# Retailer-to-Individual Customer Product Supply Strategies Under a Semireal Demand Pattern 

Zhiyi Zhuo © ${ }^{1}$, ${ }^{1}$ Shuhong Chen ( ${ }^{1}$, ${ }^{2}$ Weihua Lin, ${ }^{3}$ Hong Yan ${ }^{(1),}{ }^{4}$ and Yue $\mathbf{H e}{ }^{5}$<br>${ }^{1}$ School of Business, Wuyi University, Wuyishan 354300, Fujian, China<br>${ }^{2}$ School of Mathematics and Computer, Wuyi University, Wuyishan 354300, Fujian, China<br>${ }^{3}$ School of Mathematics and Statistics, Hanshan Normal University, Chaozhou 521041, Guangdong, China<br>${ }^{4}$ School of Management, Zhejiang Shuren University, Hangzhou 310015, Zhejiang, China<br>${ }^{5}$ Business School of Sichuan University, Chengdu 610065, China<br>Correspondence should be addressed to Hong Yan; lgthyan@connect.polyu.hk

Received 16 November 2022; Revised 19 June 2023; Accepted 29 June 2023; Published 6 November 2023
Academic Editor: Arunava Majumder
Copyright © 2023 Zhiyi Zhuo et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.


#### Abstract

In the consumer goods supply chain, there are three different modes of demand: real, false, and semireal. It is an interesting topic to discuss the optimal profit of enterprises according to these three demand patterns. This paper develops three mathematical models that can be used to investigate the factors that affect retailers with respect to the design of product supply strategies for individual customers under a semireal demand pattern and thereby addresses the problem of retailers' maximum profitability. The results of the present study show that these models may effectively help retailers develop appropriate supply strategies for individual customers under a semireal demand pattern; in turn, this may help retailers improve operational performance. The main contribution of the current study lies in the construction of mathematical models of product supply strategies for the individual customer in the off-invoice mode, the scan-back mode, and the unsold-item processing mode under the semireal demand pattern. The effectiveness of the models has been verified through numerical calculations. In concrete management practice, the mathematical model given in this paper can be used to effectively adjust the quantity of goods purchased, correct retail prices, and optimize sales discounts to maximize profits.


## 1. Introduction

Demand theory is the classical theory that studies the laws of supply and demand of goods. It focuses on the basis of supply and demand, the relationship between changes in supply and demand and the market value or price of production, and the effect of changes in supply and demand on market prices. This is demonstrated by the fact that the price of a commodity fluctuates up and down around the value of the commodity. When the supply of a commodity is greater than the demand, the price of the commodity falls or even fluctuates below the value of the commodity; when the supply of a commodity is less than the demand, the price of the commodity rises. Thus, the relationship between customer demand and commodity supply not only affects the rise and fall of commodity prices but also affects the profits of manufacturers and retailers [1]. At present, most scholars' research
on customer demand is from the perspective of marketing. The core idea of these studies is to take demand information as the strategic orientation of the enterprise and to meet the needs of customers through scientific market business strategy analysis and perfect product services. However, customer needs are very complex, and different people, different backgrounds, different economic powers, different social experiences, and different cultures can generate different customer needs, such as multiproduct monopoly market demand [2] and on-demand functions [3]. Thus, customer needs are diverse and multifaceted with some instability.

In the field of consumer goods supply chains, customer demand is the core of product supply. There are two interesting phenomena of customer demand in product marketing. The first interesting phenomenon is that people must eat to live. From this perspective, food is the most important material product to meet human survival needs [4]. However,
with the continuous progress of social productivity, the problem of food and clothing has become a thing of the past. Faced with a dazzling array of food, what to eat today has become a difficult choice for people. The second interesting phenomenon is that the original function of clothes is to keep warm, which is also the most important material product to meet the needs of human survival [5]. However, with the continuous improvement of social productivity, people's income level has been greatly improved. At this time, people buying clothes no longer have obvious adverse effects on their income level and quality of life. At this time, when people are faced with a dazzling array of clothes, they will show a state of pursuing beauty; that is, they buy clothes when they are happy and buy clothes when they are unhappy [6]. Previous studies have shown that the customer needs in these two simple life examples have not only met the basic needs of human survival but have also been sublimated to the realm of ideology. This interesting phenomenon also applies to the vast majority of products. Currently, due to factors such as a wide variety and rapid replacement, the products produced by enterprises have changed from the original basic physiological needs of solving problems to meet the needs of people's increasingly rich cultural, spiritual, and entertainment levels in terms of satisfying customer needs, which is what we call real demand [7-9], false demand [10], and semireal demand [11, 12]. However, Crawford [13] found through common commodity purchase behavior that consumers faced with a large amount of product information are forced to decompose their needs into smaller elements. These elements are neither traditionally nonessential needs nor objectively determined essential needs. Rather, they are the material and symbolic aspects of consumer demand formation under uncertainty, which we call semireal demand demands [11, 12]. We take the simple example of coffee marketing behavior to explore this semireal demand. Drinking coffee is the primary factor because of thirst, which is its essential function. This function focuses on practicality. This attribute of the product is mainly a material guarantee of people's survival needs, corresponding to the real demand [7-9]. However, in today's society, it has become a fashion for people to invite coffee, which is an induced social function. This function of the product focuses on taste and has nothing to do with human survival needs; it is the pursuit of higher spiritual enjoyment after satisfying the survival needs of human beings and therefore belongs to false demand [10]. Semireal demand $[11,12]$ is a property of both essential needs, such as the basic function of coffee being to quench thirst, and these functions are the basic functions to ensure human survival needs; at the same time, there are nonessential needs characteristics, such as the deeper functions of coffee being the function of social work, and these functions are human in the pursuit of a higher level of spiritual needs to meet the survival needs of an ideological and sociocultural process. This type of consumer arrives at the store with a genuinely needed demand, confronted with a stream of information influenced by external factors, and eventually, with the increasing number of external influences, the consumer is forced to divide the demand into ever smaller elements. These
are neither nonbasic needs nor objectively determined basic needs and belong to the state where consumer needs are ambiguous and unstable in material and spiritual terms. We found that from the perspective of the existing product lineup, intermediate-demand products with both essential demand attributes and nonessential demand characteristics exist objectively, and there are many types and quantities of products.

