Hindawi
Mathematical Problems in Engineering
Volume 2024, Article ID 3427721, 21 pages
https://doi.org/10.1155/2024/3427721

Research Article

A Framework for Valuation and Portfolio Optimization of Venture Capital Deals with Contractual Terms

Mohammadreza Valaei and Vahid Khodakarami

Industrial Engineering Department, Bu-Ali Sina University, Hamedan, Iran

Correspondence should be addressed to Vahid Khodakarami; v.khodakarami@basu.ac.ir

Received 17 April 2023; Revised 20 November 2023; Accepted 20 December 2023; Published 25 January 2024

Academic Editor: Hakim Bendjenna

Venture capitalists invest not only in the business aspect of a deal but also in its contractual terms. Therefore, the selection of deals and the combination of contractual terms pose challenging decisions for them. This paper consists of two main sections. The first section introduces a novel framework for the valuation of venture capital (VC) deals, including startups and their contractual terms. By taking into account risk situations, this section presents the valuation of combined contractual terms, including call options, liquidity preference, and participant rights. In the second section, a new multiobjective mathematical model for VC deals and contractual terms portfolio selection is developed using right-tail probability, strategy alignment, and a utility function. To solve the proposed model, three metaheuristic algorithms—Non-Dominated Sorting Genetic Algorithm (NSGA-II), Multi-Objective Binary Harmony Search Algorithm, and Dynamic Tuning Parameter Binary Harmony Search Algorithm (DTPBHS)—are applied. Based on numerical examples, DTPBHS outperforms other algorithms in the "Mean Ideal Distance" index, but NSGA-II demonstrates the best performance in the "Rate of Achievement of two objectives simultaneously" index. Furthermore, we demonstrate that the proposed utility function is more robust than the right-tail probability function under default deals conditions.

1. Introduction

Venture capitalists, a specific type of financial intermediary, identify investees, such as startups, with significant potential for growth and entrepreneurial capacity. They provide these companies with funds, networking capabilities, and business skills to capitalize on market opportunities [1–3], playing a crucial role in the survival of new ventures [4].

On the one hand, investee selection involves not only evaluating venture projects but also considering contractual terms for venture capital (VCs). When selecting investees, VCs should assess factors such as the status of technology and the market, competitive position, growth strategy, and customer management [5]. This situation introduces a significant level of uncertainty and complexity in the valuation and selection of deals and their contractual terms. Therefore, VCs need to evaluate and maximize the profitability of investments by employing reliable and flexible methods for choosing investees and determining their contractual terms.

On the other hand, the investment strategies of VC firms play a primary role in the survival of new investments [6] and the selection of investees. These strategies may encompass the degree of VC participation in portfolio companies [7], as well as the level of diversification or localization.

In this paper, we will address three questions: How to value deals that combine contractual terms in uncertain conditions? How to model the portfolio selection of deals? What constitutes a robust objective function in the context of deal defaults?

Because the contractual terms of deals have a direct impact on investee selection, we propose a multidimensional method for the valuation of deals and their components, such as call options, liquidity preference, and participant rights. We apply the stochastic real options (ROs) method, stochastic DCF, and default rates to consider uncertainty and risk, offering an alternative to using risk-adjusted discount rates. Moreover, we introduce a new multiobjective mathematical model that incorporates both VC strategic and financial objectives, considering uncertainty and constraints. The model helps in better-assessing risks and portfolio budgeting for deal selection, considering both financial and nonfinancial dimensions. Furthermore, we compare the performance of a utility selection approach to that of a right-tail selection approach when designing an index robust against deal defaults.

The main contributions of this paper include:

(i) Introducing a novel method for valuing deals and their contractual terms, incorporating default rates.
(ii) Developing a new multiobjective mathematical model for the portfolio selection of deals and contractual terms.
(iii) Employing a novel utility function for the selection of a VC portfolio and comparing its efficiency to the right-tail function in the context of a deal default scenario.

The remainder of this paper is organized as follows: In Section 2, we provide an overview of recent research on the valuation of deal contractual terms, VC strategy, and portfolio optimization. Section 3 explores the concepts of contractual terms valuation, VC portfolio selection, and strategic alignment. Additionally, we present a method for estimating the strategic misalignment of a deal with a VC strategy. In Section 4, we develop a meta-heuristic algorithm based on the harmony search (HS) algorithm to address the NP-hard computational complexity of the model for solving real-size problems. Section 5 discusses the applicability and results of the model using a numerical example, evaluating the performance of the proposed algorithms and the utility metric instead of the right-tail metric. Finally, in Section 6, we draw our conclusions and provide a summary to conclude the paper.

2. Literature Review

2.1. VC Contractual Terms Valuation. From the point of view of the information process perspective for VC decision-making, there are four stages: screening, origination, evaluation (due diligence), and negotiation [8]. Throughout these stages, qualitative and quantitative decision-making factors are hierarchically evaluated. Decision-making about valuation is one of the important challenges for VC investors because there are many uncertainties about the value of the deals. The valuation does not only include the startup's business of a deal but also contractual terms. Liquidity preference, investment amount, ownership stake, board control, option pool, prorata rights, valuation, antidilution, vesting, ownership stake, investment amount, participation, redemption rights, and dividends are negotiable and flexible contractual terms [9]. Some of these contractual terms have a direct effect on the valuation of a deal, such as the vesting situation, while others, like the option pool [10], have an indirect effect on the value of deals [11]. These contractual terms collectively influence the value of a deal. Therefore, a method is needed to evaluate both the business and contractual terms of deals under uncertain conditions.

There are several quantitative and qualitative methods for startup deal valuation, such as the Berkus method, Jordan Cooper, risk factor, VC methods, First Chicago, Chepakovich, multiples method, market-oriented approaches, score card, RO, and DCF [12–14]. Numerous of these methods are too qualitative and provide imprecise outputs (in some cases, unrealistic results) for startups.

RO valuation considers the cash flow value and adds the value of managerial flexibility to assets [15], as discussed by Li et al. [16], who proposed an RO valuation method for financial incentive allocation in infrastructural projects. They examined the effects of ambiguity on RO value based on pessimistic and optimistic approaches and concluded that the RO valuation method is better than the net present value (NPV) method for uncertainty analysis. As argued by Montajabiha et al. [17], the n-fold compound option model was used to evaluate multistages investment projects in R&D pharmaceutical projects. They also applied a robust model for R&D portfolio selection but did not utilize the value distribution function and the information on fat-tail distributions. Nigro et al. [18] assessed pharmaceutical industry projects by RO and categorized the project based on growth phases, and optimized the mathematical model, but they did not consider the uncertainty of valuations. One of the most popular methods for option pricing is the Black-Scholes model, which has been the subject of numerous discussions and examinations in academic literature [19]. There are different versions of the Black–Scholes model, such as literatures [20, 21], but the main difference in the proposed method compared to the literature is that the proposed valuation method is a combination of RO, DCF, decision tree, and Monte Carlo simulation [14].

2.2. VC Portfolio. Operation research problems usually answer only one of the "what," "where," and "when" questions [22]. This paper aims to address the combination of these questions for the VCs portfolio. Generally, VCs aim to select the best deals for success, but various risks can undermine profitability. As discussed in the review paper by Chaparro et al. [23], portfolio selection methods can be classified into 12 sectors: financial methods, probabilistic methods, option pricing theory, strategic methods, scoring methods, combinatorial optimization, behavioral methods, mapping approaches, ROs, integrated methods, information gap theory, and scenario-based approach. The paper suggests that the portfolio selection method should be based on the innovation level. When there is radical innovation, qualitative methods are preferred. As a result, the priority is behavioral and information gap theory. The second priority is integrated methods and ROs. In this paper, we consider that VCs do not invest in extreme radical innovation investees. Therefore, integrated methods and ROs are assumed to be appropriate.

According to Guo et al.'s research [24], different stakeholders have different tendencies toward portfolio selection goals, which are assumed in the proposed mathematical model. At the first level, top managers prefer to select a portfolio aligned with organizational strategies. At the second level, portfolio managers want to choose the projects that maximize portfolio return. They pay great attention to the interrelation and synergies of projects. At the third level, the project managers should meet resource limitations and risk tolerance. In this paper, the proposed model belongs to the first and second levels of stakeholders. Aouni et al. [25] used the stochastic goal programing model and the concept of utility function in the case of 15 deals in order to design the decision-making process of VCs under three different scenarios. In another article published by Aouni et al. [26], a goal programing model based on fuzzy logic VC decision was proposed. The effect of the entry of a new deal on the existence of a portfolio and the effect of mutual interaction between new deals are considered by Zhao et al. [27]. Moreover, VCs can choose their deal portfolio based on the risk-taking or risk-aversion of investors by using a tuning parameter [27].

One of the most important criteria for VC portfolio optimization is portfolio size. According to Cumming [28], four categories of factors affect the size of a portfolio: (1) Characteristics of VC funds, including the type of fund (such as public or private VC), fund duration, fundraising, and the number of VC fund managers. (2) Characteristics of entrepreneurial companies, including development stage, technology, and geographical location. (3) The character of financing, including staging, syndication, and deal size. (4) Market conditions. In an area where the number of VCs is limited, entrepreneurs are likely not in a position to select among VC financiers [29]. Overall, the number of investees in a portfolio depends on many factors, such as funding stage, VC budget, VC type, the level of diversification, geographical distance, VC experience, financed entrepreneurial firms number, VC firm age, fundraising number and successful exits number [29, 30]. For example, Huntsman and Hoban [31] have demonstrated that ten investments may not be enough to reach a reasonably steady portfolio return for a VC portfolio, and portfolio diversity is not proportional to the fund size. Also, VC fund contractual conditions usually deny each investment from investing in more than a specific percentage (usually 10%) of the fund, and it is critical to maintain the appropriate diversity level [29]. In addition, the fund availability established by each investment company is seriously and directly associated with the fund size. As the investable companies grow, they are more likely to enter new funds. So, the small fund size restricts the VCs from accessing the advantages of investment results [29]. In the proposed model, the number and amount of investments are considered, and it is assumed that the stock offering of each deal is fixed for a certain amount of money because there are limited stock volume choices available for VCs to select. This assumption reflects the fact that investees, such as startup founders, prefer to keep a certain amount of equity to preserve their decision-making power in corporate governance.

Diversification criteria are another factor for portfolio selection. There are three principal categories of diversification (specialization): industry, stage of venture, and geography. Organizational learning theory discusses that specialized investor perseverance relies on extraordinary success in the field. However, diversified VCs are more likely to sustain due to less competition in a particular industry. Portfolio diversification is often beneficial in the early stages/high-risk stages. Informal venture capital (IVC) (including lesser-known individuals, entrepreneurial friends, and angel investors forming groups of IVCs) is less profited than formal venture capital (FVC) in terms of industry and stage diversity. Insufficient investment for IVC can push them to participate in fewer investment rounds, despite successful investment selections. It may prevent them from thoroughly enjoying the diversity benefits. The previous investment experience is suggested to provide a more robust alternative for IVC diversification needs. As a result, IVC can profit more by concentrating on the typical industry by achieving an expressive investment [32]. The small funds can protect forthcoming assets under management by adopting specialized strategies with relatively small investments in a restricted number of companies with losing diversification. In contrast, small funds may invest in many portfolio companies and choose a diversification strategy [29]. Therefore, in this article, according to the importance of this concept, the diversification index is used.

As discussed by Treville et al. [33], the worth of a VC portfolio is determined by the value of the small number of top deals that are available for assessment. Therefore, it can be acknowledged that the value of a portfolio relies on the presence of fat right-tail distribution deals rather than averages (IVC or FVC investor returns are extremely skewed, high probability of low-expected NPV and low probability of high-expected NPV, and existing literature confirms it [32]). The combination of the right tail of deals for the portfolio deal's probability distribution of NPV (we call it PDNPV) is different from the summation of deal value because this problem is not always subadditivity, no smooth, and nonconvex [34–36].

The "strategy" is the key to reaching profit for VCs [5]. As a result, the alignment of investee characteristics with the VC strategies is an important issue in selecting deals. Consequently, VC investors have strategies for reaching their goals. Ayob and Dana [37] proposed three strategies for VC products in developing countries, consisting of producing low-cost, differentiated, and specialized products. Park and Bae [38] proposed a 3D integrative framework of VC strategy in developing countries. The framework consists of three dimensions: target market, product/market maturity, and technological capability. They have also proposed seven practical strategies based on the main three strategies.

Various models exist for VC portfolio selection, including stochastic goal programing [25], fuzzy goal programing [26], modern portfolio theory [39], data envelopment analysis [40], and fuzzy inference system–agent [41]. Many research studies have focused on assessing the weight of selection criteria and integrating multicriteria decision-making techniques with fuzzy theory. However, only a limited number of studies have applied mathematical models specifically to the selection of VCs' investees (as opposed to deals). This limited usage underscores the untapped potential of mathematical models in the field of deal portfolio selection.

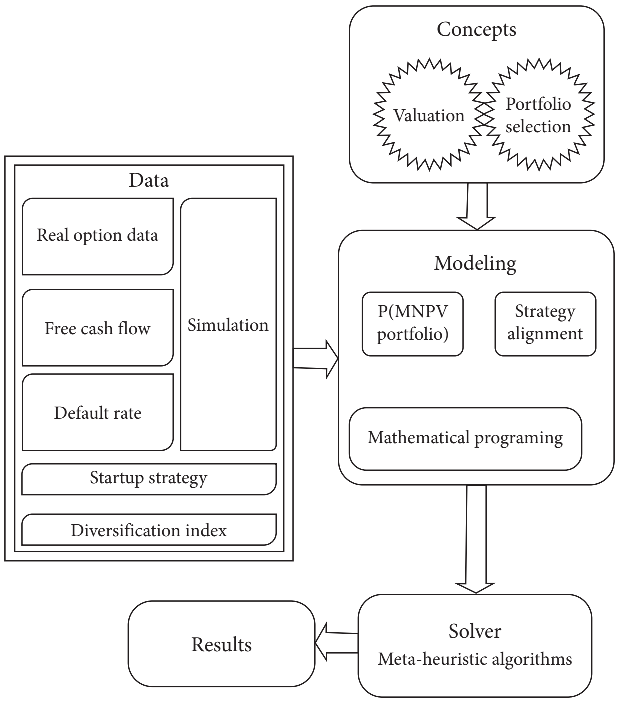

Figure 1: The framework of proposed venture capital portfolio selection.

To the best of our knowledge, no research has been conducted on the combined valuation and selection of startup deals and their contractual terms (such as call options, liquidity preference, and participant rights) for VC. Therefore, we propose a new method for valuation and a multi-objective portfolio selection mathematical model for VC, considering both financial and nonfinancial objectives. Various factors, including strategy alignment, the level of investee innovation, diversification, deal options, fund limitations, and the number of investments, are taken into account.

3. Problem Description and Formulation

The framework of this paper consists of two main concepts: valuation and selection of VC portfolio (Figure 1).

