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Research Article

Robust Secure Beamforming for Multiantenna Relay Networks with Multiple Eavesdroppers

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Physical layer security has recently emerged as a promising approach to achieve security and communication integration. The built-in security mechanism based on the characteristics of the wireless channel provides a feasible idea for the realization of "one secret at a time." It has significant potential applications in high-rate data transmission encryption, authentication, integrity protection of service data, and Internet of Things lightweight encryption. This paper investigates the robust secure transmit design problem for amplify-and-forward multiantenna relay networks in the presence of multiple single-antenna eavesdroppers. A robust artificial noise (AN) aided secure beamforming design is proposed by taking into account the imperfect channel state information (CSI) of the eavesdroppers. The goal is to jointly design the beamforming matrix and the AN covariance matrix at the relay based on the imperfect CSI of eavesdroppers, such that the worst-case secrecy rate is maximized subject to the total power and the per-antenna power constraints. Aiming at the nonconvex optimization problem, a two-level optimization algorithm based on semi-definite relaxation (SDR) and S-procedure is proposed. It is proven that there is a rank-one optimal solution for the SDR problem. Simulation results show that the proposed scheme is robust and superior to existing schemes.

1. Introduction

With the rapid development of mobile communication network, its security has become a research hotspot [1, 2]. Traditionally, information security is guaranteed through the key-based encryption technique. However, key management (including key generation and distribution) is challenging in mobile wireless networks [3]. For this reason, physical layer security, which exploits the characteristics of wireless channels to encrypt confidential messages, has recently attracted much attention [4–6]. The emerging physical layer security technology takes advantage of the diversity and time variation of wireless channels and the uniqueness and interoperability of both channels of legitimate communication and explores the built-in security mechanisms based on the characteristics of wireless channels in the physical layer. It has important application prospects in high-rate data transmission encryption, authentication, integrity protection of service data, and Internet of Things lightweight encryption [7–9]. The key idea is to safely deliver confidential information by exploiting the physical characteristics of the wireless channel [10, 11]. The research on physical layer security has been used in relay communication and point-to-point communication system [12, 13] and could be extended to various wiretap channels such as broadcast channels [14], interference channels [15], and cooperative relay channels [16, 17].

There have been several studies addressing the physical layer security problem in cooperative relay communications. For the case of single-antenna relays, secure transmit schemes have been designed by Yang et al. [17], Feng et al. [18], Xu et al. [19], Wang et al. [20], Yang et al. [21], Li et al. [22], and Park et al. [23]. Yang et al. [17] designed the relay beamforming vector to obtain an optimal secrecy rate. An intelligent reflecting surface is deployed to modulate the received confidential signal to achieve secure transmission by Feng et al. [18] and Xu et al. [19]. The hybrid of cooperative beamforming and jamming has been proposed by Wang et al. [20] and Yang et al. [21] to improve security performance, where the beamforming vector and the artificial noise (AN) covariance matrix are jointly designed for the secrecy rate maximization (SRM) problem. All these works are conducted based on perfect channel state information (CSI). Taking into account the deterministic channel error, the authors by Li et al. [22] investigated the robust design to maximize the worst-case secrecy rate. Based on the statistical CSI of the eavesdropper, three jamming strategies performed at the destination were proposed for minimizing the secrecy outage probability by Park et al. [23].

For the case of a multiantenna relay, the physical layer security problem was studied by Vishwakarma and Chockalingam [24], Jilani and Ohtsuki [25], Li et al. [26], Wang et al. [27], Yang et al. [28], Zhang et al. [29, 30], Wang et al. [31, 32]. Using the decode-and-forward (DF) protocol, the secure transmit design was explored by Vishwakarma and Chockalingam [24] and Jilani and Ohtsuki [25]. Compared to the DF protocol, the transmit design for the SRM problem is more complex when using the amplify-and-forward (AF) protocol due to the noise amplification effects at the relay. Under perfect CSI, the SRM problem was studied by Li et al. [26] and Wang et al. [27], where an iterative algorithm was proposed by Li et al. [26], and a suboptimal joint source-relay linear precoding and power allocation scheme was developed by Wang et al. [27]. Under imperfect CSI, the robust design was investigated by Yang et al. [28], Zhang et al. [29, 30], Wang et al. [31, 32]. From the quality of service view, an optimal resource allocation strategy was proposed by Yang et al. [28], and the robust joint beamforming design of the source and relay for minimizing the overall power was proposed by Zhang et al. [29]. In the study of Zhang et al. [30], Wang et al. [31, 32], the suboptimal beamforming matrix was designed in order to maximize the worst-case secrecy rate. It is noteworthy that only one eavesdropper is considered in most works on the multiantenna relay networks mentioned above. And the methods are not applied to the scenarios with multiple eavesdroppers directly, or they are applied to special channels [33].

In this paper, we consider a two-hop AF multiantenna relay network. It is assumed that the CSI of the eavesdroppers is imperfectly known at the relay, and the channel uncertainties are bound in ellipsoidal regions. Our work is different from Wang et al. [31] in the following aspects: (1) we assume that multiple eavesdroppers exist; (2) the AN is adopted to enhance the security performance; (3) the per-antenna power constraints are considered in addition to the total power constraint. The goal is to jointly design the beamforming matrix and the AN covariance matrix at the relay based on the imperfect CSI of eavesdroppers, such that the worst-case secrecy rate is maximized subject to the total power and the per-antenna power constraints. We show that the nonconvex worst-case secrecy rate maximization (WCSRM) problem can be recast into a two-level optimization. The outer part is a one-variable optimization problem, which can be handled by one-dimensional search. The inner part can be solved by the combination of the semidefinite relaxation (SDR) technique [34, 35] and the S-procedure [36]. Moreover, we prove that the SDR is tight, i.e., a rank-one solution always exists. Simulation results are provided to show the efficacy of the proposed AN-aided secure beamforming design.

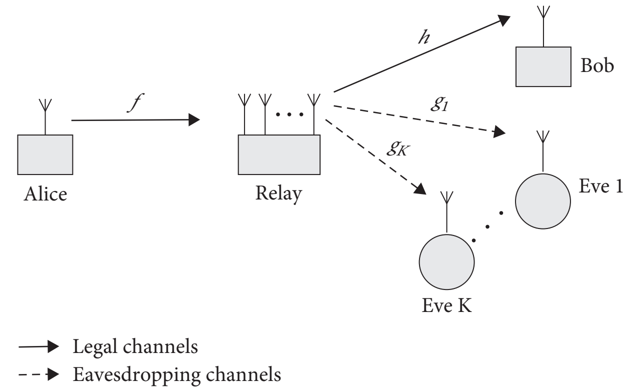

Figure 1: System model for the secure multiantenna relay network.

1.1. Notations. $\mathrm{Tr}(\cdot), (\cdot)^{-1}, (\cdot)^*, (\cdot)^H, \|\cdot\|$ and $\mathrm{Rank}(\cdot)$ denote the trace, inverse, conjugate, Hermitian transpose, Euclidean norm, and rank of a matrix, respectively. $\mathbb{C}^N$ and $\mathbb{C}^{N\times N}$ denote the spaces of $N\times 1$ complex vector and $N\times N$ complex matrix, respectively. $\otimes, \odot, \mathbf{I}_N$ and $\mathbf{1}_{N\times 1}$ stand for the Hadamard product, Kronecker product, identity matrix of dimension N and the all-one column vector of dimension N, respectively. $\mathbf{D}(\mathbf{q})$ represents a diagonal matrix with $\mathbf{q}$ on the main diagonal. $\mathrm{Re}(\cdot)$ extracts the real part of a complex variable. $\mathbf{Q}\succeq\mathbf{0}(\mathbf{Q}\succ\mathbf{0})$ means that $\mathbf{Q}$ is a positive semidefinite (definite) matrix, $\mathbf{q}=\mathrm{vec}(\mathbf{Q})$ denotes a column vector by stacking all the elements of $\mathbf{Q}$ and $\mathrm{vec}^{-1}(\mathbf{q})$ is the inverse operation of $\mathrm{vec}(\mathbf{Q})$ for recovering $\mathbf{Q}$. $\mathbf{x}\sim\mathscr{CN}$^$(\mathbf{c},\mathbf{Q})$ means that $\mathbf{x}$ is a complex circular Gaussian random vector with mean $\mathbf{c}$ and covariance $\mathbf{Q}$.

