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Research Article

Analysis of Age-Structured Mathematical Model of Malaria Transmission Dynamics via Classical and ABC Fractional Operators

Ademe Kebede Gizaw and Chernet Tuge Deressa

Department of Mathematics, College of Natural Sciences, Jimma University, Jimma, Ethiopia

Correspondence should be addressed to Ademe Kebede Gizaw; kebedeademe2020@gmail.com

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Malaria is a complex disease with many factors influencing the transmission dynamics, including age. This research analyzes the transmission dynamics of malaria by developing an age-structured mathematical model using the classical integer order and Atangana–Baleanu–Caputo fractional operators. The analysis of the model focused on several important aspects. The existence and uniqueness of solutions of fractional order were explored based on some fixed-point theorems,such as Banach and Krasnoselski. The Positivity and boundedness of the solutions were also investigated. Furthermore, through mathematical analysis techniques, we analyzed different types of stability results, and the results showed that the disease-free equilibrium point of the model is proved to be both locally and globally asymptotically stable if the basic reproduction number is less than one, whereas the endemic equilibrium point of the model is both locally and globally asymptotically stable if the basic reproduction number is greater than one. The findings from the sensitivity analysis revealed that the most sensitive parameters, essential for controlling or eliminating malaria are mosquito biting rate, density-dependent natural mortality rate, clinical recovery rate, and recruitment rate for mosquitoes. Numerical simulations are also performed to examine the behavior of the model for different values of the fractional-order alpha,and the result revealed that as the value α reduces from 1, the spread of the endemic grows slower. By incorporating these findings, this research helps to clarify the dynamics of malaria and provides information on how to create efficient control measures.

1. Introduction

Malaria, originally known as the Latin word “bad air” in ancient Roman times, is still a major life-threatening disease that happens to be vector-borne, and is one of the most deadly infectious diseases worldwide [1]. It hurts people’s health as well as economic development in many developing nations, especially in sub-Saharan Africa [2]. It is endemic in over 85 countries by the World Health Organization (WHO) [3]. By biting a person, an adult female infected with Anopheles mosquito transmits the *Plasmodium* parasite [4].

At least five species of *Plasmodium* parasites commonly cause human malaria: *P. falciparum*, *P. vivax*, *P. ovale*, *P. malariae*, and *P. knowlesi* [5]. Of these, two species—*P. falciparum* and *P. vivax*—pose the greatest threat. *P. falciparum* is responsible for the majority of infections worldwide and is the dominant species in sub-Saharan Africa [6]. While *P. vivax*, of the five malaria parasites, has the widest geographic distribution because it can survive at lower temperatures within a mosquito than the other four parasites that infect humans, it can also cause extremely severe malaria in children [7]. *P. falciparum* and *P. vivax* are the most dominant malaria parasites in Ethiopia, accounting for 60% and 40% of malaria cases, respectively [8].

The data released by the WHO indicate that there were about 247-million cases and 619,000 deaths in 2021 [1]. Approximately, 96% of the confirmed deaths were from the African region, 76% were children under 5 years of age, and 32% of pregnant women were exposed to malaria infection. This makes malaria one of the most serious health challenges. The

mathematical modeling of infectious diseases has proved to play an important role in understanding the insights of the transmission dynamics and appropriate control strategies [9]. Several mathematical models and many scientific efforts have been made to reduce the impact of malaria on humans. Ross [10] was the first to develop a mathematical model for studying the dynamics of human malaria infection.

According to Ross [10], if the mosquito population can be reduced to below a certain threshold, then malaria can be eradicated. Following the Ross model, several models were carried out by various researchers by taking into account a variety of parameters. For instance, Macdonald [11] made some modifications to Rose's model and concluded that reducing the number of mosquitoes is not enough to eradicate or mitigate the burden of malaria in areas of intense transmission.

Numerous mathematical models relating to malaria have been developed as a result of an increased understanding of the biology and epidemiology of the disease, mostly by expanding the two fundamental models created by Ross [10] and Macdonald [11]. The models included a number of features to increase their biological realism and forecast the disease's prevalence. The main factors considered in the models were human population migration and visitation [12], human age structure [13], an age-structured model of malaria transmission with acquired immunity [14], etc.

As far as the researchers are aware, all these models employ integer-order derivatives in their differential equations. Fractional calculus, a branch of applied mathematics that extends integer calculus to noninteger orders, has found numerous applications in diverse fields such as engineering, control networks, physical systems, and mathematical modeling [15–20].

In recent years, researchers have been developing mathematical models using fractional-order differential equations (FODEs) in a wide range of fields like physics, thermodynamics, viscoelasticity, electrical theory, mechatronics, medicine, chemistry, chaos theory, finance, and economics ([21] and other references cited within). The main reasons given for using fractional derivative models are that time-fractional operators enable memory effects (i.e., the response of a system is a function of its history), while space-fractional operators enable nonlocal and scale effects [22]. Furthermore, fractional-order operators enlarge the region of stability and capture the memory dynamics and genetic properties that exist in both biological and engineering systems [23]. It can also provide a better fit for real data for the different disease models [24, 25].

Many researchers make use of fractional-order derivatives to model real-life world problems, but few of them are commonly used, including Riemann–Liouville [26], Caputo [27], Caputo and Fabrizio (CF) [28], and Atangana and Baleanu (AB) [29]. All these definitions of the fractional derivatives have their advantages and disadvantages. For example, Riemann-Liouville and Caputo operators are called fractional derivatives with singular kernels [30]. They have the disadvantage that their kernel has a singularity at the endpoint of the interval. Two new nonsingular fractional derivatives with an exponential function and a Mittag–Leffler function kernel, respectively, were developed by Caputo and Fabrizio (CF) [28] and Atangana and Baleanu (AB) [29] to address this issue. The CF with an exponential kernel is limited in its ability to describe phenomena of nonexponential characters, such as anomalous relaxation, because solutions due to an exponential kernel exhibit an exponential decline comparable to the conventional integer order model [31]. Atangana and Baleanu (AB) [29] defined two fractional derivatives in the Caputo and Riemann–Liouville senses based on the generalized stretched Mittag–Leffler function to address this shortcoming.

In the setting of fractional calculus, the Mittag–Leffler function serves as a generalization of the exponential function. Its key benefit lies in its nonlocal and nonsingular behavior. The ABC fractional operators also offer a more comprehensive definition of the crossover property in the epidemic models. Thus, researchers have determined that the ABC fractional operator stands as the most suitable choice for simulating real-world occurrences, such as pandemic diseases [32]. Further exploration of this operator's applicability to models can be found in [33–36]. Building on this understanding, scholars have employed this operator to create numerous mathematical models. The outcomes of these simulations have resoundingly demonstrated the applicability and effectiveness of the ABC operator.

Motivated by and inspired from the above discussions, this paper analyzes a mathematical model of age-structured malaria disease dynamics and transmission, including children, adults, and pregnant women, using both classical (integer) order and AB fractional order operators in the Caputo sense. This model extends the integer malaria disease transmission model of Azu-Tungmah et al. [37] to a fractional-order model, incorporating an exposed mosquito class.

The paper is organized as follows: Section 2 presents the formulation of the mathematical model. Section 3 describes an integer-order mathematical model analysis of malaria. Section 4 presents a mathematical model analysis for the fractional order of malaria disease transmission. Numerical simulation results are presented in Section 5, Section 6 contains discussion, and Section 7 contains conclusions.

2. Model Formulation

This model is an extension of the integer-order model of malaria transmission dynamics proposed by Tungmah et al. [37], along with the addition of the exposed class to mosquitoes. The fractional derivative is defined as the Atangana-Baleanu fractional order derivative in the Caputo sense. The model is formulated as follows: with human and mosquito subgroups. The human subgroup is further divided into four compartments: susceptible $(S_{\mathrm{H}}(t))$, infectious under 5 years $(I_{\mathrm{I}}(t))$, infectious over 5 years $(I_{\mathrm{A}}(t))$, and infectious pregnant women $(\mathrm{I}_{\mathrm{P}}(t))$.

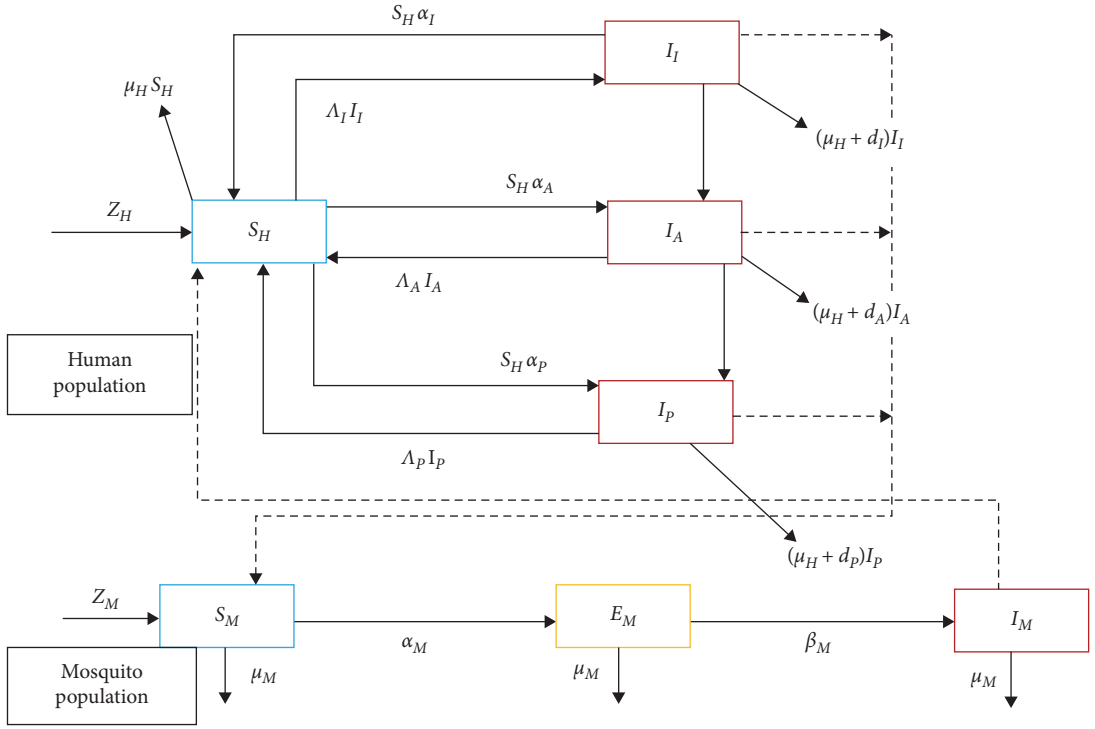

Figure 1: Flowchart for the malaria transmission dynamics.

At a per capita recruitment rate (Z_H), individuals enter the human population through the susceptible ($S_H(t)$) compartment. When human malaria infection occurs, persons under the age of five transition to compartment ($I_I(t)$), those above the age of five who are not pregnant move to the compartment ($I_A(t)$), and those who are pregnant move to the compartment ($I_P(t)$). Clinical treatment is administered to those in the infectious compartments ($I_I(t)$), ($I_A(t)$), and ($I_P(t)$) at the rates of Λ_I, Λ_A, and Λ_P before they return to the ($S_H(t)$) compartment for reinfection.

Also, at the rates (d_I), (d_A), and (d_P), respectively, infectious people can die from sickness and leave the human population. When a child turns 5-year old, they can join the infected compartment of the infectious over five compartments at the rate ($\varnothing$), and they can join the infected pregnant women compartment at the rate of Ω when they turn 5 and are also infectious over 5 years. It is anticipated that infectious pregnant women cannot enter the infectious beyond the 5-year compartment since the majority of infectious pregnant women receives clinical treatment before giving birth. Also, the mortality rate (N_H) of humans in each compartment is dependent on the population density.

Thus, the total human population $N_H(t) = S_H(t) + I_I(t) + I_A(t) + I_P(t)$. The mosquito population is divided into three compartments: susceptible mosquitoes ($S_M(t)$), exposed mosquitoes ($E_M(t)$), and infected mosquitoes ($I_M(t)$).Hence, the total mosquito population $N_M(t) = S_M(t) + E_M(t) + I_M(t)$.

Table 1: The state variables for the Model 1.

State variables	Description
$S_H(t)$	Number of susceptible humans at time t
$I_I(t)$	Number of infectious infants at time t
$I_A(t)$	Number of infectious adults at time t
$I_P(t)$	Number of infectious pregnant women at time t
$S_M(t)$	Number of susceptible mosquitoes at time t
$E_M(t)$	Number of exposed mosquitoes at time t
$I_M(t)$	Number of infectious mosquitoes at time t
$N_H(t)$	The total human population at time t
$N_M(t)$	Total mosquito population at time t

In Figure 1, the dotted arrows depict the interaction and transmission of disease between humans and mosquitoes, while the solid arrows depict the passage of individuals from one compartment to another.

3. Formulation of an Integer-Order Compartmental Malaria Model

The model's state variables are presented in Table 1, and its parameters are outlined in Table 2. Building upon these variables and parameters, we extend the classical integer model of Tungmah et al. [37] by incorporating an exposed mosquito class. The extended model is as follows:

Table 2: Variable of model.

Variables	Description	Dimension
z_H	Recruitment for the human population	Humans × Time_−1
z_M	Recruitment for the mosquitoe population	Time −1
μ_H	Density-dependent natural mortality rate for humans	Time −1
μ_M	Density-dependent natural mortality rate for adult female Anopheles mosquitoes	Time −1
d_I	Per capita disease-induced mortality rate for people under 5 years	Time −1
d_A	Per capita disease-induced mortality rate for people over 5 years	Time −1
d_P	Per capita disease-induced mortality rate for pregnant women	Time −1
Λ_I	The clinical recovery rate for people under 5 years	Time −1
Λ_A	The clinical recovery rate for people over 5 years	Time −1
Λ_P	The clinical recovery rate for pregnant women	Time −1
ϕ_I	Number of bites on people under 5 years per female mosquito per unit time	Time −1
ϕ_A	Number of bites on people over 5 years per female mosquito per unit time.	Time −1
ϕ_P	Number of bites on pregnant women per female mosquito per unit time.	Time −1
θ_{MH}	Fraction of bites that successfully infect humans	Time_−1
θ_{HM}	Fraction of bites that successfully infect mosquitoes	Time −1
β_M	Rate of progression from S_E to E_M compartment	Time −1
α_M	Rate of progression from E_M to I_M compartment	Time −1
ϕ	Rate of progression from I_I to I_A compartment	Humans × Time_−1
Ω	Rate of progression from I_A to I_P compartment	Humans × Time_−1

$$\begin{cases} S'_H(t) = Z_H + \Lambda_I I_I + \Lambda_A I_A + \Lambda_P I_P - (\alpha_I + \alpha_A + \alpha_P + \mu_H) S_H, \\ I'_I(t) = \alpha_I S_H - (\Lambda_I + \mu_H + d_I + \phi) I_I, \\ I'_A(t) = \alpha_A S_H + \phi I_I - (\mu_H + d_A + \Lambda_A + \Omega) I_A, \\ I'_P(t) = \alpha_P S_H + \Omega I_A - (\mu_H + d_P + \Lambda_P) I_P, \\ S'_M(t) = Z_M - (\alpha_M + \mu_M) S_M, \\ E'_M(t) = \alpha_M S_M - (\mu_M + \beta_M) E_M, \\ I'_M(t) = \beta_M E_M - \mu_M I_M, \end{cases} \tag{1}$$

with initial conditions $S_H(0) = S_{H0}$, $I_I(0) = I_{I0}$, $I_A(0) = I_{A0}$, $I_P(0) = I_{P0}$, $S_M(0) = S_{M0}$, $E_M(0) = E_{M0}$, and $I_M(0) = I_{M0}$.

