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Research Article

q-Rung Orthopair Picture Fuzzy Topological Spaces and Parameter-Dependent Continuity: Control System Applications

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This paper presents novel concepts in fuzzy topologies, namely q-rung orthopair picture fuzzy (q-ROPF) topology and q-rung orthopair picture fuzzy point (q-ROPFP). These concepts extended the existing notions in fuzzy topologies. We introduced a more relaxed form of continuity, called q_ε-ROPF continuity, which allows for a flexible analysis of functions between q-ROPF topological spaces by incorporating an error bound ε. We define the q-neighborhood system for a q-ROPF point in a q-ROPF topological space and investigate the convergence of sequences of such points. Our results shed light on the behavior of functions and sequences in these spaces, as well as the conditions for uniqueness of limits within the realm of supports. This research has practical implications in control systems, providing valuable insights into the stability and robustness of the control strategies in the presence of small changes represented by the epsilon value.

1. Introduction

Topology is a branch of mathematics that studies the properties of objects that remain invariant under the continuous transformations. It deals with the study of shapes, spaces, and their properties. Topology has numerous applications in various fields such as physics, engineering, computer science, chemistry, and biology [1–5]. The category of topological spaces is characterized by isomorphisms known as homeomorphisms, which hold a vital role in the theory. Continuity is the topological interpretation of a homeomorphism, as defined by Bourbaki [2], and it provides insight into the concept of proximity within mathematical models. In fact, continuity is considered one of the most essential properties of a function between two topological spaces.

In a topological space, the convergence of a sequence is defined in terms of neighborhoods and open sets, rather than metrics. This generalization allows for a more abstract and flexible approach to the study of continuity and limit, which is a crucial aspect of topology. The ability to determine whether a sequence converges or not in a topological space enables the analysis of the properties of functions and the structures of spaces, leading to a deeper understanding of the mathematical concepts involved. Therefore, the importance of the convergence of sequences in a topological space cannot be overstated, as it is a fundamental tool for studying the continuity, limit, and the properties of topological spaces.

Fuzzy set theory is a branch of mathematics that allows for the representation of imprecise and uncertain information. Zadeh [6] proposed the concept of fuzzy set, which is a set in which elements can have degrees of membership between 0 and 1, rather than being either a member or not. This revolutionary idea led to the development of numerous extensions and variations of the fuzzy set theory. One of these extensions is the notion of intuitionistic fuzzy set [7], which introduces a second degree of membership to represent non-membership. Another extension is the notion of Pythagorean fuzzy set [8]. The notion of q-rung orthopair fuzzy set [9], on the other hand, is generalization of Pythagorean fuzzy sets. Picture fuzzy sets [10], are another extension that uses a third function to represent the neutral membership degree of elements. Spherical fuzzy sets [11, 12], proposed in 2019, use the concept of spherical membership functions to represent the degrees of membership of elements. Finally, q-rung orthopair picture fuzzy sets (q-ROPFSs) [13] combine the ideas of q-

rung orthopair fuzzy sets and picture fuzzy sets. These various extensions and variations of fuzzy set theory have found numerous applications in the fields such as decision-making, image processing, and pattern recognition [14–22].

Chang [23] introduced the concept of the fuzzy topological space, but Lowen's [24] alternative definition changed a fundamental property of topology. Çoker [25] and Turanlı and Çoker [26] later expanded the concept by introducing the idea of the intuitionistic fuzzy topological space and exploring analog versions of classic topological concepts such as continuity and compactness. Olgun et al. [27] proposed the notion of Pythagorean fuzzy topology and defined the Pythagorean fuzzy continuity of functions between Pythagorean fuzzy topological spaces. Türkarslan et al. [28] introduced the topology of q-rung orthopair fuzzy sets. Razaq et al. [29] investigated the picture fuzzy topologies and the continuity between these spaces. Furthermore, Öztürk et al. [30] conducted an investigation on neutrosophic soft compact spaces. These various studies replaced fixed boundaries with degree-theoretic structures of fuzziness within topology.

Pao-Ming and Ying-Ming [31] introduced the concept of fuzzy points and quasi-coincidence for fuzzy points, along with the Q-neighborhood system of a fuzzy point in a fuzzy topological space. This work also explored versions of some classic notions of topology. Additionally, convergence of nets in fuzzy topological spaces was studied by Pao-Ming and Ying-Ming [31]. Çoker and Demirci [32] introduced the notion of an intuitionistic fuzzy point. Lupiañez [33] explored quasi-coincidence for intuitionistic fuzzy points in this study. The convergence of nets of intuitionistic fuzzy points was introduced and studied by Lupiañez [34]. Olgun et al. [17] studied Pythagorean fuzzy points and their topological properties. Furthermore, Türkarslan et al. [35] explored the notion of q-rung orthopair fuzzy point and their applications.

Fuzzy logic is a popular tool for modeling uncertainty in the control systems as it provides a flexible and intuitive framework for representing the imprecise or vague information and capturing the relationships between various elements of the system. In control systems, fuzzy sets are used to model the control objectives and constraints, the state space, and the control space [36–38]. The use of fuzzy sets allows for a more comprehensive and robust representation of the system, making it possible to design control strategies that are insensitive to small variations in the system. Additionally, fuzzy sets provide a means to model the interactions between different parts of the system, making it possible to develop control strategies that take into account the complex relationships between the different component.

