 15, as listed in Table 1. As mentioned above, these combinations provide the ARL_0 which is approximately equal to 370. For the IC process, it is assumed that the shift in process dispersion is $\delta = 1$, while for the OC process, $\delta = 1.1$, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, and 0.2 for the upward shifts, and for the downward shifts, $\delta = 0.95$, 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55, and 0.5 are chosen. The ARLs and SDALs (given in the parentheses) values for the one and two-sided THWMAV chart, along with competing charts, are presented in Tables 1–6. For each λ, the smallest ARL_1 values are shown in bold font for each shift. The main findings from these results are as follows:

(i) As λ rises, the values of L_{THWMAV} for the THWMAV charts also increase to achieve desired ARL_0 value. For instance, with $n = 5$, the value of L_{THWMAV} for upper-sided THWMAV chart is 0.205, 0.279, 0.636, and 1.305 when $\lambda = 0.05$, 0.1, 0.2, and 0.3, respectively, to obtain ARL_0 close to 370 (see Table 1).

(ii) Similarly, for the two-sided THWMAV chart, if $\lambda = 0.05$, then $L_{\mathrm{THWMAV}} = 1.2424$, if $\lambda = 0.1$, then $L_{\mathrm{THWMAV}} = 1.267$, if $\lambda = 0.2$, then $L_{\mathrm{THWMAV}} = 2.058$, and if $\lambda = 0.3$, then $L_{\mathrm{THWMAV}} = 1.795$ to achieve ARL_0 which is approximately 370 (see Table 1).

(iii) As λ increases, the SDRL_0 values for the THWMAV charts decrease. For instance, with $\mathrm{ARL}_0 = 200$, $n = 5$, for the upper-sided THWMAV chart, if $\lambda = 0.05$, then $\mathrm{SDRL}_0 = 2560.33$, if $\lambda = 0.1$, then $\mathrm{SDRL}_0 = 1878.20$, if $\lambda = 0.2$, then $\mathrm{SDRL}_0 = 828.92$, and if $\lambda = 0.2$, then $\mathrm{SDRL}_0 = 372.45$ (see Table 1). Similarly, for the upper-sided THWMAV chart, the SDRL_0 value is 2129.33 when $\lambda = 0.05$, 1747.29 when $\lambda = 0.1$, 473.58 when $\lambda = 0.2$, and 234.46 when $\lambda = 0.3$ (see Table 1).

(iv) For a specified n, as λ increases, the OC performance of the one and two-sided THWMAV charts deteriorates. For example, for $n = 5$ and $\delta = 1.1$, the upper-sided THWMAV chart has ARL_1 and SDRL_1 (OC SDRL) values are 3.92 and 7.25 when $\lambda = 0.05$; 4.01 and 7.49 when $\lambda = 0.1$; 6.66 and 12.16 when $\lambda = 0.2$; and 13.96 and 20.45 when $\lambda = 3$; respectively (see Table 2).

(v) Similarly, for the two-sided THWMAV chart, $\mathrm{ARL}_1 = 10.35$ and $\mathrm{SDRL}_1 = 17.15$ when $\lambda = 0.05$, $\mathrm{ARL}_1 = 10.87$ and $\mathrm{SDRL}_1 = 18.14$ at $\lambda = 0.1$, $\mathrm{ARL}_1 = 18.03$ and $\mathrm{SDRL}_1 = 27.45$ for $\lambda = 0.2$, and $\mathrm{ARL}_1 = 31.80$ and $\mathrm{SDRL}_1 = 36.52$ when $\lambda = 0.3$ (see Table 2).

(vi) For a given λ, the OC performances for the THWMAV charts improve as n increases. For example, with $\lambda = 0.2$ and $\delta = 1.2$, the upper-sided THWMAV chart has ARL_1 values are 3.49, 2.32,

Table 1: IC ARL and SDRL values along with values of L for one and two-sided THWMAV charts to achieve $ARL_0 \cong 200$, 370, and 500.

		$n = 5$				$n = 10$				$n = 15$			
Chart	$\lambda =$	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3
Upper-sided	L	0.03	0.06	0.429	0.986	0.031	0.074	0.427	0.983	0.023	0.059	0.426	0.979
	ARL_0	200.68	200.61	200.57	200.77	200.72	200.19	200.70	200.84	200.67	200.97	200.27	200.70
	$SDRL_0$	2560.33	1878.20	828.92	372.45	2599.42	1822.90	834.57	370.98	2484.88	1885.58	842.05	371.32
	L	0.205	0.279	0.636	1.305	0.1775	0.2793	0.6348	1.305	0.18548	0.279	0.6348	1.305
	ARL_0	370.50	369.95	370.39	370.39	370.32	369.92	370.59	370.59	370.40	370.33	370.59	370.58
	$SDRL_0$	3638.62	2640.99	1263.09	564.53	3743.12	2661.44	1262.87	566.60	3663.59	2721.22	1253.48	567.53
	L	0.29	0.366	0.746	1.478	0.282	0.364	0.747	1.478	0.2844	0.368	0.75	1.479
	ARL_0	500.36	500.02	500.59	500.59	500.34	499.82	500.94	499.84	499.95	500.92	499.56	500.27
	$SDRL_0$	4397.58	3170.29	1532.09	690.27	4488.15	3170.29	1530.88	695.58	4437.15	3130.29	1519.08	695.50
Two-sided	L	1.174	1.189	1.417	1.795	1.1606	1.182	1.418	1.795	1.163	1.186	1.418	1.795
	ARL_0	200.67	200.96	200.82	200.25	2294.30	199.99	200.77	200.24	199.02	200.48	200.53	200.43
	$SDRL_0$	2129.33	1747.29	473.58	234.46	200.63	1719.48	476.92	234.91	2345.54	1697.43	474.15	234.77
	L	1.2424	1.267	1.563	2.058	1.244	1.269	1.559	2.057	1.239	1.263	1.56	2.057
	ARL_0	370.84	370.47	370.88	370.08	370.44	370.89	370.06	370.85	370.16	369.87	369.52	370.89
	$SDRL_0$	3348.03	2400.26	741.37	361.27	3268.84	2351.07	739.23	362.93	3812.76	2410.05	740.24	363.40
	L	1.2786	1.2854	1.642	2.205	1.2781	1.287	1.6374	2.201	1.2788	1.2893	1.6376	2.2
	ARL_0	500.26	500.55	500.58	500.61	500.23	499.84	499.38	500.26	370.28	499.91	500.02	500.35
	$SDRL_0$	4014.62	3410.27	913.49	449.27	4084.79	3381.31	914.99	449.31	2513.37	3282.06	913.36	449.09

and 1.92 and $SDRL_1$ values are 4.50, 2.29, and 1.67 when $n = 5$, 10, and 15, respectively (see Tables 2, 4, and 6). Similarly, for the two-sided THWMAV chart, $ARL_1 = 7.60$, 4.64, 3.59 and $SDRL_1 = 8.78$, 4.30, 2.95 when $n = 5$, 10, 15, respectively (see Tables 3, 5, and 7).

(vii) As λ increases, the EQL values also increase; as a result, the overall performance of the one and two-sided THWMAV charts deteriorates. For example, with $n = 5$, the EQL values of for upper-sided THWMAV chart are 13.59, 13.64, 14.54, and 16.74 for $\lambda = 0.05$, 0.1, 0.2, and $\lambda = 0.3$, respectively (see Table 2). Similarly, for the two-sided THWMAV chart, EQL = 15.91 when $\lambda = 0.05$, EQL = 16.07 when $\lambda = 0.1$, EQL = 18.39 when $\lambda = 0.2$, and EQL = 22.44 when $\lambda = 0.3$ (see Table 3).

5. Comparative Study

The current section compares the upper and two-sided THWMAV charts against the upper and two-sided competing charts, such as DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts. The charts are compared with common $ARL_0 = 200$, using $\lambda = 0.05$, 0.1, 0.2, 0.3 and $n = 5$, 10, 15. The comparisons are given in the following subsections.

5.1. THWMAV *versus* EWMAV *Charts.* The THWMAV charts achieve supreme performance against the EWMAV charts in terms of smaller ARL_1 and $SDRL_1$ for single shifts and a certain range of shifts. For example, for a single shift, at $n = 5$, $\lambda = 0.05$, and $\delta = 1.1, 1.2, 1.3, \ldots$, the upper-sided THWMAV chart yields $ARL_1 = 3.92$, 2.38, 1.89, ... and $SDRL_1 = 7.25$, 2.91, 1.87, ..., while the upper-sided EWMAV chart provides $ARL_1 = 27.18$, 10.83, 6.21, ... and $SDRL_1 = 27.82$, 10.40, 5.73, ... (see Table 2 and Figure 1), respectively. Similarly, the two-sided THWMAV chart produces $ARL_1 = 10.35$, 5.36, 3.78, ... and $SDRL_1 = 17.15$, 6.30, 3.74, ..., whereas the upper-sided EWMAV delivers $ARL_1 = 41.63$, 15.17, 8.33, ... and $SDRL_1 = 40.49$, 13.46, 7.08, ... (see Table 2 and Figure 2) for the same sizes of shift. Similarly, the THWMAV charts attain better overall performance over the EWMAV charts for a certain range of shifts. For instance, when $n = 5$ and $\lambda = 0.05$, the upper-sided THWMAV chart's EQL, RARL, and PCI values are 13.59, 1.00, and 1.00, while the upper-sided EWMAV chart's EQL, RARL, and PCI values are 20.37, 1.50, and 2.75 (see Table 2), respectively. Moreover, the two-sided THWMAV chart has the EQL, RARL, and PCI values of 15.91, 1.00, and 1.00, respectively; however, the two-sided EWMAV chart achieves larger EQL, RARL, and PCI values which are 25.53, 1.60, and 2.13 (see Table 3), respectively.

5.2. THWMAV *versus* DEWMAV *Charts.* The upper and two-sided THWMAV charts reveal outstanding performance over the upper and two-sided DEWMAV charts, respectively, in the terms of the least ARL_1 and $SDRL_1$. For example, at $n = 10$, $\lambda = 0.1$, and $\delta = 1.1$, the ARL_1 and $SDRL_1$ values for the upper and two-sided THWMAV charts are 2.85, 4.01, and 7.09, 9.28, whereas the ARL_1 and $SDRL_1$ values for the upper- and two-sided DEWMAV charts are; 16.07, 15.99, and 22.96, 21.22, respectively (see Tables 4 and 5). Likewise, the one and two-sided THWMAV charts achieve superior overall performance against the one and two-sided DEWMAV chart, respectively, in terms of minimum EQL, PCI, and RARL values. For example, with $n = 10$ and $\lambda = 0.1$, the one- and two-sided THWMAV chart's EQL, RARL, and PCI values are 12.89, 1.00, and 10.00 and 13.97, 1.00, and 1.00, respectively, which are smaller than the one- and two-sided DEWMAV chart's EQL, RARL, and PCI values of 16.08, 1.25, and 2.35 and 18.51, 1.32, and 1.70 (see Tables 4 and 5), respectively.

Table 2: Performance measures for upper-sided THWMAV chart versus upper-sided competing charts when $n = 5$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.03$	0.1095	1.408	0.981	1.223	1.876	0.06	0.507	1.956	1.355	1.587	2.145
1	200.68	200.77	200.23	200.76	200.52	200.05	200.61	200.69	200.65	200.10	200.52	200.69
	(2560.33)	(1859.27)	(261.01)	(264.43)	(250.72)	(221.37)	(1878.20)	(714.61)	(203.99)	(230.54)	(222.91)	(207.67)
1.1	**3.92**	4.35	21.08	23.52	23.48	27.18	**4.01**	7.46	29.67	28.09	28.31	32.15
	(7.25)	(8.16)	(24.49)	(29.03)	(27.71)	(27.82)	(7.49)	(13.24)	(26.46)	(29.45)	(29.29)	(30.94)
1.2	**2.38**	2.56	8.60	9.42	9.22	10.83	**2.46**	3.69	12.62	11.63	11.40	12.79
	(2.91)	(3.17)	(9.06)	(11.69)	(10.57)	(10.40)	(3.00)	(4.65)	(10.82)	(11.97)	(11.27)	(11.49)
1.3	**1.89**	1.98	5.18	5.40	5.28	6.21	**1.92**	2.63	7.41	6.77	6.54	7.31
	(1.87)	(1.98)	(4.93)	(6.75)	(5.95)	(5.73)	(1.95)	(2.70)	(6.18)	(7.08)	(6.38)	(6.27)
1.4	**1.63**	1.69	3.76	3.64	3.57	4.29	**1.64**	2.13	5.09	4.54	4.40	4.91
	(1.40)	(1.46)	(3.32)	(4.39)	(3.83)	(3.74)	(1.40)	(1.94)	(4.08)	(4.77)	(4.23)	(4.02)
1.5	**1.46**	1.52	2.97	2.73	2.71	3.23	**1.48**	1.83	3.91	3.37	3.29	3.69
	(1.11)	(1.18)	(2.44)	(3.11)	(2.72)	(2.68)	(1.13)	(1.54)	(2.96)	(3.46)	(3.03)	(2.89)
1.6	**1.36**	1.40	2.49	2.23	2.23	2.61	**1.38**	1.64	3.18	2.65	2.61	2.96
	(0.95)	(0.99)	(1.95)	(2.36)	(2.08)	(2.04)	(0.97)	(1.27)	(2.34)	(2.62)	(2.31)	(2.23)
1.7	**1.28**	1.32	2.18	1.88	1.89	2.20	**1.30**	1.51	2.70	2.23	2.22	2.50
	(0.81)	(0.86)	(1.64)	(1.85)	(1.62)	(1.61)	(0.85)	(1.10)	(1.94)	(2.11)	(1.85)	(1.81)
1.8	**1.23**	1.25	1.95	1.66	1.68	1.96	**1.24**	1.41	2.39	1.91	1.92	2.18
	(0.71)	(0.76)	(1.43)	(1.48)	(1.32)	(1.36)	(0.74)	(0.95)	(1.68)	(1.69)	(1.50)	(1.51)
1.9	**1.19**	1.21	1.78	1.51	1.54	1.77	**1.19**	1.33	2.14	1.73	1.74	1.95
	(0.65)	(0.67)	(1.26)	(1.22)	(1.11)	(1.15)	(0.64)	(0.84)	(1.47)	(1.45)	(1.29)	(1.28)
2	**1.15**	1.17	1.65	1.40	1.42	1.62	**1.16**	1.27	1.95	1.56	1.57	1.76
	(0.58)	(0.61)	(1.14)	(1.03)	(0.93)	(0.99)	(0.59)	(0.76)	(1.32)	(1.20)	(1.08)	(1.10)
EQL	13.59	13.76	18.95	18.96	18.90	20.37	13.64	14.79	22.10	20.75	20.68	22.04
RARL	1.00	1.01	1.39	1.39	1.39	1.50	1.00	1.08	1.62	1.52	1.52	1.62
PCI	1.00	1.04	2.39	2.38	2.36	2.75	1.00	1.29	3.16	2.81	2.78	3.12