Applying the definitions of the force of infections as indicated in the Addawe and Lope [38] model, the force of infections for infants, adults, and pregnant women are as follows:

$$\alpha_I = \frac{\varnothing_I \theta_{MH} I_M}{N_H}, \alpha_A = \frac{\varnothing_A \theta_{MH} I_M}{N_H}, \alpha_P = \frac{\varnothing_P \theta_{MH} I_M}{N_H}. \tag{2}$$

The force of infection for mosquitoes is as follows:

$$\alpha_M = \frac{(\varnothing_I I_I + \varnothing_A I_A + \varnothing_P I_P)\theta_{HM}}{N_H}. \tag{3}$$

4. The Integer-Order Model Analysis

This section presents the key characteristics of the model system of Equation (1), including the existence and uniqueness of solutions, positivity of solutions, bounds, basic reproduction number, equilibria with their stability analysis, and sensitivity analysis.

4.1. Existence and Uniqueness of Solutions, Positivity of Solutions, and Boundedness of Model Solutions. The mathematical well-posedness of a model relies on key elements like the existence and uniqueness of solutions, the positivity of solutions and boundedness. These elements ensure the model is physically meaningful, epidemiologically sound, and generates accurate and dependable forecasts.

Theorem 1 (existence and uniqueness of solutions). *If S_{H0}, I_{I0}, I_{A0}, I_{P0}, S_{M0}, E_{M0}, and I_{M0} are positive, then there exists a unique solution $(S_H(t), I_I(t), I_A(t), I_P(t), S_M(t), E_M(t), I_M(t))$ to system (1) in R^7_+, for all $t \geq 0$.*

Proof. To demonstrate the existence of a solution for system (1), we first rewrite the system in the form:

$X' = F(X)$,

where $X = (S_H(t), I_I(t), I_A(t), I_P(t), S_M(t), E_M(t), I_M(t))^T \in R^7_+$ and $F(X)$ is given by

$$F(X)=\begin{bmatrix} F_1(X)\\ F_2(X)\\ F_3(X)\\ F_4(X)\\ F_5(X)\\ F_6(X)\\ F_7(X)\end{bmatrix}=\begin{pmatrix} Z_{\mathrm{H}}+\Lambda_{\mathrm{I}}I_{\mathrm{I}}+\Lambda_{\mathrm{A}}I_{\mathrm{A}}+\Lambda_{\mathrm{P}}I_{\mathrm{P}}-\left(\frac{(\varnothing_{\mathrm{I}}+\varnothing_{\mathrm{A}}+\varnothing_{P})\mu_{\mathrm{H}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}+\mu_{\mathrm{H}}\right)S_{\mathrm{H}},\\ \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}-(\Lambda_{\mathrm{I}}+\mu_{\mathrm{H}}+d_{\mathrm{I}}+\phi)I_{\mathrm{I}},\\ \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}+\phi I_{\mathrm{I}}-(\mu_{\mathrm{H}}+d_{\mathrm{A}}+\Lambda_{\mathrm{A}}+\Omega)I_{\mathrm{A}},\\ \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}+\Omega I_{\mathrm{A}}-(\mu_{\mathrm{H}}+d_{\mathrm{P}}+\Lambda_{\mathrm{P}})I_{\mathrm{P}},\\ Z_{\mathrm{M}}-\frac{(\varnothing_{\mathrm{I}}I_{\mathrm{I}}+\varnothing_{\mathrm{A}}I_{\mathrm{A}}+\varnothing_{\mathrm{P}}I_{\mathrm{P}})\mu_{\mathrm{H}}S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}-\mu_{\mathrm{M}}S_{\mathrm{M}},\\ \frac{(\varnothing_{I}I_{I}+\varnothing_{\mathrm{A}}I_{\mathrm{A}}+\varnothing_{\mathrm{P}}I_{\mathrm{P}})\mu_{\mathrm{H}}S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}-(\mu_{\mathrm{M}}+\beta_{\mathrm{M}})E_{\mathrm{M}},\\ \beta_{\mathrm{M}}E_{\mathrm{M}}-\mu_{\mathrm{M}}I_{\mathrm{M}}.\end{pmatrix} \tag{4}$$

Note that, since $F_1(X)$, $F_2(X),\ldots,F_7(X)$ are all C^1, the existence of at least one solution for system (1) is guaranteed. Moreover, we obtain:

$\|F(X_1)-F(X_2)\|_1<L\|X_1-X_2\|_1$, where $X_1,X_2\in R_+^7$, with $L=\max\{B_1,B_2,\ldots,B_7\}$, where $B_1=\|\frac{(\varnothing_{\mathrm{I}}+\varnothing_{\mathrm{A}}+\varnothing_{\mathrm{P}})\mu_{\mathrm{H}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}+\mu_{\mathrm{H}}\|$, $B_2=\|\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}+\alpha_{\mathrm{P}}+\mu_{\mathrm{H}}\|$, $B_3=\mu_{\mathrm{H}}+d_{\mathrm{A}}+\Lambda_{\mathrm{A}}+\Omega$, $B_4=\mu_{\mathrm{H}}+d_{\mathrm{P}}+\Lambda_{\mathrm{P}}$, $B_5=\|\frac{(\varnothing_{\mathrm{I}}I_{\mathrm{I}}+\varnothing_{\mathrm{A}}I_{\mathrm{A}}+\varnothing_{\mathrm{P}}I_{\mathrm{P}})\mu_{\mathrm{H}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}+\mu_{\mathrm{M}}\|$, $B_6=\mu_{\mathrm{M}}+\beta_{\mathrm{M}}$, $B_7=\mu_{\mathrm{M}}$.

Thus, the function F is locally Lipschitzian in $X(t)$.

Consequently, it then follows through the Cauchy–Lipschitz theorem [39] that system (1) has a unique local solution. □

Theorem 2 (nonnegativity of model solutions). *If the initial data S_{H0}, I_{I0}, I_{A0}, I_{P0}, S_{M0}, E_{M0}, and I_{M0} are nonnegative, the solution $(S_{\mathrm{H}}(t),\ I_{\mathrm{I}}(t),\ I_{\mathrm{A}}(t),\ I_{\mathrm{P}}(t),\ S_{\mathrm{M}}(t),\ E_{\mathrm{M}}(t),\ I_{\mathrm{M}}(t))$ to system Equation (1) is nonnegative for all time $t\geq 0$.*

Proof. The purpose of this subsection is to demonstrate how all solutions of the model Equation (1) remain nonnegative if their initial data are nonnegative.

To prove this, from the first Equation in (1), we get

$S_{\mathrm{H}}'(t)+g(t)S_{\mathrm{H}}=f(t)$, where $f(t)=Z_{\mathrm{H}}+\Lambda_{\mathrm{I}}I_{\mathrm{I}}+\Lambda_{\mathrm{A}}I_{\mathrm{A}}+\Lambda_{\mathrm{P}}I_{\mathrm{P}}$ and $g(t)=\frac{(\varnothing_{\mathrm{I}}+\varnothing_{\mathrm{A}}+\varnothing_{\mathrm{P}})\mu_{\mathrm{H}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}+\mu_{\mathrm{H}}$.

Thus, the general solution to the first Equation in (1) is as follows:

$$S_{\mathrm{H}}(t)=e^{-\int g(t)dt}\left[\int_0^t e^{\int g(s)ds}(Z_{\mathrm{H}}+\Lambda_{\mathrm{I}}I_{\mathrm{I}}(s)+\Lambda_{\mathrm{A}}I_{\mathrm{A}}(s)+\Lambda_{\mathrm{P}}I_{\mathrm{P}}(s))ds+S_{\mathrm{H}}(0)\right]. \tag{5}$$

Therefore, the positivity of the solutions $I_{\mathrm{I}}(t)$, $I_{\mathrm{A}}(t)$, and $I_{\mathrm{P}}(\mathrm{t})$ for all $t>0$, allows us to guarantee the positivity of $S_{\mathrm{H}}(t)$. Now, we confirm that $I_{\mathrm{I}}(t)$ given in system (1) is nonnegative for all $t\geq 0$. Suppose that the positivity does not hold, therefore there must be a $t_0>0$ such that $I_{\mathrm{I}}(t_0)=0$, $I_{\mathrm{I}}'(t_0)\leq 0$, and $I_{\mathrm{I}}(t)>0$ for all $t\in[0,t_0)$, because the initial condition $I_{\mathrm{I0}}>0$.Thus, $I_{\mathrm{I}}(t)$ must be negative for some t_0. However, in the interval $[0,\mathrm{t}_0)$ the function $I_{\mathrm{I}}(t)$ is positive, and at t_0, $I_{\mathrm{I}}'(t)$ is nonpositive. Thus, from the second equation of model (1), it follows that for t_0,

$$I_{\mathrm{I}}'(t_0)=\frac{\varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}I_{\mathrm{M}}(t_0)}{N_{\mathrm{H}}}S_{\mathrm{H}}(t_0)\geq 0. \tag{6}$$

This contradicts that $I_{\mathrm{I}}'(t_0)\leq 0$. Hence, we must have $I_{\mathrm{I}}(t)>0$, for all $t\geq 0$.

Similarly, it can be shown that $I_{\mathrm{A}}(t)>0$ and $I_{\mathrm{P}}(t)>0$ for all $t\geq 0$. Hence, we concluded that the nonnegativity of the solutions $I_{\mathrm{I}}(t)$, $I_{\mathrm{A}}(t)$, and $I_{\mathrm{P}}(t)$ for all $t\geq 0$, allows us to guarantee the nonnegative of $S_{\mathrm{H}}(t)$.

The fifth equation in Equation (1) can be rewritten as follows:

$\frac{dS_{\mathrm{M}}(t)}{dt}+h(t)S_{\mathrm{M}}(t)=Z_{\mathrm{M}}$, where

$$h(t)=\frac{(\varnothing_{\mathrm{I}}I_{\mathrm{I}}(t)+\varnothing_{\mathrm{A}}\mathrm{I}_{\mathrm{A}}(t)+\varnothing_{\mathrm{P}}I_{\mathrm{P}}(t))\mu_{\mathrm{H}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}+\mu_{\mathrm{M}}. \tag{7}$$

As a result, $S_M(t) = e^{-\int_0^t h(s)ds}(Z_H \int_0^t e^{\int_0^t h(s)ds} ds + S_M(0)) \geq 0$, since exponential functions are always nonnegative. It can also be shown that $E_M(t) \geq 0$ and $I_M(t) \geq 0$ are nonnegative for all $t_0 > 0$. This completes the proof of Theorem 2.□

Theorem 3 (boundedness of model solutions). *All solutions* $(S_H, I_I, I_A, I_P, S_M, E_M, I_M) \in R_+^7$ *of the malaria model Equation (1) are bounded, meaning that*

(i) *if* $N_H(t) = S_H(t) + I_I(t) + I_A(t) + I_P(t)$, *then* $\text{Lim}_{t\to\infty} \text{Sup} N_H(t) \leq \frac{Z_H}{\mu_H}$.
(ii) *if* $N_M(t) = S_M(t) + E_M(t) + I_M(t)$, *then* $\text{Lim}_{t\to\infty} \text{Sup} N_H(t) \leq \frac{Z_H}{\mu_H}$.

Proof. The human population and the mosquito population are the two segments of model Equation (1). $N_H(t) = S_H(t) + I_I(t) + I_A(t) + I_P(t)$ represents the total human population. Using the first four equations in the model and differentiating both sides of $N_H(t)$ about time, we obtain

$$\frac{dN_H(t)}{dt} = Z_H - \mu_H N_H - \left(d_1 I_I + d_2 I_A + d_p I_P\right). \tag{8}$$

This implies,

$$\frac{dN_H(t)}{dt} \leq Z_H - \mu_H N_H. \tag{9}$$

Therefore,

$$N_H(t) \leq \frac{Z_H}{\mu_H} + \left(N_{H0} - \frac{Z_H}{\mu_H}\right) e^{-\mu_{Ht}}. \tag{10}$$

So, as $t \to \infty$, the human population N_H approaches $\frac{Z_H}{\mu_H}$, and

$$N_H(t) = S_H(t) + I_I(t) + I_A(t) + I_P(t) \leq \frac{Z_H}{\mu_H}. \tag{11}$$

At $t = 0$, Equation (11) yields $N_H(0) \geq 0$ and $N_H(0) \leq \frac{Z_H}{\mu_H}$.

Hence, the bounded region of the system (1) for the human population is, therefore,given by:

$$\Omega_H = \left\{(S_H, I_I, I_A, I_P) \in R_+^4 : 0 \leq S_H + I_I + I_A + I_P \leq \frac{Z_H}{\mu_H}\right\}. \tag{12}$$

Using the last three equations in model (1), differentiating both sides of $N_M(t)$ with respect to time, and solving, we obtain:

$$N_M(t) = S_M(t) + E_M(t) + I_M(t) \leq \frac{Z_M}{\mu_M}. \tag{13}$$

Now, if $t = 0$, Equation (13) implies $N_M(0) \geq 0$ and $N_M(0) \leq \frac{Z_M}{\mu_M}$.

Thus, the bounded region of the system (1) for the mosquito population is given by:

$$\Omega_M = \left\{(S_M, E_M, I_M) \in R_+^3 : 0 \leq S_M + E_M + I_M \leq \frac{Z_M}{\mu_M}\right\}. \tag{14}$$

Hence, the biologically feasible region of the model system (1) is given by:

$$\Omega = \Omega_H \times \Omega_M \subset R_+^7, \tag{15}$$

where

$$\Omega_H = \left\{(S_H, I_I, I_A, I_P) \in R_+^4 : 0 \leq S_H + I_I + I_A + I_P \leq \frac{Z_H}{\mu_H}\right\}, \tag{16}$$

and

$$\Omega_M = \left\{(S_M, E_M, I_M) \in R_+^3 : 0 \leq S_M + E_M + I_M \leq \frac{Z_M}{\mu_M}\right\}. \tag{17}$$

Within this region, the model is epidemiologically and mathematically well-posed, ensuring a unique, positive, and bounded solution in R_+^7. □

4.2. Basic Reproduction Number and Equilibria with Their Stability Analysis. The disease-free equilibrium (DFE) of model system (1) occurs when there is an absence of malaria in the population, characterized mathematically by $I_I = I_A = I_P = E_M = I_M = 0$. To determine the DFE point, we set the right-hand side of each equation in system (1) equal to zero, leading to

$$E_0 = \left(\frac{Z_H}{\mu_H}, 0, 0, 0, \frac{Z_M}{\mu_M}, 0, 0\right). \tag{18}$$

The epidemiological concept of the basic reproduction number, denoted by $\mathcal{R}_0$ [40], is a key indicator of a disease's transmissibility. It represents the average number of secondary infections caused by a single infected individual in a susceptible population. If $\mathcal{R}_0$ is less than one, then the disease cannot invade the population and the infection will eventually die out. The time it takes for this to happen generally depends on how much smaller $\mathcal{R}_0$ is from one. If $\mathcal{R}_0$ is greater than one, then an invasion is possible and the infection can spread through the population. Generally, the larger

the value of $\mathscr{R}_0$, the more severe and potentially widespread the epidemic will be [41].