2. System Model and Problem Formulation

As shown in Figure 1, we consider a two-hop AF relay network consisting of one source (Alice), one relay, one legitimate destination (Bob), and multiple eavesdroppers (Eves). Each of the nodes is equipped with a single antenna, except that the relay is equipped with N $(N\geqslant 2)$ antennas. There are no direct links between Alice and Bob, nor between Alice and Eve, due to the weak quality of the channels. Alice intends to transmit the confidential information to Bob, assisted by the trusted relay, while keeping it secret from the Eves. This model corresponds to the following scenario: each user belongs to the same system but subscribes to different services, e.g., pay-TV services. When a relay node sends a subscription service to a paying user, other users who do not subscribe to the service cannot access the service and are treated as eavesdroppers.

All the nodes work in a half-duplex mode. Hence, one round of information exchange includes two phases. In the first phase, the signals are transmitted from Alice to the relay. The signal vector received at the relay can be written as follows:

$$\mathbf{y}_r = \mathbf{f}s + \mathbf{n}_r, \tag{1}$$

where $\mathbf{f} \in \mathbb{C}^N$ denotes the channel vector from Alice to the relay; $s \sim \mathscr{CN}(0, P_s)$ is the confidential information with average power P_s; and $\mathbf{n}_r \sim \mathscr{CN}(\mathbf{0}, \mathbf{I}_N)$ denotes the additive white Gaussian noise (AWGN) vector received at the relay. In the second phase, the relay forwards the signals multiplied by a beamforming matrix. At the same time, the AN is transmitted to confuse the Eves. Hence, the signal vector to be transmitted by the relay can be expressed as follows:

$$\mathbf{x} = \mathbf{Af}s + \mathbf{An}_r + \mathbf{v}, \tag{2}$$

where $\mathbf{A} \in \mathbb{C}^{N\times N}$ is the beamforming matrix, $\mathbf{v} \sim \mathscr{CN}(\mathbf{0}, \Omega)$ is the artificial noise vector with $\Omega \succeq \mathbf{0}$ being the AN covariance matrix. Note that the channel vector $\mathbf{f}$, the AWGN vector $\mathbf{n}_r$ and the AN vector $\mathbf{v}$ are mutually independent. From Equation (2), the total power of all antennas and the power of the nth antenna are computed, respectively, as follows:

$$P_r = \mathrm{Tr}(P_s\mathbf{Aff}^H\mathbf{A}^H) + \mathrm{Tr}(\mathbf{AA}^H) + \mathrm{Tr}(\Omega), \tag{3a}$$

$$P_n = \mathbf{e}_n^T(P_s\mathbf{Aff}^H\mathbf{A}^H + \mathbf{AA}^H + \Omega)\mathbf{e}_n, \forall n \in \mathscr{N}, \tag{3b}$$

where $\mathbf{e}_n$ is a unit vector with the nth entry being one and $N \triangleq \{1, \ldots, N\}$. The signals received at Bob and the kth Eve can be written, respectively, as follows:

$$y_b = \mathbf{h}^H\mathbf{Af}s + \mathbf{h}^H\mathbf{An}_r + \mathbf{h}^H\mathbf{v} + n_b, \tag{4a}$$

$$y_k = \mathbf{g}_k^H\mathbf{Af}s + \mathbf{g}_k^H\mathbf{An}_r + \mathbf{g}_k^H\mathbf{v} + n_k, \forall k \in \mathscr{K}, \tag{4b}$$

where $\mathbf{h} \in \mathbb{C}^N$ is the channel vector between the relay and Bob; $\mathbf{g}_k \in \mathbb{C}^N, \forall k \in \mathscr{K}$, is the channel vector between the relay and the kth Eve, $\mathscr{K} \triangleq \{1, \ldots, K\}$; and $n_b \sim \mathscr{CN}(0, 1)$ and $n_k \sim \mathscr{CN}(0, 1)$ are the AWGN terms at the receivers.

According to Equation (4), the received signal-to-interference-plus-noise ratios (SINRs) at Bob and the kth Eve are, respectively,

$$\mathrm{SINR}_b = \frac{P_s|\mathbf{h}^H\mathbf{Af}|^2}{1 + \mathbf{h}^H\Omega\mathbf{h} + \|\mathbf{h}^H\mathbf{A}\|^2}, \tag{5a}$$

$$\mathrm{SINR}_{e,k} = \frac{P_s\left|\mathbf{g}_k^H\mathbf{Af}\right|^2}{1 + \mathbf{g}_k^H\Omega\mathbf{g}_k + \|\mathbf{g}_k^H\mathbf{A}\|^2}, \ k \in \mathscr{K}. \tag{5b}$$

Similar to the model by Wang et al. [31], only partial CSI of Eves is available at the relay. The worst-case ellipsoidal error model is adopted to characterize the imperfect CSI of Eves. In this model, the actual channel vector of the kth Eve takes the form the following:

$$\mathbf{g}_k = \widehat{\mathbf{g}}_k + \Delta\mathbf{g}_k, \forall k \in \mathscr{K}, \tag{6}$$

and

$$\Delta\mathbf{g}_k \in \mathscr{G}_k \triangleq \left\{\Delta\mathbf{g}_k | \Delta\mathbf{g}_k^H\Sigma_k\Delta\mathbf{g}_k \leqslant \varepsilon_k^2\right\}, \forall k \in \mathscr{K}, \tag{7}$$

where $\widehat{\mathbf{g}}_k$ is the estimated channel vector from the relay to the kth Eve; $\Delta\mathbf{g}_k$ is the corresponding error vector; $\Sigma_k \succ \mathbf{0}$ defines the shape of the uncertainty region G_k; and $\varepsilon_k \geqslant 0$ controls the size of the region.

In this paper, we investigate the WCSRM problem. The objective is to jointly design the beamforming matrix and the AN covariance matrix at the relay in order to maximize the worst-case secrecy rate. Let R_s^* denote the optimal worst-case secrecy rate. The WCSRM problem is subject to the total power constraint, and the per-antenna power constraints can be formulated as follows:

$$\begin{aligned} R_s^* = \max_{\mathbf{A},\Omega\succeq\mathbf{0}} \quad & \min_{k\in K}\left\{\frac{1}{2}\log(1+\mathrm{SINR}_b) - \frac{1}{2}\max_{\Delta\mathbf{g}_k\in G_k}\log\left(1+\mathrm{SINR}_{e,k}\right)\right\}, \\ \text{s.t.} \quad & P_r \leqslant P_{\max},\ P_n \leqslant \rho_n, \forall n \in \mathscr{N}, \end{aligned} \tag{8}$$

where 1/2 is inserted due to the fact that the relay transmission is divided into two phases; $\log(\cdot)$ denotes the logarithmic function; $P_{\max}$ and ρ_n are the maximum total power of all the antennas and the maximum power of the nth antenna at the relay, respectively. The optimization problem (Equation (8)) is nonconvex and it is difficult to get the solution. In the following, we focus on handling this WCSRM problem.