	$\lambda = 0.2$						$\lambda = 0.3$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.429$	1.245	2.352	1.738	1.954	2.355	0.986	1.877	2.474	1.965	2.162	2.449
1	200.57	200.66	200.56	200.16	200.56	200.10	200.77	200.83	200.55	200.99	200.39	200.27
	(828.92)	(294.71)	(195.00)	(210.95)	(207.38)	(201.96)	(372.45)	(208.90)	(197.29)	(206.10)	(203.68)	(200.27)
1.1	**6.66**	18.30	34.12	32.60	33.64	37.58	**13.96**	28.59	36.85	35.86	37.35	41.65
	(12.16)	(23.28)	(28.39)	(31.87)	(32.88)	(35.86)	(20.45)	(26.33)	(32.30)	(34.75)	(36.13)	(40.34)
1.2	**3.49**	7.51	14.76	13.28	13.42	14.77	**5.89**	12.08	15.38	14.23	14.61	16.24
	(4.50)	(8.36)	(11.47)	(12.14)	(12.25)	(13.25)	(7.11)	(10.66)	(12.46)	(12.80)	(13.31)	(14.91)
1.3	**2.48**	4.58	8.57	7.72	7.53	8.14	**3.77**	7.13	8.79	8.06	8.08	8.84
	(2.57)	(4.46)	(6.42)	(6.93)	(6.62)	(6.85)	(3.83)	(6.07)	(6.68)	(6.96)	(6.94)	(7.63)
1.4	**2.01**	3.37	5.91	5.19	5.08	5.45	**2.87**	4.91	6.06	5.46	5.40	5.84
	(1.82)	(3.01)	(4.29)	(4.65)	(4.33)	(4.34)	(2.62)	(4.01)	(4.43)	(4.57)	(4.41)	(4.73)
1.5	**1.76**	2.72	4.51	3.87	3.78	4.10	**2.36**	3.75	4.55	4.11	4.06	4.28
	(1.47)	(2.27)	(3.19)	(3.45)	(3.14)	(3.14)	(2.00)	(2.88)	(3.21)	(3.39)	(3.21)	(3.29)

Table 2: Continued.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.6	1.59	2.32	3.63	3.08	3.03	3.25	**2.04**	3.09	3.62	3.28	3.22	3.38
	(1.21)	(1.83)	(2.51)	(2.71)	(2.45)	(2.38)	(1.63)	(2.30)	(2.49)	(2.66)	(2.47)	(2.48)
1.7	**1.46**	2.04	3.04	2.57	2.54	2.71	**1.83**	2.62	3.04	2.69	2.67	2.81
	(1.04)	(1.55)	(2.05)	(2.21)	(1.98)	(1.91)	(1.38)	(1.90)	(2.03)	(2.14)	(1.97)	(1.99)
1.8	**1.37**	1.84	2.63	2.21	2.19	2.35	**1.65**	2.31	2.62	2.31	2.31	2.43
	(0.91)	(1.35)	(1.74)	(1.83)	(1.64)	(1.59)	(1.19)	(1.63)	(1.72)	(1.78)	(1.65)	(1.64)
1.9	**1.30**	1.68	2.33	1.94	1.93	2.07	**1.54**	2.09	2.32	2.05	2.04	2.15
	(0.81)	(1.19)	(1.53)	(1.55)	(1.37)	(1.35)	(1.07)	(1.45)	(1.51)	(1.54)	(1.40)	(1.39)
2	**1.25**	1.57	2.10	1.74	1.76	1.89	**1.45**	1.90	2.09	1.84	1.84	1.93
	(0.73)	(1.07)	(1.36)	(1.32)	(1.19)	(1.17)	(0.97)	(1.30)	(1.32)	(1.32)	(1.21)	(1.20)
EQL	14.54	18.08	23.82	22.35	22.42	23.55	16.74	21.68	24.31	23.27	23.43	24.62
RARL	1.00	1.24	1.64	1.54	1.54	1.62	1.00	1.30	1.45	1.39	1.40	1.47
PCI	1.00	1.66	2.77	2.46	2.45	2.67	1.00	1.61	1.94	1.77	1.78	1.92

Table 3: Performance measures for two-sided THWMAV chart versus two-sided competing charts when $n = 5$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 1.174$	1.259	2.124	1.517	1.71	2.281	1.189	1.468	2.517	1.792	1.997	2.482
0.5	1.70 (1.08)	1.81 (1.14)	3.25 (1.42)	2.02 (1.33)	2.12 (1.20)	2.89 (1.34)	**1.72** (1.09)	2.13 (1.26)	3.87 (1.45)	2.50 (1.55)	2.60 (1.37)	3.30 (1.47)
0.55	**2.01** (1.30)	2.15 (1.36)	3.86 (1.75)	2.57 (1.86)	2.65 (1.63)	3.54 (1.75)	**2.03** (1.31)	2.52 (1.49)	4.64 (1.88)	3.18 (2.08)	3.25 (1.83)	4.08 (1.94)
0.6	**2.41** (1.60)	2.57 (1.67)	4.65 (2.26)	3.33 (2.58)	3.38 (2.26)	4.43 (2.37)	**2.44** (1.61)	3.03 (1.83)	5.66 (2.52)	4.12 (2.81)	4.11 (2.47)	5.10 (2.62)
0.65	**2.89** (2.01)	3.10 (2.11)	5.75 (3.06)	4.48 (3.65)	4.42 (3.14)	5.65 (3.25)	**2.93** (2.03)	3.66 (2.34)	7.15 (3.49)	5.47 (3.80)	5.35 (3.37)	6.55 (3.66)
0.7	**3.54** (2.64)	3.83 (2.80)	7.45 (4.40)	6.22 (5.23)	5.97 (4.48)	7.51 (4.64)	**3.59** (2.66)	4.59 (3.18)	9.35 (4.95)	7.35 (5.25)	7.15 (4.71)	8.77 (5.32)
0.75	**4.43** (3.67)	4.84 (3.95)	10.12 (6.53)	8.79 (7.56)	8.41 (6.62)	10.35 (6.85)	**4.50** (3.71)	5.93 (4.60)	12.85 (7.32)	10.27 (7.39)	9.95 (6.85)	12.27 (8.15)
0.8	**5.82** (5.58)	6.45 (6.07)	14.84 (10.35)	13.18 (11.56)	12.52 (10.30)	15.35 (10.98)	**5.93** (5.68)	8.17 (7.25)	18.70 (11.33)	14.93 (11.07)	14.84 (10.92)	18.67 (13.70)
0.85	**8.01** (9.10)	9.20 (10.25)	23.76 (17.80)	20.89 (18.86)	20.40 (17.60)	24.96 (19.71)	**8.20** (9.32)	12.42 (12.92)	29.37 (19.27)	23.57 (18.87)	24.29 (19.59)	31.10 (25.79)
0.9	**12.54** (18.33)	15.15 (21.31)	43.33 (35.28)	37.74 (36.69)	37.80 (36.38)	46.97 (41.96)	**12.98** (18.86)	22.40 (27.72)	52.39 (37.60)	43.72 (39.8)	46.36 (42.77)	60.44 (55.86)
0.95	**24.53** (53.57)	32.59 (66.55)	101.29 (90.22)	89.86 (102.33)	93.25 (104.06)	113.52 (114.85)	**26.02** (56.35)	54.87 (87.61)	115.48 (90.85)	104.79 (110.27)	113.31 (117.54)	135.50 (135.10)
1	200.67 (2129.33)	200.88 (956.93)	200.79 (173.26)	200.87 (259.41)	200.39 (242.68)	200.41 (215.69)	200.96 (1747.29)	200.12 (416.68)	200.59 (156.97)	200.81 (226.81)	200.58 (219.5)	200.40 (203.54)
1.1	**10.35** (17.15)	12.64 (20.64)	41.29 (38.11)	36.45 (40.12)	36.50 (39.56)	41.63 (40.49)	**10.87** (18.14)	20.07 (29.45)	48.12 (37.83)	41.54 (41.70)	42.74 (42.71)	48.19 (46.41)
1.2	**5.36** (6.30)	6.16 (7.27)	15.55 (13.74)	14.41 (15.41)	13.57 (13.79)	15.17 (13.46)	**5.51** (6.61)	8.13 (9.27)	19.11 (14.35)	16.05 (14.86)	15.72 (14.33)	17.20 (14.9)
1.3	**3.78** (3.74)	4.13 (4.05)	8.64 (7.25)	8.07 (8.68)	7.49 (7.53)	8.33 (7.08)	**3.82** (3.74)	5.06 (4.88)	10.63 (7.85)	9.18 (8.49)	8.64 (7.65)	9.33 (7.62)
1.4	**2.96** (2.61)	3.18 (2.78)	5.87 (4.65)	5.24 (5.62)	4.97 (4.90)	5.51 (4.54)	**2.99** (2.63)	3.73 (3.24)	7.12 (5.10)	6.07 (5.71)	5.73 (5.07)	6.11 (4.77)
1.5	**2.48** (2.05)	2.62 (2.15)	4.42 (3.33)	3.80 (4.02)	3.63 (3.47)	4.06 (3.21)	**2.49** (2.04)	3.02 (2.46)	5.28 (3.72)	4.42 (4.21)	4.19 (3.64)	4.49 (3.42)
1.6	**2.15** (1.67)	2.27 (1.78)	3.57 (2.58)	3.00 (3.06)	2.87 (2.60)	3.25 (2.47)	**2.17** (1.69)	2.56 (1.99)	4.20 (2.87)	3.42 (3.20)	3.28 (2.77)	3.54 (2.62)
1.7	**1.92** (1.45)	2.02 (1.53)	3.03 (2.14)	2.45 (2.41)	2.37 (2.04)	2.68 (1.96)	**1.95** (1.46)	2.25 (1.69)	3.51 (2.35)	2.79 (2.57)	2.72 (2.23)	2.93 (2.08)
1.8	**1.75** (1.28)	1.83 (1.34)	2.63 (1.83)	2.10 (1.94)	2.06 (1.68)	2.33 (1.63)	**1.76** (1.28)	2.02 (1.49)	3.01 (1.99)	2.37 (2.11)	2.30 (1.82)	2.51 (1.72)
1.9	**1.62** (1.14)	1.68 (1.20)	2.35 (1.62)	1.85 (1.61)	1.83 (1.41)	2.05 (1.36)	**1.65** (1.16)	1.84 (1.32)	2.66 (1.75)	2.07 (1.76)	2.05 (1.56)	2.21 (1.47)

Table 3: Continued.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
2	**1.53**	1.58	2.14	1.66	1.65	1.86	**1.53**	1.69	2.37	1.86	1.82	1.99
	(1.04)	(1.10)	(1.47)	(1.34)	(1.18)	(1.18)	(1.05)	(1.19)	(1.54)	(1.51)	(1.31)	(1.27)
EQL	15.91	16.72	25.49	23.55	23.33	25.53	16.07	18.93	28.29	25.50	25.69	28.03
RARL	1.00	1.05	1.60	1.48	1.47	1.60	1.00	1.18	1.76	1.59	1.60	1.74
PCI	1.00	1.09	2.18	1.87	1.82	2.13	1.00	1.32	2.55	2.09	2.08	2.39

	$\lambda = 0.2$						$\lambda = 0.3$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 1.417$	1.996	2.737	2.106	2.295	2.647	1.795	2.465	2.785	2.299	2.473	2.719
0.5	2.05	3.02	4.06	3.03	3.09	3.74	**2.67**	3.81	4.10	3.30	3.37	4.13
	(1.24)	(1.41)	(1.51)	(1.63)	(1.46)	(1.68)	(1.38)	(1.44)	(1.69)	(1.58)	(1.51)	(2.01)
0.55	**2.44**	3.58	4.95	3.82	3.84	4.68	**3.17**	4.54	5.10	4.10	4.20	5.30
	(1.47)	(1.71)	(2.04)	(2.11)	(1.91)	(2.30)	(1.64)	(1.86)	(2.35)	(2.04)	(2.02)	(2.93)
0.6	**2.92**	4.31	6.17	4.84	4.83	5.97	**3.79**	5.53	6.53	5.14	5.30	7.01
	(1.80)	(2.18)	(2.81)	(2.76)	(2.57)	(3.25)	(2.05)	(2.49)	(3.36)	(2.69)	(2.81)	(4.36)
0.65	**3.52**	5.31	7.89	6.24	6.21	7.89	**4.66**	6.97	8.58	6.62	6.94	9.67
	(2.28)	(2.93)	(3.93)	(3.65)	(3.54)	(4.78)	(2.72)	(3.44)	(4.85)	(3.69)	(4.09)	(6.80)
0.7	**4.36**	6.82	10.49	8.29	8.37	10.96	**5.89**	9.11	11.77	8.90	9.61	14.12
	(3.05)	(4.15)	(5.69)	(5.05)	(5.17)	(7.47)	(3.76)	(4.90)	(7.33)	(5.51)	(6.40)	(11.03)
0.75	**5.69**	9.22	14.61	11.49	11.86	16.21	**7.89**	12.51	17.02	12.69	14.22	21.92
	(4.48)	(6.17)	(8.58)	(7.49)	(8.09)	(12.28)	(5.64)	(7.23)	(11.63)	(8.86)	(10.67)	(18.6)
0.8	**7.75**	13.46	21.55	17.08	18.25	26.06	**11.32**	18.24	26.20	19.86	22.94	35.75
	(6.95)	(9.94)	(13.86)	(12.27)	(13.93)	(21.91)	(9.06)	(11.21)	(19.68)	(15.63)	(19.15)	(32.70)
0.85	**11.51**	21.66	34.58	28.39	31.30	45.32	**18.24**	28.66	43.38	34.47	40.39	61.44
	(12.20)	(17.29)	(24.66)	(23.44)	(27.00)	(41.42)	(16.14)	(18.96)	(35.40)	(30.54)	(36.93)	(58.96)
0.9	**20.48**	39.98	63.29	55.43	62.00	86.30	**33.84**	51.18	80.62	67.99	78.99	109.85
	(26.48)	(34.87)	(50.21)	(51.89)	(58.91)	(83.27)	(33.55)	(36.90)	(71.35)	(65.24)	(76.63)	(107.33)
0.95	**49.82**	95.40	135.56	127.89	138.04	163.75	**83.81**	113.63	158.03	145.46	157.30	183.22
	(85.20)	(91.88)	(118.23)	(130.45)	(139.35)	(161.68)	(94.32)	(89.22)	(149.11)	(145.37)	(156.33)	(181.47)
1	200.82	200.98	200.12	200.88	200.53	200.66	200.25	200.97	200.32	200.36	200.87	200.82
	(473.58)	(191.48)	(179.54)	(208.51)	(205.27)	(200.48)	(234.46)	(156.84)	(191.42)	(203.00)	(201.71)	(200.94)
1.1	**18.03**	38.19	50.92	48.52	50.75	58.06	**31.80**	47.69	55.37	54.28	57.82	64.63
	(27.45)	(37.77)	(41.57)	(47.42)	(49.96)	(56.31)	(36.52)	(37.65)	(49.02)	(53.53)	(56.78)	(63.17)
1.2	**7.60**	14.22	19.70	17.76	17.94	19.99	**11.81**	18.67	20.42	19.24	19.90	22.62
	(8.78)	(13.33)	(14.67)	(15.54)	(15.90)	(17.85)	(12.07)	(14.44)	(16.31)	(17.13)	(18.03)	(20.96)
1.3	**4.81**	7.90	11.15	9.85	9.63	10.41	**6.74**	10.48	11.09	10.31	10.25	11.33
	(4.69)	(6.91)	(7.91)	(8.18)	(8.03)	(8.65)	(6.14)	(7.86)	(8.26)	(8.51)	(8.58)	(9.71)
1.4	**3.62**	5.42	7.46	6.59	6.32	6.69	**4.75**	6.98	7.34	6.78	6.68	7.09
	(3.16)	(4.38)	(5.11)	(5.43)	(5.12)	(5.23)	(3.98)	(5.09)	(5.24)	(5.33)	(5.28)	(5.67)
1.5	**2.90**	4.12	5.51	4.83	4.65	4.87	**3.70**	5.15	5.36	4.98	4.83	5.08
	(2.36)	(3.15)	(3.70)	(4.01)	(3.66)	(3.65)	(2.91)	(3.64)	(3.68)	(3.88)	(3.67)	(3.86)
1.6	**2.50**	3.37	4.36	3.80	3.63	3.79	**3.03**	4.13	4.25	3.89	3.78	3.93
	(1.96)	(2.48)	(2.89)	(3.16)	(2.80)	(2.75)	(2.29)	(2.84)	(2.84)	(3.02)	(2.83)	(2.86)

Table 3: Continued.