Theorem 4. *The basic reproduction number of the model Equation (1) is given by*

$$R_0 = \rho(FV^{-1}) = \sqrt{\frac{\theta_{HM}\theta_{MH}\mu_H\beta_M Z_M\left(\begin{array}{c}A_2A_3\varnothing_I^2 + A_1A_3\varnothing_A^2 + A_1A_2\varnothing_P^2 + A_1\Omega\varnothing_A\varnothing_P\\ +A_3\varnothing_I\varnothing_A\phi + \Omega\varnothing_I\varnothing_P\phi\end{array}\right)}{A_1A_2A_3(\mu_M+\beta_M)\mu_M^2 Z_H}}. \tag{19}$$

where ρ represents the spectral radius (the dominant eigenvalue in magnitude) of $\rho(FV^{-1})$.

Proof. We determine the basic reproduction number, denoted by $\mathscr{R}_0$, for system (1) using the next-generation matrix approach, as described by van den Driessche and Watmough [13]. The calculation of $\mathscr{R}_0$ begins with rewriting the infective classes of the model equations in the form:

$$X'(t) = \mathscr{F}(t, X(t)) - \nu(t, X(t)), \tag{20}$$

where,

$$\begin{cases} X(t) = (I_I, I_A, I_P, E_M, I_M)^T, \\ \mathscr{F}(t, X(t)) = \begin{bmatrix} \frac{\mu_H\varnothing_I\theta_{MH}}{N_H}S_H I_M \\ \frac{\mu_H\varnothing_A\theta_{MH}}{N_H}S_H I_M \\ \frac{\mu_H\varnothing_P\theta_{MH}}{N_H}S_H I_M \\ \frac{(\varnothing_I I_I + \varnothing_A I_A + \varnothing_P I_P)\theta_{HM}}{N_H}S_M \\ 0 \end{bmatrix}, \\ \nu(t, X(t)) = \begin{bmatrix} -(\phi + \Lambda_I + \mu_H + d_I)I_I \\ \phi I_I - (\mu_H + d_A + \Lambda_A + \Omega)I_A \\ \Omega I_A - (\mu_H + d_P + \Lambda_P)I_P \\ -(\mu_M + \beta_M)E_M \\ \beta_M E_M - \mu_M I_M \end{bmatrix}. \end{cases} \tag{21}$$

Thus, we obtain

$$\mathrm{F} = \text{Jacobian of } \mathscr{F} \text{ at DFE} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\varnothing_I\theta_{MH}}{N_H}\frac{Z_H}{\mu_H} \\ 0 & 0 & 0 & 0 & \frac{\varnothing_A\theta_{MH}}{N_H}\frac{Z_H}{\mu_H} \\ 0 & 0 & 0 & 0 & \frac{\varnothing_P\theta_{MH}}{N_H}\frac{Z_H}{\mu_H} \\ \frac{\varnothing_I\theta_{HM}}{N_H}\frac{Z_M}{\mu_M} & \frac{\varnothing_A\theta_{HM}}{N_H}\frac{Z_M}{\mu_M} & \frac{\varnothing_P\theta_{HM}}{N_H}\frac{Z_M}{\mu_M} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{22}$$

and

$$V = \text{Jacobian of } \nu \text{ at DFE} = \begin{bmatrix} -A_1 & 0 & 0 & 0 & 0 \\ \phi & -A_2 & 0 & 0 & 0 \\ 0 & \mathrm{D} & -\mathrm{A}_3 & 0 & 0 \\ 0 & 0 & 0 & -A_4 & 0 \\ 0 & 0 & 0 & A_6 & -A_5 \end{bmatrix}, \tag{23}$$

where

$$\begin{aligned} A_1 &= (\phi + \Lambda_I + \mu_H + d_I), A_2 = (\mu_H + d_A + \Lambda_A + \Omega), A_3 \\ &= (\mu_H + d_P + \Lambda_P), A_4 = (\mu_M + \beta_M), A_5 = \mu_M \end{aligned} \tag{24}$$

Thus,

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & \frac{-A_6F}{A_4A_5} & \frac{-F}{A_5}, \\ 0 & 0 & 0 & \frac{-A_6G}{A_4A_5} & \frac{-G}{A_5}, \\ 0 & 0 & 0 & \frac{-A_6H}{A_4A_5} & \frac{-H}{A_5}, \\ \frac{-A_2A_3M - A_3N\phi - DP\phi}{A_1A_2A_3} & \frac{-A_3N - DP}{A_2A_3} & \frac{-P}{A_3} & 0 & 0, \\ 0 & 0 & 0 & 0 & 0. \end{bmatrix} \tag{25}$$

The eigenvalues of FV^{-1} are $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and

$$\lambda_{4,5} = \pm\sqrt{\frac{\theta_{\mathrm{HM}}\theta_{\mathrm{MH}}\mu_{\mathrm{H}}Z_{\mathrm{M}}\beta_{\mathrm{M}}\left(\begin{array}{c}A_2A_3\varnothing_{\mathrm{I}}^2 + A_1A_3\varnothing_{\mathrm{A}}^2 + A_1A_2\varnothing_{\mathrm{P}}^2 + A_1\Omega\varnothing_{\mathrm{A}}\varnothing_{\mathrm{P}} \\ +A_3\varnothing_{\mathrm{I}}\varnothing_{\mathrm{A}}\phi + \Omega\varnothing_{\mathrm{I}}\varnothing_{\mathrm{P}}\phi\end{array}\right)}{A_1A_2A_3(\mu_{\mathrm{M}} + \beta_{\mathrm{M}})\mu_{\mathrm{M}}^2Z_{\mathrm{H}}}}. \tag{26}$$

Thus,

$$R_0 = \rho(FV^{-1}) = \sqrt{\frac{\theta_{\mathrm{HM}}\theta_{\mathrm{MH}}\mu_{\mathrm{H}}Z_{\mathrm{M}}\beta_{\mathrm{M}}\left(\begin{array}{c}A_2A_3\varnothing_{\mathrm{I}}^2 + A_1A_3\varnothing_{\mathrm{A}}^2 + A_1A_2\varnothing_{\mathrm{P}}^2 + A_1\Omega\varnothing_{\mathrm{A}}\varnothing_{\mathrm{P}} \\ +A_3\varnothing_{\mathrm{I}}\varnothing_{\mathrm{A}}\phi + \Omega\varnothing_{\mathrm{I}}\varnothing_{\mathrm{P}}\phi\end{array}\right)}{A_1A_2A_3(\mu_{\mathrm{M}} + \beta_{\mathrm{M}})\mu_{\mathrm{M}}^2Z_{\mathrm{H}}}}. \tag{27}$$

This completes the proof of the theorem. □

Theorem 5. *The disease free equilibrium point of system (1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. To determine the stability of the DFE, we calculate the eigenvalues of the Jacobian matrix at that equilibrium. If all eigenvalues have negative real parts, the equilibrium is stable. Conversely, if any eigenvalue has a nonnegative real part, the equilibrium is unstable.

The Jacobian matrix of the model Equation (1) evaluated at DFE is:

$$J(E_0) = \begin{bmatrix} -\mu_H & \Lambda_{\mathrm{I}} & \Lambda_A & \Lambda_P & 0 & 0 & -(\varnothing_{\mathrm{I}} + \varnothing_A + \varnothing_P)\theta_{\mathrm{MH}} \\ 0 & -A_1 & 0 & 0 & 0 & 0 & \varnothing_{\mathrm{I}}\theta_{\mathrm{MH}} \\ 0 & \phi & -A_2 & 0 & 0 & 0 & \varnothing_A\theta_{\mathrm{MH}} \\ 0 & 0 & \Omega & -A_3 & 0 & 0 & \varnothing_{\mathrm{P}}\theta_{\mathrm{MH}} \\ 0 & -\dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & -\dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & -\dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & -\mu_{\mathrm{M}} & 0 & 0 \\ 0 & \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_M} & \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & 0 & -(\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_M & -\mu_M \end{bmatrix}. \tag{28}$$

By inspection, it is easy to see that two eigenvalues of Equation (28) are $\lambda_1 = -\mu_{\mathrm{H}}$ and $\lambda_5 = -\mu_{\mathrm{M}}$, while the remaining eigenvalues are obtained from the following 5 x 5 matrix:

$$J(E_0) = \begin{bmatrix} -A_1 & 0 & 0 & 0 & \varnothing_I\theta_{\mathrm{MH}} \\ \phi & -A_2 & 0 & 0 & \varnothing_A\theta_{\mathrm{MH}} \\ 0 & \Omega & -A_3 & 0 & \varnothing_{\mathrm{P}}\theta_{\mathrm{MH}} \\ \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}\dfrac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} & -(\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) & 0 \\ 0 & 0 & 0 & \beta_{\mathrm{M}} & -\mu_{\mathrm{M}} \end{bmatrix}. \tag{29}$$

Using the elementary matrix row operations in matrix Equation (29), we obtain

$$J^*(E_0) = \begin{bmatrix} -A_1 & 0 & 0 & 0 & \varnothing_I\theta_{MH} \\ 0 & -A_2 & 0 & 0 & \dfrac{A_1\varnothing_A\theta_{MH} + \varnothing\varnothing_I\theta_{MH}}{A_1} \\ 0 & 0 & -A_3 & 0 & \dfrac{A_1\Omega\varnothing_A\theta_{MH} + A_1A_2\varnothing_P\theta_{MH} + \Omega\varnothing_I\theta_{MH}\varnothing}{A_1A_2} \\ 0 & 0 & 0 & -A_4 & \dfrac{(\mu_M + \beta_M)\mu_M R_0^2}{\beta_M} \\ 0 & 0 & 0 & 0 & -A_5 \end{bmatrix}. \tag{30}$$

where

$$\lambda_2 = -A_1 = -(\phi + \Lambda_I + \mu_H + d_I), \tag{31}$$

$$\lambda_3 = -A_2 = -(\mu_H + d_A + \Lambda_A + \Omega), \tag{32}$$

$$\lambda_4 = -A_3 = -(\mu_H + d_P + \Lambda_P), \tag{33}$$

$$\lambda_6 = -A_4 = -\mu_M(1 - R_0^2), \tag{34}$$

$$\lambda_7 = -A_5 = -\mu_M. \tag{35}$$

Consequently, the eigenvalues of matrix Equation (28), $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_7$, are all negative. λ_6 is negative if $R_0 < 1$ and positive if $R_0 > 1$.Thus, the malaria model Equation (1) is locally asymptotically stable at the DFE if $R_0 < 1$ and unstable if $R_0 > 1$. □

4.3. Global Asymptotic Stability of Disease-Free Equilibrium. Chavez et al. [42] technique is used in model Equation (1) to establish global asymptotic stability at the DFE point. The process can be summed up as follows: the proposed model Equation (1) is divided into the two subsystems specified by:

$$\frac{dX_1}{dt} = F(X_1, X_2), \tag{36}$$

$$\frac{dX_2}{dt} = G(X_1, X_2). \tag{37}$$

The number of uninfected and infected people are represented in the system (1) by the variables X_1 and X_2, respectively, where $X_1 = (S_H, S_M) \in R_+^2$ and $X_2 = (I_A, I_I, I_P, E_M, I_M) \in R_+^5$. E^0 stands for the DFE point and is defined as $E^0 = (X_0, 0)$.

The two conditions listed below must be met for there to be global stability at the disease-free equilibrium point.

(1) If $\frac{dX_1}{dt} = F(X_0, 0)$, then X_0 is globally asymptotically stable.
(2) $G(X_1, X_2) = AX_2 - \widehat{G}(X_1, X_2)$, where $\widehat{G}(X_1, X_2) \geq 0$ for $(X_1, X_2) \in \Omega$.

At the second condition, $A = D_{X_2}G(X_0, 0)$ is a Metzler matrix that is the off-diagonal entries are nonnegative and Ω is the feasible region. Then, the following statement holds.

Lemma 1. *If $R_0 < 1$, then the equilibrium point $E^0 = (X_0, 0)$ of the system (1) is globally asymptotically stable, provided that conditions 1 and 2 hold.*

Theorem 6. *For the system (1), the DFE (E_0) is globally asymptotically stable (GAS) if* $R_0 < 1$.

Proof. Let $X_1 = (S_H, S_M) \in R_+^2$ and $X_2 = (I_A, I_I, I_P, E_M, I_M) \in R_+^5$. We group system (1) into:
$\frac{dX_1}{dt} = F(X_1, 0), \frac{dX_2}{dt} = G(X_1, X_2)$, where:

$$\frac{dX_1}{dt} = F(X_1, 0) = \begin{cases} \dfrac{dS_H}{dt} = Z_H - \mu_H S_H, \\ \dfrac{dS_M}{dt} = Z_H - \mu_H, \end{cases} \tag{38}$$

and

$$\frac{dX_2}{dt} = G(X_1, X_2) = \begin{pmatrix} \dfrac{\mu_H\varnothing_I\theta_{MH}I_M}{Z_H}S_H - A_1I_I \\ \dfrac{\mu_H\varnothing_A\theta_{MH}I_M}{Z_H}S_H + \phi I_I - A_2I_A \\ \dfrac{\mu_H\varnothing_P\theta_{MH}I_M}{Z_H}S_H + \Omega I_A - A_3I_P \\ \dfrac{(\varnothing_I I_I + \varnothing_A I_A + \varnothing_P I_P)\mu_H S_M\theta_{HM}}{Z_H} - (\mu_M + \beta_M)E_M \\ \beta_M E_M - \mu_M I_M \end{pmatrix}. \tag{39}$$

Now, $X_0 = (S_{1H}, S_{1M}) = (\frac{Z_H}{\mu_H}, \frac{Z_M}{\mu_M})$ is the disease-free equilibrium point of the reduced system (38); we show that X_0 is a globally stable equilibrium in Ω.

To do this, we solve Equation (38); solving the first Equation in (38); gives

$S_{\mathrm{H}} = \frac{Z_{\mathrm{H}}}{\mu_{\mathrm{H}}} + (S_{0H} - \frac{Z_{\mathrm{H}}}{\mu_{\mathrm{H}}})e^{-\mu t}$ which converges to $\frac{Z_{\mathrm{H}}}{\mu_{\mathrm{H}}}$ as $t \to \infty$.

Next, by solving the second Equation in (38), we get

$S_{\mathrm{M}} = \frac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}} + (S_{0\mathrm{M}} - \frac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}})e^{-\mu t}$ which approaches $\frac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}}$ as $t \to \infty$.