3. Robust AN-Aided Transmission Design

In this section, we propose a two-level optimization algorithm to solve the WCSRM problem (Equation (8)) and obtain the secure beamforming matrix and AN covariance matrix. Substituting the expressions of SINR_b and $\mathrm{SINR}_{e,k}$ in Equation (5) as well as the expressions of P_r and P_n in Equation (3) into Equation (8), we can rewrite Equation (8) as follows:

$$\begin{aligned} R_s^* = \max_{\mathbf{A},\Omega\succeq\mathbf{0}} \quad & \frac{1}{2}\log\left(1+\frac{P_s|\mathbf{h}^H\mathbf{A}\mathbf{f}|^2}{1+\mathbf{h}^H\Omega\mathbf{h}+\|\mathbf{h}^H\mathbf{A}\|^2}\right)-\frac{1}{2}\log\left(\frac{1}{\tau}\right) \\ \text{s.t.} \quad & \max_{\Delta\mathbf{g}_k\in G_k}\log\left(1+\frac{P_s|\mathbf{g}_k^H\mathbf{A}\mathbf{f}|^2}{1+\mathbf{g}_k^H\Omega\mathbf{g}_k+\|\mathbf{g}_k^H\mathbf{A}\|^2}\right)\leqslant\log\left(\frac{1}{\tau}\right),\forall k\in\mathcal{K}, \\ & \mathrm{Tr}(P_s\mathbf{A}\mathbf{f}\mathbf{f}^H\mathbf{A}^H+\mathbf{A}\mathbf{A}^H+\Omega)\leqslant P_{\max}, \\ & \mathbf{e}_n^T(P_s\mathbf{A}\mathbf{f}\mathbf{f}^H\mathbf{A}^H+\mathbf{A}\mathbf{A}^H+\Omega)\mathbf{e}_n\leqslant\rho_n,\forall n\in\mathcal{N}, \end{aligned} \tag{9}$$

where τ is the introduced auxiliary variable. Applying the matrix identities $\mathrm{Tr}(\mathbf{B}^H\mathbf{C}\mathbf{D}\mathbf{F})=\mathrm{vec}(\mathbf{B})^H(\mathbf{F}^T\otimes\mathbf{C})\mathrm{vec}(\mathbf{D})$ and $\mathrm{Tr}(\mathbf{B}\mathbf{C})=\mathrm{Tr}(\mathbf{C}\mathbf{B})$, Equation (12) can be equivalently expressed as follows:

$$R_s^* = \max_{\mathbf{a},\Omega\succeq\mathbf{0}}\frac{1}{2}\log\left(1+\frac{\mathbf{a}^H\mathbf{B}_1\mathbf{a}}{1+\mathrm{Tr}(\mathbf{h}\mathbf{h}^H\Omega)+\mathbf{a}^H\mathbf{B}_2\mathbf{a}}\right)+\frac{1}{2}\log(\tau), \tag{10a}$$

$$\text{s.t.}\ \max_{\Delta\mathbf{g}_k\in G_k}\log\left(1+\frac{\mathbf{a}^H\mathbf{C}_{1,k}\mathbf{a}}{1+\mathrm{Tr}\left(\mathbf{g}_k\mathbf{g}_k^H\Omega\right)+\mathbf{a}^H\mathbf{C}_{2,k}\mathbf{a}}\right)\leqslant\log\left(\frac{1}{\tau}\right),\quad \forall k\in K, \tag{10b}$$

$$\mathbf{a}^H\mathbf{D}_1\mathbf{a}+\mathrm{Tr}(\Omega)\leqslant P_{\max}, \tag{10c}$$

$$\mathbf{a}^H\mathbf{D}_{2,n}\mathbf{a}+\mathrm{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega)\leqslant\rho_n,\forall n\in N, \tag{10d}$$

where $\mathbf{a}=\mathrm{vec}(\mathbf{A})$; $\mathbf{B}_1=P_s(\mathbf{f}^*\otimes\mathbf{h})(\mathbf{f}^*\otimes\mathbf{h})^H$; $\mathbf{B}_2=\mathbf{I}_N\otimes(\mathbf{h}\mathbf{h}^H)$; $\mathbf{C}_{1,k}=P_s(\mathbf{f}^*\otimes\mathbf{g}_k)(\mathbf{f}^*\otimes\mathbf{g}_k)^H$; $\mathbf{C}_{2,k}=\mathbf{I}_N\otimes\left(\mathbf{g}_k\mathbf{g}_k^H\right)$; $\mathbf{D}_1=(P_s\mathbf{f}^*\mathbf{f}^T+\mathbf{I}_N)\otimes\mathbf{I}_N$; $\mathbf{D}_{2,n}=(P_s\mathbf{f}^*\mathbf{f}^T+\mathbf{I}_N)\otimes(\mathbf{e}_n\mathbf{e}_n^T)$.

3.1. Two-Level Optimization Formulation. Equation (10) is nonconvex and can not be solved directly. However, if τ is fixed in Equation (10), due to the monotonicity of $\log(\cdot)$ function, we just need to consider the following optimization problem

$$\begin{aligned} \gamma(\tau) = \max_{\mathbf{a},\Omega\succeq\mathbf{0}} \quad & \frac{\mathbf{a}^H\mathbf{B}_1\mathbf{a}}{1+\mathrm{Tr}(\mathbf{h}\mathbf{h}^H\Omega)+\mathbf{a}^H\mathbf{B}_2\mathbf{a}}, \\ \text{s.t.} \quad & (10\mathrm{b}),(10\mathrm{c})\ \text{and}\ (10\mathrm{d}). \end{aligned} \tag{11}$$

which has the same optimal solution to Equation (10). Equation (11) can be converted into an SDP, which can be solved efficiently using CVX [37], as it will be shown later in the next subsection. Motivated by this, Equation (10) can be equivalently reformulated as a two-level optimization problem [14, 21]. The inner part is Equation (11) with fixed τ. The outer part can be written as follows:

$$\begin{aligned} R_s^* = \max_{\tau} \quad & \frac{1}{2}\log(1+\gamma(\tau))+\frac{1}{2}\log(\tau) \\ \text{s.t.} \quad & \tau_{\min}\leqslant\tau\leqslant\tau_{\max}, \end{aligned} \tag{12}$$

where $\tau_{\min}$ and $\tau_{\max}$, which will be determined later in this subsection, are the lower bound and the upper bound on τ, respectively, and $\gamma(\tau)$ is the optimal objective value of the inner-level optimization problem (Equation (11)) for a given τ. Equation (12) is a single-variable optimization problem with respect to τ. It can be solved through the one-dimensional linear search over $\tau\in[\tau_{\min},\tau_{\max}]$. During the search process, the $\gamma(\tau)$ is obtained by solving the inner optimization problem (Equation (11)). In the next subsection, we will concentrate on handling the inner-level optimization problem (Equation (11)).

To complete the description of the two-level optimization problem, we determine the values of $\tau_{\min}$ and $\tau_{\max}$ in Equation (12). Given the optimization problem (Equation (10)), the auxiliary variable τ is inside the interval $[\tau_{\min},\tau_{\max}]$. Let us observe the constraints in Equation (10b). Due to the monotonicity of $\log(\cdot)$ function, we simplify Equation (10b) as follows:

$$1+\max_{\Delta\mathbf{g}_k\in G_k}\frac{\mathbf{a}^H\mathbf{C}_{1,k}\mathbf{a}}{1+\mathrm{Tr}\left(\mathbf{g}_k\mathbf{g}_k^H\Omega\right)+\mathbf{a}^H\mathbf{C}_{2,k}\mathbf{a}}\leqslant\frac{1}{\tau},\ \forall k\in K. \tag{13}$$

The second term on the left-hand side of the above inequality is nonnegative, and its minimal value 0 is achieved when the optimization variable $\mathbf{a}$ is set to $\mathbf{0}$. Hence, the relation $1\leqslant 1/\tau$ holds true, meaning that $\tau_{\max}=1$.