We have sorted out the latest relevant research on demand in the field of product supply chains. In the literature on this topic, Fathollahi-Fard et al. [14] provide a systematic assessment of the submissions of supply chain, production, and operation systems for the sustainable development goals (SDGs) from the perspective of journal special issue editors. Kaviyani-Charati et al. [15] considered two two-tier Stackelberg models for nonagile and agile conditions in the presence of strategic customers and finally determined the optimal production and order quantities and prices with and without agile capabilities. Akbari-Kasgari et al. [16] studied a resilient and sustainable closed-loop supply chain network for copper mines to explore the economic, environmental, and social objectives of a copper mining network by designing a resilience strategy with alternate suppliers, ultimately achieving a reduction in the impact of earthquakes on the mining industry. Shahsavar et al. [17] modeled the biorecycling management of different plastic wastes in cities based on a new integrated multicriteria decision-making (MCDM) approach, and the results showed that urban management still needs an efficient and robust decision-making framework to achieve the concept of green cities and SDGs. Sadri et al. [18] explored a new approach based on green and intelligence components using network data envelopment analysis in the field of port performance evaluation. Ghoushchi et al. [19] used a MCDM model to propose a new approach on how to select the best landfill for medical waste, and the results showed that the criteria used to determine the best landfill selection for medical waste include three categories: environmental, economic, and social. Bhuniya et al. [20] investigated different business strategy models based on trade credit, revenue sharing contracts, variable demand and productivity, and theoretically established global optimization. Bachar et al. [21] developed a flexible production model to rework repairable defective products and outsource them to prevent backlogs. Dey et al. [22], on the other hand, focused on variable lead times and variances under controlled productivity and advertisingdependent demand and quantified the benefits of such reduced lead times on commonly used lot sizes, productivity, safety margins, reorder points, advertising costs, and supplier setup costs. Das et al. [23], on the other hand, found that market demand depends on the selling price and greenness level of the items by studying the coordination of a twochannel supply chain taking into account the greenness level of the items. Barman and De [24] advocate a multi-item deteriorating inventory model that does not allow for shortages by proposing a single-stage EOQ model for deteriorating items where the demand function depends on the nonlinear selling price, nonlinear time, and inventory quantity, with the objective of determining the selling price and
length of time for each item until the inventory reaches zero. Barman et al. [25] propose a production inventory system with a manufacturer-retailer supply chain that deals with noninstantaneously deteriorating products and find that the fixed markup policy offered by the retailer is a signal to the manufacturer to resolve the game between channel members in the supply chain. Barman et al. [26] further explored and compared the optimal pricing strategy with and without government subsidies to maximize the overall profit of the supply chain, and the results of the study showed that members of the dual-channel green supply chain make better decisions with and without government subsidies and improve the market competitiveness of green products.

We found that these studies did not consider the results of consumer product demand attributes. We find that technological advances have led to a wide range of products, and due to the increasing standard of living, consumers are considering not only price as a single decision variable but also more psychological factors, such as status symbols, fashion and beauty, and even social factors. This is also the main factor of market demand segmentation in this paper. The difference between consumer product demand attributes and traditional marketing theory research is that we do not distinguish three different demand modes by the price of the product but by the situation, motivation, and psychological factors of consumers when they buy products. As a basis for the division, that is, what is the main psychological purpose of consumers when purchasing products, we explore the product supply strategies of manufacturers and retailers. This is also the innovation of our research to break through the existing theory. We believe that the comprehensive impact mechanism of various unconscious or subconscious consumption habits on people's main evolutionary needs and motivations and consumer psychology on customer needs under the relationship between market supply and demand is an important problem that urgently needs to be solved. This paper studies the retailer's design and planning of product supply strategies for individual customers under the mode of intermediate demand. This research fills an important theoretical gap in the field of consumer goods supply chains and has certain theoretical research value.

## 2. Variable Function

2.1. Semireal-Demand Function. The demand function is used to represent the correlation between the quantity of this good demanded and the various factors that affect that quantity demanded. At this point, modeling various demand functions has been seen as a fundamental tool for revealing the application of demand theory in various fields. For example, Petruzzi and Dada [27] view demand as a stochastic function of prices so that demand functions can be modeled in additive or multiplicative form. Mills [28] constructs an additive function of demand $D(p, \varepsilon)=y(p)+\varepsilon$, which represents a phenomenal demand curve. Karlin and Carr [29] constructed a multiplicative function of demand $D(p, \varepsilon)=y(p) \varepsilon$, which represents an elastic demand curve. However, these
classical demand models only consider a single decision variable, which does not truly reflect people's choices when faced with various goods. Thus, we also extend the study of the demand function to the range of function values of real demand [7-9], false demand [10], and semireal demand [11, 12] and define a new formulation of the semireal demand function:

$$
\begin{equation*}
D_{h}=\frac{3\left(1-d_{0}\right)}{2}(a-b p+\varepsilon), a, b>0, \varepsilon \in U\left(\mu, \sigma^{2}\right) \tag{1}
\end{equation*}
$$

