Hindawi Publishing Corporation
Modelling and Simulation in Engineering
Volume 2013, Article ID 491843, 7 pages
http://dx.doi.org/10.1155/2013/491843

Research Article

Theoretical Model for Predicting Moisture Ratio during Drying of Spherical Particles in a Rotary Dryer

F. T. Ademiluyi and M. F. N. Abowei

Department of Chemical/Petrochemical Engineering, Rivers State University of Science & Technology, P.O. Box 5080, Port Harcourt 5005, Nigeria

Correspondence should be addressed to F. T. Ademiluyi; ademuluyi@yahoo.com

Received 3 April 2012; Accepted 8 January 2013

Academic Editor: Agostino Bruzzone

A mathematical model was developed for predicting the drying kinetics of spherical particles in a rotary dryer. Drying experiments were carried out by drying fermented ground cassava particles in a bench scale rotary dryer at inlet air temperatures of 115–230°C, air velocities of 0.83 m/s–1.55 m/s, feed mass of 50–500 g, drum drive speed of 8 rpm, and feed drive speed of 100 rpm to validate the model. The data obtained from the experiments were used to calculate the experimental moisture ratio which compared well with the theoretical moisture ratio calculated from the newly developed Abowei-Ademiluyi model. The comparisons and correlations of the results indicate that validation and performance of the established model are rather reasonable.

1. Introduction

Rotary drying is a very complicated process that can be applied not only to thermal drying but also movement of particles within the dryer. Several authors have carried out investigations on the steady state modeling of the rotary drying process. Static models are in general differential equations, and they are suitable for investigation of static distributions. Myklestad [1] was the first to obtain an expression to predict product moisture content throughout a rotary dryer based on drying air temperature, initial moisture content, and product feed rate. Thin layer drying equations contribute to the understanding of the heat and mass transfer phenomena in agricultural products and computer simulations for designing new and improving existing commercial drying processes [2]. They are used to estimate drying times of several products and also to generalize drying curves. In thin layer drying model, the rate of change in material moisture content in the falling rate drying period is proportional to the instantaneous difference between material moisture content and the expected material moisture content when it comes into equilibrium with the drying air [3].

Many authors have developed semiempirical models based on the diffusion theory to predict the drying kinetics of moist substances in thin layer as shown in Table 1 (where MR is the moisture ratio). The constants a, b, c, k, k_o, K_1, and n in eight models by most authors have been found to be functions of inlet air temperature, inlet air velocity, humidity, and so forth, the mass of feed was not accounted for by all the authors and in the drying of substances with high moisture content like fermented ground cassava, dairy products, and some pharmaceutical product in rotary dryer, and the mass of feed should be accounted for in the thin layer drying equation. It was observed that although several models have been proposed, there is not a general theory to describe the mechanism of rotary drying, and it seems that specific models for an equipment and material are more useful than general models [4].

Therefore the objective of this study is to develop a theoretical model for predicting the drying kinetics of spherical particles in a rotary dryer accounting for the quantity of materials to be dried in the model.

2. Materials and Method

2.1. Theoretical Development of Thin Layer Drying Equation. Fick's diffusion equation (1) has been accepted for describing

Table 1: Mathematical models given by various authors for the drying curves.

Model number	Model name	Model equation	References
1	Newton	$\mathrm{MR} = \exp(-kt)$	[5]
2	Page	$\mathrm{MR} = \exp(-kt^n)$	[6, 7]
3	Modified page	$\mathrm{MR} = \exp(-(kt)^n)$	[8]
4	Henderson and Pabis	$\mathrm{MR} = a\exp(-kt)$	[9]
5	Logarithmic	$\mathrm{MR} = a\exp(-kt) + c$	[10]
6	Two-term	$\mathrm{MR} = a\exp(-k_o t) + b\exp(-k_1 t)$	[11]
7	Wang and Singh	$\mathrm{MR} = 1 + at + bt^2$	[12]
8	Ademiluyi-modified page model	$\mathrm{MR} = a\exp(-(kt)^n)$	[13–15]

