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Research Article

Q-Rung Interval-Valued Probabilistic Dual Hesitant Fuzzy Sets: A New Tool for Multiattribute Group Decision-Making

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This paper aims at proposing a novel multiattribute group decision-making (MAGDM) method in complex decision-making environments. To this end, we first introduce a tool, called q-rung interval-valued probabilistic dual hesitant fuzzy sets (q-RIVPDHFSs), for decision makers to express their evaluation information over a set of finite alternatives in MAGDM procedures. The q-RIVPDHFS consists of some possible membership and nonmembership degrees, along with their interval-valued probabilistic information. Due to this structure, q-RIVPDHFSs are more powerful and flexible than the traditional q-rung probabilistic q-rung dual hesitant fuzzy sets, in which probabilistic information of membership and nonmembership degree is denoted by crisp numbers. Second, some other related concepts of q-RIVPDHFSs, such as operational laws, comparison method, distance measure, and aggregation operators, are introduced. Third, based on these novel concepts, two MAGDM methods (Algorithms 1 and 2) are put forward. Last but not least, a practical decision-making example is provided to show the effectiveness of our proposed MAGDM method. We also compare our Algorithms 1 and 2 with some existing decision-making methods to explain why our methods are more powerful and useful.

1. Introduction

Decision-making is one of the commonest activities in economics, management, social communication, and even daily. As a matter of fact, large numbers of decision-making issues in actual life can be regarded as multiattribute group decision-making (MAGDM), wherein multiexperts evaluate all possible alternatives under multiple attributes, and the final ranking order of alternatives is derived by decision makers' preference information [1–7]. A MAGDM procedure can be roughly divided into four parties, i.e., decision-making problem definition, decision makers' opinions expression, ranking order of alternatives determination, and decision-making advice implementation. The main tasks of the four parties are presented as follows: (1) decision-making problem definition: decision makers identify all potential alternatives and criteria, which is essential for selecting an optimal alternative; (2) decision makers' opinions expression: decision makers express their opinions on the performance of each alternative under every criterion in different information representation forms; (3) ranking order of alternatives determination: decision makers determine the ranking order of alternatives based on a certain methodology and decision makers' evaluation matrices; (4) decision-making advice implementation: based on the ranking order of alternatives, decision makers select the best or suboptimal alternatives.

It is widely acknowledged that, among the above-mentioned four parties of MAGDM procedures, the second and third parties, i.e., decision makers' opinions expression and ranking order of alternatives determination, are the most important. In fact, it is not an easy job to depict decision makers' evaluation information, owing to not only the increasing complexity of MAGDM issues but also the fuzziness, hesitation, and uncertainty of human beings' cognitive processes. In addition, the fact that decision makers usually have different expertise and come from different backgrounds also makes it almost impossible to denote experts' complex preferences by using crisp numbers. The fuzzy sets theory, originated by Prof. Zadeh [8], has facilitated the method to appropriately express decision makers' evaluation information over alternatives. Afterward, quite a few extensions of fuzzy sets, such as intuitionistic fuzzy sets (IFSs) [9], hesitant fuzzy sets [10], interval-valued hesitant fuzzy sets [11], hesitant fuzzy linguistic terms sets [12], dual hesitant fuzzy sets [13], Pythagorean fuzzy sets (PFSs) [14], and generalized orthopair fuzzy sets [15], have been proposed, which exhibit high efficiency in representing decision makers' preference opinions. For example, PFSs satisfy the constraint that the square sum of membership degree (MD) and nonmembership degree (NMD) need to be smaller than or equal to 1. Compared to IFSs, PFSs can describe larger information space and are more suitable to depict decision makers' evaluations in the complex decision-making environment. In [16], Tsao and Chen proposed a parametric likelihood measure based on the beta distribution and developed a likelihood-oriented methodology under PFSs. Wan et al. [17] introduced a novel ranking method for Pythagorean fuzzy numbers and applied it in MAGDM. Albahri et al. [18] proposed a Pythagorean fuzzy dynamic MAGDM method for COVID-19 vaccine dose recipients. In [19], Wan et al. introduced a Pythagorean fuzzy mathematical programming method to solve MAGDM problems. Garg et al. [20] introduced a series of complex Pythagorean fuzzy Archimedean Bonferroni mean operators and applied them in MAGDM. Wan et al. [21] proposed a three-phase MAGDM method under PFSs and applied it in haze management. The state-of-the-art intuitionistic fuzzy sets, hesitant fuzzy sets, hesitant fuzzy term sets, Pythagorean fuzzy sets, and generalized orthopair fuzzy sets-based MAGDM methods can be found in [22–26], respectively, where researchers and readers in this field can have an overlook of how these information representation tools have been applied in solving MAGDM problems.

Recently, the q-rung probabilistic dual hesitant fuzzy sets (q-RPDHFSs) [27] have been proposed to address decision makers' complicated evaluation values. The q-RPDHFS is an extension of Xu et al.'s [28] q-rung dual hesitant fuzzy set (q-RDHFS). The q-RDHFS has been proven to be powerful and effective in describing decision makers' evaluation information in the process of MAGDM and it has been widely applied in solving realistic decision-making problems. In [29], Kou et al. introduced a series of power Hamy mean under q-RDHFSs, and by integrating entropy weights, a novel MAGDM was proposed and applied in hospital medical quality evaluation. Li et al. [30] proposed Archimedean operational rules and extended power average operators for q-RDHFSs and proposed a new MAGDM method.

Afterward, Feng et al. [31] used interval values rather than crisp numbers to denote the membership and nonmembership degrees, introduced interval-valued q-RDHFSs, studied their desirable properties, investigated their applications in decision-making, and put forward a novel MAGDM approach. By integrating linguistic term sets with q-RDHFSs, Wang et al. [32] proposed the concept of q-rung dual hesitant uncertain linguistic sets and based on which a novel MAGDM method was developed and applied in enterprise informatization-level evaluation. Although q-RDHFS is effective to depict decision makers' evaluation values in MAGDM, its drawback is also obvious, i.e., it only considers multiple MDs and NMDs, but ignores their importance degrees. Motived by probabilistic hesitant fuzzy sets [33] and probabilistic dual hesitant fuzzy sets [34], the q-RPDHFSs have been proposed. Compared to q-RDHFSs, q-RPDHFSs consider not only multiple MDs and NMDs but also their corresponding importance degrees. Hence, q-RPDHFSs are more powerful and useful to address practical MAGDM problems. In reference [27], authors studied aggregation operators for q-rung probabilistic dual hesitant fuzzy information and applied them to an investment project selection problem. Q-RPDHFSs require decision makers to implement crisp numbers to denote the probabilistic information of their provided membership and no-membership degrees, which, however, is sometimes inadequate and insufficient for dealing with practical MAGDM problems. In realistic decision-making contexts, decision makers prefer using interval values rather than crisp numbers to express probabilistic values of membership and nonmembership degrees. Some researchers and scholars have noticed this phenomenon and similar studies have been conducted [35–37]. For instance, authors in [35] generalized the probabilistic linguistic sets [38] to interval-valued probabilistic linguistic sets by taking the interval-valued probabilistic value of each possible linguistic terms into account. In [36], Song et al. considered interval-valued probabilistic information in probabilistic hesitant fuzzy sets and gave the definition of interval-valued probabilistic hesitant fuzzy sets (IVPHFSs). In [37], Liu and Cheng used interval-valued rather than crisp numbers to denote probabilistic information of each member in dual hesitant fuzzy sets, and proposed the so-called interval-valued probabilistic dual hesitant fuzzy sets (IVPDHFSs).

Above-mentioned studies motivated us to consider interval-valued probabilities in q-RPDHFSs and propose the concept of q-rung interval-valued probabilistic dual hesitant fuzzy sets (q-RIVPDHFSs). The advantages of q-RIVPDHFSs over some existing fuzzy sets theories are obvious. Compared with q-RPDHFSs, our q-RIVPDHFSs take interval-valued probabilistic information into consideration, and hence, they are more powerful to handle decision makers' evaluation preferences. Compared with interval-valued dual hesitant fuzzy sets, the restraint of our q-RIVPDHFSs is laxer and, hence, they can deal with more complicated realistic MAGDM problems. Compared with interval-valued hesitant fuzzy sets, our q-RIVPDHFSs consider both membership and nonmembership degrees, which indicates that q-RIVPDHFSs can comprehensively express decision makers' evaluation information by simultaneously reflecting decision makers' positive and negative opinions. Afterward, other related concepts, such as operational laws, comparison methods, distance measures, aggregation operators of q-RIVPDHFSs are proposed one after another.

For the third part of MAGDM procedures, the TOPSIS method is an effective tool that help decision makers obtain the ranking orders of alternatives according to their preference values. As aforementioned, decision-making environments are becoming more and more complicated, it is necessary to extend the classical TOPSIS into different fuzzy set situations to study practical MAGDM methods. For instance, Hu et al. [39] studied a new TOPSIS under hesitant fuzzy sets, introduced the so-called HF-TOPSIS and applied it in supplier selection for an automotive company. Liu and Rodriguez [40] extended TOPSIS into hesitant fuzzy linguistic terms to propose a new MAGDM method. Akram et al. [41] and Alkan and Kahraman [42] investigated TOPSIS method under Pythagorean fuzzy sets and q-rung orthopair fuzzy sets and applied it to healthcare technology purchase and government strategies against COVID-19 pandemic evaluation. Some other new extensions of the classical TOPSIS can be found in [43–47]. Based on the above analysis, it is necessary and worth extending the classical TOPSIS to q-RIVPDHFSs and introducing a novel decision-making method.

Based on the above analysis, the contributions and novelties of this study are fourfold:

(1) A new information representation tool, called q-RIVPDHFSs, is proposed to express decision makers' complicated evaluation values. The q-RIVPDHFSs not only inherent the advantages of q-RPDHFSs, but also consider interval-valued possibilities. Hence, q-RIVPDHFSs are suitable to handle realistic MAGDM problems.

(2) Some aggregation operators for q-RIVPDHFSs are introduced and based on which a novel MAGDM method is developed (we call it Algorithm 1 in the following sections). The properties of these operators are studied and the main calculation process of the decision-making method is illustrated.

(3) An extended TOPSIS under q-RIVPDHFSs is presented and its detailed calculation process is demonstrated (we call it Algorithm 2 in the following sections).

(4) The effectiveness of the two approaches is validated through numerical examples. We use the methods to solve some real MAGDM methods so that their rightness can be clearly witnessed. In addition, a comparative analysis with some existing approaches is conducted to show the advantages of our proposed methods.

The rest of this paper is organized as follows. Section 2 reviews basic preliminaries. In Section 3, we first explain why we need q-RIVPDHFS, and subsequently, the concept of q-RIVPDHFS and some other related notions are proposed. Section 4 presents two algorithms to solve MAGDM methods and their detailed steps are demonstrated. Section 5 applies the proposed methods in real and practical MAGDM problems. Conclusion remarks are provided in Section 6.

2. Preliminaries

This section reviews the concepts of q-RPDHFSs and interval values, which are important for the following sections.

2.1. Q-Rung Probabilistic Dual Hesitant Fuzzy Sets.

Definition 1 (see [27]). Let X be a given fixed set, then a q-rung probabilistic dual hesitant fuzzy set (q-RPDHFS) D defined on X is expressed as:

$$A = \{x, h_D(x)|p_D(x), g_D(x)|t_D(x)|x \in X\}, \tag{1}$$

where $h_D(x)$ and $g_D(x)$ are two sets of some values in [0, 1], denoting the possible MDs and NMDs of the element $x \in X$ to set A. In addition, $p_D(x)$ and $t_D(x)$ are the probabilistic information of $h_D(x)$ and $g_D(x)$, respectively, such that

$$0 \le \gamma, \eta \le 1, \left(\gamma^+\right)^q + \left(\eta^+\right)^q \le 1\,(q > 1), 0 \le p_i, t_j \le 1, \sum_{i=1}^{\#h} p_i = 1, \sum_{j=1}^{\#g} t_i = 1, \tag{2}$$

where $\gamma \in h_D(x)$, $\eta \in g_D(x)$, $\gamma^+ = \cup_{\gamma \in h_D(x)} \max\{\gamma\}$, $\eta^+ = \cup_{\eta \in g_D(x)} \max\{\eta\}$, $p_i \in p_D(x)$, and $t_i \in t_D(x)$, $\#h$ and $\#g$ denote the numbers of values in h and g, respectively. For the sake of easy description, we call the ordered pair $d(x) = (h_D(x)|p_D(x), g_D(x)|t_D(x))$ a q-rung probabilistic dual hesitant fuzzy element (q-RPDHFE), which can be denoted as $d = (h|p_h, g|t_g)$ for simplification. Especially, when $q = 1$, D reduces to a probabilistic dual hesitant fuzzy set and d is reduced to a probabilistic dual hesitant fuzzy element. When $q = 2$, then D reduces to a probabilistic dual hesitant Pythagorean fuzzy set and d is reduced to a probabilistic dual hesitant Pythagorean fuzzy element.