In Figure 1, we apply valuation and portfolio selection concepts to design a new model for deal contractual terms valuation and portfolio optimization. For valuation, the input data involve free cash flows (FCFs) probability function, default rates in different periods, the data related to ROs, and contractual terms. The inserted data for portfolio selection includes VC strategies, diversification index, the minimum NPV required for VC, valuation of deals and their contractual terms, budget constraints, and the maximum number of investees. Then, a solver method like a Meta-heuristic algorithm will determine the portfolio's compound.

3.1. The Valuation of VC Contractual Terms. In this paper, it is assumed that there are several deals with a variety of contractual terms. Therefore, we not only evaluate the NPV of a deal but also assess the effect of deals with contractual terms between the founder and the investors. We consider all kinds of deal risks into three groups: hard factors, soft factors, and scenario factors.

The factors with sufficient data about future scenarios and probabilities are classified as hard factors. In this group, these factors will be estimated using decision trees and probability distribution functions. The second group consists of scenario factors, where the scenarios are identifiable, but their probabilities cannot be determined directly. Here, we employ the RO method to account for them. In the third group, soft factors are typically examined qualitatively, including individual and managerial attributes of the investor, the motivation and experience of the founders, the team members' ability to recover in case of partial failures, and their resilience in the face of economic, political, social, and legal conditions [42]. The evaluation of these soft factors is carried out using expert-elicited Bayesian network (EBN) analysis, as described by Valaei and Khodakarami [42]. The output of EBNs is the default rate in each period of a deal's life.

In the following, By using an example, we will present how to calculate the value of a stochastic European call option by using a Monte Carlo Simulation for a startup deal. Then, the formula for the valuation of contractual terms, specifically for liquidity preference and participant rights, is proposed.

In this paper, the price of a European call option is calculated based on the Black–Scholes model (Equation (1)) (it is evident that various alternative methods exist for RO pricing that can be used, including Binomial Models) [43]. The inputs of the model consist of the current underlying asset price, the strike underlying asset price, the time to maturity, the risk-free rate, and the volatility of the underlying asset [43].

$$O_t = S \cdot N(d_1) - Xe^{-\alpha t} \cdot N(d_2); \quad d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(\alpha + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}; \quad d_2 = d_1 - \sigma\sqrt{t}, \tag{1}$$

The parameters of the valuation model are described as follows:

(O_t) Call option premium at time t*—the value of a call option with expiration time "t";

(V_t) Future FCF value*—future FCF value of deal at time "t";

(R_t) Realized FCF value *—the FCF value of a deal at time "t";

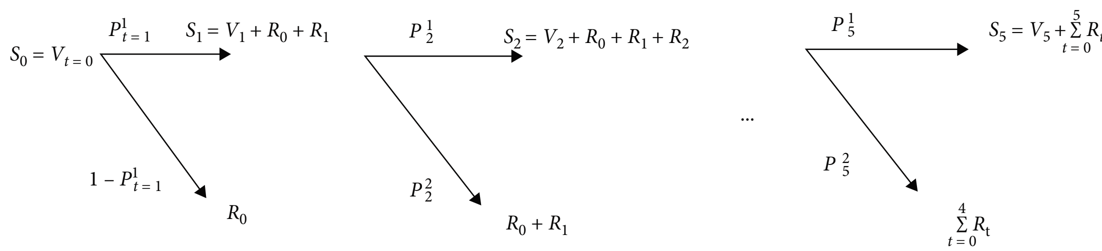

Figure 2: A simple NPV decision tree of a deal.

(S_t) FCF value of the deal at time "t" *—overall past and future value of FCF at time "t" related to stock ownership. It sums of V_t and $\sum R_t$;

(X_t) Strike price of deal *—the exercise price is the value at which the option can be exercised at time "t";

(g) Time to maturity of call option;

(α) Risk-free rate;

(σ) Volatility—the underlying asset volatility;

$N()$ Cumulative normal density function N (0.1);

(O_g) Call option premium *—the value of a call option with expiration time g;

($P^2{}_t$) Failure probability—default rate at time "t";

($P^1{}_t$) No failure probability at time "t" ($P^1{}_t = 1 - P^2{}_t$);

(w) Share ownership percentage;

(L_g) liquidity preference multiple;

*If there is a sign of "′" such as O'_t, it shows the present value of each parameter.

For instance, there is a probability distribution of net present value (PDNPV) for a deal. Because the NPV of a deal is not a single value, the NPV of derivative tools, such as a call option, will not be a singular value either. Therefore, in this paper, we employ the Monte Carlo simulation method to generate PDNPV for deals and their options, reflecting the premium cost in the cash flow of a deal.

In this paper, the value of the current underlying asset is a probability distribution function. Assuming that the NPV motion follows a random walk based on Geometric Brownian Motion, as described in Equation (2) [44], We calculate the standard NPV of a new deal (without considering the default rate) with a call option using Equation (3).

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\varepsilon\right], \tag{2}$$

$$\text{Deal NPV}_t = \begin{cases} -O_t & S_t - X_t < 0 \\ S'_5 - X'_t - O_t & S_t - X_t \geq 0 \end{cases}. \tag{3}$$

Based on Equation (3), if the price of the deal is higher than the strike price, the call option will be exercised. Otherwise, the investment will not continue, and the deal will be abandoned, resulting in the NPV of the deal will be negative, equal to the option premium. For instance, Figure 2 shows the PDNPV of a deal with a European call option and a time to maturity of 4 years, demonstrated using a Monte Carlo simulation (The Monte Carlo simulation is done by Crystal Ball software). If there is a failure probability in each period ($1 - P_t$), the NPV of the startup is shown in Figure 2.

The NPV of a startup deal with a call option (with expiration g year) and the default rate is shown in Equation (4).

$$\text{Deal NPV} = \begin{cases} -O_g & \left(\prod_{t=1}^{T=g} P^1_t\right)\left(V_g + \sum_{t=1}^{g} R_t\right) - X_g < 0 \\ \left(\prod_{t=1}^{T=5} P^1_t\right)\left(V'_{T=5} + \sum_{t=1}^{T=5} R'_t\right) + \sum_{\tau=g+1}^{T}\left(\left(\prod_{t=2}^{\tau} P^1_{t-1}\right)(1 - P^1_\tau)\left(\sum_{t=1}^{\tau-1} R'_t\right)\right) - X'_g - O_g & \left(\prod_{t=1}^{T=g} P^1_t\right)\left(V_g + \sum_{t=1}^{g} R_t\right) - X_g \geq 0 \end{cases}, \tag{4}$$

The NPV of a startup with a call option (with expiration g), default rate, and liquidity preference is shown in Equation (5).

$$\text{Deal NPV}=\begin{cases} -O_g & \left(\prod_{t=1}^{T=g} P_t^1\right)\left(V_g+\sum_{t=1}^{g} R_t\right)-X_g<0 \\ \max\left\{\begin{array}{c}\left(\prod_{t=1}^{T=5} P_t^1\right)\left(V'_{T=5}+\sum_{t=1}^{T=5} R'_t\right), \\ \left(\prod_{t=1}^{T=5} P_t^1\right)\cdot\min\left\{\left(L_g\times X'_g\right),\left(\dfrac{\left(V'_{T=5}+\sum_{t=1}^{T=5} R'_t\right)}{w}\right)\right\}\end{array}\right\}+ & \\ \sum_{\tau=g+1}^{T=5}\left(\left(\prod_{t=2}^{\tau} P_{t-1}^1\right)\left(1-P_\tau^1\right)\cdot\min\left\{\left(L_g\times X'_g\right),\left(\dfrac{\sum_{t=1}^{\tau-1} R'_t}{w}\right)\right\}\right)-X'_g-O_g & \left(\prod_{t=1}^{T=g} P_t^1\right)\left(V_g+\sum_{t=1}^{g} R_t\right)-X_g\geq 0,\quad g<5 \end{cases}. \tag{5}$$

The NPV of a startup with a call option (with expiration g), default rate, liquidity preference, and participant right is shown in Equation (6). To assess the impact of certain independent contractual terms, please refer to reference [11].

$$\text{Deal NPV}=\begin{cases} -O_g & \left(\prod_{t=1}^{T=g} P_t^1\right)\left(V_g+\sum_{t=1}^{g} R_t\right)-X_g<0 \\ \left(\left(\prod_{t=1}^{T=5} P_t^1\right)\cdot\left(\min\left\{\left(L_g\times X'_g\right),\left(\dfrac{V'_5+\sum_{t=1}^{T=5} R'_t}{w}\right)\right\}+w\times\max\left(0,\dfrac{V'_5+\sum_{t=1}^{T=5} R'_t}{w}-\left(L_g\times X'_g\right)\right)\right)\right)+ & \\ \sum_{\tau=g+1}^{T=5}\left(\left(\prod_{t=2}^{\tau} P_{t-1}^1\right)\left(1-P_\tau^1\right)\left(\min\left\{\left(L_g\times X'_g\right),\left(\dfrac{\sum_{t=1}^{\tau-1} R'_t}{w}\right)\right\}+w\times\max\left\{0,\left(\dfrac{\sum_{t=1}^{\tau-1} R'_t}{w}\right)-\left(L_g\times X'_g\right)\right\}\right)\right)-X'_g-O_g & \left(\prod_{t=1}^{T=g} P_t^1\right)\left(V_g+\sum_{t=1}^{g} R_t\right)-X_g\geq 0,\quad g<5 \end{cases}. \tag{6}$$

3.2. VC Portfolio Selection. The mathematical model in this paper is based on the multidimensional multiple-choice knapsack problem (MMKP) [45] and the portfolio selection problem under the value-at-risk measure [36, 46], which does not have the subadditivity property. MMKP is a generalization of the ordinary knapsack problem [47], and it is a more complex variant of the binary knapsack problem and computationally is an NP-Hard problem [45, 48, 49]. Moreover, the NP-Hard complexity of portfolio selection problem with Value at Risk objective (such as right tail probability) is proved by Benati and Rizzi [46]. Since the real-world investment problems are composed of different objectives, it is proposed a multiobjective stochastic version of the multidimensional multichoice knapsack problem. Thus, the proposed model is a multiobjective binary model to select a set of deals and their RO, maximizing the right tail of a cumulative probability of the portfolio NPV and minimizing the strategic misalignment of the selected deals.

In this section, the proposed mathematical model is introduced. The following assumptions are made:

(i) The cash flow predictions of each deal are independent of each other.
(ii) It is assumed that the stock offering of deals is fixed for a certain amount of fixed money.
(iii) The portfolio's budget is deterministic, and it is divided into consuming and reserving budgets.
(iv) The focus of this paper is limited to a specific geographical area (without geographical diversification).
(v) The FCFs are predicted stochastically.

In the following, decision variables are determined, and the abbreviations and parameters used in the mathematical model are defined.

3.2.1. Model Indices. Investee: i;

Industry: j;

Deals: u (all kinds of contractual terms for each deal are considered independent variables).

3.2.2. Model Parameters. Z_1: The first objective function is designed for calculating the right tail of the portfolio NPV distribution;

Z_2: The second objective function aims to quantify the strategy misalignment;

MNPV: The minimum attractive NPV for the VC portfolio;

D_{ji}: The total amount of strategy misalignment for investee i in industry j;

d_{ji}^{y}: The amount of strategy misalignment for investee i in industry j regarding strategic dimension y;

U: The maximum number of company strategy dimensions;

C_{jiu}: The investment cost for the investee i in industry j with deal u;

P_{jiu}: The present value of future cost for the investee i in industry j with deal u;

B: The maximum available budget;

E: The maximum reserved available budget;

N: The maximum number of the deals that VC can manage;

L: The minimum amount of Herfindahl diversification index for VC portfolio;

H: The maximum amount of Herfindahl diversification index for VC portfolio;

k: The maximum number of industries;

n_j: The maximum number of investee in industry j;

T_{ji}: The maximum number of deals for the investee i in industry j;

NPV_{jiu}: NPV of the investee i in industry j with deal u;

$\Phi_{jiu}(\mathrm{MNPV})$: The complementary cumulative distribution function of the NPV of investee i in industry j with deal u for the MNPV;

$f^{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{u=1}^{T_{ji}}\chi_{jiu}}(h)$: $f(h)$ is a joint probability distribution function for the sum of the probability distribution function of $\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{u=1}^{T_{ji}}\chi_{jiu}$.

3.2.3. Decision Variables. χ_{jiu}: Binary variable for selecting the deal i in industry j;

α_{jiu}: Auxiliary binary variable for selecting the deal i in industry j.

To evaluate the strategic misalignment between investees and VCs, a distinct version of the index, influenced by literatures [50, 51] and incorporating the Herfindahl index [52, 53], is employed. This index measures the Euclidean Distance for each dimension. The formulations are as follows (Equation (7)):

$$D_{ji} = \sqrt{\sum_{y=1}^{U=9} \frac{\left(d_{ji}^{y}\right)^2}{U-1}},\quad d_{ji}^{y} = \frac{\text{Investee i strategy index in dimantion of } y - \text{VC strategy index in dimantion of } y}{\text{Maximun strategy index in dimantion of } y - \text{Minimum strategy index in dimantion of } y}, \tag{7}$$

where D_{ji} represents the overall strategic misalignment of deal i in industry j, and d_{ji}^{y} is the strategic misalignment of deal i in industry j at dimension y.

As given in Table 1, For instance, if a VC with an "import substitution" strategy intends to invest in a deal with an "early market entry" strategy, conflicts may arise in strategic alignment [38].

In order to show the formulas, in Equations (8)–(17), we illustrate the complete mathematical model as follows:

$$\text{Max } Z_1:$$
$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}} \Phi_{jiu}(\mathrm{MNPV})\alpha_{jiu} + \sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}} \int_{-\infty}^{\mathrm{MNPV}} \left(\Phi_{jiu}(\mathrm{MNPV}-h)\cdot\chi_{jiu}\right) f^{\left(\chi_{1,1,1},\ldots,\chi_{k,n_j,T_{ji}}\right)}(h)\, dh, \tag{8}$$

$$\text{Min } Z_2: \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} D_{ji}\left(\sum_{u=1}^{T_{ji}}\chi_{jiu}\right)}{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{u=1}^{T_{ji}}\chi_{jiu}}, \tag{9}$$

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}} C_{jiu}\left(\chi_{jiu}+\alpha_{jiu}\right) \le B, \tag{10}$$

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}} P_{jiu}\left(\chi_{jiu}+\alpha_{jiu}\right) \le E, \tag{11}$$

$$1 \le \sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{u=1}^{T_{ji}} \left(\chi_{jiu}+\alpha_{jiu}\right) \le N, \tag{12}$$

TABLE 1: Strategy alignment measurement [38].

