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Research Article

Innovative Solutions for the Kadomtsev–Petviashvili Equation via the New Iterative Method

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This research paper presents a new iterative method (NIM) for obtaining the solution to the potential Kadomtsev–Petviashvili (PKP) equation. NIM is a promising approach to solving complex mathematical problems, and its effectiveness and efficiency are highlighted through its application to the PKP equation. The results obtained through the use of NIM are compared to the exact solutions of the PKP equation, and it is found that the NIM approach provides results that are in close agreement with the exact solutions. This demonstrates the utility and accuracy of NIM and makes it a valuable tool for solving similar mathematical problems in the future. Furthermore, the lack of discretization in the NIM approach makes it a more convenient method for solving the PKP equation compared to traditional approaches that require discretization. Overall, the findings of this research paper suggest that NIM is a highly effective and convenient method for obtaining approximate analytical solutions to complex mathematical problems, such as the $(2+1)$-dimensional PKP equation.

1. Introduction

Nonlinear phenomena are of paramount importance in numerous scientific and engineering disciplines. Despite their widespread significance, the models of real-world problems that incorporate nonlinearity are often challenging to solve, either theoretically or numerically. This has led to a growing interest in developing better and more efficient methods for determining solutions to nonlinear models, whether they be approximate or exact, analytical or numerical.

In recent years, significant progress has been made in this direction, with researchers devoting a great deal of attention to the search for new and improved methods for solving nonlinear problems. This has led to a wealth of studies and publications that explore various approaches to solving nonlinear models (such as those by [1–10]). These efforts are aimed at improving our understanding of nonlinearity and finding ways to effectively model and solve real-world problems that incorporate nonlinear dynamics.

Despite the progress that has been made, much work remains to be done. Nonlinear models can still be difficult to solve, and the search for more effective and efficient methods for determining their solutions continues to be a focus of research and development efforts in various scientific and engineering disciplines. Nevertheless, the progress that has been made in recent years highlights the promise of continued progress in the future as researchers continue to work toward finding better and more efficient methods for solving nonlinear problems.

The $(2+1)$-dimensional potential Kadomtsev–Petviashvili (PKP) equation is an important partial differential equation in the field of nonlinear wave phenomena. It is a generalization of the $(2+1)$-dimensional KP equation and describes the propagation of nonlinear waves in a two-dimensional space with an additional degree of freedom.

The importance of PKP lies in its ability to model a wide range of physical phenomena, such as plasma physics, fluid dynamics, and nonlinear optics. For example, PKP has been used to describe the interaction of Langmuir waves and ion-acoustic waves in plasma physics, the propagation of waves in shallow water, and the dynamics of optical pulses in nonlinear media.

Moreover, PKP possesses several interesting mathematical properties, such as soliton solutions, integrability, and infinite-dimensional symmetries. These properties make

PKP an attractive subject of study in the field of mathematical physics, where it has been investigated extensively using a variety of techniques, including inverse scattering transform, Darboux transformation, and the Hirota bilinear method.

Overall, the PKP equation plays an important role in both the physical and mathematical sciences and continues to be an active area of research [11–13].

The PKP equation is a partial differential equation that has applications in various fields of physics, including fluid dynamics, plasma physics, and nonlinear optics. Here are some examples of how PKP equations are used in the real world.

Fluid Dynamics: The PKP equation is used to describe the dynamics of fluids in various contexts, including ocean currents, atmospheric circulation, and turbulence. For example, the PKP equation has been used to study the behavior of waves in shallow water and to model the flow of viscous fluids in channels and pipes.

Plasma Physics: The PKP equation is also used in the study of plasma physics, particularly in the context of magnetohydrodynamics (MHDs). MHD is a branch of plasma physics that studies the behavior of ionized gases in the presence of magnetic fields. The PKP equation has been used to describe the dynamics of MHD waves and to study the stability of plasma configurations.

Nonlinear Optics: The PKP equation has applications in the field of nonlinear optics, which studies the behavior of light in nonlinear media. Nonlinear optics is used in a variety of applications, including telecommunications, laser technology, and medical imaging. The PKP equation has been used to model the propagation of ultrashort pulses of light in optical fibers and to study the dynamics of solitons in nonlinear media [14–16].

In 2001, Senthilvelan [17] solved the potential PKP equation by applying the homogenous balance method (HBM). Later, Li and Zhang [18] made improvements to the steps of the HBM and applied the method to the study of the PKP equation. Through their research, they were able to obtain various exact solutions to the equation, including soliton, multisoliton, and rational-type solutions.