The basic operations of q-RPDHFEs are presented as follows:

Definition 2 (see [27]). Let $d = (h|p_h, g|t_g)$, $d_1 = (h_1|p_{h_1}, g_1|t_{g_1})$, and $d_2 = (h_2|p_{h_2}, g_2|t_{g_2})$ be any three q-RPDHFEs and λ be a positive real number, then:

(1) $d_1 \oplus d_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left\{ \left(\gamma_1^q + \gamma_2^q - \gamma_1^q \gamma_2^q\right)^{1/q} |p_{\gamma_1} p_{\gamma_2} \right\}, \left\{ \eta_1 \eta_2 | t_{\eta_1} t_{\eta_2} \right\} \right\}$;

(2) $d_1 \otimes d_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left\{ \gamma_1 \gamma_2 | p_{\gamma_1} p_{\gamma_2} \right\}, \left\{ \left(\eta_1^q + \eta_2^q - \eta_1^q \eta_2^q\right)^{1/q} | t_{\eta_1} t_{\eta_2} \right\} \right\}$;

Step 1. Standardize the original decision matrix. Attributes in a MAGDM problem can be usually classified into two kinds, i.e., benefit type and cost type. Hence, the original q-rung interval-valued probabilistic dual hesitant fuzzy matrix should be standardized according to the following formula: $\tilde{d}_{ij}' = \begin{cases} (h_{ij}|\tilde{p}_{h_{ij}}, g_{ij}|\tilde{t}_{g_{ij}}) \\ (g_{ij}|\tilde{t}_{g_{ij}}, h_{ij}|\tilde{p}_{h_{ij}},) \end{cases}$
Step 2. Utilize the q-RIVPDHFWA or the q-RIVPDHFWG operator to calculate the comprehensive evaluation value of $x_i\,(i = 1, 2, \ldots, m)$, i.e., $\tilde{d}_i' = q - \text{RIVPDHFWA}\,(\tilde{d}_{i1}', \tilde{d}_{i2}', \ldots, \tilde{d}_{in}')$.
or $\tilde{d}_i' = q - \text{RIVPDHFWG}\,(\tilde{d}_{i1}', \tilde{d}_{i2}', \ldots, \tilde{d}_{in}')$.
Step 3. Compute the score and deviation values of each alternative's comprehensive evaluation value.
Step 4. Rank the alternatives according to their score values and select the optimal alternative.

Algorithm 1: An aggregation operator-based MAGDM method.

Step 1. Standardize the original decision matrix, which is same as the Step 1 presented in Subsection 4.
Step 2. Arrange IVPMEs and IVPNMEs of each q-RIVPDHFE in ascending order according to Remark 1. After this step, the decision matrix becomes a normalized decision matrix.
Step 3. For attribute a_i, extend the shorter evaluation values of alternatives according to Remark 1 until they have the same numbers of IVPMEs and IVPNMEs. For convenient description, we denote the new decision matrix as $\tilde{D}_{ij}'' = (\tilde{d}_{ij}'')_{m\times n}$. After this step, with regard to attribute a_j, the series of evaluation values $\tilde{d}_{1j}'', \tilde{d}_{2j}'', \ldots, \tilde{d}_{nj}''$ have the same numbers of IVPMEs and IVPNMEs.
Step 4. Determine the q-RIVPDHFPIS and q-RIVPDHFNIS by $x^+ = \left\{(\tilde{d}_1'')^+, (\tilde{d}_2'')^+, \ldots, (\tilde{d}_n'')^+\right\}$,
where $(\tilde{d}_j'')^+ = \left\{\begin{matrix} \max_s\left\{\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}} + \tilde{p}^U_{\gamma^{(s)}}],\ s = 1, 2, \ldots, m\right\}, \\ \min_l\left\{\eta^{(l)}|[\tilde{t}^L_{\eta^{(l)}}, \tilde{t}^U_{\eta^{(l)}}],\ l = 1, 2, \ldots, m\right\} \end{matrix}\right\}$, $x^- = \left\{(\tilde{d}_1'')^-, (\tilde{d}_2'')^+, \ldots, (\tilde{d}_n'')^+\right\}$,
where $(\tilde{d}_j'')^- = \left\{\begin{matrix} \max_s\left\{\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}} + \tilde{p}^U_{\gamma^{(s)}}],\ s = 1, 2, \ldots, m\right\}, \\ \min_l\left\{\eta^{(l)}|[\tilde{t}^L_{\eta^{(l)}}, \tilde{t}^U_{\eta^{(l)}}],\ l = 1, 2, \ldots, m\right\} \end{matrix}\right\}$.
Step 6. Calculate the weighted distance between the alternative $x_i\,(i = 1, 2, \ldots, m)$ and the q-RIVPDHFPIS x^+ and the q-RIVPDHFNIS x^-.
Step 7. Calculate the relative importance degree of $x_i\,(i = 1, 2, \ldots, m)$ by $CI_i = (d(x_i, x^-)/(d(x_i, x^+) + d(x_i, x^-)))$.
In addition, the bigger the value CI_i is, the better the alternative x_i.
Step 8. Rank alternatives according to $CI_i\,(i = 1, 2, \ldots, m)$, the final ranking result of alternatives is derived.

Algorithm 2: A TOPSIS-based MAGDM method.

(3) $\lambda\, d = \cup_{\gamma\in h,\eta\in g}\left\{\left\{(1-(1-\gamma^q)^\lambda)^{1/q}|p_\gamma\right\}, \left\{\eta^\lambda|t_\eta\right\}\right\}$;
(4) $d^\lambda = \cup_{\gamma\in h,\eta\in g}\left\{\left\{\gamma^\lambda|p_\gamma\right\}, \left\{(1-(1-\eta^q)^\lambda)^{1/q}|t_\eta\right\}\right\}$.

The method to rank any two q-RPDHFEs is presented as follows:

Definition 3 (See [27]). Let $d = (h|p_h, g|t_g)$ be a q-RPDHFE, then the score function of d is expressed as

$$S(d) = \sum_{\gamma\in h}\gamma^q p_\gamma - \sum_{\eta\in g}\eta^q t_\eta. \tag{3}$$

And the accuracy function of d is expressed as:

$$H(d) = \sum_{\gamma\in h}\gamma^q p_\gamma + \sum_{\eta\in g}\eta^q t_\eta. \tag{4}$$

For any two q-RPDHFEs $d_1 = (h_1|p_{h_1}, g_1|t_{g_1})$ and $d_2 = (h_2|p_{h_2}, g_2|t_{g_2})$, then we have.

(1) if $S(d_1) > S(d_2)$, then $d_1 > d_2$;
(2) if $S(d_1) = S(d_2)$, then

if $H(d_1) > H(d_2)$, then $d_1 > d_2$;
if $H(d_1) = H(d_2)$, then $d_1 = d_2$.

2.2. The Concept of Interval Values

Definition 4 (See [48]). Let $\tilde{a}$ be an interval value and $\tilde{a} = [a^L, a^U] = \left\{x|a^L \le x \le a^U\right\}$. Let $\tilde{a}_1 = [a_1^L, a_1^U]$ and $\tilde{a}_2 = [a_2^L, a_2^U]$ be any two interval values; if $a_1^L, a_2^U \ge 0$ and $0 \le \lambda \le 1$, then we have

(1) $\tilde{a}_1 + \tilde{a}_2 = [a_1^L + a_2^L, b_1^L + b_2^L]$;
(2) $\tilde{a}^\lambda = [(a^L)^\lambda, (a^U)^\lambda]$;
(3) $\lambda\tilde{a} = [\lambda a^L, \lambda a^U]$;
(4) $\tilde{a}_1 \cdot \tilde{a}_2 = [a_1^L a_2^L, b_1^L b_2^L]$.

3. Q-Rung Interval-Valued Probabilistic Dual Hesitant Fuzzy Sets

It is noted that in q-RPDHFS, probabilistic information is denoted by crisp numbers. However, in many real decision-making situations, instead of crisp numbers, DMs prefer to employ interval values to depict probabilistic information. Hence, it is necessary to propose the concept of q-RIVPDHFSs. In this section, we first explain the necessity of proposing q-RIVPDHFSs. Then, the definition of q-RIVPDHFSs is provided. Afterward, some other related notions, such as operational rules, ranking methods, aggregation operators, and distance measures of q-RIVPDHFSs are also developed.

3.1. Necessity of Proposing q-RIVPDHFS. In the classical q-RPDHFS, probabilistic information of membership and membership degrees is denoted by a crisp number. However, in some realistic situations, it is difficult for decision makers to provide crisp probabilistic values. Actually, in most decision-making problems, decision maker prefers to use interval values to denote the probabilistic information. Hence, q-RPDHFS is insufficient and inadequate to handle some real decision-making issues. The following example to better demonstrate this phenomenon is provided.

Example 1. Two professors are invited to evaluate a doctoral thesis under the criteria of standardability. One professor is 30% sure about the MD 0.4 and feels 70% confident in MD 0.8. In addition, he/she is 50% sure about the NMD 0.6 and feels 50% confident about the NMD 0.8. Another professor is 20% to 60% confident in the MD 0.4, and feels 30% to 40% in the MD 0.9. In addition, he/she feels 10% to 30% in the NMD 0.1, 10% to 20% in the NMD 0.4, and he /she is 40% to 50% confident in the NMD 0.8.

It is obvious that the first professor's evaluation value can be denoted as $d = \{\{0.3|0.4, 0.8|7\}, \{0.6|0.5, 0.8|5\}\}$, which is a q-RPDHFE. However, it is impossible to employ a q-RPDHFE to depict the evaluation value of the second professor. This is because the probabilistic information that provided by the second professor is interval values. In other words, q-RPDHFSs fail to handle decision-making problems wherein probabilistic information is in the form of interval values. Motived by interval-valued probabilistic dual hesitant fuzzy sets, it is necessary to propose the interval-valued form q-RPDHFSs, i.e., q-RIVPDHFSs.

3.2. The Definition of q-RIVPDHFSs

Definition 5. Let X be a given fixed set, then the mathematical expression of a q-rung interval-valued probabilistic dual hesitant fuzzy set (q-RIVPDHFS) $\widetilde{D}$ defined on X is expressed as:

$$\widetilde{D} = \left\{x, h_{\widetilde{D}}(x)|\widetilde{p}_{\widetilde{D}}(x), g_{\widetilde{D}}(x)|\widetilde{t}_{\widetilde{D}}(x)|x \in X\right\}, \quad (5)$$

where $h_{\widetilde{D}}(x)$ and $g_{\widetilde{D}}(x)$ are two sets of some values in [0, 1], denoting the possible MDs and NMDs of the element $x \in X$ to set A. In addition, $\widetilde{p}_{\widetilde{D}}(x) = [\widetilde{p}^{U}_{\widetilde{D}}(x), \widetilde{p}^{L}_{\widetilde{D}}(x)]$ ($\widetilde{p}^{U}_{\widetilde{D}}(x) = \inf \widetilde{p}_{\widetilde{D}}(x)$ and $\widetilde{p}^{L}_{\widetilde{D}}(x) = \sup \widetilde{p}_{\widetilde{D}}(x)$) and $\widetilde{t}_{\widetilde{D}}(x) = [\widetilde{t}^{U}_{\widetilde{D}}(x), \widetilde{t}^{L}_{\widetilde{D}}(x)]$ ($\widetilde{t}^{U}_{\widetilde{D}}(x) = \inf \widetilde{t}_{\widetilde{D}}(x)$ and $\widetilde{t}^{L}_{\widetilde{D}}(x) = \sup \widetilde{t}_{\widetilde{D}}(x)$) are two interval values, denoting the interval-valued probabilistic information of $h_{\widetilde{D}}(x)$ and $g_{\widetilde{D}}(x)$, respectively, such that:

$$0 \le \gamma, \eta \le 1,\ (\gamma^{+})^{q} + (\eta^{+})^{q} \le 1\,(q > 1),\ \widetilde{p}_{\widetilde{D}}(x), \widetilde{t}_{\widetilde{D}}(x) \subseteq [0, 1],$$
$$\sum_{i=1}^{\#h} \left(\widetilde{p}^{L}_{\widetilde{D}}(x)\right)_{i} = 1, \sum_{j=1}^{\#g} \left(\widetilde{t}^{L}_{\widetilde{D}}(x)\right)_{j} = 1, \quad (6)$$

where $\gamma \in h_{\widetilde{D}}(x)$, $\eta \in g_{\widetilde{D}}(x)$, $\gamma^{+} = \cup_{\gamma \in h_{\widetilde{D}}(x)} \max\{\gamma\}$, $\eta^{+} = \cup_{\eta \in g_{\widetilde{D}}(x)} \max\{\eta\}$, $\widetilde{p}^{L}_{\widetilde{D}}(x) = \mathrm{Sup}(\widetilde{p}_{\widetilde{D}}(x))_{i}$, $(\widetilde{t}^{L}_{\widetilde{D}}(x))_{i} = \sup \widetilde{t}_{\widetilde{D}}(x)$, and $\#h$ and $\#g$ denote the numbers of values in h and g, respectively. For the sake of easy description, we call the ordered pair $\widetilde{d}(x) = (h_{\widetilde{D}}(x)|p_{\widetilde{D}}(x), g_{\widetilde{D}}(x)|t_{\widetilde{D}}(x))$ a q-rung interval-valued probabilistic dual hesitant fuzzy element (q-RIVPDHFE), which can be denoted as $\widetilde{d} = (h|\widetilde{p}_{h}, g|\widetilde{t}_{g})$ for simplification. In addition, we can $h|\widetilde{p}_{h}$ and $g|\widetilde{t}_{g}$ the interval-valued probabilistic membership elements (IVPMEs) and interval-valued probabilistic nonmembership elements (IVPNMEs), respectively. Especially, when $q = 1$, then $\widetilde{D}$ reduces to an interval-valued probabilistic dual hesitant fuzzy set (IVPDHFS) and $\widetilde{d}$ is reduced to an interval-valued probabilistic dual hesitant fuzzy element. When $q = 2$, then $\widetilde{D}$ reduces to an interval-valued probabilistic dual hesitant Pythagorean fuzzy set and $\widetilde{d}$ is reduced to an interval-valued probabilistic dual hesitant Pythagorean fuzzy element. In addition, it is noted that q-RPDHFE is a special case of q-RIVPDHFE, where the upper bound and lower bound of any interval-valued probabilistic value are equal.

Example 2 (Continued to Example 1). The evaluation value of the second professor can be denoted as $\widetilde{d} = \{\{0.4|[0.2, 0.6], 0.9|[0.3, 0.4]\}, \{0.1|[0.1, 0.3], 0.4| [0.1 , 0.2], 0.8|[0.4, 0.5]\}$. In addition, as $0.9 + 0.8 = 1.7 > 1$, the evaluation value cannot be handled by interval-valued probabilistic dual hesitant fuzzy set, which also illustrates the powerfulness of our proposed q-RIVPDHFS.