	Strategies							Strategic Misalignment Measurement
	Reactive imitation	Proactive localization	Import substitution*	Creative imitation	Early market entry*	Global niche	Global innovation	d_{ji}^{y*}
Dimension								
Product-market								
Maturity	Existing:1	Emerging:0	**Existing:1**	Emerging (local/exsting (global):0	**Emerging:0**	Existing:1	Emerging:0	$0-1=-1$
Growth potential	Low:1	Moderate:2	**Low:1**	Moderate/high:2 /3	**Very high:4**	High:3	Very High:4	$\frac{4-1}{4-1}=1$
Market uncertainty	Low:1	Moderate:2	**Low:1**	Moderate:2	**High:3**	Moderate:2	Very High: 4	$\frac{3-1}{4-1}=0.66$
Technological capability								
Technological capability, Tech. Capabilities (pioneer/follower)	Local follower:0	Local follower:0	**Local pioneer:1**	Local pioneer/global follower:0	**Global follower:0**	Global pioneer:0	Global pioneer:0	$0-1=-1$
R&D investment	Low:1	Low:1	**Moderate:2**	Moderate:2	**Moderate:2**	High:3	High:3	$\frac{2-2}{3-1}=0$
Technological partners	–	Global:1	**Local:0**	Local/ global:0 or 1	**Global:1**	Global:1	Global:1	$1-0=1$
Priority of R&D	–	–	**Cost reduction:1**	Product quality/cost reduction:0 or 1	**Entry timing:0**	Product quality	Entry timing	$0-1=-1$
Target market								
Target market	Local:0	Local:0	**Local:0**	Local/global:0 or 1	**Global:1**	Global:1	Global:1	$1-0=1$
Degree of internationalization	Low:1	Low:1	**Low:1**	Moderate:2	**High:3**	High:3	High:3	$\frac{3-1}{3-1}=1$

*For example, if the assumed strategy of a VC is Import Substitution and the strategy of an investee is Early Market Entry, the misalignment strategy for the selection is calculated in the last column. Bold columns are examples of strategies and are relevant.

Table 2: Coding of solutions.

Industry 1								Industry 2								Industry n							
Investee 1				Investee 2				Investee 1				Investee 2				Investee 1				Investee 2			
Deal 1	Deal 2	Deal 3	Deal 4	Deal 1	Deal 2	Deal 3	Deal 4	Deal 1	Deal 2	Deal 3	Deal 4	Deal 1	Deal 2	Deal 3	Deal 4	Deal 1	Deal 2	Deal 3	Deal 4	Deal 1	Deal 2	Deal 3	Deal 4
0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$L \le 1 - \left(\frac{\sum_{j=1}^{k} \left(\sum_{i=1}^{n_j} \sum_{u=1}^{T_{ji}} \left(\chi_{jiu} + \alpha_{jiu}\right) \right)^2}{\left(\sum_{j=1}^{k} \sum_{i=1}^{n_j} \sum_{u=1}^{T_{ji}} \left(\chi_{jiu} + \alpha_{jiu}\right) \right)^2} \right) \le H, \tag{13}$$

$$\sum_{u=1}^{T_{ji}} \left(\chi_{jiu} + \alpha_{jiu}\right) \le 1 j = 1, \ldots, k \quad i = 1, \ldots, n_j, \tag{14}$$

$$\chi_{jiu} + \alpha_{jiu} \le 1 j = 1, \ldots, k\,, \;\; i = 1, \ldots, n_j\,, \quad u = 1, \ldots T_{ji}, \tag{15}$$

$$\sum_{j=1}^{k} \sum_{i=1}^{n_j} \sum_{u=1}^{T_{ji}} \alpha_{jiu} = 1, \tag{16}$$

$$\chi_{jiu} = \{0, 1\}. \tag{17}$$

As we aim to capitalize on the asymmetric returns of deals, the first objective (8) employs the right tail of PDNPV at the point of MNPV. This aligns with the opportunistic strategy of seeking the highest investment return. Proceeding from the left side of the distribution, we utilize numerical integration with a trapezoidal base and calculate the cumulative probabilistic distribution functions until reaching the value of MNPV (its proof is presented in the Appendix [54]). The second objective (9) focuses on estimating the strategic misalignment of a portfolio, resulting in a value of one or greater.

Constraints (10) and (11) are related to budget considerations. Constraint (10) pertains to the maximum budget available for selecting deals right now, while Constraint (11) entails reserving a budget for the future cost of the selected deal, including the exercise of options in subsequent years.

Additionally, Constraint (12) establishes boundaries for both the maximum and minimum number of investees that can be managed, resembling a cardinality constraint [55]. In this research, industrial diversification is considered due to the importance of the industrial type of investees that operate. A diversification index called the Herfindahl index [52, 53] is applied in Constraint (13) on the portfolio selection model. It is proposed that the upper and lower bounds of the constraint be determined based on successful and similar VC practices (see [56]).

Since VCs can choose only one deal for each investee, we use Constraint (14). Constraint (15) is intended to decide between the primary and auxiliary variable choices. In the first part of the primary objective function, you must select one of the deals. Therefore, we add Constraint (16) to the model. Constraint (17) represents binary variables. Each variable shows the value of a deal with individual contractual terms (consisting of options).

4. Algorithm Design

The number of variables in the proposed problem could increase significantly when a VC has several investees with different contractual terms. The exact algorithms, such as branch and bond algorithms and dynamic programing algorithms, cannot handle large-scale problems, and solving them requires a significant amount of CPU time due to their NP-hardness [57]. Given these challenging, several meta-heuristic algorithms have been developed for optimization in the last few decades [58]. As discussed by Zou et al. [59], the HS algorithm is a well-organized method for solving knapsack problems, and it can find better solutions compared to other metaheuristics, such as genetic algorithm in a stochastic multiobjective problem under the same situation [60, 61]. The HS method is an emerging meta-heuristic optimization algorithm employed to cope with numerous challenging tasks during the past decade. The HS algorithm is inspired by the underlying principles of musicians' improvization of harmony, which can be seen as a search process for the global optimum in optimization problems that are evaluated by an objective function [62]. The musical harmony in HS is similar to a variable vector, and the best harmony achieved in the end can be seen as the optimal global solution [58].

4.1. Dynamic Parameters Tuning Multiobjective HS. A variety of harmony algorithms have been proposed, including an efficient HS algorithm [63], an improved HS algorithm [64], multiobjective binary differential evolution HS (MOBDEHS) [65], and multiobjective HS algorithm [66]. A new version of the multiobjective binary HS (MBHS) algorithm [66] is the dynamic parameters tuning multiobjective harmony search algorithm (DPTMOHS), which uses dynamically tuned parameters to create nondominant solutions. The algorithm for dynamically tuning the parameters has been designed based on reference [60], and the Pseudocode of the proposed algorithm (DPTMOHS) is shown in the coding done by MATLAB 2013a software on a Core i7, 8 GB. Before delving into the detailed explanation of the algorithm, it is necessary to present the coding method of the problem, as shown in Table 2.

As depicted in Figure 3, a deal is selected from the harmony memory (based on the elitism parameter) if a new random number is less than the harmony memory consideration rate (HMCR). Subsequently, at the next level, if a new random number is less than the pitch adjustment rate (PAR) (the mutation parameter), a deal is randomly selected from the neighborhood of the last deal position (with all deals sorted in ascending order). If the algorithm does not enter the process of HMCR and PAR, it utilizes a random mechanism, as shown in Figure 3.

```
For each X_i ∈ [ i = 1, ..., n ] do
  If  rand () =< HMCR then
        Select from memory of harmony
     If rand () =< PAR  then
        Select the neighbouring deal of your choice.
     End
  Else
     If rand ()>0.5
        Randomly select a deal
     Else
        Do not select any deal
     End
  End
End
```

FIGURE 3: Pseudocode of algorithm.

To handle the constraint mentioned in the model within the algorithm, a pruning approach is employed for newly generated solutions. In essence, if an infeasible solution is generated, the algorithm promptly discards it.

4.2. Comparing Metric. There are various metrics for comparing approximate algorithms when accuracy is unmeasurable. In this research, we employ three metrics to compare the three algorithms. These metrics include the rate of achievement of two objectives simultaneously. The relative achieved spread (RAS), which measures the rate of achieving the two best objectives simultaneously, the mean ideal distance (MID, which quantifies the distance between Pareto solutions and the best answer), and the spread of nondominated solutions (SDS, indicating the range of nonexhaustive solutions) [67]. In Equations (18)–(20), Z_{1s} and Z_{2s} are normalized using the method described in literature [67], and n shows the number of solutions in Pareto solutions.

$$\mathrm{MID} = \frac{\sum_{s=1}^{n} C_s}{n} \quad C_s = \sqrt{(Z_{1s} - Z_1^*)^2 \quad + \quad (Z_{2s} - Z_2^*)^2}, \tag{18}$$

$$\mathrm{SDS} = \sqrt{\frac{\sum_{s=1}^{n} (\mathrm{MID} - C_s)^2}{n-1}}, \tag{19}$$

$$\mathrm{RAS} = \frac{\sum_{s=1}^{n} \left(\frac{Z_{1s} - \min\{Z_{1s}, Z_{2s}\}}{\min\{Z_{1s}, Z_{2s}\}} \right) + \left(\frac{Z_{2s} - \min\{Z_{1s}, Z_{2s}\}}{\min\{Z_{1s}, Z_{2s}\}} \right)}{n}. \tag{20}$$

5. Computational Tests

5.1. Generating Test Problems. To test and verify the algorithms in the proposed model, we generate practical testing by using 10 deals with 40 premium call options (Table 3). The strategy misalignment of each candidate deal is randomly generated from a uniform distribution (1, 5), as demonstrated in Table 4. The cumulative probability distributions of NPV are summarized in Table 5.

5.2. Comparing Results. To assess the performance of the DPTMOHS algorithm, we utilize two series of test problems: one for small size and another for medium size.

5.2.1. Small-Size Test Problem. For the small-sized problem, new test problems were designed to evaluate the capability of the DPTMOHS algorithm in finding Pareto solutions. Three groups of problems, with sizes 8, 10, and 12 and each having three levels of MNPV (with values of 50, 75, and 100), were created. The results, based on quality metrics (the proportion of Pareto solutions [68]), are presented for 1,000 iterations of the algorithm in Table 6. The table illustrates that the DPTMOHS algorithm can effectively find Pareto solutions, demonstrating the efficacy of the proposed algorithm in identifying optimal points for the problem (the data are provided in Table 7).

5.2.2. Medium-Size Test Problem. The algorithm of Nondominated Sorting Genetic Algorithm II (NSGA-II) [69, 70] and MBHS algorithm [66] are used for comparison with the DPTMOHS algorithm. These algorithms need to adjust their parameters (DPTMOHS does not need to parameter tuning). In this paper, the NSGA-II (the maximum number of iteration parameters is not considered because the termination condition is determined based on the number of objective evaluations) and MBHS (the harmony memory size (HMS) parameter in the classic harmony search algorithm has been omitted in this study. This omission is based on the assumption that what is crucial for VCs is to generate a range of dominant solutions rather than focusing on a single solution. Furthermore, due to the utilization of dynamic parameter tuning, the count of nondominated solutions may fluctuate throughout the optimization process) parameters [66] are tuned by the Taguchi method for tuning parameters [60]. The tables are selected for standard orthogonal array L_{16} (4^2) and L_9 (3^3) experiments, in which three repetitions are performed (the data are provided in Tables 8 and 9). The levels of experiments were conducted using the Taguchi method, and their results were obtained based on the signal-to-noise (S/N) ratio. Then, the best-tuned parameters of the algorithm were calculated using Minitab 17 software [60]. Tables 10 and 11 show the best parameters of the MBHS and NSGA-II algorithms.

After determining the parameters for both the MBHS and NSGA-II algorithms, their performance will be assessed against that of the DPTMOHS algorithm, utilizing RAS, MID, and SDS metrics (see Table 12).

To determine whether there exists a statistically significant difference in the performance of the algorithms (Table 12), we employ the Kruskal–Wallis test (Figure 4) [67] and a paired t-test (Table 13) [68] with a confidence interval of 0.95.

The results of the Kruskal–Wallis test indicate no significant difference in the SDS index (P-value = 0.321). However, there are notable distinctions in the RAS (P-value = 0.0001) and MID (P-value = 0.0002) indices. Subsequent paired t-test results demonstrate that the MID of the DPTMOHS algorithm outperforms that of NSGA-II. Conversely, the RAS of the NSGA-II algorithm surpasses that of the DPTMOHS algorithm.

As outlined, the probability of failure for any VC deal is very high. Therefore, we propose a new approach for the first

Table 3: Estimation of startup cash flow.

Startup	Probability function	Parameters	Cash flow					
			Zero year	First year	Second year	Third year	Fourth year	Fifth year
Deal 1	Uniform	Worst case	−30	0.66	0.75	0.8	0.83	0.85
		Best case		3	12	20	30	42
Deal 2	Uniform	Worst case	−35	0.5	1.52	1.75	1.4	1.2
		Best case		2	10.5	28	35	42
Deal 3	Uniform	Worst case	−50	2	2	2	1.8	2
		Best case		8	18	32	45	72
Deal 4	Uniform	Worst case	−80	0	5.15	5.33	3.5	7.4
		Best case		18	26	48	48	185
Deal 5	Uniform	Worst case	−75	15	7	3.67	3.06	1.2
		Best case		60	63	60	50.3	43.2
Deal 6	Uniform	Worst case	−50	0.3	0.875	1.4	1.16	1
		Best case		3	14	35	42	49
Deal 7	Uniform	Worst case	−30	0.66	0.75	0.8	0.83	0.85
		Best case		3	12	20	30	42
Deal 8	Uniform	Worst case	−40	1.3	1.5	1.6	1.5	1.71
		Best case		12	24	40	54	84
Deal 9	Uniform	Worst case	−95	3	3.25	3.2	2.3	5.2
		Best case		18	52	80	84	259
Deal 10	Uniform	Worst case	−94	10	5.25	2.94	1.71	1.02
		Best case		90	48	73.5	61.8	50.4

Table 4: Strategy misalignment of each candidate.

	Deal 1	Deal 2	Deal 3	Deal 4	Deal 5	Deal 6	Deal 7	Deal 8	Deal 9	Deal 10
Strategy misalignment	5	2	2	1	4	3	2	1	2	4

objective of portfolio selection, which involves replacing the right tail of the probability function. This involves a utility function based on the probability of the right tail of PDNPV relative to the probability of negative PDNPV (see Equation (21)).

$$\frac{\left(\begin{array}{c}\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}}\Phi_{jiu}(\text{MNPV})\alpha_{jiu}+\\ \sum_{j=1}^{k}\sum_{i=1}^{n_j}\int_{-\infty}^{\text{MNPV}}\left(\Phi_{jiu}(\text{MNPV}-h)\cdot\chi_{jiu}\right)f^{\left(\chi_{1,1,1},\dots,\chi_{k,n_j,T_{ji}}\right)}(h)\,dh\end{array}\right)}{\left(1-\begin{array}{c}\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sum_{t=1}^{T_{ji}}\Phi_{jiu}(\text{MNPV})\alpha_{jiu}+\\ \sum_{j=1}^{k}\sum_{i=1}^{n_j}\int_{-\infty}^{0}\left(\Phi_{jiu}(0-h)\cdot\chi_{jiu}\right)f^{\left(\chi_{1,1,1},\dots,\chi_{k,n_j,T_{ji}}\right)}(h)\,dh\end{array}\right)}. \tag{21}$$

To facilitate a comparison between the two approaches, we have designed 20 test problems (refer to Table 14). After implementing the DPTMOHS algorithm for each of these test problems, the efficiency of each approach is assessed through the proposed stress test, known as the deal failure rate. In this test, we assume that the NPV of each selected deal in the portfolio fails randomly (using Monte Carlo simulation). The failure is determined by selecting from the negative area of the NPV distribution function for each deal.