	λ = 0.05						λ = 0.1					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.7	**2.20**	2.87	3.61	3.07	2.99	3.10	**2.64**	3.43	3.49	3.17	3.10	3.23
	(1.66)	(2.07)	(2.33)	(2.54)	(2.27)	(2.17)	(1.94)	(2.32)	(2.29)	(2.43)	(2.25)	(2.28)
1.8	**1.96**	2.51	3.09	2.62	2.56	2.67	**2.31**	2.96	2.99	2.70	2.64	2.73
	(1.44)	(1.77)	(1.99)	(2.14)	(1.90)	(1.82)	(1.66)	(1.98)	(1.94)	(2.03)	(1.87)	(1.86)
1.9	**1.79**	2.24	2.70	2.26	2.22	2.34	**2.08**	2.63	2.61	2.35	2.32	2.39
	(1.29)	(1.57)	(1.70)	(1.79)	(1.60)	(1.52)	(1.48)	(1.73)	(1.66)	(1.75)	(1.59)	(1.57)
2	**1.67**	2.04	2.41	2.02	1.99	2.08	**1.92**	2.36	2.33	2.09	2.07	2.14
	(1.18)	(1.41)	(1.51)	(1.55)	(1.38)	(1.31)	(1.34)	(1.55)	(1.45)	(1.51)	(1.38)	(1.34)
EQL	18.39	24.41	29.92	27.92	28.53	31.71	22.44	27.99	31.75	29.80	30.99	34.79
RARL	1.00	1.33	1.63	1.52	1.55	1.72	1.00	1.25	1.41	1.33	1.38	1.55
PCI	1.00	1.54	2.11	1.82	1.84	2.14	1.00	1.41	1.62	1.42	1.47	1.76

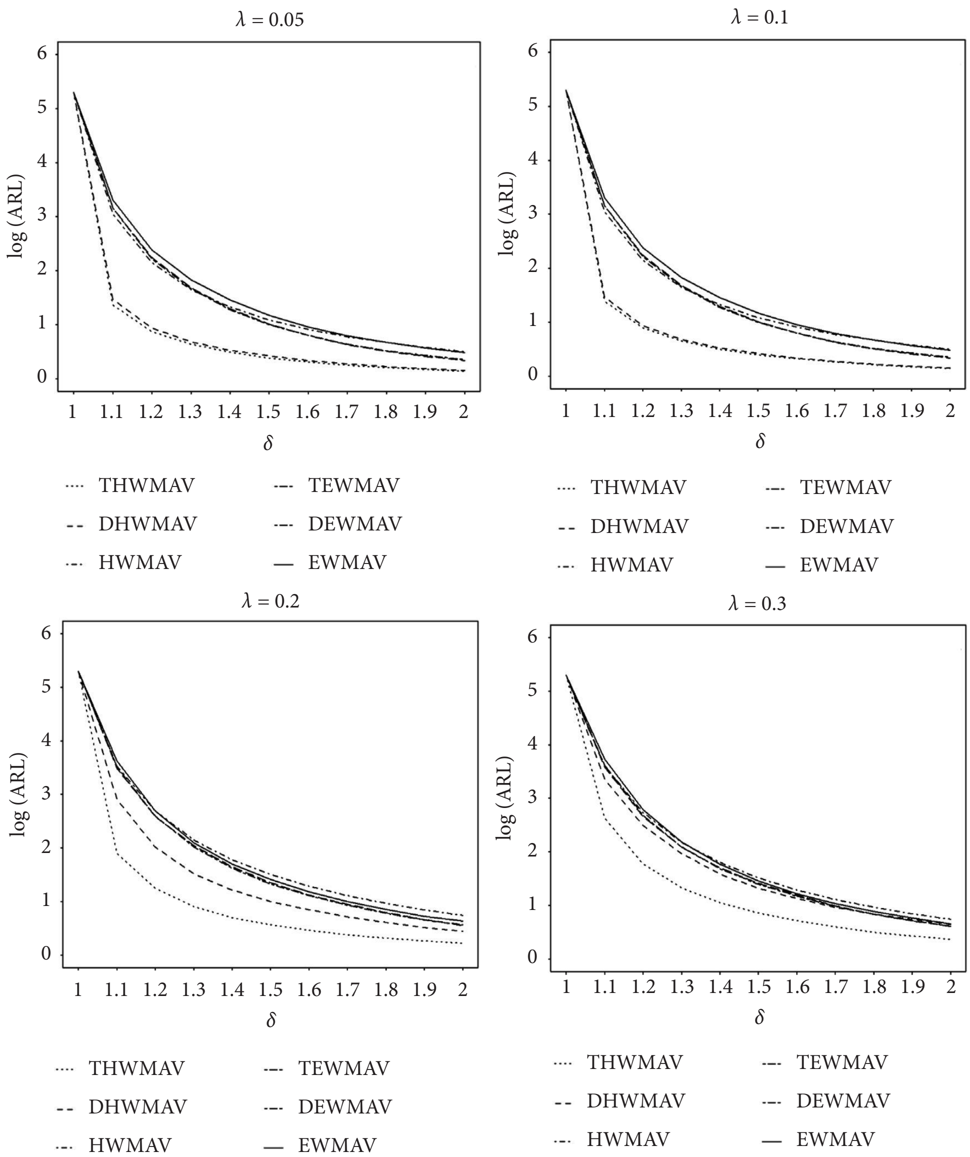

Figure 1: ARL plots for the upper-sided THWMAV chart versus upper-sided competing charts when $n = 5$.

5.3. THWMAV *versus* TEWMAV *Charts.* The upper-sided THWMAV chart shows excellent performance against the upper-sided TEWMAV chart by keeping the lower ARL_1 and $SDRL_1$ values. Similarly, the two-sided THWMAV chart achieves dominant performance against the two-sided TEWMAV chart. In detail, when $n = 15$, $\lambda = 0.05$, and $\delta = 1.1, 1.2, 1.3, 1.4, 1.5$, then the upper-sided THWMAV chart bears the ARL_1 and $SDRL_1$ values of 2.31, 1.51, 1.25, 1.13, 1.07 and 2.78, 1.21, 0.76, 0.53, 0.37, respectively, while, the upper-sided TEWMAV chart achieves ARL_1 and $SDRL_1$ values of 9.44, 3.38, 1.96, 1.44, 1.22 and 11.53, 3.92, 1.89, 1.08, 0.68, respectively (see Table 6). Similarly, for the same n, λ, and δ values, the ARL_1 and $SDRL_1$ for the two-sided THWMAV chart are 5.56, 2.93, 2.04, 1.61, 1.37 and 6.33, 2.46, 1.52, 1.11, 0.85, respectively, whereas the ARL_1 and $SDRL_1$ values of the two-sided TEWMAV chart are larger, i.e., 14.70, 5.07, 2.72, 1.84, 1.45, and 15.14, 5.20, 2.59, 1.52, 0.98, respectively (see Table 7). Besides, at $n = 15$ and $\lambda = 0.1$, the EQL = 12.69, RARL = 1.00, and PCI = 1.00 values of the upper-sided THWMAV chart indicate a better overall detection ability as compared to the upper-sided TEWMAV chart because EQL = 14.64, RARL = 1.15, and PCI = 1.76 of the upper-sided TEWMAV chart are larger. Correspondingly, the two-sided THWMAV chart's EQL = 13.29, RARL = 1.00, and PCI = 1.00 are smaller relative to the two-sided TEWMAV chart's EQL = 16.15, RARL = 1.21, and PCI = 1.53, respectively (see Table 6).

5.4. THWMAV *versus* HWMAV *Charts.* The upper-sided THWMAV chart achieves superior performance against the HWMAV chart in terms of minimum ARL_1 and $SDRL_1$. For

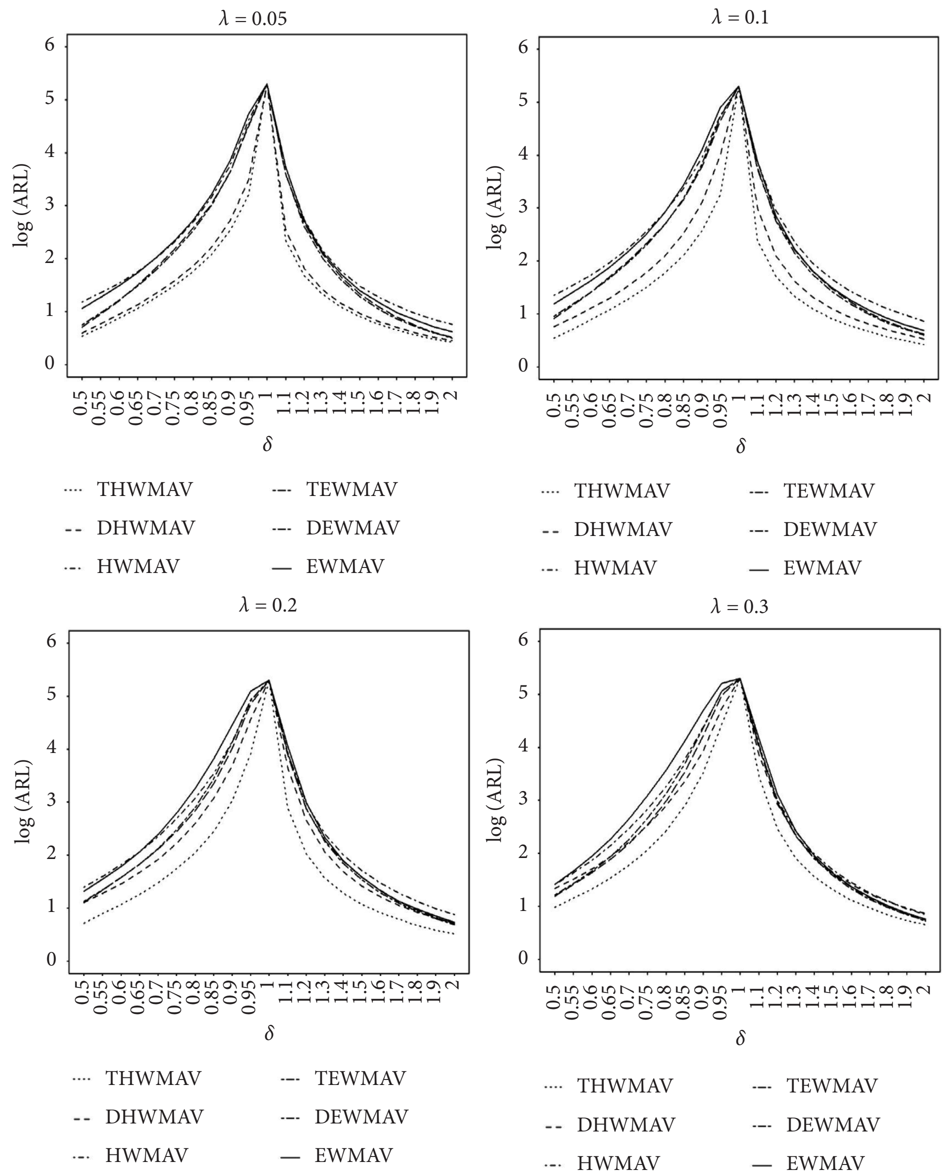

FIGURE 2: ARL plots for the two-sided THWMAV chart versus two-sided competing charts when $n = 5$.

example, for $n = 10$, $\lambda = 0.2$, and $\delta = 1.2$, the upper-sided THWMAV chart provides $ARL_1 = 2.32$ and $SDRL_1 = 2.29$, whereas the upper-sided HWMAV chart provides $ARL_1 = 8.25$ and $SDRL_1 = 5.94$ (see Table 4). Similarly, the two-sided THWMAV chart also gets excellent detection ability against the two-sided HWMAV chart in terms of smaller ARL_1 and $SDRL_1$. For instance, at $n = 10$, $\lambda = 0.2$, and $\delta = 0.9$, the upper-sided THWMAV chart's $ARL_1 = 11.12$ and $SDRL_1 = 12.99$ are smaller as compared to the two-sided HWMAV chart's $ARL_1 = 32.47$ and $SDRL_1 = 23.41$ values (see Table 5). Moreover, the upper-sided THWMAV chart has a superior overall performance to the upper-sided HWMAV chart. In detail, at $n = 10$ and $\lambda = 0.2$, the upper-sided chart has EQL = 13.33, RARL = 1.00, and PCI = 1.00 values, whereas the upper-sided HWMAV chart's EQL = 18.06, RARL = 1.36, and PCI = 2.27 are larger in comparison (see Table 4). Likewise, the two-sided THWMAV chart reveals improved detection ability against the two-sided HWMAV chart, as it offers smaller EQL, RARL, and PCI values of 15.10, 1.00, 1.00, as compared to the EQL, RARL, and PCI values for the upper-sided HWMAV chart, which are: 21.20, 1.40, 1.88 (see Table 5).

