Nonlinear Dynamics of a Quantum Cournot Duopoly with Bounded Rationality and Relative Profit Maximization

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1. Introduction

Nonlinear economic models contain many complex phenomena. If there is confusion in an economic model, especially in a duopoly game, it can cause chaos and unpredictability in the market, such as [1–3]. Delaying or avoiding bifurcation and chaos is essential. Many scholars have attempted to describe games in the quantum domain, in particular the quantum scheme proposed by Li et al. [4, 5]. Some new and exciting dynamical results have been discovered in this research on quantum games, which are different from the classical game models [6–10].

In recent years, classical dynamic Cournot games have continued to be of interest internationally. Elsadany [11] considered players seeking to maximise their relative profits rather than their own absolute profits and proposed the classical Cournot duopoly game based on bounded rationality and relative profit maximization. Subsequently, Pecora and Sodini [12] studied the Cournot duopoly game in a continuous time frame and discovered the dynamic behaviour of competitors when deciding on output decisions and compared how different time delays affect the stability of the economy. Cerboni Baiardi and Naimzada [13] examined competition between quantity-setting players in a nonlinear deterministic duopoly environment distinguished by an isoelectric demand curve. They observed a double instability of the Cournot-Nash equilibrium as a result of both the number of players and the percentage of imitators. Among participants, Andaluz et al. [14] presented a nonlinear Cournot duopoly game showing isoelectric demand in general. Lian and Zheng [15] discussed the dynamic interactions of players in Cournot markets. The market outcome of the stage game is presented, showing transfer probabilities and finding the steady state of the system.

However, in the classical dynamic Gourod duopoly game model described above, there is the dilemma that Nash equilibrium is inferior to Pareto optimal. Quantum game theory may provide an effective solution to this dilemma [4, 5]. Moreover, even if both players behave “selfishly” in a quantum game, as they do in a classical game, they actually cooperate due to the quantum entanglement between them. It is not entirely rational for players to play a quantum game. They can determine the stability and dynamic behavior of the game model based on quantum properties (e.g., quantum entanglement). Several new questions will arise as a result. It is not entirely rational for players to play...
a quantum game. They can determine the stability and dynamic behavior of the game model based on quantum properties (e.g., quantum entanglement). Several new questions will arise as a result. This has been studied by many scholars and different conclusions have been obtained. Based on heterogeneous players, Shi et al. [16, 17] proposed a dynamic quantum Cournot duopoly game. It is shown that the stability region increases with quantum entanglement. Zhang et al. [18] found that the stability region is influenced by the difference in cost coefficients between the quadratic and linear cost models and quantum entanglement. Lo and Yeung [19, 20] proposed that positive quantum entanglement makes the interval of equilibrium widen and bifurcation and chaos are delayed rather than advanced. Therefore, slight changes in the game can have an opposite effect on the outcome. This paper provides a comparable solution for both players.

The paper follows the following structure. In Section 2, a quantum Cournot duopoly game model based on relative profit maximization and bounded rationality is proposed. In Section 3, quantum entanglement is discussed theoretically with equilibrium stability and nonlinear dynamics. In Section 4, the numerical simulations fully demonstrate the model’s nonlinear dynamics. Section 5 contains the conclusion.

2. The Model

Elsadany [11] proposes a game in which two players produce (cross-sectional) differentiated products of varieties 1 and 2 and concludes that the absolute profit of each player is, respectively,

\[ \phi_1(q_1, q_2) = q_1(a - q_1 - b_1q_2) - c_1q_1 - d_1q_1q_2, \]
\[ \phi_2(q_1, q_2) = q_2(a - q_2 - b_2q_1) - c_2d_2 - d_2q_1q_2, \]  

(1)

where \( b_1, c_1, d_1 \) are used to measure the product level difference, marginal cost, and adjustment rate of player \( i \) (\( i = 1, 2 \)), respectively.