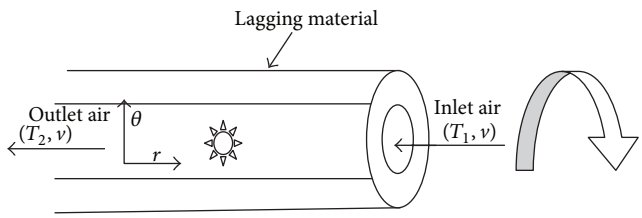

Figure 1: Showing hypothetical profile of moisture diffusion from a spherical particle in a rotary dryer.

the drying characteristics of biological and chemical products in the falling rate period [16] as follows:

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial M}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial M}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial M}{\partial z}\right), \quad (1)$$

where D is the diffusion coefficient, M is moisture content (dry basis) at any time t, and t is drying time. The equation of diffusion for a spherical particle at constant diffusivity and radial (as shown in Figure 1) flux takes the following form:

$$\frac{\partial M}{\partial t} = D\left(\frac{\partial^2 M}{\partial r^2} + \frac{2}{r}\frac{\partial M}{\partial r}\right). \quad (2)$$

In order to solve (2) the following assumptions were adopted:

(1) moisture movement is only diffusion and unidirectional;

(2) diffusion coefficient D is independent of moisture concentration;

(3) drying process is isothermal, that is, adiabatic dryer;

(4) material to be dried is spherical in shape;

(5) shrinkage is neglected.

Using $-\lambda^2$ as a separation constant we obtain from (2)

$$\frac{1}{D}\frac{dM}{dt} = -\lambda^2, \quad (3)$$

$$\left(\frac{\partial^2 M}{\partial r^2} + \frac{2}{r}\frac{\partial M}{\partial r}\right) = -\lambda^2. \quad (4)$$

Integrating (3) using separation of variables gives

$$T = C_1 e^{-\lambda^2 Dt}. \quad (5)$$

Equation (4) is of the form

$$r^2 R'' + 2rR' + \lambda^2 r^2 R = 0. \quad (6)$$

Equation (6) is a Bessel equation of order zero, the solution of which is [17]

$$R(r) = C_2(\lambda r)^{1/2} J_{1/2}(\lambda r) + C_3(\lambda r)^{1/2} J_{-1/2}(\lambda r). \quad (7)$$

But

$$J_{1/2}(\lambda r) = \sqrt{\frac{2}{\pi(\lambda r)}}\sin(\lambda r),$$
$$J_{-1/2}(\lambda r) = \sqrt{\frac{2}{\pi(\lambda r)}}\cos(\lambda r), \quad (8)$$

$$R(r) = \sqrt{\frac{2}{\pi\lambda}}\left[\frac{C_2}{r}\sin(\lambda r) + \frac{C_3}{r}\cos(\lambda r)\right]. \quad (9)$$

Combining (5) and (9) gives $M(r \cdot t) = R(r)T(t)$, so that

$$M(r,t) = C_1 e^{-\lambda^2 Dt}\sqrt{\frac{2}{\pi\lambda}}\left[\frac{C_2}{r}\sin(\lambda r) + \frac{C_3}{r}\cos(\lambda r)\right] \quad (10)$$

and applying the boundary conditions in (11). The solution to (10) in the case of a sphere is expressed as

$$\frac{\partial M}{\partial r} = 0, \quad r = 0,\ r \geq 0,$$
$$M = M_e, \quad r = a,\ t \geq 0, \quad (11)$$
$$M = M_o, \quad 0 \leq r \leq a,\ t = 0,$$

$$\mathrm{MR} = \frac{M_i - M_e}{M_o - M_e} = \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[\frac{-n^2\pi^2 D}{r^2}\right]t, \quad (12)$$

where r is the radius of sphere, MR is moisture ratio, M_o is initial moisture content (% db), M_e is equilibrium moisture content (% db), M_i is moisture content at time t (% db), and t is drying time (hr).