To assess whether there is a significant difference between the two approaches, the Paired Wilcoxon Rank test (in this test, there is no assumption of normal data distribution) is employed with a confidence interval of 0.95 (P-value = 0.00001) [67]. The results show a significant difference between the two methods, as illustrated in Table 9.

5.3. Discussion. As pointed out earlier, both the value of a business's cash flows and the contractual terms have a significant impact on the benefit to an investor when considering a startup. In this research, we illustrate how to evaluate a combination of specific contractual terms. Utilizing RO theory, decision trees, and PDNPV, we propose a practical formulation for valuing call options, liquidity preferences, and participant rights in VC contracts. We then present a practical mathematical model for selecting a combination of a deal portfolio based on the right tail of PDNPV and strategy alignment. To solve the model, we compare the performance of NSGA-II, DPTMOHS, and MBHS algorithms. The results of statistical hypothesis tests indicate no significant difference

Table 5: Cumulative probability distribution of NPV.

Cumulative distribution function of net present value

Probability	Deal 1	Deal 1−1	Deal 1−2	Deal 1−3	Deal 1−4	Deal 2	Deal 2−1	Deal 2−2	Deal 2−3	Deal 2−4	Deal 3	Deal 3−1	Deal 3−2	Deal 3−3	Deal 3−4	Deal 4	Deal 4−1	Deal 4−2	Deal 4−3	Deal 4−4	Deal 5	Deal 5−1	Deal 5−2	Deal 5−3	Deal 5−4
1%	−30	−39	−39	−37	−36	−33	−37	−36	−33	−33	−50	−77	−79	−75	−71	−79	−120	−118	−111	−107	−72	−115	−112	−105	−101
2%	−29	−37	−36	−34	−32	−33	−34	−33	−30	−30	−50	−74	−75	−69	−64	−79	−113	−110	−101	−99	−71	−109	−105	−99	−94
3%	−29	−36	−34	−32	−30	−32	−33	−31	−28	−28	−50	−71	−72	−65	−58	−78	−108	−106	−96	−91	−70	−105	−100	−94	−88
4%	−29	−35	−33	−30	−28	−32	−31	−29	−26	−26	−50	−70	−70	−62	−53	−78	−105	−101	−91	−85	−69	−101	−96	−89	−84
5%	−29	−33	−32	−29	−27	−32	−30	−28	−25	−25	−50	−68	−67	−59	−50	−78	−102	−97	−86	−80	−69	−98	−93	−85	−82
6%	−29	−32	−30	−27	−26	−31	−29	−27	−24	−24	−50	−66	−65	−57	−50	−77	−99	−94	−82	−80	−68	−95	−90	−82	−78
7%	−29	−31	−29	−26	−25	−31	−28	−26	−23	−23	−50	−65	−63	−53	−50	−77	−97	−90	−80	−80	−67	−93	−87	−80	−76
8%	−29	−31	−28	−25	−25	−31	−26	−24	−22	−22	−50	−64	−61	−50	−50	−77	−94	−87	−78	−78	−67	−90	−85	−77	−75
9%	−29	−30	−27	−24	−24	−30	−25	−23	−21	−21	−50	−62	−59	−50	−49	−76	−92	−84	−75	−77	−66	−88	−82	−75	−75
10%	−29	−29	−26	−24	−23	−30	−24	−22	−20	−21	−50	−61	−56	−49	−48	−76	−89	−82	−74	−75	−65	−87	−80	−75	−75
11%	−28	−28	−25	−23	−23	−30	−23	−21	−20	−20	−50	−60	−54	−47	−47	−76	−87	−79	−72	−74	−65	−85	−77	−74	−75
12%	−28	−28	−24	−22	−22	−30	−23	−20	−19	−20	−50	−59	−52	−47	−47	−75	−85	−77	−70	−73	−64	−83	−75	−73	−73
13%	−28	−27	−23	−22	−22	−29	−21	−19	−19	−19	−50	−58	−50	−46	−46	−75	−82	−74	−69	−71	−64	−82	−74	−71	−72
14%	−28	−26	−22	−21	−22	−29	−20	−18	−18	−19	−50	−56	−48	−45	−46	−75	−80	−72	−68	−70	−63	−80	−73	−70	−71
15%	−28	−25	−21	−21	−21	−29	−19	−18	−17	−18	−50	−55	−46	−44	−45	−75	−78	−70	−67	−69	−62	−78	−71	−68	−70
16%	−28	−24	−20	−20	−21	−29	−18	−17	−17	−18	−49	−54	−44	−43	−44	−74	−76	−68	−66	−68	−62	−77	−70	−67	−69
17%	−28	−23	−20	−20	−21	−28	−17	−16	−17	−18	−49	−52	−43	−42	−44	−74	−74	−66	−65	−67	−61	−75	−68	−66	−68
18%	−28	−22	−19	−20	−20	−28	−16	−16	−16	−17	−49	−50	−42	−42	−43	−74	−72	−64	−64	−66	−61	−74	−66	−65	−67
19%	−28	−20	−18	−19	−20	−28	−15	−15	−16	−17	−49	−47	−40	−41	−43	−73	−69	−63	−63	−65	−60	−72	−65	−64	−66
20%	−28	−19	−18	−19	−20	−28	−14	−15	−15	−17	−49	−44	−39	−41	−42	−73	−66	−62	−62	−65	−60	−70	−64	−63	−65
21%	−27	−18	−17	−18	−20	−28	−13	−14	−15	−16	−49	−41	−38	−40	−42	−73	−64	−60	−61	−64	−59	−69	−63	−62	−64
22%	−27	−17	−17	−18	−19	−27	−12	−14	−15	−16	−49	−38	−38	−40	−41	−72	−61	−59	−60	−63	−58	−68	−62	−61	−63
23%	−27	−16	−17	−18	−19	−27	−12	−13	−14	−16	−49	−35	−37	−39	−41	−72	−59	−58	−59	−62	−58	−66	−60	−60	−63
24%	−27	−15	−16	−18	−19	−27	−11	−13	−14	−15	−49	−34	−36	−39	−41	−72	−56	−57	−58	−61	−57	−65	−59	−59	−62
25%	−27	−15	−16	−17	−18	−27	−11	−13	−14	−15	−49	−33	−36	−38	−40	−71	−54	−56	−57	−60	−57	−63	−58	−59	−61
26%	−27	−14	−16	−17	−18	−26	−10	−12	−14	−15	−49	−31	−35	−38	−40	−71	−53	−54	−56	−59	−56	−62	−57	−58	−61
27%	−27	−13	−15	−17	−18	−26	−10	−12	−13	−15	−49	−30	−34	−37	−39	−70	−51	−53	−55	−59	−55	−60	−56	−57	−60
28%	−27	−13	−15	−16	−18	−26	−9	−12	−13	−14	−49	−30	−34	−37	−39	−70	−50	−52	−55	−58	−54	−59	−55	−56	−59
29%	−26	−13	−15	−16	−18	−26	−9	−11	−13	−14	−49	−29	−33	−36	−39	−70	−48	−51	−54	−57	−54	−57	−54	−55	−58
30%	−26	−12	−14	−16	−17	−25	−9	−11	−12	−14	−49	−28	−33	−36	−38	−69	−47	−50	−53	−57	−53	−56	−53	−55	−58
31%	−26	−12	−14	−16	−17	−25	−8	−11	−12	−14	−49	−27	−33	−36	−38	−69	−45	−50	−52	−56	−53	−54	−52	−54	−57
32%	−26	−11	−14	−16	−17	−25	−8	−10	−12	−13	−49	−27	−32	−35	−38	−68	−44	−49	−51	−55	−52	−53	−51	−53	−56
33%	−26	−11	−14	−15	−17	−25	−8	−10	−12	−13	−49	−26	−32	−35	−37	−68	−43	−48	−51	−54	−51	−51	−50	−52	−56
34%	−26	−11	−13	−15	−16	−24	−7	−10	−11	−13	−48	−26	−31	−34	−37	−67	−42	−47	−50	−54	−51	−50	−49	−52	−55
35%	−25	−10	−13	−15	−16	−24	−7	−10	−11	−13	−48	−25	−31	−34	−37	−67	−40	−46	−49	−53	−50	−49	−49	−51	−55
36%	−25	−10	−13	−15	−16	−24	−7	−9	−11	−12	−48	−24	−30	−34	−36	−66	−39	−45	−48	−52	−50	−47	−48	−50	−54
37%	−25	−10	−13	−15	−16	−24	−7	−9	−11	−12	−48	−24	−30	−33	−36	−66	−38	−44	−47	−51	−49	−46	−47	−50	−53
38%	−25	−10	−12	−14	−16	−23	−6	−9	−11	−12	−48	−24	−30	−33	−36	−65	−37	−43	−47	−51	−48	−45	−46	−49	−53
39%	−25	−9	−12	−14	−15	−23	−6	−9	−10	−12	−48	−23	−29	−33	−35	−65	−36	−42	−46	−50	−47	−44	−45	−48	−52
40%	−25	−9	−12	−14	−15	−23	−6	−8	−10	−12	−48	−23	−29	−32	−35	−64	−35	−42	−45	−49	−46	−43	−45	−48	−51
41%	−24	−9	−12	−14	−15	−23	−6	−8	−10	−11	−48	−22	−28	−32	−35	−64	−34	−41	−45	−49	−46	−42	−44	−47	−51
42%	−24	−9	−11	−14	−15	−22	−5	−8	−10	−11	−48	−22	−28	−32	−35	−63	−33	−40	−44	−48	−45	−41	−43	−46	−50
43%	−24	−8	−11	−13	−15	−22	−5	−8	−10	−11	−48	−22	−28	−31	−34	−62	−32	−39	−43	−47	−44	−40	−43	−46	−50

Probability	Deal 6	Deal 6−1	Deal 6−2	Deal 6−3	Deal 6−4	Deal 7	Deal 7−1	Deal 7−2	Deal 7−3	Deal 7−4	Deal 8	Deal 8−1	Deal 8−2	Deal 8−3	Deal 8−4	Deal 9	Deal 9−1	Deal 9−2	Deal 9−3	Deal 9−4	Deal 10	Deal 10−1	Deal 10−2	Deal 10−3	Deal 10−4
1%	−40	−26	−25	−23	−24	−27	−31	−30	−27	−27	−22	−27	−27	−27	−26	−92	−162	−154	−139	−138	−84	−130	−125	−115	−115
2%	−39	−24	−22	−21	−22	−27	−29	−27	−25	−24	−19	−25	−25	−25	−24	−91	−153	−145	−131	−128	−83	−121	−117	−107	−105
3%	−38	−21	−20	−19	−20	−26	−27	−25	−23	−23	−18	−24	−24	−24	−23	−90	−147	−139	−124	−120	−81	−115	−111	−101	−99
4%	−37	−19	−19	−18	−19	−26	−26	−24	−22	−21	−16	−23	−23	−23	−22	−89	−144	−133	−119	−113	−80	−111	−106	−95	−95
5%	−37	−18	−18	−17	−18	−26	−25	−23	−21	−21	−15	−23	−22	−22	−22	−88	−139	−128	−115	−109	−80	−107	−101	−91	−90
6%	−36	−17	−17	−16	−17	−25	−24	−22	−20	−20	−14	−22	−22	−21	−21	−87	−135	−124	−111	−105	−79	−104	−97	−89	−90
7%	−36	−15	−15	−15	−16	−25	−23	−21	−19	−19	−13	−21	−21	−21	−21	−86	−132	−120	−107	−101	−78	−101	−94	−87	−89
8%	−35	−13	−15	−15	−15	−25	−22	−20	−18	−19	−13	−21	−21	−20	−20	−86	−128	−116	−104	−98	−77	−98	−91	−84	−87
9%	−35	−12	−14	−14	−15	−25	−21	−19	−18	−18	−12	−21	−20	−20	−20	−85	−125	−112	−100	−95	−76	−96	−89	−82	−85
10%	−34	−11	−13	−13	−14	−24	−20	−18	−17	−18	−11	−20	−20	−20	−19	−84	−122	−109	−96	−95	−75	−93	−87	−80	−83
11%	−34	−9	−12	−13	−14	−24	−20	−18	−17	−17	−11	−20	−19	−19	−19	−84	−120	−106	−95	−95	−74	−91	−84	−78	−81
12%	−33	−8	−11	−12	−13	−24	−19	−17	−16	−17	−10	−19	−19	−19	−19	−83	−116	−103	−95	−95	−74	−88	−82	−76	−79
13%	−33	−7	−10	−11	−12	−23	−18	−16	−16	−16	−10	−19	−19	−19	−18	−82	−114	−100	−95	−95	−73	−86	−80	−74	−78
14%	−33	−6	−10	−11	−12	−23	−17	−16	−15	−16	−9	−19	−18	−18	−18	−82	−111	−97	−95	−95	−72	−84	−78	−73	−76
15%	−32	−5	−9	−10	−12	−23	−16	−15	−15	−16	−9	−19	−18	−18	−18	−81	−109	−95	−95	−95	−71	−82	−76	−72	−75
16%	−32	−4	−8	−10	−11	−23	−16	−15	−15	−15	−8	−18	−18	−18	−18	−80	−107	−95	−95	−95	−70	−80	−74	−70	−73
17%	−32	−4	−7	−9	−11	−23	−15	−14	−14	−15	−8	−18	−18	−18	−17	−80	−104	−95	−93	−95	−69	−79	−73	−69	−72
18%	−31	−3	−7	−9	−10	−22	−14	−14	−14	−15	−7	−18	−17	−17	−17	−79	−102	−94	−92	−95	−69	−77	−71	−68	−71
19%	−31	−3	−6	−8	−10	−22	−13	−13	−13	−14	−7	−17	−17	−17	−17	−78	−100	−92	−90	−93	−68	−75	−69	−66	−70
20%	−31	−2	−6	−8	−9	−22	−13	−13	−13	−14	−7	−17	−17	−17	−17	−78	−97	−90	−88	−92	−67	−73	−68	−65	−69
21%	−30	−2	−5	−8	−9	−22	−12	−12	−13	−14	−6	−17	−17	−17	−16	−77	−95	−88	−87	−90	−66	−71	−66	−64	−67
22%	−30	−2	−5	−7	−9	−21	−11	−12	−13	−13	−6	−17	−16	−16	−16	−76	−93	−87	−86	−89	−66	−70	−65	−63	−66
23%	−30	−2	−4	−7	−8	−21	−11	−11	−12	−13	−5	−16	−16	−16	−16	−75	−91	−85	−84	−88	−65	−68	−64	−62	−65
24%	−29	−1	−4	−7	−8	−21	−10	−11	−12	−13	−5	−16	−16	−16	−16	−75	−90	−83	−83	−86	−64	−66	−63	−60	−64
25%	−29	−1	−4	−6	−8	−21	−10	−11	−12	−13	−5	−16	−16	−16	−16	−74	−88	−82	−82	−85	−63	−65	−61	−59	−63
26%	−29	−1	−4	−6	−7	−21	−9	−10	−11	−12	−4	−16	−16	−16	−15	−73	−86	−80	−80	−84	−62	−63	−60	−58	−62
27%	−29	−1	−3	−6	−7	−20	−9	−10	−11	−12	−4	−16	−15	−15	−15	−72	−84	−79	−79	−82	−61	−60	−59	−57	−62
28%	−28	−1	−3	−5	−7	−20	−8	−10	−11	−12	−4	−15	−15	−15	−15	−71	−82	−77	−78	−81	−61	−59	−58	−57	−61
29%	−28	−1	−3	−5	−7	−20	−8	−10	−11	−12	−3	−15	−15	−15	−15	−71	−80	−76	−77	−80	−60	−57	−56	−56	−60
30%	−28	−1	−3	−5	−6	−20	−8	−9	−10	−11	−3	−15	−15	−15	−15	−70	−79	−75	−76	−79	−59	−56	−55	−55	−59
31%	−28	−1	−2	−4	−6	−19	−7	−9	−10	−11	−2	−15	−15	−15	−14	−69	−76	−73	−75	−77	−58	−54	−54	−54	−58
32%	−27	−1	−2	−4	−6	−19	−7	−9	−10	−11	−2	−15	−15	−15	−14	−68	−75	−72	−74	−76	−57	−52	−53	−53	−57
33%	−27	−1	−2	−4	−6	−19	−7	−8	−10	−11	−2	−14	−14	−14	−14	−68	−73	−70	−73	−75	−56	−51	−52	−52	−56
34%	−27	−1	−2	−4	−5	−19	−6	−8	−10	−11	−2	−14	−14	−14	−14	−67	−71	−69	−71	−74	−55	−50	−51	−51	−55
35%	−27	0	−2	−4	−5	−18	−6	−8	−9	−10	−1	−14	−14	−14	−14	−66	−69	−67	−70	−73	−55	−48	−49	−51	−55
36%	−26	0	−2	−3	−5	−18	−6	−8	−9	−10	−1	−14	−14	−14	−14	−65	−68	−66	−69	−72	−54	−46	−48	−50	−54
37%	−26	0	−2	−3	−5	−18	−6	−8	−9	−10	0	−14	−14	−14	−13	−63	−66	−65	−68	−71	−53	−45	−48	−49	−53
38%	−26	0	−1	−3	−4	−18	−5	−7	−9	−10	0	−14	−14	−13	−13	−62	−64	−64	−66	−69	−52	−43	−47	−48	−52
39%	−25	0	−1	−3	−4	−18	−5	−7	−9	−10	0	−13	−13	−13	−13	−62	−62	−62	−65	−68	−51	−42	−46	−48	−51
40%	−25	0	−1	−3	−4	−17	−5	−7	−8	−9	1	−13	−13	−13	−13	−61	−61	−61	−64	−67	−50	−41	−45	−47	−51
41%	−25	0	−1	−3	−4	−17	−5	−7	−8	−9	1	−13	−13	−13	−13	−60	−59	−60	−63	−67	−49	−39	−44	−46	−50
42%	−25	0	−1	−2	−4	−17	−5	−7	−8	−9	1	−13	−13	−13	−13	−59	−57	−58	−62	−65	−48	−38	−43	−45	−49
43%	−24	0	−1	−2	−3	−17	−4	−6	−8	−9	2	−13	−13	−13	−12	−57	−56	−57	−61	−64	−47	−37	−42	−44	−48