5.5. THWMAV versus DHWMAV Charts. At $n = 5$, $\lambda = 0.1$, and $\delta = 1.2$, the THWMAV charts achieve better detection ability than the DHWMAV charts. For instance, the upper and two-sided THWMAV charts ARL_1 and $SDRL_1$ values are equal to 2.46 and 5.51, and 3.00 and 6.61, respectively, while upper and two-sided DHWMAV charts ARL_1 and $SDRL_1$ values are 3.69 and 8.13, and 4.65 and 9.27, respectively (see Tables 2 and 3 and Figures 1 and 2). It indicates the inferior performance for the upper and two-sided DHWMAV charts. In addition, with $n = 5$, $\lambda = 0.1$, the EQL,

Table 4: Performance measures for upper-sided THWMAV chart versus upper-sided competing charts when $n = 10$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.031$	0.114	1.408	0.981	1.222	1.876	0.074	0.507	1.956	1.354	1.586	2.144
1	200.72	200.01	200.47	200.27	200.91	200.27	200.19	200.42	200.65	200.66	200.40	200.83
	(2599.42)	(1795.33)	(259.91)	(263.78)	(249.72)	(220.25)	(1822.90)	(714.75)	(202.76)	(229.83)	(222.08)	(208.28)
1.1	**2.76**	3.00	12.02	13.21	12.97	15.50	**2.85**	4.62	17.54	16.08	16.07	18.49
	(3.84)	(4.26)	(13.11)	(16.28)	(14.93)	(15.13)	(4.01)	(6.63)	(15.08)	(16.36)	(15.99)	(16.95)
1.2	**1.78**	1.87	4.85	4.86	4.77	5.81	**1.83**	2.46	6.95	6.15	5.96	6.91
	(1.67)	(1.77)	(4.42)	(5.89)	(5.18)	(5.08)	(1.71)	(2.44)	(5.53)	(6.27)	(5.59)	(5.65)
1.3	**1.43**	1.49	3.03	2.75	2.75	3.40	**1.45**	1.81	4.11	3.41	3.41	3.92
	(1.06)	(1.13)	(2.42)	(3.07)	(2.67)	(2.72)	(1.08)	(1.48)	(3.01)	(3.41)	(3.07)	(2.98)
1.4	**1.26**	1.29	2.27	1.91	1.95	2.39	**1.28**	1.50	2.91	2.31	2.33	2.71
	(0.78)	(0.82)	(1.67)	(1.83)	(1.66)	(1.72)	(0.80)	(1.08)	(2.01)	(2.15)	(1.92)	(1.90)
1.5	**1.16**	1.19	1.85	1.52	1.57	1.88	**1.17**	1.32	2.28	1.77	1.80	2.09
	(0.60)	(0.64)	(1.31)	(1.21)	(1.15)	(1.21)	(0.62)	(0.84)	(1.53)	(1.47)	(1.32)	(1.36)
1.6	**1.10**	1.12	1.58	1.31	1.36	1.58	**1.11**	1.21	1.90	1.48	1.52	1.74
	(0.47)	(0.50)	(1.06)	(0.86)	(0.81)	(0.91)	(0.48)	(0.66)	(1.24)	(1.06)	(0.97)	(1.03)
1.7	**1.06**	1.08	1.41	1.20	1.23	1.41	**1.07**	1.14	1.64	1.30	1.34	1.51
	(0.37)	(0.41)	(0.87)	(0.64)	(0.61)	(0.72)	(0.39)	(0.54)	(1.04)	(0.77)	(0.74)	(0.81)
1.8	**1.05**	1.05	1.29	1.13	1.15	1.28	**1.05**	1.10	1.47	1.20	1.24	1.37
	(0.31)	(0.33)	(0.73)	(0.47)	(0.47)	(0.58)	(0.32)	(0.45)	(0.88)	(0.60)	(0.59)	(0.66)
1.9	**1.03**	1.04	1.21	1.09	1.10	1.21	**1.03**	1.07	1.35	1.14	1.16	1.27
	(0.25)	(0.27)	(0.61)	(0.37)	(0.37)	(0.49)	(0.25)	(0.36)	(0.76)	(0.46)	(0.47)	(0.55)
2	**1.02**	1.02	1.15	1.06	1.07	1.15	**1.02**	1.05	1.24	1.10	1.11	1.20
	(0.21)	(0.22)	(0.52)	(0.30)	(0.31)	(0.40)	(0.22)	(0.30)	(0.64)	(0.38)	(0.38)	(0.46)
EQL	12.88	12.92	15.45	15.15	15.19	16.09	12.89	13.42	17.11	16.07	16.08	16.96
RARL	1.00	1.00	1.20	1.18	1.18	1.25	1.00	1.04	1.33	1.25	1.25	1.32
PCI	1.00	1.03	1.88	1.77	1.78	2.10	1.00	1.18	2.41	2.05	2.06	2.35
	$\lambda = 0.2$						$\lambda = 0.3$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.427$	1.244	2.353	1.738	1.953	2.356	0.983	1.877	2.476	1.965	2.161	2.45
1	200.70	200.70	200.07	200.92	200.87	200.31	200.84	200.99	200.48	200.23	199.98	200.19
	(834.57)	(292.14)	(194.23)	(212.08)	(209.11)	(201.28)	(370.98)	(207.77)	(197.15)	(204.69)	(202.90)	(200.16)
1.1	**4.24**	10.34	20.68	18.63	19.10	22.05	**7.91**	16.79	22.43	20.49	21.50	25.08
	(6.10)	(12.13)	(16.33)	(17.32)	(17.88)	(20.37)	(10.24)	(14.92)	(18.62)	(18.95)	(20.15)	(23.56)
1.2	**2.32**	4.29	8.25	7.14	7.09	7.91	**3.51**	6.64	8.56	7.55	7.70	8.66
	(2.29)	(4.03)	(5.94)	(6.16)	(5.97)	(6.43)	(3.42)	(5.40)	(6.33)	(6.18)	(6.37)	(7.25)
1.3	**1.74**	2.74	4.77	4.04	3.98	4.40	**2.38**	3.95	4.88	4.28	4.26	4.66
	(1.41)	(2.23)	(3.24)	(3.49)	(3.19)	(3.24)	(1.97)	(2.94)	(3.34)	(3.42)	(3.26)	(3.50)
1.4	**1.45**	2.09	3.31	2.72	2.71	2.99	**1.84**	2.82	3.35	2.90	2.90	3.16
	(1.02)	(1.56)	(2.15)	(2.27)	(2.06)	(2.06)	(1.39)	(1.96)	(2.16)	(2.23)	(2.09)	(2.18)
1.5	**1.29**	1.72	2.55	2.05	2.07	2.29	**1.56**	2.22	2.57	2.21	2.20	2.39
	(0.79)	(1.21)	(1.60)	(1.61)	(1.45)	(1.47)	(1.08)	(1.50)	(1.58)	(1.62)	(1.49)	(1.52)

Table 4: Continued.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.6	**1.19**	1.50	2.10	1.69	1.70	1.88	**1.38**	1.86	2.09	1.77	1.82	1.94
	(0.63)	(0.99)	(1.28)	(1.20)	(1.10)	(1.11)	(0.87)	(1.22)	(1.24)	(1.20)	(1.14)	(1.15)
1.7	**1.13**	1.35	1.79	1.44	1.48	1.62	**1.26**	1.61	1.77	1.53	1.56	1.67
	(0.51)	(0.82)	(1.07)	(0.91)	(0.85)	(0.88)	(0.71)	(1.02)	(1.02)	(0.94)	(0.88)	(0.92)
1.8	**1.09**	1.25	1.57	1.30	1.34	1.44	**1.18**	1.44	1.57	1.37	1.40	1.48
	(0.42)	(0.68)	(0.90)	(0.71)	(0.69)	(0.72)	(0.60)	(0.86)	(0.86)	(0.76)	(0.72)	(0.75)
1.9	**1.06**	1.17	1.43	1.21	1.24	1.33	**1.13**	1.32	1.41	1.26	1.28	1.35
	(0.35)	(0.57)	(0.78)	(0.57)	(0.55)	(0.60)	(0.49)	(0.74)	(0.72)	(0.61)	(0.59)	(0.63)
2	**1.04**	1.12	1.31	1.15	1.17	1.24	**1.09**	1.24	1.31	1.18	1.21	1.26
	(0.29)	(0.48)	(0.66)	(0.46)	(0.45)	(0.51)	(0.41)	(0.63)	(0.62)	(0.49)	(0.50)	(0.53)
EQL	13.33	14.99	18.06	16.94	17.02	17.81	14.34	16.89	18.36	17.40	17.57	18.43
RARL	1.00	1.12	1.36	1.27	1.28	1.34	1.00	1.18	1.28	1.21	1.22	1.29
PCI	1.00	1.44	2.27	1.93	1.95	2.17	1.00	1.47	1.74	1.53	1.56	1.71

Table 5: Performance measures for two-sided THWMAV chart versus two-sided competing charts when $n = 10$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 1.1606$	1.256	2.122	1.513	1.708	2.277	1.182	1.4645	2.515	1.795	1.992	2.481
0.5	**1.07**	1.10	1.71	1.12	1.19	1.53	**1.08**	1.18	2.12	1.24	1.34	1.70
	(0.38)	(0.44)	(0.95)	(0.37)	(0.42)	(0.62)	(0.39)	(0.57)	(0.99)	(0.52)	(0.55)	(0.67)
0.55	**1.20**	1.25	2.10	1.28	1.38	1.82	**1.21**	1.40	2.56	1.49	1.61	2.04
	(0.61)	(0.68)	(1.11)	(0.60)	(0.63)	(0.79)	(0.62)	(0.82)	(1.09)	(0.78)	(0.76)	(0.86)
0.6	**1.40**	1.48	2.57	1.56	1.66	2.22	**1.42**	1.70	3.06	1.88	1.98	2.51
	(0.85)	(0.92)	(1.27)	(0.95)	(0.90)	(1.04)	(0.87)	(1.06)	(1.27)	(1.15)	(1.05)	(1.13)
0.65	**1.70**	1.82	3.15	2.02	2.11	2.79	**1.73**	2.10	3.76	2.48	2.54	3.17
	(1.11)	(1.18)	(1.53)	(1.44)	(1.30)	(1.42)	(1.13)	(1.31)	(1.57)	(1.66)	(1.48)	(1.55)
0.7	**2.10**	2.26	3.93	2.79	2.81	3.64	**2.13**	2.61	4.74	3.40	3.40	4.17
	(1.42)	(1.51)	(1.99)	(2.22)	(1.93)	(2.01)	(1.44)	(1.65)	(2.17)	(2.43)	(2.13)	(2.19)
0.75	**2.66**	2.86	5.18	4.02	3.95	4.99	**2.71**	3.35	6.41	4.88	4.74	5.75
	(1.89)	(2.00)	(2.88)	(3.39)	(2.94)	(2.99)	(1.92)	(2.22)	(3.25)	(3.58)	(3.14)	(3.32)
0.8	**3.46**	3.76	7.42	6.23	5.98	7.38	**3.52**	4.53	9.26	7.41	7.16	8.58
	(2.69)	(2.87)	(4.62)	(5.50)	(4.73)	(4.81)	(2.72)	(3.31)	(5.20)	(5.54)	(4.96)	(5.45)
0.85	**4.81**	5.35	11.84	10.48	10.03	12.05	**4.92**	6.71	14.92	12.06	11.74	14.25
	(4.38)	(4.82)	(8.31)	(9.51)	(8.37)	(8.63)	(4.47)	(5.78)	(9.12)	(9.22)	(8.65)	(10.2)
0.9	**7.51**	8.85	22.63	20.01	19.49	23.40	**7.83**	11.85	28.02	22.80	23.07	28.93
	(8.72)	(9.97)	(17.53)	(18.67)	(17.36)	(18.87)	(9.03)	(12.57)	(18.76)	(18.66)	(19.11)	(24.16)
0.95	**15.15**	19.42	61.08	52.64	53.99	66.54	**16.11**	30.89	71.97	62.09	66.45	83.66
	(26.58)	(32.62)	(52.46)	(55.62)	(56.37)	(63.62)	(27.93)	(44.24)	(54.56)	(61.24)	(65.83)	(80.84)
1	200.63	200.74	200.74	200.56	200.62	200.36	199.99	200.25	200.71	200.19	200.76	200.26
	(2294.30)	(970.60)	(173.30)	(259.34)	(243.90)	(215.96)	(1719.48)	(418.32)	(156.60)	(227.19)	(219.08)	(206.20)
1.1	**6.88**	8.13	22.63	20.44	19.71	22.47	**7.09**	11.30	27.45	23.14	22.96	26.17
	(9.01)	(10.57)	(19.91)	(21.41)	(19.79)	(20.16)	(9.28)	(13.81)	(20.34)	(21.18)	(21.22)	(23.19)
1.2	**3.63**	3.94	8.13	7.36	6.96	7.90	**3.68**	4.85	10.02	8.57	8.08	8.89
	(3.37)	(3.69)	(6.46)	(7.74)	(6.70)	(6.37)	(3.44)	(4.40)	(7.06)	(7.67)	(6.86)	(6.87)
1.3	**2.53**	2.72	4.67	3.93	3.79	4.36	**2.58**	3.14	5.63	4.65	4.44	4.85
	(2.04)	(2.19)	(3.34)	(4.02)	(3.47)	(3.29)	(2.07)	(2.50)	(3.71)	(4.21)	(3.66)	(3.54)
1.4	**1.99**	2.09	3.30	2.60	2.56	2.96	**2.02**	2.37	3.85	3.02	2.94	3.24
	(1.49)	(1.57)	(2.23)	(2.53)	(2.18)	(2.08)	(1.51)	(1.75)	(2.44)	(2.69)	(2.33)	(2.21)
1.5	**1.67**	1.74	2.56	1.95	1.93	2.24	**1.69**	1.93	2.94	2.22	2.20	2.45
	(1.18)	(1.23)	(1.70)	(1.70)	(1.48)	(1.47)	(1.19)	(1.37)	(1.81)	(1.87)	(1.63)	(1.58)
1.6	**1.46**	1.51	2.12	1.58	1.61	1.83	**1.47**	1.65	2.40	1.77	1.77	2.00
	(0.96)	(1.01)	(1.39)	(1.20)	(1.08)	(1.10)	(0.98)	(1.12)	(1.48)	(1.36)	(1.20)	(1.20)
1.7	**1.33**	1.37	1.81	1.38	1.41	1.59	**1.34**	1.47	2.03	1.52	1.52	1.69
	(0.80)	(0.86)	(1.17)	(0.90)	(0.83)	(0.88)	(0.82)	(0.95)	(1.26)	(1.03)	(0.91)	(0.94)
1.8	**1.23**	1.26	1.59	1.26	1.27	1.41	**1.24**	1.34	1.76	1.35	1.36	1.51
	(0.68)	(0.71)	(1.01)	(0.69)	(0.63)	(0.7)	(0.69)	(0.80)	(1.08)	(0.80)	(0.73)	(0.78)
1.9	**1.17**	1.19	1.44	1.17	1.19	1.31	**1.17**	1.24	1.58	1.24	1.25	1.37
	(0.57)	(0.61)	(0.87)	(0.54)	(0.51)	(0.60)	(0.57)	(0.68)	(0.95)	(0.63)	(0.58)	(0.64)

Table 5: Continued.