3.3. Operational Rules of q-RIVPDHFEs. Based on the operational rules for q-RPDHFEs presented in Definition 2 and the operations for interval values presented in Definition 4, we give the following operational laws for q-RIVPDHFEs.

Definition 6. For any three q-RIVPDHFEs, $\widetilde{d} = (h|\widetilde{p}_{h}, g|\widetilde{t}_{g})$, $\widetilde{d}_{1} = (h_{1}|\widetilde{p}_{h_{1}}, g_{1}|\widetilde{t}_{g_{1}})$, and $\widetilde{d}_{2} = (h_{2}|\widetilde{p}_{h_{2}}, g_{2}|\widetilde{t}_{g_{2}})$, and $\lambda > 0$ be a positive real number, then

(1) $\widetilde{d}_{1} \oplus \widetilde{d}_{2} = \cup_{\gamma_{1} \in h_{1}, \gamma_{2} \in h_{2}, \eta_{1} \in g_{1}, \eta_{2} \in g_{2}} \left\{\left\{(\gamma_{1}^{q} + \gamma_{2}^{q} - \gamma_{1}^{q}\gamma_{2}^{q})^{1/q}| [\widetilde{p}^{L}_{\gamma_{1}}\widetilde{p}^{L}_{\gamma_{2}}, \widetilde{p}^{U}_{\gamma_{1}}\widetilde{p}^{U}_{\gamma_{2}}]\right\}, \left\{\eta_{1}\eta_{2}|[\widetilde{t}^{L}_{\eta_{1}}\widetilde{t}^{L}_{\eta_{2}}, \widetilde{t}^{U}_{\eta_{1}}\widetilde{t}^{U}_{\eta_{2}}]\right\}\right\}$;

(2) $\widetilde{d}_{1} \otimes \widetilde{d}_{2} = \cup_{\gamma_{1} \in h_{1}, \gamma_{2} \in h_{2}, \eta_{1} \in g_{1}, \eta_{2} \in g_{2}} \{\{\gamma_{1}\gamma_{2}|[\widetilde{p}^{L}_{\gamma_{1}}\widetilde{p}^{L}_{\gamma_{2}}, \widetilde{p}^{U}_{\gamma_{1}}\widetilde{p}^{U}_{\gamma_{2}}]\}, \left\{(\eta_{1}^{q} + \eta_{2}^{q} - \eta_{1}^{q}\eta_{2}^{q})^{1/q}|[\widetilde{t}^{L}_{\eta_{1}}\widetilde{t}^{L}_{\eta_{2}}, \widetilde{t}^{U}_{\eta_{1}}\widetilde{t}^{U}_{\eta_{2}}]\right\}\}$;

(3) $\lambda\widetilde{d} = \cup_{\gamma \in h, \eta \in g} \left\{\left\{(1 - (1 - \gamma^{q})^{\lambda})^{1/q}|[\widetilde{p}^{L}_{\gamma}, \widetilde{p}^{U}_{\gamma}]\right\}, \{\eta^{\lambda}|[\widetilde{t}^{L}_{\eta}, \widetilde{t}^{U}_{\eta}]\}\right\}$;

(4) $\tilde{d}^{\lambda} = \cup_{\gamma\in h,\eta\in g}\left\{\left\{\gamma^{\lambda}|[\tilde{p}^{L}_{\gamma}, \tilde{p}^{U}_{\gamma}]\right\}, \quad \left\{(1-(1-\gamma^{q})^{\lambda})^{1/q}\right.\right.$ $|[\tilde{t}^{L}_{\eta}, \tilde{t}^{U}_{\eta}]\}\}$.

Example 3. Let $\tilde{d}_1 = \{\{0.7|[0.4, 0.5], 0.8|[0.3, 0.5]\}, \{0.3|[0.1, 0.2], 0.4|[0.1, 0.2], 0.7|[0.3, 0.6]\}$ and $\tilde{d}_2 = \{\{0.5|[0.3, 0.7], 0.6|[0.1, 0.3]\}, \{0.4|[0.1, 0.2], 0.5|[0.6, 0.8]\}\}$ be any two q-RIVPDHFEs, then we have the following results

$$\tilde{d}_1 \oplus \tilde{d}_2 = \left\{\begin{array}{c}\left\{\begin{array}{c}0.7519|[0.12, 0.35], 0.7856|[0.04, 0.15],\\ 0.8306|[0.09, 0.35], 0.8515|[0.03, 0.15]\end{array}\right\},\\ \left\{\begin{array}{c}0.12|[0.01, 0.04], 0.15|[0.06, 0.16], 0.16|[0.01, 0.04],\\ 0.2|[0.06, 0.16], 0.28|[0.03, 0.12], 0.35|[0.18, 0.48]\end{array}\right\}\end{array}\right\},$$
$$\tilde{d}_1 \otimes \tilde{d}_2 = \left\{\begin{array}{c}\left\{\begin{array}{c}0.35|[0.12, 0.35], 0.42|[0.04, 0.15],\\ 0.4|[0.09, 0.35], 0.48|[0.03, 0.15]\end{array}\right\},\\ \left\{\begin{array}{c}0.4469|[0.01, 0.04], 0.5297|[0.06, 0.16], 0.4985|[0.01, 0.04],\\ 0.5657|[0.06, 0.16], 0.7275|[0.03, 0.12], 0.7519|[0.18, 0.48]\end{array}\right\}\end{array}\right\}, \tag{7}$$
$$2\tilde{d}_1 = \left\{\begin{array}{c}\{0.8283|[0.4, 0.5], 0.9133|[0.3, 0.5]\},\\ \{0.09|[0.1, 0.2], 0.16|[0.1, 0.2], 0.49|[0.3, 0.6]\}\end{array}\right\},$$
$$\tilde{d}_1^{2} = \left\{\begin{array}{c}\{0.49|[0.4, 0.5], 0.64|[0.3, 0.5]\},\\ \{0.3763|[0.1, 0.2], 0.4985|[0.1, 0.2], 0.8283|[0.3, 0.6]\}\end{array}\right\}.$$

3.4. Ranking Method for q-RIVPDHFEs. To compare any two q-RIVPDHFEs, a ranking method is provided as follows:

Definition 7. Let $\tilde{d} = (h|\tilde{p}_h, g|\tilde{t}_g)$ be a q-RIVPDHFE, then the score function $\tilde{d}$ is defined as:

$$S(\tilde{d}) = \sum_{\gamma\in h}^{\#h}\left(\gamma^{q}\tilde{p}^{L}_{\gamma} + \gamma^{q}\tilde{p}^{U}_{\gamma}\right) - \sum_{\eta\in g}^{\#g}\left(\eta^{q}\tilde{t}^{L}_{\eta} + \eta^{q}\tilde{t}^{U}_{\eta}\right). \tag{8}$$

Based on the score function, the deviation function of $\tilde{d}$ is expressed as follows:

Definition 8. Let $\tilde{d} = (h|\tilde{p}_h, g|\tilde{t}_g)$ be a q-RIVPDHFE, and $S(\tilde{d})$ be its score function. The deviation function of $\tilde{d}$ is expressed as:

$$\Phi(\tilde{d}) = \left(\sum_{\gamma\in h}^{\#h}\left(\left(\gamma^{q} - S(\tilde{d})\right)^{2}\left(\tilde{p}^{L}_{\gamma} + \tilde{p}^{U}_{\gamma}\right)\right) + \sum_{\eta\in g}^{\#g}\left(\left(\eta^{q} - S(\tilde{d})\right)^{2}\left(\tilde{t}^{L}_{\eta} + \tilde{t}^{U}_{\eta}\right)\right)\right)^{1/2}. \tag{9}$$

Based on the score and deviation functions, a method for ranking any two q-RIVPDHFEs is presented as follows:

Definition 9. Let $\tilde{d}_1 = (h_1|\tilde{p}_{h_1}, g_1|\tilde{t}_{g_1})$ and $\tilde{d}_2 = (h_2|\tilde{p}_{h_2}, g_2|\tilde{t}_{g_2})$ be any two q-RIVPDHFEs, $S(\tilde{d}_1)$ and $S(\tilde{d}_2)$ be the score function of $\tilde{d}_1$ and $\tilde{d}_2$, and $\Phi(\tilde{d}_1)$ and $\Phi(\tilde{d}_2)$ be the deviation function of $\tilde{d}_1$ and $\tilde{d}_2$. Then, we have

(1) If $S(\tilde{d}_1) > S(\tilde{d}_2)$, then $\tilde{d}_1 > \tilde{d}_2$;
(2) If $S(\tilde{d}_1) < S(\tilde{d}_2)$, then $\tilde{d}_1 < \tilde{d}_2$;
(3) If $S(\tilde{d}_1) = S(\tilde{d}_2)$, then

If $\Phi(\tilde{d}_1) < \Phi(\tilde{d}_2)$, then $\tilde{d}_1 > \tilde{d}_2$;
If $\Phi(\tilde{d}_1) > \Phi(\tilde{d}_2)$, then $\tilde{d}_1 < \tilde{d}_2$;
If $\Phi(\tilde{d}_1) = \Phi(\tilde{d}_2)$, then $\tilde{d}_1 = \tilde{d}_2$.

3.5. Aggregation Operators for q-RIVPDHFEs

Definition 10. Let $\tilde{d}_j = (h_j|\tilde{p}_{h_j}, g_j|\tilde{t}_{g_j})\,(j = 1, 2, \ldots, n)$ be a collection of q-RIVPDHFEs, and $w = (w_1, w_2, \ldots, w_n)^T$ be the corresponding weight vector, such that $\sum_{j=1}^{n} w_j = 1$ and $1 \geq w_j \geq 0$, then the q-rung interval-valued probabilistic dual hesitant fuzzy weighted average (q-RIVPDHFWA) operator is defined as

$$q - \text{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \oplus_{j=1}^{n} w_j\tilde{d}_j. \tag{10}$$

Specifically, if $w = (1/n, 1/n, \ldots, 1/n)^T$, then the q-RIVPDHFWA is reduced to the q-rung interval-valued probabilistic dual hesitant fuzzy average operator.

Theorem 1. *Let* $\tilde{d}_j = (h_j|\tilde{p}_{h_j}, g_j|\tilde{t}_{g_j})\,(j = 1, 2, \ldots, n)$ *be a collection of q-RIVPDHFEs, then the aggregated result by q-RIVPDHFWA operator is still a q-RIVPDHFE and*

$$\begin{aligned} &q-\text{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = \cup_{\gamma_j \in h_j, \eta_j \in g_j}, \\ &\left\{\left\{\left(1 - \prod_{j=1}^{n}\left(1-\gamma_j^q\right)^{w_j}\right)^{1/q} | \left[\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^L, \prod_{j=1}^{n}\tilde{p}_{\gamma_j}^U\right]\right\}, \left\{\prod_{j=1}^{n}\eta_j^{w_j} | \left[\prod_{j=1}^{n}\tilde{t}_{\eta_j}^L, \prod_{j=1}^{n}\tilde{t}_{\eta_j}^U\right]\right\}\right\}. \end{aligned} \tag{11}$$

Proof. (i) It is obvious that Eq. (11) holds for $n = 1$;

(ii) For $n = 2$, according to Definition 6, we can obtain:

$$\begin{aligned} w_1\tilde{d}_1 &= \cup_{\gamma_1 \in h_1, \eta_1 \in g_1}\left\{\left\{\left(1-\left(1-\gamma_1^q\right)^{w_1}\right)^{1/q} | \left[\tilde{p}_{\gamma_1}^L, \tilde{p}_{\gamma_1}^U\right]\right\}, \left\{\eta_1^{w_1} | \left[\tilde{t}_{\eta_1}^L, \tilde{t}_{\eta_1}^U\right]\right\}\right\}, \\ w_2\tilde{d}_2 &= \cup_{\gamma_2 \in h_2, \eta_2 \in g_2}\left\{\left\{\left(1-\left(1-\gamma_2^q\right)^{w_2}\right)^{1/q} | \left[\tilde{p}_{\gamma_2}^L, \tilde{p}_{\gamma_2}^U\right]\right\}, \left\{\eta_2^{w_2} | \left[\tilde{t}_{\eta_2}^L, \tilde{t}_{\eta_2}^U\right]\right\}\right\}. \end{aligned} \tag{12}$$

Hence,

$$\begin{aligned} &q-\text{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2\right) = w_1\tilde{d}_1 \oplus w_2\tilde{d}_2 = \cup_{\gamma_1 \in h_1, \eta_1 \in g_1, \gamma_2 \in h_2, \eta_2 \in g_2}, \\ &\left\{\left\{\left(1-\left(1-\gamma_1^q\right)^{w_1}\left(1-\gamma_2^q\right)^{w_2}\right)^{1/q} | \left[\tilde{p}_{\gamma_1}^L\tilde{p}_{\gamma_2}^L, \tilde{p}_{\gamma_1}^U\tilde{p}_{\gamma_2}^U\right]\right\}, \left\{\eta_1^{w_1}\eta_2^{w_2} | \left[\tilde{t}_{\eta_1}^L\tilde{t}_{\eta_2}^L, \tilde{t}_{\eta_1}^U\tilde{t}_{\eta_2}^U\right]\right\}\right\}, \end{aligned} \tag{13}$$

which means that (11) holds for <i>n</i ≥ 2.