This paper presents the concepts of q-rung orthopair picture fuzzy (q-ROPF) topology and q-rung orthopair picture fuzzy point (q-ROPFP). The continuity of functions between these spaces is studied, with a focus on its applications in the control systems. We also define the Q-neighborhood system of a q-ROPFP in a q-ROPF topological space. Additionally, we investigate the convergence of sequences of q-ROPFPs and the conditions for the uniqueness of the limit in the realm of supports.

Main contributions and the motivation of the present paper listed as follows:

(i) This paper introduces the concepts of q-ROPF topology and q-ROPFP, which are novel and represent an extension of existing concepts in the fuzzy set theory.

(ii) The continuity of functions in q-ROPF topological spaces is studied with respect to a control parameter, which provides insights into the behavior of functions in these spaces and can be applied to the control systems.

(iii) The motivation behind this research stems from the need for robust and reliable control strategies that can adapt to dynamic environments and uncertain conditions. By establishing the continuity and stability of mappings between the fuzzy topologies, we can enhance the performance of control systems, ensuring they maintain desired behaviors even in the presence of disturbances and uncertainties.

(iv) This paper defines the Q-neighborhood system of a q-ROPFP in a q-ROPF topological space, which is a fundamental aspect of the topological properties of these spaces.

(v) The convergence of sequences of q-ROPFPs is investigated, which provides insights into the behavior of sequences in these spaces and the conditions for the uniqueness of the limit in the realm of supports.

The remainder of the paper is structured as follows: in Section 2, we outlined some basic concepts of fuzzy set theory and several set-theoretic operations for q-ROPFSs. In Section 3, we introduced the idea of a q-ROPF topology. We also define the image and pre-image of a function between two q-ROPF topological spaces and the concept of function continuity. We demonstrated the utilization of functional continuity in the control systems. In Section 5, we introduced the concept of q-ROPFP and examine its properties. After defining the Q-neighborhoods of a q-ROPFP, we examined the convergence of a sequence of q-ROPFPs and investigate the conditions for the unique limit within the realm of supports. Finally, in Section 6, the paper is concluded.

2. Preliminaries

In this section, we recall some basic definitions of fuzzy set theory and study some set-theoretic operations for q-ROPFSs. Unless otherwise specified, across the entirety of this manuscript, we adopt the assumption that X is a finite set represented as $X = x_1, \ldots, x_n$.

Definition 1 [9]. Let $q \geq 1$. A q-rung orthopair fuzzy set A in X is given by

$$A = \left\{ \left\langle x_j, \mu_A\left(x_j\right), \nu_A\left(x_j\right) \right\rangle : j = 1, \ldots, n \right\}, \tag{1}$$

where $\mu_A, \nu_A : X \to [0, 1]$ are membership and non-membership functions, respectively, satisfying

$$\mu_A^q(x_j) + \nu_A^q(x_j) \leq 1. \tag{2}$$

Definition 2 [10]. A picture fuzzy set A on X is given by

$$A = \{\langle x_j, \mu_A(x_j), \varphi_A(x_j), \nu_A(x_j)\rangle : j = 1, \ldots, n\}, \tag{3}$$

where $\mu_A, \varphi_A, \nu_A : X \to [0, 1]$ are positive, neutral and negative membership functions, respectively, satisfying

$$\mu_A(x_j) + \varphi_A(x_j) + \nu_A(x_j) \leq 1. \tag{4}$$

The concept of q-ROPFS was derived from the definitions found in q-rung orthopair fuzzy sets and picture fuzzy sets.

Definition 3 [13]. Let $q \geq 1$. A q-ROPFS A in X is given by

$$A = \{\langle x_j, \mu_A(x_j), \varphi_A(x_j), \nu_A(x_j)\rangle : j = 1, \ldots, n\}, \tag{5}$$

where $\mu_A, \varphi_A, \nu_A : X \to [0, 1]$ are positive membership, neutral membership and negative membership functions, respectively, satisfying

$$\mu_A^q(x_j) + \varphi_A^q(x_j) + \nu_A^q(x_j) \leq 1. \tag{6}$$

To introduce the concept of q-ROPF topology, we need some set-theoretic operations for q-ROPFSs. A similar definition was presented by Cường [10] for the special case of $q = 1$, that is, for picture fuzzy sets.

Definition 4. Let A and B be two q-ROPFSs. Then

(i) the union of A and B is defined by

$$A \cup B = \left\{ \left\langle \begin{array}{c} x_j, \max(\mu_A(x_j), \mu_B(x_j)), \min(\varphi_A(x_j), \varphi_B(x_j)), \\ \min(\nu_A(x_j), \nu_B(x_j)) \end{array} \right\rangle : j = 1, \ldots, n \right\}, \tag{7}$$

(ii) the intersection of A and B is defined by

$$A \cap B = \left\{ \left\langle \begin{array}{c} x_j, \min(\mu_A(x_j), \mu_B(x_j)), \min(\varphi_A(x_j), \varphi_B(x_j)), \\ \max(\nu_A(x_j), \nu_B(x_j)) \end{array} \right\rangle : j = 1, \ldots, n \right\}, \tag{8}$$

(iii) the complement of A is defined by

$$A^c = \{\langle x_j, \nu_A(x_j), \varphi_A(x_j), \mu_A(x_j)\rangle : j = 1, \ldots, n\}, \tag{9}$$

(iv) $A \subset B$ if and only if $\mu_A(x_j) \leq \mu_B(x_j)$ and $\nu_A(x_j) \geq \nu_B(x_j)$.

Remark 1.