Table 5: Continued.

Probability	Cumulative distribution function of net present value																																																	
	Deal 1	Deal 1−1	Deal 1−2	Deal 1−3	Deal 1−4	Deal 2	Deal 2−1	Deal 2−2	Deal 2−3	Deal 2−4	Deal 3	Deal 3−1	Deal 3−2	Deal 3−3	Deal 3−4	Deal 4	Deal 4−1	Deal 4−2	Deal 4−3	Deal 4−4	Deal 5	Deal 5−1	Deal 5−2	Deal 5−3	Deal 5−4	Deal 6	Deal 6−1	Deal 6−2	Deal 6−3	Deal 6−4	Deal 7	Deal 7−1	Deal 7−2	Deal 7−3	Deal 7−4	Deal 8	Deal 8−1	Deal 8−2	Deal 8−3	Deal 8−4	Deal 9	Deal 9−1	Deal 9−2	Deal 9−3	Deal 9−4	Deal 10	Deal 10−1	Deal 10−2	Deal 10−3	Deal 10−4
44%	−24	−8	−11	−13	−14	−22	−5	−8	−9	−11	−47	−21	−28	−31	−34	−62	−31	−38	−43	−47	−43	−39	−42	−45	−49	−24	0	−1	−2	−3	−16	−4	−6	−8	−9	2	−13	−13	−12	−12	−56	−54	−55	−59	−63	−46	−36	−41	−44	−47
45%	−24	−8	−11	−13	−14	−21	−5	−7	−9	−11	−47	−21	−27	−31	−34	−61	−30	−38	−42	−46	−43	−38	−42	−45	−49	−24	0	−1	−2	−3	−16	−4	−6	−7	−8	2	−12	−12	−12	−12	−55	−53	−54	−59	−62	−45	−35	−40	−43	−47
46%	−23	−8	−11	−13	−14	−21	−5	−7	−9	−10	−47	−21	−27	−30	−33	−61	−29	−37	−41	−45	−42	−37	−41	−44	−48	−24	0	−1	−2	−3	−16	−4	−6	−7	−8	3	−12	−12	−12	−12	−54	−51	−53	−58	−61	−44	−34	−39	−42	−46
47%	−23	−8	−10	−13	−14	−21	−5	−7	−9	−10	−47	−20	−26	−30	−33	−60	−28	−36	−40	−45	−41	−36	−40	−43	−47	−23	0	−1	−2	−3	−15	−4	−6	−7	−8	3	−12	−12	−12	−12	−53	−50	−52	−57	−60	−42	−32	−38	−41	−45
48%	−23	−7	−10	−12	−14	−21	−4	−7	−9	−10	−47	−20	−26	−30	−33	−59	−28	−35	−40	−44	−40	−36	−39	−43	−47	−23	0	−1	−2	−3	−15	−4	−6	−7	−8	3	−12	−12	−12	−12	−52	−48	−51	−56	−59	−41	−31	−38	−40	−44
49%	−23	−7	−10	−12	−13	−20	−4	−7	−8	−10	−47	−20	−26	−29	−32	−59	−27	−35	−39	−43	−39	−35	−39	−42	−46	−23	0	−1	−2	−3	−15	−3	−5	−7	−8	4	−12	−12	−12	−11	−50	−46	−50	−55	−58	−40	−30	−37	−40	−44
50%	−22	−7	−10	−12	−13	−20	−4	−7	−8	−10	−47	−19	−26	−29	−32	−58	−26	−34	−38	−43	−38	−34	−38	−41	−45	−23	0	−1	−2	−3	−15	−3	−5	−7	−8	4	−12	−12	−12	−11	−49	−45	−48	−54	−57	−39	−29	−36	−39	−43
51%	−22	−7	−10	−12	−13	−19	−4	−6	−8	−9	−46	−19	−25	−29	−32	−57	−25	−33	−37	−42	−37	−33	−37	−41	−45	−22	0	−1	−1	−2	−14	−3	−5	−6	−7	4	−11	−11	−11	−11	−48	−44	−47	−53	−56	−38	−28	−35	−38	−42
52%	−22	−7	−10	−12	−13	−19	−4	−6	−8	−9	−46	−19	−25	−28	−31	−56	−25	−33	−37	−41	−36	−32	−37	−40	−44	−22	0	0	−1	−2	−14	−3	−5	−6	−7	5	−11	−11	−11	−11	−46	−42	−46	−52	−55	−37	−27	−34	−37	−41
53%	−22	−7	−9	−11	−13	−19	−4	−6	−8	−9	−46	−18	−25	−28	−31	−56	−24	−32	−36	−41	−35	−31	−36	−40	−43	−22	0	0	−1	−2	−14	−3	−5	−6	−7	5	−11	−11	−11	−11	−45	−40	−45	−50	−54	−35	−26	−33	−36	−40
54%	−21	−6	−9	−11	−13	−18	−4	−6	−8	−9	−46	−18	−24	−28	−31	−55	−23	−31	−35	−40	−34	−31	−35	−39	−43	−22	0	0	−1	−2	−13	−3	−5	−6	−7	5	−11	−11	−11	−11	−44	−39	−44	−49	−53	−34	−26	−32	−36	−40
55%	−21	−6	−9	−11	−12	−18	−3	−6	−7	−9	−46	−18	−24	−28	−31	−54	−22	−30	−35	−39	−33	−30	−35	−38	−42	−21	0	0	−1	−2	−13	−3	−4	−6	−7	6	−11	−11	−11	−11	−42	−38	−43	−48	−52	−32	−25	−31	−35	−39
56%	−21	−6	−9	−11	−12	−18	−3	−6	−7	−9	−45	−17	−24	−27	−30	−53	−22	−30	−34	−39	−31	−29	−34	−38	−42	−21	0	0	−1	−2	−13	−2	−4	−6	−6	6	−11	−11	−11	−10	−40	−36	−42	−47	−51	−31	−24	−31	−34	−38
57%	−21	−6	−9	−11	−12	−17	−3	−5	−7	−8	−45	−17	−23	−27	−30	−52	−21	−29	−33	−38	−30	−28	−33	−37	−41	−21	0	0	−1	−2	−12	−2	−4	−5	−6	6	−10	−10	−10	−10	−38	−35	−41	−46	−51	−29	−23	−30	−33	−37
58%	−20	−6	−9	−11	−12	−17	−3	−5	−7	−8	−45	−17	−23	−27	−30	−51	−20	−28	−33	−37	−29	−28	−33	−37	−40	−20	0	0	−1	−2	−12	−2	−4	−5	−6	7	−10	−10	−10	−10	−37	−34	−40	−45	−49	−28	−22	−29	−33	−36
59%	−20	−6	−8	−10	−12	−17	−3	−5	−7	−8	−45	−16	−23	−26	−29	−50	−19	−28	−32	−37	−28	−27	−32	−36	−40	−20	0	0	−1	−2	−12	−2	−4	−5	−6	7	−10	−10	−10	−10	−35	−32	−39	−44	−48	−26	−21	−28	−32	−36
60%	−19	−6	−8	−10	−11	−16	−3	−5	−7	−8	−44	−16	−23	−26	−29	−49	−19	−27	−31	−36	−26	−26	−31	−35	−39	−20	0	0	−1	−1	−11	−2	−4	−5	−6	7	−10	−10	−10	−10	−33	−31	−38	−43	−47	−25	−20	−27	−31	−35
61%	−19	−5	−8	−10	−11	−16	−3	−5	−6	−8	−44	−16	−22	−26	−29	−48	−18	−26	−31	−36	−25	−25	−31	−35	−39	−19	0	0	−1	−1	−11	−2	−4	−5	−6	8	−10	−10	−10	−10	−31	−30	−37	−42	−46	−23	−20	−27	−31	−34
62%	−19	−5	−8	−10	−11	−15	−3	−5	−6	−7	−44	−16	−22	−25	−28	−47	−18	−26	−30	−35	−23	−24	−30	−34	−38	−19	0	0	−1	−1	−11	−2	−3	−5	−6	8	−10	−9	−9	−10	−29	−29	−36	−41	−45	−22	−19	−26	−30	−33
63%	−18	−5	−8	−10	−11	−15	−2	−4	−6	−7	−43	−15	−22	−25	−28	−46	−17	−25	−30	−34	−22	−24	−29	−33	−37	−19	0	0	−1	−1	−10	−2	−3	−5	−5	8	−9	−9	−9	−9	−27	−27	−34	−39	−44	−20	−18	−25	−29	−33
64%	−18	−5	−7	−9	−11	−14	−2	−4	−6	−7	−43	−15	−21	−25	−28	−45	−16	−24	−29	−34	−21	−23	−28	−33	−37	−19	0	0	−1	−1	−10	−2	−3	−4	−5	9	−9	−9	−9	−9	−25	−26	−33	−38	−43	−18	−17	−24	−28	−32
65%	−18	−5	−7	−9	−11	−14	−2	−4	−6	−7	−43	−15	−21	−24	−28	−43	−16	−24	−28	−33	−19	−22	−28	−32	−36	−18	0	0	0	−1	−9	−2	−3	−4	−5	9	−9	−9	−9	−9	−23	−25	−32	−37	−42	−17	−17	−23	−27	−31
66%	−17	−5	−7	−9	−10	−13	−2	−4	−5	−7	−42	−14	−21	−24	−27	−42	−15	−23	−28	−32	−17	−21	−27	−31	−35	−18	0	0	0	−1	−9	−1	−3	−4	−5	9	−9	−9	−9	−9	−21	−24	−31	−36	−41	−15	−16	−22	−26	−30
67%	−17	−4	−7	−9	−10	−13	−2	−4	−5	−6	−42	−14	−20	−24	−27	−41	−15	−22	−27	−31	−16	−21	−26	−31	−34	−18	0	0	0	−1	−8	−1	−3	−4	−5	10	−8	−9	−9	−9	−18	−23	−30	−35	−40	−13	−15	−22	−26	−29
68%	−16	−4	−7	−9	−10	−12	−2	−4	−5	−6	−41	−14	−20	−23	−27	−39	−14	−22	−26	−31	−14	−20	−25	−30	−33	−17	0	0	0	−1	−8	−1	−3	−4	−4	10	−8	−8	−8	−8	−16	−22	−29	−34	−39	−11	−14	−21	−25	−29
69%	−16	−4	−7	−8	−10	−12	−2	−4	−5	−6	−41	−14	−20	−23	−26	−37	−13	−21	−26	−30	−12	−19	−25	−29	−33	−17	0	0	0	−1	−8	−1	−2	−4	−4	10	−8	−8	−8	−8	−13	−21	−28	−33	−37	−9	−13	−20	−24	−28
70%	−15	−4	−6	−8	−10	−11	−2	−3	−5	−6	−40	−13	−19	−23	−26	−36	−13	−20	−25	−29	−10	−18	−24	−28	−32	−17	0	0	0	−1	−7	−1	−2	−3	−4	11	−8	−8	−8	−8	−11	−20	−27	−32	−36	−7	−13	−19	−23	−27
71%	−15	−4	−6	−8	−9	−10	−2	−3	−5	−6	−40	−13	−19	−22	−26	−34	−12	−20	−24	−29	−8	−18	−23	−27	−31	−16	0	0	0	−1	−7	−1	−2	−3	−4	11	−8	−8	−8	−8	−8	−19	−25	−31	−35	−5	−12	−18	−22	−26
72%	−14	−4	−6	−8	−9	−10	−1	−3	−4	−5	−39	−13	−19	−22	−25	−32	−12	−19	−24	−28	−6	−17	−22	−27	−30	−16	0	0	0	−1	−6	−1	−2	−3	−4	12	−7	−8	−8	−8	−6	−18	−24	−29	−34	−2	−11	−18	−21	−25
73%	−13	−4	−6	−8	−9	−9	−1	−3	−4	−5	−38	−12	−18	−22	−25	−30	−11	−18	−23	−27	−4	−16	−22	−26	−29	−15	0	0	0	−1	−6	−1	−2	−3	−4	12	−7	−7	−7	−7	−3	−16	−23	−28	−32	1	−10	−17	−20	−24
74%	−13	−3	−6	−7	−9	−8	−1	−3	−4	−5	−38	−12	−18	−21	−24	−27	−11	−18	−22	−26	−2	−15	−21	−25	−28	−15	0	0	0	0	−5	−1	−2	−3	−3	12	−7	−7	−7	−7	0	−15	−22	−27	−31	3	−10	−16	−19	−23
75%	−12	−3	−6	−7	−8	−8	−1	−3	−4	−5	−37	−12	−18	−21	−24	−25	−10	−17	−22	−26	1	−15	−20	−24	−27	−15	0	0	0	0	−4	−1	−2	−3	−3	13	−7	−7	−7	−7	3	−14	−21	−25	−29	6	−9	−15	−18	−22
76%	−11	−3	−5	−7	−8	−7	−1	−3	−4	−5	−36	−11	−17	−21	−24	−23	−10	−17	−21	−25	3	−14	−19	−22	−26	−14	0	0	0	0	−4	−1	−2	−2	−3	13	−6	−7	−7	−7	7	−13	−19	−24	−28	9	−8	−14	−17	−20
77%	−10	−3	−5	−7	−8	−6	−1	−2	−3	−4	−35	−11	−17	−20	−23	−20	−9	−16	−20	−24	6	−13	−18	−21	−25	−14	0	0	0	0	−3	−1	−1	−2	−3	14	−6	−6	−7	−7	11	−12	−18	−22	−26	12	−7	−13	−16	−19
78%	−10	−3	−5	−6	−8	−5	−1	−2	−3	−4	−34	−11	−17	−20	−23	−17	−9	−15	−19	−23	9	−12	−17	−20	−24	−13	0	0	0	0	−2	−1	−1	−2	−3	14	−6	−6	−6	−6	15	−11	−16	−21	−25	14	−7	−12	−14	−18
79%	−9	−3	−5	−6	−7	−4	−1	−2	−3	−4	−33	−11	−16	−20	−22	−14	−8	−15	−18	−23	12	−11	−16	−19	−22	−13	0	0	0	0	−2	0	−1	−2	−2	15	−5	−6	−6	−6	19	−9	−15	−19	−23	18	−6	−11	−13	−17
80%	−8	−2	−4	−6	−7	−3	−1	−2	−3	−4	−32	−10	−16	−19	−22	−11	−8	−14	−17	−22	16	−10	−14	−17	−20	−12	0	0	0	0	−1	0	−1	−2	−2	15	−5	−6	−6	−6	24	−8	−14	−17	−21	22	−5	−10	−12	−15
81%	−7	−2	−4	−6	−7	−2	−1	−2	−2	−3	−30	−10	−15	−19	−21	−7	−7	−13	−16	−21	19	−9	−13	−16	−18	−12	0	0	0	0	0	0	−1	−2	−2	16	−5	−5	−5	−6	29	−7	−12	−15	−19	25	−4	−9	−9	−13
82%	−5	−2	−4	−5	−6	−1	−1	−2	−2	−3	−29	−9	−15	−18	−21	−2	−7	−13	−16	−20	23	−8	−11	−14	−16	−11	0	0	0	0	1	0	−1	−1	−2	16	−4	−5	−5	−5	35	−6	−11	−13	−17	29	−3	−8	−7	−11
83%	−4	−2	−4	−5	−6	0	0	−1	−2	−3	−27	−9	−15	−18	−21	2	−6	−12	−15	−19	27	−7	−9	−11	−13	−11	0	0	0	0	2	0	−1	−1	−1	17	−4	−5	−5	−5	41	−5	−9	−11	−14	33	−2	−6	−4	−9
84%	−3	−2	−3	−5	−6	2	0	−1	−2	−2	−25	−9	−14	−17	−20	6	−6	−11	−14	−18	32	−5	−6	−8	−10	−10	0	0	0	0	3	0	0	−1	−1	17	−3	−4	−4	−5	47	−3	−6	−8	−10	38	−1	−4	0	−5
85%	−1	−2	−3	−4	−5	3	0	−1	−1	−2	−23	−8	−14	−17	−19	12	−5	−10	−12	−17	38	−3	−1	−3	−5	−10	0	0	0	0	4	0	0	−1	−1	18	−3	−4	−4	−4	54	−2	−3	−4	−6	44	0	−1	5	0
86%	0	−1	−3	−4	−5	5	0	−1	−1	−1	−21	−8	−13	−16	−19	18	−4	−9	−11	−15	43	−1	4	2	0	−9	0	0	0	0	5	0	0	0	0	19	−2	−3	−4	−4	62	0	3	4	2	50	5	4	10	6