Thus, these asymptotic dynamics are independent of initial conditions in Ω. Hence, the convergence of solutions of Equation (38) is global in Ω.

Next,

$$G(X_1, X_2) = AX_2 - \widehat{G}(X_1, X_2) = \begin{pmatrix} -\mathrm{A}_1 & 0 & 0 & 0 & \varnothing_{\mathrm{I}}\theta_{\mathrm{MH}} \\ \phi & -\mathrm{A}_2 & 0 & 0 & \varnothing_{\mathrm{A}}\theta_{\mathrm{MH}} \\ 0 & \Omega & -\mathrm{A}_3 & 0 & \varnothing_{\mathrm{P}}\theta_{\mathrm{MH}} \\ \frac{\varnothing_{\mathrm{I}}\mu_{\mathrm{H}}\theta_{\mathrm{HM}}}{\mathrm{Z}_{\mathrm{H}}}\frac{\mathrm{Z}_{\mathrm{M}}}{\mu_{\mathrm{M}}} & \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}}{\mathrm{Z}_{\mathrm{H}}}\frac{\mathrm{Z}_{\mathrm{M}}}{\mu_{\mathrm{M}}} & \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}}{\mathrm{Z}_{\mathrm{H}}}\frac{\mathrm{Z}_{\mathrm{M}}}{\mu_{\mathrm{M}}} & -(\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) & 0 \\ 0 & 0 & 0 & \beta_{\mathrm{M}} & -\mu_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \mathrm{I}_{\mathrm{I}} \\ \mathrm{I}_{\mathrm{A}} \\ \mathrm{I}_{\mathrm{P}} \\ \mathrm{E}_{\mathrm{M}} \\ \mathrm{I}_{\mathrm{M}} \end{pmatrix}$$
$$- \begin{pmatrix} \varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}}}{\mathrm{Z}_{\mathrm{H}}}\mathrm{S}_{\mathrm{H}}, \\ \varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}}}{\mathrm{Z}_{\mathrm{H}}}\mathrm{S}_{\mathrm{H}}, \\ \varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}}}{\mathrm{Z}_{\mathrm{H}}}\mathrm{S}_{\mathrm{H}}, \\ \varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}\mathrm{I}_{\mathrm{I}} + \varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}\mathrm{I}_{\mathrm{A}} + \varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}\mathrm{I}_{\mathrm{P}} - \frac{(\varnothing_{\mathrm{I}}\mathrm{I}_{\mathrm{I}} + \varnothing_{\mathrm{A}}\mathrm{I}_{\mathrm{A}} + \varnothing_{\mathrm{P}}\mathrm{I}_{\mathrm{P}})\mu_{\mathrm{H}}\mathrm{S}_{\mathrm{M}}\theta_{\mathrm{HM}}}{\mathrm{Z}_{\mathrm{H}}} \\ 0 \end{pmatrix}, \tag{40}$$

where A is the Jacobian of $G(X_1, X_2)$ taken to $(I_I, I_A, I_P, E_M, I_M)$ and evaluated at $(X_0, 0)$,which is an M-matrix that is the off-diagonal entries are nonnegative and Ω is the feasible region.

Note that $\widehat{G}(X_1, X_2) = AX_2 - G(X_1, X_2)$, where

$$\widehat{G}(X_1, X_2) = \begin{bmatrix} \widehat{G}_1(X_1, X_2) \\ \widehat{G}_2(X_1, X_2) \\ \widehat{G}_3(X_1, X_2) \\ \widehat{G}_4(X_1, X_2) \\ \widehat{G}_5(X_1, X_2) \end{bmatrix} = \begin{bmatrix} \varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}I_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}, \\ \varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}I_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}, \\ \varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}I_{\mathrm{M}} - \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}, \\ \varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}I_I + \varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}I_{\mathrm{A}} + \varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}I_{\mathrm{P}} - \frac{(\varnothing_{\mathrm{I}}I_{\mathrm{I}} + \varnothing_{\mathrm{A}}I_{\mathrm{A}} + \varnothing_{\mathrm{P}}I_{\mathrm{P}})\mu_{\mathrm{H}}S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} \\ 0 \end{bmatrix}. \tag{41}$$

Thus, if the human population is at an equilibrium level, it follows that $\widehat{G}(X_1, X_2) \geq 0$ for $(X_1, X_2) \in D_1$ since $\varnothing_{\mathrm{I}}\theta_{\mathrm{MH}}I_{\mathrm{M}} \geq \frac{\mu_{\mathrm{H}}\varnothing_I\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}$, $\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}I_{\mathrm{M}} \geq \frac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}$, $\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}I_{\mathrm{M}} \geq \frac{\mu_H\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}}I_{\mathrm{M}}}{Z_{\mathrm{H}}}S_{\mathrm{H}}$, and $\varnothing_{\mathrm{I}}\theta_{\mathrm{HM}}I_{\mathrm{I}} + \varnothing_{\mathrm{A}}\theta_{\mathrm{HM}}I_{\mathrm{A}} + \varnothing_{\mathrm{P}}\theta_{\mathrm{HM}}I_{\mathrm{P}} \geq \frac{(\varnothing_{\mathrm{I}}I_{\mathrm{I}} + \varnothing_{\mathrm{A}}I_{\mathrm{A}} + \varnothing_{\mathrm{P}}I_{\mathrm{P}})\mu_{\mathrm{H}}S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}}$.

Thus, by Lemma 1, the DFE E_0 is globally asymptotically stable for $R_0 < 1$. □

4.4. Endemic Equilibrium and Its Stability. In the scenario where malaria permeates the population ($I_{\mathrm{I}} > 0, I_{\mathrm{A}} > 0, I_{\mathrm{P}} > 0$, $E_{\mathrm{M}} > 0$, and $I_{\mathrm{M}} > 0$), model (1) accommodates an equilibrium point aptly coined the malaria endemic equilibrium point, denoted by $E^* = (S^*_{\mathrm{H}}, I^*_{\mathrm{I}}, I^*_{\mathrm{A}}.I^*_{\mathrm{P}}, S^*_{\mathrm{M}}, E^*_{\mathrm{M}}, I^*_{\mathrm{M}})$.

The Endemic Equilibrium point of model system (1), denoted by $E^* = (S^*_{\mathrm{H}}, I^*_{\mathrm{I}}, I^*_{\mathrm{A}}.I^*_{\mathrm{P}}, S^*_{\mathrm{M}}, E^*_{\mathrm{M}}, I^*_{\mathrm{M}})$, is obtained by solving the following system of equations:

$$\begin{cases} Z_{\mathrm{H}} + \Lambda_{\mathrm{I}} I_{\mathrm{I}}^* + \Lambda_{\mathrm{A}} I_{\mathrm{A}}^* + \Lambda_{\mathrm{P}} I_{\mathrm{P}}^* - \left(\dfrac{(\varnothing_{\mathrm{I}} + \varnothing_{\mathrm{A}} + \varnothing_{\mathrm{P}}) \mu_{\mathrm{H}} \theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} + \mu_{\mathrm{H}} \right) S_{\mathrm{H}}^* = 0, \\ \dfrac{\mu_{\mathrm{H}} \varnothing_{\mathrm{I}} \theta_{\mathrm{MH}} I_{\mathrm{M}}^*}{Z_{\mathrm{H}}} S_{\mathrm{H}}^* - A_1 I_{\mathrm{I}}^* = 0, \\ \dfrac{\mu_{\mathrm{H}} \varnothing_{A} \theta_{\mathrm{MH}} I_{M}^*}{Z_{\mathrm{H}}} S_{\mathrm{H}}^* + \phi I_{\mathrm{I}}^* - A_2 I_{\mathrm{A}}^* = 0, \\ \dfrac{\mu_{\mathrm{H}} \varnothing_{\mathrm{P}} \theta_{\mathrm{MH}} I_{\mathrm{M}}^*}{Z_{\mathrm{H}}} S_{\mathrm{H}}^* + \Omega I_{\mathrm{A}}^* - A_3 I_{\mathrm{P}}^* = 0, \\ Z_{\mathrm{M}} - \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}}^* + \varnothing_{\mathrm{A}} I_{\mathrm{A}}^* + \varnothing_{\mathrm{P}} I_{\mathrm{P}}^*) \mu_{\mathrm{H}} S_{\mathrm{M}}^* \theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} - \mu_{\mathrm{M}} S_{\mathrm{M}}^* = 0, \\ \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}}^* + \varnothing_{\mathrm{A}} I_{\mathrm{A}}^* + \varnothing_{\mathrm{P}} I_{\mathrm{P}}^*) \mu_{\mathrm{H}} S_{\mathrm{M}}^* \theta_{HM}}{Z_{H}} - (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) E_{\mathrm{M}}^* = 0, \\ \beta_{\mathrm{M}} E_{\mathrm{M}}^* - \mu_{\mathrm{M}} I_{\mathrm{M}}^* = 0. \end{cases} \tag{42}$$

Thus, the Endemic Equilibrium point of the model system (1) is $E^* = (S_H^*, I_I^*, I_A^*.I_P^*, S_M^*, \mathrm{E}_{\mathrm{M}}^*, \mathrm{I}_{\mathrm{M}}^*)$, where

$$\begin{cases} S_{\mathrm{H}}^* = \dfrac{Z_{\mathrm{H}}(R_0^2 \mu_{\mathrm{M}}(\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_H + Z_{\mathrm{M}} \mathrm{A}_5 \theta_{\mathrm{MH}} \beta_{\mathrm{M}})}{R_0^2 \mu_{\mathrm{H}}(\mathrm{A}_5 \mu_{\mathrm{H}} \theta_{\mathrm{MH}} \beta_M Z_{\mathrm{M}} + \beta_{\mathrm{M}} Z_{\mathrm{H}}(\mu_{\mathrm{M}} + \beta_{\mathrm{M}}))}, \\ I_{\mathrm{A}}^* = \dfrac{(\varnothing_{\mathrm{A}} \mathrm{A}_1 + \phi \varnothing_{\mathrm{I}}) \theta_{\mathrm{MH}} (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{M}} Z_{\mathrm{H}} (R_0^2 - 1)}{\mathrm{A}_1 \mathrm{A}_2 R_0^2 (\mathrm{A}_5 Z_{\mathrm{M}} \theta_{\mathrm{MH}} \beta_{\mathrm{M}} + \mu_{\mathrm{M}} (\beta_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}})}, \\ I_{\mathrm{P}}^* = \dfrac{(\mu_{\mathrm{M}} + \beta_{\mathrm{M}})(\mathrm{A}_1 \mathrm{A}_2 \varnothing_{\mathrm{P}} + \Omega(\varnothing_{\mathrm{A}} \mathrm{A}_1 + \phi \varnothing_{\mathrm{I}})) \beta_{\mathrm{MH}} Z_{\mathrm{M}} Z_H (R_0^2 - 1)}{\mathrm{A}_1 \mathrm{A}_2 \mathrm{A}_3 R_0^2 (\mathrm{A}_5 Z_{\mathrm{M}} \theta_{\mathrm{MH}} \beta_{\mathrm{M}} + \mu_{\mathrm{M}} (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}})}, \\ S_{\mathrm{M}}^* = \dfrac{\mathrm{Z}_{\mathrm{M}} (\mathrm{A}_5 Z_{\mathrm{M}} \theta_{\mathrm{MH}} \beta_{\mathrm{M}} + \mu_{\mathrm{M}} (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}})}{\mu_{\mathrm{M}} (R_0^2 \mu_{\mathrm{M}} (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}} + Z_{\mathrm{M}} \mathrm{A}_5 \theta_{\mathrm{MH}} \beta_{\mathrm{M}})}, \\ E_{\mathrm{M}}^* = \dfrac{\mu_{\mathrm{M}} Z_{\mathrm{M}} Z_{\mathrm{H}} (R_0^2 - 1)}{R_0^2 \mu_{\mathrm{M}} (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}} + Z_M \mathrm{A}_5 \theta_{\mathrm{MH}} \beta_{\mathrm{M}}}, \\ I_{\mathrm{M}}^* = \dfrac{\beta_{\mathrm{M}} Z_{\mathrm{M}} Z_{\mathrm{H}} (R_0^2 - 1)}{R_0^2 \mu_{\mathrm{M}} (\beta_{\mathrm{M}} + \beta_{\mathrm{M}}) Z_{\mathrm{H}} + Z_{\mathrm{M}} \mathrm{A}_5 \theta_{\mathrm{MH}} \beta_{\mathrm{M}}}. \end{cases} \tag{43}$$

This implies that the only scenario where the force of infections is positive at the endemic equilibrium point is one where $R_0 > 1$. Thus, we have proved the following theorem.

Theorem 7. *The malaria model (1) has a unique endemic equilibrium in a region Ω, if $R_0 > 1$.*

Theorem 8 (see [43]). *(Krasovkil–LaSalle Theorem (Extension of Lyapunov's Theorem)). Consider the autonomous system $x' = F(x)$, where x^* is an equilibrium point, i.e., $\mathrm{F}(x^*) = 0$.*

Suppose there exists a continuously differentiable function $L : R^{\mathrm{n}} \to R$ that is positive definite on the entire space, radially unbounded, and that satisfies:

$L'(x) \leq 0$ and $\forall x \in R^n$.

Define the invariant set $\Omega = \{x \in R^n | L'(x) = 0\}$. If Ω contains only the equilibrium x^*,then x^* is globally stable.