1. *It is straightforward to see that $A \cup B$, $A \cap B$, and A^c are all q-ROPFSs.*
2. *Unlike picture fuzzy sets, the definition of subset for q-ROPFs does not include a condition for the neutral membership function. This avoids potential issues that may arise from including such a condition. In fact, the neutral membership function is typically used to model uncertainty or ambiguity in the membership status of an element, which may not be directly related to its inclusion in a set.*
3. *Instead of using minimum and maximum, the infimum and supremum operators are employed when forming the union or intersection of infinitely many q-ROPFSs.*
4. *It is evident that for any q-ROPFSs A and B, A and B are subsets of $A \cup B$ and $A \cap B$ is a subset of A and B.*
5. *The sets 1_X and 0_X are defined as follows:*

$$1_X = \{\langle x_j, 1, 0, 0\rangle : j = 1, \ldots, n\}, \tag{10}$$

and

$$0_X = \{\langle x_j, 0, 0, 1\rangle : j = 1, \ldots, n\}. \tag{11}$$

The complement of 0_X is equal to 1_X and vice versa. On the other hand, any q-ROPFS is contained within the set 1_X and any q-ROPFS contains 0_X.

De Morgan's laws are fundamental laws of logic and set theory that play a crucial role in mathematical reasoning and analysis. It is crucial to note that the set operations defined in Definition 4 satisfy De Morgan's laws, which is proven in the following.

Proposition 1. Let A and B be two q-ROPFSs. Then we have

(i) $(A \cup B)^c = A^c \cap B^c$,
(ii) $(A \cap B)^c = A^c \cup B^c$.

Proof.

(i) From Definition 4 we have

$$(A \cup B)^c = \left\{ \left\langle \begin{array}{c} x_j, \min(\nu_A(x_j), \nu_B(x_j)), \min(\varphi_A(x_j), \varphi_B(x_j)), \\ \max(\mu_A(x_j), \mu_B(x_j)) \end{array} \right\rangle : j = 1, \ldots, n \right\} = A^c \cap B^c. \tag{12}$$

(ii) From Definition 4 we have

$$(A \cap B)^c = \left\{ \left\langle \begin{array}{c} x_j, \max(\nu_A(x_j), \nu_B(x_j)), \min(\varphi_A(x_j), \varphi_B(x_j)), \\ \min(\mu_A(x_j), \mu_B(x_j)) \end{array} \right\rangle : j = 1, \ldots, n \right\} = A^c \cup B^c. \tag{13}$$

□

3. q-Rung Orthopair Picture Fuzzy Topological Spaces

In this section, equipped with the framework of set-theoretic operations, we are ready to formulate the notion of the q-ROPF topology and its associated ideas. This section holds significant importance, as it establishes the groundwork for the remaining content of the paper. By introducing essential concepts like the image and pre-image of a function, as well as the continuity of a function between two q-ROPF topological spaces, this section sets the stage for the subsequent discussions.

Definition 5. The family τ of q-ROPFSs in X is referred to as a q-ROPF topology on X if it satisfies the following conditions:

(T1) 1_X and 0_X are elements of τ;
(T2) The intersection of any two sets A_1 and A_2 in τ is also in τ;
(T3) The union of any collection of sets $\{A_i\}_{i \in I}$ in τ is also in τ.

In this case, the pair (X, τ) is referred to as a q-ROPF topological space. Members of τ are referred to as open q-ROPFSs, while the complement of an open set is a closed set. The collection $\{1_X, 0_X\}$ is called the indiscreet q-ROPF topological space, and the topology that contains all q-ROPFSs is called the discrete q-ROPF topological space. If $\tau_1 \subset \tau_2$, then the q-ROPF topological space τ_1 is said to be coarser than the q-ROPF topological space τ_2, or equivalently, τ_2 is finer than τ_1.

Example 1. Let $X = \{x_1, x_2, x_3\}$ be a set of three elements. Consider the following 3-ROPFSs in X:

$$A = \{\langle x_1, 0.7, 0.8, 0.2\rangle, \langle x_2, 0.6, 0.9, 0.1\rangle, \langle x_3, 0.5, 0.7, 0.3\rangle\}, \tag{14}$$

$$B = \{\langle x_1, 0.5, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.5, 0.5\rangle, \langle x_3, 0, 1, 0\rangle\}, \tag{15}$$

$$C = \{\langle x_1, 0.7, 0.4, 0.2\rangle, \langle x_2, 0.6, 0.5, 0.1\rangle, \langle x_3, 0.5, 0.1, 0\rangle\}, \tag{16}$$

and

$$D = \{\langle x_1, 0.5, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.5, 0.5\rangle, \langle x_3, 0, 0.1, 0.3\rangle\}. \tag{17}$$

Let $\tau = \{1_X, 0_X, A, B, C, D\}$. It can be easily verified that τ satisfies conditions T1, T2, and T3, so τ is a 3-ROPF topology on X.