(iii) We assume that equation (11) holds for $n = k$, i.e.,

$$\begin{aligned} &q-\text{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_k\right) = \cup_{\gamma_j \in h_j, \eta_j \in g_j}, \\ &\left\{\left\{\left(1 - \prod_{j=1}^{k}\left(1-\gamma_j^q\right)^{w_j}\right)^{1/q} | \left[\prod_{j=1}^{k}\tilde{p}_{\gamma_j}^L, \prod_{j=1}^{k}\tilde{p}_{\gamma_j}^U\right]\right\}, \left\{\prod_{j=1}^{k}\eta_j^{w_j} | \left[\prod_{j=1}^{k}\tilde{t}_{\eta_j}^L, \prod_{j=1}^{k}\tilde{t}_{\eta_j}^U\right]\right\}\right\}. \end{aligned} \tag{14}$$

(iv) For $n = k + 1$, then we have

$$\begin{aligned} &q-\text{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_{k+1}\right) = \oplus_{j=1}^{k} w_j\tilde{d}_j \oplus w_{k+1}\tilde{d}_{k+1} = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \\ &\left\{\left\{\left(1 - \prod_{j=1 j=1}^{k}\left(1-\gamma_j^q\right)^{w_j}\right)^{1/q} | \left[\prod_{j=1 j=1}^{k}\tilde{p}_{\gamma_j}^L, \prod_{j=1 j=1}^{k}\tilde{p}_{\gamma_j}^U\right]\right\}, \left\{\prod_{j=1 j=1}^{k}\eta_j^{w_j} | \left[\prod_{j=1 j=1}^{k}\tilde{t}_{\eta_j}^L, \prod_{j=1 j=1}^{k}\tilde{t}_{\eta_j}^U\right]\right\}\right\} \\ &\oplus \cup_{\gamma_{k+1} \in h_{k+1}, \eta_{k+1} \in g_{k+1}}\left\{\left\{\left(1-\left(1-\gamma_{k+1}^q\right)^{w_{k+1}}\right)^{1/q} | \left[\tilde{p}_{\gamma_{k+1}}^L, \tilde{p}_{\gamma_{k+1}}^U\right]\right\}, \left\{\eta_{k+1}^{w_{k+1}} | \left[\tilde{t}_{\eta_{k+1}}^L, \tilde{t}_{\eta_{k+1}}^U\right]\right\}\right\} = \cup_{\gamma_j \in h_j, \eta_j \in g_j} \\ &\left\{\left\{\left(1 - \prod_{j=1}^{k+1}\left(1-\gamma_j^q\right)^{w_j}\right)^{1/q} | \left[\prod_{j=1}^{k+1}\tilde{p}_{\gamma_j}^L, \prod_{j=1}^{k+1}\tilde{p}_{\gamma_j}^U\right]\right\}, \left\{\prod_{j=1}^{k+1}\eta_j^{w_j} | \left[\prod_{j=1}^{k+1}\tilde{t}_{\eta_j}^L, \prod_{j=1}^{k+1}\tilde{t}_{\eta_j}^U\right]\right\}\right\}, \end{aligned} \tag{15}$$

which indicates that equation (11) holds for $n = k + 1$. Hence, the proof of Theorem 1 is completed.

In the following, we will give some properties of the proposed q-RIVPDHFWA operator. □

Theorem 2 (Monotonicity). *Let* $\tilde{d}_j = (h_j|\tilde{p}_{h_j}, g_j|\tilde{t}_{g_j})$ *and* $\tilde{d}_j^* = (h_j^*|\tilde{p}_{h_j}^*, g_j^*|\tilde{t}_{g_j}^*)\,(j = 1, 2, \ldots, n)$ *be two sets of q-RIVPDHFEs. For the elements in* $\tilde{d}_j$ *and* $\tilde{d}_j^*$, *if* $\gamma_j \le \gamma_j^*$, $\eta_j \ge \eta_j^*$, $\tilde{p}_{h_j} = \tilde{p}_{h_j}^*$ *and* $\tilde{t}_{g_j} = \tilde{t}_{g_j}^*$, *then*

$$q - \mathrm{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) \le q - \mathrm{RIVPDHFWA}\left(\tilde{d}_1^*, \tilde{d}_2^*, \ldots, \tilde{d}_3^*\right). \tag{16}$$

Proof. For any $j = 1, 2, \ldots, n$, there is $\gamma_j \le \gamma_j^*$. Then, for the aggregated result of the q-RIVPDHFWA operator, we have $(1 - \prod_{j=1}^n (1 - \gamma_j^q)^{w_j})^{1/q} \le (1 - \prod_{j=1}^n (1 - \gamma_{j^*}^q)^{w_j})^{1/q}$. Similarly, we have $\prod_{j=1}^n \eta_j^{w_j} \ge \prod_{j=1}^n \eta_{j^*}^{w_j}$. According to the ranking method of any two q-RIVPDHFEs provided in Subsection 3.3, $q - \mathrm{RIVPDHFWA}\,(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n) \le q-$ $\mathrm{RIVPDHFWA}\,(\tilde{d}_1^*, \tilde{d}_2^*, \ldots, \tilde{d}_3^*)$ can be obtained. □

Theorem 3 (Boundedness). *Let* $\tilde{d}_j = (h_j|\tilde{p}_{h_j}, g_j|\tilde{t}_{g_j})$, $\tilde{d}_j^- = (h_j^-|\tilde{p}_{h_j^-}, g_j^-|\tilde{t}_{g_j^-})$, $\tilde{d}_j^+ = (h_j^+|\tilde{p}_{h_j^+}, g_j^+|\tilde{t}_{g_j^+})\,(j = 1, 2, \ldots, n)$ *be three sets of q-RIVPDHFEs. If every element in* h_j^- *satisfies* $\gamma_j^- = \min\,(h_j)$, $\eta_j^- = \max(g_j)$, $(\tilde{p}_{h_j}^L)^- = \min(\tilde{p}_{h_j}^L)$, $(\tilde{p}_{h_j}^U)^- = \min(\tilde{p}_{h_j}^U)$, $(\tilde{t}_{h_j}^L)^- = \max(\tilde{p}_{h_j}^L)$, *and* $(\tilde{t}_{h_j}^U)^+ = \max(\tilde{p}_{h_j}^U)$, *every element in* h_j^+ *satisfies* $\gamma_j^+ = \max(h_j)$, $\eta_j^+ = \min(g_j)$, *and* $(\tilde{p}_{h_j}^L)^+ = \max(\tilde{p}_{h_j}^L)$, $(\tilde{p}_{h_j}^U)^+ = \max(\tilde{p}_{h_j}^U)$, $(\tilde{t}_{h_j}^L)^+ = \min(\tilde{p}_{h_j}^L)$, *and* $(\tilde{t}_{h_j}^U)^+ = \min(\tilde{p}_{h_j}^U)$, *then*

$$\begin{aligned} &q - \mathrm{RIVPDHFWA}\left(\tilde{d}^-, \tilde{d}^-, \ldots, \tilde{d}^-\right) \le q - \mathrm{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right), \\ &\le q - \mathrm{RIVPDHFWA}\left(\tilde{d}^+, \tilde{d}^+, \ldots, \tilde{d}^+\right). \end{aligned} \tag{17}$$

Proof. For any $j = 1, 2, \ldots, n$, it is obvious that $\gamma_j^- \le \gamma_j \le \gamma_j^+$. Thus, according to the aggregated results,

$$\left(1 - \prod_{j=1}^{n}\left(1 - \left(\gamma_j^-\right)^q\right)^{w_j}\right)^{1/q} \le \left(1 - \prod_{j=1}^{n}\left(1 - \gamma_j^q\right)^{w_j}\right)^{1/q} \le \left(1 - \prod_{j=1}^{n}\left(1 - \left(\gamma_j^+\right)^q\right)^{w_j}\right)^{1/q}. \tag{18}$$

Similarly, we can get $\prod_{j=1}^n (\eta_j^-)^{w_j} \ge \prod_{j=1}^n \eta_j^{w_j} \ge \prod_{j=1}^n (\eta_j^+)^{w_j}$. Then, as $(\tilde{p}_{h_j}^L)^- \le \tilde{p}_{h_j}^L \le (\tilde{p}_{h_j}^L)^+$, we have $\prod_{j=1}^n (\tilde{p}_{h_j}^L)^- \le \prod_{j=1}^n \tilde{p}_{\gamma_j}^L \le \prod_{j=1}^n (\tilde{p}_{h_j}^L)^+$. Similarly, $\prod_{j=1}^n (\tilde{p}_{h_j}^U)^- \le \prod_{j=1}^n \tilde{p}_{\gamma_j}^U \le \prod_{j=1}^n (\tilde{p}_{h_j}^U)^+$. In addition, as $(\tilde{t}_{h_j}^L)^- \ge \tilde{t}_{h_j}^L \ge (\tilde{t}_{h_j}^L)^+$, $\prod_{j=1}^n (\tilde{t}_{h_j}^L)^- \ge \prod_{j=1}^n \tilde{t}_{\gamma_j}^L \ge \prod_{j=1}^n (\tilde{t}_{h_j}^L)^+$, and $\prod_{j=1}^n (\tilde{t}_{h_j}^U)^- \ge \prod_{j=1}^n \tilde{t}_{\gamma_j}^U \ge \prod_{j=1}^n (\tilde{t}_{h_j}^U)^+$ can be gained. Finally, according to Subsection 3.3, Theorem 2 can be proved. □

Theorem 4 (Commutativity). *Let* $\tilde{d}_j = (h_j|\tilde{p}_{h_j}, g_j|\tilde{t}_{g_j})$ *and* $\tilde{d}_j' = (h_j'|\tilde{p}_{h_j'}, g_j'|\tilde{t}_{g_j}')\,(j = 1, 2, \ldots, n)$ *be two sets of q-RIVPDHFEs, and* $\tilde{d}_j^P$ *is any permutation of* $\tilde{d}_j$, *then*

$$q - \mathrm{RIVPDHFWA}\left(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_n\right) = q - \mathrm{RIVPDHFWA}\left(\tilde{d}_1', \tilde{d}_2', \ldots, \tilde{d}_n'\right). \tag{19}$$

The proof of Theorem 4 is trivial and we omit it here.

In the following, we will discuss the special cases of the q-RIVPDHFWA operator with respect to the parameter q.

Case 1. If $q = 1$, the $q - \mathrm{RIVPDHFWA}$ will reduce to the intuitionistic interval-valued probabilistic dual hesitant fuzzy weighted average (IIVPDHFWA) operator, that is:

$$q-\text{RIVPDHFWA}_{q=1}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\oplus_{j=1}^{n}w_j\tilde{d}_j=\cup_{\gamma_j\in h_j,\eta_j\in g_j},$$
$$\left\{\left\{1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{w_j}\middle|\left[\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{L},\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{U}\right]\right\},\left\{\prod_{j=1}^{n}\eta_j^{w_j}\middle|\left[\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{L},\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{U}\right]\right\}\right\}. \tag{20}$$

Case 2. If $q=2$, the $q-$RIVPDHFWA will reduce to the Pythagorean interval-valued probabilistic dual hesitant fuzzy weighted average (PIVPDHFWA) operator, that is:

$$q-\text{RIVPDHFWA}_{q=2}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\oplus_{j=1}^{n}w_j\tilde{d}_j=\cup_{\gamma_j\in h_j,\eta_j\in g_j},$$
$$\left\{\left\{\left(1-\prod_{j=1}^{n}\left(1-\gamma_j^2\right)^{w_j}\right)^{1/2}\middle|\left[\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{L},\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{U}\right]\right\},\left\{\prod_{j=1}^{n}\eta_j^{w_j}\middle|\left[\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{L},\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{U}\right]\right\}\right\}. \tag{21}$$

Case 3. If $q=3$, the $q-$RIVPDHFWA will reduce to the Fermatean interval-valued probabilistic dual hesitant fuzzy weighted average (FIVPDHFWA) operator, that is:

$$q-\text{RIVPDHFWA}_{q=3}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\oplus_{j=1}^{n}w_j\tilde{d}_j=\cup_{\gamma_j\in h_j,\eta_j\in g_j},$$
$$\left\{\left\{\left(1-\prod_{j=1}^{n}\left(1-\gamma_j^3\right)^{w_j}\right)^{1/3}\middle|\left[\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{L},\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{U}\right]\right\},\left\{\prod_{j=1}^{n}\eta_j^{w_j}\middle|\left[\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{L},\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{U}\right]\right\}\right\}. \tag{22}$$

In the following, we continue to propose the geometric average operator of the q-RIVPDHFEs.

Definition 11. Let $\tilde{d}_j=(h_j|\tilde{p}_{h_j},g_j|\tilde{t}_{g_j})$ be a collection of q-RIVPDHFEs, and $w=(w_1,w_2,\ldots,w_n)^T$ be the corresponding weight vector, such that $\sum_{j=1}^{n}w_j=1$ and $1\geq w_j\geq 0$, then the q-rung interval-valued probabilistic dual hesitant fuzzy weighted geometric (q-RIVPDHFWG) operator is defined as:

$$q-\text{RIVPDHFWG}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\otimes_{j=1}^{n}\tilde{d}_j^{w_j}=\cup_{\gamma_j\in h_j,\eta_j\in g_j}. \tag{23}$$

Specifically, if $w=(1/n,1/n,\ldots,1/n)^T$, then the q-RIVPDHFWG is reduced to the q-rung interval-valued probabilistic dual hesitant fuzzy geometric operator.

Based on the operational rules presented in Definition 6, the following theorem can be obtained.

Theorem 5. *Let $\tilde{d}_j=(h_j|\tilde{p}_{h_j},g_j|\tilde{t}_{g_j})(j=1,2,\ldots,n)$ be a collection of q-RIVPDHFEs, then the aggregated result by q-RIVPDHFWG operator is still a q-RIVPDHFE and*

$$q-\text{RIVPDHFWG}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\left\{\left\{\prod_{j=1}^{n}\gamma_j^{w_j}\middle|\left[\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{L},\prod_{j=1}^{n}\tilde{p}_{\gamma_j}^{U}\right]\right\},\left\{\left(1-\prod_{j=1}^{n}\left(1-\eta_j^q\right)^{w_j}\right)^{1/q}\middle|\left[\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{L},\prod_{j=1}^{n}\tilde{t}_{\eta_j}^{U}\right]\right\}\right\}. \tag{24}$$

The proof of Theorem 6 is similar to that of Theorem 1, which is omitted here. In addition, the q-RIVPDHFWG also has the following properties.