From the work of Abowei [18], the mass of hydrocarbon M was accounted for when modeling one-dimensional diffusion of oil spill in water and obtain a general solution (see (13)) to predict the diffusion of known quantity of crude oil in water. This equation is analogous to the diffusion equation (12) describing diffusion of moisture in porous spherical particles as follows:

$$C_p = \frac{M_p}{A\left[4\pi D_m t\right]^{1/2}} e^{-x^2/4D_m t} \left(\mathrm{g/cm^3}\right), \tag{13}$$

where M_p is the quantity of oil spilled and C_p is the concentration of oil spilled at any time. A is the area where oil is spill; D_m is the diffusion coefficient, and t is the time.

Comparing (12) with (13), the term $M_p/A[4\pi D_m t]^{1/2}$ in (12) is analogous to the term $(6/\pi^2)\sum_{n=1}^{\infty}(1/n^2)$ in (13), and hence (12) can be rewritten as

$$\mathrm{MR} = \frac{M_p}{\rho A\left[4\pi D_m t\right]^{1/2}} \exp\left[\frac{-n^2\pi^2 D_m}{r^2}\right] t, \tag{14}$$

where MR is the moisture ratio. A is surface area available for moisture transfer which for the rotary dryer is $\pi(R^2 + RL)$, M_p in equation (14) is now the mass of fermented ground cassava which is analogous to the M_p in equation (13), ρ is the average density of sample to be dried, and the density is added t_p equation (14) to make the equation dimensionless, since moisture ratio MR is dimensionless, so that (14) becomes

$$\mathrm{MR} = \frac{M_p}{\rho\left(\pi\left(R^2 + RL\right)\right)\left[4\pi D_m t\right]^{1/2}} \exp\left[\frac{-n^2\pi^2 D_m}{r^2}\right] t, \tag{15}$$

where r is the average radius of particle to be dried R is the radius of rotary dryer drum, L is the length of the rotary dryer, and $D_m = D$ is diffusion coefficient $= D_o \exp(E_a/RT)$, where E_a is the activation energy. Equation (15) is the new theoretical Abowei-Ademiluyi model for predicting the drying of any spherical particles in a rotary dryer. Equation (15) was simulated to obtain the theoretically determined moisture ratio.

2.1.1. Dimensional Analysis Approach. In order to remove moisture from a moist material in a rotary dryer, the moisture ratio (MR) can be taken to be a function of the change in temperature ΔT, the quantity of fermented ground cassava to be dried M_p, the latent heat λ, diameter of particle D to be dried, inlet air velocity v, and drum speed N, so that mathematically the moisture ratio MR is dimensionless as

$$\mathrm{MR} = \phi\left\lfloor \Delta T, M_p, \lambda, D, V, N \right\rfloor, \tag{16}$$

where ϕ is a correction factor.

Applying dimensional analysis we have

$$\frac{M}{M} = \phi\left[T^a, M^b, \left(\frac{FL}{M}\right)^c, L^d, \left(\frac{L}{\theta}\right)^e, \left(\frac{1}{\theta}\right)^f\right]. \tag{17}$$

Applying the Buckingham π method gives

$$\sum M : 0 = b - c, \tag{18}$$

$$\sum T : 0 = a, \tag{19}$$

$$\sum L : 0 = d + e + c, \tag{20}$$

$$\sum \theta : 0 = -e - f, \tag{21}$$

$$\sum F : 0 = c. \tag{22}$$

Solving (18) to (22) gives $a = 0$, $b = 0$, $c = 0$, $d = -1$, $e = 1$, and $f = -1$ which

$$\mathrm{MR} = \phi\left[\frac{VN}{D}\right]. \tag{23}$$

So that

$$\phi = \frac{(\mathrm{MR})\,D}{VN}. \tag{24}$$

The dimensionless constant ϕ can be evaluated theoretically and experimentally by substituting for MR in (14) into (24) to give the correction factor as

$$\phi = \frac{\left(M_p/\rho A\left[4\pi D_m t\right]^{1/2}\right)\exp\left\lfloor -n^2\pi^2 D_m/r^2 \right\rfloor tD}{VN}. \tag{25}$$