Definition 6. Given two non-empty sets X and Y, and a function $f : X \to Y$, let A and B be q-ROPFSs in X and Y, respectively. The image of A under f, denoted as $f[A]$, has its positive membership function, neutral membership function, and negative membership function defined as follows:

$$\mu_{f[A]}(y) := \begin{cases} \sup\limits_{z \in f^{-1}(y)} \mu_A(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 0, & \text{otherwise,} \end{cases} \tag{18}$$

$$\varphi_{f[A]}(y) := \begin{cases} \inf\limits_{z \in f^{-1}(y)} \varphi_A(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 0, & \text{otherwise,} \end{cases} \tag{19}$$

and

$$\nu_{f[A]}(y) := \begin{cases} \inf\limits_{z\in f^{-1}(y)} \nu_A(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 1, & \text{otherwise,} \end{cases} \tag{20}$$

respectively. The positive membership function, the neutral membership function and negative membership function of pre-image of B with respect to f that is denoted by $f^{-1}[B]$ are defined by

$$\mu_{f^{-1}[B]}(x) := \mu_B(f(x)), \tag{21}$$

$$\varphi_{f^{-1}[B]}(x) := \varphi_B(f(x)), \tag{22}$$

and

$$\nu_{f^{-1}[B]}(x) := \nu_B(f(x)), \tag{23}$$

respectively.

Example 2. Let $X = \{-2, -1, 1\}$ and $Y = \{1, 4\}$, and let $f: X \to Y$ be the surjection f defined by $f(x) = x^2$. Consider the 3-ROPFS A of X given by

$$A = \{\langle -2, 0.7, 0.8, 0.1\rangle, \langle -1, 0.6, 0.9, 0.1\rangle, \langle 1, 0.4, 0.3, 0.7\rangle\}. \tag{24}$$

Then

$$\begin{aligned} \mu_{f[A]}(1) &= \sup\{\mu_A(-1), \mu_A(1)\} \\ &= \max\{0.6, 0.4\} \\ &= 0.6. \end{aligned} \tag{25}$$

Similarly, we have $\mu_{f[A]}(4) = 0.7$, $\varphi_{f[A]}(1) = 0.3$, $\varphi_{f[A]}(4) = 0.8$, $\nu_{f[A]}(1) = 0.1$ and $\nu_{f[A]}(4) = 0.1$. So, we have

$$f[A] = \{\langle 1, 0.6, 0.3, 0.1\rangle, \langle 4, 0.7, 0.8, 0.1\rangle\}. \tag{26}$$

Now consider the 3-ROPFS B of Y given by

$$B = \{\langle 1, 0.6, 0.4, 0.2\rangle, \langle 4, 0.1, 0.5, 0.6\rangle\}. \tag{27}$$

Then we get

$$\mu_{f^{-1}[B]}(-2) = \mu_B(f(-2)) = \mu_B(4) = 0.1. \tag{28}$$

Similarly, we obtain $\mu_{f^{-1}[B]}(-1) = 0.6$, $\mu_{f^{-1}[B]}(1) = 0.6$, $\varphi_{f^{-1}[B]}(-2) = 0.5$, $\varphi_{f^{-1}[B]}(-1) = 0.4$, $\varphi_{f^{-1}[B]}(1) = 0.4$, $\nu_{f^{-1}[B]}(-2) = 0.6$, $\nu_{f^{-1}[B]}(-1) = 0.2$, and $\nu_{f^{-1}[B]}(1) = 0.2$. Hence, we have

$$f^{-1}[B] = \{\langle -2, 0.1, 0.5, 0.6\rangle, \langle -1, 0.6, 0.4, 0.2\rangle, \langle 1, 0.6, 0.4, 0.2\rangle\}. \tag{29}$$

The following proposition highlights the preservation of q-ROPFS structure under the function mapping.

Proposition 2. Let X and Y be two non-empty sets, let $f: X \to Y$ be a function and let A and B be q-ROPFSs in X and Y, respectively. Then $f[A]$ is a q-ROPFS in Y and $f^{-1}[B]$ is a q-ROPFS in X.

Proof. If $f^{-1}(y) = \varnothing$, then from Definition 6 we have

$$\mu^q_{f[A]}(y) + \varphi^q_{f[A]}(y) + \nu^q_{f[A]}(y) = 0 + 0 + 1 \le 1. \tag{30}$$

Otherwise we have

$$\begin{aligned} \mu^q_{f[A]}(y) + \varphi^q_{f[A]}(y) + \nu^q_{f[A]}(y) &= \left(\sup_{z\in f^{-1}(y)} \mu_A(z)\right)^q \\ &\quad + \left(\inf_{z\in f^{-1}(y)} \varphi_A(z)\right)^q \\ &\quad + \left(\inf_{z\in f^{-1}(y)} \nu_A(z)\right)^q \\ &= \sup_{z\in f^{-1}(y)} \mu^q_A(z) + \inf_{z\in f^{-1}(y)} \varphi^q_A(z) \\ &\quad + \inf_{z\in f^{-1}(y)} \nu^q_A(z) \\ &\le \sup_{z\in f^{-1}(y)} \left(1 - \varphi^q_A(z) - \nu^q_A(z)\right) \\ &\quad + \inf_{z\in f^{-1}(y)} \left(\varphi^q_A(z) + \nu^q_A(z)\right) \\ &= 1. \end{aligned} \tag{31}$$

Hence $f[A]$ is a q-ROPFS. On the other hand since B is a q-ROPFS we obtain

$$\begin{aligned} &\mu^q_{f^{-1}[B]}(x) + \varphi^q_{f^{-1}[B]}(x) + \nu^q_{f^{-1}[B]}(x) = \mu^q_B(f(x)) \\ &\quad + \varphi^q_B(f(x)) + \nu^q_B(f(x)) \le 1. \end{aligned} \tag{32}$$

□

4. Continuity and Applications in Control Systems

In this section, we explore the application of q-ROPF topologies in the context of control systems. First, we examine the concept of q_ε-ROPF continuity, which is a measure of how smoothly a function maps elements from one q-ROPF topology to another. This is important in the design of control strategies, as it ensures that small changes in the state space result in small changes in the control space. In the second subsection, we will apply the notion of q_ε-ROPF continuity to a specific example of a control system, demonstrating how q-ROPF topologies can be used to model the state space and the control space, and to design a stable and robust control strategy.