Theorem 6 (Monotonicity). *Let $\tilde{d}_j=(h_j|\tilde{p}_{h_j},g_j|\tilde{t}_{g_j})$ and $\tilde{d}_j^*=(h_j^*|\tilde{p}_{h_j}^*,g_j^*|\tilde{t}_{g_j}^*)(j=1,2,\ldots,n)$ be two sets of q-RIVPDHFEs. For the elements in $\tilde{d}_j$ and $\tilde{d}_j^*$, if $\gamma_j\leq\gamma_j^*$, $\eta_j\geq\eta_j^*$, $\tilde{p}_{h_j}=\tilde{p}_{h_j}^*$, and $\tilde{t}_{g_j}=\tilde{t}_{g_j}^*$, then*

$$q-\mathrm{RIVPDHFWG}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)\le q-\mathrm{RIVPDHFWG}\left(\tilde{d}_1^*,\tilde{d}_2^*,\ldots,\tilde{d}_3^*\right). \tag{25}$$

Theorem 7 (Boundedness). *Let* $\tilde{d}_j=(h_j|\tilde{p}_{h_j},g_j|\tilde{t}_{g_j})$, $\tilde{d}_j^-=(h_j^-|\tilde{p}_{h_j^-},g_j^-|\tilde{t}_{g_j^-})$, *and* $\tilde{d}_j^+=(h_j^+|\tilde{p}_{h_j^+},g_j^+|\tilde{t}_{g_j^+})$ $(j=1,2,\ldots,n)$ *be three sets of q-RIVPDHFEs. If every element in* h_j^- *satisfies* $\gamma_j^-=\min(h_j)$, $\eta_j^-=\max(g_j)$, *and* $(\tilde{p}_{h_j}^L)^-=\min(\tilde{p}_{h_j}^L)$, $(\tilde{p}_{h_j}^U)^-=\min(\tilde{p}_{h_j}^U)$, $(\tilde{t}_{h_j}^L)^-=\max(\tilde{p}_{h_j}^L)$, *and* $(\tilde{t}_{h_j}^U)^+=\max(\tilde{p}_{h_j}^U)$; *every element in* h_j^+ *satisfies* $\gamma_j^+=\max(h_j)$, $\eta_j^+=\min(g_j)$, *and* $(\tilde{p}_{h_j}^L)^+=\max(\tilde{p}_{h_j}^L)$, $(\tilde{p}_{h_j}^U)^+=\max(\tilde{p}_{h_j}^U)$, $(\tilde{t}_{h_j}^L)^+=\min(\tilde{p}_{h_j}^L)$, *and* $(\tilde{t}_{h_j}^U)^+=\min(\tilde{p}_{h_j}^U)$, *then*

$$q-\mathrm{RIVPDHFWG}\left(\tilde{d}^-,\tilde{d}^-,\ldots,\tilde{d}^-\right)\le q-\mathrm{RIVPDHFWG}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right),\le q-\mathrm{RIVPDHFWG}\left(\tilde{d}^+,\tilde{d}^+,\ldots,\tilde{d}^+\right). \tag{26}$$

Theorem 8 (Commutativity). *Let* $\tilde{d}_j=(h_j|\tilde{p}_{h_j},g_j|\tilde{t}_{g_j})$ *and* $\tilde{d}_j'=(h_j'|\tilde{p}_{h_j'}',g_j'|\tilde{t}_{g_j}')$ $(j=1,2,\ldots,n)$ *be two sets of q-RIVPDHFEs, and* $\tilde{d}_j'$ *is any permutation of* $\tilde{d}_j$, *then*

$$q-\mathrm{RIVPDHFWG}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=q-\mathrm{RIVPDHFWG}\left(\tilde{d}_1',\tilde{d}_2',\ldots,\tilde{d}_n'\right). \tag{27}$$

The proofs of Theorems 6 and 7 are similar to those of Theorems 2 and 3. In addition, Theorem 8 is trivial.

There are some special cases of the q-RIVPDHFWG operator with respect to the parameter q.

Case 4. If $q=1$, the $q-\mathrm{RIVPDHFWG}$ will reduce to the intuitionistic interval-valued probabilistic dual hesitant fuzzy weighted geometric (IIVPDHFWG) operator, that is:

$$\begin{aligned}&q-\mathrm{RIVPDHFWG}_{q=1}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\otimes_{j=1}^n\tilde{d}_j^{w_j}=\cup_{\gamma_j\in h_j,\eta_j\in g_j}\\&\left\{\left\{\prod_{j=1}^n\gamma_j^{w_j}|\left[\prod_{j=1}^n\tilde{p}_{\gamma_j}^L,\prod_{j=1}^n\tilde{p}_{\gamma_j}^U\right]\right\},\left\{1-\prod_{j=1}^n\left(1-\eta_j\right)^{w_j}|\left[\prod_{j=1}^n\tilde{t}_{\eta_j}^L,\prod_{j=1}^n\tilde{t}_{\eta_j}^U\right]\right\}\right\}.\end{aligned} \tag{28}$$

Case 5. If $q=2$, the $q-\mathrm{RIVPDHFWG}$ will reduce to the Pythagorean interval-valued probabilistic dual hesitant fuzzy weighted geometric (PIVPDHFWG) operator, that is:

$$\begin{aligned}&q-\mathrm{RIVPDHFWG}_{q=2}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\otimes_{j=1}^n\tilde{d}_j^{w_j}=\cup_{\gamma_j\in h_j,\eta_j\in g_j},\\&\left\{\left\{\prod_{j=1}^n\gamma_j^{w_j}|\left[\prod_{j=1}^n\tilde{p}_{\gamma_j}^L,\prod_{j=1}^n\tilde{p}_{\gamma_j}^U\right]\right\},\left\{\left(1-\prod_{j=1}^n\left(1-\eta_j^2\right)^{w_j}\right)^{1/2}|\left[\prod_{j=1}^n\tilde{t}_{\eta_j}^L,\prod_{j=1}^n\tilde{t}_{\eta_j}^U\right]\right\}\right\}.\end{aligned} \tag{29}$$

Case 6. If $q=3$, the $q-\mathrm{RIVPDHFWG}$ will reduce to the Fermatean interval-valued probabilistic dual hesitant fuzzy weighted geometric (FIVPDHFWG) operator, that is:

$$q-\mathrm{RIVPDHFWG}_{q=3}\left(\tilde{d}_1,\tilde{d}_2,\ldots,\tilde{d}_n\right)=\oplus_{j=1}^{n}w_j\tilde{d}_j=\cup_{\gamma_j\in h_j,\eta_j\in g_j}$$
$$\left\{\left\{\prod_{j=1}^{n}\gamma_j^{w_j}\left[\prod_{j=1}^{n}\tilde{p}^L_{\gamma_j},\prod_{j=1}^{n}\tilde{p}^U_{\gamma_j}\right]\right\},\left\{\left(1-\prod_{j=1}^{n}\left(1-\eta_j^3\right)^{w_j}\right)^{1/3}|\left[\prod_{j=1}^{n}\tilde{t}^L_{\eta_j},\prod_{j=1}^{n}\tilde{t}^U_{\eta_j}\right]\right\}\right\}. \tag{30}$$

3.6. Distance Measure between Two q-RIVPDHFEs. In this subsection, we aim at proposing a distance measure between two q-RIVPDHFEs. Before doing so, we first propose the concept of normalized q-RIVPDHFE.

Definition 12. Let $\tilde{d}=(h|\tilde{p}_h,g|\tilde{t}_g)$ be a q-RIVPDHFE, then $\tilde{d}$ is called a normalized q-RIVPDHFE if and only if all IVPMEs and IVPNMEs are in ascending order.

Remark 1. It is noted that a q-RIVPDHFE can be called a normalized q-RIVPDHFE if and only if all IVPMEs and IVPNMEs are in ascending order. Hence, if a q-RIVPDHFE is nonnormalized, then a transformation method is necessary, which is presented as follows: Let $\tilde{d}=(h|\tilde{p}_h,g|\tilde{t}_g)$ be a q-RIVPDHFE $\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}}]$ and $\gamma^{(k)}|[\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}}]$ be any two IVPMEs of $\tilde{d}$ (where $\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}}]$, $\gamma^{(k)}|[\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}}]\in h|\tilde{p}_h$) and $\eta^{(l)}|[\tilde{t}^L_{\eta^{(l)}},\tilde{t}^U_{\eta^{(l)}}]$ and $\eta^{(m)}|[\tilde{t}^L_{\eta^{(m)}},\tilde{t}^U_{\eta^{(m)}}]$ be any two IVPNMEs of $\tilde{d}$ (where $\eta^{(l)}|[\tilde{t}^L_{\eta^{(l)}},\tilde{t}^U_{\eta^{(l)}}]$, $\eta^{(m)}|[\tilde{t}^L_{\eta^{(m)}},\tilde{t}^U_{\eta^{(m)}}]\in g|\tilde{t}_g$), where $s,k=1,2,\ldots,\#h$, $l,m=1,2,\ldots,\#g$, and $\#h$ and $\#g$ denote the numbers of IVPMEs and IVPNMEs in d, respectively. Then,

(1) if $\gamma^{(s)}(\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}})>\gamma^{(k)}(\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}})$, then $\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}}]>\gamma^{(k)}|[\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}}]$;

(2) if $\gamma^{(s)}(\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}})=\gamma^{(k)}(\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}})$, then if $\gamma^{(s)}>\gamma^{(k)}$, then $\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}}]>\gamma^{(k)}|[\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}}]$; If $\gamma^{(s)}=\gamma^{(k)}$, then $\gamma^{(s)}|[\tilde{p}^L_{\gamma^{(s)}}+\tilde{p}^U_{\gamma^{(s)}}]=\gamma^{(k)}|[\tilde{p}^L_{\gamma^{(k)}}+\tilde{p}^U_{\gamma^{(k)}}]$.

The two IVPNMEs $\eta^{(l)}|[\tilde{t}^L_{\eta^{(l)}},\tilde{t}^U_{\eta^{(l)}}]$ and $\eta^{(m)}|[\tilde{t}^L_{\eta^{(m)}},\tilde{t}^U_{\eta^{(m)}}]$ can be compared in a similar way. For a q-RIVPDHFE $\tilde{d}=(h|\tilde{p}_h,g|\tilde{t}_g)$, if all IVPMEs and IVPNMEs of $\tilde{d}$ are in ascending order, then we call $\tilde{d}$ normalized q-RIVPDHFE.

Based on the above definition, the distance measure between any two q-RIVPDHFE is defined as follows.

Definition 13. Let $\tilde{d}_1=(h_1|\tilde{p}_{h_1},g_1|\tilde{t}_{g_1})$ and $\tilde{d}_2=(h_2|\tilde{p}_{h_2},g_2|\tilde{t}_{g_2})$ be any two normalized q-RIVPDHFEs, then the distance between $\tilde{d}_1$ and $\tilde{d}_2$ is presented as follows:

$$d\left(\tilde{d}_1,\tilde{d}_2\right)=\frac{\sum_{i=1}^{n}\left|\left(\gamma_1^{\sigma(i)}\right)^q\left(\tilde{p}^L_{\gamma_1^{\sigma(i)}}+\tilde{p}^U_{\gamma_1^{\sigma(i)}}\right)-\left(\gamma_2^{\sigma(i)}\right)^q\left(\tilde{p}^L_{\gamma_2^{\sigma(i)}}+\tilde{p}^U_{\gamma_2^{\sigma(i)}}\right)\right|+\sum_{j=1}^{n}\left|\left(\eta_1^{\sigma(j)}\right)^q\left(\tilde{t}^L_{\eta_1^{\sigma(j)}}+\tilde{t}^U_{\eta_1^{\sigma(j)}}\right)-\left(\eta_2^{\sigma(j)}\right)^q\left(\tilde{t}^L_{\eta_2^{\sigma(j)}}+\tilde{t}^U_{\eta_2^{\sigma(j)}}\right)\right|}{\#h+\#g}, \tag{31}$$

where $\gamma_1^{\sigma(i)}|[\tilde{p}^L_{\gamma_1^{\sigma(i)}},\tilde{p}^U_{\gamma_1^{\sigma(i)}}]\in h_1|\tilde{p}_{h_1}$, $\eta_1^{\sigma(j)}|[\tilde{t}^L_{\eta_1^{\sigma(j)}},\tilde{t}^U_{\eta_1^{\sigma(j)}}]\in g_1|\tilde{t}_{g_1}$, $\gamma_2^{\sigma(i)}|[\tilde{p}^L_{\gamma_2^{\sigma(i)}},\tilde{p}^U_{\gamma_2^{\sigma(i)}}]\in h_2|\tilde{p}_{h_2}$, $\eta_2^{\sigma(j)}|[\tilde{t}^L_{\eta_2^{\sigma(j)}},\tilde{t}^U_{\eta_2^{\sigma(j)}}]\in g_2|\tilde{t}_{g_2}$, $\gamma_1^{\sigma(i)}|[\tilde{p}^L_{\gamma_1^{\sigma(i)}},\tilde{p}^U_{\gamma_1^{\sigma(i)}}]\le\gamma_1^{\sigma(i+1)}|[\tilde{p}^L_{\gamma_1^{\sigma(i+1)}},\tilde{p}^U_{\gamma_1^{\sigma(i+1)}}]$, $\eta_1^{\sigma(j)}|[\tilde{t}^L_{\eta_1^{\sigma(j)}},\tilde{t}^U_{\eta_1^{\sigma(j)}}]\le\eta_1^{(j+1)}|[\tilde{t}^L_{\eta_1^{\sigma(j+1)}},\tilde{t}^U_{\eta_1^{\sigma(j+1)}}]$, $\gamma_2^{\sigma(i)}|[\tilde{p}^L_{\gamma_2^{\sigma(i)}},\tilde{p}^U_{\gamma_2^{\sigma(i)}}]\le\gamma_2^{\sigma(i+1)}|[\tilde{p}^L_{\gamma_2^{\sigma(i+1)}},\tilde{p}^U_{\gamma_2^{\sigma(i+1)}}]$, and $\eta_2^{\sigma(j)}|[\tilde{t}^L_{\eta_2^{\sigma(j)}},\tilde{t}^U_{\eta_2^{\sigma(j)}}]\le\eta_2^{\sigma(j+1)}|[\tilde{t}^L_{\eta_2^{\sigma(j+1)}},\tilde{t}^U_{\eta_2^{\sigma(j+1)}}]$. In addition, $\#h$ and $\#g$ represent the numbers of IVPMEs and IVPNMEs of $\tilde{d}_1$ and $\tilde{d}_2$, respectively.