Theorem 9. *The endemic equilibrium $E^* = (S_{\mathrm{H}}^*, I_{\mathrm{I}}^*, I_{\mathrm{A}}^*, I_{\mathrm{P}}^*, S_{\mathrm{M}}^*, E_{\mathrm{M}}^*, I_{\mathrm{M}}^*)$ of the model system (1) is globally asymptotically stable in Ω if $R_0 > 1$.*

Proof. This section deals with the global stability of model system (1) in the domain Ω. To do this, we define the Lyapunov Function as follows:

$$\begin{aligned} &L(S_H, I_{\mathrm{I}}, I_{\mathrm{A}}, I_{\mathrm{P}}, S_{\mathrm{M}}, E_{\mathrm{M}}, I_{\mathrm{M}}) \\ &\quad = \frac{1}{2}((S_{\mathrm{H}} - S_{\mathrm{H}}^*) + (I_{\mathrm{I}} - I_{\mathrm{I}}^*) + (I_{\mathrm{A}} - I_{\mathrm{A}}^*) + (I_{\mathrm{P}} - I_{\mathrm{P}}^*))^2 \\ &\qquad + \frac{1}{2}((S_{\mathrm{M}} - S_{\mathrm{M}}^*) + (E_{\mathrm{M}} - E_{\mathrm{M}}^*) + (I_{\mathrm{M}} - I_{\mathrm{M}}^*))^2. \end{aligned} \tag{44}$$

Let $L : R_+^7 \to R$. Then, Lis continuously differentiable function and also,

(i) $L(S_H^*, I_I^*, I_A^*.I_P^*, S_M^*, E_M^*, I_M^*) = 0$,

(ii) $L(S_H^*, I_I^*, I_A^*.I_P^*, S_M^*, E_M^*, I_M^*) > 0$ for all $(S_H^*, I_I^*, I_A^*.I_P^*, S_M^*, E_M^*, I_M^*) \neq (S_H, I_I, I_A, I_P, S_M, E_M, I_M)$, and calculating the time derivative of L along the trajectories of system (1), we obtain:

$$\begin{aligned}\frac{dL}{dt} = {} & ((S_H - S_H^*) + (I_I - I_I^*) + (I_A - I_A^*) \\ & + (I_P - I_P^*))\left(\frac{dS_H}{dt} + \frac{dI_I}{dt} + \frac{dI_A}{dt} + \frac{dI_P}{dt}\right) + ((S_M - S_M^*) \\ & + (E_M - E_M^*) + (I_M - I_M^*))\left(\frac{dS_M}{dt} + \frac{dE_M}{dt} + \frac{dI_M}{dt}\right).\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{dL}{dt} = {} & (N_H - (S_H^* + I_I^* + I_A^* + I_P^*))(Z_H - \mu_H N_H \\ & - (d_1 I_I + d_2 I_A + d_p I_P)) + (N_M - (S_M^* + E_M^* \\ & + I_M^*))(Z_M - \mu_M N_M).\end{aligned} \tag{46}$$

From the first four equations of system (42), that is,

$$\begin{cases} Z_H + \Lambda_I I_I^* + \Lambda_A I_A^* + \Lambda_P I_P^* - \left(\dfrac{(\varnothing_I + \varnothing_A + \varnothing_P)\mu_H \theta_{MH} I_M}{Z_H} + \mu_H\right) S_H^* = 0, \\ \dfrac{\mu_H \varnothing_I \theta_{MH} I_M^*}{Z_H} S_H^* - A_1 I_I^* = 0, \\ \dfrac{\mu_H \varnothing_A \theta_{MH} I_M^*}{Z_H} S_H^* + \phi I_I^* - A_2 I_A^* = 0, \\ \dfrac{\mu_H \varnothing_P \theta_{MH} I_M^*}{Z_H} S_H^* + \Omega I_A^* - A_3 I_P^* = 0. \end{cases} \tag{47}$$

We get,

$$S_H^* + I_I^* + I_A^* + I_P^* = \frac{Z_H - (d_1 I_I^* + d_2 I_A^* + d_p I_P^*)}{\mu_H}. \tag{48}$$

Once again, from the last three equations in (42), that is,

$$\begin{cases} Z_M - \dfrac{(\varnothing_I I_I^* + \varnothing_A I_A^* + \varnothing_P I_P^*)\mu_H S_M^* \theta_{HM}}{Z_H} - \mu_M S_M^* = 0, \\ \dfrac{(\varnothing_I I_I^* + \varnothing_A I_A^* + \varnothing_P I_P^*)\mu_H S_M^* \theta_{HM}}{Z_H} - (\mu_M + \beta_M) E_M^* = 0, \\ \beta_M E_M^* - \mu_M I_M^* = 0. \end{cases} \tag{49}$$

We obtain,

$$S_M^* + E_M^* + I_M^* = N_M - \frac{Z_M}{\mu_M}. \tag{50}$$

Now substituting Equations (48) and (50) into Equation (46), we have

$$\begin{aligned}\frac{dL}{dt} = {} & \left(N_H - \frac{Z_H - (d_1 I_I^* + d_2 I_A^* + d_p I_P^*)}{\mu_H}\right)(Z_H - \mu_H N_H \\ & - (d_1 I_I + d_2 I_A + d_p I_P)) + \left(N_M - \frac{Z_M}{\mu_M}\right)(Z_M - \mu_M N_M).\end{aligned} \tag{51}$$

$$\frac{dL}{dt} \leq \left(N_H - \frac{Z_H}{\mu_H}\right)(Z_H - \mu_H N_H) + \left(N_M - \frac{Z_M}{\mu_M}\right)(Z_M - \mu_M N_M), \tag{52}$$

$$\frac{dL}{dt} \leq -\left(\frac{(Z_H - \mu_H N_H)^2}{\mu_H} + \frac{(Z_M - \mu_M N_M)^2}{\mu_M}\right). \tag{53}$$

Thus, we have that $\frac{dL}{dt} = 0$ if and only if $Z_H = \mu_H N_H$ and $Z_M = \mu_M N_M$ hold. The largest closed and bounded invariant set in $\{(S_H, I_I, I_A, I_P, S_M, E_M, I_M) \in R_+^7 : \frac{dL}{dt} = 0\}$ is the singleton $\{E^*\}$, where E^* is the endemic equilibrium point. As a result, when $R_0 > 1$ in the region Ω, the unique equilibrium point E^* is globally asymptotically stable, according to the LaSalle invariance principle. This completes the proof of the theorem. □

4.5. Sensitivity Analysis. This section examines the sensitivity analysis of a mathematical model of malaria with age-structure to different parameters and their impact on the

Table 3: The sensitivity indices for R_0 of the reproduction number of the model (1).

Parameter	Value	Source	Sensitivity index
Z_H	414521	[37]	−0.49929
Z_M	134267979835	[37]	0.49928
μ_H	0.016	[37]	0.47322
μ_M	0.058176	[37]	−1.09840
D_I	0.020605	[37]	−0.00903
D_A	0.19113	[37]	−0.227690
D_P	0.49273	[37]	−0.00014
Λ_I	0.11855	[37]	−0.05197
Λ_A	0.14348	[37]	−0.17090
Λ_P	0.14154	[37]	−0.000040
θ_{MH}	0.00016937	[37]	0.49929
θ_{HM}	0.00454	[37]	0.49930
ϕ_I	0.33575	[37]	0.17693
ϕ_A	0.98982	[37]	0.82142
ϕ_P	0.012704	[37]	0.000022
ϕ	0.10743	[37]	0.00621
Ω	0.016744	[37]	−0.01980
β_M	0.2328	Estimated	0.09982

system dynamics. By analyzing age-specific parameters, vector-related parameters, intervention parameters, and long-term outcome parameters, we better understand the factors that affect malaria transmission and develop targeted strategies for controlling and eliminating the disease. The normalized sensitivity index of a variable to a parameter is the ratio of the relative change in the variable to the relative change in the parameter [44].

That is the formula

$$\pi_u^{R_0} = \frac{\partial R_0}{\partial u} \times \frac{u}{R_0}, \tag{54}$$

gives the sensitivity index R_0 in relation to a parameter, let's say u.

The sensitivity index with negative signs indicates that for an increase in the corresponding parameters, there is a decrease in the value of the reproduction number and vice versa. Table 3 shows that the density-dependent natural mortality rate for adult female Anopheles mosquito's μ_M has got highest sensitivity index of −1.0984. This means that decreasing the density-dependent natural mortality rate for adult female Anopheles mosquito's by 10% would increase R_0 by 10.884%. The second highest index 0.82142 is that of the number of bites on people over 5 years per female mosquito per unit of time ϕ_A.That is increasing ϕ_A by 10% will increase R_0 by 8.2142%. The parameters Z_M, θ_{MH}, and θ_{HM} have sensitivity index of 0.49928, 049929, and 0.4993, respectively. By lowering these parameters by 10%, R_0 is reduced by 4.9928%, 4.9929%, and 4.993%, respectively.

5. Fractional-Order Malaria Model

In this section, we review some fundamental definitions from fractional calculus, as well as a few well-known theorems that will be used throughout the paper.

Definition 1 (see [27]). The gamma function of $\gamma > 0$ is defined as follows:

$$\Gamma(\gamma) = \int_0^\infty \mathrm{x}^{\gamma-1}\mathrm{e}^{-\mathrm{x}}dx. \tag{55}$$

Definition 2 (see [26]). Let $\alpha, \beta > 0$. The function $E_{\alpha,\beta}(z)$ is defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}. \tag{56}$$

Note that the following relations hold as a result of the definition provided in Equation (56):

(i) $E_{\alpha,1}(z) = E_\alpha(x) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}, \alpha > 0,$
(ii) $E_{\alpha,\beta}(z) = \frac{1}{z}(E_{\alpha,\beta-a}(z) - \frac{1}{\Gamma(\beta-a)}).$

Definition 3 (see [29]). Atangana–Baleanu fractional derivative in the Caputo sense.

Let $\mathrm{f} \in \mathrm{C}^1(\mathrm{a}, \mathrm{b}), \mathrm{b} > a, \alpha \in [0, 1]$. The Atangana–Baleanu fractional derivative of f of order α in Caputo sense with base point a is defined as follows:

$${}^{\mathrm{ABC}}_{\alpha}D_t^{\alpha} f(t) = \frac{B(\alpha)}{(1-\alpha)} \int_\alpha^t E_\alpha\left(-\alpha\frac{(t-x)^\alpha}{(1-\alpha)}\right) f'(x)dx, \tag{57}$$

where $B(\alpha)$ is the normalization function given by $B(\alpha) = 1 - \alpha + \frac{\alpha}{\Gamma(\alpha)}$, characterized by $B(0) = B(1) = 1$.

Definition 4 (see [29]). The fractional integral associateof the fractional derivative of Atangana–Baleanu is defined as follows:

$${}^{\mathrm{ABC}}_{\alpha}I_t^{\alpha} f(t) = \frac{(1-\alpha)}{B(\alpha)} f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_\alpha^t (t-q)^{\alpha-1} f(q)dq. \tag{58}$$

Theorem 10. *Let $f : [a, b] \to R$ be a bounded and continuous function then the following results hold as in [29], $\|{}^{\mathrm{ABC}}_{\alpha}D_t^{\alpha} f(t)\| \le \frac{(1-\alpha)}{M(\alpha)} \|f(t)\|$, where $\|f(t)\| = \max_{a\le t\le b} |f(t)|$.*

Further, the Atangana–Baleanu derivative fulfill the Lipschitz condition [29].

$$\| {}^{\mathrm{ABC}}_{a}D_t^{\alpha} f_1(t) - {}^{\mathrm{ABC}}_{a}D_t^{\alpha} f_2(t)\| \le L\|f_1(t) - f_2(t)\|, \tag{59}$$

where $0<\alpha\le 1$ is the order of fractional derivative.

Theorem 11 (see [29]). *The Laplace transform of the Atangana–Baleanu fractional derivative in Caputo sense is given as*

$$\begin{aligned} &L\{ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} f(t)\}(s) \\ &= L\left\{\frac{B(\alpha)}{(1-\alpha)}\int_0^t E_{\alpha}\left(-\alpha\frac{(t-x)^{\alpha}}{(1-\alpha)}\right) f'(x)dx\right\} \\ &= \frac{B(\alpha)(s^{\alpha}F(s) - s^{\alpha-1}f(0))}{s^{\alpha}(1-\alpha)+\alpha}, s>0. \end{aligned} \tag{60}$$

The ABC fractional derivative of the model Equation (1) is given as follows:

$$\begin{cases} {}^{\mathrm{ABC}}_{0}D_t^{\alpha} S_{\mathrm{H}}(t) = Z_{\mathrm{H}} + \Lambda_{\mathrm{I}} I_{\mathrm{I}} + \Lambda_{\mathrm{A}} \mathrm{I}_{\mathrm{A}} + \Lambda_{\mathrm{P}} I_{\mathrm{P}} - \dfrac{(\varnothing_{\mathrm{I}} + \varnothing_{\mathrm{A}} + \varnothing_{\mathrm{P}})\mu_{\mathrm{H}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}} S_{\mathrm{H}}}{Z_{\mathrm{H}}} - \mu_{\mathrm{H}} S_{\mathrm{H}}, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{I}}(t) = \dfrac{\mu_{\mathrm{H}}\varnothing_I\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} S_{\mathrm{H}} - A_1 I_{\mathrm{I}}, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{A}}(t) = \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{A}}\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} S_{\mathrm{H}} + \phi I_{\mathrm{I}} - A_2 I_{\mathrm{A}}, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{P}}(t) = \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} S_{\mathrm{H}} + \Omega \mathrm{I}_{\mathrm{A}} - \mathrm{A}_3 I_{\mathrm{P}}, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} S_{\mathrm{M}}(t) = Z_{\mathrm{M}} - \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}} + \varnothing_A \mathrm{I}_{\mathrm{A}} + \varnothing_{\mathrm{P}} I_{\mathrm{P}})\mu_{\mathrm{H}} S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} - \mu_{\mathrm{M}} S_{\mathrm{M}}, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} E_{\mathrm{M}}(t) = \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}}^{*} + \varnothing_A \mathrm{I}_{A} + \varnothing_{\mathrm{P}} I_{\mathrm{P}})\mu_{\mathrm{H}} S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} - (\mu_{\mathrm{M}} + \beta_{\mathrm{M}}) E_{\mathrm{M},} \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{M}}(t) = \beta_{\mathrm{M}} E_{\mathrm{M}} - \mu_{\mathrm{M}} \mathrm{I}_{\mathrm{M}}, \end{cases} \tag{61}$$

With $S_{\mathrm{H}}(0)\ge 0$, $I_{\mathrm{I}}(0)\ge 0$, $I_{\mathrm{A}}(0)\ge 0$, $I_{\mathrm{P}}(0)\ge 0$, $S_{\mathrm{M}}(0)\ge 0$, $E_{\mathrm{M}}(0)\ge 0$ and

$$I_{\mathrm{M}}(0)\ge 0, \tag{62}$$

where ${}^{\mathrm{ABC}}_{0}D_t^{\alpha}$ is the Atangana–Baleanu Caputo fractional derivative of order α.

Lemma 2. *(Generalized Mean Value Theorem see [45]). Supposing that $g(t)\in C[a,b]$ and ${}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(t)\in C[a,b]$ for $0<\alpha\le 1$, then $g(t) = g(k) + \frac{1}{\Gamma(\alpha)}{}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(\tau)(t-k)^{\alpha}$, with $0\le\tau\le t$, $\forall t\in[a,b]$.*

Remark 1. Suppose that $g(t)\in C[0,b]$ and ${}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(t)\in C[a, b]$ for $0<\alpha\le 1$ from Lemma 2 one candeduce that

(i) if ${}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(t)\ge 0$ and $\forall t\in(0,b]$, then thefunction $g(t)$ is non-decreasing and
(ii) if ${}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(t)\le 0$ and $\forall t\in(0,b]$, then the function $g(t)$ is non-increasing.