In an independent investigation, Li and Zhang [19] successfully derived novel soliton-like solutions for the PKP equation through the application of symbolic computation. Additionally, Kaya and El-Sayed [20] employed the Adomian decomposition method (ADM) to ascertain the solution of the PKP equation. Simultaneously, Batiha and Batiha [21] utilized the variational iteration method (VIM) to uncover solutions for the $(2+1)$-dimensional PKP equation. These diverse approaches underscore the range of methodologies employed in addressing the PKP equation, showcasing the potential of different methods in resolving intricate nonlinear problems.

The new iterative method (NIM), presented by Daftardar-Gejji and Jafari [22] in 2006, emerged as a potent mathematical tool for tackling both linear and nonlinear functional equations. Demonstrating its efficacy, NIM has been successfully applied to solve a diverse array of nonlinear equations, encompassing integral equations, algebraic equations, as well as ordinary or partial differential equations of both fractional and integer order.

NIM stands out due to its simplicity in implementation and comprehensibility, rendering it accessible to a wide spectrum of researchers and practitioners. In contrast to well-established methods like the ADM [23], the homotopy perturbation method [24], and the VIM [25], NIM has demonstrated superior performance and increased efficiency. This has contributed to its popularity among individuals addressing intricate nonlinear problems [26].

In this study, our aim is to use the NIM to obtain a numerical solution for the $(2+1)$-dimensional PKP Equation, which is given in the following form:

$$u_{xt}+\frac{3}{2}u_x u_{xx}+\frac{1}{4}u_{xxxx}+\frac{3}{4}u_{yy}=0. \tag{1}$$

The initial conditions are already known and will be used in our analysis. In order to validate the effectiveness of NIM, we will compare the results obtained through its application with the exact solution of the PKP equation.

2. Overview of the NIM

This section provides an outline of the NIM numerical method, detailing its approach as follows [27–30]:

$$\mathbf{y}=f+\mathbb{L}(\mathbf{y})+\mathbb{N}(\mathbf{y}). \tag{2}$$

In the presented equation, f represents a given function, while $\mathbb{L}$ and $\mathbb{N}$ denote linear and nonlinear operators, respectively. The solution to Equation (2) is expressed as follows:

$$\mathbf{y}=\sum_{i=0}^{\infty}\mathbf{y}_i. \tag{3}$$

Suppose we have the following:

$$H_0=\mathbb{N}(\mathbf{y}_0), \tag{4}$$

$$H_m=\mathbb{N}\left(\sum_{i=0}^{m}\mathbf{y}_i\right)-\mathbb{N}\left(\sum_{i=0}^{m-1}\mathbf{y}_i\right). \tag{5}$$

Then, we get the following:

$$\begin{aligned}
H_0 &= \mathbb{N}(\mathbf{y}_0),\\
H_1 &= \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1)-\mathbb{N}(\mathbf{y}_0),\\
H_2 &= \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2)-\mathbb{N}(\mathbf{y}_0+\mathbf{y}_1),\\
H_3 &= \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2+\mathbf{y}_3)-\mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2)+\cdots.
\end{aligned} \tag{6}$$

Therefore, $\mathbb{N}(\mathbf{y})$ can be decomposed as follows:

$$\mathbb{N}\left(\sum_{i=0}^{\infty}\mathbf{y}_i\right) = \mathbb{N}(\mathbf{y}_0) + \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1) - \mathbb{N}(\mathbf{y}_0) + \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2) - \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1) + \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2+\mathbf{y}_3) - \mathbb{N}(\mathbf{y}_0+\mathbf{y}_1+\mathbf{y}_2) + \cdots. \tag{7}$$

The recurrence relation is written in the following form:

$$\begin{aligned} \mathbf{y}_0 &= f \\ \mathbf{y}_1 &= \mathbb{L}(\mathbf{y}_0) + H_0 \\ \mathbf{y}_{m+1} &= \mathbb{L}(\mathbf{y}_m) + H_m,\ m = 1, 2, \cdots. \end{aligned} \tag{8}$$

By knowing that $\mathbb{L}$ is linear we will have the following:

$$\sum_{i=0}^{m}\mathbb{L}(\mathbf{y}_i) = \mathbb{L}\left(\sum_{i=0}^{m}\mathbf{y}_i\right). \tag{9}$$