4.1. q_ε-ROPF Continuity. In this subsection we define the q_ε-ROPF continuity of a function that is defined between two q-ROPF topological spaces.

Definition 7. Let (X, τ_1) and (Y, τ_2) be two q-ROPF topological spaces and let $f: X \to Y$ be a function. If for any open q-ROPFS B of Y we have $f^{-1}[B]$ is an open q-ROPFS of X, then f is said to be q-ROPF continuous.

The condition that for any q-ROPFS in the domain space, the preimage of any open set in the codomain space must also be an open set in the domain space can be restrictive in certain real-life applications, such as control systems, where small deviations in the input may cause larger changes in the output, and these changes may not be accurately captured by the classical definition of q-ROPF continuity. To overcome this limitation, we define the concept of q_ε-ROPF continuity. This definition allows for small deviations in the preimage, measured by the positive parameter ε, and provides a more flexible and practical approach to modeling control systems and other real-life applications.

Definition 8. Let (X, τ_1) and (Y, τ_2) be two q-ROPF topological spaces and let $f: X \to Y$ be a function. Let d be a metric on $\mathbb{R}^3$ and let $0 < \varepsilon \leq \sqrt{2}/2$. If, for any open q-ROPFS B in Y, there exists an open q-ROPFS A in X such that, for any $x \in X$, the point $(\mu_B(f(x)), \varphi_B(f(x)), \nu_B(f(x)))$ is within or on the sphere of center $(\mu_A(x), \varphi_A(x), \nu_A(x))$ and radius ε, then f is said to be q_ε-ROPF continuous with respect to d. If d is the Euclidean metric, then the function is referred to as Euclidean q_ε-ROPF continuous.

4.2. Application of q-ROPF Continuity in Control Systems. In our research work, we propose a novel application method that utilizes the concept of q_ε-ROPF continuity in the context of fuzzy control systems. It is important to note that our contribution lies in applying this method specifically to the current environment of fuzzy control systems. By using the q-ROPF fuzzy topologies, we can model the state space and the control space of the system, and analyze the continuity of the mapping between them. This information then be used to design a control strategy that ensures the stability of the system.

(i) Consider a set $X = \{x_1, x_2, x_3\}$ and define the 3-ROPFSs

$$\begin{aligned} A_1 &= \{\langle x_1, 0.58, 0.40, 0.10\rangle, \langle x_2, 0.40, 0.43, 0.05\rangle, \langle x_3, 0, 0.01, 1\rangle\}, \\ A_2 &= \{\langle x_1, 0.25, 0.60, 0\rangle, \langle x_2, 0.05, 0, 0.90\rangle, \langle x_3, 0.56, 0.30, 0.05\rangle\}, \\ A_3 &= \{\langle x_1, 0.25, 0.40, 0.10\rangle, \langle x_2, 0.05, 0, 0.90\rangle, \langle x_3, 0, 0.01, 1\rangle\}, \\ A_4 &= \{\langle x_1, 0.58, 0.40, 0\rangle, \langle x_2, 0.40, 0, 0.05\rangle, \langle x_3, 0.56, 0.01, 0.05\rangle\}. \end{aligned} \tag{33}$$

The family $\{A_i\}_{i=1}^4$ forms a 3-ROPF topology on X. We call this topology τ_1.

(ii) Consider a set $Y = \{y_1, y_2, y_3\}$ and define the 3-ROPFSs

$$\begin{aligned} B_1 &= \{\langle y_1, 0.66, 0.33, 0\rangle, \langle y_2, 0.33, 0.33, 0\rangle, \langle y_3, 0, 0, 1\rangle\}, \\ B_2 &= \{\langle y_1, 0.33, 0.66, 0\rangle, \langle y_2, 0, 0, 1\rangle, \langle y_3, 0.66, 0.33, 0\rangle\}, \\ B_3 &= \{\langle y_1, 0.33, 0.33, 0\rangle, \langle y_2, 0, 0, 1\rangle, \langle y_3, 0, 0, 1\rangle\}, \\ B_4 &= \{\langle y_1, 0.66, 0.33, 0\rangle, \langle y_2, 0.33, 0, 0\rangle, \langle y_3, 0.66, 0, 0\rangle\}. \end{aligned} \tag{34}$$

The family $\{B_i\}_{i=1}^4$ defines a 3-ROPF topology on Y. We call this topology τ_2.

(iii) Representing control systems using 3-ROPF topologies allow us to model the relationship between the state space and the control space in a more abstract and mathematical way. By defining a 3-ROPF topology on the state space and the control space, we can capture important information about the behavior of the system, such as the continuity and stability of the mapping from the state space to the control space. This information is critical for designing effective control strategies that ensure that small changes in the state space result in small changes in the control space.

(iv) Consider a scenario where we want to control a dynamic system, a robotic arm. The current state of the arm, including its position, velocity, and acceleration, can be represented by a set of three elements, modeled using the 3-ROPF topology τ_1. Based on the desired position, velocity, and acceleration values that the arm should track, another 3-ROPF topology, τ_2, is established to represent the desired behavior of the system. The elements of the sets correspond to different points in the workspace of the arm, and the open sets correspond to the regions in which the arm can move. By mapping the positions of the arm in τ_1 to those in τ_2 using a continuous function, we can model the behavior of the robotic arm as it moves and adjusts its position in response to different inputs or conditions. This can help us better understand and control the behavior of the arm in real-world situations.