2.2. Experimental Work. In order to validate the model, fermented ground cassava particles was dried in a bench scale rotary dryer (Figure 2). The developed theoretical model (15) was simulated with Microsoft Excel 2007 using the following data:

(i) average density product ($\mathrm{kg/m^3}$) = 400, [19],

(ii) drying time (120–1200 secs)-step 60,

(iii) $r = 0.0175$ m, $R = 0.0508$ m, $L = 0.46$ m.

The diffusion coefficients (D_T, D_M, and D_V in $\mathrm{m^2/s}$) in (26)–(28) were obtained experimentally at different inlet air temperature (T in °C), inlet air velocity (V in m/s), and mass of feed (M in kg) from previous work [15] on fermented ground cassava as follows:

$$D_T = 9.747 \times 10^{-8} \exp\left[\frac{-13892}{8.314T}\right] \quad r^2 = 0.994, \tag{26}$$

$$D_M = 8.938 \times 10^{-10} + 5.937 \times 10^{-11} * \frac{\log(M)}{M} \quad r^2 = 0.986, \tag{27}$$

$$D_V = 4.702 \times 10^{-9} + \left(-8.49 \times 10^{-9}\right)\exp(-V) \quad r^2 = 0.990. \tag{28}$$

2.2.1. Sample Preparation. The cassava cultivar used in this study is TMS 30572 obtained from Rivers State Agricultural

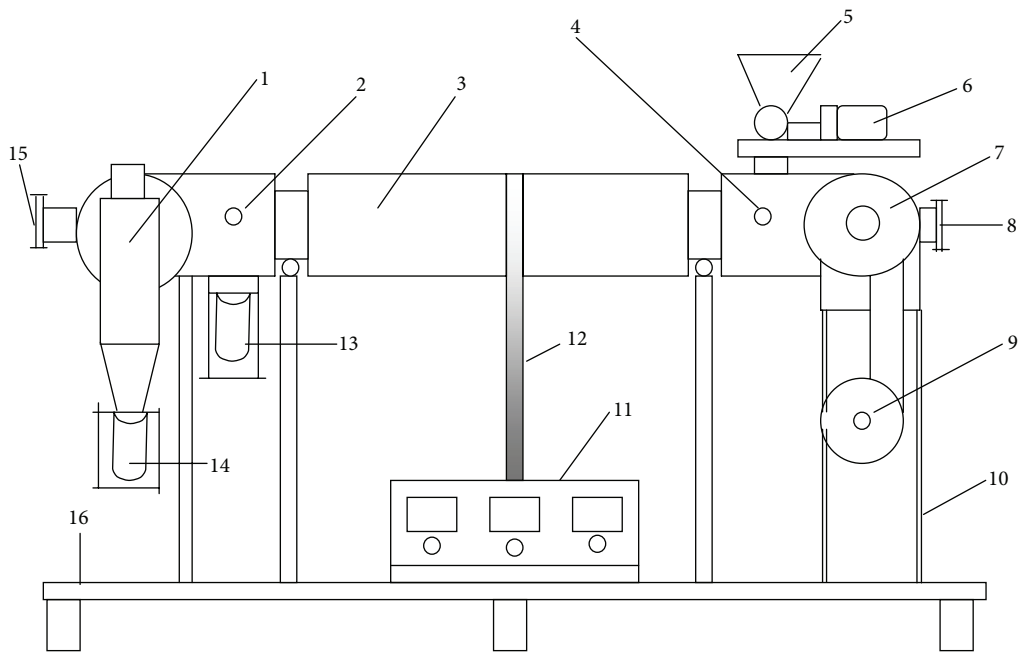

Figure 2: Schematic diagram of bench scale rotary dryer: (1) cyclone, (2 and 4) probe connections, (3) rotary drum, (5) feed hopper, (6) feed drive, (7) electric heater arrangement, (8 and 15) sight glasses, (9) air blower and orifice plate control, (10) support, (11) control box, (12) chain drive, (13 and 14) dried product receivers, and (16) steel table.

Development Project farm (ADP) at Rumuokoro, Port Harcourt. The choice of this cassava cultivar TMS 30572 was based on its preference by farmers, because of its high yield and suitability for gari processing [20]. The cassava cultivar was peeled, washed, grated, and packed in sack for pressing. The dewatered mash was allowed to ferment naturally for 72 hrs; sieved with a mesh of 3.5 mm, and then dried in a bench rotary dryer (Figure 2).