So,

$$\begin{aligned} \sum_{i=0}^{m+1}\mathbf{y}_i &= \sum_{i=0}^{m}\mathbb{L}(\mathbf{y}_i) + \mathbb{N}\left(\sum_{i=0}^{m}\mathbf{y}_i\right). \\ &= \mathbb{L}\left(\sum_{i=0}^{m}\mathbf{y}_i\right) + \mathbb{N}\left(\sum_{i=0}^{m}\mathbf{y}_i\right),\ m = 1, 2, \cdots \end{aligned} \tag{10}$$

thus,

$$\sum_{i=0}^{\infty}\mathbf{y}_i = f + \mathbb{L}\left(\sum_{i=0}^{\infty}\mathbf{y}_i\right) + \mathbb{N}\left(\sum_{i=0}^{\infty}\mathbf{y}_i\right). \tag{11}$$

So, the solution will be given as follows:

$$\mathbf{y} = \sum_{i=0}^{k-1}\mathbf{y}_i. \tag{12}$$

3. Convergence Analysis of NIM

Theorem 1. *For any n and given real constants $L > 0$ and $\|u_i\| \le M < \frac{1}{e}$ for $i = 1, 2, \ldots$, if the nonlinear operator N is $C^{(\infty)}$ in the neighborhood of u_0 and $\|N^{(n)}(u_0)\| \le L$, then the series $\sum_{n=0}^{\infty}H_n$ is absolutely convergent, and $\|H_n\| \le LM^n e^{n-1}(e-1)$ for $n = 1, 2, \ldots$.*

Proof.

$$\|H_n\| \le LM^n \sum_{i_n=1}^{\infty}\sum_{i_{n-1}=0}^{\infty}\cdots\sum_{i_1=0}^{\infty}\left(\prod_{j=1}^{n}\frac{1}{i_j!}\right) = LM^n e^{n-1}(e-1). \tag{13}$$

Therefore, the series $\sum_{n=1}^{\infty}\|H_n\|$ is bounded by the convergent series $LM(e-1)\sum_{n=1}^{\infty}(Me)^{n-1}$, where $M < 1/e$. Consequently, $\sum_{n=0}^{\infty}H_n$ converges absolutely, as established by the comparison test. □

Since demonstrating the boundedness of u_i for all i poses challenges, a more practical result is provided in the following theorem. This theorem specifies conditions on $N^{(k)}(u_0)$ that are sufficient to ensure the convergence of the series.

Theorem 2. *If the series $\sum_{n=0}^{\infty}H_n$ converges absolutely, given that N is $C^{(\infty)}$ and $\|N^{(n)}(u_0)\| \le M \le e^{-1}$ for all n.*

Proof. Let's examine the recurrence relation

$$\varepsilon_n = \varepsilon_0 \exp(\varepsilon_{n-1}),\ n = 1, 2, 3, \ldots, \tag{14}$$

where $\varepsilon_0 = M$. Define $\eta_n = \varepsilon_n - \varepsilon_{n-1}, n = 1, 2, 3, \ldots$. We can observe that

$$\|H_n\| \le \eta_n,\ n = 1, 2, 3, \ldots. \tag{15}$$

Now, let

$$\sigma_n = \sum_{i=1}^{n}\eta_i = \varepsilon_n - \varepsilon_0. \tag{16}$$

Note that $\varepsilon_0 = e^{-1} > 0$, $\varepsilon_1 = \varepsilon_0 \exp(\varepsilon_0) > \varepsilon_0$, and $\varepsilon_2 = \varepsilon_0 \exp(\varepsilon_1) > \varepsilon_0 \exp(\varepsilon_0) = \varepsilon_1$. In general, $\varepsilon_n > \varepsilon_{n-1} > 0$. Hence $\sum\eta_n$ is a series of positive real numbers. Observe that

$$\begin{aligned} 0 < \varepsilon_0 &= M = e^{-1} < 1, \\ 0 < \varepsilon_1 &= \varepsilon_0 \exp(\varepsilon_0) < \varepsilon_0 e^1 = e^{-1}e^1 = 1, \\ 0 < \varepsilon_2 &= \varepsilon_0 \exp(\varepsilon_1) < \varepsilon_0 e^1 = 1. \end{aligned} \tag{17}$$

In general, $0 < \varepsilon_n < 1$. Hence, $\sigma = \varepsilon_n - \varepsilon_0 < 1$. This implies that $\{\sigma_n\}_{n=1}^{\infty}$ is bounded above by 1, and hence convergent. Therefore, $\sum H_n$ is absolutely convergent by the comparison test. □

4. Examination of the PKP Equation with Numerical Applications

This paper focuses on the $(2+1)$-dimensional PKP equation, defined as follows:

$$u_{xt} + \frac{3}{2}u_x u_{xx} + \frac{1}{4}u_{xxxx} + \frac{3}{4}u_{yy} = 0. \tag{18}$$

The initial conditions for the PKP equation are given as follows:

$$\begin{aligned} u(x,0,t) &= 1 + 2k\alpha \tanh[k(\alpha x - ct)], \\ u_y(x,0,t) &= 2\alpha\beta k^2 \operatorname{sech}^2[k(\alpha x - ct)], \end{aligned} \tag{19}$$

where $c = (k^2\alpha^3 + \frac{3\beta^3}{4\alpha})$.