Remark 2. In Definition 13, it is noted that when computing the distance between two q-RIVPDHFEs, they should have the same numbers of IVPMEs and IVPNMEs. However, this requirement cannot be always satisfied in most practical decision-making situations. Hence, we propose the following method to extend the shorter q-RIVPDHFEs. Let $\tilde{d}_1=(h_1|\tilde{p}_{h_1},g_1|\tilde{t}_{g_1})$ and $\tilde{d}_2=(h_2|\tilde{p}_{h_2},g_2|\tilde{t}_{g_2})$ be any two normalized q-RIVPDHFEs, which can be denoted as:

$$\begin{array}{l}\widetilde{d}_1 = \left\{\begin{array}{l}\left\{\gamma_1^{\sigma(1)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(1)}}, \widetilde{p}^U_{\gamma_1^{\sigma(1)}}\right], \gamma_1^{\sigma(2)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(2)}}, \widetilde{p}^U_{\gamma_1^{\sigma(2)}}\right], \ldots, \gamma_1^{\sigma(\#h_1)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(\#h_1)}}, \widetilde{p}^U_{\gamma_1^{\sigma(\#h_1)}}\right]\right\}, \\ \left\{\eta_1^{\sigma(1)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(1)}}, \widetilde{t}^U_{\eta_1^{\sigma(1)}}\right], \eta_1^{\sigma(2)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(2)}}, \widetilde{t}^U_{\eta_1^{\sigma(2)}}\right], \ldots, \eta_1^{\sigma(\#g_1)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(\#g_1)}}, \widetilde{t}^U_{\eta_1^{\sigma(\#g_1)}}\right]\right\}\end{array}\right\}, \\ \widetilde{d}_2 = \left\{\begin{array}{l}\left\{\gamma_2^{\sigma(1)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(1)}}, \widetilde{p}^U_{\gamma_2^{\sigma(1)}}\right], \gamma_2^{\sigma(2)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(2)}}, \widetilde{p}^U_{\gamma_2^{\sigma(2)}}\right], \ldots, \gamma_2^{\sigma(\#h_2)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(\#h_2)}}, \widetilde{p}^U_{\gamma_2^{\sigma(\#h_2)}}\right]\right\}, \\ \left\{\eta_2^{\sigma(1)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(1)}}, \widetilde{t}^U_{\eta_2^{\sigma(1)}}\right], \eta_2^{\sigma(2)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(2)}}, \widetilde{t}^U_{\eta_2^{\sigma(2)}}\right], \ldots, \eta_2^{\sigma(\#g_2)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(\#g_2)}}, \widetilde{t}^U_{\eta_2^{\sigma(\#g_2)}}\right]\right\}\end{array}\right\}.\end{array} \tag{32}$$

Without loss of generality, we assume $\#h_1 > \#h_2$ and $\#g_1 < \#g_2$, then we can generalize $\widetilde{d}_1$ and $\widetilde{d}_2$ into

$$\begin{array}{l}\widetilde{d}_1' = \left(h_1'|\widetilde{p}_{h_1'}, g_1'|\widetilde{t}_{g_1'}\right) = \left\{\left\{\gamma_1^{\sigma(1)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(1)}}, \widetilde{p}^U_{\gamma_1^{\sigma(1)}}\right], \gamma_1^{\sigma(2)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(2)}}, \widetilde{p}^U_{\gamma_1^{\sigma(2)}}\right], \ldots, \gamma_1^{\sigma(\#h_1)}|\left[\widetilde{p}^L_{\gamma_1^{\sigma(\#h_1)}}, \widetilde{p}^U_{\gamma_1^{\sigma(\#h_1)}}\right]\right\},\right. \\ \left.\left\{\eta_1^{\sigma(1)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(1)}}, \widetilde{t}^U_{\eta_1^{\sigma(1)}}\right], \eta_1^{\sigma(2)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(2)}}, \widetilde{t}^U_{\eta_1^{\sigma(2)}}\right], \ldots, \eta_1^{\sigma(\#g_1)}|\left[\widetilde{t}^L_{\eta_1^{\sigma(\#g_1)}}, \widetilde{t}^U_{\eta_1^{\sigma(\#g_1)}}\right], \eta_1^{\sigma(\#g_1)}|0, \ldots, \eta_1^{\sigma(\#g_1)}|0\right\}\right\}, \\ \widetilde{d}_2' = \left(h_2'|\widetilde{p}_{h_2'}, g_2'|\widetilde{t}_{g_2'}\right) = \left\{\left\{\gamma_2^{\sigma(1)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(1)}}, \widetilde{p}^U_{\gamma_2^{\sigma(1)}}\right], \gamma_2^{\sigma(2)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(2)}}, \widetilde{p}^U_{\gamma_2^{\sigma(2)}}\right], \ldots, \gamma_2^{\sigma(\#h_2)}|\left[\widetilde{p}^L_{\gamma_2^{\sigma(\#h_2)}}, \widetilde{p}^U_{\gamma_2^{\sigma(\#h_2)}}\right], \gamma_2^{\sigma(\#h_2)}|0, \ldots, \gamma_2^{\sigma(\#h_2)}|0\right\},\right. \\ \left.\left\{\eta_2^{\sigma(1)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(1)}}, \widetilde{t}^U_{\eta_2^{\sigma(1)}}\right], \eta_2^{\sigma(2)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(2)}}, \widetilde{t}^U_{\eta_2^{\sigma(2)}}\right], \ldots, \eta_2^{\sigma(\#g_2)}|\left[\widetilde{t}^L_{\eta_2^{\sigma(\#g_2)}}, \widetilde{t}^U_{\eta_2^{\sigma(\#g_2)}}\right]\right\}\right\},\end{array} \tag{33}$$

where $\#h_2' = \#h_1' = \#h_1$ and $\#g_1' = \#g_2' = \#g_2$.

4. Two MAGDM Methods under q-RIVPDHFSs Condition

In this subsection, we propose two methods, namely, Algorithms 1 and 2 to deal with MAGDM problems under q-RIVPDHFSs. To this end, we first introduce the basic structure of a typical MAGDM problem in which attributes' evaluation values are in the form of q-RIVPDHFEs. Subsequently, we introduce an aggregation operator-based method and a TOPSIS-based method to handle the MAGDM problem. The main steps of these two approaches are provided in detail.

4.1. Structure of a Typical MAGDM Problem with q-RIVPDHFSs. A typical MAGDM problem under q-rung interval-valued probabilistic dual hesitant fuzzy situation can be described as follow: we assume that there are m alternatives, which are denoted as $X = \{x_1, x_2, \ldots, x_m\}$. A set of decision makers are invited to evaluate the capability of the m alternatives. In order to evaluate the alternatives comprehensively, decision makers provide their evaluated values under n attributes, which can be denoted as $A = \{a_1, a_2, \ldots, a_n\}$. As different attributes probably have different importance, we assume the weight vector of the n attributes to be $w = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{j=1}^n w_j = 1$ and $0 \le w_j \le 1$. Based on the proposed q-RIVPDHFSs, decision makers utilize a q-RIVPDHFE $\widetilde{d}_{ij} = (h_{ij}|\widetilde{p}_{h_{ij}}, g_{ij}|\widetilde{t}_{g_{ij}})$ to denote their evaluation information over attribute a_j $(j = 1, 2 \ldots, n)$ of alternative x_i $(i = 1, 2, \ldots, m)$. At last, a q-rung interval-valued probabilistic dual hesitant fuzzy decision matrix $\widetilde{D}$ can be obtained, which is shown as follows:

$$\widetilde{D} = \left(\widetilde{d}_{ij}\right)_{m\times n} = \begin{bmatrix} \left(h_{11}|\widetilde{p}_{h_{11}}, g_{11}|\widetilde{t}_{g_{11}}\right) & \left(h_{12}|\widetilde{p}_{h_{12}}, g_{12}|\widetilde{t}_{g_{12}}\right) & \cdots & \left(h_{1n}|\widetilde{p}_{h_{1n}}, g_{1n}|\widetilde{t}_{g_{1n}}\right) \\ \left(h_{21}|\widetilde{p}_{h_{21}}, g_{21}|\widetilde{t}_{g_{21}}\right) & \left(h_{22}|\widetilde{p}_{h_{22}}, g_{22}|\widetilde{t}_{g_{22}}\right) & \cdots & \left(h_{2n}|\widetilde{p}_{h_{2n}}, g_{2n}|\widetilde{t}_{g_{2n}}\right) \\ \cdots & \cdots & \ddots & \cdots \\ \left(h_{m1}|\widetilde{p}_{h_{m1}}, g_{m1}|\widetilde{t}_{g_{m1}}\right) & \left(h_{m2}|\widetilde{p}_{h_{m2}}, g_{m2}|\widetilde{t}_{g_{m2}}\right) & \cdots & \left(h_{mn}|\widetilde{p}_{mn}, g_{mn}|\widetilde{t}_{g_{mn}}\right) \end{bmatrix}. \tag{34}$$

5. Illustrative Examples

Medical tourism is internationally recognized as a low-carbon and energy-saving sunrise industry. The volume of world medical tourism increased to about 10 USD billion in 2012 and is expected to increase to 33 USD billion by 2020 [49]. According to the definition by the World Tourism Organization (UNWTO), medical tourism combines well-being tourism with medical recuperation and rehabilitation, which refers to a new tourism service with the theme of medical treatment, nursing, and recovery of health [50]. In many regions, the demand for medical tourism is quite high, especially in developing and less developed countries where many patients have to endure inconveniences of low medical quality, long waiting time for medical facilities, low medical service level, and high medical costs [51]. As a result, such patients may move to treatment areas or countries that can meet their needs. In most cases, traveling abroad means that tourists can benefit from more affordable or better treatment because they prefer medical tourism destinations that provide foreign medical tourists with advanced medical technology, high-quality medical services, and infrastructure. In view of the potential and profitability of the medical tourism business, more and more regions or countries are developing diversified medical service products to actively promote their medical tourism business. Recently, to understand the decision-making process of medical tourists and potential consumers in choosing medical tourism destination brands, Yu et al. [52] investigated the attributes of medical tourism destinations most concerned by medical tourism consumers. The researchers found that the multiple attributes of medical tourism destinations, i.e., professionalism of the medical staff (G_1), the convenience of the information collection process (G_2), and personal information security (G_3), and procedural convenience (G_4), will produce a positive impact on customer participation. Table 1 gives brief explanations of these attributes.

Based on the above analysis, let us consider a medical location selection problem. There is a world-famous medical tourism city. To better promote the local medical tourism market, the place decides to further carry out publicity plans to enhance its influence in the global medical tourism market. Particularly, considering that medical tourism is closely related to the physical and mental health of customers, trust related to potential risks related to medical activities (such as medical accidents or distrust of medical staff and facilities) is very critical. Therefore, relevant departments will evaluate the major local medical institutions serving international medical consumers and select high-quality ones as iconic brands for publicity. After screening, the performance of four medical institutions $\{A_1, A_2, A_3, A_4\}$ on the four attributes $\{G_1, G_2, G_3, G_4\}$ mentioned above will be examined to measure their ability to provide a highly satisfactory tourist experience. Several field experts are invited to form a special group to produce the evaluation of these candidates. Experts agree to express their opinions by q-RIVPDHFSs, and their assessments are summarized in the decision matrix displayed in Table 2. The weight vector of the four attributes is $w = (0.3, 0.2, 0.1, 0.4)^T$. Subsequently, the four medical tourism brands will be evaluated according to the decision matrix, and then the follow-up publicity plan will be formulated.

5.1. The Decision-Making Process by Using Algorithm 1

Step 1. Normalize the original decision matrix. It is noted that all the attributes are benefit type. Hence, the original decision matrix does not need to be normalized.

Step 2. Utilize the q-RIVPDHFWA operator to aggregate the attribute values of the alternative x_i. Then, the comprehensive evaluation values of all candidate alternatives are obtained. As the overall evaluation values are complicated, we omit them in order to save space.

Step 3. Compute the scores of the overall evaluation values according to Definition 7, we can get:

$$S(\tilde{d}_1') = 0.1987, S(\tilde{d}_2') = 0.2753, S(\tilde{d}_3') = 0.1395, S(\tilde{d}_4') = 0.0366. \tag{35}$$

Step 4. Rank alternatives and we can get $x_2 \succ x_1 \succ x_3 \succ x_4$, and A_2 is the optimal alternative.

5.2. The Decision-Making Process by Using Algorithm 2

Step 1. As all attributes are benefit types, the original decision matrix does not need to be normalized.

Step 2. Arrange IVPMEs and IVPNMEs of each q-RIVPDHFE in ascending order according to Remark 1.