The relative profit of a player is defined by Elsadany as the difference between the absolute profit of the player and the average of the absolute profits of the other players. Players 1 and 2 each have a relative profit of \( \Phi_1 \) and \( \Phi_2 \) denote their relative profits. The following equation gives the relative profits of the two players:

\[ \Phi_1(q_1, q_2) = \phi_1(q_1, q_2) - \phi_2(q_1, q_2) = q_1(a - q_1 - (b + d_1)q_2) - q_2(a - q_2 - (b + d_2)q_1) - c_1q_1 + c_2q_2, \]
\[ \Phi_2(q_1, q_2) = \phi_2(q_1, q_2) - \phi_1(q_1, q_2) = q_2(a - q_2 - (b + d_2)q_1) - q_1(a - q_1 - (b + d_1)q_2) - c_2q_2 + c_1q_1. \]  

(2)

Based on the Li-Du-Massar quantum scheme [4], the Cournot duopoly game described above can be recast as a quantum version with bound rationality and relative profit maximization. Let \( \hat{Q}_j = i(\hat{b}_{j}^\dagger - \hat{b}_j)/\sqrt{2} \) be “momentum” operator of \( j \)’s electromagnetic field. Set the final measurement be corresponding to the observables \( \hat{Y}_j = (\hat{b}_{j}^\dagger + \hat{b}_j)/\sqrt{2} \) (the “position” operators) of player \( j \), where \( \hat{b}_j^\dagger (\hat{b}_j) \) is the creation (annihilation) operator of player \( j \)’s electromagnetic field. The following steps are available.

(a) The game starts from \( |00\rangle \). The entangling operator \( F(y) \) is given by

\[ \hat{F}(y) = \exp \left\{ -y \left( \hat{b}_1^\dagger \hat{b}_2^\dagger - \hat{b}_1 \hat{b}_2 \right) \right\} \]
\[ = \exp \left\{ iy(\hat{Y}_1 \hat{Q}_2 + \hat{Y}_2 \hat{P}_1) \right\}. \]  

(3)

Using the squeeze parameter \( y \geq 0 \), we can measure the degree of entanglement. Thus, the initial state is
\[ |\psi_0\rangle = \exp \left\{-y\left(\tilde{b}_1^\dagger \tilde{b}_2 - \tilde{b}_1 \tilde{b}_2^\dagger\right)\right\}|00\rangle. \quad (4) \]

(2) Through unitary operations, the two players execute their strategic moves

\[ \tilde{D}_1(y_1) = \exp \left\{y_1\left(\tilde{b}_1^\dagger - \tilde{b}_1\right)\right\}, \quad (5) \]

and

\[ \tilde{D}_2(y_2) = \exp \left\{y_2\left(\tilde{b}_2^\dagger - \tilde{b}_2\right)\right\}. \quad (6) \]

(3) The two players’ states are measured after a disentanglement operation \( \tilde{Q}(y)^\dagger \). The final state is carried out by \( |\psi_f\rangle = \tilde{F}(\gamma)^\dagger \tilde{D}_1(y_1) \otimes \tilde{D}_2(y_2) \tilde{F}(\gamma)|00\rangle \). Detailed calculation gives

\[ \tilde{F}(\gamma)^\dagger \tilde{D}_1(y_1) \tilde{F}(\gamma) = \exp \left\{-iy_1(\tilde{Q}_1\cosh \gamma + \tilde{Q}_2\sinh \gamma)\right\}, \]

\[ \tilde{F}(\gamma < \text{list aend} >)^\dagger \tilde{D}_2(y_2) \tilde{F}(\gamma) = \exp \left\{-iy_2(\tilde{Q}_2\cosh \gamma + \tilde{Q}_1\sinh \gamma)\right\}. \quad (7) \]

Therefore,

\[ \tilde{F}(\gamma)^\dagger [\tilde{D}_1(y_1) \otimes \tilde{D}_2(y_2)] \tilde{F}(\gamma)|00\rangle \]

\[ = \exp \left\{-i(y_1\cosh \gamma + y_2\sinh \gamma)\tilde{Q}_1\right\}|01\rangle, \]

\[ \times \exp \left\{-i(y_2\cosh \gamma + y_1\sinh \gamma)\tilde{Q}_2\right\}|02\rangle. \quad (8) \]

We obtain the final state

\[ |\psi_f\rangle = \exp \left\{-i(y_1\cosh \gamma + x_2\sinh \gamma)\tilde{Q}_1\right\}|01\rangle, \]

\[ \otimes \exp \left\{-i(y_2\cosh \gamma + y_1\sinh \gamma)\tilde{Q}_2\right\}|02\rangle. \quad (9) \]