	λ = 0.05						λ = 0.1					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
2	**1.12**	1.14	1.33	1.12	1.13	1.23	**1.12**	1.17	1.44	1.17	1.19	1.28
	(0.49)	(0.52)	(0.76)	(0.43)	(0.41)	(0.50)	(0.49)	(0.58)	(0.83)	(0.50)	(0.49)	(0.55)
EQL	13.92	14.33	18.72	17.39	17.31	18.53	13.97	15.33	20.34	18.45	18.51	19.91
RARL	1.00	1.03	1.34	1.25	1.24	1.33	1.00	1.10	1.46	1.32	1.32	1.42
PCI	1.00	1.07	1.87	1.52	1.51	1.77	1.00	1.23	2.17	1.71	1.70	1.97

	λ = 0.2						λ = 0.3					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	L = 1.418	1.995	2.734	2.104	2.294	2.646	1.795	2.463	2.783	2.299	2.469	2.719
0.5	**1.16**	1.58	2.21	1.45	1.57	1.86	**1.40**	2.07	2.12	1.60	1.71	1.95
	(0.54)	(0.90)	(0.90)	(0.66)	(0.66)	(0.73)	(0.79)	(1.00)	(0.82)	(0.72)	(0.69)	(0.77)
0.55	**1.36**	1.94	2.64	1.78	1.89	2.25	**1.72**	2.50	2.54	1.97	2.06	2.38
	(0.79)	(1.08)	(1.00)	(0.92)	(0.86)	(0.93)	(1.01)	(1.10)	(0.97)	(0.95)	(0.89)	(1.01)
0.6	**1.65**	2.39	3.16	2.26	2.35	2.79	**2.11**	3.00	3.10	2.48	2.56	2.98
	(1.03)	(1.26)	(1.22)	(1.27)	(1.15)	(1.24)	(1.21)	(1.27)	(1.26)	(1.26)	(1.18)	(1.38)
0.65	**2.03**	2.93	3.93	2.97	3.01	3.56	**2.61**	3.68	3.93	3.21	3.26	3.89
	(1.28)	(1.51)	(1.62)	(1.75)	(1.57)	(1.73)	(1.46)	(1.57)	(1.75)	(1.70)	(1.61)	(2.01)
0.7	**2.53**	3.68	5.07	3.99	3.99	4.74	**3.27**	4.64	5.20	4.26	4.30	5.33
	(1.62)	(1.94)	(2.32)	(2.42)	(2.21)	(2.54)	(1.84)	(2.15)	(2.61)	(2.33)	(2.30)	(3.15)
0.75	**3.24**	4.80	6.97	5.60	5.52	6.74	**4.22**	6.25	7.36	5.91	6.07	7.91
	(2.17)	(2.74)	(3.56)	(3.48)	(3.27)	(4.10)	(2.54)	(3.21)	(4.18)	(3.44)	(3.63)	(5.37)
0.8	**4.36**	6.80	10.24	8.22	8.29	10.46	**5.87**	9.03	11.26	8.80	9.38	13.02
	(3.22)	(4.39)	(5.78)	(5.31)	(5.35)	(7.24)	(3.99)	(5.14)	(7.10)	(5.71)	(6.44)	(10.21)
0.85	6.41	10.79	16.69	13.41	13.98	18.62	**9.17**	14.54	19.33	15.05	16.69	24.40
	(5.58)	(7.89)	(10.53)	(9.51)	(10.39)	(14.99)	(7.20)	(9.01)	(13.96)	(11.37)	(13.53)	(21.49)
0.9	**11.12**	20.70	32.47	26.92	29.26	40.73	**17.39**	27.36	39.64	32.06	36.59	54.32
	(12.00)	(17.07)	(23.41)	(22.54)	(25.43)	(37.28)	(15.93)	(18.57)	(32.37)	(28.46)	(33.74)	(51.76)
0.95	**27.98**	56.64	85.00	77.77	86.32	110.16	**48.15**	70.52	103.87	92.62	103.40	131.68
	(41.86)	(52.44)	(70.57)	(76.38)	(85.02)	(108.06)	(51.66)	(53.64)	(94.2)	(91.15)	(101.81)	(130.24)
1	200.77	200.77	200.07	200.10	200.66	200.44	200.24	200.98	200.53	200.59	200.22	200.74
	(476.92)	(190.97)	(178.46)	(210.58)	(207.31)	(202.23)	(234.91)	(155.69)	(190.27)	(206.88)	(203.39)	(201.98)
1.1	**10.50**	20.57	29.19	26.30	27.23	32.15	**17.22**	26.94	31.92	29.19	31.26	37.26
	(13.08)	(19.29)	(22.15)	(23.56)	(24.97)	(30.03)	(18.04)	(20.31)	(26.90)	(26.96)	(29.34)	(35.19)
1.2	**4.64**	7.48	10.60	9.20	9.05	10.09	**6.39**	9.80	10.82	9.61	9.78	11.23
	(4.30)	(6.18)	(7.27)	(7.32)	(7.19)	(8.06)	(5.52)	(7.03)	(7.84)	(7.56)	(7.91)	(9.53)
1.3	**3.06**	4.33	5.93	5.11	4.92	5.30	**3.87**	5.53	5.85	5.27	5.17	5.68
	(2.44)	(3.17)	(3.79)	(4.05)	(3.72)	(3.81)	(2.95)	(3.71)	(3.88)	(3.94)	(3.80)	(4.26)
1.4	**2.29**	3.09	4.03	3.37	3.30	3.52	**2.82**	3.78	3.91	3.51	3.44	3.69
	(1.69)	(2.15)	(2.47)	(2.68)	(2.40)	(2.38)	(2.00)	(2.40)	(2.47)	(2.56)	(2.39)	(2.54)
1.5	**1.88**	2.44	3.03	2.47	2.44	2.61	**2.22**	2.91	2.94	2.59	2.57	2.72
	(1.34)	(1.65)	(1.81)	(1.89)	(1.69)	(1.66)	(1.55)	(1.82)	(1.78)	(1.84)	(1.71)	(1.74)
1.6	**1.62**	2.02	2.45	1.97	1.96	2.10	**1.87**	2.36	2.36	2.05	2.05	2.18
	(1.10)	(1.34)	(1.43)	(1.43)	(1.28)	(1.26)	(1.26)	(1.47)	(1.37)	(1.41)	(1.28)	(1.31)

TABLE 5: Continued.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.7	**1.44**	1.73	2.06	1.65	1.66	1.79	**1.62**	2.00	1.98	1.72	1.73	1.83
	(0.92)	(1.13)	(1.20)	(1.10)	(1.00)	(1.00)	(1.07)	(1.24)	(1.14)	(1.11)	(1.01)	(1.04)
1.8	**1.31**	1.53	1.78	1.45	1.47	1.57	**1.45**	1.75	1.73	1.51	1.53	1.60
	(0.78)	(0.97)	(1.02)	(0.87)	(0.80)	(0.81)	(0.91)	(1.08)	(0.96)	(0.89)	(0.83)	(0.84)
1.9	**1.23**	1.40	1.59	1.31	1.34	1.42	**1.33**	1.56	1.53	1.37	1.38	1.45
	(0.66)	(0.84)	(0.89)	(0.70)	(0.66)	(0.68)	(0.78)	(0.94)	(0.81)	(0.72)	(0.68)	(0.71)
2	**1.16**	1.30	1.44	1.22	1.24	1.32	**1.24**	1.42	1.41	1.26	1.28	1.33
	(0.57)	(0.73)	(0.78)	(0.57)	(0.54)	(0.59)	(0.67)	(0.83)	(0.70)	(0.59)	(0.57)	(0.60)
EQL	15.10	18.11	21.20	19.67	20.07	22.00	17.06	20.16	22.22	20.74	21.39	23.90
RARL	1.00	1.20	1.40	1.30	1.33	1.46	1.00	1.18	1.30	1.22	1.25	1.40
PCI	1.00	1.42	1.88	1.56	1.59	1.83	1.00	1.36	1.49	1.29	1.33	1.55

Table 6: Performance measures for upper-sided THWMAV chart versus upper-sided competing charts when $n = 15$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.023$	0.109	1.409	0.984	1.222	1.876	0.059	0.506	1.956	1.355	1.585	2.142
1	200.67	199.83	199.92	200.76	200.94	200.93	200.97	200.27	200.56	200.16	200.67	200.87
	(2484.88)	(1819.65)	(257.18)	(263.01)	(251.74)	(220.27)	(1885.58)	(724.21)	(201.78)	(227.90)	(221.81)	(207.76)
1.1	**2.31**	2.49	8.78	9.44	9.23	11.22	**2.38**	3.63	12.99	11.74	11.55	13.45
	(2.78)	(3.02)	(9.01)	(11.53)	(10.4)	(10.44)	(2.88)	(4.48)	(10.79)	(11.77)	(11.10)	(11.78)
1.2	**1.51**	1.59	3.62	3.38	3.38	4.21	**1.57**	2.01	5.06	4.30	4.23	4.96
	(1.21)	(1.29)	(3.00)	(3.92)	(3.43)	(3.44)	(1.28)	(1.75)	(3.77)	(4.34)	(3.83)	(3.81)
1.3	**1.25**	1.28	2.34	1.96	1.99	2.49	**1.27**	1.50	3.05	2.39	2.42	2.86
	(0.76)	(0.81)	(1.73)	(1.89)	(1.69)	(1.79)	(0.80)	(1.07)	(2.07)	(2.22)	(1.98)	(2.00)
1.4	**1.13**	1.15	1.77	1.44	1.49	1.81	**1.14**	1.27	2.21	1.69	1.72	2.02
	(0.53)	(0.56)	(1.21)	(1.08)	(1.00)	(1.12)	(0.55)	(0.76)	(1.45)	(1.33)	(1.21)	(1.26)
1.5	**1.07**	1.08	1.46	1.22	1.26	1.47	**1.07**	1.15	1.75	1.35	1.40	1.62
	(0.37)	(0.40)	(0.93)	(0.68)	(0.65)	(0.78)	(0.39)	(0.55)	(1.11)	(0.85)	(0.81)	(0.89)
1.6	**1.04**	1.04	1.27	1.12	1.14	1.29	**1.04**	1.08	1.48	1.20	1.23	1.38
	(0.27)	(0.30)	(0.71)	(0.45)	(0.45)	(0.58)	(0.28)	(0.41)	(0.88)	(0.58)	(0.57)	(0.66)
1.7	**1.02**	1.02	1.17	1.07	1.08	1.18	**1.02**	1.05	1.30	1.11	1.13	1.24
	(0.20)	(0.22)	(0.56)	(0.32)	(0.32)	(0.44)	(0.21)	(0.31)	(0.70)	(0.41)	(0.41)	(0.51)
1.8	**1.01**	1.01	1.10	1.04	1.05	1.12	**1.01**	1.03	1.19	1.06	1.08	1.15
	(0.15)	(0.17)	(0.43)	(0.22)	(0.24)	(0.35)	(0.15)	(0.22)	(0.56)	(0.29)	(0.32)	(0.40)
1.9	**1.01**	1.01	1.06	1.02	1.03	1.07	**1.01**	1.02	1.12	1.04	1.05	1.10
	(0.12)	(0.12)	(0.33)	(0.16)	(0.18)	(0.27)	(0.12)	(0.18)	(0.44)	(0.22)	(0.24)	(0.32)
2	**1.00**	1.01	1.04	1.01	1.02	1.05	**1.00**	1.01	1.07	1.02	1.03	1.06
	(0.08)	(0.10)	(0.26)	(0.12)	(0.14)	(0.21)	(0.09)	(0.13)	(0.34)	(0.16)	(0.18)	(0.25)
EQL	12.66	12.66	14.26	14.06	14.08	14.75	12.69	12.99	15.43	14.64	14.68	15.32
RARL	1.00	1.00	1.13	1.11	1.11	1.17	1.00	1.02	1.22	1.15	1.16	1.21
PCI	1.00	1.02	1.65	1.55	1.55	1.82	1.00	1.13	2.07	1.76	1.77	2.01
	$\lambda = 0.2$						$\lambda = 0.3$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 0.426$	1.242	2.353	1.738	1.95	2.354	0.979	1.877	2.475	1.964	2.161	2.449
1	200.27	200.96	200.15	200.18	200.64	200.50	200.70	200.83	199.94	200.15	200.75	200.27
	(842.05)	(292.73)	(193.32)	(211.04)	(207.26)	(202.05)	(371.32)	(206.84)	(195.79)	(206.02)	(204.47)	(200.54)
1.1	**3.36**	7.56	15.48	13.59	13.77	15.99	**5.87**	12.39	16.63	14.79	15.44	18.24
	(4.13)	(8.20)	(11.77)	(12.10)	(12.38)	(14.21)	(6.83)	(10.59)	(13.25)	(13.13)	(13.87)	(16.81)
1.2	**1.92**	3.23	6.00	5.07	5.02	5.64	**2.71**	4.85	6.20	5.37	5.44	6.09
	(1.67)	(2.74)	(4.08)	(4.33)	(4.03)	(4.24)	(2.36)	(3.70)	(4.30)	(4.26)	(4.21)	(4.70)
1.3	**1.46**	2.16	3.51	2.84	2.85	3.17	**1.88**	2.94	3.57	3.03	3.07	3.38
	(1.02)	(1.62)	(2.23)	(2.36)	(2.14)	(2.17)	(1.41)	(2.02)	(2.28)	(2.32)	(2.19)	(2.32)
1.4	**1.24**	1.66	2.49	1.95	1.98	2.22	**1.49**	2.15	2.49	2.11	2.14	2.31
	(0.72)	(1.14)	(1.51)	(1.48)	(1.35)	(1.37)	(1.01)	(1.42)	(1.49)	(1.49)	(1.40)	(1.44)
1.5	**1.13**	1.39	1.93	1.53	1.58	1.73	**1.29**	1.70	1.91	1.62	1.66	1.79
	(0.52)	(0.86)	(1.14)	(0.99)	(0.94)	(0.97)	(0.75)	(1.09)	(1.10)	(1.02)	(0.96)	(1.01)

Table 6: Continued.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.6	**1.07**	1.23	1.59	1.30	1.34	1.46	**1.17**	1.45	1.59	1.37	1.41	1.50
	(0.38)	(0.66)	(0.91)	(0.69)	(0.67)	(0.72)	(0.57)	(0.87)	(0.86)	(0.74)	(0.71)	(0.75)
1.7	**1.04**	1.14	1.38	1.18	1.20	1.30	**1.10**	1.28	1.38	1.22	1.25	1.32
	(0.29)	(0.51)	(0.73)	(0.51)	(0.50)	(0.57)	(0.44)	(0.68)	(0.68)	(0.54)	(0.54)	(0.58)
1.8	**1.02**	1.08	1.25	1.11	1.13	1.19	**1.06**	1.18	1.24	1.14	1.16	1.21
	(0.22)	(0.39)	(0.59)	(0.37)	(0.38)	(0.45)	(0.33)	(0.54)	(0.54)	(0.41)	(0.42)	(0.47)
1.9	**1.01**	1.05	1.16	1.06	1.08	1.13	**1.03**	1.11	1.16	1.09	1.10	1.14
	(0.17)	(0.30)	(0.46)	(0.28)	(0.30)	(0.35)	(0.25)	(0.43)	(0.43)	(0.31)	(0.33)	(0.38)
2	**1.01**	1.03	1.10	1.04	1.05	1.08	**1.02**	1.07	1.10	1.06	1.06	1.09
	(0.12)	(0.23)	(0.36)	(0.21)	(0.23)	(0.29)	(0.19)	(0.34)	(0.34)	(0.25)	(0.26)	(0.30)
EQL	12.91	14.01	16.11	15.21	15.30	15.89	13.57	15.28	16.27	15.54	15.71	16.31
RARL	1.00	1.09	1.25	1.18	1.19	1.23	1.00	1.13	1.20	1.14	1.16	1.20
PCI	1.00	1.33	2.01	1.71	1.73	1.91	1.00	1.38	1.61	1.42	1.45	1.59

Table 7: Performance measures for two-sided THWMAV chart versus two-sided competing charts when $n = 15$ for $ARL_0 \cong 200$.