The exact solution to the PKP equation has been determined and was first published by Senthilvelan [17]. The traveling wave solution is expressed as follows [20]:

$$u(x,y,t) = 1 + 2k\alpha \tanh\left[k\left(\alpha x + \beta y - \left(k^2\alpha^3 + \frac{3\beta^3}{4\alpha}\right)t\right)\right]. \tag{20}$$

To solve Equation (18) by NIM, we integrate Equation (18) twice with respect to y from 0 to y and use Equation (19) to get the following:

$$\begin{aligned} u &= 1 + 2\alpha k \tanh(km) + 2\alpha\beta k^2 y \operatorname{sech}^2(km) \\ &+ \int_0^y \int_0^y \left(-\frac{4}{3}u_{xt} - 2u_x u_{xx} - \frac{1}{3}u_{xxx}\right) dy\, dy, \end{aligned} \tag{21}$$

where $m = \alpha x - t(\frac{3\beta^3}{4\alpha} + \alpha^3 k^2)$.

By using Equation (8), we get the following:

$$u_0 = 1 + 2\alpha k \tanh(km) + 2\alpha\beta k^2 y \operatorname{sech}^2(km), \tag{22}$$

$$\begin{aligned} u_1 &= \frac{1}{2}y^2\Bigg(16\alpha^5k^5 \tanh(k(m))\operatorname{sech}^4(k(m)) + \frac{1}{3}(4\alpha^4k^4\operatorname{sech}^4(k(m)) \\ &- 8\alpha^4k^4 \tanh^2(k(m))\operatorname{sech}^2(k(m))) + \frac{16}{3}\alpha^2k^3\left(\alpha^3(-k^2) - \frac{3\beta^3}{4\alpha}\right)\tanh(k(m))\operatorname{sech}^2(k(m))\Bigg), \\ &\vdots \end{aligned} \tag{23}$$

Thus,

$$\begin{aligned} \sum_{i=0}^{1} u_i &= 1 + 2\alpha k \tanh(km) + 2\alpha\beta k^2 y \operatorname{sech}^2(km) \\ &+ \frac{1}{2}y^2\Bigg(16\alpha^5k^5 \tanh(k(m))\operatorname{sech}^4(k(m)) + \frac{1}{3}(4\alpha^4k^4\operatorname{sech}^4(k(m)) \\ &- 8\alpha^4k^4 \tanh^2(k(m))\operatorname{sech}^2(k(m))) + \frac{16}{3}\alpha^2k^3\left(\alpha^3(-k^2) - \frac{3\beta^3}{4\alpha}\right)\tanh(k(m))\operatorname{sech}^2(k(m))\Bigg), \end{aligned} \tag{24}$$

where $m = \alpha x - t(\frac{3\beta^3}{4\alpha} + \alpha^3 k^2)$.

Figure 1 illustrates the comparison between the 1-iterate of NIM and the exact solution given by Equation (20).

Table 1 presents a comparison between the first iteration of NIM and the exact solution given by Equation (20).

To validate the accuracy of NIM, we showcase the solution of the PKP Equation (18) through ADM, as previously demonstrated in [20]. In Table 2, we present a comprehensive comparison between the ADM solution [20] and the exact solution provided by Equation (20). This analysis allows us to assess the precision and reliability of NIM in solving complex mathematical problems.

We will now use the NIM to find an approximate analytical solution to the PKP equation as follows:

$$u_{xt} + \frac{3}{2}u_x u_{xx} + \frac{1}{4}u_{xxxx} + \frac{3}{4}u_{yy} = 0, \tag{25}$$

with the initial conditions are as follows:

$$\begin{aligned} u(x,0,t) &= 1 + 2k\alpha \tan[k(\alpha x - ct)], \\ u_y(x,0,t) &= 2\alpha\beta k^2 \sec^2[k(\alpha x - ct)], \end{aligned} \tag{26}$$

where $c = (k^2\alpha^3 + \frac{3\beta^3}{4\alpha})$.