The final measurement of quantum entanglement, i.e., \( y \neq 0 \), derives the yield of each of the two players:

\[ \frac{\partial \phi^Q_1(y_1, y_2)}{\partial y_1} = (a - c_1)\cosh \gamma + (b_2 - b_1)y_2\cosh 2\gamma + (-a + c_2)\sinh \gamma \]

\[ + y_1(-2 + (b_2 - b_1)\sinh 2\gamma), \]

\[ \frac{\partial \phi^Q_2(y_1, y_2)}{\partial y_2} = (a - c_2)\cosh \gamma + (b_1 - b_2)y_1\cosh 2\gamma + (-a + c_1)\sinh \gamma \]

\[ + y_2(-2 + (b_1 - b_2)\sinh 2\gamma), \quad (11) \]

where \( b_1 = b + d_1 \) and \( b_2 = b + d_2 \).

From the above analysis, we obtain a novel discrete dynamical system with quantum entanglement after considering quantum properties (for convenience, let \( y_1 = x_n \), \( y_2 = y_n \)).

\[ x_{n+1} = x_n + \alpha_1 x_n \left(\begin{array}{c}
-2x_n + (b_2 - b_1)y_n \cosh 2\gamma + (c_2 - a)\sinh 2\gamma \\
+ \cosh \gamma (a - c_1 + 2(b_2 - b_1)x_n \sinh \gamma)
\end{array}\right), \quad (12) \]

\[ y_{n+1} = y_n + \alpha_2 y_n \left(\begin{array}{c}
-2y_n + (b_1 - b_2)x_n \cosh 2\gamma + (c_1 - a)\sinh 2\gamma \\
+ \cosh \gamma (a - c_2 + 2(b_1 - b_2)y_n \sinh \gamma)
\end{array}\right). \]
3. Quantum Equilibrium Points and Local Dynamics

This section presents a theoretical study of the complex dynamic behavior of model (12). It is easy to show that (12) has the following quantum equilibrium points:

\[ E_0 = (0,0), E_1 = \left( 0, \frac{(c_2 - a) \cosh y + (a - c_1) \sinh y}{-2 + (b_1 - b_2) \sinh 2y} \right), \]

\[ E_2 = \left( \frac{\sinh 2y((c_2 - a) \cosh y + (a - c_1) \sinh y)}{b_1 - b_2}, 0 \right), \]

\[ E^* = (x^*, y^*), \]

where

\[ x^* = \frac{(A_{11} - A_{12}) \cosh y + (A_{22} - A_{21}) \sinh y}{4 + (b_1 - b_2)^2}, \]

\[ y^* = \frac{(A_{12} - A_{11}) \cosh y + (A_{21} - A_{22}) \sinh y}{4 + (b_2 - b_1)^2}, \]

\[ A_{11} = a(2 - b_1 + b_2), A_{12} = 2c_1 - (b_1 - b_2)c_2, \]

\[ A_{21} = a(2 - b_2 + b_1), A_{22} = 2c_2 - (b_2 - b_1)c_1. \]  

Equilibrium points \( E_0, E_1, E_2 \) are called quantum boundary equilibrium points, and the equilibrium point \( E^* \) is called the quantum Cournot-Nash equilibrium point. Since the output of the yield must be positive, the following discussion is based on the condition that the quantum Cournot-Nash equilibrium point is positive.

Since the boundary equilibrium point is almost impossible to occur in a practical situation, we will study the dynamical behaviour of \( E^* \). Let \( A \) be the Jacobian matrix of (12) at \( E^* \)

\[ A = \begin{pmatrix}
1 - \alpha_1 x^* (2 + (b_1 - b_2) \sinh 2y) & \alpha_1 (b_2 - b_1) x^* \cosh 2y \\
\alpha_2 (b_1 - b_2) y^* \cosh 2y & 1 - \alpha_2 y^* (2 + (b_2 - b_1) \sinh 2y)
\end{pmatrix}. \]  