	$\lambda = 0.05$						$\lambda = 0.1$					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 1.163$	1.2553	2.122	1.513	1.71	2.277	1.186	1.466	2.515	1.796	1.993	2.478
0.5	**1.01** (0.11)	1.01 (0.13)	1.18 (0.54)	1.01 (0.12)	1.03 (0.17)	1.16 (0.37)	**1.01** (0.11)	1.02 (0.21)	1.40 (0.72)	1.04 (0.20)	1.08 (0.27)	1.25 (0.45)
0.55	**1.04** (0.28)	1.05 (0.32)	1.45 (0.81)	1.06 (0.26)	1.11 (0.32)	1.35 (0.52)	**1.04** (0.29)	1.10 (0.43)	1.78 (0.92)	1.14 (0.38)	1.21 (0.44)	1.48 (0.58)
0.6	**1.13** (0.50)	1.17 (0.56)	1.82 (1.02)	1.18 (0.47)	1.26 (0.52)	1.61 (0.69)	**1.14** (0.52)	1.27 (0.69)	2.22 (1.05)	1.33 (0.64)	1.43 (0.65)	1.78 (0.75)
0.65	**1.31** (0.76)	1.38 (0.82)	2.30 (1.21)	1.43 (0.81)	1.51 (0.78)	1.99 (0.92)	**1.33** (0.77)	1.55 (0.95)	2.75 (1.20)	1.68 (0.99)	1.77 (0.92)	2.23 (1.01)
0.7	**1.61** (1.05)	1.71 (1.11)	2.90 (1.46)	1.85 (1.31)	1.95 (1.19)	2.54 (1.29)	**1.64** (1.06)	1.96 (1.24)	3.45 (1.50)	2.25 (1.52)	2.32 (1.36)	2.87 (1.41)
0.75	**2.04** (1.39)	2.17 (1.46)	3.77 (1.97)	2.64 (2.12)	2.67 (1.86)	3.46 (1.94)	**2.07** (1.41)	2.51 (1.61)	4.54 (2.12)	3.24 (2.36)	3.23 (2.05)	3.94 (2.13)
0.8	**2.68** (1.95)	2.88 (2.05)	5.25 (3.02)	4.14 (3.60)	4.05 (3.09)	5.05 (3.11)	**2.73** (1.98)	3.40 (2.31)	6.45 (3.39)	4.98 (3.77)	4.84 (3.30)	5.78 (3.44)
0.85	**3.77** (3.08)	4.10 (3.31)	8.20 (5.41)	7.05 (6.43)	6.82 (5.59)	8.22 (5.63)	**3.84** (3.13)	4.96 (3.84)	10.30 (6.08)	8.31 (6.42)	8.04 (5.79)	9.51 (6.31)
0.9	**5.87** (6.05)	6.63 (6.75)	15.76 (11.81)	14.13 (13.13)	13.56 (11.80)	16.10 (12.29)	**6.05** (6.23)	8.59 (8.29)	19.73 (12.82)	16.16 (12.80)	15.79 (12.43)	19.22 (14.98)
0.95	**11.99** (18.92)	14.86 (22.49)	44.91 (37.87)	38.86 (39.36)	39.39 (39.26)	47.89 (44.43)	**12.68** (19.88)	22.50 (29.84)	53.62 (39.19)	45.37 (42.54)	47.62 (45.65)	59.78 (55.78)
1	199.02 (2345.54)	200.43 (950.48)	199.75 (173.11)	200.36 (259.54)	200.42 (246.45)	199.93 (216.08)	200.48 (1697.43)	200.68 (418.10)	200.42 (157.24)	200.03 (226.73)	200.40 (218.14)	200.82 (205.66)
1.1	**5.56** (6.33)	6.44 (7.35)	16.08 (13.57)	14.70 (15.14)	14.09 (13.78)	15.93 (13.63)	**5.76** (6.59)	8.42 (9.27)	19.72 (14.22)	16.71 (14.71)	16.17 (14.21)	18.24 (15.29)
1.2	**2.93** (2.46)	3.13 (2.62)	5.82 (4.27)	5.07 (5.20)	4.86 (4.49)	5.56 (4.25)	**2.99** (2.50)	3.74 (3.08)	7.14 (4.79)	5.92 (5.26)	5.63 (4.66)	6.23 (4.58)
1.3	**2.04** (1.52)	2.17 (1.62)	3.47 (2.30)	2.72 (2.59)	2.67 (2.24)	3.10 (2.16)	**2.08** (1.55)	2.45 (1.79)	4.10 (2.52)	3.19 (2.83)	3.09 (2.42)	3.47 (2.33)
1.4	**1.61** (1.11)	1.68 (1.17)	2.47 (1.60)	1.84 (1.52)	1.83 (1.33)	2.16 (1.36)	**1.63** (1.13)	1.86 (1.30)	2.86 (1.72)	2.10 (1.71)	2.08 (1.49)	2.37 (1.47)
1.5	**1.37** (0.85)	1.41 (0.90)	1.93 (1.25)	1.45 (0.98)	1.47 (0.89)	1.70 (0.96)	**1.38** (0.87)	1.53 (1.01)	2.21 (1.33)	1.60 (1.13)	1.61 (1.01)	1.83 (1.03)
1.6	**1.22** (0.66)	1.25 (0.70)	1.61 (1.01)	1.25 (0.67)	1.27 (0.63)	1.43 (0.71)	**1.23** (0.67)	1.32 (0.78)	1.79 (1.08)	1.34 (0.78)	1.37 (0.72)	1.52 (0.77)
1.7	**1.14** (0.52)	1.15 (0.55)	1.38 (0.81)	1.14 (0.47)	1.16 (0.45)	1.27 (0.54)	**1.14** (0.52)	1.20 (0.62)	1.53 (0.90)	1.20 (0.56)	1.22 (0.53)	1.34 (0.61)
1.8	**1.08** (0.41)	1.09 (0.43)	1.25 (0.67)	1.08 (0.34)	1.10 (0.34)	1.18 (0.43)	**1.09** (0.41)	1.12 (0.48)	1.35 (0.74)	1.12 (0.41)	1.14 (0.4)	1.22 (0.47)
1.9	**1.05** (0.31)	1.05 (0.33)	1.17 (0.54)	1.05 (0.25)	1.06 (0.26)	1.12 (0.34)	**1.05** (0.32)	1.08 (0.39)	1.24 (0.62)	1.07 (0.30)	1.09 (0.31)	1.14 (0.38)

Table 7: Continued.

	λ = 0.05						λ = 0.1					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
2	**1.03**	1.03	1.11	1.03	1.04	1.08	**1.03**	1.05	1.15	1.04	1.06	1.10
	(0.25)	(0.26)	(0.43)	(0.19)	(0.20)	(0.28)	(0.25)	(0.30)	(0.49)	(0.23)	(0.24)	(0.31)
EQL	13.16	13.48	16.36	15.45	15.39	16.21	13.29	14.15	17.57	16.15	16.16	17.15
RARL	1.00	1.02	1.24	1.17	1.17	1.23	1.00	1.06	1.32	1.21	1.22	1.29
PCI	1.00	1.05	1.69	1.38	1.38	1.60	1.00	1.17	1.95	1.53	1.53	1.75

	λ = 0.2						λ = 0.3					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
δ	$L = 1.418$	1.994	2.736	2.202	2.294	2.645	1.795	2.463	2.785	2.297	2.47	2.719
0.5	**1.07**	1.37	1.47	1.17	1.25	1.39	**1.07**	1.37	1.47	1.17	1.25	1.39
	(0.36)	(0.70)	(0.59)	(0.39)	(0.45)	(0.52)	(0.36)	(0.70)	(0.59)	(0.39)	(0.45)	(0.52)
0.55	**1.23**	1.73	1.80	1.38	1.48	1.67	**1.23**	1.73	1.80	1.38	1.48	1.67
	(0.63)	(0.92)	(0.75)	(0.58)	(0.59)	(0.66)	(0.63)	(0.92)	(0.75)	(0.58)	(0.59)	(0.66)
0.6	**1.51**	2.18	2.22	1.71	1.80	2.05	**1.51**	2.18	2.22	1.71	1.80	2.05
	(0.89)	(1.06)	(0.89)	(0.82)	(0.78)	(0.86)	(0.89)	(1.06)	(0.89)	(0.82)	(0.78)	(0.86)
0.65	**1.89**	2.70	2.76	2.18	2.27	2.60	**1.89**	2.70	2.76	2.18	2.27	2.60
	(1.12)	(1.21)	(1.12)	(1.13)	(1.05)	(1.19)	(1.12)	(1.21)	(1.12)	(1.13)	(1.05)	(1.19)
0.7	**2.41**	3.39	3.56	2.90	2.95	3.46	**2.41**	3.39	3.56	2.90	2.95	3.46
	(1.39)	(1.49)	(1.58)	(1.57)	(1.48)	(1.77)	(1.39)	(1.49)	(1.58)	(1.57)	(1.48)	(1.77)
0.75	**3.12**	4.45	4.91	4.03	4.10	4.95	**3.12**	4.45	4.91	4.03	4.10	4.95
	(1.81)	(2.10)	(2.48)	(2.29)	(2.24)	(2.91)	(1.81)	(2.10)	(2.48)	(2.29)	(2.24)	(2.91)
0.8	**4.27**	6.30	7.38	5.96	6.13	7.86	**4.27**	6.30	7.38	5.96	6.13	7.86
	(2.67)	(3.35)	(4.27)	(3.58)	(3.77)	(5.45)	(2.67)	(3.35)	(4.27)	(3.58)	(3.77)	(5.45)
0.85	**6.50**	10.04	12.53	9.93	10.60	14.48	**6.50**	10.04	12.53	9.93	10.60	14.48
	(4.68)	(6.01)	(8.32)	(6.79)	(7.68)	(11.69)	(4.68)	(6.01)	(8.32)	(6.79)	(7.68)	(11.69)
0.9	**12.10**	19.26	26.30	20.82	23.43	34.19	**12.10**	19.26	26.30	20.82	23.43	34.19
	(10.42)	(12.71)	(20.28)	(17.20)	(20.44)	(31.54)	(10.42)	(12.71)	(20.28)	(17.20)	(20.44)	(31.54)
0.95	**34.87**	52.53	77.90	67.35	76.22	101.27	**34.87**	52.53	77.90	67.35	76.22	101.27
	(35.92)	(38.63)	(68.91)	(65.03)	(73.91)	(99.67)	(35.92)	(38.63)	(68.91)	(65.03)	(73.91)	(99.67)
1	200.43	200.55	200.39	200.60	200.93	200.50	200.43	200.55	200.39	200.60	200.93	200.50
	(234.77)	(156.29)	(191.41)	(205.54)	(203.46)	(201.16)	(234.77)	(156.29)	(191.41)	(205.54)	(203.46)	(201.16)
1.1	**12.38**	19.34	22.79	20.31	21.47	26.16	**12.38**	19.34	22.79	20.31	21.47	26.16
	(12.19)	(14.14)	(18.24)	(17.85)	(19.3)	(24.19)	(12.19)	(14.14)	(18.24)	(17.85)	(19.30)	(24.19)
1.2	**4.71**	7.00	7.66	6.72	6.72	7.59	**4.71**	7.00	7.66	6.72	6.72	7.59
	(3.69)	(4.76)	(5.21)	(4.97)	(5.04)	(5.9)	(3.69)	(4.76)	(5.21)	(4.97)	(5.04)	(5.9)
1.3	**2.94**	4.02	4.20	3.71	3.66	3.94	**2.94**	4.02	4.20	3.71	3.66	3.94
	(2.06)	(2.49)	(2.59)	(2.66)	(2.51)	(2.69)	(2.06)	(2.49)	(2.59)	(2.66)	(2.51)	(2.69)
1.4	**2.17**	2.82	2.85	2.47	2.48	2.64	**2.17**	2.82	2.85	2.47	2.48	2.64
	(1.47)	(1.70)	(1.67)	(1.71)	(1.60)	(1.64)	(1.47)	(1.70)	(1.67)	(1.71)	(1.60)	(1.64)
1.5	**1.72**	2.18	2.17	1.87	1.88	1.99	**1.72**	2.18	2.17	1.87	1.88	1.99
	(1.13)	(1.33)	(1.22)	(1.21)	(1.12)	(1.13)	(1.13)	(1.33)	(1.22)	(1.21)	(1.12)	(1.13)
1.6	**1.45**	1.77	1.77	1.52	1.55	1.64	**1.45**	1.77	1.77	1.52	1.55	1.64
	(0.91)	(1.08)	(0.97)	(0.88)	(0.83)	(0.84)	(0.91)	(1.08)	(0.97)	(0.88)	(0.83)	(0.84)

Table 7: Continued.

	λ = 0.05						λ = 0.1					
	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV	THWMAV	DHWMAV	HWMAV	TEWMAV	DEWMAV	EWMAV
1.7	**1.29**	1.51	1.50	1.32	1.35	1.42	**1.29**	1.51	1.50	1.32	1.35	1.42
	(0.72)	(0.90)	(0.77)	(0.66)	(0.63)	(0.66)	(0.72)	(0.90)	(0.77)	(0.66)	(0.63)	(0.66)
1.8	**1.18**	1.34	1.33	1.20	1.23	1.28	**1.18**	1.34	1.33	1.20	1.23	1.28
	(0.58)	(0.73)	(0.62)	(0.51)	(0.50)	(0.53)	(0.58)	(0.73)	(0.62)	(0.51)	(0.50)	(0.53)
1.9	**1.11**	1.22	1.22	1.12	1.15	1.19	**1.11**	1.22	1.22	1.12	1.15	1.19
	(0.46)	(0.59)	(0.50)	(0.38)	(0.40)	(0.43)	(0.46)	(0.59)	(0.50)	(0.38)	(0.40)	(0.43)
2	**1.07**	1.15	1.15	1.08	1.10	1.13	**1.07**	1.15	1.15	1.08	1.10	1.13
	(0.36)	(0.49)	(0.41)	(0.31)	(0.31)	(0.36)	(0.36)	(0.49)	(0.41)	(0.31)	(0.31)	(0.36)
EQL	13.99	15.99	18.18	16.96	17.21	18.58	15.29	17.44	18.84	17.67	18.17	19.94
RARL	1.00	1.14	1.30	1.21	1.23	1.33	1.00	1.14	1.23	1.16	1.19	1.30
PCI	1.00	1.34	1.75	1.44	1.46	1.68	1.00	1.32	1.43	1.23	1.27	1.46

RARL, and PCI values of the upper and two-sided THWMAV charts are 13.64, 1.00, 1.00, and 16.07, 1.00, 1.00, respectively, which suggests the better overall detection ability for the upper and two-sided THWMAV charts against the upper and two-sided DHWMAV charts that have the EQL, RARL, and PCI values of 14.79, 1.08, 1.29, and 18.93, 1.18, 1.32, respectively (see Tables 2 and 3).