According to Equation (10a), the relation $\tau\geqslant(1+\gamma_b)^{-1}$ must hold true for guaranteeing the nonnegative secrecy rate with γ_b being the maximum value of SINR_b. The maximum value γ_b can be achieved under the scenario, where all the power is used to transmit the information to Bob neglecting the security problem. Hence, we have the following:

$$\begin{aligned} \gamma_b = \max_{\mathbf{a}} \quad & \frac{\mathbf{a}^H\mathbf{B}_1\mathbf{a}}{1+\mathbf{a}^H\mathbf{B}_2\mathbf{a}} \\ \text{s.t.} \quad & \mathbf{a}^H\mathbf{D}_1\mathbf{a}\leqslant P_{\max}. \end{aligned} \tag{14}$$

Equation (14) can be rewritten as a generalized Rayleigh quotient problem [38]. Using the results of [38], we have $\gamma_b=P_s\mathbf{u}^H(\mathbf{D}_1/P_{\max}+\mathbf{B}_2)^{-1}\mathbf{u}$, where $\mathbf{u}=\mathbf{f}^*\otimes\mathbf{h}$. Based on the derivation above, we have the following:

$$\tau_{\min}=(1+P_s(\mathbf{f}^*\otimes\mathbf{h})^H(\mathbf{D}_1/P_{\max}+\mathbf{B}_2)^{-1}(\mathbf{f}^*\otimes\mathbf{h}))^{-1}. \tag{15}$$

3.2. Tight SDR of Inner Optimization Problem. Equation (11) is a fractional quadratically constrained quadratic problem, which is difficult to solve. To circumvent this difficulty, we resort to the SDR technique [34]. Let us define $\overline{\mathbf{A}} = \mathbf{a}\mathbf{a}^H$ and drop the nonconvex constraint $\mathrm{Rank}(\overline{\mathbf{A}}) = 1$. Then, we can get a relaxed version of Equation (11) as follows:

$$\phi(\alpha) = \max_{\overline{\mathbf{A}}\geq \mathbf{0}, \Omega \geq \mathbf{0}} \frac{\mathrm{Tr}(\mathbf{B}_1\overline{\mathbf{A}})}{1 + \mathrm{Tr}(\mathbf{h}\mathbf{h}^H\Omega) + \mathrm{Tr}(\mathbf{B}_2\overline{\mathbf{A}})}, \tag{16a}$$

$$\begin{aligned}\text{s.t. } &\mathrm{Tr}(\mathbf{C}_{1,k}\overline{\mathbf{A}}) - \alpha\mathrm{Tr}(\mathbf{C}_{2,k}\overline{\mathbf{A}}) - \alpha\mathbf{g}_k^H\Omega\mathbf{g}_k \\ &\leqslant \alpha, \forall \Delta\mathbf{g}_k \in G_k, \forall k \in K,\end{aligned} \tag{16b}$$

$$\mathrm{Tr}(\mathbf{D}_1\overline{\mathbf{A}}) + \mathrm{Tr}(\Omega) \leqslant P_{\max}, \tag{16c}$$

$$\mathrm{Tr}(\mathbf{D}_{2,n}\overline{\mathbf{A}}) + \mathrm{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega) \leqslant \rho_n, \forall n \in N. \tag{16d}$$

Here $\alpha = 1/\tau - 1$. Note that $\phi(\alpha) \geqslant \gamma(\tau)$, because the nonconvex rank-one constraint is dropped in Equation (16). It can be seen that there are infinitely many constraints in Equation (16b), which make it hard to solve Equation (16). It is necessary to convert these constraints into quadratic forms in terms of $\Delta\mathbf{g}_k$ for the use of the S-procedure [36] later.

To proceed, we define the following notations: $\tilde{\mathbf{f}} \triangleq \mathbf{f} \otimes \mathbf{1}_{N\times 1}$, $\overline{\mathbf{g}}_k \triangleq \mathbf{L}\widehat{\mathbf{g}}_k$, $\Delta\overline{\mathbf{g}}_k \triangleq \mathbf{L}\Delta\mathbf{g}_k$, $\breve{\mathbf{g}}_k \triangleq \mathbf{L}\mathbf{g}_k = \overline{\mathbf{g}}_k + \Delta\overline{\mathbf{g}}_k, \forall k \in K$, with $\mathbf{L} \triangleq \mathbf{1}_{N\times 1} \otimes \mathbf{I}_N$. Besides, $\mathbf{E} \triangleq \mathbf{I}_N \otimes \mathbf{E}_N$, with $\mathbf{E}_N$ an all-one $N \times N$ matrix.

First, we deal with the first term, denoted by ℓ_1, on the left-hand side (LHS) of the inequality in Equation (16b). It can be easily shown that $\mathbf{f}^* \otimes \mathbf{g}_k = \tilde{\mathbf{f}}^* \odot \breve{\mathbf{g}}_k$, $\forall k \in K$. Then, we can written ℓ_1 as follows:

$$\begin{aligned}\ell_1 &= P_s(\mathbf{f}^* \otimes \mathbf{g}_k)^H\overline{\mathbf{A}}(\mathbf{f}^* \otimes \mathbf{g}_k) \\ &= P_s\left(\widetilde{\mathbf{f}}^* \odot \breve{\mathbf{g}}_k\right)^H\overline{\mathbf{A}}\left(\widetilde{\mathbf{f}}^* \odot \breve{\mathbf{g}}_k\right) \\ &= P_s\breve{\mathbf{g}}_k^H\mathbf{D}\left(\widetilde{\mathbf{f}}\right)\overline{\mathbf{A}}\mathbf{D}\left(\widetilde{\mathbf{f}}^*\right)\breve{\mathbf{g}}_k.\end{aligned} \tag{17}$$

After some mathematical manipulations, the ℓ_1 can be rewritten as follows:

$$\ell_1 = \Delta\mathbf{g}_k^H\mathbf{T}_{1,k}(\overline{\mathbf{A}})\Delta\mathbf{g}_k + 2\mathrm{Re}\left(\mathbf{t}_{1,k}^H(\overline{\mathbf{A}})\Delta\mathbf{g}_k\right) + c_{1,k}(\overline{\mathbf{A}}), \tag{18}$$

where

$$\begin{aligned}\mathbf{T}_{1,k}(\overline{\mathbf{A}}) &= P_s\mathbf{L}^T\mathbf{D}\left(\widetilde{\mathbf{f}}\right)\overline{\mathbf{A}}\mathbf{D}\left(\widetilde{\mathbf{f}}^*\right)\mathbf{L}, \\ \mathbf{t}_{1,k}^H(\overline{\mathbf{A}}) &= P_s\left(\widetilde{\mathbf{f}}^* \odot \overline{\mathbf{g}}_k\right)^H\overline{\mathbf{A}}\mathbf{D}\left(\widetilde{\mathbf{f}}^*\right)\mathbf{L}, \\ c_{1,k}(\overline{\mathbf{A}}) &= P_s\left(\widetilde{\mathbf{f}}^* \odot \overline{\mathbf{g}}_k\right)^H\overline{\mathbf{A}}\left(\widetilde{\mathbf{f}}^*\ \overline{\mathbf{g}}_k\right).\end{aligned} \tag{19}$$

Next, we proceed with the second term, denoted by ℓ_2, on the LHS of the inequality in Equation (16b). It can be shown that $\mathbf{I}_N \otimes (\mathbf{g}_k\mathbf{g}_k^H) = \mathbf{E} \odot (\breve{\mathbf{g}}_k\breve{\mathbf{g}}_k^H), \forall k \in K$. Then, the ℓ_2 can be expressed as follows:

$$\begin{aligned}\ell_2 &= \alpha\mathrm{Tr}\left(\left(\mathbf{I}_N \otimes \left(\mathbf{g}_k\mathbf{g}_k^H\right)\right)\overline{\mathbf{A}}\right) \\ &= \alpha\mathrm{Tr}\left(\left(\mathbf{E} \odot \left(\breve{\mathbf{g}}_k\breve{\mathbf{g}}_k^H\right)\right)\overline{\mathbf{A}}\right) \\ &= \alpha\breve{\mathbf{g}}_k^H\left(\mathbf{E} \odot \overline{\mathbf{A}}\right)\breve{\mathbf{g}}_k.\end{aligned} \tag{20}$$