The exact solution to Equation (25) can be found in [17] and is given by the following:

$$u(x,y,t) = 1 + 2k\alpha \tan\left[k\left(\alpha x + \beta y + \left(k^2\alpha^3 - \frac{3\beta^3}{4\alpha}\right)t\right)\right]. \tag{27}$$

To solve Equation (25) by NIM, we integrate Equation (25) and use Equation (26) to get the following:

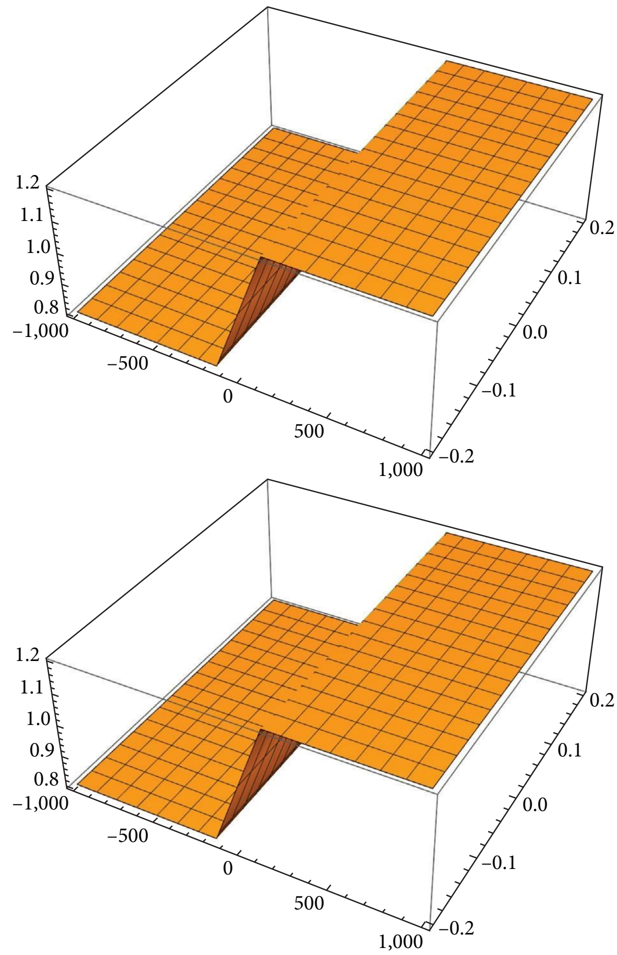

FIGURE 1: A comparison between the exact solution and the numerical results of $u(x, y, t)$ is depicted, where the numerical solution is obtained through the 1-iterate NIM solution. The comparison is conducted for the parameter values $\alpha = 1$, $\beta = 0.1$, $k = 0.1$, and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

TABLE 1: Comparison between the exact solution and the numerical results for the numerical solution $u(x, y, t)$ obtained through the 1-iterate NIM solution is presented for the values $\alpha = 1, \beta = 0.1, k = 0.1$ and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

x_i/y_i	0.1	0.2	0.3	0.4	0.5
0.1	6.40477×10^{-9}	2.64440×10^{-8}	6.13530×10^{-8}	1.12362×10^{-7}	1.80704×10^{-7}
0.2	1.25927×10^{-8}	5.11935×10^{-8}	1.17031×10^{-7}	2.11330×10^{-7}	3.35310×10^{-7}
0.3	1.87615×10^{-8}	7.58620×10^{-8}	1.72528×10^{-7}	3.09961×10^{-7}	4.89370×10^{-7}
0.4	2.48999×10^{-8}	1.00414×10^{-7}	2.27741×10^{-7}	4.08101×10^{-7}	6.42644×10^{-7}
0.5	3.09997×10^{-8}	1.24807×10^{-7}	2.82605×10^{-7}	5.05611×10^{-7}	7.94914×10^{-7}

TABLE 2: Comparison between the exact solution and the numerical results obtained through the ADM solution [20] is presented for the values $\alpha = 1, \beta = 0.1, k = 0.1$ and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

x_i/y_i	0.1	0.2	0.3	0.4	0.5
0.1	6.40478×10^{-9}	2.64439×10^{-8}	6.13527×10^{-8}	1.12364×10^{-7}	1.80706×10^{-7}
0.2	1.25928×10^{-8}	5.11930×10^{-8}	1.17029×10^{-7}	2.11326×10^{-7}	3.35301×10^{-7}
0.3	1.87611×10^{-8}	7.58615×10^{-8}	1.72522×10^{-7}	3.09956×10^{-7}	4.89368×10^{-7}
0.4	2.48998×10^{-8}	1.00411×10^{-7}	2.27744×10^{-7}	4.08099×10^{-7}	6.42668×10^{-7}
0.5	3.09995×10^{-8}	1.24803×10^{-7}	2.82608×10^{-7}	5.05603×10^{-7}	7.94960×10^{-7}