We found that although the normal distribution and the uniform distribution will show different differences in the results, in essence, the function modeling ideas are the same, so we set $\varepsilon$ as a function that obeys the uniform distribution on the interval $[-(a-b p), a-b p]$. Then, by the nature of the uniform distribution function, it is known that

$$
\begin{equation*}
f(\varepsilon)=\frac{1}{2(a-b p)}, \varepsilon \in[-(a-b p), a-b p] . \tag{2}
\end{equation*}
$$

2.2. Retailer Profit Function. In the course of product operation, the retailer must first purchase wholesale products from the manufacturer. If the quantity $(q)$ of wholesale products purchased by the retailer exceeds the quantity demanded by the customer, there will be surplus products that will occupy the inventory space. In this type of situation in which supply exceeds demand, it is impossible to achieve profit optimization. For example, when the fruit is first marketed, the market demand is high due to small production and thus the price is high, while after a large number of marketings, the market demand tends to level off and the production is large, resulting in low prices. On the other hand, if the quantity of wholesale products purchased is insufficient to allow the retailer to keep up with customer demand, a situation is created in which demand exceeds supply. Maximizing profit is also impossible in this situation since the business fails to profit from customers whose demand cannot be met.

In the situation of oversupply, the quantity of purchased wholesale goods is $q$, the quantity of demand is $p D(p, \varepsilon)$, and the surplus quantity $q-D(p, \varepsilon)$ is processed according to the cost of $h$ RMB yuan per unit. Here, $h \geq-c$ can take a negative value. It represents the residual of the unit product. Thus, in this situation, the retailer's profit is $\prod(q, p)=p D(p$, $\varepsilon)-c q-h[q-D(p, \varepsilon)]$.

Similarly, in the situation of short supply, the quantity of purchased wholesale goods is $q$, the selling price is $p q$, and the loss per unit of the shortage is $S$. At this time, the retailer's profit is $\Pi(q, p)=p q-c q-S[D(p, \varepsilon)-q]$. Taking into consideration the two situations above, the relevant retailer profit is as follows:
$\Pi(q, p)=\left\{\begin{array}{ll}p D(p, \varepsilon)-c q-h[q-D(p, \varepsilon)], & q \geq D(p, \varepsilon) \\ p q-c q-S[D(p, \varepsilon)-q], & q<D(p, \varepsilon)\end{array}\right.$.

Table 1: Parameter index design.

| Symbol | Description |
| :--- | :---: |
| $\alpha$ | Discounted price per product in the off-invoice model |
| $p$ | Retail price set by the retailer for the product |
| $q$ | Retailer's order quantity for the product |
| $u$ | Product utility efficiency |
| $v$ | Product grade efficiency |
| $w_{1}$ | Weight of product utility efficiency |
| $w_{2}$ | Weight of product grade efficiency |
| $d_{0}$ | Minimum value of weight of product utility efficiency |
| $p_{m}$ | Maximum efficiency or reservation price |
| $D_{h}$ | Semireal demand function |
| $\Pi_{R}$ | Retailer profit |
| $a$ | Price |
| $b$ | Demand |

## 3. Parameter Index Design

According to the abovementioned variable functions, this paper attempts to build a mathematical model in which the manufacturer plans a product supply strategy for the individual customer under the false demand pattern. The necessary parameter indexes are designed before modeling (Table 1).

Based on the above-designed variables and their range of values, this paper will strive to establish a mathematical model that describes the retailer's product supply strategy when planning for the individual customer under the real demand pattern. The corresponding optimal purchase quantity and optimal pricing will also be solved.

## 4. Mathematical Model Construction and Analysis

In this section, three mathematical models are built to represent the retailer's planning of product supply strategies for the individual customer under the semireal demand pattern. These models are referred to as the off-invoice mode, the scan-back mode, and the unsold-item processing mode. The corresponding optimal production volume $q$ and optimal pricing $p$ needed to achieve the retailer's optimal profit are solved. To make the study more intuitive, we have made a model diagram (Figure 1).
4.1. Off-Invoice Mode. In this mode of supply, the traditional means of promotion is that when a manufacturer supplies a retailer, each product is sold at an additional discount $\alpha$, which is directly deducted from the purchasing payment. Therefore, the wholesale price of the products actually purchased by the retailer is $w_{0}-\alpha$. At this point, if the retailer's purchase volume is $q \leq D_{h}$, demand exceeds supply. It follows that all the goods will be sold and that the sales income for the retailer is $p q$. If the retailer's purchase volume is $q>D_{h}$, supply exceeds demand. In that case, the retailer's sales income is $p D_{h}$, and the retailer's profit is as follows:

$$
\Pi_{R h}=\left\{\begin{array}{l}
-q\left(w_{0}-\alpha\right)+p q, q \leq D_{h}  \tag{4}\\
-q\left(w_{0}-\alpha\right)+p D_{h}, q>D_{h}
\end{array}\right.
$$