(v) The function $f: X \to Y$ maps the current state of the robotic arm to the desired state by assigning each state element to a corresponding control value, as given by $f(x_1) = y_1, f(x_2) = y_2, f(x_3) = y_3$. This function serves as a link between the state space and the control space of the system. Its design is based on the specific control objectives and constraints of the robotic arm and aims to track a desired trajectory.

(vi) f is Euclidean $3_{0.15}$-ROPF continuous with respect to the 3-ROPF topologies τ_1 and τ_2. It is easy to see that for any open 3-ROPFS B in Y, there exists an open q-ROPFS A such that, for any $x \in X$, the point $(\mu_B(f(x)), \varphi_B(f(x)), \nu_B(f(x)))$ is within or on the sphere of center $(\mu_A(x), \varphi_A(x), \nu_A(x))$ and radius 0.15. For example, we have

$$\begin{aligned}\left(\mu_{B_1}(f(x_1)), \varphi_{B_1}(f(x_1)), \nu_{B_1}(f(x_1))\right) &= \left(\mu_{B_1}(y_1), \varphi_{B_1}(y_1), \nu_{B_1}(y_1)\right)\\ &= (0.66, 0.33, 0),\end{aligned} \tag{35}$$

$$\begin{aligned}\left(\mu_{B_1}(f(x_2)), \varphi_{B_1}(f(x_2)), \nu_{B_1}(f(x_2))\right) &= \left(\mu_{B_1}(y_2), \varphi_{B_1}(y_2), \nu_{B_1}(y_2)\right)\\ &= (0.33, 0.33, 0),\end{aligned} \tag{36}$$

and

$$\begin{aligned}\left(\mu_{B_1}(f(x_3)), \varphi_{B_1}(f(x_3)), \nu_{B_1}(f(x_3))\right) &= \left(\mu_{B_1}(y_3), \varphi_{B_1}(y_3), \nu_{B_1}(y_3)\right)\\ &= (0, 0, 1).\end{aligned} \tag{37}$$

On the other hand it is clear that

$$\begin{aligned}\left(\mu_{A_1}(x_1), \varphi_{A_1}(x_1), \nu_{A_1}(x_1)\right) &= (0.58, 0.40, 0.10),\\ \left(\mu_{A_1}(x_2), \varphi_{A_1}(x_2), \nu_{A_1}(x_2)\right) &= (0.40, 0.43, 0.05),\\ \left(\mu_{A_1}(x_3), \varphi_{A_1}(x_3), \nu_{A_1}(x_3)\right) &= (0, 0.01, 1).\end{aligned} \tag{38}$$

Hence, the point $\left(\mu_{B_1}(f(x_i)), \varphi_{B_1}(f(x_i)), \nu_{B_1}(f(x_i))\right)$ is within the sphere of center $\left(\mu_{A_1}(x_i), \varphi_{A_1}(x_i), \nu_{A_1}(x_i)\right)$ and radius 0.15 for each $i = 1, 2, 3$. It means that for any small change in the state space represented by τ_1, there is a corresponding change in the control space represented by τ_2 controlled by $\varepsilon = 0.15$. This implies that the mapping from the state space to the control space is smooth and well-behaved, satisfying the continuity requirements of the system. To provide a clearer understanding of the stability and continuity of the mapping, consider the following: if the robotic arm moves slightly in the state space, the corresponding behavior in the control space should reflect this change consistently. For example, if the arm's position in the state space shifts from x_1 to x_1', the mapping function f should ensure a corresponding shift in the control space from y_1 to y_1'. Similarly, if the arm's position moves from x_2 to x_2' or x_3 to x_3', the mapping function should consistently reflect those changes in the control space.

(vii) It is concluded that the control system is stable and robust. The function f maps the state space represented by τ_1 to the control space represented by τ_2 in a continuous manner, satisfying the continuity requirements of the system. This implies that the mapping between the state space and the control space is smooth and well-behaved. The continuity property of the function f ensures that small changes in the state space result in corresponding changes in the control space, making the control strategy stable and reliable.

5. q-ROPFPs and Convergence

In this section, we introduce the novel concept of a q-ROPFP, which extends the classical concept of a fuzzy point. We explore the properties of q-ROPFPs, defining a new type of convergence in q-ROPF topological spaces. Specifically, we define the Q-neighborhood system of a q-ROPFP and examine the convergence of sequences of q-ROPFPs, aiming to identify the conditions for the uniqueness of the limit in the realm of supports.

Definition 9. Let X be a set, let $s \in X$ and let $\alpha \in [0, 1)$, $\gamma \in [0, 1)$ and $\beta \in (0, 1]$ be real numbers such that $\alpha^q + \gamma^q + \beta^q \leq 1$. Then the q-ROPFS

$$[s]_{\alpha,\gamma,\beta} = \left\{\left\langle x_j, c_\alpha(x_j), c_\gamma(x_j), \left(1 - c_{1-\beta}\right)(x_j)\right\rangle : j = 1, \ldots, n\right\}, \tag{39}$$

is called a q-ROPFP in X where

$$c_t(x) := \begin{cases} t, & x = t \\ 0, & x \neq t \end{cases}. \tag{40}$$

It is clear that

$$(1 - c_{1-t})(x) = \begin{cases} t, & x = t \\ 1, & x \neq t. \end{cases} \tag{41}$$

We simply use the notation

$$[s] = \left(c_\alpha, c_\gamma, 1 - c_{1-\beta}\right), \tag{42}$$

when there is no confusion. The ordinary point s is called the support of $[s]$. We say that a q-ROPFP $[s]$ is included in a q-ROPFS A if $[s] \subset A$ and we write $[s] \in A$.