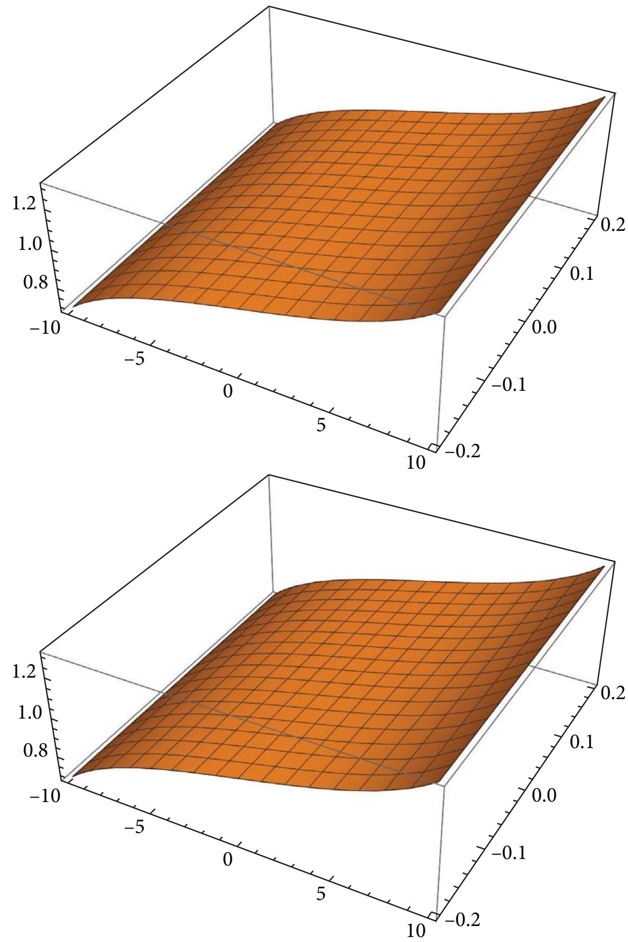

Figure 2: Comparison of the exact solution with the numerical results for the 1-iteration NIM solution of $u(x, y, t)$, obtained for the parameter values $\alpha = 1$, $\beta = 0.1$, $k = 0.1$, and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

$$u = 1 + 2\alpha k \tan(km) + 2\alpha\beta k^2 y \sec^2(km) + \int_0^y \int_0^y \left(-\frac{4}{3}u_{xt} - 2u_x u_{xx} - \frac{1}{3}u_{xxx}\right) dy\, dy, \tag{28}$$

where $m = \alpha x - t\left(\frac{3\beta^3}{4\alpha} + \alpha^3 k^2\right)$.

By using Equation (8), we get the following:

$$u_0 = 1 + 2\alpha k \tan(km) + 2\alpha\beta k^2 y \sec^2(km), \tag{29}$$

$$\begin{aligned} u_1 &= \frac{1}{2}y^2\Bigg(-16\alpha^5k^5\tan(k(m))\sec^4(k(m)) + \frac{1}{3}(-4\alpha^4k^4\sec^4(k(m)) \\ &\quad - 8\alpha^4k^4\tan^2(k(m))\sec^2(k(m))) - \frac{16}{3}\alpha^2k^3\left(\alpha^3(-k^2) - \frac{3\beta^3}{4\alpha}\right)\tan(k(m))\sec^2(k(m))\Bigg), \\ &\;\;\vdots \end{aligned} \tag{30}$$

Thus,

$$\begin{aligned} \sum_{i=0}^{1} u_i &= 1 + 2\alpha k \tan(km) + 2\alpha\beta k^2 y \sec^2(km) \\ &\quad + \frac{1}{2}y^2\Bigg(-16\alpha^5k^5\tan(k(m))\sec^4(k(m)) + \frac{1}{3}(-4\alpha^4k^4\sec^4(k(m)) \\ &\quad - 8\alpha^4k^4\tan^2(k(m))\sec^2(k(m))) - \frac{16}{3}\alpha^2k^3\left(\alpha^3(-k^2) - \frac{3\beta^3}{4\alpha}\right)\tan(k(m))\sec^2(k(m))\Bigg), \end{aligned} \tag{31}$$