Table 5: Continued.

Probability	Cumulative distribution function of net present value																								
	Deal 1	Deal 1−1	Deal 1−2	Deal 1−3	Deal 1−4	Deal 2	Deal 2−1	Deal 2−2	Deal 2−3	Deal 2−4	Deal 3	Deal 3−1	Deal 3−2	Deal 3−3	Deal 3−4	Deal 4	Deal 4−1	Deal 4−2	Deal 4−3	Deal 4−4	Deal 5	Deal 5−1	Deal 5−2	Deal 5−3	Deal 5−4
87%	2	−1	−2	−3	−4	7	0	−1	−1	−1	−18	−8	−13	−16	−18	25	−4	−8	−9	−13	48	5	11	8	6
88%	4	−1	−2	−3	−4	9	0	0	0	1	−14	−7	−12	−15	−17	32	−3	−7	−7	−10	56	12	17	14	12
89%	6	−1	−2	−2	−3	11	0	0	3	3	−10	−7	−11	−14	−17	39	−2	−6	−2	−5	63	20	25	21	19
90%	9	−1	−1	−1	−1	13	0	2	5	5	−6	−6	−11	−14	−16	48	−1	−2	7	3	71	29	35	29	27
91%	12	0	1	2	2	16	1	5	8	8	−1	−6	−10	−13	−15	57	9	10	16	14	81	40	45	38	37
92%	16	0	4	6	6	19	4	8	11	11	6	−5	−9	−11	−14	68	19	20	30	29	93	50	57	47	48
93%	20	3	8	10	12	23	7	11	14	14	14	−4	−8	−10	−12	83	33	34	44	44	106	64	70	60	60
94%	26	7	13	16	18	27	11	15	19	19	25	−3	−6	−7	−10	101	49	49	60	59	120	81	85	75	75
95%	34	13	22	23	24	31	16	20	23	24	38	−1	−2	3	−3	121	71	72	83	79	138	98	102	92	95
96%	43	23	33	34	35	37	23	27	29	30	60	13	17	24	18	147	100	101	109	109	171	129	128	117	129
97%	56	38	45	47	48	46	31	37	38	38	92	43	48	53	50	187	135	141	146	151	203	164	166	152	164
98%	76	60	65	66	71	60	44	50	52	51	156	93	101	106	102	253	202	202	210	209	267	225	223	205	214
99%	122	104	115	114	112	83	66	76	72	74	308	250	236	257	228	382	336	338	376	340	399	363	318	316	316
100%	942	934	763	805	935	320	266	396	450	305	5,417	5,389	2,494	3,565	4,436	3,034	3,001	2,835	3,455	2,554	2,318	2,283	2,820	1,567	1,684
Cost	**30**	**7**	**10**	**13**	**15**	**35**	**5**	**8**	**10**	**11**	**50**	**18**	**25**	**29**	**32**	**80**	**27**	**36**	**42**	**47**	**75**	**33**	**40**	**46**	**50**

Probability	Deal 6	Deal 6−1	Deal 6−2	Deal 6−3	Deal 6−4	Deal 7	Deal 7−1	Deal 7−2	Deal 7−3	Deal 7−4	Deal 8	Deal 8−1	Deal 8−2	Deal 8−3	Deal 8−4	Deal 9	Deal 9−1	Deal 9−2	Deal 9−3	Deal 9−4	Deal 10	Deal 10−1	Deal 10−2	Deal 10−3	Deal 10−4
87%	−8	0	0	0	0	6	0	0	0	1	19	−1	−3	−3	−4	69	8	11	13	11	55	11	11	16	12
88%	−7	0	0	0	0	7	0	1	2	3	20	−1	−2	−3	−3	78	17	20	21	19	62	18	17	22	18
89%	−6	0	0	0	0	9	0	3	4	4	21	0	−2	−2	−3	88	26	29	32	29	70	27	24	29	25
90%	−6	0	0	0	0	11	2	5	5	5	22	0	−1	−2	−2	100	37	42	45	38	78	35	33	38	35
91%	−5	0	0	0	0	13	4	7	7	7	22	0	−1	−1	−2	113	51	56	58	52	89	44	44	47	45
92%	−4	0	0	0	0	15	6	9	9	9	23	0	0	−1	−1	128	64	74	73	68	100	57	53	59	55
93%	−2	0	0	0	0	18	8	11	12	11	25	0	0	0	−1	146	85	91	90	88	114	71	64	73	65
94%	−1	0	0	0	0	20	12	15	15	14	26	0	0	0	−1	168	105	114	109	109	129	89	77	87	79
95%	1	0	0	0	1	24	15	18	18	18	27	0	0	0	0	192	130	140	136	135	150	105	95	104	96
96%	3	0	1	2	3	29	20	22	22	22	29	0	0	0	0	231	168	178	178	179	177	132	118	132	112
97%	5	1	3	4	5	35	27	28	28	28	30	0	0	0	0	285	216	222	233	227	214	165	154	169	140
98%	9	5	5	7	8	46	38	36	38	34	33	1	0	1	1	359	292	297	312	317	263	215	203	219	200
99%	15	12	11	12	13	61	54	54	53	48	36	2	2	4	4	518	441	436	506	455	380	327	290	307	281
100%	52	47	44	48	45	275	264	242	254	205	60	22	15	24	41	2,602	2,546	5,858	3,413	2,582	1,324	1,242	1,850	1,166	1,108
Cost	**50**	**1**	**2**	**3**	**5**	**30**	**5**	**7**	**8**	**10**	**40**	**19**	**21**	**22**	**23**	**95**	**47**	**55**	**62**	**67**	**90**	**33**	**41**	**47**	**51**

Table 6: The percentage of finding Pareto solutions in algorithms.

Size	MNPV	Exact algorithm* (%)	DPTMOHS algorithm (%)
8	50	100	100
8	75	100	100
8	100	100	100
10	50	100	100
10	75	100	100
10	100	100	100
12	50	100	100
12	75	100	100
12	100	100	66

*This algorithm solves the problem by counting the entire feasible space.

Table 7: The Pareto solution of algorithms MOBDEHS for five iterations.

Solutions	Deal 1	Deal 2	Deal 3	Deal 4	Deal 5	Deal 6	Deal 7	Deal 8	Deal 9	Deal 10	Deal 11	Deal 12	Objective 1	Objective 2
	0	0	0	0	0	1	1	1					0.073353	2
	1	0	0	0	0	1	1	1					0.11078383	2.75
1	1	0	0	0	1	1	1	1					0.137424105	3.2
	1	0	1	0	1	1	0	1					0.145801031	3.8
	1	1	0	1	1	1	0	0					0.153424909	4.4
	0	0	0	0	0	1	1	1					0.051881	2
2	1	0	0	0	0	1	1	1					0.08084466	2.75
	1	0	0	0	1	1	1	1					0.100283076	3.2
	1	0	1	0	1	1	0	1					0.109298719	3.8
	0	0	0	0	0	1	0	1					0.021	2
	0	1	1	0	0	0	0	0					0.0399	2.5
3	0	0	0	1	1	0	0	0					0.0401	2.8333
	0	0	0	1	0	1	1	0					0.040931	3
	1	1	0	0	0	1	1	0					0.06071436	3.5
	1	1	0	0	1	1	0	0					0.07017149	4.25
	0	0	0	0	0	1	0	1	1	1			0.11150892	2
4	1	0	0	0	0	1	0	1	1	1			0.14675165	2.6
	1	0	0	1	0	1	0	0	1	1			0.154373822	3.2
	0	0	0	0	0	1	0	1	1	1			0.06389883	2
5	1	0	0	0	0	1	0	1	1	1			0.091650821	2.6
	1	0	0	0	1	1	0	1	0	1			0.100513936	3.2
	1	0	1	0	1	1	0	1	0	0			0.109298719	3.8
	0	0	0	0	0	1	0	1	1	1			0.04275226	2
	1	0	0	0	0	1	0	1	1	1			0.062626546	2.6
6	1	0	0	0	1	1	0	1	0	1			0.071408062	3.2
	1	0	1	0	1	1	0	1	0	0			0.080257478	3.8
	1	1	0	1	1	1	0	0	0	0			0.088969144	4.4
	0	0	0	0	0	0	1	1	0	1	1	0	0.11936526	2
7	1	0	0	0	0	1	0	1	1	1	0	0	0.14675165	2.6
	1	1	0	1	0	1	0	0	0	1	0	0	0.153449543	3.8
	0	0	0	0	0	1	1	1	0	0	1	0	0.09078712	2
8	1	0	0	0	0	1	0	1	1	1	0	0	0.091650821	2.6
	1	0	0	0	0	0	0	1	0	1	1	0	0.09976591	2.75
	1	1	0	0	0	0	1	1	0	0	1	0	0.117723069	3.2
	0	0	0	0	0	1	1	1	0	0	1	0	0.06229335	2
9	1	0	0	0	0	1	0	1	1	1	0	0	0.062626546	2.6
	1	0	0	0	0	0	0	1	1	0	1	0	0.07148536	2.75
	1	1	0	0	0	0	1	1	0	0	1	0	0.089625689	3.2

Table 8: Taguchi levels for MBHS.

Parameter	Level 1	Level 2	Level 3	Level 4
HMCR	0.7	0.9	0.95	0.99
PAR	0.1	0.3	0.4	0.5

Table 9: Taguchi levels for NSGA-II.

Parameter	Level 1	Level 2	Level 3
nPoP	20	30	40
Pcrossover	0.6	0.7	0.8
Pmutation	0.2	0.3	0.4

Table 10: The best level of MBHS algorithm.

Parameter	HMCR	PAR
Best level	0.95	0.4

Table 11: The best level of NSGA-II algorithm.

Parameter	nPoP (population size)	Pcrossover (crossover percentage)	Pmutation (mutation percentage)
Best level	20	0.7	0.4

Table 12: Comparing the performance of NSGA-II, DPTMOHS, and MBHS algorithms.