Theorem 12. *For $\forall(t\ge 0)$, the solutions of a system in Equation (61) with a positive initial conditions are positive.*

Proof. From model (61), we get

$$\begin{cases} {}^{\mathrm{ABC}}_{0}D_t^{\alpha} S_{\mathrm{H}}(t)|_{S_{\mathrm{H}}=0} = Z_{\mathrm{H}} + \Lambda_{\mathrm{I}} I_{\mathrm{I}} + \Lambda_{\mathrm{A}} I_{\mathrm{A}} + \Lambda_{\mathrm{P}} I_{\mathrm{P}} > 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{I}}(t)|_{I_{\mathrm{I}}=0} = \dfrac{\mu_{\mathrm{H}}\varnothing_I\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_H} S_{\mathrm{H}} \ge 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{A}(t)|_{I_{\mathrm{A}}=0} = \dfrac{\mu_{\mathrm{H}}\varnothing_A\theta_{\mathrm{MH}} I_{M}}{Z_H} S_{\mathrm{H}} + \phi I_{\mathrm{I}} \ge 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{p}}(t)|_{I_{\mathrm{p}}=0} = \dfrac{\mu_{\mathrm{H}}\varnothing_{\mathrm{P}}\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} S_{\mathrm{H}} + \Omega I_{\mathrm{A}} \ge 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} S_{\mathrm{M}}(t)|_{S_{\mathrm{M}}=0} = Z_{\mathrm{M}} > 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} E_{\mathrm{M}}(t)|_{E_{\mathrm{M}}=0} = \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}} + \varnothing_{\mathrm{A}} I_{\mathrm{A}} + \varnothing_{\mathrm{P}} I_{\mathrm{P}})\mu_{\mathrm{H}} S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} \ge 0, \\ {}^{\mathrm{ABC}}_{0}D_t^{\alpha} I_{\mathrm{M}}(t)|_{I_{\mathrm{M}}=0} = \beta_{\mathrm{M}} E_{\mathrm{M}} \ge 0. \end{cases} \tag{63}$$

As a result, the feasible region provided by Ω is positivity invariant for model (61), which is inferred from Lemma 2 and remark 1. As a result, the solution remains inside Ω. □

Theorem 13. *The biologically feasible region $\Omega = \Omega_{\mathrm{H}}\times\Omega_{\mathrm{M}}\subset R_+^7$ is positively invariant with respect to the initial conditions in R_+^7 for the system (61).*

Proof. Adding the first four equations of system (61), we obtain the total human population:

$$
\begin{aligned}
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}N_{\mathrm{H}}(t) &= Z_{\mathrm{H}} - \mu_{\mathrm{H}}N_{\mathrm{H}} - \left(\mathrm{d}_1 I_{\mathrm{I}} + d_2 I_{\mathrm{A}} + d_{\mathrm{p}} I_{\mathrm{P}}\right) \\
&\leq Z_{\mathrm{H}} - \mu_{\mathrm{H}}N_{\mathrm{H}}(t).
\end{aligned} \tag{64}
$$

Similarly, adding the last three equations in system (61), we obtain the total mosquito population:

$$
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}N_{\mathrm{M}}(t) = Z_{\mathrm{M}} - \mu_{\mathrm{M}}N_{\mathrm{M}}(\mathrm{t}). \tag{65}
$$

Applying the Laplace transform to Equation (65), we get:

$$
\mathscr{L}\{\,{}^{\mathrm{ABC}}_{0}D_t^{\alpha}N_{\mathrm{H}}(t)\}(\mathrm{s}) \leq \mathscr{L}\{Z_{\mathrm{H}} - \mu_{\mathrm{H}}N_{\mathrm{H}}(t)\}(\mathrm{s}). \tag{66}
$$

Using Theorem 11, we have: $\frac{\mathrm{B}(\alpha)(\mathrm{s}^{\alpha}\mathscr{L}[N_{\mathrm{H}}(t)](\mathrm{s}) - \mathrm{s}^{\alpha-1}N_{\mathrm{H}}(0))}{\mathrm{s}^{\alpha}(1-\alpha)+\alpha} \leq \frac{Z_{\mathrm{H}}}{\mathrm{s}} + \frac{\mathrm{s}^{\alpha-1}N_{\mathrm{H}}(0)}{\mathrm{s}^{\alpha}(1-\alpha)+\alpha}$, where $N_{\mathrm{H}}(0)$ represents the initial value of the total human population.

Therefore,

$$
\begin{aligned}
\mathscr{L}\{N_H(t)\}(\mathrm{s}) \leq &\left(\frac{\mathrm{s}^{\alpha-1}N_{\mathrm{H}}(0)B(\alpha)}{B(\alpha)\mathrm{s}^{\alpha} + \mu_{\mathrm{H}}(\mathrm{s}^{\alpha}(1-\alpha)+\alpha)}\right) \\
&+ \frac{Z_{\mathrm{H}}}{\mathrm{s}}\left(\frac{\mathrm{s}^{\alpha}(1-\alpha)+\alpha}{B(\alpha)\mathrm{s}^{\alpha} + \mu_{\mathrm{H}}(\mathrm{s}^{\alpha}(1-\alpha)+\alpha)}\right).
\end{aligned} \tag{67}
$$

Therefore,

$$
\begin{aligned}
\mathscr{L}\{N_{\mathrm{H}}(t)\}(\mathrm{s}) \leq &\frac{Z_{\mathrm{H}}\alpha}{(B(\alpha) + \mu_{\mathrm{H}}(1-\alpha))}\left(\frac{\mathrm{s}^{\alpha-(\alpha+1)}}{\mathrm{s}^{\alpha} + \frac{\mu_{\mathrm{H}}\alpha}{B(\alpha)+\mu_{\mathrm{H}}(1-\alpha)}}\right) \\
&+\left(\frac{\mathrm{Z_H}(1-\alpha)}{\mathrm{B}(\alpha) + \mu_{\mathrm{H}}(1-\alpha)} + \frac{N_H(0)\mathrm{B}(\alpha)}{\mathrm{B}(\alpha) + \mu_{\mathrm{H}}(1-\alpha)}\right)\frac{\mathrm{s}^{\alpha-1}}{\mathrm{s}^{\alpha} + \frac{\mu_{\mathrm{H}}\alpha}{\mathrm{B}(\alpha)+\mu_{\mathrm{H}}(1-\alpha)}}.
\end{aligned} \tag{68}
$$

Applying the inverse Laplace transform on both sides of Equation (68), we get:

$$
\begin{aligned}
N_H(t) \leq &\frac{\mathrm{Z_H}\alpha\mathrm{t}^{\alpha}}{(\mathrm{B}(\alpha) + \mu_{\mathrm{H}}(1-\alpha))}\mathrm{E}_{\alpha,\alpha+1}(-\mathrm{kt}^{\alpha}) \\
&+\left(\frac{\mathrm{Z_H}(1-\alpha)}{\mathrm{B}(\alpha) + \mu_{\mathrm{H}}(1-\alpha)} + \frac{N_H(0)\mathrm{B}(\alpha)}{\mathrm{B}(\alpha) + \mu_{\mathrm{H}}(1-\alpha)}\right)\mathrm{E}_{\alpha,1}(-\mathrm{kt}^{\alpha}),
\end{aligned} \tag{69}
$$

where $k = \frac{\mu_{\mathrm{H}}\alpha}{B(\alpha)+\alpha_{\mathrm{H}}(1-\alpha)}$. From Mitage–Leffler property $E_{\alpha,\beta}(z) = \frac{1}{z}\left(E_{\alpha,\beta-a}(z) - \frac{1}{\Gamma(\beta-a)}\right)$, we get

$$
\mathrm{E}_{\alpha,\alpha+1}(-\mathrm{kt}^{\alpha}) = \frac{1}{-\mathrm{kt}^{\alpha}}\left(E_{\alpha,1}(-\mathrm{kt}^{\alpha}) - 1\right). \tag{70}
$$

Thus,

$N_{\mathrm{H}}(t) \leq \frac{Z_{\mathrm{H}}}{-\mu_{\mathrm{H}}}\left(E_{\alpha,1}(-\mathrm{kt}^{\alpha}) - 1\right) + \left(\frac{Z_{\mathrm{H}}(1-\alpha)}{B(\alpha)+\mu_{\mathrm{H}}(1-\alpha)} + \frac{N_{\mathrm{H}}(0)B(\alpha)}{B(\alpha)+\mu_{\mathrm{H}}(1-\alpha)}\right)$

$E_{\alpha,1}(-\mathrm{kt}^{\alpha}) = \frac{Z_{\mathrm{H}}}{\mu_{\mathrm{H}}}$ since $E_{\alpha,1}(-\mathrm{kt}^{\alpha}) \to 0$ as $t \to \infty$.

Therefore, the epidemiologically feasible region for the human population is as follows:

$$
\Omega_H = \left\{(S_{\mathrm{H}}, I_I, I_{\mathrm{A}}, I_{\mathrm{P}}) \in R_+^4 : 0 \leq S_{\mathrm{H}} + I_{\mathrm{I}} + I_{\mathrm{A}} + I_{\mathrm{P}} \leq \frac{Z_{\mathrm{H}}}{\mu_{\mathrm{H}}}\right\}. \tag{71}
$$

Similarly, it can be shown that the feasible region for the mosquito population is as follows:

$$
\Omega_{\mathrm{M}} = \left\{(S_{\mathrm{M}}, E_{\mathrm{M}}, I_{\mathrm{M}}) \in R_+^3 : 0 \leq S_M + E_{\mathrm{M}} + I_{\mathrm{M}} \leq \frac{Z_{\mathrm{M}}}{\mu_{\mathrm{M}}}\right\}. \tag{72}
$$

This establishes that the biologically feasible region $\Omega = \Omega_{\mathrm{H}} \times \Omega_{\mathrm{M}} \subset R_+^7$ is positively invariant with respect to initial conditions in R_+^7 for the system (61). □

5.1. Existence and Uniqueness Solutions of the Fractional Malaria Model. This section demonstrates the existence and uniqueness of solutions for the fractional ABC malaria model in Equation (1), employing fixed point theory. To facilitate this analysis, we reformulate Equation (61) into the equivalent form:

$$
\begin{cases}
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}S_{\mathrm{H}}(t) = F_1(t, S_{\mathrm{H}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}I_{\mathrm{I}}(t) = F_2(t, I_{\mathrm{I}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}I_{\mathrm{A}}(t) = F_3(t, I_{\mathrm{A}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}I_{\mathrm{P}}(t) = F_4(t, I_{\mathrm{P}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_{\mathrm{t}}^{\alpha}S_{\mathrm{M}}(t) = F_5(t, S_{\mathrm{M}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_{\mathrm{t}}^{\alpha}E_{\mathrm{M}}(t) = F_6(t, E_{\mathrm{M}}(t)), \\
{}^{\mathrm{ABC}}_{0}D_t^{\alpha}I_{\mathrm{M}}(t) = F_7(t, I_{\mathrm{M}}(t)),
\end{cases} \tag{73}
$$

with initial conditions $S_{\mathrm{H}}(0) = S_{\mathrm{H0}}$, $I_{\mathrm{I}}(0) = I_{\mathrm{I0}}$, $I_{\mathrm{A}}(0) = I_{\mathrm{A0}}$, $I_{\mathrm{P}}(0) = I_{\mathrm{P0}}$, $S_{\mathrm{M}}(0) = S_{\mathrm{M0}}$, $E_{\mathrm{M}}(0) = E_{\mathrm{M0}}$, and $I_{\mathrm{M}}(0) = I_{\mathrm{M0}}$.

Note that

$$\begin{cases} F_1(t, S_H(t)) = Z_H + \Lambda_I I_{\mathrm{I}} + \Lambda_{\mathrm{A}} \mathrm{I}_{\mathrm{A}} + \Lambda_P I_{\mathrm{P}} - \dfrac{(\varnothing_I + \varnothing_A + \varnothing_P)\mu_{\mathrm{H}}\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}}\mathrm{S}_{\mathrm{H}}}{Z_{\mathrm{H}}} - \mu_{\mathrm{H}} S_{\mathrm{H}}, \\ F_2(t, I_I(t)) = \dfrac{\mu_{\mathrm{H}}\varnothing_I\theta_{\mathrm{MH}} I_{\mathrm{M}}}{Z_{\mathrm{H}}} S_{\mathrm{H}} - A_1 I_I, \\ F_3(t, I_{\mathrm{A}}(t)) = \dfrac{\mu_{\mathrm{H}}\varnothing_A\theta_{\mathrm{MH}}\mathrm{I}_{\mathrm{M}}}{Z_H} S_{\mathrm{H}} + \phi I_I - A_2 I_{\mathrm{A}}, \\ F_4(t, I_{\mathrm{P}}(t)) = \dfrac{\mu_H\varnothing_{\mathrm{P}}\theta_{MH}\mathrm{I}_{\mathrm{M}}}{Z_H} S_{\mathrm{H}} + \Omega I_A - \mathrm{A}_3 I_P, \\ F_5(t, S_{\mathrm{M}}(t)) = Z_{\mathrm{M}} - \dfrac{(\varnothing_I I_I + \varnothing_A I_{\mathrm{A}} + \varnothing_P I_{\mathrm{P}})\mu_{\mathrm{H}} S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} - \mu_{\mathrm{M}} S_{\mathrm{M}}, \\ F_6(t, E_{\mathrm{M}}(t)) = \dfrac{(\varnothing_{\mathrm{I}} I_{\mathrm{I}}^{*} + \varnothing_{\mathrm{A}} I_{\mathrm{A}} + \varnothing_P I_{\mathrm{P}})\mu_{\mathrm{H}} S_{\mathrm{M}}\theta_{\mathrm{HM}}}{Z_{\mathrm{H}}} - (\mu_{\mathrm{M}} + \beta_{\mathrm{M}})\mathrm{E}_{\mathrm{M},} \\ F_7(t, I_{\mathrm{M}}(t)) = \beta_{\mathrm{M}}\mathrm{E}_{\mathrm{M}} - \mu_{\mathrm{M}}\mathrm{I}_{\mathrm{M}}. \end{cases} \tag{74}$$

Using Laplace transformation on both sides of the first equation in Equation (73), we get:

$$\mathscr{L}[\,{}^{\mathrm{ABC}}_{0}D_t^{\alpha} S_{\mathrm{H}}(t)](s) = \mathscr{L}[F_1(t, S_{\mathrm{H}}(t))](s), s>0, \tag{75}$$

And according to Theorem 11, we have

$$\frac{B(\alpha)(\mathrm{s}^{\alpha}\mathscr{L}[S_H(t)](\mathrm{s}) - \mathrm{s}^{\alpha-1} S_H(0))}{\mathrm{s}^{\alpha}(1-\alpha) + \alpha} = \mathscr{L}[g(\mathrm{t})](\mathrm{s}), \tag{76}$$

where $\mathrm{g(t)} = F_1(t, S_H(t))$, which is equivalent,

$$\mathscr{L}[S_{\mathrm{H}}(t)](\mathrm{s}) = \frac{1}{\mathrm{S}} S_H(0) + \frac{1-\alpha}{\mathrm{B}(\alpha)}\mathscr{L}[\mathrm{g(t)}](\mathrm{s}) + \frac{\alpha}{s^{\alpha}B(\alpha)}\mathscr{L}[\mathrm{g(t)}](\mathrm{s}). \tag{77}$$

Applying the inverse Laplace transform on both sides of Equation (78), we get the follwing equation:

$$S_H(t) = S_H(0) + \frac{1-\alpha}{B(\alpha)} g(\mathrm{t}) + \mathscr{L}^{-1}\left\{\frac{\alpha}{s^{\alpha}B(\alpha)}[g(\mathrm{t})](\mathrm{s})\right\}(t). \tag{78}$$

Now, the last term in Equation (79) can be written as follows:

$$\mathscr{L}^{-1}\left\{\frac{\alpha}{s^{\alpha}B(\alpha)}[\mathrm{g(t)}](\mathrm{s})\right\}(t) = \mathscr{L}^{-1}\{F(s)\mathrm{G(s)}\}(t), \tag{79}$$

where $F(s) = \frac{\alpha}{s^{\alpha}B(\alpha)} = \frac{\alpha}{B(\alpha)}\mathscr{L}[\frac{t^{\alpha-1}}{s^{\alpha}}]$ and $\mathrm{G(s)} = \mathscr{L}[\mathrm{g(t)}](\mathrm{s})$.