Table 3: The comparison between the absolute errors of the 1-iterate of NIM and the exact solution (defined by Equation (27)) is shown for $\alpha = 1, \beta = 0.1, k = 0.1$, and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

x_i/y_i	0.1	0.2	0.3	0.4	0.5
0.1	1.881112×10^{-8}	7.767271×10^{-8}	1.802273×10^{-7}	3.301182×10^{-7}	5.309873×10^{-7}
0.2	3.704053×10^{-8}	1.505953×10^{-7}	3.443144×10^{-7}	6.218481×10^{-7}	9.868464×10^{-7}
0.3	5.531921×10^{-8}	2.237181×10^{-7}	5.088582×10^{-7}	9.144043×10^{-7}	1.444022×10^{-6}
0.4	7.367144×10^{-8}	2.971382×10^{-7}	6.740804×10^{-7}	1.208182×10^{-6}	1.903113×10^{-6}
0.5	9.212181×10^{-8}	3.709553×10^{-7}	8.401994×10^{-7}	1.503562×10^{-6}	2.364744×10^{-6}

Table 4: The comparison between the absolute errors of the VIM [21] and the exact solution (defined by Equation (27)) is shown for $\alpha = 1$, $\beta = 0.1, k = 0.1$, and $c = k^2\alpha^3 + \frac{3\beta^3}{4\alpha}$.

x_i/y_i	0.1	0.2	0.3	0.4	0.5
0.1	1.88110×10^{-8}	7.76721×10^{-8}	1.802272×10^{-7}	3.301181×10^{-7}	5.309874×10^{-7}
0.2	3.704055×10^{-8}	1.505955×10^{-7}	3.443146×10^{-7}	6.218483×10^{-7}	9.868466×10^{-7}
0.3	5.531924×10^{-8}	2.237185×10^{-7}	5.088581×10^{-7}	9.144040×10^{-7}	1.444020×10^{-6}
0.4	7.367142×10^{-8}	2.971380×10^{-7}	6.740805×10^{-7}	1.208183×10^{-6}	1.903115×10^{-6}
0.5	9.212180×10^{-8}	3.709555×10^{-7}	8.401996×10^{-7}	1.503560×10^{-6}	2.364746×10^{-6}

where $m = \alpha x - t(\frac{3\beta^3}{4\alpha} + \alpha^3 k^2)$.

Figure 2 presents the comparison between the 1-iteration of NIM and the exact solution defined by Equation (27).

Table 3 displays a contrast between the initial iteration of NIM and the exact solution defined by Equation (27).

Table 4 showcases a comparison between the VIM [21] and the exact solution provided by Equation (27).

5. Conclusions

The development of the NIM presents a significant breakthrough in the field of differential equations. Unlike traditional numerical methods, NIM does not rely on linearization or make any limiting assumptions, making it highly versatile and applicable to a wide range of linear and nonlinear differential equations. The successful application of NIM in deriving numerical solutions for the $(2+1)$-dimensional PKP equation demonstrates its remarkable efficacy and ease-of-use. Furthermore, the comparison with the exact solution showcases the accuracy and precision of NIM. The potential applications of NIM are vast, as it offers a powerful and efficient tool for uncovering analytical and numerical solutions for a diverse range of differential equations in various fields such as physics, engineering, and mathematics. Overall, NIM has opened up new possibilities for researchers and professionals in the field of differential equations, making complex problems more accessible and solvable.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The author declares that there is no conflicts of interest.

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References

[1] J. Chen and Y. Zhou, "New exact solutions for a generalized Fisher equation," *Mathematical Methods in the Applied Sciences*, vol. 43, no. 3, pp. 1539–1549, 2020.

[2] H. Liu, Z. Chen, and X. Wang, "New exact solutions for the KdV–Burgers equation using the exp-function method," *Communications in Theoretical Physics*, vol. 68, no. 11, pp. 1361–1367, 2017.

[3] A. Yildirim and S. Aktas, "New exact solutions for the modified Korteweg–de Vries equation via the first integral method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 79, Article ID 104994, 2019.

[4] M. B. Almatrafi, "Solitary wave solutions to a fractional model using the improved modified extended tanh-function method," *Fractal and Fractional*, vol. 7, no. 3, Article ID 252, 2023.

[5] M. B. Almatrafi, "Abundant traveling wave and numerical solutions for Novikov–Veselov system with their stability and accuracy," *Applicable Analysis*, vol. 102, no. 8, pp. 2389–2402, 2023.

[6] A. R. Alharbi and M. B. Almatrafi, "Exact solitary wave and numerical solutions for geophysical KdV equation," *Journal of King Saud University-Science*, vol. 34, no. 6, Article ID 102087, 2022.

[7] A. Alharbi and M. B. Almatrafi, "Exact and numerical solitary wave structures to the variant Boussinesq system," *Symmetry*, vol. 12, no. 9, Article ID 1473, 2020.