5.6. Main Findings of the Study. Some essential findings of this study can be given as follows:

(i) The one and two-sided THWMAV charts undoubtedly boost the efficiency of the process dispersion monitoring.

(ii) The one and two-sided THWMAV charts perform better against the competing DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts for all parametric choices.

(iii) The one and two-sided THWMAV charts reveal better overall detection ability against the competing DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts.

(iv) The performance for the one and two-sided THWMAV charts deteriorates as the value of λ increases.

(v) The OC performance for the one and two-sided THWMAV charts improves as the sample size n increases.

(vi) The overall performance for the one and two-sided THWMAV charts degrades as the value of λ increases.

6. Application of THWMAV Charts

This section analyzes the dataset of the wind station, Dhahran (26°32′ N, 50°14′E, Saudi Arabia), to demonstrate the practical application of the one and two-sided THWMAV charts. This dataset is considered from the study of Riaz et al. [21], which addresses the daily power generated by the wind station during the winter (15 Nov. to 29 Feb. 2020). Table 8 presents the dataset consisting of 21 subgroups, each of size $n = 5$. The A – D (Anderson–Darling) test determines normal distribution as a goodness of fit to the dataset. The A – D test provides the p value as 0.353, which suggests that the underlying process of the daily power generating at the wind station follows the normal distribution having a mean of 0 and standard deviation of 1.1, i.e., $N(0, 1.1)$. This means that this dataset can be regarded as random samples from the IC process having a standard deviation of 1.1, i.e., $\sigma_0 = 1.1$. To implement the upper-sided THWMAV chart, following Riaz et al. [21], the shift of moderate size, i.e., $\delta = 1.2$, is introduced artificially in the dataset at subgroup number 16 so that OC process standard deviation σ_1 is equal to 1.32. At $\text{ARL}_0 = 200$, the design parameters for the upper-sided THWMAV, DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts are chosen such as $(\lambda, L_{\text{THWMAV}}) = (0.2, 0.429)$, $(\lambda, L_{\text{DHWMAV}}) = (0.2, 1.245)$, $(\lambda, L_{\text{HWMAV}}) = (0.2, 2.352)$, $(\lambda, L_{\text{TEWMAV}}) = (0.2, 1.738)$, $(\lambda, L_{\text{DEWMAV}}) = (0.2, 1.954)$, and $(\lambda, L_{\text{EWMAV}}) = (0.1, 2.355)$, respectively. Using these design parameters, respectively, the upper-sided THWMAV, DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts are designed, and the statistics TH_t, DH_t, H_t, TE_t, DE_t and E_t, respectively, are plotted against the subgroup number t. The upper-sided THWMAV, DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts are depicted in Figure 3, which indicates that the upper-sided THWMAV chart detects OC signal at the 18th subgroup number, while the upper-sided DHWMAV, DEWMAV, and EWMAV charts detect the OC signal at subgroup number 21, 21, 19, respectively, and the HWMAV and TEWMAV charts do not detect any OC signal (see Table 8 and Figure 3). This indicates that the upper-sided THWMAV chart is more sensitive than the upper-sided competing charts.

Similarly, to construct the two-sided THWMAV chart, the artificial shift of size 1.25 is introduced in the dataset at the start of the process. In this case, the OC process dispersion σ_1 is equal to 1.375. At $\text{ARL}_0 \cong 200$, the plotting statistics TH_t, DH_t, H_t, TE_t, DE_t, and E_t for the two-sided charts are constructed using the design parameters for the two-sided THWMAV, DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts, i.e., $(\lambda, L_{\text{THWMAV}}) = (0.1, 1.189)$, $(\lambda, L_{\text{DHWMAV}}) = (0.1, 1.468)$, $(\lambda, L_{\text{HWMAV}}) = (0.1, 2.517)$, $(\lambda, L_{\text{TEWMAV}}) = (0.1, 1.792)$ $(\lambda, L_{\text{DEWMAV}}) = (0.1, 1.997)$, and $(\lambda, L_{\text{EWMAV}}) = (0.1, 2.482)$, respectively. The two-sided charts are depicted in Figure 4, which demonstrate that the two-sided THWMAV chart triggers the OC signal at the 8th subgroup number, while the two-sided DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts trigger the OC signal at the 8th, 11th, 15th, 12th, and 10th subgroup numbers, respectively (see Table 8 and Figure 4). In addition, the two-sided THWMAV chart overall triggers 14 OC signals, while the two-sided DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts overall detect 14, 8, 7, 10, and 10 OC signals, respectively. This reveals that the two-sided THWMAV chart is more sensitive than the two-sided competing charts.

Table 8: Wind farm data along with plotting statistics for one and two-sided THWMAV charts versus one and two-sided competing charts.

						Upper-sided charts at $\lambda = 0.1$ and $\delta = 1.2$							Two-sided charts at $\lambda = 0.2$ and $\delta = 1.25$						
t	X_{1t}	X_{2t}	X_{3t}	X_{4t}	X_{5t}	V_t	TH_t	DH_t	H_t	TE_t	DH_t	E_t	V_t	TH_t	DH_t	H_t	TE_t	DH_t	E_t
1	0.6243	0.3376	−0.9172	0.0709	0.5376	−1.0988	−0.0088	−0.0440	−0.2198	−0.0088	−0.0440	−0.2198	−0.6268	−0.0006	−0.0063	−0.0627	−0.0006	−0.0063	−0.0627
2	0.0718	0.3996	−0.0596	1.3659	0.3227	−1.3047	−1.1005	−1.1071	−1.1400	−0.0315	−0.1225	−0.4367	−0.8592	−0.0026	−0.6291	−0.6500	−0.0026	−0.0199	−0.1423
3	−0.0353	−1.2937	−0.0296	0.0709	0.6338	−0.8406	−1.1988	−1.1873	−1.1295	−0.0655	−0.2015	−0.5175	−0.3334	−0.0057	−0.7389	−0.7020	−0.0057	−0.0340	−0.1614
4	−0.4403	−1.4717	0.0898	−0.4838	1.2130	−0.0752	−1.0733	−1.0411	−0.8801	−0.1018	−0.2470	−0.4290	0.5466	−0.0091	−0.5949	−0.4912	−0.0091	−0.0397	−0.0906
5	−0.7443	−1.8528	−0.1009	0.9281	1.8177	1.0366	−0.8149	−0.7552	−0.4565	−0.1264	−0.2248	−0.1359	1.8487	−0.0107	−0.2965	−0.1015	−0.0107	−0.0254	0.1033
6	−1.1330	−1.6470	0.6685	1.3644	−0.6021	0.6235	−0.4479	−0.4133	−0.2405	−0.1365	−0.1766	0.0160	1.3620	−0.0096	0.1276	0.2399	−0.0096	0.0001	0.2292
7	−1.6013	−1.7784	0.6058	1.5148	0.2711	1.0580	−0.2659	−0.2232	−0.0096	−0.1285	−0.0964	0.2244	1.8739	−0.0047	0.3385	0.4781	−0.0047	0.0394	0.3036
8	−1.3388	−1.4219	1.3121	1.1986	0.0989	0.7815	−0.0790	−0.0512	0.0876	−0.1048	−0.0100	0.3358	1.5478	0.0044	0.5546	0.6449	0.0044	0.0864	0.3691
9	−0.1471	−0.7052	0.6952	1.8248	−0.2702	−0.0153	0.0222	0.0210	0.0150	−0.0748	0.0451	0.2656	0.6161	0.0169	0.6694	0.6646	0.0169	0.1297	0.4298
10	0.4738	−1.2046	1.3146	1.5710	0.2134	0.2250	0.0207	0.0302	0.0778	−0.0423	0.0876	0.2574	0.8957	0.0325	0.6663	0.6871	0.0325	0.1725	0.5573
11	−0.6251	−0.5070	2.4901	1.6751	−0.0464	0.9765	0.0558	0.0867	0.2411	−0.0038	0.1503	0.4013	1.7777	0.0515	0.6980	0.7962	0.0515	0.2232	0.6794
12	−0.9566	−0.1195	−0.1200	0.4168	0.2475	−1.4091	0.1224	0.0686	−0.2001	0.0226	0.1281	0.0392	−0.9766	0.0716	0.7686	0.6100	0.0716	0.2522	0.5138
13	−1.2295	0.8355	−0.9625	−0.2670	−0.4800	−0.5697	−0.0085	−0.0261	−0.1138	0.0353	0.0860	−0.0826	−0.0237	0.0917	0.6327	0.5731	0.0917	0.2730	0.4600
14	−0.5131	−0.8745	1.6919	−0.6865	−1.8279	0.7237	−0.0453	−0.0411	−0.0200	0.0451	0.0845	0.0787	1.4799	0.1128	0.5973	0.6775	0.1128	0.3019	0.5620
15	0.6651	−0.3131	3.1758	−0.4326	−0.1728	1.2290	−0.0268	0.0137	0.2163	0.0620	0.1293	0.3087	2.0763	0.1358	0.6663	0.7945	0.1358	0.3431	0.7134
16	0.8609	1.2055	0.7537	−0.5934	0.6171	−0.5034	0.0431	0.0254	−0.0627	0.0761	0.1327	0.1463	−0.4073	0.1591	0.7354	0.6316	0.1591	0.3689	0.6014
17	−0.0632	−3.9879	−0.3150	−0.0427	−0.2883	2.4264	0.0324	0.1096	0.4957	0.1062	0.2267	0.6023	2.6094	0.1844	0.6942	0.8683	0.1844	0.4122	0.8022
18	−1.1750	−0.0632	−0.2966	0.2486	0.1488	−0.9105	0.1465	0.1124	−0.0581	0.1332	0.2413	0.2998	−0.8242	0.2095	0.7725	0.6274	0.2095	0.4350	0.6395
19	−1.8194	−0.1347	2.6720	−0.3043	0.6523	2.2484	0.1130	0.1819	0.5263	0.1728	0.3309	0.6895	2.4255	0.2359	0.7163	0.8717	0.2359	0.4733	0.8181
20	−1.2295	−1.6588	1.8223	−1.1769	0.3310	1.7330	0.2213	0.2701	0.5139	0.2271	0.4444	0.8982	1.8933	0.2641	0.8009	0.9002	0.2641	0.5185	0.9256
21	−0.3456	−1.3377	0.7569	0.2470	0.6539	0.0687	0.2836	0.2766	0.2420	0.2821	0.5020	0.7323	0.1796	0.2929	0.8384	0.7785	0.2929	0.5518	0.8510

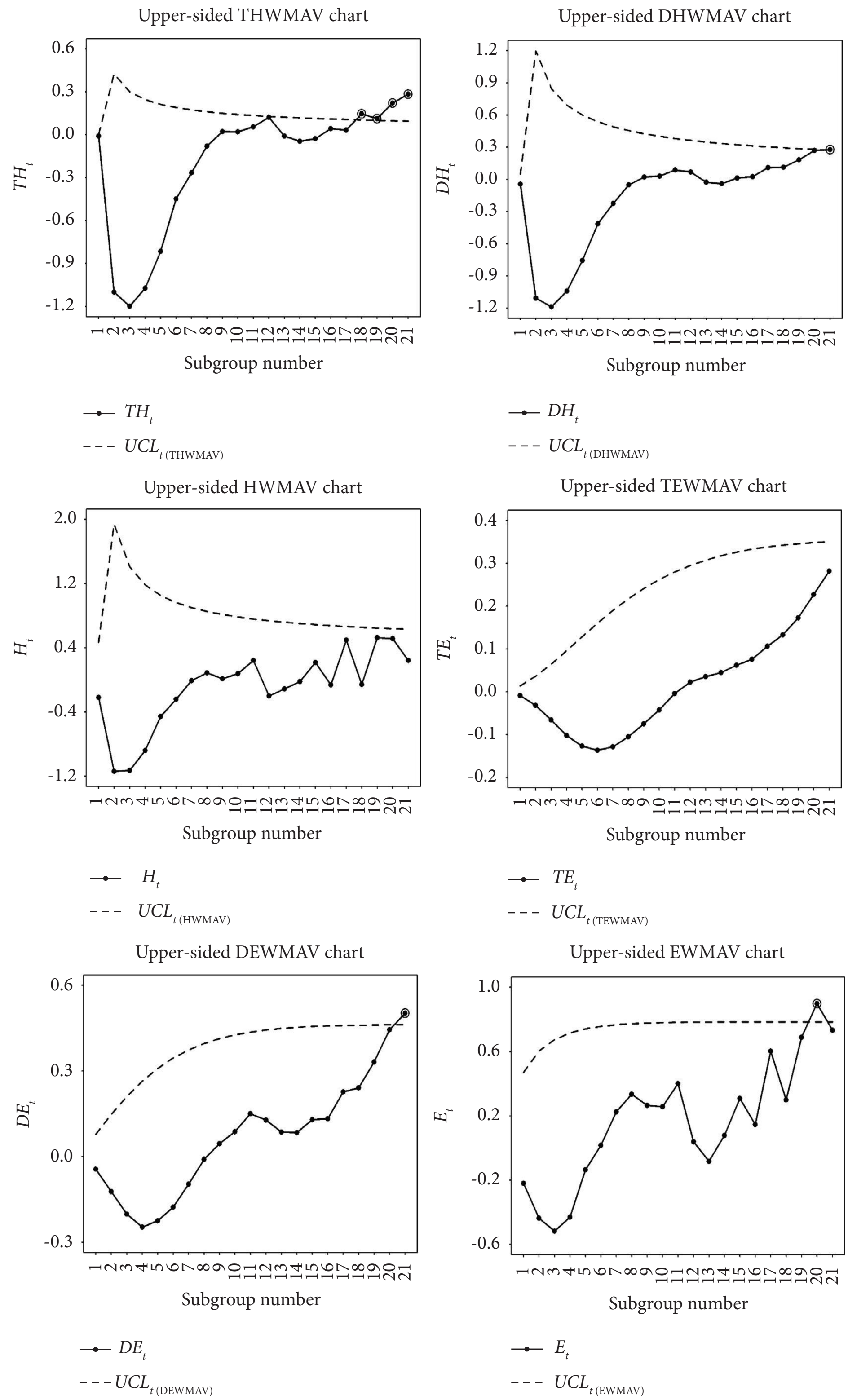

Figure 3: The upper-sided THWMAV chart versus upper-sided competing charts for wind farm data at $\lambda = 0.1$ and $\delta = 1.2$.