Thus, using the convolution theorem Equation (79) yields the following equation:

$$\mathscr{L}^{-1}\left\{\frac{\alpha}{s^{\alpha}B(\alpha)}[\mathrm{g(t)}](\mathrm{s})\right\}(t) = \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau. \tag{80}$$

Therefore, using Equation (80), Equation (78) takes the following form:

$$S_{\mathrm{H}}(t) - S_{\mathrm{H}}(0) = \frac{1-\alpha}{\mathrm{B}(\alpha)} F_1(t, S_{\mathrm{H}}(t)) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau. \tag{81}$$

Similarly, Equations (2)–(7) in Equation (73) can be written as follows:

$$\begin{cases} I_{\mathrm{I}}(t) - I_{\mathrm{I}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_2(t, I_{\mathrm{I}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau, \\ I_{\mathrm{A}}(t) - I_{\mathrm{A}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_3(t, I_{\mathrm{A}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau, \\ I_{\mathrm{P}}(t) - I_{\mathrm{P}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_4(t, I_{\mathrm{P}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau, \\ S_M(t) - S_{\mathrm{M}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_5(t, S_{\mathrm{M}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau, \\ E_{\mathrm{M}}(t) - E_{\mathrm{M}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_6(t, E_{\mathrm{M}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau, \\ I_{\mathrm{M}}(t) - I_{\mathrm{M}}(0) = \dfrac{1-\alpha}{B(\alpha)} F_7(t, I_{\mathrm{M}}(t)) + \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t g(\tau)(t-\tau)^{\alpha-1}d\tau. \end{cases} \tag{82}$$

Now let's redefine system (73) in a more general form as follows:

$${}^{\mathrm{ABC}}_{0}D_t^{\alpha} g(t) = f(t, g(t)), g(0) = g_0 \geq 0, \tag{83}$$

where

$$g(t) = (S_{\mathrm{H}}(t), I_I(t), I_{\mathrm{A}}(t), I_{\mathrm{P}}(t), S_{\mathrm{M}}(t), E_{\mathrm{M}}(t), I_{\mathrm{M}}(t)), \tag{84}$$

$$g_0 = (S_{\mathrm{H0}}, I_{\mathrm{I0}}, I_{\mathrm{A0}}, I_{\mathrm{P0}}, S_{\mathrm{M0}}, E_{\mathrm{M0}}, I_{\mathrm{M0}}). \tag{85}$$

and,

$$\begin{aligned} f(t, g(t)) = {} & (F_1(t, g(t)), F_2(t, g(t)), F_3(t, g(t)), F_4(t, g(t)), \\ & F_5(t, g(t)), F_6(t, g(t)), F_7(t, g(t))). \end{aligned} \tag{86}$$

Note that for fractional analysis of the Malaria model (83), let us define Banach space $J = R \times R \times R \times R \times R \times R \times R$, $R = [0, \tau]$ under the norm defined by:

$$\begin{aligned} \|M\| = \|S_{\mathrm{H}}, I_{\mathrm{I}}, I_{\mathrm{A}}, \mathrm{I_P}, \mathrm{S_M}, E_{\mathrm{M}}, I_{\mathrm{M}}\| = \max_{t\in[0,\tau]} & |S_{\mathrm{H}}(t) + \mathrm{I_I}(t) \\ & + I_{\mathrm{A}}(t) + I_{\mathrm{P}}(t) + S_{\mathrm{M}}(t) + \mathrm{E_M}(t) + \mathrm{I_M}(t)|. \end{aligned} \tag{87}$$

The following theorem will be utilized for our primary finding.

Theorem 14 (see [46]). *Let N be a convex, closed, and nonempty subset of a Banach space B. Suppose that F and G are mappings from N into N, satisfying the following conditions:*

(i) *$Fu + Gv \in N$ for all $u, v \in N$.*
(ii) *F is continuous and compact.*
(iii) *G is a contraction mapping.*

Then, the operator equation $F\xi + G\xi = \xi$ has at least one solution in N.

Now, if we set $S_H(t) = g(t)$ and $F_1(t, S_H(t)) = f(t, g(t))$, and applying Equation (81), then Equation (83) can be expressed as follows:

$$\begin{aligned} g(t) = {} & g(0) + (1-\alpha)/B(\alpha).f(t, g(t)) \\ & + \alpha/B(\alpha).\Gamma(\alpha)\int_0^t \mathrm{f}(\tau, \mathrm{g}(\tau))(t-\tau)^{\alpha-1}d\tau. \end{aligned} \tag{88}$$

We now investigate two hypotheses based on Lipschitzian and a few growth condition assumptions to demonstrate the existence and uniqueness of solutions of fractional malaria model Equation (83).

A_1: There are two constants, a and b, such that

$$|f(t, g(t))| \le a|g(t)| + b, t \in [0, \tau]. \tag{89}$$

A_2: There exists constant L_M, for every $g_1, g_2 \in R$, such that

$$|f(t, g_1) - f(t, g_2)| \le L_M|g_1 - g_2|, \tag{90}$$

$$t \in [0, \tau]. \tag{91}$$

Let us define two operators F and G from Equations (83) and (88) as follows:

$$\begin{cases} Fg = g(0) + \dfrac{1-\alpha}{B(\alpha)}f(t, g(t)), \text{and} \\ Gg = \dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t \mathrm{f}(\tau, \mathrm{g}(\tau))(t-\tau)^{\alpha-1}d\tau. \end{cases} \tag{92}$$

Theorem 15. *If A_1 and A_2 holds, then Equation (83) has at least one solution which means that consider system (1) has one solution if*

$$\frac{(1-\alpha)}{B(\alpha)}L_M < 1. \tag{93}$$

Proof. To show that F is a contraction, let $g_1 \in B$, where $B = \{g \in J : \|g\| \le r, r > 0\}$ is a closed convex set. Using the definition of F in Equation (92), we get:

$$\begin{aligned} |Fg - Fg_1| = {} & \left| g(0) + \frac{1-\alpha}{B(\alpha)}f(t, g(t)) \right. \\ & \left. - \left(g_1(0) + \frac{1-\alpha}{B(\alpha)}f(t, g_1(t)) \right) \right|, \end{aligned} \tag{94}$$

$$\begin{aligned} |Fg - Fg_1| = {} & \frac{1-\alpha}{B(\alpha)}|f(t, g(t)) - f(t, g_1(t))| \\ \le {} & \frac{1-\alpha}{B(\alpha)}L_M|g - g_1|. \end{aligned} \tag{95}$$

Thus, F is a contraction.

To show that G is relatively compact, we have to show that G is bounded and continuous. For this, we proceed as follows: it is obvious that G is continuous as F is continuous, also for $g \in J$, we have

$$\|Gg\| = \max_{\mathrm{t}\in[0,\tau]} \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \left| \int_0^t \mathrm{f}(\tau, \mathrm{g}(\tau))(t-\tau)^{\alpha-1}d\tau \right| \tag{96}$$

$$\|Gg\| \le \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \max_{\mathrm{t}\in[0,\tau]} \int_0^t |\,\mathrm{f}(\tau, \mathrm{g}(\tau))|(t-\tau)^{\alpha-1}d\tau \tag{97}$$

$$\|Gg\| \le \frac{t^\alpha}{B(\alpha)\Gamma(\alpha)}[ar + b]. \tag{98}$$

Hence, Equation (98) shows that G is bounded. Let $t_1 > t_2 \in [0, \tau]$, such that

$$\|Gg(t_1) - Gg(t_2)\| = \left\| \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t \mathrm{f}(\tau, \mathrm{g}(\tau))(t_1 - \tau)^{\alpha-1} d\tau - \left(\frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t \mathrm{f}(\tau, \mathrm{g}(\tau))(t_2 - \tau)^{\alpha-1} d\tau \right) \right\|. \tag{99}$$

$$|Gg(t_1) - Gg(t_2)| = \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \left| \int_0^t ((t_1 - \tau)^{\alpha-1} - (t_2 - \tau)^{\alpha-1}) \mathrm{f}(\tau, \mathrm{g}(\tau)) d\tau \right|. \tag{100}$$

$$|Gg(t_1) - Gg(t_2)| \leq \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t ((t_1 - \tau)^{\alpha-1} - (t_2 - \tau)^{\alpha-1}) |\mathrm{f}(\tau, \mathrm{g}(\tau))| d\tau. \tag{101}$$

$$|Gg(t_1) - Gg(t_2)| \leq \frac{[ar + b]}{B(\alpha)\Gamma(\alpha)} [t_1^\alpha - t_2^\alpha]. \tag{102}$$

As t_1 approaches t_2, the right hand side of Equation (102) tends to zero. Since G is continuous, $|Gg(t_1) - Gg(t_2)|$ also approaches 0 as t_1 approaches t_2.

Therefore, G is bounded and continuous, which implies that it is also uniformly continuous and bounded. By the Arzelà–Ascoli theorem, G is relatively compact and completely continuous. Invoking Theorem 14, we conclude that the integral Equation (82) has at least one solution, and consequently, the system itself has at least one solution.

To address the question of uniqueness, we provide the following result. □

Theorem 16 (uniqueness). *The model given by Equation (83) has a unique solution provided that the following conditions satisfy*

$$\omega = \left(\frac{1 - \alpha}{F(\alpha)} + \frac{\mathrm{t}^\alpha}{F(\alpha)\Gamma(\alpha)} \right) L_{\mathrm{M}} < 1, \tag{103}$$

under Assumption 2.

Proof. To prove Theorem 16, let us assume that $J = [0, T]$ and consider the operator ψ: $C(J, R^7) \to C(J, R^7)$.

Thus, using Equation (88) we have

$$\psi(g(t)) = g(0) + (1 - \alpha)/B(\alpha).f(\tau, g(\tau)) + \alpha/B(\alpha).\Gamma(\alpha) \int_0^\tau \mathrm{f}(\tau, \mathrm{g}(\tau))(t - \tau)^{n-1} d\tau. \tag{104}$$

Let $g_1, g_2 \in C(J, R^7)$ and $\tau \in [0, t]$. Then, we have

$$\|\psi(g_1(t)) - \psi(g_2(t))\| = \left\| \frac{1 - \alpha}{B(\alpha)} (f(t, g_1(t)) - f(t, g_2(t))) + \alpha/B(\alpha).\Gamma(\alpha) \int_0^t (\mathrm{f}(\tau, \mathrm{g}_1(\tau)) - \mathrm{f}(\tau, \mathrm{g}_2(\tau)))(t - \tau)^{n-1} d\tau \right\|. \tag{105}$$

Additionally, after some algebraic simplification, we can employ the Lipschitz condition and the ideas of triangular inequality given in Equation (105).

$$\|\psi(g_1(t)) - \psi(g_2(t))\| \leq \max_{\mathrm{t} \in [0,\tau]} \frac{1 - \alpha}{B(\alpha)} |f(t, g_1(t)) - f(t, g_2(t))| + \max_{\mathrm{t} \in [0,\tau]} \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \left| \int_0^t (t - \tau)^{n-1} |\mathrm{f}(\tau, \mathrm{g}_1(\tau)) - \mathrm{f}(\tau, \mathrm{g}_2(\tau))| d\tau \right|. \tag{106}$$

Thus, we eventually have:

$$\|\psi(g_1(t)) - \psi(g_2(t))\| \leq \left(\frac{1 - \alpha}{B(\alpha)} + \frac{t^\alpha}{B(\alpha)\Gamma(\alpha)} \right) L_{\mathrm{M}} \|\mathrm{g}_1 - \mathrm{g}_2\|, \tag{107}$$

where $\omega = (\frac{1-\alpha}{B(\alpha)} + \frac{t^\alpha}{B(\alpha)\Gamma(\alpha)}) L_M$.

Therefore, the operator ψ becomes a contraction if condition Equation (107) holds on $\mathrm{C}(\mathrm{J}, \mathrm{R}^7)$. Consequently, the Banach fixed point theorem ensures that system (103) possesses a unique solution. □

5.2. Numerical Iterative Scheme and Simulations. We use the method revised in [47], which combines the two-step Lagrange polynomial and the fundamental theorem of fractional calculus, to approximate the Atangana–Baleanu fractional integral. To obtain an iterative strategy, apply the aforementioned technique to the system (88).

At $t = t_{n+1}$ and $n = 0, 1, 2, \cdots$, we have

$$\begin{aligned} g(t_{n+1}) - g(0) &= \frac{(1 - \alpha)}{B(\alpha)} f(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^{t_{n+1}} \mathrm{f}(\tau, \mathrm{g}(\tau))(t_{n+1} - \tau)^{n-1} d\tau \\ &= \frac{1 - \alpha}{B(\alpha)} f(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{i=0}^{n} \int_{t_i}^{t_{i+1}} \mathrm{f}(\tau, \mathrm{g}(\tau))(t_{n+1} - \tau)^{n-1} d\tau. \end{aligned} \tag{108}$$

With the help of interpolation polynomial, we approximate the function $f(\tau, g(\tau))$ over $[t_i, t_{i+1}]$.

$$f(\tau, g(\tau)) \cong \mathrm{p_k}(\tau) = \frac{\mathrm{f}(t_i, \mathrm{g}(t_i))}{h}(\tau - t_{i-1}) - \frac{\mathrm{f}(t_{i-1}, g(t_{i-1}))}{h}(\tau - \mathrm{t_i}). \tag{109}$$

Using Equation (109) Equation (108) takes the form:

$$\begin{aligned} g(t_{n+1}) = {} & g(0) + \frac{1-\alpha}{B(\alpha)} f(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \left(\sum_{i=0}^{n} \frac{\mathrm{f}(t_i, g(t_i))}{h} \int_{t_i}^{t_{i+1}} (\tau - \mathrm{t_{i-1}})(t_{n+1} - \tau)^{n-1} d\tau \right. \\ & \left. - \frac{\mathrm{f}(t_{i-1}, g(t_{i-1}))}{h} \int_{t_i}^{t_{i+1}} (\tau - \mathrm{t}_i)(t_{n+1} - \tau)^{n-1} \right) d\tau. \end{aligned} \tag{110}$$

Solving the integrals involved in Equation (110), we obtain the following approximate solution, which represents:

$$g(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} f(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \begin{pmatrix} \frac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)), \\ -\frac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)). \end{pmatrix} \tag{111}$$

Hence, we have the following recursive formulas for the proposed malaria model Equation (61):

$$S_H(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_1(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \begin{pmatrix} \frac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\frac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{pmatrix}, \tag{112}$$

$$I_{\mathrm{I}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_2(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \begin{pmatrix} \frac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\frac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{pmatrix}, \tag{113}$$

$$I_{\mathrm{A}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_3(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \begin{pmatrix} \frac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\frac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{pmatrix}, \tag{114}$$

$$I_{\mathrm{P}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_4(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \left(\begin{array}{l} \dfrac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\dfrac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{array} \right), \tag{115}$$

$$S_{\mathrm{M}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_5(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \left(\begin{array}{l} \dfrac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\dfrac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{array} \right), \tag{116}$$

$$E_{\mathrm{M}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_6(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \left(\begin{array}{l} \dfrac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\dfrac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{array} \right), \tag{117}$$

$$I_{\mathrm{M}}(t_{n+1}) = g(t_0) + \frac{1-\alpha}{B(\alpha)} F_7(t_n, g(t_n)) + \frac{\alpha}{B(\alpha)} \sum_{i=0}^{n} \left(\begin{array}{l} \dfrac{h^\alpha f(t_i, g(t_i))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^\alpha (n-i+2+\alpha) - (n-i)^\alpha (n-i+2+2\alpha)) \\ -\dfrac{h^\alpha f(t_{i-1}, g(t_{i-1}))}{B(\alpha)\Gamma(\alpha)} ((n+1-i)^{\alpha+1} - (n-i)^\alpha (n-i+1+\alpha)) \end{array} \right). \tag{118}$$

6. Results and Discussion

This section presents the findings of the proposed classical (integer-order) malaria model (1) and the numerical solutions of the fractional malaria model (61). This model, encompassing human and mosquito subgroups, is formulated as follows: the human subgroup is divided into four compartments: susceptible $(S_{\mathrm{H}}(t))$ individuals, infectious individuals under 5 years $(I_{\mathrm{I}}(t))$ of old, infectious individuals over 5 years $(I_{\mathrm{A}}(t))$ of old, and infectious pregnant women $(\mathrm{I_P}(t))$.The mosquito population is divided into three compartments: susceptible mosquitoes $(\mathrm{S_M}(t))$, exposed mosquitoes $(\mathrm{E_M(t)})$, and infected mosquitoes $(I_{\mathrm{M}}(t))$. This study utilizes the Atangana–Baleanu fractional differential operator in the Caputo sense for the numerical solution.