The following theorem provides a key insight into the structure of q-ROPFSs, showing that they are composed of the union of all the q-ROPFPs that are contained by them.

Theorem 1. A q-ROPFS A is the union of all the q-ROPFPs that are contained by A, i.e., $A = \bigcup_{[s]\in A} [s]$.

Proof. Let us say $B = \bigcup_{[s]\in A} [s]$. If $[s] \in A$, then we have for any $x \in X$ that $\mu_{[s]}(x) \le \mu_A(x)$ which yields that

$$\mu_B(x) = \sup_{[s]\in A} \mu_{[s]}(x) \le \mu_A(x). \tag{43}$$

Similarly, we have for any $x \in X$ that $\varphi_B(x) \le \varphi_A(x)$ and $\nu_B(x) \ge \nu_A(x)$. So, $B \subset A$. On the other hand, for any $x \in X$ we get $[x]_{\mu_A(x),\varphi_A(x),\nu_A(x)} \in A$. Therefore we have

$$\begin{aligned} \mu_B(x) &= \sup_{\substack{[s]\in A \\ s\in X}} \mu_{[s]}(x) \\ &= \sup_{[x]\in A} \mu_{[x]}(x) \\ &\ge \mu_A(x). \end{aligned} \tag{44}$$

Similarly, we have for any $x \in X$ that $\varphi_B(x) \ge \varphi_A(x)$ and $\nu_B(x) \le \nu_A(x)$. So, $A \subset B$. □

The following theroem asserts that the q-ROPFP structure is maintained when functions are applied.

Theorem 2. Let $f : X \to Y$ be a function and let $[s]_{\alpha,\gamma,\beta}$ be a q-ROPFP in X. Then the image of $[s]_{\alpha,\gamma,\beta}$ is the q-ROPFP

$$f[u] := f\left[[u]_{\alpha,\gamma,\beta}\right] = [f(u)]_{\alpha,\gamma,\beta}, \tag{45}$$

in Y with the support $f(u)$.

Proof. From Definition 6 we obtain

$$\begin{aligned} \mu_{f\left[[u]_{\alpha,\gamma,\beta}\right]}(y) &= \begin{cases} \sup\limits_{z\in f^{-1}(y)} \mu_{[u]_{\alpha,\gamma,\beta}}(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \alpha, & \text{if } u \in f^{-1}(y) \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \alpha, & \text{if } f(u) = y \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \tag{46}$$

$$\begin{aligned} \varphi_{f\left[[u]_{\alpha,\gamma,\beta}\right]}(y) &= \begin{cases} \inf\limits_{z\in f^{-1}(y)} \varphi_{[u]_{\alpha,\gamma,\beta}}(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \gamma, & \text{if } u \in f^{-1}(y) \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \gamma, & \text{if } f(u) = y \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{47}$$

and

$$\begin{aligned} \upsilon_{f\left[[u]_{\alpha,\gamma,\beta}\right]}(y) &= \begin{cases} \inf\limits_{z\in f^{-1}(y)} \nu_{[u]_{\alpha,\gamma,\beta}}(z), & \text{if } f^{-1}(y) \text{ is non-empty} \\ 1, & \text{otherwise,} \end{cases} \\ &= \begin{cases} \beta, & \text{if } u \in f^{-1}(y) \\ 1, & \text{otherwise} \end{cases} \\ &= \begin{cases} \beta, & \text{if } f(u) = y \\ 1, & \text{otherwise.} \end{cases} \end{aligned} \tag{48}$$

□

The following definitions introduces the concept of quasi-coincidence between a q-ROPFP and a q-ROPFS.

Definition 10. A q-ROPFP $[s]_{\alpha,\gamma,\beta}$ is said to be quasi-coincident with a q-ROPFS A if $\alpha^q + \mu_A^q(u) > 1$, $\gamma^q + \varphi_A^q(u) > 1$ and $\beta^q + \nu_A^q(u) < 1$. In this case, we write $[s]_{\alpha,\gamma,\beta} * A$ or in brief $[s] * A$.

In topology, a fundamental concept is the notion of a neighborhood of a point, which is a set containing the point and some points around it. In a q-ROPF topological space, we can define a similar concept of a Q-neighborhood of a q-ROPFP, where the neighborhood is a fuzzy set that is quasi-coincident with the given q-ROPFP. This leads to the definition of the Q-neighborhood system of a q-ROPFP, which is a family of Q-neighborhoods that captures the local behavior of the q-ROPFP in the q-ROPF topological space.

Definition 11. A fuzzy set A in a q-ROPF topological space (X, τ) is called a Q-neighbourhood of a q-ROPFP $[s]$ if there exists an open q-ROPFS $B \subset A$ such that $[s] * B$. The family of Q-neighbourhoods of $[s]$ is called the Q-neighbourhood system of $[s]$ and denoted by $\mathcal{N}_*[s]$.

A key property of topological spaces is their separation axioms, which provide information on how distinct points can be separated from each other by open sets. One such

separation axiom is the Hausdorff property, which has an analog in q-ROPF topological spaces. The following definition introduces the q-ROPF Hausdorff property.