[8] A. R. Alharbi and M. B. Almatrafi, "New exact and numerical solutions with their stability for Ito integro-differential equation via Riccati–Bernoulli sub-ODE method," *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 1447–1456, 2020.

[9] K. Batiha and B. Batiha, "A new algorithm for solving linear ordinary differential equations," *World Applied Sciences Journal*, vol. 15, no. 12, pp. 1774–1779, 2011.

[10] B. Batiha, "Numerical solution of a class of singular second-order IVPs by variational iteration method," *International Journal of Mathematical Analysis*, vol. 3, no. 40, pp. 1953–1968, 2009.

[11] H. Chen and H. Q. Zhang, "Traveling wave solutions to the $(2+1)$-dimensional potential Kadomtsev–Petviashvili equation," *Nonlinear Dynamics*, vol. 89, no. 1, pp. 113–123, 2017.

[12] Z. Dai and Y. Chen, "Homoclinic breather solutions for the $(2+1)$-dimensional potential Kadomtsev–Petviashvili equation," *Journal of Physics A: Mathematical and Theoretical*, vol. 46, no. 13, Article ID 135201, 2013.

[13] A. M. Wazwaz, *Partial Differential Equations and Solitary Waves Theory*, Springer Science & Business Media, 2012.

[14] A. Constantin and D. Lannes, "The hydrodynamical relevance of the Painlevé–Kuratowski–Petrovsky equation," *Archive for Rational Mechanics and Analysis*, vol. 185, no. 1, pp. 131–150, 2007.

[15] D. Biskamp, *Magnetohydrodynamic Turbulence*, Vol. 614, Cambridge University Press, 2003.

[16] G. P. Agrawal, *Nonlinear Fiber Optics*, Vol. 6, Academic Press, 2012.

[17] M. Senthilvelan, "On the extended applications of homogenous balance method," *Applied Mathematics and Computation*, vol. 123, no. 3, pp. 381–388, 2001.

[18] D.-S. Li and H.-Q. Zhang, "Symbolic computation and various exact solutions of potential Kadomstev–Petviashvili equation," *Applied Mathematics and Computation*, vol. 145, no. 2-3, pp. 351–359, 2003.

[19] D.-S. Li and H.-Q. Zhang, "New soliton-like solutions to the potential Kadomstev–Petviashvili (PKP) equation," *Applied Mathematics and Computation*, vol. 146, no. 2-3, pp. 381–384, 2003.

[20] D. Kaya and S. M. El-Sayed, "Numerical soliton-like solutions of the potential Kadomtsev–Petviashvili equation by the decomposition method," *Physics Letters A*, vol. 320, no. 2-3, pp. 192–199, 2003.

[21] B. Batiha and K. Batiha, "An analytic study of the (2 + 1)-dimensional potential Kadomtsev–Petviashvili equation," *Advances In Theoretical and Applied Mechanics*, vol. 3, no. 11, pp. 513–520, 2010.

[22] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.

[23] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*, Kluwer, 1994.

[24] J.-H. He, "Homotopy perturbation technique," *Computer Methods in Applied Mechanics and Engineering*, vol. 178, no. 3-4, pp. 257–262, 1999.

[25] J.-H. He, "Approximate analytical solution for seepage flow with fractional derivatives in porous media," *Computer Methods in Applied Mechanics and Engineering*, vol. 167, no. 1-2, pp. 57–68, 1998.

[26] V. Daftardar-Gejji and S. Bhalekar, "Solving fractional diffusion-wave equations using a new iterative method," *Fractional Calculus and Applied Analysis*, vol. 11, no. 2, pp. 193–202, 2008.

[27] B. Batiha and F. Ghanim, "Solving strongly nonlinear oscillators by new numerical method," *International Journal of Pure and Apllied Mathematics*, vol. 116, no. 1, pp. 115–124, 2017.

[28] B. Batiha and F. Ghanim, "Numerical implementation of Daftardar-Gejji and Jafari method to the quadratic Riccati equation," *Buletinul Academiei de Ştiinţe a Republicii Moldova. Matematica*, vol. 3, no. 97, pp. 21–29, 2021.

[29] B. Batiha, "New solution of the sine-Gordon equation by the Daftardar-Gejji and Jafari method," *Symmetry*, vol. 14, no. 1, Article ID 57, 2022.

[30] B. Batiha, F. Ghanim, O. Alayed et al., "Solving multispecies Lotka–Volterra equations by the Daftardar-Gejji and Jafari method," *International Journal of Mathematics and Mathematical Sciences*, vol. 2022, Article ID 1839796, 7 pages, 2022.