We find that the profit function of the retailer in question is a segmented function after a categorical discussion. Therefore, we combine the expression of the demand function (1) and the expression of the density function (2) to obtain a model of the demand function of the new retailer in the offinvoice mode model using probability statistics [26]:

$$
\begin{align*}
\Pi_{R h}= & -w_{0} q+p \int_{0}^{q} D_{h}(\varepsilon) f(\varepsilon) d \varepsilon+p \int_{q}^{2(a-b p)} q f(\varepsilon) d \varepsilon+\alpha q \\
= & \left(-w_{0}+\alpha\right) q+p \int_{0}^{q} \frac{3\left(1-d_{0}\right)}{2} \cdot \varepsilon \cdot \frac{1}{2(a-b p)} d \varepsilon \\
& +p q\left(1-\frac{q}{2(a-b p)}\right) \\
= & \left(-w_{0}+\alpha+p\right) q-\frac{p q^{2}\left(1+3 d_{0}\right)}{8(a-b p)} . \tag{5}
\end{align*}
$$

To calculate the retailer's optimal purchase volume and optimal pricing in this mode of supply, the optimization method should be used to obtain the derivative of the average profit function for the purchase volume $q$. The derivative should be zero. The details are shown below.

$$
\begin{equation*}
\frac{\partial \Pi_{R h}}{\partial q}=-w_{0}+\alpha+p-\frac{p q\left(1+3 d_{0}\right)}{4(a-b p)}=0 \tag{6}
\end{equation*}
$$

From this, we obtain

$$
\begin{equation*}
q=\frac{4(a-b p)\left(-w_{0}+\alpha+p\right)}{\left(1+3 d_{0}\right) p} \tag{7}
\end{equation*}
$$

From the previous equation,

$$
\begin{align*}
& \frac{\partial \Pi_{R h}}{\partial p}=q-\frac{q^{2}\left(1+3 d_{0}\right)}{8(a-b p)}-\frac{b p q^{2}\left(1+3 d_{0}\right)}{8(a-b p)^{2}} \\
& =\left[8 b^{2} p^{2}-16 a b p+8 a^{2}-a q\left(1+3 d_{0}\right)\right] \frac{q}{8(a-b p)^{2}}=0 . \tag{8}
\end{align*}
$$

Based on Vieta's theorem, we obtain

$$
\begin{align*}
\Delta & =16^{2} a^{2} b^{2}-4 \times 8 b^{2} \times\left(8 a^{2}-a q\left(1+3 d_{0}\right)\right)  \tag{9}\\
& =32 b^{2} a d\left(1+3 d_{0}\right)
\end{align*}
$$



Figure 1: Model diagram.

Using the root formula, we obtain

$$
\begin{equation*}
p=\frac{16 a b+4 b \sqrt{2 a q\left(1+3 d_{0}\right)}}{16 b^{2}}=\frac{4 a+\sqrt{2 a q\left(1+3 d_{0}\right)}}{4 b} . \tag{10}
\end{equation*}
$$

The $q$ value obtained by Solution (7) is the retailer's optimal purchase quantity, and the $p$ value obtained by Solution (10) is the retailer's optimal pricing.
4.2. Scan-Back Mode. In this mode of supply, the manufacturer's giving out profit to the retailer does not take place directly through the deduction of money at the time of purchase. Instead, it occurs after the retailer sells the product. Suppose the manufacturer sets the wholesale price to the retailer at $w_{0}$ and gives the retailer a rebate of $\beta$ per unit product sold. The retailer's purchase volume is $q$. If the
retailer's purchase volume is greater than the demand quantity, supply exceeds demand. In this situation, the rebate is $(p+\beta) D_{h}$, where $p$ represents the retailer's retail price. If the retailer's purchase volume cannot keep up with demand, demand exceeds supply. In this situation, the rebate is $(p+\beta) q$, and the retailer's profit is as follows:

$$
\Pi_{R h}=\left\{\begin{array}{l}
-w_{0} q+(p+\beta) q, q \leq D_{h}  \tag{11}\\
-w_{0} q+(p+\beta) D_{h}, q>D_{h}
\end{array}\right.
$$

We find that the profit function of the retailer in question is a segmented function after a categorical discussion. Therefore, we combine the expression of the demand function (1) and the expression of the density function (2) to obtain a model of the demand function of the new retailer in the scanback mode model using probability statistics [30]:

$$
\begin{align*}
\Pi_{R h} & =-w_{0} q+(p+\beta) \int_{0}^{q} D_{h}(\varepsilon) f(\varepsilon) d \varepsilon+(p+\beta) \int_{q}^{2(a-b p)} q f(\varepsilon) d \varepsilon \\
& =-w_{0} q+(p+\beta) \int_{0}^{q} \frac{3\left(1-d_{0}\right)}{2} \cdot \varepsilon \cdot \frac{1}{2(a-b p)} d \varepsilon+(p+\beta) q \int_{q}^{2(a-b p)} \frac{1}{2(a-b p)} d \varepsilon \\
& =-w_{0} q+\frac{3\left(1-d_{0}\right)(p+\beta) q^{2}}{8(a-b p)}+(p+\beta) q-\frac{(p+\beta) q^{2}}{2(a-b p)}  \tag{12}\\
& =\left(-w_{0}+p+\beta\right) q-\frac{\left(1+3 d_{0}\right)(p+\beta) q^{2}}{8(a-b p)}
\end{align*}
$$

To calculate the retailer's optimal purchase volume and optimal pricing in this supply mode, the optimization method should be used to obtain the derivative of the profit function for the purchase volume $q$. The derivative should be zero. The detailed calculation is as follows:

$$
\begin{equation*}
\frac{\partial \Pi_{R h}}{\partial q}=\left(-w_{0}+p+\beta\right)-\frac{\left(1+3 d_{0}\right)(p+\beta) q}{4(a-b p)}=0 \tag{13}
\end{equation*}
$$