After some mathematical manipulations, the ℓ_2 is given by the following:

$$\ell_2 = -\Delta\mathbf{g}_k^H\mathbf{T}_{2,k}(\overline{\mathbf{A}})\Delta\mathbf{g}_k - 2\mathrm{Re}\left(\mathbf{t}_{2,k}^H(\overline{\mathbf{A}})\Delta\mathbf{g}_k\right) - c_{2,k}(\overline{\mathbf{A}}), \tag{21}$$

where

$$\begin{aligned}\mathbf{T}_{2,k}(\overline{\mathbf{A}}) &= \alpha\mathbf{L}^T(\mathbf{E} \odot \overline{\mathbf{A}})\mathbf{L}, \\ \mathbf{t}_{2,k}^H(\overline{\mathbf{A}}) &= \alpha\overline{\mathbf{g}}_k^H(\mathbf{E} \odot \overline{\mathbf{A}})\mathbf{L}, \\ c_{2,k}(\overline{\mathbf{A}}) &= \alpha\overline{\mathbf{g}}_k^H(\mathbf{E} \odot \overline{\mathbf{A}})\overline{\mathbf{g}}_k.\end{aligned} \tag{22}$$

Finally, we consider the third term, denoted by ℓ_3, on the LHS of the inequality in Equation (16b). It is obvious that the ℓ_3 has the following quadratic forms:

$$\ell_3 = -\Delta\mathbf{g}_k^H(\alpha\Omega)\Delta\mathbf{g}_k - 2\mathrm{Re}\left(\alpha\widehat{\mathbf{g}}_k^H\Omega\Delta\mathbf{g}_k\right) - \alpha\widehat{\mathbf{g}}_k^H\Omega\widehat{\mathbf{g}}_k. \tag{23}$$

According to Equations (18), (21), and (23), the constraints in Equation (16b) are equivalent to the following implication:

$$\begin{aligned}&\Delta\mathbf{g}_k^H\Sigma_k\Delta\mathbf{g}_k \leqslant \varepsilon_k^2 \Rightarrow \Delta\mathbf{g}_k^H\mathbf{T}_k(\overline{\mathbf{A}}, \Omega)\Delta\mathbf{g}_k \\ &\quad + 2\mathrm{Re}\left(\mathbf{t}_k^H(\overline{\mathbf{A}}, \Omega)\Delta\mathbf{g}_k\right) + c_k(\overline{\mathbf{A}}, \Omega) - \alpha \leqslant 0, \forall k \in \mathscr{K},\end{aligned} \tag{24}$$

where

$$\begin{aligned}\mathbf{T}_k(\overline{\mathbf{A}}, \Omega) &= \mathbf{T}_{1,k}(\overline{\mathbf{A}}) - \mathbf{T}_{2,k}(\overline{\mathbf{A}}) - \alpha\Omega, \\ \mathbf{t}_k^H(\overline{\mathbf{A}}, \Omega) &= \mathbf{t}_{1,k}^H(\overline{\mathbf{A}}) - \mathbf{t}_{2,k}^H(\overline{\mathbf{A}}) - \alpha\widehat{\mathbf{g}}_k^H\Omega, \\ c_k(\overline{\mathbf{A}}, \Omega) &= c_{1,k}(\overline{\mathbf{A}}) - c_{2,k}(\overline{\mathbf{A}}) - \alpha\widehat{\mathbf{g}}_k^H\Omega\widehat{\mathbf{g}}_k.\end{aligned} \tag{25}$$

By applying the S-procedure [36], the constraints in Equations (24) and (16b) can be equivalently rewritten as the following linear matrix inequalities:

$$\mathbf{F}_k\left(\alpha,\overline{\mathbf{A}},\Omega,\mu_k\right)\triangleq\begin{bmatrix}\mu_k\Sigma_k-\mathbf{T}_k\left(\overline{\mathbf{A}},\Omega\right) & -\mathbf{t}_k\left(\overline{\mathbf{A}},\Omega\right)\\ -\mathbf{t}_k^H\left(\overline{\mathbf{A}},\Omega\right) & -\mu_k\varepsilon_k^2-c_k\left(\overline{\mathbf{A}},\Omega\right)+\alpha\end{bmatrix}\succeq\mathbf{0},\forall k\in\mathcal{K},\tag{26}$$

where $\mu_k\geqslant 0$ is an auxiliary variable. Now, by replacing Equation (16b) with Equation (26), Equation (16) can be reformulated as follows:

$$\begin{aligned}\phi(\alpha)=\max_{\overline{\mathbf{A}}\succeq\mathbf{0},\Omega\succeq\mathbf{0}}\quad & \frac{\operatorname{Tr}\left(\mathbf{B}_1\overline{\mathbf{A}}\right)}{1+\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\Omega)+\operatorname{Tr}\left(\mathbf{B}_2\overline{\mathbf{A}}\right)}\\ \text{s.t.}\quad & \mathbf{F}_k\left(\alpha,\overline{\mathbf{A}},\Omega,\mu_k\right)\succeq\mathbf{0},\ \mu_k\geqslant 0,\ \forall k\in\mathcal{K},\\ & \operatorname{Tr}\left(\mathbf{D}_1\overline{\mathbf{A}}\right)+\operatorname{Tr}(\Omega)\leqslant P_{\max},\\ & \operatorname{Tr}\left(\mathbf{D}_{2,n}\overline{\mathbf{A}}\right)+\operatorname{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega)\leqslant\rho_n,\forall n\in\mathcal{N}.\end{aligned}\tag{27}$$

By applying the Charnes–Cooper transformation [39], the quasi-convex problem (Equation (27)) can be recast into a convex SDP. Specifically, let $\overline{\mathbf{A}}=\mathbf{W}/\eta,\Omega=\mathbf{S}/\eta$, $\mu_k=\nu_k/\eta,\eta>0$. Equation (27) can be equivalently converted into the following:

$$\begin{aligned}\phi(\alpha)=\max_{\mathbf{W}\succeq\mathbf{0},\mathbf{S}\succeq\mathbf{0}}\quad & \operatorname{Tr}(\mathbf{B}_1\mathbf{W})\\ \text{s.t.}\quad & \eta+\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\mathbf{S})+\operatorname{Tr}(\mathbf{B}_2\mathbf{W})=1,\\ & \Psi_k(\alpha,\mathbf{W},\mathbf{S},\nu_k)\succeq\mathbf{0},\ \nu_k\geqslant 0,\ \forall k\in\mathcal{K},\\ & \operatorname{Tr}(\mathbf{D}_1\mathbf{W})+\operatorname{Tr}(\mathbf{S})\leqslant P_{\max}\eta,\ \eta>0,\\ & \operatorname{Tr}\left(\mathbf{D}_{2,n}\mathbf{W}\right)+\operatorname{Tr}(\mathbf{e}_n\mathbf{e}_n^T\mathbf{S})\leqslant\rho_n\eta,\ \forall n\in\mathcal{N},\end{aligned}\tag{28}$$

where

$$\Psi_k(\alpha,\mathbf{W},\mathbf{S},\nu_k)\triangleq\begin{bmatrix}\nu_k\Sigma_k-\mathbf{T}_k(\mathbf{W},\mathbf{S}) & -\mathbf{t}_k(\mathbf{W},\mathbf{S})\\ -\mathbf{t}_k^H(\mathbf{W},\mathbf{S}) & -\nu_k\varepsilon_k^2-c_k(\mathbf{W},\mathbf{S})+\alpha\eta\end{bmatrix}.\tag{29}$$

The resulting problem (Equation (28)) is a convex SDP, which can be efficiently solved by existing conic optimization software, e.g., CVX [37]. When the optimal solution ($\mathbf{W}^*$, $\mathbf{S}^*$, η^*) of Equation (28) is obtained, the optimal solution ($\overline{\mathbf{A}}^*$, $\mathbf{S}^*$) of Equation (27) can be recovered through the variable change $\left(\overline{\mathbf{A}}^*,\Omega^*\right)=(\mathbf{W}^*/\eta^*,\mathbf{S}^*/\eta^*)$.