Step 3. For attribute a_i, extend the shorter evaluation values of alternatives according to Remark 2 until they have the same numbers of IVPMEs and IVPNMEs. After the above two steps, the new decision matrix is listed in Table 3.

Step 4. Determine the q-RIVPDHFPIS x^+ and q-RIVPDHFNIS x^-, and we can obtain

$$\begin{aligned}
x^+ &= \left\{\begin{array}{c}
\{\{0.7|[0.5,0.1],0.7|[0.3,0.6]\},\{0.6|[0.3,0.5],0.9|[0.2,0.5]\}\},\\
\{\{0.6|[0.2,0.4],0.8|[0.6,0.7]\},\{0.5|[0.7,1],0.7|[0.4,0.6]\}\},\\
\{\{0.6|[0.9,1],0.9|[0.4,0.8]\},\{0.8|[0.1,0.2],0.9|[0.3,0.8]\}\},\\
\{\{0.3|[0.3,1],0.8|[0.3,0.7]\},\{0.4|[0.7,1],0.6|[0.3,0.6]\}\}
\end{array}\right\},\\
x^- &= \left\{\begin{array}{c}
\{\{0.1|[0.1,0.7],0.4|0\},\{0.2|[0.2,0.5],0.5|[0.1,0.4]\}\},\\
\{\{0.4|[0.1,0.3],0.7|[0.1,0.6]\},\{0.4|[0.3,0.6],0.5|0\}\},\\
\{\{0.4|[0.1,0.2],0.6|0\},\{0.4|[0.2,0.3],0.5|[0.1,0.5]\}\},\\
\{\{0.2|[0.1,0.2],0.3|0\},\{0.1|[0.4,0.7],0.4|0\}\}
\end{array}\right\}.
\end{aligned} \tag{36}$$

Table 1: The attributes of medical tourism destinations brand.

Attributes	Explanations
G_1: Professionalism of medical staff (doctors and nurses)	The professional skills and foreign language skills of medical staff are high
G_2: Convenience of the information collection process	The specialties of the medical staff and information about medical facilities can be easily found; information about medical facilities can be provided in foreign languages
G_3: Personal information security	The measures implemented by medical facilities to protect personal information are strict
G_4: Procedural convenience	Making an appointment with medical facilities is convenient and easy

Table 2: The Q-rung interval-valued probabilistic dual hesitant fuzzy decision matrix D.

	a_1
x_1	$\{\{0.7\|[0.3, 0.6], 0.6\|[0.3, 0.4]\}, \{0.2\|[0.2, 0.5], 0.4\|[0.4, 0.5]\}\}$
x_2	$\{\{0.4\|[0.9, 1]\}, \{0.3\|[0.4, 0.5], 0.5\|[0.3, 0.5]\}\}$
x_3	$\{\{0.1\|[0.1, 0.7], 0.4\|[0.2, 0.3]\}, \{0.3\|[0.2, 0.6], 0.5\|[0.1, 0.4]\}\}$
x_4	$\{\{0.7\|[0.5, 1]\}, \{0.9\|[0.2, 0.5], 0.6\|[0.3, 0.5]\}\}$
	a_2
x_1	$\{\{0.6\|[0.2, 0.4], 0.7\|[0.1, 0.6]\}, \{0.5\|[0.7, 1]\}\}$
x_2	$\{\{0.8\|[0.6, 0.7], 0.6\|[0.2, 0.3]\}, \{0.3\|[0.5, 1]\}\}$
x_3	$\{\{0.8\|[0.4, 0.8], 0.6\|[0.1, 0.2]\}, \{0.4\|[0.3, 0.6], 0.6\|[0.2, 0.4]\}\}$
x_4	$\{\{0.4\|[0.1, 0.3], 0.6\|[0.4, 0.7]\}, \{0.7\|[0.4, 0.6], 0.8\|[0.1, 0.4]\}\}$
	a_3
x_1	$\{\{0.9\|[0.4, 0.8], 0.4\|[0.1, 0.2]\}, \{0.8\|[0.1, 0.2], 0.3\|[0.7, 0.8]\}\}$
x_2	$\{\{0.6\|[0.9, 1]\}, \{0.4\|[0.2, 0.3], 0.6\|[0.5, 0.7]\}\}$
x_3	$\{\{0.5\|[0.2, 0.4], 0.9\|[0.1, 0.6]\}, \{0.7\|[0.1, 0.2], 0.9\|[0.3, 0.8]\}\}$
x_4	$\{\{0.3\|[0.8, 1]\}, \{0.5\|[0.1, 0.5], 0.3\|[0.2, 0.5]\}\}$
	a_4
x_1	$\{\{0.2\|[0.5, 0.6], 0.4\|[0.2, 0.4]\}, \{0.4\|[0.7, 1]\}\}$
x_2	$\{\{0.7\|[0.3, 0.8], 0.2\|[0.1, 0.2]\}, \{0.6\|[0.2, 0.3], 0.1\|[0.4, 0.7]\}\}$
x_3	$\{\{0.3\|[0.3, 1]\}, \{0.7\|[0.1, 0.3], 0.1\|[0.6, 0.7]\}\}$
x_4	$\{\{0.8\|[0.3, 0.7], 0.2\|[0.2, 0.3]\}, \{0.7\|[0.2, 0.4], 0.6\|[0.3, 0.6]\}\}$

Table 3: The normalized decision matrix.

	a_1
x_1	$\{\{0.6\|[0.3, 0.4], 0.7\|[0.3, 0.6]\}, \{0.2\|[0.2, 0.5], 0.4\|[0.4, 0.5]\}\}$
x_2	$\{\{0.4\|[0.9, 1], 0.4\|0\}, \{0.3\|[0.4, 0.5], 0.5\|[0.3, 0.5]\}\}$
x_3	$\{\{0.1\|[0.1, 0.7], 0.4\|[0.2, 0.3]\}, \{0.3\|[0.2, 0.6], 0.5\|[0.1, 0.4]\}\}$
x_4	$\{\{0.7\|[0.5, 1], 0.7\|0\}, \{0.6\|[0.3, 0.5], 0.9\|[0.2, 0.5]\}\}$
	a_2
x_1	$\{\{0.6\|[0.2, 0.4], 0.7\|[0.1, 0.6]\}, \{0.5\|[0.7, 1], 0.5\|0\}\}$
x_2	$\{\{0.6\|[0.2, 0.3], 0.8\|[0.6, 0.7]\}, \{0.3\|[0.5, 1], 0.3\|0\}\}$
x_3	$\{\{0.6\|[0.1, 0.2], 0.8\|[0.4, 0.8]\}, \{0.4\|[0.3, 0.6], 0.6\|[0.2, 0.4]\}\}$
x_4	$\{\{0.4\|[0.1, 0.3], 0.6\|[0.4, 0.7]\}, \{0.8\|[0.1, 0.4], 0.7\|[0.4, 0.6]\}\}$
	a_3
x_1	$\{\{0.4\|[0.1, 0.2], 0.9\|[0.4, 0.8]\}, \{0.8\|[0.1, 0.2], 0.3\|[0.7, 0.8]\}\}$
x_2	$\{\{0.6\|[0.9, 1], 0.6\|0\}, \{0.4\|[0.2, 0.3], 0.6\|[0.5, 0.7]\}\}$
x_3	$\{\{0.5\|[0.2, 0.4], 0.9\|[0.1, 0.6]\}, \{0.7\|[0.1, 0.2], 0.9\|[0.3, 0.8]\}\}$
x_4	$\{\{0.3\|[0.8, 1], 0.3\|0\}, \{0.3\|[0.2, 0.5], 0.5\|[0.1, 0.5]\}\}$
	a_4
x_1	$\{\{0.2\|[0.5, 0.6], 0.4\|[0.2, 0.4]\}, \{0.4\|[0.7, 1], 0.4\|0\}\}$
x_2	$\{\{0.2\|[0.1, 0.2], 0.7\|[0.3, 0.8]\}, \{0.1\|[0.4, 0.7], 0.6\|[0.2, 0.3]\}\}$
x_3	$\{\{0.3\|[0.3, 1], 0.3\|0\}, \{0.1\|[0.6, 0.7], 0.7\|[0.1, 0.3]\}\}$
x_4	$\{\{0.2\|[0.2, 0.3], 0.8\|[0.3, 0.7]\}, \{0.7\|[0.2, 0.4], 0.6\|[0.3, 0.6]\}\}$

Table 4: Distance of each alternative between q-RIVPDHFPIS x^+ and q-RIVPDHFNIS x^-.

	$d(x_i, x^+)$	$d(x_i, x^-)$	$CI_i\,(i = 1, 2, 3, 4)$
x_1	0.2104	0.0888	0.2967
x_2	0.1960	0.1024	0.3433
x_3	0.2149	0.0819	0.2759
x_4	0.1172	0.2042	0.6353

Table 5: The original decision matrix in Example 4.

	G_1	G_2	G_3
x_1	$\{\{0.7\|0.2, 0.6\|0.2, 0.5\|0.6\}, \{0.2\|1\}\}$	$\{\{0.7\|1\}, \{0.5\|1\}\}$	$\{\{0.2\|1\}, \{0.2\|1\}\}$
x_2	$\{\{0.1\|1\}, \{0.4\|1\}\}$	$\{\{0.3\|1\}, \{0.7\|1\}\}$	$\{\{0.7\|1\}, \{0.3\|0.5, 0.2\|0.5\}\}$
x_3	$\{\{0.6\|1\}, \{0.5\|1\}\}$	$\{\{0.6\|1\}, \{0.2\|1\}\}$	$\{\{0.1\|1\}, \{0.7\|1\}\}$
x_4	$\{\{0.05\|0.7, 0.2\|0.3\}, \{0.5\|1\}\}$	$\{\{0.3\|1\}, \{0.6\|0.5, 0.5\|0.5\}\}$	$\{\{0.8\|1\}, \{0.5\|1\}\}$

Table 6: Decision-making results of Example 4 by employing different MAGDM methods.

	Score values $S(d_i)\,(i = 1, 2, 3, 4)$	The final ranking results
Li et al.'s [27] method ($q = 3$ and $L = (1, 1, 1)$)	$S(d_1) = -0.0373, S(d_2) = -0.2261$ $S(d_3) = -0.1033$, $S(d_4) = -0.1563$	$x_1 \succ x_3 \succ x_4 \succ x_2$
Algorithm 1 ($q = 3$)	$S(d_1) = 0.2531$, $S(d_2) = 0.2984$ $S(d_3) = -0.0090$, $S(d_4) = 0.3471$	$x_4 \succ x_2 \succ x_1 \succ x_3$
Algorithm 2 ($q = 3$)	$S(d_1) = 0.1941$, $S(d_2) = 0.3729$ $S(d_3) = 0.4085$, $S(d_4) = 0.5434$	$x_4 \succ x_3 \succ x_2 \succ x_1$

Step 5. Calculate the weighted distance between each alternative and q-RIVPDHFPIS x^+ and q-RIVPDHFNIS x^-. Additionally, based on the distance between each alternative and the q-RIVPDHFPIS x^+ and q-RIVPDHFNIS x^-, the relative importance degree of each alternative can be obtained. These results are listed in Table 4.

Step 6. According to $CI_i\,(i = 1, 2, 3, 4)$, we can obtain the ranking result of alternatives, i.e., $x_4 \succ x_2 \succ x_1 \succ x_3$. Therefore, the optimal alternative is x_4.

5.3. Comparative Analysis. In this subsection, to better demonstrate the advantages of our proposed MAGDM methods, we conduct comparative analysis. More specifically, we compare our proposed methods with that introduced by Li et al. [27] based on the q-RPDHFSs that presented by Liu and Chen [37] based on interval-valued probabilistic dual hesitant fuzzy sets to illustrate the advantages of our method.

5.3.1. Comparison with Li et al.'s [27] Method. The method proposed by Li et al. [27] is based on q-RPDHFSs, which provide decision makers with large information space to express their evaluation values. In addition, decision makers are allowed to give several set of values for MDs and NMDs, and can assign a probability value to each value. However, in real problems, decision makers are more inclined to use an interval value to express the probability value, rather than a single real value. In Examples 1 and 2, we provide illustrative instances to explain why we need q-RIVPDHFSs. In addition, to better show the advantages and superiorities of our proposed method over that introduced by Li et al. [27], we provide the following examples.

Example 4 **(**Revised from [27]**).** Let us consider an investment projection selection problem. An enterprise now wants to invest its idle money in an investment project to make some profits. After elementary evaluation, four possible investment alternatives are taken into consideration, which is denoted as $\{x_1, x_2, x_3, x_4\}$. To comprehensively evaluate the four alternatives, the following three attributes are taken into account, i.e., the quality of product and service (G_1), social and environmental impacts (G_2), and economic benefits (G_3). The weight vector of attributes is $w = (0.3, 0.2, 0.5)^T$. Decision makers are invited to provide q-RDHFEs to express their evaluation values, and the original decision matrix is listed in Table 5. As pointed out in Definition 5, q-RPDHFE is a special case of q-RIVPDHFE. As a matter of fact, it is easy to convert a q-RPDHFE into a q-RIVPDHFE. For example, let $d = \{\{0.7|0.2, 0.6|0.2, 0.5|0.6\}, \{0.2|1\}\}$ be a q-RPDHFE, then it can be converted into $d = \{\{0.7|[0.2, 0.2], 0.6|[0.2, 0.2], 0.5|[0.6, 0.6]\}, \{0.2|\,[1, 1]\}\}$. Hence, our proposed methods can be applied to solve MAGDM problems where decision makers' evaluation values are expressed by q-RPDHFEs. We use Li et al.'s [27] method and our proposed Algorithms 1 and 2 to solve Example 5 and present the decision-making results in Table 6.

From Table 6, it is found that our proposed Algorithms 1 and 2 can successfully solve Example 4, which indicates that our proposed methods can deal with decision-making problems in q-rung probabilistic dual hesitant fuzzy environment. However, Li et al. [27] is not as flexible as the Algorithms 1 and 2 introduced in this study. To better demonstrate the advantages of our proposed Algorithms 1 and 2, the following example is provided.