We derive the following:

$$
\begin{equation*}
q=\frac{4(a-b p)\left(-w_{0}+p+\beta\right)}{\left(1+3 d_{0}\right)(p+\beta)} \tag{14}
\end{equation*}
$$

From the above,

$$
\begin{align*}
& \frac{\partial \Pi_{R h}}{\partial p}=q-\frac{q^{2}\left(1+3 d_{0}\right)}{8(a-b p)}-\frac{b\left(1+3 d_{0}\right)(p+\beta) q^{2}}{8(a-b p)^{2}} \\
& =\left[8 b^{2} p^{2}-16 a b p+8 a^{2}-\left(1+3 d_{0}\right) q(a+b \beta)\right] \cdot \frac{q}{8(a-b p)^{2}}=0 . \tag{15}
\end{align*}
$$

Based on Vieta's theorem, we derive

$$
\begin{equation*}
\Delta=16 b^{2}\left(2\left(1+3 d_{0}\right)(a+b \beta) q\right) \tag{16}
\end{equation*}
$$

Therefore, we obtain

$$
\begin{align*}
p & =\frac{16 a b+\sqrt{16 b^{2} \cdot 2\left(1+3 d_{0}\right)(a+b \beta) q}}{16 b^{2}}  \tag{17}\\
& =\frac{4 a+\sqrt{2 q\left(1+3 d_{0}\right)(a+b \beta)}}{4 b} .
\end{align*}
$$

$$
\begin{align*}
\Pi_{R h}= & -w_{0} q+p \int_{0}^{q} D_{h}(\varepsilon) f(\varepsilon) d \varepsilon+p \int_{q}^{2(a-b p)} q f(\varepsilon) d \varepsilon+\gamma \int_{0}^{q}\left(q-D_{h}(\varepsilon)\right) f(\varepsilon) d \varepsilon \\
= & -w_{0} q+p \int_{0}^{q} \frac{3\left(1-d_{0}\right)}{2} \cdot \varepsilon \cdot \frac{1}{2(a-b p)} d \varepsilon+p q \int_{q}^{2(a-b p)} \frac{1}{2(a-b p)} d \varepsilon \\
& +\gamma \int_{0}^{q}\left[q-\frac{3\left(1-d_{0}\right)}{2} \varepsilon\right] \cdot \frac{1}{2(a-b p)} d \varepsilon  \tag{19}\\
= & (-w 0+p) q+\frac{3 p q^{2}\left(1-d_{0}\right)}{8(a-b p)}-\frac{p q^{2}}{2(a-b p)}+\frac{\gamma q^{2}}{2(a-b p)}-\frac{3\left(1-d_{0}\right) \gamma q^{2}}{8(a-b p)} \\
= & \left(-w_{0}+p\right) q-\frac{\left(1+3 d_{0}\right)(p-\gamma) q^{2}}{8(a-b p)} .
\end{align*}
$$

To calculate the retailer's optimal purchase volume and optimal pricing in this mode of supply, the optimization method should be used to obtain the derivative of the profit function for the purchase volume $q$. The derivative should be zero. The details are as follows:

$$
\begin{equation*}
\frac{\partial \Pi_{R h}}{\partial q}=-w_{0}+p+\frac{\left(1+3 d_{0}\right)(\gamma-p) q}{4(a-b p)}=0 . \tag{20}
\end{equation*}
$$

From the above,

$$
\begin{align*}
& \frac{\partial \Pi_{R h}}{\partial p}=q-\frac{q^{2}\left(1+3 d_{0}\right)}{8(a-b p)}-\frac{b\left(1+3 d_{0}\right)(p-\gamma) q^{2}}{8(a-b p)^{2}} \\
& =\left[8 b^{2} p^{2}-16 a b p+8 a^{2}-\left(1+3 d_{0}\right)(a-b \gamma) q\right] \cdot \frac{q}{8(a-b p)^{2}}=0 \tag{22}
\end{align*}
$$

Based on Vieta's theorem, we derive

$$
\begin{equation*}
\Delta=32 b^{2}\left(1+3 d_{0}\right)(a-b \gamma) q . \tag{23}
\end{equation*}
$$

Therefore, we obtain

$$
\begin{align*}
p & =\frac{16 a b+\sqrt{32 b^{2}\left(1+3 d_{0}\right)(a-b \gamma) q}}{16 b^{2}} \\
& =\frac{4 a+\sqrt{2\left(1+3 d_{0}\right)(a-b \gamma) q}}{4 b} \tag{24}
\end{align*}
$$

The $q$ value obtained from Solution (21) is the retailer's optimal purchase volume, and the $p$ value obtained from Solution (24) is the retailer's optimal pricing.

## 5. Analysis of Numerical Examples

Because the relationship between the variables in the mathematical model in which the retailer designs and plans a supply strategy for individual customers and group customers to yield optimal profit under the real demand pattern is complex, neither the optimal production volume (sales volume) $q^{*}$ nor the optimal pricing $p^{*}$ has an explicit solution. Thus, to test whether the model truly reflects the practical problem of product supply strategies under different modes of demand and whether the model accurately grasps the interrelationship between variables and the objective laws of obedience, this paper will verify the optimal solution of the model by calculating numerical examples. We perform numerical calculations with Mathematica 8.0.