7. Summary, Conclusions, and Recommendations

This paper introduced the one and two-sided triple homogenously weighted moving average charts, denoted as THWMAV charts, to detect the process dispersion shifts. The upper-sided THWMAV chart monitors the upward shifts in the process dispersion. In contrast, the two-sided THWMAV chart monitors both upward and downward shifts in the process dispersion parameter. Extensive Monte

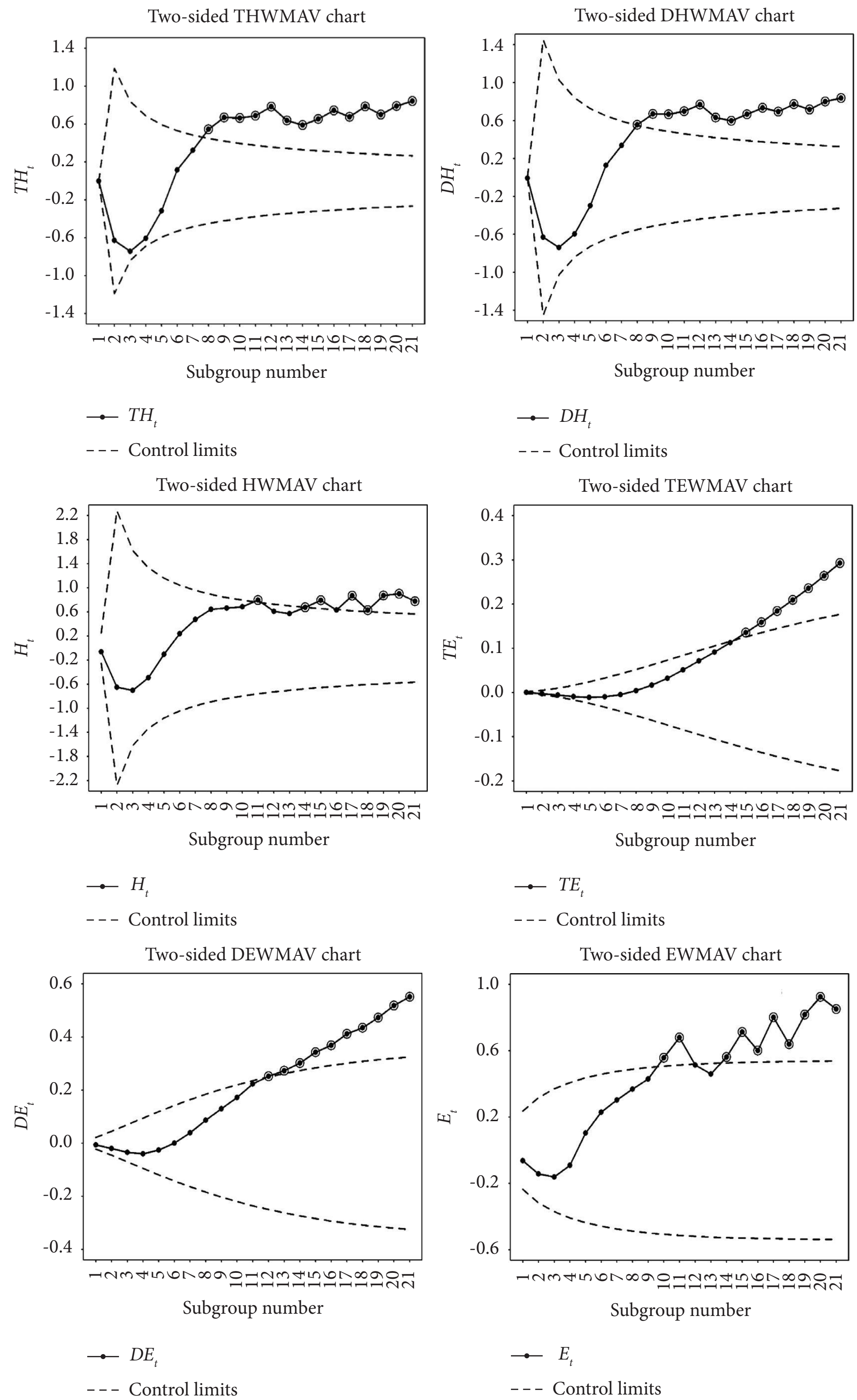

Figure 4: The two-sided THWMAV chart versus two-sided competing charts for wind farm data at $\lambda = 0.2$ and $\delta = 1.25$.

Carlo simulations are used to compute approximate run-length indicators, such as ARL and SDRL for the one and two-sided THWMAV charts. Also, to investigate the overall performance of the THWMAV charts, EQL, RARL, and PCI measures are computed. A comprehensive comparative study is performed, and the one and two-sided THWMAV charts are compared to the competing one and two-sided DHWMAV, HWMAV, TEWMAV, DEWMAV, and EWMAV charts. The comparison demonstrated that the THWMAV charts have better detection ability than the competing charts. Finally, the dataset of the wind farm is also analyzed to support the one and two-sided THWMAV charts, which also reveals that the THWMAV charts are more sensitive in detecting process dispersion shifts. Finally, several issues are recommended for future studies of the THWMAV charts. For example, the concepts of the THWMAV charts can be used with the idea of neutrosophic statistics [32, 33], in the high-yield processes [34], in multivariate process cases [35], etc.

Appendix

A. Process Variable and Transformation

Let X_{it} $(i = 1, 2, \ldots, n,\ t = 1, 2, \ldots)$ denote the process variable having independently identically normal distribution with mean μ_0 and dispersion $\delta\sigma_0$, where δ denotes the amount of shift in process dispersion. When the process is IC then $\delta = 1$; otherwise, $\delta < 1$ for downward shifts and $\delta > 1$ for upward shifts. Let $\overline{X}_t = 1/n \sum_{i=1}^{n} X_{it}$ be the sample mean and $S_t^2 = 1/n - 1 \sum_{i=1}^{n} (X_{it} - \overline{X}_t)^2$ denote the sample variance for the t^{th} subgroup having size n; then, for the IC process, S_t^2 has a chi-square distribution with degrees of freedom equal to $n-1$, i.e., $S_t^2 \sim \sigma_0^2/n - 1\chi^2_{(n-1)}$. In this case, according to Quesenberry [28], the transformation $V_t = \Phi^{-1}(G((n-1)S_t^2/\sigma_0^2; n-1))$ is known as the independently identically standard normal random variable, i.e., $V_t \sim N(0,1)$ for $t = 1, 2, \ldots$. So,

$$E(V_t) = 0, \operatorname{Var}(V_t) = 1, \operatorname{Cov}(V_t, V_s) = 0, \quad \text{for } t = 1, 2, \ldots, t \neq s. \tag{A.1}$$

For the IC process, the expected value of $\overline{V}_t$ is given as

$$\begin{aligned} E(\overline{V}_t) &= \frac{1}{t} E\left(\sum_{j=1}^{t} V_j\right) \\ &= \frac{1}{t} E(V_1 + V_2 + \ldots + V_t) \\ &= \frac{1}{t}[E(V_1) + E(V_2) + \ldots + E(V_t)] = 0. \end{aligned} \tag{A.2}$$

Similarly, the variance of $\overline{V}_t$ is given as

$$\begin{aligned} \operatorname{Var}(\overline{V}_t) &= \frac{1}{t^2} \operatorname{Var}\left(\sum_{j=1}^{t} V_j\right) \\ &= \frac{1}{t^2}\left[\sum_{j=1}^{t} \operatorname{Var}(V_j) + 2 \sum_{j<k} \operatorname{Cov}(V_j, V_k)\right] \\ &= \frac{1}{t^2}[1 + 1 + \ldots + 1] = \frac{1}{t}. \end{aligned} \tag{A.3}$$

Likewise, the covariance of V_t and $\overline{V}_{t-1}$, i.e., $\operatorname{Cov}(V_t, \overline{V}_{t-1})$, is given as

$$\begin{aligned} \operatorname{Cov}(V_t, \overline{V}_{t-1}) &= \operatorname{Cov}\left(V_t, \frac{V_1 + V_2 + \ldots + V_{t-1}}{t-1}\right) \\ &= \frac{1}{t-1}[\operatorname{Cov}(V_t, V_1) + \operatorname{Cov}(V_t, V_2) + \ldots + \operatorname{Cov}(V_t, V_{t-1})] \\ &= \frac{1}{t-1}[0 + 0 + \ldots + 0] = 0. \end{aligned} \tag{A.4}$$

A_1. Mean and Variance of HWMAV *Chart.* The plotting statistic of the HWMAV chart is defined as

$$H_t = \lambda V_t + (1-\lambda)\overline{V}_{t-1}. \tag{A.5}$$

For the stable process, the mean of H_t is given as

$$\begin{aligned} E(H_t) &= E[\lambda V_t + (1-\lambda)\overline{V}_{t-1}] \\ &= \lambda E(V_t) + (1-\lambda) E(\overline{V}_{t-1}) \\ &= \lambda(0) + (1-\lambda)(0) = 0 \end{aligned} \tag{A.6}$$

(cf. equations A.1 and A.2).

Similarly, the variance of H_t is given as

$$\begin{aligned} \operatorname{Var}(H_t) &= \operatorname{Var}[\lambda V_t + (1-\lambda)\overline{V}_{t-1}] \\ &= \lambda^2 \operatorname{Var}(V_t) + (1-\lambda)^2 \operatorname{Var}(\overline{V}_{t-1}) + 2\lambda(1-\lambda)\operatorname{Cov}(V_t, \overline{V}_{t-1}) \\ &= \lambda^2(1) + (1-\lambda)^2 \frac{1}{t-1} + 2\lambda(1-\lambda)(0) \end{aligned} \tag{A.7}$$

(cf. equations A.1, A.3, and A.4)

$$= \left\{\lambda^2 + \frac{(1-\lambda)^2}{t-1}\right\},$$
$$\text{or } \mathrm{Var}(H_t) = \lambda^2, \text{if } t = 1,\ \mathrm{Var}(H_t) = \left\{\lambda^2 + \frac{(1-\lambda)^2}{t-1}\right\}, \text{if } t > 1. \tag{A.8}$$

A_2. *Mean and Variance of* DHWMAV *Chart.* The DHWMAV statistic DH_t is given as

$$\begin{aligned}\mathrm{DH}_t &= \lambda H_t + (1-\lambda)\overline{V}_{t-1}\\ &= \lambda\left[\lambda V_t + (1-\lambda)\overline{V}_{t-1}\right] + (1-\lambda)\overline{V}_{t-1}\end{aligned} \tag{A.9}$$

(cf. equation A.5)

$$= \lambda^2 V_t + \left(1-\lambda^2\right)\overline{V}_{t-1}. \tag{A.10}$$

For the IC process, the mean of DH_t is given as

$$\begin{aligned}E(\mathrm{DH}_t) &= E\left[\lambda^2 V_t + \left(1-\lambda^2\right)\overline{V}_{t-1}\right]\\ &= \lambda^2 E(V_t) + \left(1-\lambda^2\right)E\left(\overline{V}_{t-1}\right)\\ &= \lambda^2(0) + \left(1-\lambda^2\right)(0) = 0\end{aligned} \tag{A.11}$$

(cf. equations A.1 and A.2).

In the same way, the variance of DH_t is given as

$$\begin{aligned}\mathrm{Var}(\mathrm{DH}_t) &= \mathrm{Var}\left[\lambda^2 V_t + \left(1-\lambda^2\right)\overline{V}_{t-1}\right]\\ &= \lambda^4 \mathrm{Var}(V_t) + \left(1-\lambda^2\right)^2 \mathrm{Var}\left(\overline{V}_{t-1}\right) + 2\lambda^2\left(1-\lambda^2\right)\mathrm{Cov}\left(V_t, \overline{V}_{t-1}\right)\\ &= \lambda^4(1) + \left(1-\lambda^2\right)^2\left(\frac{1}{t-1}\right) + 2\lambda^2\left(1-\lambda^2\right)(0)\end{aligned} \tag{A.12}$$

(cf. equations A.1, A.3, and A.4)

$$= \left\{\lambda^4 + \frac{\left(1-\lambda^2\right)^2}{t-1}\right\},$$
$$\text{or } Var(\mathrm{DH}_t) = \lambda^4, \text{if } t = 1, \mathrm{Var}(\mathrm{DH}_t) = \left\{\lambda^4 + \frac{\left(1-\lambda^2\right)^2}{t-1}\right\}, \text{if } t > 1. \tag{A.13}$$

A_3. *Mean and Variance of* THWMAV *Chart.* The plotting statistic for the THWMAV chart is given as

$$\begin{aligned}TH_t &= \lambda DH_t + (1-\lambda)\overline{V}_{t-1}\\ &= \lambda\left[\lambda^2 V_t + \left(1-\lambda^2\right)\overline{V}_{t-1}\right] + (1-\lambda)\overline{V}_{t-1}\end{aligned} \tag{A.14}$$

(cf. equation A.9)

$$= \lambda^3 V_t + \left(1-\lambda^3\right)\overline{V}_{t-1}. \tag{A.15}$$

For the IC process, the mean of TH_t is given as

$$\begin{aligned}E(\mathrm{TH}_t) &= E\left[\lambda^3 V_t + \left(1-\lambda^3\right)\overline{V}_{t-1}\right]\\ &= \lambda^3 E(V_t) + \left(1-\lambda^3\right)E\left(\overline{V}_{t-1}\right)\\ &= \lambda^3(0) + \left(1-\lambda^3\right)(0) = 0\end{aligned} \tag{A.16}$$

(cf. equations A.1 and A.2).

Similarly, the IC variance of TH_t is defined as

$$\begin{aligned}\operatorname{Var}(\mathrm{TH}_t) &= \operatorname{Var}\left[\lambda^3 V_t + \left(1-\lambda^3\right)\overline{V}_{t-1}\right] \\ &= \lambda^6 \operatorname{Var}(V_t) + \left(1-\lambda^3\right)^2 \operatorname{Var}\left(\overline{V}_{t-1}\right) + 2\lambda^3\left(1-\lambda^3\right)\operatorname{Cov}\left(V_t, \overline{V}_{t-1}\right) \\ &= \lambda^6(1) + \left(1-\lambda^3\right)^2\left(\frac{1}{t-1}\right) + 2\lambda^3\left(1-\lambda^3\right)(0)\end{aligned} \tag{A.17}$$

(cf. equations A.1, A.3, and A.4)

$$\begin{aligned}&= \left\{\lambda^6 + \frac{\left(1-\lambda^3\right)^2}{t-1}\right\}, \\ &\text{or } \operatorname{Var}(TH_t) = \lambda^6, \text{if } t=1, \operatorname{Var}(TH_t) = \left\{\lambda^6 + \frac{\left(1-\lambda^3\right)^2}{t-1}\right\}, \text{if } t>1.\end{aligned} \tag{A.18}$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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