Now, the inner-level optimization problem (Equation (12)) has been solved by using the SDR technique and the S-procedure. In the following, we will show that there always exists a rank-one optimal solution for Equation (27). That is to say, the relaxation problem (Equation (16)), which is equivalent to Equation (27) is tight to Equation (12).

We consider the following power minimization problem as follows:

$$\min_{\overline{\mathbf{A}}\succeq\mathbf{0},\Omega\succeq\mathbf{0}}\ \operatorname{Tr}\left(\mathbf{D}_1\overline{\mathbf{A}}\right),\tag{30a}$$

$$\text{s.t. }\Psi_k\left(\alpha,\overline{\mathbf{A}},\Omega,\mu_k\right)\succeq\mathbf{0},\ \mu_k\geqslant 0,\ \forall k\in\mathcal{K},\tag{30b}$$

$$\operatorname{Tr}\left(\mathbf{D}_1\overline{\mathbf{A}}\right)+\operatorname{Tr}(\Omega)\leqslant P_{\max},\tag{30c}$$

$$\operatorname{Tr}\left(\mathbf{D}_{2,n}\overline{\mathbf{A}}\right)+\operatorname{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega)\leqslant\rho_n,\ \forall n\in\mathcal{N},\tag{30d}$$

$$\operatorname{Tr}\left((\phi(\alpha)\mathbf{B}_2-\mathbf{B}_1)\overline{\mathbf{A}}\right)+\phi(\alpha)\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\Omega)+\phi(\alpha)\leqslant 0,\tag{30e}$$

where $\phi(\alpha)$ is the optimal objective value of Equation (27). The constraint in Equation (30e) is converted from $\operatorname{Tr}\left(\mathbf{B}_1\overline{\mathbf{A}}\right)/1+\mathbf{h}^H\Omega\mathbf{h}+\operatorname{Tr}\left(\mathbf{B}_2\overline{\mathbf{A}}\right)\geqslant\phi(\alpha)$. For Equation (30), we can derive two important properties listed in Propositions 1 and 2.

Proposition 1. *Any optimal solution of Equation (30), denoted by $\left(\overline{\mathbf{A}}^\dagger,\Omega^\dagger\right)$, is also the optimal solution of Equation (27).*

Proof. By checking the constraints of Equations (30) and (27), we can find that $\left(\overline{\mathbf{A}}^\dagger,\Omega^\dagger\right)$ is also the feasible solution of Equation (27). Hence, $\operatorname{Tr}\left(\mathbf{B}_1\overline{\mathbf{A}}^\dagger\right)/1+\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\Omega^\dagger)+\operatorname{Tr}\left(\mathbf{B}_2\overline{\mathbf{A}}^\dagger\right)\leqslant\phi(\alpha)$. From Equation (30e), we have $\operatorname{Tr}\left(\mathbf{B}_1\overline{\mathbf{A}}^\dagger\right)/1+\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\Omega^\dagger)+\operatorname{Tr}\left(\mathbf{B}_2\overline{\mathbf{A}}^\dagger\right)\geqslant\phi(\alpha)$. As a result, $\operatorname{Tr}\left(\mathbf{B}_1\overline{\mathbf{A}}^\dagger\right)/1+\operatorname{Tr}(\mathbf{h}\mathbf{h}^H\Omega^\dagger)+\operatorname{Tr}\left(\mathbf{B}_2\overline{\mathbf{A}}^\dagger\right)=\phi(\alpha)$ must hold true, which implies that $\left(\overline{\mathbf{A}}^\dagger,\Omega^\dagger\right)$ is optimal to Equation (27). □

Proposition 2. *When $\phi(\alpha)>0$, the optimal solution $\overline{\mathbf{A}}^\dagger$ of Equation (30) is rank-one.*

The proof of Proposition 2 is given in Appendix.

From Propositions 1 and 2, we can derive the following theorem.

Theorem 1. *When $\phi(\alpha)>0$, there always exists an optimal solution $\left(\overline{\mathbf{A}}^\dagger,\Omega^\dagger\right)$ for Equation (27) such that* $\operatorname{Rank}\left(\overline{\mathbf{A}}^\dagger\right)=1$. *Furthermore, the rank-one solution can always be obtained by solving Equations (27) and (30).*

According to Theorem 1, we can get the optimal beamforming matrix $\mathbf{A}^*=\operatorname{vec}^{-1}(\mathbf{a})$ of Equation (8) with $\overline{\mathbf{A}}^\dagger=\mathbf{a}\mathbf{a}^H$ being the rank-one optimal solution of Equation (27).

4. Simulation Results

In this section, we use Monte Carlo simulations to evaluate the worst-case secrecy rate performance of the proposed AN-aided beamforming design. The CVX [37] is used to solve the optimization problem (Equation (28)). Each curve is obtained by averaging over 1,000 independent channel realizations. In all the simulations, the channel coefficients are independently generated following the complex Gaussian distribution with zero-mean and unit covariance. We set

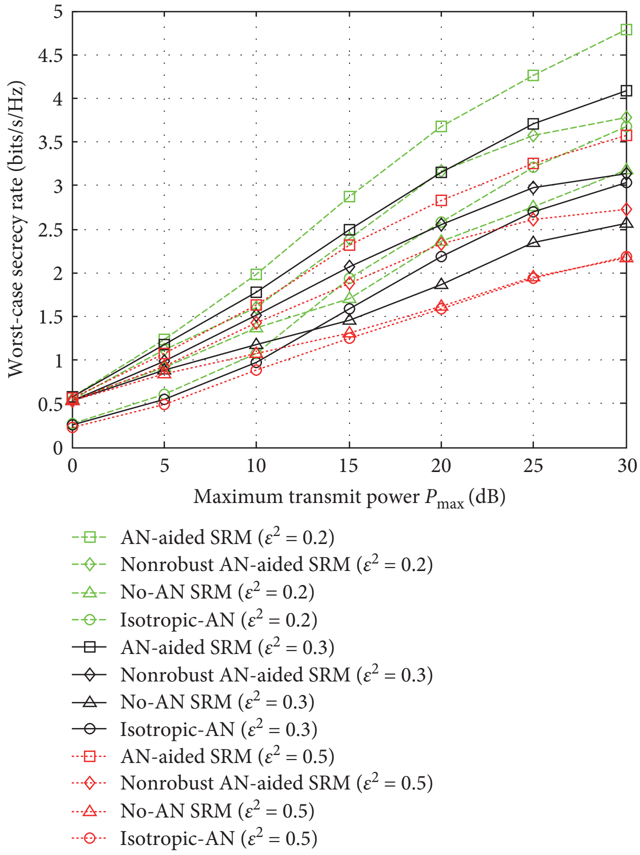

FIGURE 2: Worst-case secrecy rate versus the maximum transmit power P_{max} with $N = 6, K = 3$ and $\varepsilon^2 = 0.2$, 0.3, and 0.5.