Definition 12. A q-ROPF topological space (X, τ) is said to be Hausdorff if any q-ROPFPs $[s_1]$ and $[s_2]$ satisfying $s_1 \neq s_2$ there exist Q-neighbourhoods V_1 and V_2 of $[s_1]$ and $[s_2]$, respectively, such that $V_1 \cap V_2 = 0_X$.

In the context of q-ROPF topological spaces, convergence of a sequence of q-ROPFPs to a particular q-ROPFP can be defined using Q-neighbourhoods. This notion is important because it provides a framework for studying the convergence of sequences in q-ROPF topological spaces, which has important implications in many areas of mathematics. Furthermore, the definition of convergence involving Q-neighbourhoods allows us to capture the local behavior of sequences.

Definition 13. A sequence $\{[s_n]\}$ of q-ROPFPs of (X, τ) is said to be convergent to a q-ROPFP $[s]$ if for any Q-neighbourhood V of $[s]$ there exists a positive integer n_0 such that $[s_n] * V$ whenever $n \geq n_0$. In this case, we write

$$\lim_n [s_n] = [s]. \tag{49}$$

When working with q-ROPF topological spaces, it is important to understand how sequences of q-ROPFPs behave in terms of convergence. The following theorem provides a useful result in this regard.

Theorem 3. In a q-ROPF topological space (X, τ) if a sequence $([s]_n)$ of q-ROPFPs is convergent to $[s]_{\alpha_1, \gamma_1, \beta_1}$, then it is convergent to $[s]_{\alpha_2, \gamma_2, \beta_2}$ whenever $\alpha_1 > \alpha_2$, $\gamma_1 > \gamma_2$ and $\beta_1 < \beta_2$.

Proof. Let $\lim_n [s]_n = [s]_{\alpha_1, \gamma_1, \beta_1}$. Then for any Q-neighbourhood V of $[s]_{\alpha_1, \gamma_1, \beta_1}$ there exists a positive integer n_0 such that $[s_n] * V$ whenever $n \geq n_0$. Assume that U be a Q-neighbourhood of $[s]_{\alpha_2, \gamma_2, \beta_2}$. Since $\alpha_1 > \alpha_2$, $\gamma_1 > \gamma_2$ and $\beta_1 < \beta_2$ we have U is also a Q-neighbourhood of $[s]_{\alpha_1, \gamma_1, \beta_1}$. Hence $[s_n] * U$ whenever $n \geq n_0$. $\lim_n [s]_n = [s]_{\alpha_2, \gamma_2, \beta_2}$. □

Theorem 3 demonstrates that a sequence of q-ROPFPs can converge to distinct q-ROPFSs that share the same support. In contrast, the following theorem asserts that a sequence of q-ROPFPs cannot converge to q-ROPFSs with different supports in a Hausdorff q-ROPF topological space.

Theorem 4. Let (X, τ) be a q-rung orthopair picture topological space. If (X, τ) is a Hausdorff space, then any sequence of q-ROPFPs of X does not converge to two q-ROPFPs $[s_1]$ and $[s_2]$ such that $s_1 \neq s_2$.

Proof. Let $([x]_n)$ be a sequence of q-ROPFPs such that $\lim_n [x_n] = [s_1]$ and $\lim_n [x_n] = [s_2]$ where $s_1 \neq s_2$. Then for arbitrary Q-neighbourhoods V_1 and V_2 of $[s_1]$ and $[s_2]$, respectively, there exists a positive integer n_1 such that $[x_n] * V_1$ and $[x_n] * V_2$ whenever $n \geq n_0$. Therefore we have

$$\mu_{V_1}(x_n) > 0, \mu_{V_2}(x_n) > 0, \tag{50}$$

$$\varphi_{V_1}(x_n) > 0, \varphi_{V_2}(x_n) > 0, \tag{51}$$

and

$$\nu_{V_1}(x_n) < 1, \nu_{V_2}(x_n) < 1 \tag{52}$$

whenever $n \geq n_0$. So we have $V_1 \cap V_2 \neq O_X$ which contradicts with the fact that (X, τ) is Hausdorff. □

6. Conclusion

In this paper, we have introduced the concept of q-ROPF topological space and studied some of their basic properties. We have defined various notions such as Q-neighbourhoods, quasi-coincidence, convergence, and Hausdorffness. We have also introduced the concept of q_ε-ROPF continuity, which is a weaker form of continuity in which the continuity condition holds only up to a certain error bound. We have applied the concept of this continuity to a control system problem, in which we have shown how to design a controller that ensures that the output of a system remains within a certain error bound of the desired trajectory. Finally, we have also investigated the convergence of sequences of q-ROPFPs in Hausdorff q-rung orthopair picture topological spaces, and have proven that such sequences cannot converge to two distinct q-ROPFPs with different supports. This study paves the way for various future research possibilities. One intriguing avenue would involve examining the properties of continuous fuzzy mappings and their interplay with different types of continuity. Furthermore, exploring the relationships between continuity and other mathematical domains, such as fuzzy topology and fuzzy analysis, would be of great interest. Last, applying our findings to the other control system challenges and assessing the efficacy of continuous fuzzy controllers in real-world scenarios would provide valuable insights. As a part of our future work, we will investigate the relationship between q_ε-ROPF continuity and Q-neighborhoods in the realm of q-ROPFPs.

Data Availability

No underlying data supporting the results of this study were generated or analyzed, and no publicly archived datasets were generated during the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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