The characteristic equation of the Jacobian matrix \( A \) at \( E^* \) is \( \lambda^2 - P\lambda + Q = 0 \), where

\[ P = -2(-1 + \alpha_1 x^* + \alpha_2 y^*) - (b_1 - b_2) (\alpha_1 x^* + \alpha_2 y^*) \sinh 2y, \]

\[ Q = (b_1 - b_2) (\alpha_1 \alpha_2 (-b_1 + b_2) x^* y^* \cosh 4y + (-\alpha_1 x^* + \alpha_2 y^*) \sinh 2y) \]

\[ + (-1 + 2\alpha_1 x^*) (-1 + 2\alpha_2 y^*). \]  

According to the theory of nonlinear dynamics [23], when \( |\lambda_1| < 1 \) and \( |\lambda_2| < 1 \), a fixed point \( E^* \) is the sink point and locally asymptotically stable; when \( |\lambda_1| > 1 \) and \( |\lambda_2| > 1 \), \( E^* \) is the source point and locally unstable; when \( |\lambda_1| > 1 \) and \( |\lambda_2| < 1 \) or \( |\lambda_1| < 1 \) and \( |\lambda_2| > 1 \), \( E^* \) is the saddle; and when \( |\lambda_1| = 1 \) or \( |\lambda_2| = 1 \), \( E^* \) is the nonhyperbolic point. Let \( G(\lambda) = \lambda^2 - P\lambda + Q \). By using the Jury's condition [24], it can be derived that all eigenvalues \( \lambda \) of \( A \) satisfy \( |\lambda| < 1 \) when and only when \( |P| - 1 < Q < 1 \), at which point \( E^* \) is locally stable. Simplified and written in the following form:

\[ \begin{align*}
(a) & \quad G(1) = 1 - P + Q > 0, \\
(b) & \quad G(-1) = 1 + P + Q > 0, \\
(c) & \quad Q < 1.
\end{align*} \]  

To obtain the condition that \( E^* \) is a nonhyperbolic point, we make \( G(1) = 0 \), or \( G(-1) = 0 \), or \( Q = 1 \). Condition (a) \( G(1) = a_1 a_2 (4 + (b_1 - b_2)^2) x^* y^* > 0 \) can always be satisfied. Condition (b) obtains

\[ a_1 < \frac{-4 + 4\alpha_2 y^* + 2\alpha_2 (b_2 - b_1) y^* \sinh 2y}{x^* (-4 + 4\alpha_2 y^* + \alpha_2 (b_2 - b_1) y^* + 2(b_2 - b_1) \sinh 2y)}, \]

and \( a_1 = 4 - 4\alpha_1 y^* + 2\alpha_2 (b_1 - b_2) y^* \sinh 2y/x^* (4 - 4\alpha_2 y^* + \alpha_5 (b_1 - b_2)^2 y^* \cosh 4y + 2(b_2 - b_1) \sinh 2y) \) is satisfied when \(-1\) is an eigenvalue of the Jacobian matrix.

Condition (c) \( Q < 1 \) can be simplified into
\[
\alpha_t < \frac{\alpha_2 y^*(2 + (b_2 - b_1) \sinh 2\gamma)}{x^*(-2 + \alpha_2 (4 + b_1 - b_2)^2) y^* + (b_2 - b_1) \sinh 2\gamma}.
\]  

(18)

Based on inequality (17) and Definition 1, we can make the following Proposition 1 in regards to the local stability of \( E^* \).

Proposition 1. Based on the case that the quantum Cournot–Nash equilibrium point \( E^* \) is positive, the condition for local asymptotic stability is

\[
0 < \alpha_t < \frac{4 - 4\alpha_1 y^* + 2\alpha_2 (b_1 - b_2) y^* \sinh 2\gamma}{x^* (4 - 4\alpha_2 y^* + \alpha_2 (b_1 - b_2)^2) y^* \cosh 2\gamma + 4(b_1 - b_2) \sinh 2\gamma},
\]

or

\[
0 < \alpha_t < \frac{\alpha_2 y^* (2 + (b_2 - b_1) \sinh 2\gamma)}{x^* (2 + \alpha_2 (4 + b_1 - b_2)^2) y^* + (b_2 - b_1) \sinh 2\gamma}.
\]

(19)  

(20)