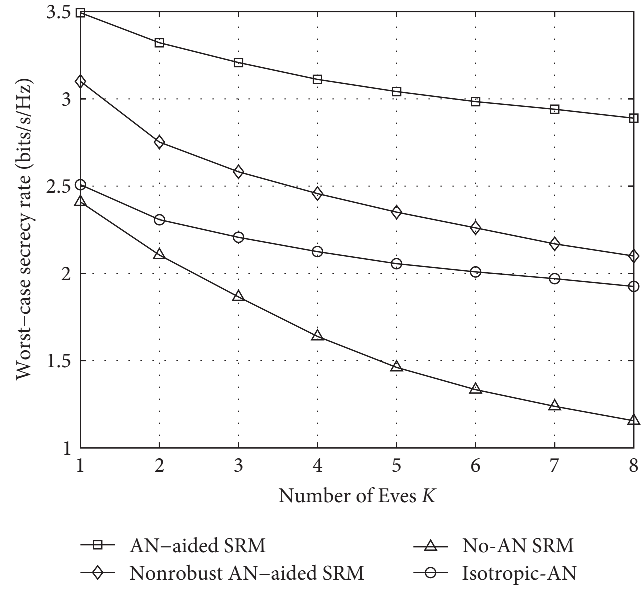

FIGURE 3: Worst-case secrecy rate versus the number of Eves K with $P_{max} = 20$ dB, $N = 6$, and $\varepsilon^2 = 0.3$.

the signal power of Alice as $P_s = 20$ dB. The relay is equipped with $N = 6$ antennas, and the maximum transmit power is $P_{max} = 20$ dB unless specified otherwise. It is assumed that each antenna's maximum transmit power is $\rho = \rho_n = P_{max}/N$, $\forall n$. Without loss of generality, we set $\Sigma = \Sigma_k = \mathbf{I}_N$ and $\varepsilon^2 = \varepsilon_k^2, \forall k$.

We compare the performance of our proposed design (labeled "AN-aided SRM") with the ones of the following three designs: (1) the "Nonrobust AN-aided SRM," where the estimated CSI with errors is considered as the perfect CSI; (2) the "No-AN SRM" which is the same as the "AN-aided SRM," except that the AN covariance matrix is set as zero; (3) the "Isotropic AN," where the AN is transmitted in the null space of the main channel, and the power is equally allocated between the beamforming matrix and isotropic AN [14, 40]. Note that the "Isotropic AN" design does not take into account the per-antenna power constraints. Hence, the obtained solution $(\overline{\mathbf{A}}, \Omega)$ in the "Isotropic AN" design should be multiplied by the scaling factor $\min\{1, \{\rho_n/(\mathrm{Tr}(\mathbf{D}_{2,n}\overline{\mathbf{A}}) + \mathrm{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega))\}_1^N\}$ for satisfying the per-antenna constraints in Equation (10d).

In Figure 2, we present the worst-case secrecy rate versus the maximum transmit power P_{max} for different designs with $K = 3$ and $\varepsilon^2 = 0.2$, 0.3, and 0.5. As can be clearly seen from Figure 2, the proposed AN-aided SRM design surpasses the other three methods over the whole region of powers tested. Moreover, we find that the nonrobust design is sensitive to the imperfect CSI, i.e., it has lower performance than the robust method. This demonstrates the necessity to design the robust method. When the power is small, e.g., $P_{max} = 0$ dB, all the designs approach the same performance nearly, but the rate gap between the proposed design and the other methods becomes wider with the increasing power. This indicates that the AN can improve the secrecy performance; also, the proposed AN pattern is more efficient for confusing the Eves than the isotropic AN. We also compared the case with different impact parameter values $\varepsilon^2 = 0.2$, 0.3, and 0.5, and as can be clearly seen from Figure 2, the worst-case secrecy rate decreases as ε^2 increases.

Figure 3 plots the worst-case secrecy rate against the number of Eves with $P_{max} = 20$ dB and $\varepsilon^2 = 0.3$. As expected, we can observe that the worst-case secrecy rates of all the designs decrease with an increasing number of Eves. The performance of the No-AN SRM design drops dramatically. By contrast, the proposed AN-aided SRM design always shows the best performance under the different number of Eves. This confirms that the AN can enhance security significantly. Even though both the AN-aided SRM design and the Isotropic-AN use the AN, the former has better performance than the latter. This is attributed to the fact that AN-aided SRM design fully uses the CSI of all channels, including Eves' channels with errors and Bob's channel, to optimize the beamforming matrix and the AN covariance matrix. On the contrary, for the Isotropic-AN, the relay can not fully exploit the degrees of freedom of its own since the beamforming matrix and the AN pattern are fixed according to the CSI of Bob.

To investigate the impact of imperfect CSI of Eves on the worst-case secrecy rate performance, in Figure 4, we show the worst-case secrecy rate behaviors of different designs with fixed maximum transmit power and the fixed number

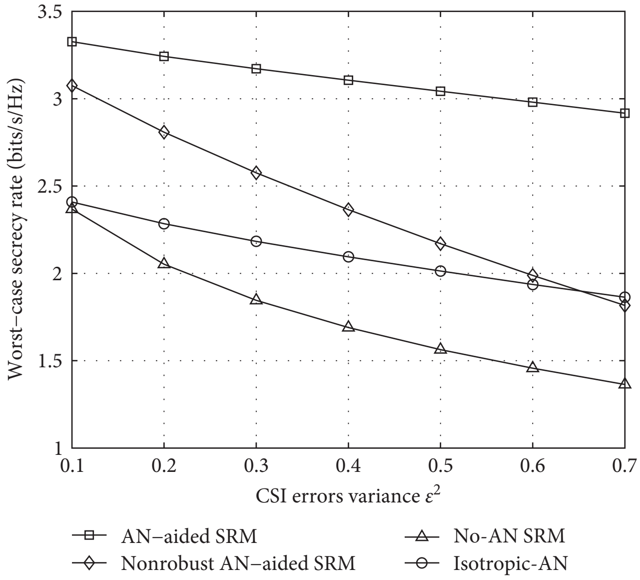

Figure 4: Worst-case secrecy rate versus the CSI errors variance ε^2 with $P_{\max} = 20$ dB, $N = 6$ and $K = 3$.

of Eves, i.e., $P_{\max} = 20$ dB and $K = 3$. As is expected, the worst-case secrecy rate is a monotonically decreasing function of ε^2. This is because the relay has to allocate more power to the artificial noise for jamming the Eves when the CSI error variance ε^2 increases. As a result, the signal power becomes less, and the worst-case secrecy rate decreases correspondingly. In addition, the performance of the Nonrobust AN-aided SRM design degrades rapidly when the CSI errors variance ε^2 becomes larger. By contrast, the proposed AN-aided SRM design provides substantial performance improvements compared to the other designs due to the joint optimization of the beamforming matrix and AN covariance matrix. This phenomenon again confirms the robustness and effectiveness of the proposed design.

5. Conclusion

In this paper, we explore robust and secure transmission for AF multiantenna relay networks in the presence of multiple single-antenna eavesdroppers. The beamforming matrix and the AN covariance matrix of the relay are jointly optimized to maximize the worst-case secrecy rate under total power and per-antenna power constraints. Due to the intractability of the resulting WCSRM problem, it is converted into a two-level optimization. The outer part can be handled by 1D search, while the inner part can be handled by SDR technique. The simulation results have effectively showcased the performance enhancements and robustness achieved by the proposed AN-aided transmit design.