Assuming that the production cost of a certain product manufactured by Company J is 70 and that the highest retail price of the product in the market is found to be 100, the retail price of the product sold by the retailer to the individual customer needs to satisfy $70<p \leq 100$. The basic parameters are set as follows:

$$
\begin{align*}
& \alpha=2, a=90, w_{0}=70, d_{0}=0.2, b=0.8, p_{m}=100 \\
& \gamma=65, u=0.9, v=0.8 \tag{25}
\end{align*}
$$

5.1. Off-Invoice Mode. It is assumed that to encourage sales, the manufacturer offers an additional discount of $\alpha$ on each product sold to the retailer and that this discount is deducted directly from the initial purchase payment. The actual wholesale price to the retailer is then $w_{0}-\alpha$. Using Mathematica 8.0 to substitute these basic parameters of Equations (6) and (7), we obtain the optimal retail price and optimal order quantity for the retailer.

$$
\begin{equation*}
p_{1}^{*}=96.48, q_{1}^{*}=7.26 \tag{26}
\end{equation*}
$$

The results of the solution show that the obtained $p_{1}^{*}=$ 96.48 satisfies the price constraint $70<p \leq 100$ for the retailer. With Mathematica 8.0, we obtain a graph of the relationship between the retail price and the retailer's order quantity to the manufacturer (Figure 2). We find that in the off-invoice model, the relationship between the optimal fixed value and the optimal order quantity has an inverted parabolic relationship. The higher the pricing set by the retailer


Figure 2: Optimal pricing change trend with optimal order quantity under the off-invoice mode.
for the individual customer is, the fewer orders the retailer will receive. This shows that in this distribution mode, the larger the value of the preferential value $a$ is, the greater the optimal price $p$ is proportional to the optimal order quantity $q$. We believe that this result is consistent with the commodity price law. When the retail price is low, the profit of individual products is lower, and more customers can accept this price. As the retail price increases and does not exceed the price expected by the individual customer, the individual customer is still willing to buy, and the retailer will gain increasingly more profit at this time. However, when the retail price exceeds the expected psychological price of individual customers, there will be fewer and fewer individual customers; at this time, although the profit of individual products increased, the reduction of individual customers instead led to a decline in the profits of retailers. To further verify the validity of the model, we revalidated it by changing the values of the parameters, and the results obtained from the validation were all greater than 0 , thus proving that the model is correct and valid.
5.2. Scan-Back Mode. It is assumed that the manufacturer, on the basis of the given wholesale price, offers the retailer a rebate of $\beta$ for each unit sold and that the retailer's purchase volume is $q$. If $q$ is greater than demand, that is, if supply exceeds demand, the rebate would be $(p+\beta) D_{h}$, where $p$ represents the retailer's retail price. If $q$ is less than demand, that is, if demand exceeds supply, the rebate would be $(p+$ $\beta$ ) $q$. Using Mathematica 8.0 to substitute the basic parameters into Equations (12) and (13), we obtain the following:

$$
\begin{equation*}
p_{2}^{*}=96.50, q_{2}^{*}=7.24 \tag{27}
\end{equation*}
$$

The results of the solution show that the obtained $p_{2}^{*}=$ 96.50 satisfies the price constraint $70<p \leq 100$ for the retailer. With Mathematica 8.0, we obtain a graph of the relationship between the retail price and the retailer's order quantity to the manufacturer (Figure 3). We find that in the off-invoice model, the relationship between the optimal fixed value and the optimal order quantity has an inverted parabolic relationship. The higher the pricing set by the retailer for the individual customer is, the fewer orders the retailer


Figure 3: Optimal pricing change trend with optimal order quantity under scan-back mode.
will receive. This shows that in this distribution mode, the larger the rebate value $\beta$ is, the better the price $p$ is proportional to the best order quantity $q$. We believe that this result is consistent with the commodity price law. When the retail price is low, the profit of individual products is lower, and more customers can accept this price. As the retail price increases and does not exceed the price expected by the individual customer, the individual customer is still willing to buy, and the retailer will gain increasingly more profit at this time. However, when the retail price exceeds the expected psychological price of individual customers, there will be fewer and fewer individual customers; at this time, although the profit of individual products increased, the reduction of individual customers instead led to a decline in the profits of retailers. To further verify the validity of the model, we revalidated it by changing the values of the parameters, and the results obtained from the validation were all greater than 0 , thus proving that the model is correct and valid.
5.3. Unsold-Item Processing Mode. It is assumed that the manufacturer buys back the unsold items from the retailer at a discounted price; in this situation, the retailer purchased the product at a unit price of $w_{0}$, and the purchase volume is $q$. If the demand quantity $D_{h}$ is greater than $q$, all of the products will be sold. Assume that the retail price is $p$ and that the sales income to the retailer is $p q$. If demand is less than $q, q-D_{h}$ products are not sold after the amount $D_{h}$ is sold. The manufacturer buys back the unsold items at the price of $\gamma$ per unit. In this case, the total price paid by the manufacturer is $p D_{h}+\gamma\left(q-D_{h}\right)$. Using Mathematica 8.0 to substitute the basic parameters into Equations (20) and (21), we derive the following:

$$
\begin{equation*}
p_{3}^{*}=92.99, q_{3}^{*}=32.05 \tag{28}
\end{equation*}
$$

The results of the solution show that the obtained $p_{2}^{*}=$ 96.50 satisfies the price constraint $70<p \leq 100$ for the retailer. With Mathematica 8.0, we obtain a graph of the relationship between the retail price and the retailer's order quantity to the manufacturer (Figure 4). We find that in the off-invoice model, the relationship between the optimal fixed