Table 7: The original decision matrix in Example 5.

	G_1	G_2	G_3
x_1	$\{\{0.7\|0.2, 0.6\|0.2, 0.5\|0.6\}, \{0.2\|1\}\}$	$\{\{0.7\|[0.31]\}, \{0.5\|1\}\}$	$\{\{0.2\|1\}, \{0.2\|1\}\}$
x_2	$\{\{0.1\|1\}, \{0.4\|1\}\}$	$\{\{0.3\|1\}, \{0.7\|1\}\}$	$\{\{0.7\|1\}, \{0.3\|0.5, 0.2\|0.5\}\}$
x_3	$\{\{0.6\|1\}, \{0.5\|1\}\}$	$\{\{0.6\|1\}, \{0.2\|1\}\}$	$\{\{0.1\|1\}, \{0.7\|1\}\}$
x_4	$\{\{0.05\|[0.50.7], 0.2\|0.3\}, \{0.5\|1\}\}$	$\{\{0.3\|1\}, \{0.6\|0.5, 0.5\|0.5\}\}$	$\{\{0.8\|1\}, \{0.5\|1\}\}$

Table 8: Decision-making results of Example 5 by employing different MAGDM methods.

	Score values $S(d_i)\,(i = 1, 2, 3, 4)$	The final ranking results
Li et al.'s [27] method ($q = 3$ and $L = (1,1,1)$)	Cannot be calculated	None
Algorithm 1 ($q = 3$)	$S(d_1) = 0.1548$, $S(d_2) = 0.2984$ $S(d_3) = -0.0090$, $S(d_4) = 0.2860$	$x_2 \succ x_4 \succ x_1 \succ x_3$
Algorithm 2 ($q = 3$)	$S(d_1) = 0.1648$, $S(d_2) = 0.3871$ $S(d_3) = 0.4241$, $S(d_4) = 0.5641$	$x_4 \succ x_3 \succ x_2 \succ x_1$

Table 9: The original decision matrix of Example 5.

	G_1 (MC)
USA	$\{\{0.4686\|0.6, 0.5527\|0.2, 0.5071\|0.2\}, \{0.3671\|1\}\}$
CAN	$\{\{0.5164\|0.5, 0.4598\|0.5\}, \{0.3173\|1\}\}$
RUS	$\{\{0.4977\|0.54, 0.6015\|0.36, 0.4778\|0.06, 0.5858\|0.4\}, \{0.2590\|1\}\}$
DNK	$\{\{0.1523\|0.7, 0.0878\|0.3\}, \{0.5940\|0.9, 0.6249\|0.1\}\}$
CHN	$\{\{0.2191\|0.7, 0.2583\|0.3\}, \{0.6074\|0.6, 0.6544\|0.4\}\}$
NOR	$\{\{0.2526\|0.48, 0.2064\|0.32, 0.2814\|0.12, 0.2370\|0.08\}, \{0.5950\|1\}\}$
	G_2 (DD)
USA	$\{\{0.7131\|1\}, \{0.2148\|0.8, 0.2454\|0.2\}\}$
CAN	$\{\{0.2779\|1\}, \{0.6320\|0.6, 0.6947\|0.4\}\}$
RUS	$\{\{0.4651\|0.5, 0.4411\|0.5\}, \{0.2285\|0.5, 0.2\|0.5\}\}$
DNK	$\{\{0.3206\|0.3, 0.2893\|0.3, 0.4271\|0.2, 0.4008\|0.2\}, \{0.3469\|0.5, 0.3262\|0.5\}\}$
CHN	$\{\{0.5211\|1\}, \{0.3682\|0.5, 0.2921\|0.5\}\}$
NOR	$\{\{0.2427\|0.36, 0.3042\|0.24, 0.2033\|0.24, 0.2681\|0.16\}, \{0.7319\|1\}\}$
	G_3 (EI)
USA	$\{\{0.3039\|0.6, 0.4457\|0.4\}, \{0.3298\|0.42, 0.3103\|0.28, 0.2887\|0.18, 0.2716\|0.12\}\}$
CAN	$\left\{\begin{array}{l}\{0.4772\|0.56, 0.5251\|0.24, 0.4500\|0.14, 0.5004\|0.06\},\\ \{0.2880\|0.35, 0.2512\|0.35, 0.3099\|0.15, 0.2703\|0.15\}\end{array}\right\}$
RUS	$\{\{0.1342\|1\}, \{0.7319\|0.6, 0.6553\|0.4\}\}$
DNK	$\{\{0.5716\|0.7, 0.5207\|0.3\}, \{0.3537\|0.5, 0.2604\|0.5\}\}$
CHN	$\{\{0.7710\|0.54, 0.7107\|0.36, 0.7124\|0.06, 0.6367\|0.04\}, \{0.1491\|1\}\}$
NOR	$\{\{0.2681\|0.8, 0.3\|0.2\}, \{0.6164\|1\}\}$
	G_4 (MR)
USA	$\{\{0.6370\|0.35, 0.6\|0.35, 0.6697\|0.15, 0.6361\|0.15\}, \{0.2826\|1\}\}$
CAN	$\{\{0.3\|0.7, 0.2686\|0.3\}, \{0.4749\|1\}\}$
RUS	$\{\{0.3145\|0.42, 0.3779\|0.28, 0.3773\|0.18, 0.4349\|0.12\}, \{0.4580\|1\}\}$
DNK	$\{\{0.1679\|0.48, 0.2349\|0.32, 0.1351\|0.12, 0.2047\|0.08\}, \{0.4182\|0.6, 0.4778\|0.4\}\}$
CHN	$\{\{0.2438\|0.55, 0.1776\|0.45\}, \{0.4524\|0.45, 0.4145\|0.45, 0.4295\|0.05, 0.3935\|0.05\}\}$
NOR	$\{\{0.3630\|1\}, \{0.5233\|0.35, 0.4924\|0.35, 0.4142\|0.15, 0.3897\|0.15\}\}$

Example 5 (Continued to Example 4). In some real decision-making problems, decision makers prefer to use q-RIVPDHFEs rather than q-RPDHFEs to express their evaluation values. Hence, we make revisions in Example 4. We change the original decision matrix listed in Table 5. We assume the attribute value of G_2 of an alternative x_1 is changed from $\{\{0.7|1\}, \{0.5|1\}\}$ into $\{\{0.7|[0.31]\}, \{0.5|1\}\}$. In addition, the value of an attribute of G_1 of an alternative x_1 is changed from $\{\{0.05|0.7, 0.2|0.3\}, \{0.5|1\}\}$ into $\{\{0.05|[0.50.7], 0.2|0.3\}, \{0.5|1\}\}$. The other attribute values keep unchanged. The new decision matrix is listed in Table 7. We use Li et al.'s [27] method and our proposed Algorithms 1 and 2 to solve Example 4 and the results are presented in Table 8

As we can see from Table 8, our proposed Algorithms 1 and 2 can successfully solve Example 5, however, Li et al.'s [27] method is powerless to handle such a problem. This is because Li et al.'s [27] method is based on q-RPDHFS and hence it is incapable to handle MAGDM problems wherein probabilistic information is denoted by interval values.

Table 10: The score values and ranking results of Example 5 by utilizing different methods.

Method	USA	CAN	RUS	DNK	CHN	NOR	Ranking
Liu and Cheng's [37] method based on the three-phased MCGDM framework with IVPDHFSs	1	0.7305	0.6614	0.6439	0.8371	0	USA ≻ CHN ≻ CAN, ≻ RUS ≻ DNK ≻ NOR
Our method based on Algorithm 1 ($q = 1$)	0.6076	−0.1287	0.4586	−0.4420	−0.1879	−0.4997	USA ≻ RUS ≻ CAN, ≻ CHN ≻ DNK ≻ NOR
Our method based on Algorithm 2 ($q = 1$)	0.3842	0.3496	0.3684	0.2936	0.3300	0.3767	USA ≻ NOR ≻ RUS, ≻ CAN ≻ CHN ≻ DNK

Table 11: The decision results of revised Example 5 by utilizing different methods.

Method	USA	CAN	RUS	DNK	CHN	NOR	Ranking
Liu and Cheng's [37] method based on the three-phased MCGDM framework with IVPDHFSs	Cannot be calculated						None
Our method based on Algorithm 1 ($q = 3$)	0.3986	0.0844	0.2818	−0.1416	−0.0221	−0.2824	USA≻RUS≻CAN ≻CHN≻DNK≻NOR
Our method based on Algorithm 2 ($q = 3$)	0.3184	0.2909	0.2840	0.1779	0.2310	0.2975	USA≻NOR≻CAN ≻RUS≻CHN≻DNK

Hence, the decision-making approaches presented in this study are more powerful and flexible than that introduced by Li et al. [27].

5.3.2. Comparison with Liu and Cheng's [37] Method. We continue to compare our proposed method with that introduced by Liu and Cheng [37] based on IVPDHFSs. The IVPDHFS is constructed by a set of MDs and NMDs as well as their probabilistic information, which is similar to q-RIVPDHFS. However, the constraint of q-RIVPDHFS is laxer than that of IVPDHFS. In other words, q-RIVPDHFS is more powerful and flexible than IVPDHFS. In addition, in order to demonstrate the advantages of our proposed decision-making method over that introduced d by Liu and Cheng [37], the following example is provided.

Example 6 (Revised from [37]). The geopolitics of the Arctic is becoming increasingly prominent with global warming. In order to achieve peaceful and sustainable development, the complex political and geographical environment urges us to assess the geopolitical risks in the Arctic. Hao et al. have studied geopolitical risks in the Arctic. The study established a committee of experts to assess the risks of arctic resource development operations in six countries: Russia (RUS), Canada (CAN), the United States (USA), Denmark (DNK), Norway (NOR), and China (CHN) and in four dimensions: G_1 potential military conflicts (MCs), G_2 diplomatic disputes (DDs), G_3 dependence on energy imports (EIs), and G_4 control over marine routes (MRs). The weight vector of attributes is $w = (0.2895, 0.1711, 0.0658, 0.4737)^T$. Decision makers used probabilistic dual hesitant fuzzy sets (PDHFS) to express their evaluation values and the original decision matrix is listed in Table 9. We use our proposed methods and Liu and Cheng's [37] method to solve Example 5 and present the decision results in Table 10.

It is noted that the evaluation values in Example 5 is based on PDHFS, which is a special case of IVPDHFS proposed by Liu and Cheng [37]. Hence, both our proposed methods and Liu and Cheng's [37] method can solve this example. As can be seen from Table 6, the ranking results of the method proposed by Liu and Cheng [37] are basically consistent with the ranking results of Algorithm 1 in this paper, in which USA has the highest risk of arctic investment, and NOR has the lowest risk. The ranking results of Algorithm 2 in this paper are somewhat different from the first two methods. Although USA still has the highest risk of arctic investment, DNK has the lowest risk, and NOR has the second highest risk. In summary, it can be seen from the ranking results that our method has the same effect as the method of Liu and Cheng [37]. However, our proposed methods are more powerful and flexible than that introduced by Liu and Cheng [37]. This is because Liu and Cheng's [37] method is based on IVPDHFS, which is a special case of our q-RIVPDHFS. In other words, our proposed methods have wider application than Liu and Cheng's [37] method. For example, in Example 5, the attribute value G_4 of CAN is changed from $\{\{0.3|0.7, 0.2686|0.3\}, \{0.4749|1\}\}$ to $\{\{0.6|0.7, 0.2686|0.3\}, \{0.4749|1\}\}$. We use Liu and Cheng's method [37] and our proposed methods to solve the revised example and the decision results are presented in Table 11. As seen from Table 11, Liu and Cheng's [37] method fails to handle the revised example because the revised evaluation value cannot be handled by IVPDHFS. Therefore, our proposed methods are more powerful than that developed by Liu and Cheng [37].

6. Conclusion

The main contribution of this paper is to propose two new MAGDM methods to deal with decision-making problems under complicated and uncertain environments. In order to do this, we first introduce a new concept, called q-RIVPDHFSs, to represent decision makers' complex and uncertain evaluation information. Compared with q-RPDHFSs, the newly developed q-RIVPDHFSs allow interval-valued probabilistic information, which makes them more powerful and useful to handle decision makers' fuzzy cognitive information in MAGDM procedures. Afterward, some other related notions, such as operational rules, comparison methods, distance measures, and aggregation operators, have been proposed and discussed. Based on these new concepts, we introduced two methods to resolve fuzzy MAGDM problems. The methods include the one based on aggregation operator and the one based on TOPSIS method. Last but not least, we conducted numerical experiments to demonstrate the validity and advantages of our proposed methods.

In future works, we plan to continue our studies from two aspects. First, we shall investigate more applications of our methods in more down-to-earth MAGDM problems, such as supplier selection [53, 54], investment projection selection [55, 56], best airline selection [57, 58], low-carbon supplier selection [59, 60], and healthcare waste treatment technology selection [61, 62]. Second, we shall study more aggregation operators for fusing q-rung interval-valued probabilistic dual hesitant fuzzy information. In the present article, we have only introduced the simple weighted average and geometric operators for q-RIVPDHFEs. In future works, we will propose new operators for q-RIVPDHFEs by extending the power Bonferroni mean [63], power Heronian mean [64], power Maclaurin symmetric mean [65], power Muirhead mean [66], etc., into q-RIVPDHFSs. Finally, we did not consider decision makers' consensus in this study. As a matter of fact, decision makers' consensus is very important in decision-making process. A high consensus degree can guarantee the quality of the final decision results. Therefore, consensus-based decision-making methods have become a promising research topic [67–70]. In the future, we will consider novel MAGDM methods based on consensus under q-RIVPDHFSs.

Data Availability

All the data are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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