Appendix

It can be verified that the convex problem (Equation (8)) satisfies Slater's conditions [36]. Hence, the strong duality holds between Equation (30) and its dual problem, and the optimal solutions satisfy the Karush–Kuhn–Tucker (KKT) conditions. The Lagrangian function of Equation (30) is given by the following:

$$\begin{aligned}\mathscr{L} = {} & \mathrm{Tr}\big(\mathbf{D}_1\overline{\mathbf{A}}\big) - \mathrm{Tr}\big(\mathbf{X}\overline{\mathbf{A}}\big) - \mathrm{Tr}(\mathbf{Y}\Omega) - \sum_{k=1}^{K}\mathrm{Tr}\big(\mathbf{Z}_k\mathbf{F}_k\big(\alpha, \overline{\mathbf{A}}, \Omega, \mu_k\big)\big) - \sum_{k=1}^{K}\delta_k\mu_k \\ & + \theta\big(\mathrm{Tr}\big(\mathbf{D}_1\overline{\mathbf{A}}\big) + \mathrm{Tr}(\Omega) - P_{\max}\big) + \sum_{n=1}^{N}\psi_n\big(\mathrm{Tr}\big(\mathbf{D}_{2,n}\overline{\mathbf{A}}\big) + \mathrm{Tr}(\mathbf{e}_n\mathbf{e}_n^T\Omega) - \rho_n\big) \\ & + \zeta\big(\mathrm{Tr}\big((\phi(\alpha)\mathbf{B}_2 - \mathbf{B}_1)\overline{\mathbf{A}}\big) + \phi(\alpha)\mathrm{Tr}(\Omega\mathbf{h}\mathbf{h}^H) + \phi(\alpha)\big),\end{aligned} \tag{31}$$

where $\mathbf{X} \succeq \mathbf{0}$, $\mathbf{Y} \succeq \mathbf{0}$, $\mathbf{Z}_k \succeq \mathbf{0}$, $\delta_k \geqslant 0, \forall k \in \mathscr{K}$, $\theta \geqslant 0, \psi_n, \forall n \in \mathscr{N}$, and $\zeta \geqslant 0$ are the optimal dual variables associated with the constraints $\overline{\mathbf{A}} \succeq \mathbf{0}, \Omega \succeq \mathbf{0}, \mathbf{F}_k(\alpha, \overline{\mathbf{A}}, \Omega, \mu_k) \succeq \mathbf{0}$, $\nu_k \geqslant 0, \forall k \in \mathscr{K}$, Equations (30c), (30d), and (30e), respectively.

For ease of expression, we first rewrite $\mathbf{F}_k(\alpha, \overline{\mathbf{A}}, \Omega, \mu_k)$ as follows:

$$\begin{aligned}\mathbf{F}_k\big(\alpha, \overline{\mathbf{A}}, \Omega, \mu_k\big) = {} & \mathrm{blkdiag}\big(\mu_k\Sigma_k, -\mu_k\varepsilon_k^2 + \alpha\big) \\ & - \mathbf{P}_{1,k}^H\overline{\mathbf{A}}\mathbf{P}_{1,k} + \alpha\mathbf{P}_{2,k}^H\big(\mathbf{E} \odot \overline{\mathbf{A}}\big)\mathbf{P}_{2,k} \\ & + \alpha\mathbf{P}_{3,k}^H\Omega\mathbf{P}_{3,k},\end{aligned} \tag{32}$$

where blkdiag(.) represents the block diagonal matrix; $\mathbf{P}_{1,k} = \big[\mathbf{D}\big(\tilde{\mathbf{f}}^*\big)\mathbf{L}\big(\tilde{\mathbf{f}}^* \odot \overline{\mathbf{g}}_k\big)\big]$; $\mathbf{P}_{2,k} = [\mathbf{L}\overline{\mathbf{g}}_k]$; and $\mathbf{P}_{3,k} = [\mathbf{I}_N\ \widehat{\mathbf{g}}_k]$.

Substituting Equation (32) into Equation (31), we can get some KKT conditions related to the proof as follows:

$$\begin{aligned}\mathbf{X} = {} & (1 + \theta)\mathbf{D}_1 + \sum_{n=1}^{N}\psi_n\mathbf{D}_{2,n} + \zeta(\phi(\alpha)\mathbf{B}_2 - \mathbf{B}_1) \\ & + \sum_{k=1}^{K}\Big(\mathbf{P}_{1,k}\mathbf{Z}_k\mathbf{P}_{1,k}^H - \alpha\Big(\mathbf{E} \odot \Big(\mathbf{P}_{2,k}\mathbf{Z}_k\mathbf{P}_{2,k}^H\Big)\Big)\Big),\end{aligned} \tag{33a}$$

$$\mathbf{Y} = -\alpha \sum_{k=1}^{K} \mathbf{P}_{3,k}\mathbf{Z}_k\mathbf{P}_{3,k}^H + \theta\mathbf{I}_N + \sum_{n=1}^{N} \psi_n \mathbf{e}_n\mathbf{e}_n^T + \zeta\phi(\alpha)\mathbf{h}\mathbf{h}^H, \tag{33b}$$

$$\mathbf{X}\overline{\mathbf{A}} = \mathbf{0},\ \overline{\mathbf{A}} \geq \mathbf{0}. \tag{33c}$$

According to Equation (33b), the Kronecker product of $\mathbf{I}_N$ and $\mathbf{Y}$ can be expressed as follows:

$$\begin{aligned}\mathbf{I}_N \otimes \mathbf{Y} = &-\alpha \sum_{k=1}^{K} \mathbf{I}_N \otimes \left(\mathbf{P}_{3,k}\mathbf{Z}_k\mathbf{P}_{3,k}^H\right) + \theta\mathbf{I}_N \otimes \mathbf{I}_N \\ &+ \sum_{n=1}^{N} \psi_n \mathbf{I}_N \otimes (\mathbf{e}_n\mathbf{e}_n^T) + \zeta\phi(\alpha)\mathbf{I}_N \otimes (\mathbf{h}\mathbf{h}^H).\end{aligned} \tag{34}$$

It can be easily shown that $\mathbf{I}_N \otimes \left(\mathbf{P}_{3,k}\mathbf{Z}_k\mathbf{P}_{3,k}^H\right) = \mathbf{E} \odot \left(\mathbf{P}_{2,k}\mathbf{Z}_k\mathbf{P}_{2,k}^H\right)$. Hence, subtracting Equation (34) from Equation (33a), we can get the following:

$$\mathbf{X} = \mathbf{Q} - \zeta\mathbf{B}_1, \tag{35}$$

where

$$\begin{aligned}\mathbf{Q} = &(1+\theta)P_s\left(\left(\mathbf{f}^*\mathbf{f}^T\right) \otimes \mathbf{I}_N\right) + \mathbf{I}_{N^2} + \sum_{k=1}^{K} \mathbf{P}_{1,k}\mathbf{Z}_k\mathbf{P}_{1,k}^H \\ &+ \sum_{n=1}^{N} \psi_n P_s\left(\mathbf{f}^*\mathbf{f}^T\right) \otimes (\mathbf{e}_n\mathbf{e}_n^T) + \mathbf{I}_N \otimes \mathbf{Y}.\end{aligned} \tag{36}$$

By postmultiplying both sides of Equation (35) by $\overline{\mathbf{A}}$, and using the Equation (33c), we can obtain the following:

$$\mathbf{Q}\overline{\mathbf{A}} = \zeta\mathbf{B}_1\overline{\mathbf{A}}. \tag{37}$$

It can be found that $\mathbf{I}_{N^2} \succ \mathbf{0}$, and all the other term on the right-hand side of Equation (36) are positive semidefinite matrices, therefore $\mathbf{Q} \succ \mathbf{0}$. Hence, we have the following relation:

$$\mathrm{Rank}\left(\overline{\mathbf{A}}\right) = \mathrm{Rank}\left(\mathbf{Q}\overline{\mathbf{A}}\right) = \mathrm{Rank}\left(\zeta\mathbf{B}_1\overline{\mathbf{A}}\right) \leqslant \mathrm{Rank}(\mathbf{B}_1) = 1. \tag{38}$$

Since $\phi(\alpha) > 0$, the constraint in Equation (30e) will be violated if $\overline{\mathbf{A}} = \mathbf{0}$. As a result, $\mathrm{Rank}\left(\overline{\mathbf{A}}\right) = 1$ holds true, which completes the proof.

Data Availability

The data of the models can be available by request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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