Test number	Size	Budgets	MNPV	NSGA-II			DPTMOHS			MBHS		
		B–E		MID	SDS	RAS	MID	SDS	RAS	MID	SDS	RAS
1	30	150–200	25	0.417	0.062	0.646	0.428	0.080	0.688	0.495	0.086	0.796
2	30	150–200	50	0.641	0.029	0.896	0.440	0.068	0.650	0.514	0.077	0.882
3	30	150–200	75	0.358	0.087	0.453	0.437	0.070	0.615	0.557	0.076	0.645
4	30	150–200	100	0.452	0.084	0.674	0.458	0.088	0.687	0.424	0.144	0.706
5	30	150–200	125	0.458	0.065	0.669	0.537	0.208	0.463	0.587	0.115	1.011
6	30	300–400	25	0.593	0.042	0.872	0.385	0.067	0.508	0.633	0.125	1.095
7	30	300–400	50	0.626	0.041	0.933	0.390	0.073	0.524	0.631	0.124	1.106
8	30	300–400	75	0.482	0.100	0.775	0.460	0.079	0.683	0.621	0.132	0.965
9	30	300–400	100	0.618	0.039	0.920	0.620	0.035	1.032	0.625	0.148	1.069
10	30	300–400	125	0.472	0.103	0.804	0.380	0.074	0.532	0.625	0.148	0.983
11	40	150–200	25	0.390	0.091	0.573	0.317	0.079	0.388	0.178	0.033	0.161
12	40	150–200	50	0.421	0.075	0.577	0.423	0.088	0.646	0.360	0.099	0.269
13	40	150–200	75	0.501	0.040	0.658	0.354	0.075	0.487	0.380	0.097	0.544
14	40	150–200	100	0.457	0.044	0.588	0.302	0.089	0.378	0.336	0.118	0.476
15	40	150–200	125	0.419	0.089	0.682	0.290	0.033	0.319	0.446	0.120	0.768
16	40	300–400	25	0.434	0.063	0.720	0.315	0.059	0.382	0.292	0.054	0.322
17	40	300–400	50	0.418	0.055	0.536	0.337	0.077	0.423	0.449	0.058	0.615
18	40	300–400	75	0.464	0.120	0.830	0.324	0.081	0.437	0.583	0.045	0.848
19	40	300–400	100	0.472	0.092	0.739	0.322	0.082	0.441	0.450	0.059	0.626
20	40	300–400	125	0.434	0.107	0.634	0.313	0.078	0.437	0.453	0.043	0.469
21	50	150–200	25	0.308	0.115	0.433	0.262	0.077	0.304	0.387	0.099	0.379
22	50	150–200	50	0.521	0.077	0.873	0.227	0.075	0.222	0.528	0.143	1.115
23	50	150–200	75	0.352	0.261	0.731	0.194	0.103	0.209	0.461	0.141	0.851
24	50	150–200	100	0.380	0.051	0.437	0.275	0.300	0.508	0.479	0.113	0.731
25	50	150–200	125	0.492	0.154	0.899	0.143	0.021	0.140	0.433	0.081	0.550
26	50	300–400	25	0.261	0.064	0.328	0.271	0.048	0.288	0.407	0.074	0.507
27	50	300–400	50	0.486	0.133	0.949	0.362	0.047	0.451	0.395	0.031	0.479
28	50	300–400	75	0.521	0.252	1.141	0.361	0.129	0.512	0.368	0.041	0.476
29	50	300–400	100	0.673	0.103	1.607	0.343	0.064	0.451	0.415	0.034	0.506
30	50	300–400	125	0.468	0.183	1.099	0.318	0.058	0.348	0.447	0.101	0.631

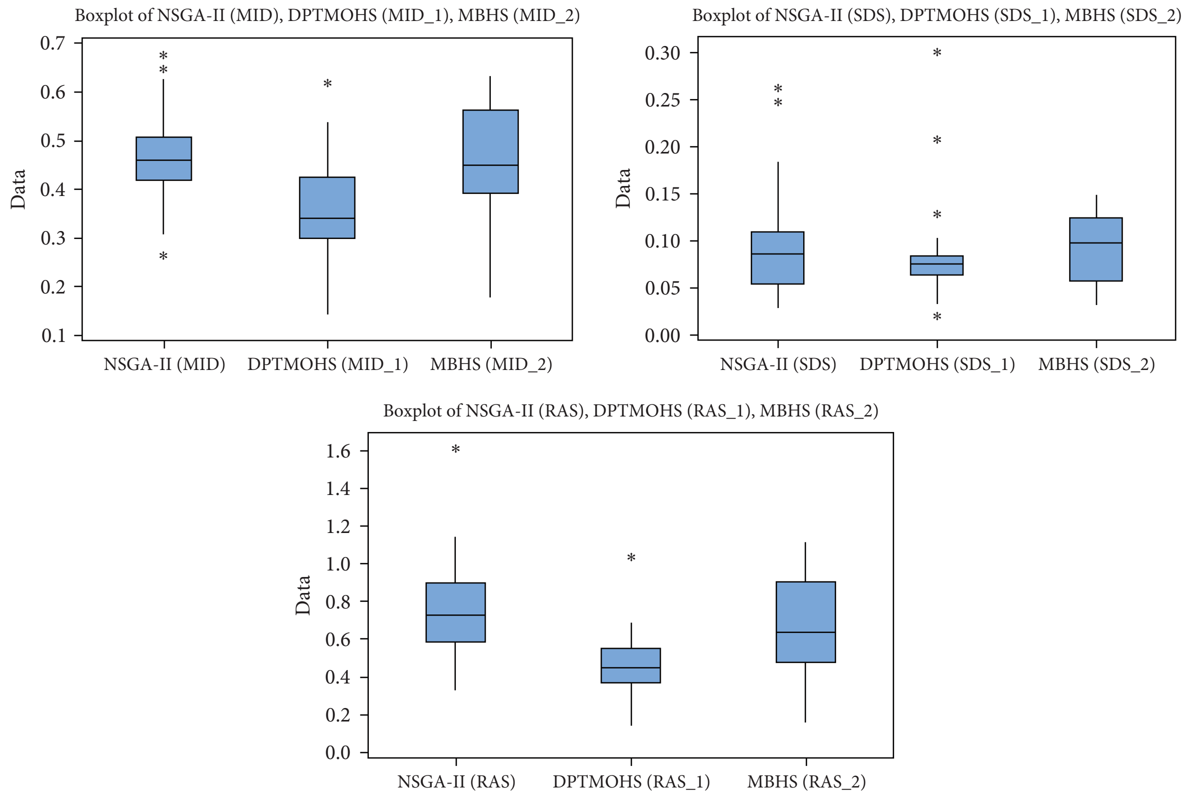

FIGURE 4: Boxplot of the RAS, MID, SDS indicators (Kruskal–Wallis test) for three algorithm.

TABLE 13: Paired t-test results to compare DPTMOHS and NSGA-II.

Indicator	Test results					Interpretation
	Paired T for MID − MID_1					
		N	Mean	StDev	SE mean	
	MID	30	0.4664	0.0952	0.0174	
	MID_1	30	0.3528	0.0977	0.0178	
MID	Difference	30	0.1135	0.1082	0.0197	Null hypothesis is rejected
	95% CI for mean difference: (0.0731, 0.1539)					
	T-test of mean difference = 0 (vs. ≠ 0): T-value = 5.75 P-value = 0.00					
	Paired T for SDS − SDS_1					
		N	Mean	StDev	SE mean	
	SDS	30	0.0941	0.0568	0.0104	
	SDS_1	30	0.0835	0.0522	0.0095	
SDS	Difference	30	0.0106	0.0782	0.0143	Null hypothesis is not rejected
	95% CI for mean difference: (−1.0186, 0.0398)					
	T-test of mean difference = 0 (vs. ≠ 0): T-value = 0.74 P-value = 0.463					
	Paired T for RAS − RAS_1					
		N	Mean	StDev	SE mean	
	RAS	30	0.7558	0.2520	0.0460	
	RAS_1	30	0.4717	0.1797	0.0325	
RAS	Difference	30	0.1135	0.1082	0.0197	Null hypothesis is rejected
	95% CI for mean difference: (0.1724, 0.3958)					
	T-test of mean difference = 0 (vs. ≠ 0): T-value = 5.2 P-value = 0.00					

Table 14: Comparison of right-tail and utility function approach.

Test problems	Number of variables	MNPV	E	B	Right tail probability	Utility function
1	40	25	150	200	52.69	66.03
2	40	25	300	400	58,54	96.60
3	40	50	150	200	59.02	57.34
4	40	50	300	400	63.92	85.25
8	40	100	300	400	99.82	94.30
9	40	125	150	200	53.08	81.31
10	40	125	300	400	53.63	95.56
11	50	25	150	200	54,09	79.48
12	50	25	300	400	73.01	73.18
13	50	50	150	200	69.31	79.71
14	50	50	300	400	55.88	108.34
15	50	75	150	200	69.85	80.43
16	50	75	300	400	83.99	148.61
17	50	100	150	200	55.00	74.17
18	50	100	300	400	87.21	133.07
19	50	125	150	200	63.90	74.38
20	50	125	300	400	39.65	65.44
Mean					**64.35**	**87.42**

in the SDS index (P-value = 0.321). However, the RAS (P-value = 0.0001) and MID (P-value = 0.0002) indexes show distinctions. According to the results of the paired t-test, the MID of the DPTMOHS algorithm is superior to that of NSGA-II, while the RAS of NSGA-II surpasses that of the DPTMOHS algorithm.

The proposed approach can be beneficial for VCs aiming to choose investees with potentially higher returns, given certain probability conditions [32]. Furthermore, we introduce a utility function that is more robust in the right tail of the PDNPV approach for portfolio selection optimization. All these methods prove highly useful for VC decision-making and for private equity investors.

6. Conclusion

VCs employ various quantitative and qualitative models for deal valuation and portfolio selection. This study introduces a novel framework for valuing and selecting deals, incorporating a combination of contractual terms, such as call options, liquidity preferences, and participant rights, within VC portfolios. VCs aim to select investee portfolios that not only yield the highest possible returns but also align with the strategic goals of their portfolio. Thus, we have proposed a multiobjective mathematical model that considers both financial and nonfinancial dimensions in VC portfolio selection. In this context, we utilize a numerical integrated method where the NPV of each deal is stochastic, determining the probability of the right tail side of the stochastic NPV portfolio. Given the NP-hard computational complexity of the model, we employ the DPTMOHS algorithm—a metaheuristic based on harmony search—to address and solve the proposed multiobjective model, followed by a thorough assessment of the algorithm's performance. Furthermore, we introduce a new robust utility function tailored for VC firms. This function demonstrates superior resilience compared to the right tail side of the NPV probability function approach when dealing with default. The proposed function empowers VC decision-makers to make more informed and resilient decisions in the face of potential failures. We encourage future researchers to explore exact algorithms to discover the best and optimum Pareto solutions. Furthermore, the integration of various types of contractual terms would significantly contribute to the advancement of this field. Moreover, we propose the development of bi-level mathematical models for deal selection, incorporating diversification indices across various investment stages and geographical locations.

Appendix

$$
\begin{aligned}
\Phi(\mathrm{MNPV}) &= Pr\{S_N > \mathrm{MNPV}\} = Pr\left\{\left(\sum_{i=1}^{N}\mathrm{npv}_i\right) > \mathrm{MNPV}\right\},\\
&= Pr\{\mathrm{npv}_1 > \mathrm{MNPV}\} + Pr\{\mathrm{npv}_1 + \mathrm{npv}_2 > \mathrm{MNPV} \cap \mathrm{npv}_1 < \mathrm{MNPV}\}\\
&\quad + Pr\{\mathrm{npv}_1 + \mathrm{npv}_2 + \mathrm{npv}_3 > \mathrm{MNPV} \cap \mathrm{npv}_1 + \mathrm{npv}_2 < \mathrm{MNPV}\}\\
&\quad + \cdots + Pr\{\mathrm{npv}_1 + \mathrm{npv}_2 + \cdots + \mathrm{npv}_N > \mathrm{MNPV} \cap \mathrm{npv}_1 + \mathrm{npv}_2 + \cdots + \mathrm{npv}_{N-1} < \mathrm{MNPV}\}\\
&\quad \Phi(\mathrm{MNPV}) = \{1 - F_{\mathrm{npv}_1}(\mathrm{MNPV})\}\\
&\quad + \int_{-\infty}^{\infty} Pr\{\mathrm{npv}_2 > \mathrm{MNPV} - \mathrm{npv}_1 \cap \mathrm{npv}_1 < \mathrm{MNPV} | \mathrm{npv}_1 = h\}\, f_{\mathrm{npv}_1}(h)\, dh\\
&\quad + \int_{-\infty}^{\infty} Pr\{\mathrm{npv}_3 > \mathrm{MNPV} - S_2 \cap S_2 < \mathrm{MNPV} | S_2 = h\}\, f_{S_2}(h)\, dh + \cdots\\
&\quad + \int_{-\infty}^{\infty} Pr\{\mathrm{npv}_{N-1} > \mathrm{MNPV} - S_{N-1} \cap S_{N-1} < \mathrm{MNPV} | S_{N-1} = h\}\, f_{S_{N-1}}(h)\, dh\\
&\quad \Phi(\mathrm{MNPV}) = \{1 - F_{\mathrm{npv}_1}(\mathrm{MNPV})\} + \int_{-\infty}^{\mathrm{MNPV}} Pr\{\mathrm{npv}_2 > \mathrm{MNPV} - h\}\, f_{\mathrm{npv}_1}(h)\, dh\\
&\quad \int_{-\infty}^{\mathrm{MNPV}} Pr\{\mathrm{npv}_3 > \mathrm{MNPV} - h\}\, f_{\mathrm{npv}_1+\mathrm{npv}_2}(h)\, dh + \cdots + \int_{-\infty}^{TNPV} Pr\{\mathrm{npv}_N > \mathrm{MNPV} - h\}\, f_{\mathrm{npv}_1+\cdots+\mathrm{npv}_{N-1}}(h)\, dh.\\
&\quad \Phi(\mathrm{MNPV}) = \{1 - F_{\mathrm{npv}_1}(\mathrm{MNPV})\} + \int_{-\infty}^{\mathrm{MNPV}} \{1 - F_{\mathrm{npv}_1}(\mathrm{MNPV} - h)\}\, f_{\mathrm{npv}_1}(h)\, dh + \int_{-\infty}^{\mathrm{MNPV}} \{1 - F_{\mathrm{npv}_2}(\mathrm{MNPV} - h)\}\, f_{S_2}(h)\, dh\\
&\quad + \cdots + \int_{-\infty}^{TNPV} \{1 - F_{\mathrm{npv}_{N-1}}(\mathrm{MNPV} - h)\}\, f_{S_{N-1}}(h)\, dh\\
&= \{1 - F_{\mathrm{npv}_1}(\mathrm{MNPV})\} + \sum_{i=1}^{N-1} \int_{-\infty}^{\mathrm{MNPV}} \{1 - F_{\mathrm{npv}_{N-1}}(\mathrm{MNPV} - h)\} f^i(h)\, dh\ , \quad f^i(h) = f_{S_i}(h)
\end{aligned}
\quad . \tag{A.1}
$$

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Disclosure

This paper is part of one of the coauthor's PhD thesis in the Industrial Engineering Department at Bu-Ali-Sina University [71].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Mohammadreza Valaei has contributed to the conceptualization, methodology, software, validation, formal analysis, investigation, data curation, writing—original draft, and Vahid Khodakarami has contributed to the writing—review & editing, supervision, project administration, funding acquisition.