Figure 4: Optimal pricing change trend with optimal order quantity under the unsold-item processing mode.
value and the optimal order quantity has an inverted parabolic relationship. The higher the price set by the retailer for the individual customer is, the more orders the retailer will receive. This shows that in this distribution mode, the larger the value of the recovery value $\gamma$, the optimal price $p$ is proportional to the optimal order quantity $q$. We believe that this result is consistent with the commodity price law. When the retail price is low, the profit of individual products is lower, and more customers can accept this price. As the retail price increases and does not exceed the price expected by the individual customer, the individual customer is still willing to buy, and the retailer will gain increasingly more profit at this time. However, when the retail price exceeds the expected psychological price of individual customers, there will be fewer and fewer individual customers; at this time, although the profit of individual products increased, the reduction of individual customers instead led to a decline in the profits of retailers. To further verify the validity of the model, we revalidated it by changing the values of the parameters, and the results obtained from the validation were all greater than 0 , thus proving that the model is correct and valid.

## 6. Managerial Insights

We study a model of a retailer's product supply strategy for individual customers in an intermediate demand model, addressing the retailer's profit maximization problem. Its management insights have the following three main components:
(1) Rational pricing. According to these mathematical models, retailers can always follow up the market demand to make reasonable pricing to maximize profits.
(2) Scientific stocking. Based on these models, retailers can observe the market demand for the products they sell in real time and adjust the quantity of goods they buy scientifically to maximize profits, taking into account the existing pricing.
(3) Discount optimization. According to these mathematical models, the retailer can observe the market demand for the goods sold in real time, combine the
retail price and the quantity of goods purchased, and optimize the discount price of the goods in time to maximize profits.

## 7. Conclusion

Customer demand in the consumer goods supply chain has long been a hot topic and has received much attention in both industry and management circles. However, the existing studies are the results under demand uncertainty and ignore the existence of three different patterns of customer demand in management practice: real, false, and semireal. We investigate the retailer's product supply strategy under one of these three demand patterns, the semireal demand pattern, and obtain some management-valuable marketing strategy results by considering the optimal order quantity, optimal pricing, and optimal discounts for the retailer under three distribution modes: off-invoice, scan-back, and unsolditem processing.

This study, based on optimization theory, proposes an optimal production volume and optimal pricing model of retailers' product supply strategies for individual customers under the semireal demand pattern. We construct three optimal profit models developed by retailers under the semireal demand model for individual customers, which cannot only guide retailers on how to price products under different sales strategies, how much profit is given, and even for purchases. Quantities are also given clear guidance. The numerical results not only provide references for the retailer's planning of product supply strategies in cases of oversupply and short supply but also assist the retailer in making more accurate product sales decisions for individual customers under a semireal demand pattern. We believe that although our data are hypothetical data, the model is valid for all the corresponding parallel sets.

Our theoretical contribution is that we have developed a model of a retailer's product supply strategy to a single customer under the semireal demand pattern. The numerical results verify the correctness of the model, thus revealing the intrinsic mechanism of the firm's product operation decision under the semireal demand model. The study verifies the optimal pricing and optimal production strategies to achieve optimal product profitability through mathematical modeling, which has some practical implications in management practice. The results of the study not only provide a theoretical basis for research on the application of demand theory in the consumer goods supply chain but also help retailers make more accurate product sales decisions for individual customers in a semireal demand pattern. In the future, we can consider using the multiplicative form of the demand function to construct function models of different demand patterns and then further improve the theoretical research on different demand patterns in consumer goods supply chains by comparing two types of profit function models. At the same time, the model can be replaced with a real case for a more in-depth analysis to achieve accurate marketing.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Zhiyi Zhuo designed the research, performed the research, and wrote the paper; Shuhong Chen constructed the mathematical models; Weihua Lin literature review and conclusion discussion; Hong Yan conceived the idea, design the methodology, and wrote the paper; Yue He numerical analysis and discussion of results; All authors participated in discussing and revising the manuscript.

## Acknowledgments

This work was supported by the National Natural Science Foundation of Fujian Province (No. 2021J05251) and the Foundation of Wuyi University (Nos. YJ202118; YJ202012).