The following discussion produces the necessary conditions for generating flip bifurcation and Neimark–Sacker bifurcation. By [23], the Jacobian matrix \( A \) with eigenvalues \( \lambda_1 = 1, |\lambda_2| \neq 1 \) is a necessary condition for generating Flip bifurcation. From the previous analysis, we obtained (Replacing \( \alpha_1 \) with \( \alpha_1' \) is to avoid confusion)

\[
\alpha_t' = \frac{4 - 4\alpha_1 y^* + 2\alpha_2 (b_1 - b_2) y^* \sinh 2\gamma}{x^* (4 - 4\alpha_2 y^* + \alpha_2 (b_1 - b_2)^2) y^* \cosh 2\gamma + 4(b_1 - b_2) \sinh 2\gamma},
\]

and

\[
\alpha_t \neq \frac{\alpha_2 y^* (2 + (b_2 - b_1) \sinh 2\gamma)}{x^* (2 + \alpha_2 (4 + b_1 - b_2)^2) y^* + (b_2 - b_1) \sinh 2\gamma}.
\]

(21)  

(22)

For the roots of \( G(\lambda) = 0 \) to be imaginary roots of mode 1, we need to satisfy

\[
\begin{cases}
(a) G(1) = 1 - P + Q > 0, \\
(b) G(-1) = 1 + P + Q > 0, \\
(c) Q = 1.
\end{cases}
\]

(23)

Definition 1. [23] Let \( F_a \) be a one parameter family of map of \( \mathbb{R}^2 \) satisfying

(i) \( F_a(0) = 0 \) for \( a \) near 0;
(ii) \( DF_a(0) \) has two complex eigenvalues \( \lambda(a), \bar{\lambda}(a) \) for a near 0 with \( |\lambda(0)| = 1 \);
(iii) \( (d | \lambda(a)/d\alpha)|_{a=0} > 0 \);
(iv) \( \lambda = \lambda(0) \) is not an mth root of unity for \( m = 1, 2, 3, 4 \).

Therefore at a near 0, \( F_a \) produces an Neimark–Sacker bifurcation at the fixed point.

From Proposition 1, we have

\[
0 < \alpha_t < \alpha_t'' \quad \text{and} \quad \alpha_t'' = \frac{\alpha_2 y^* (2 + (b_2 - b_1) \sinh 2\gamma)}{x^* (2 + \alpha_2 (4 + b_1 - b_2)^2) y^* + (b_2 - b_1) \sinh 2\gamma}.
\]

(24)

Let \(-2 < P < 2 \) and \( P \neq 0, -1 \), which guarantees the generation of \( \lambda_1, \lambda_2 \neq 1 \) for all \( m = 1, 2, 3, 4 \), avoid the resonance condition. Thus there is

\[
-4 < -2(\alpha_1 x^* + \alpha_2 y^*) - (b_1 - b_2)(\alpha_1 x^* - \alpha_2 y^*) \sinh 2\gamma < 0,
\]

and

\[
\alpha_t \neq \frac{\alpha_2 y^* (2 + (b_2 - b_1) \sinh 2\gamma)}{x^* (2 + (b_1 - b_2) \sinh 2\gamma)}
\]

\[
\alpha_t \neq 2 - 2\alpha_2 y^* + \alpha_2 (b_1 - b_2) y^* \sinh 2\gamma.
\]

(25)  

(26)

The above discussion complements our proof of the following Proposition 2.
Proposition 2. When condition (22) is satisfied, $\alpha^f_1$ is the flip bifurcation critical point for the quantum Cournot–Nash equilibrium point $E^*$ at (2).

When conditions (25) and (26) are satisfied, $\alpha^{ns}_1$ is the Neimark–Sacker bifurcation critical point of the quantum Cournot–Nash equilibrium point $E^*$ at (12).

4. Numerical Simulations

We will verify the theory’s validity through numerical simulations in this section to ensure its correctness. With different parameter values, the impact of quantum entanglement on the dynamic properties of the quantum Cournot duopoly game is visualized. For example, stability region, bifurcation and phase diagrams, maximum Lyapunov exponents (MLE), and singular attractors.