Acknowledgments

This research was funded by the Iran National Science Foundation under grant number (98017405).

References

[1] Y. Pu and S. Fang, "The optimal portfolio size of venture capital under staged financing," *Procedia Computer Science*, vol. 91, pp. 85–93, 2016.

[2] A. Knill, "Should venture capitalists put all their eggs in one basket? Diversification versus pure-play strategies in venture capital," *Financial Management*, vol. 38, no. 3, pp. 441–486, 2009.

[3] D. Klonowski, *The Venture Capital Investment Process*, Springer, 2010.

[4] E. Afful-Dadzie and A. Afful-Dadzie, "A decision making model for selecting start-up businesses in a government venture capital scheme," *Management Decision*, vol. 54, no. 3, pp. 714–734, 2016.

[5] X. Zhang, "Venture capital investment selection decision-making base on fuzzy theory," *Physics Procedia*, vol. 25, pp. 1369–1375, 2012.

[6] D. Dimov and D. De Clercq, "Venture capital investment strategy and portfolio failure rate: a longitudinal study," *Entrepreneurship Theory and Practice*, vol. 30, no. 2, pp. 207–223, 2006.

[7] B. Elango, V. H. Fried, R. D. Hisrich, and A. Polonchek, "How venture capital firms differ," *Journal of Business Venturing*, vol. 10, no. 2, pp. 157–179, 1995.

[8] D. Cumming, *Venture Capital: Investment Strategies, Structures, and Policies*, John Wiley & Sons, 2010.

[9] P. A. Gompers, W. Gornall, S. N. Kaplan, and I. A. Strebulaev, "How do venture capitalists make decisions?" *Journal of Financial Economics*, vol. 135, no. 1, pp. 169–190, 2020.

[10] A. Weissmann, "Impact of real options on venture valuations," 2015.

[11] W. Gornall and I. A. Strebulaev, "A valuation model of venture capital-backed companies with multiple financing rounds," Available at SSRN 3725240, 2021.

[12] B. Payne, "Scorecard valuation methodology," 2010 Angel Valuation Survey, 2011.

[13] G. Behrmann, *Internet Company Valuation: A Study of Valuation Methods and Their Accuracy*, EBS Universität für Wirtschaft und Recht, Hamburg, 2016.

[14] L. Yan, Z. Hong, and H. Lucheng, "Review on methods of new technology valuation," in *2010 International Conference on*

E-Business and E-Government, pp. 1932–1935, IEEE, Guangzhou, China, 2010.

[15] K. Guo, L. Zhang, and T. Wang, "Optimal scheme in energy performance contracting under uncertainty: a real option perspective," *Journal of Cleaner Production*, vol. 231, pp. 240–253, 2019.

[16] S. Li, D. Hu, J. Cai, and H. Cai, "Real option-based optimization for financial incentive allocation in infrastructure projects under public-private partnerships," *Frontiers of Engineering Management*, vol. 7, pp. 413–425, 2020.

[17] M. Montajabiha, A. A. Khamseh, and B. Afshar-Nadjafi, "A robust algorithm for project portfolio selection problem using real options valuation," *International Journal of Managing Projects in Business*, vol. 10, no. 2, pp. 386–403, 2017.

[18] G. L. Nigro, G. Enea, and A. Morreale, "A user friendly real option based model to optimize pharmaceutical R&D portfolio," *International Journal of Applied Operational Research*, vol. 5, pp. 83–95, 2013.

[19] J. D. MacBeth and L. J. Merville, "An empirical examination of the Black-Scholes call option pricing model," *The Journal of Finance*, vol. 34, no. 5, pp. 1173–1186, 1979.

[20] P. Morales-Bañuelos, N. Muriel, and G. Fernández-Anaya, "A modified Black-Scholes-Merton model for option pricing," *Mathematics*, vol. 10, no. 9, Article ID 1492, 2022.

[21] S.-P. Zhu and X.-J. He, "A modified Black–Scholes pricing formula for European options with bounded underlying prices," *Computers & Mathematics with Applications*, vol. 75, no. 5, pp. 1635–1647, 2018.

[22] M. Barbati, S. Corrente, and S. Greco, "A general space-time model for combinatorial optimization problems (and not only)," *Omega*, vol. 96, Article ID 102067, 2020.

[23] X. A. F. Chaparro, L. A. de Vasconcelos Gomes, and P. T. de Souza Nascimento, "The evolution of project portfolio selection methods: from incremental to radical innovation," *Revista de Gestão*, vol. 26, no. 3, pp. 212–236, 2019.

[24] Y. Guo, L. Wang, S. Li, Z. Chen, and Y. Cheng, "Balancing strategic contributions and financial returns: a project portfolio selection model under uncertainty," *Soft Computing*, vol. 22, no. 16, pp. 5547–5559, 2018.

[25] B. Aouni, C. Colapinto, and D. La Torre, "A cardinality constrained stochastic goal programming model with satisfaction functions for venture capital investment decision making," *Annals of Operations Research*, vol. 205, no. 1, pp. 77–88, 2013.

[26] B. Aouni, C. Colapinto, and D. La Torre, "A fuzzy goal programming model for venture capital investment decision making," *INFOR: Information Systems and Operational Research*, vol. 52, no. 3, pp. 138–146, 2014.

[27] X. Zhao, W. Zhang, and J. Wang, "Risk-hedged venture capital investment recommendation," in *Proceedings of the 9th ACM Conference on Recommender Systems*, pp. 75–82, ACM, 2015.

[28] D. J. Cumming, "The determinants of venture capital portfolio size: empirical evidence," *The Journal of Business*, vol. 79, no. 3, pp. 1083–1126, 2006.

[29] M. Nitani and A. Riding, "Fund size and the syndication of venture capital investments," *Venture Capital*, vol. 15, no. 1, pp. 53–75, 2013.

[30] S. Manigart, K. De Waele, M. Wright et al., "Determinants of required return in venture capital investments: a five-country study," *Journal of Business Venturing*, vol. 17, no. 4, pp. 291–312, 2002.

[31] B. Huntsman and J. P. Hoban Jr., "Investment in new enterprise: some empirical observations on risk, return, and market structure," *Financial Management*, vol. 9, no. 2, pp. 44–51, 1980.

[32] A. Retterath and S. Kavadias, "How to hit home runs: portfolio strategies and returns in formal and informal venture capital," Available at SSRN 3527412, 2020.

[33] S. de Treville, J. S. Petty, and S. Wager, "Economies of extremes: lessons from venture-capital decision making," *Journal of Operations Management*, vol. 32, no. 6, pp. 387–398, 2014.

[34] J. Charnes, *Financial Modeling with Crystal Ball and Excel,+ Website*, John Wiley & Sons, 2012.

[35] N. Larsen, H. Mausser, and S. Uryasev, *Algorithms for Optimization of Value-at-Risk*, Springer, 2002.

[36] F. Hooshmand, Z. Anoushirvani, and S. A. MirHassani, "Model and efficient algorithm for the portfolio selection problem with real-world constraints under value-at-risk measure," *International Transactions in Operational Research*, vol. 30, no. 5, pp. 2665–2690, 2023.

[37] A. H. Ayob and L.-P. Dana, "Product strategies for export ventures: an empirical investigation among SMEs in an emerging economy," *Jurnal Pengurusan (UKM Journal of Management)*, vol. 50, pp. 25–33, 2017.

[38] S. Park and Z.-T. Bae, "New venture strategies in a developing country: identifying a typology and examining growth patterns through case studies," *Journal of Business Venturing*, vol. 19, no. 1, pp. 81–105, 2004.

[39] H. Zhong, C. Liu, J. Zhong, and H. Xiong, "Which startup to invest in: a personalized portfolio strategy," *Annals of Operations Research*, vol. 263, no. 1-2, pp. 339–360, 2018.

[40] T. A. Herberger and F. Reinle, "A framework for screening and portfolio selection in corporate venture capital," *Studies in Economics and Finance*, vol. 37, no. 3, pp. 475–495, 2020.

[41] S. A. Hasheminejad, M. Bagherpour, S. Nouri, and M. S. Pishvaee, "Syndicated venture capital portfolio companies selection: a fuzzy inference system—agent-based approach," *International Journal of Computer Mathematics*, vol. 98, no. 11, pp. 2186–2201, 2021.

[42] M. Valaei and V. Khodakarami, "A new multi-dimensional framework for start-ups lifespan assessment using Bayesian networks," *Journal of Risk and Financial Management*, vol. 16, no. 2, Article ID 88, 2023.

[43] F. Black and M. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy*, vol. 81, no. 3, pp. 637–654, 1973.

[44] K. Reddy and V. Clinton, "Simulating stock prices using geometric brownian motion: evidence from Australian companies," *Australasian Accounting, Business and Finance Journal*, vol. 10, no. 3, pp. 23–47, 2016.

[45] M. Hifi, M. Michrafy, and A. Sbihi, "Heuristic algorithms for the multiple-choice multidimensional knapsack problem," *Journal of the Operational Research Society*, vol. 55, no. 12, pp. 1323–1332, 2004.

[46] S. Benati and R. Rizzi, "A mixed integer linear programming formulation of the optimal mean/value-at-risk portfolio problem," *European Journal of Operational Research*, vol. 176, no. 1, pp. 423–434, 2007.

[47] K. Hans, P. Ulrich, and P. David, *Knapsack Problems*, Spinger-Verlag, Berlin, 2004.

[48] X. Kong, L. Gao, H. Ouyang, and S. Li, "Solving large-scale multidimensional knapsack problems with a new binary harmony search algorithm," *Computers & Operations Research*, vol. 63, pp. 7–22, 2015.

[49] V. Cacchiani, M. Iori, A. Locatelli, and S. Martello, "Knapsack problems—an overview of recent advances. Part II: multiple,

multidimensional, and quadratic knapsack problems," *Computers & Operations Research*, vol. 143, Article ID 105693, 2022.
[50] P. B. Soares, M. R. N. Guimarães, and F. F. de Lara, "Analysis of the internal strategic alignment of the production strategy: case study in an auto parts company of the municipality of Sorocaba/SP," *Gestão & Produção*, vol. 26, no. 4, Article ID e2762, 2019.
[51] G. J. C. da Silveira, "Market priorities, manufacturing configuration, and business performance: an empirical analysis of the order-winners framework," *Journal of Operations Management*, vol. 23, no. 6, pp. 662–675, 2005.
[52] A. Buchner, A. Mohamed, and A. Schwienbacher, "Diversification, risk, and returns in venture capital," *Journal of Business Venturing*, vol. 32, no. 5, pp. 519–535, 2017.
[53] R. Cressy, A. Malipiero, and F. Munari, "Does VC fund diversification pay off? An empirical investigation of the effects of VC portfolio diversification on fund performance," *International Entrepreneurship and Management Journal*, vol. 10, no. 1, pp. 139–163, 2014.
[54] D. W. Coit, "Cold-standby redundancy optimization for nonrepairable systems," *IIE Transactions*, vol. 33, no. 6, pp. 471–478, 2001.
[55] M. A. Akbay, C. B. Kalayci, and O. Polat, "A parallel variable neighborhood search algorithm with quadratic programming for cardinality constrained portfolio optimization," *Knowledge-Based Systems*, vol. 198, Article ID 105944, 2020.
[56] S. Hong, K. Serfes, and V. Thiele, "Competition in the venture capital market and the success of startup companies: theory and evidence," *Journal of Economics & Management Strategy*, vol. 29, no. 4, pp. 741–791, 2020.
[57] R.-J. Luo, S.-F. Ji, and B.-L. Zhu, "A Pareto evolutionary algorithm based on incremental learning for a kind of multiobjective multidimensional knapsack problem," *Computers & Industrial Engineering*, vol. 135, pp. 537–559, 2019.
[58] X. Kong, L. Gao, H. Ouyang, and S. Li, "A simplified binary harmony search algorithm for large scale 0–1 knapsack problems," *Expert Systems with Applications*, vol. 42, no. 12, pp. 5337–5355, 2015.
[59] D. Zou, L. Gao, S. Li, and J. Wu, "Solving 0–1 knapsack problem by a novel global harmony search algorithm," *Applied Soft Computing*, vol. 11, no. 2, pp. 1556–1564, 2011.
[60] M. R. Valaei and J. Behnamian, "Allocation and sequencing in 1-out-of-*N* heterogeneous cold-standby systems: multiobjective harmony search with dynamic parameters tuning," *Reliability Engineering & System Safety*, vol. 157, pp. 78–86, 2017.
[61] M. Valaei, "A new meta-heuristic algorithm based on harmony search to optimize the reliability of parallel-systems by considering the allocation of redundant series components," in *Industrial Engineering*, A. Kheirkhah, Ed., Bual Sina University, Hemedan, 2015.
[62] Z. W. Geem, J. H. Kim, and G. V. Loganathan, "A new heuristic optimization algorithm: harmony search," *SIMULATION*, vol. 76, no. 2, pp. 60–68, 2001.
[63] N. Nahas and D. Thien-My, "Harmony search algorithm: application to the redundancy optimization problem," *Engineering Optimization*, vol. 42, no. 9, pp. 845–861, 2010.
[64] M. Mahdavi, M. Fesanghary, and E. Damangir, "An improved harmony search algorithm for solving optimization problems," *Applied Mathematics and Computation*, vol. 188, no. 2, pp. 1567–1579, 2007.
[65] L. Wang, L. An, H.-Q. Ni, W. Ye, P. M. Pardalos, and M.-R. Fei, "Pareto-based multi-objective node placement of industrial wireless sensor networks using binary differential evolution harmony search," *Advances in Manufacturing*, vol. 4, no. 1, pp. 66–78, 2016.
[66] L. Wang, Y. Mao, Q. Niu, and M. Fei, "A multi-objective binary harmony search algorithm," in *Advances in Swarm Intelligence. ICSI 2011*, vol. 6729 of *Lecture Notes in Computer Science*, pp. 74–81, Springer, Berlin Heidelberg, 2011.
[67] J. Behnamian and Z. Gharabaghli, "Multi-objective outpatient scheduling in health centers considering resource constraints and service quality: a robust optimization approach," *Journal of Combinatorial Optimization*, vol. 45, no. 2, Article ID 80, 2023.
[68] D. Abbasi, M. Ashrafi, and S. H. Ghodsypour, "A multi objective-BSC model for new product development project portfolio selection," *Expert Systems with Applications*, vol. 162, Article ID 113757, 2020.
[69] X. Li, B. Li, T. Jin, and P. Zheng, "Uncertain random portfolio optimization with non-dominated sorting genetic algorithm-II and optimal solution criterion," *Artificial Intelligence Review*, vol. 56, no. 8, pp. 8511–8546, 2023.
[70] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.
[71] M. Valaei, *Valuation and optimization of venture capital deals portfolio based on risk approach*, PhD thesis, Industerial engineering Department https://eng.basu.ac.ir/en/ie-seminars, Bu-Ali Sina University, Hamedan, Iran, 2023.