## References

[1] K. Marx, F. Engels, and C. J. Arthur, The German Ideology, Lawrence \& Wishart, London, 1970.
[2] T. Schaefers, M. Leban, and F. Vogt, "On-demand features: consumer reactions to tangibility and pricing structure," Journal of Business Research, vol. 139, pp. 751-761, 2022.
[3] K. H. Yang, "Efficient demands in a multi-product monopoly," Journal of Economic Theory, vol. 197, Article ID 105330, 2021.
[4] Y. Aviv, M. M. Wei, and F. Zhang, "Responsive pricing of fashion products: the effects of demand learning and strategic consumer behavior," Management Science, vol. 65, no. 7, pp. 2947-3448, 2019.
[5] N. Perullo, Taste as Experience: The Philosophy and Aesthetics of Food, Columbia University Press, 2016.
[6] E. O. Wilson, "The meaning of human existence," in Darwin's Bridge: Uniting the Humanities and Sciences, J. Carroll, D. P. McAdams, and E. O. Wilson, Eds., pp. 3-8, Oxford University Press, 2016.
[7] Z. Zhuo, S. Chen, H. Yan, and Y. He, "Models for retailers' supply strategies aimed at individual customers under the essential demand pattern," Chiang Mai Journal of Science, vol. 48, no. 1, pp. 263-275, 2021.
[8] Z. Zhuo, S. Chen, and K. Y. Chau, "A new model of manufacturer's optimal product supply strategy in the context of precision marketing: based on real demand pattern," Mathematical Problems in Engineering, vol. 2020, Article ID 6031563, 11 pages, 2020.
[9] Z. Zhuo, "Research on optimal product supply strategies for manufacturer-to-group customer under a real demand pattern," International Journal of Operational Research, vol. 44, no. 4, pp. 550-561, 2022.
[10] Z. Zhuo, "New mathematical model of retailer-to-individual customer optimal product supply strategies under false demand pattern: customer discount mode," Journal of Mathematics Research, vol. 12, no. 1, pp. 36-42, 2020.
[11] Z. Zhuo, H. Yan, and Y. He, "Constructing optimal product supply strategy model for manufacturers based on semi-
essential demand pattern," Technology Economics, vol. 39, no. 12, pp. 110-116, 2020.
[12] Z. Zhuo, K. Y. Chau, S. Huang, and Y. K. Ip, "Mathematical modeling of optimal product supply strategies for manufac-turer-to-group customers based on semi-real demand patterns," International Journal of Engineering Business Management, vol. 12, 2020.
[13] M. Crawford, "The world in a shopping mall," in Variations on a Theme Park: The New American City and the End of Public Space, M. Sorkin, Ed., pp. 3-30, Hill \& Wang, New York, 2004.
[14] A. M. Fathollahi-Fard, M. A. Dulebenets, G. Tian, and M. Hajiaghaei-Keshteli, "Sustainable supply chain network design," Environmental Science and Pollution Research, vol. 13, 2022.
[15] M. Kaviyani-Charati, S. H. Ghodsypour, and M. HajiaghaeiKeshteli, "Impact of adopting quick response and agility on supply chain competition with strategic customer behavior," Scientia Iranica, vol. 29, no. 1, pp. 387-411, 2022.
[16] M. Akbari-Kasgari, H. Khademi-Zare, M. B. Fakhrzad, M. Hajiaghaei-Keshteli, and M. Honarvar, "Designing a resilient and sustainable closed-loop supply chain network in copper industry," Clean Technologies and Environmental Policy, vol. 24, pp. 1553-1580, 2022.
[17] M. M. Shahsavar, M. Akrami, Z. Kian et al., "Bio-recovery of municipal plastic waste management based on an integrated decision-making framework," Journal of Industrial and Engineering Chemistry, vol. 108, pp. 215-234, 2022.
[18] E. Sadri, F. Harsej, M. Hajiaghaei-Keshteli, and J. Siyahbalaii, "Evaluation of the components of intelligence and greenness in Iranian ports based on network data envelopment analysis (DEA) approach," Journal of Modelling in Management, vol. 17, no. 3, pp. 1008-1027, 2021.
[19] S. J. Ghoushchi, S. R. Bonab, A. M. Ghiaci, G. Haseli, H. Tomaskova, and M. Hajiaghaei-Keshteli, "Landfill site selection for medical waste using an integrated SWARAWASPAS framework based on spherical fuzzy set," Sustainability, vol. 13, no. 24, Article ID 13950, 2021.
[20] S. Bhuniya, S. Pareek, and B. Sarkar, "A sustainable game strategic supply chain model with multi-factor dependent demand and mark-up under revenue sharing contract," Complex \& Intelligent Systems, vol. 9, pp. 2101-2128, 2023.
[21] R. K. Bachar, S. Bhuniya, S. K. Ghosh et al., "Product outsourcing policy for a sustainable flexible manufacturing system with reworking and green investment," Mathematical Biosciences and Engineering, vol. 20, no. 1, pp. 1376-1401, 2023.
[22] B. K. Dey, S. Bhuniya, and B. Sarkar, "Involvement of controllable lead time and variable demand for a smart manufacturing system under a supply chain management," Expert Systems with Applications, vol. 184, Article ID 115464, 2021.
[23] R. Das, A. Barman, B. Roy, and P. K. De, "Pricing and greening strategies in a dual-channel supply chain with cost and profit sharing contracts," Environment, Development and Sustainability, vol. 25, pp. 5053-5086, 2023.
[24] A. Barman and P. K. De, "A multi-item deteriorating inventory model under stock level-dependent, time-varying, and pricesensitive demand," in Recent Trends in Applied Mathematics, S. R. Mishra, T. N. Dhamala, and O. D. Makinde, Eds., pp. 112, Springer, Singapore, 2021.
[25] A. Barman, R. Das, and P. K. De, "An analysis of optimal pricing strategy and inventory scheduling policy for a non-
instantaneous deteriorating item in a two-layer supply chain," Applied Intelligence, vol. 52, no. 4, pp. 4626-4650, 2022.
[26] A. Barman, P. K. De, A. K. Chakraborty, C. P. Lim, and R. Das, "Optimal pricing policy in a three-layer dual-channel supply chain under government subsidy in green manufacturing," Mathematics and Computers in Simulation, vol. 204, pp. 401-429, 2023.
[27] N. C. Petruzzi and M. Dada, "Pricing and the newsvendor problem: a review with extensions," Operations Research, vol. 47, no. 2, pp. 175-344, 1999.
[28] E. S. Mills, "Uncertainty and price theory," The Quarterly Journal of Economics, vol. 73, no. 1, pp. 116-130, 1959.
[29] S. Karlin and C. R. Carr, "Prices and optimal inventory policy," Studies in Applied Probability and Management Science, vol. 4, pp. 159-172, 1962.
[30] F. Pukelsheim, "The three sigma rule," The American Statistician, vol. 48, no. 2, pp. 88-91, 1994.