Example 1. We choose the speed of adjustment $\alpha_1$ as the bifurcation parameter and the other parameters as $b_1 = 0.1, b_2 = 0.2, a = 5.8, c_1 = 0.1, c_2 = 0.1$, with initial condition $(x, y) = (3, 2.651)$, at which $\lambda_1 = -1, \lambda_2 = -0.0660073$, the model (12) undergoes flip bifurcation. Figure 1 shows the relationship between the quantum entanglement $\gamma$ and the stability region, where the stability region becomes larger as $\gamma$ increases, i.e., the bifurcation point of flip gradually increases, and the flip bifurcation is delayed. Figure 2(a) shows the bifurcation

![Figure 1: Stability region in the plane of $(\gamma, \alpha_1)$ for $\alpha_2 = 0.2, b_1 = 0.1, b_2 = 0.2, a = 5.8, c_1 = 0.1, c_2 = 0.1$.](image1.png)

![Figure 2: Bifurcation diagram and MLE with $\alpha_2 = 0.2, b_1 = 0.1, b_2 = 0.2, a = 5.8, c_1 = 0.1, c_2 = 0.1, \alpha_1 \in [0.3, 0.6]$ and with initial conditions $(x, y) = (3, 2.651)$. (a) Bifurcation diagram for $x_n$ with $\gamma = 0$ (in blue) and $\gamma = 0.2$ (in red); (b) MLE for $\gamma = 0$ (in blue) and $\gamma = 0.2$ (in red).](image2.png)
Example 2. We choose the speed of adjustment $\alpha_1$ as the bifurcation parameter, the other parameters are $a_3 = 0.3, b_2 = 0.8, b_2 = 0.4, c_2 = 0.2, c_1 = 0.2, a = 6$ and the initial condition $(x, y) = (2.3, 3.35)$. The critical values $\alpha_1 = 0.431388$ and $\lambda_1 = -0.923854 + 0.382745i, \lambda_2 = -0.923854 - 0.382745i$ at $y = 0.02$ satisfy Proposition 2 and (12) undergoes Neimark–Sacker bifurcation. Figure 4 shows the quantum entanglement $y$ with respect to the stability region. Intuitively, it shows that as $y$ increases, the bifurcation point gradually increases and the stability region expands. However, the bifurcation point is clearly more sensitive to $y$ than in Example 1. Figures 5(a) and 5(b) show the bifurcation diagram about $\alpha_1$ in $[0.35, 0.55]$ for $y = 0.02$ and $y = 0$, respectively. Figure 5(c) shows maximum Lyapunov exponents about $\alpha_1$ in $[0.35, 0.55]$ for $y = 0.02$ (in red) and $y = 0$ (in blue). Compared to Example 1, its stable region expands while the chaos-generating region shrinks as $y$ increases, but does not delay at $\alpha_1 = 0.55$. Figure 6(a) illustrates the invariant curve of (12) becoming smaller at $y = 0.02$. Figures 6(b)–6(f) show the orbits of periods-7, 14 and 28 until the chaotic attractor appears. Thus when $y$ is increased, the Neimark–Sacker bifurcation also occurs with a delay, but is more different from Flip. This is related to the initial conditions we have chosen.
Figure 5: Bifurcation diagrams and MLE with $\alpha_1 = 0.3, b_1 = 0.8, b_2 = 0.4, a = 6, c_1 = 0.2, c_2 = 0.2, \alpha_1 \in [0.35, 0.55]$ and with initial conditions $(x, y) = (2.3, 3.35)$. (a) Bifurcation diagram for $x_n$ with $\gamma = 0$; (b) Bifurcation diagram for $x_n$ with $\gamma = 0.02$; (c) MLE for $\gamma = 0.02$ (in red) and $\gamma = 0$ (in blue).

Figure 6: Continued.
5. Conclusion

With relative profit maximization and bounded rationality, this paper establishes the quantum Cournot duopoly game. Quantum entanglement is examined for its effect on the system's stability and dynamic behavior.

Based on the results, this classical model is a particular case of the quantum form. Quantum entanglement delays the onset of bifurcation behavior and expands the stability region with increasing entanglement in Flip bifurcation and Neimark–Sacker bifurcation. Furthermore, complex dynamical processes such as stability regions, bifurcation diagrams, maximum Lyapunov exponents and phase diagrams (including periodic orbits and chaotic attractors) are described using numerical simulation methods. Due to the introduction of quantum entanglement, the two players actually "cooperate." The classical model is a subset of the quantum model. It provides a more flexible way for players to control the production output, for example, by simply setting γ to nonzero or zero and choosing to regulate the steady, chaotic state of the outcome. This is closely related to profit maximization.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