The analysis of the proposed model includes the following:

(i) Evaluation of the existence and uniqueness of solutions
(ii) Stability analysis
(iii) Numerical simulations.

Sensitivity analysis revealed that the density-dependent natural mortality rate for adult female Anopheles mosquitoes (μ_{M}) has the highest negative sensitivity index, at −1.0984, among other parameters. This means that decreasing this mortality rate by 10% would lead to a 10.884% increase in $\mathscr{R}_0$. The highest positive sensitivity index, 0.82142, belongs to the number of bites on people over 5 years per female mosquito per unit of time (ϕ_{A}). This indicates that increasing ϕ_{A} by 10% would lead to an 8.2142% increase in $\mathscr{R}_0$. Subsequently, the parameters Z_M, θ_{MH}, and θ_{HM} exhibit sensitivity indices of 0.49928, 0.49929, and 0.4993, respectively. This observation aligns with Tungmah et al. [37], who found that lowering these parameters by 10% results in corresponding reductions of $\mathscr{R}_0$ by 4.9928%, 4.9929%, and 4.993%, respectively.

To gain a deeper understanding of the fractional model's Equation (61) dynamics across various scenarios and parameter combinations, numerical simulations were performed. Using the values of the proposed parameters given in Table 3, the numerical solutions of the fractional malaria model Equation (61) for different values of the fractional order $\alpha = 0.6, 0.7, 0.8, 0.9, 1$ are displayed in Figures 2. These solutions were generated using Equation (110) with a setp size of $h = 0.005$. The figures demonstrate that different values of fractional orders have a significant impact on the system's dynamics. They also indicate that as α approaches 1, the

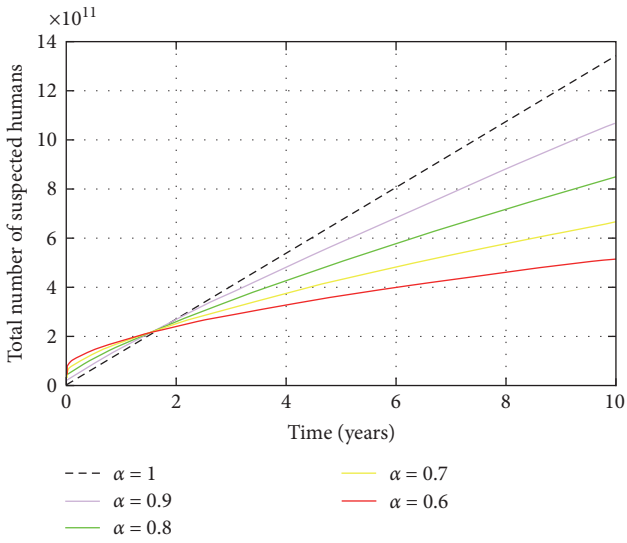

Figure 2: Total number of suspected humans for different values of α.

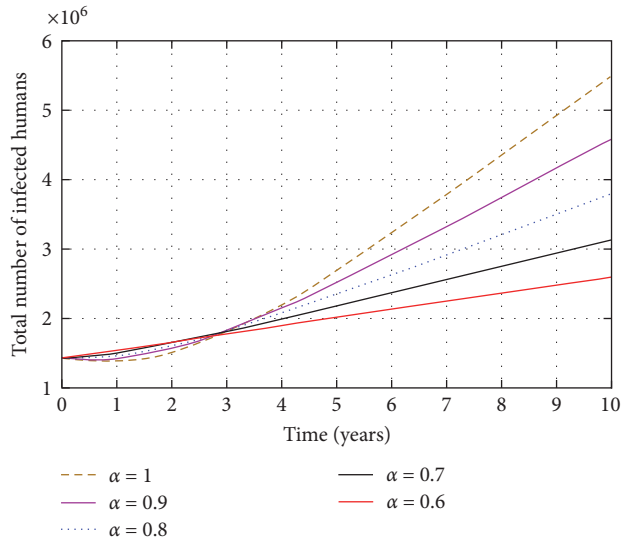

Figure 3: Total number of infected infants for different values of α.

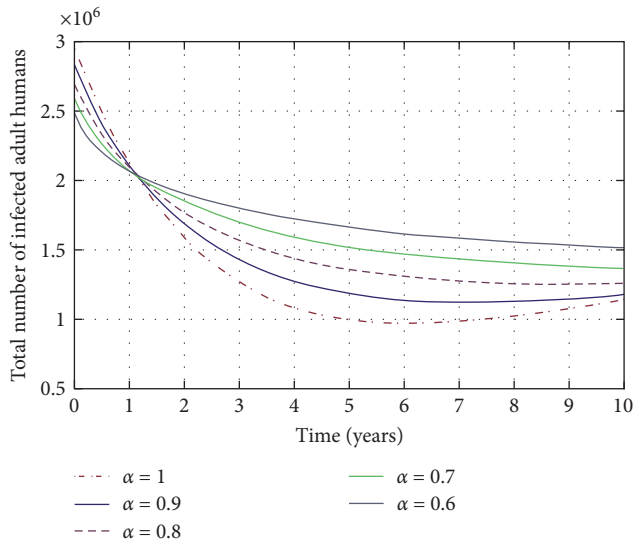

Figure 4: Total number of infected adult humans for different values of α.

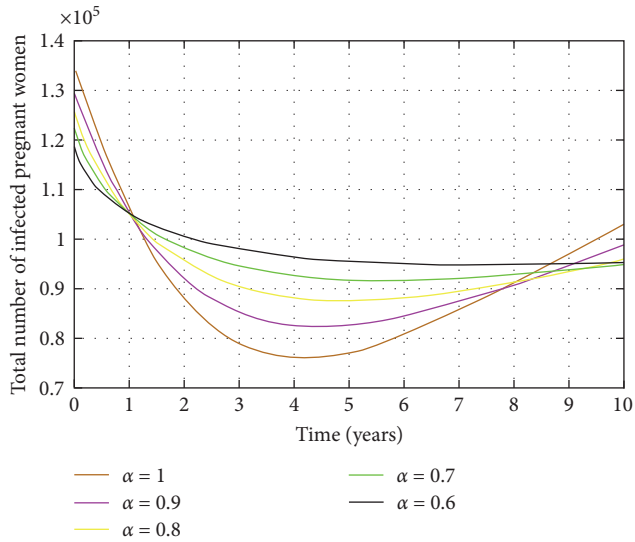

Figure 5: Total number of infected pregnant women for different values of α.

approximate solutions converges toward the classical (or integer) order solution.

The graph of the susceptible human populations over time for the fractional malaria model (61) is depicted in Figures 2–8.This population increases mildly until individuals become infected with the disease and transition to other compartments within the fractional model system (61). Figures 3–5, respectively, represent the behavior of infected infants, infected adults, and infected women populations over time with different fractional values. Figures 6– 8, in turn, represent the behavior of susceptible mosquito populations, exposed mosquitoes, and infected mosquitoes, respectively, over time for different fractional order values of the proposed fractional malaria model Equation (61). Figure 3 reveals a surprising pattern: during years 1–3, with increasing α, the number of infected infants declines. However, after year 3, a distinct shift occurs; the number of infected infants exhibits a direct positive correlation with α. In simpler terms, as α increases beyond year 3, the number of infected infants also rises. In contrast, Figure 4 shows infected adults increasing with α in the first year, followed by a decrease as α continues to rise. Figure 5 shows that the population of infected pregnant women oscillates as the values of the fractional order α vary, exhibiting a sinusoidal pattern. Figures 6–8 share similar structures

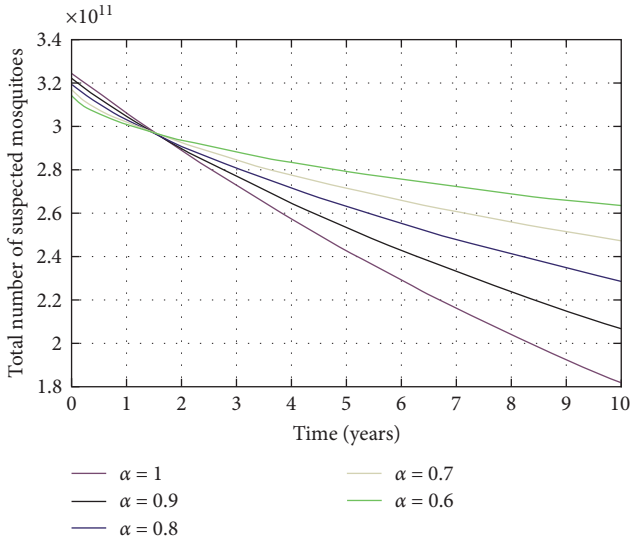

Figure 6: Total number of suspected mosquitoes for different values of α.

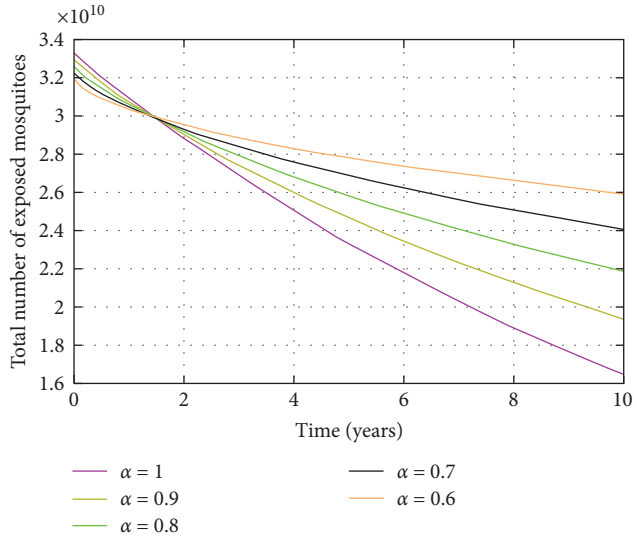

Figure 7: Total number of exposed mosquitoes for different values of α.

between years 1 and 2. However, as fractional order α increases in these years, the values of both exposed and infected mosquitoes rise, as shown in Figures 7 and 8, respectively. Conversely, Figure 6 shows a decline in suspected mosquitoes. Beyond year 2, further increases in α lead to differing trends: suspected mosquitoes decrease (Figure 6), exposed mosquitoes increase (Figure 7), and infected mosquitoes continue to rise (Figure 8).

7. Conclusions

This research analyzed the disease transmission dynamics of malaria by developing an age-structured mathematical model using the classical integer order and Atangana–Baleanu–Caputo fractional operators sense. The analysis of the model focused on several important aspects. The existence and uniqueness of solutions of fractional order model were investigated based on some fixed point theorems such as Banach and Krasnoselski, providing a solid foundation for the subsequent analysis. Positivity and boundedness of the solutions were also investigated, ensuring the practicality and reliability of the model.

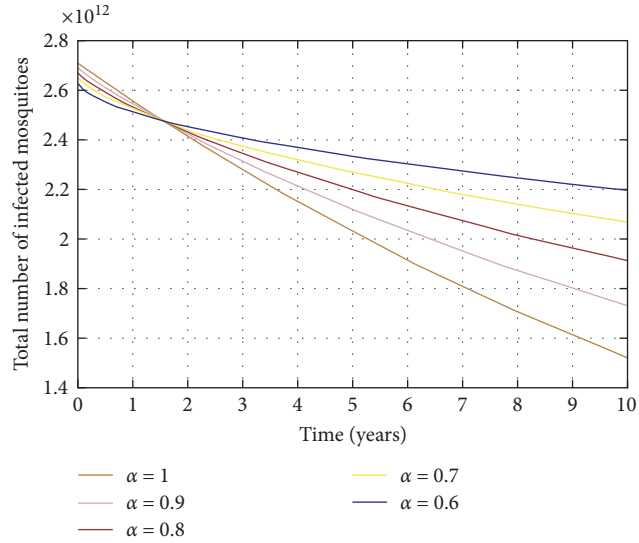

Figure 8: Total number of infected mosquitoes for different values of α.

Furthermore, the model's equilibria were discovered, and the results showed that the disease-free and endemic equilibrium points are found to be locally and globally asymptotically stable for $R_0 < 1$ and $R_0 > 1$, respectively. The sensitivity analysis revealed that the most sensitive parameters essential for controlling or eliminating malaria are mosquito biting rate, density-dependent natural mortality rate, clinical recovery rate, and recruitment rate for mosquitoes. These findings align with [37], highlighting the importance of targeting these parameters for effective control measures.

For numerical simulations, the combination of two-step Lagrange polynomial and fundamental theorem of fractional calculus and the Toufik–Atangana numerical method were employed. Several simulations were performed on the model, yielding various graphical results that aligned with the theoretical results.

Future work can expand this model by incorporating additional factors, such as environmental variables, socioeconomic factors, and vector behavior, to gain a deeper understanding of the complex interactions influencing malaria transmission dynamics. Real data from Jimma, Ethiopia, could be used for calibration and validation, allowing for a comparison of results obtained using both Caputo and ABC fractional operators.

Data Availability

All the necessary data were included in the main text.

Conflicts of Interest

The authors declare that there is no conflict of interest about the publication of this paper.

Authors' Contributions

AKG proposed the main idea of this paper. CTD supervised the work from the first draft to revision, and approval of the final manuscript for submission.

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