2.2.2. Experimental Procedure. At the beginning of each experiment, the dryer was allowed to reach steady state at the desired airflow rate, inlet air temperature, feed drive speed, and drum drive speed. When steady state condition had been attained, the fermented ground cassava mash of known moisture content was introduced into the dryer feed hopper. The drying conditions used in the experiments are inlet air temperatures of 115°C, 140°C, 190°C, and 230°C, air velocities of 0.83, 1.02, 1.397, and 1.55 m/s, mass of feed of 50 g, 100 g, 200 g, and 500 g, feed drive speeds 100 rpm, and drum drive speeds of 8 rpm. The decrease in mass of fermented mash was monitored with time per pass. The initial moisture content of samples was determined separately before start of experiment. The weight loss during drying was used to calculate the moisture content. The drying data obtained were used to calculate the experimental moisture ratio (MR) to predict the kinetics of drying fermented ground cassava.

3. Results and Discussion

3.1. Simulated Theoretical Results. The theoretical moisture ratio results are presented in Figures 3, 5, and 7. The theoretical moisture ratio decreases with drying time as inlet air temperature, inlet air velocity, and mass of feed increases. A similar profile is also exhibited in Figures 4, 6, and 8 for the experimental moisture ratio. The theoretical moisture ratio plots show a typical drying curve generally obtained during drying of moist materials [3, 21].

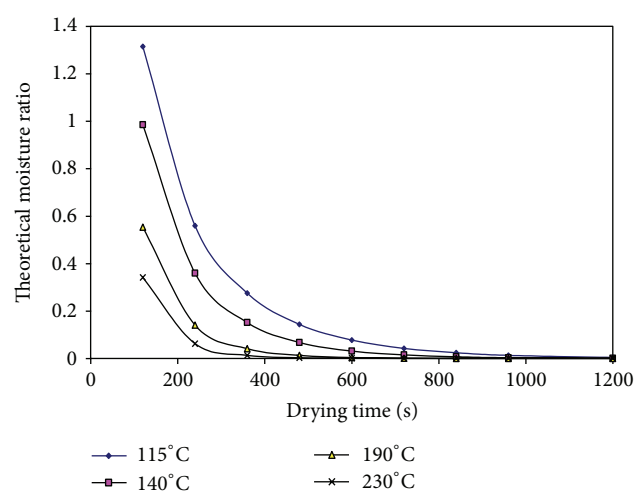

Figure 3: Variation of moisture ratio with time at different air inlet temperatures using the theoretical model.

It can be observed from the theoretical moisture ratio plots that the Abowei-Ademiluyi model does not give values for moisture ratio at $t = 0$, and this will not be a problem, since the initial moisture content from which the moisture ratio at $t = 0$ (i.e., M_o) was calculated is always known at start of drying. Hence the Abowei-Ademiluyi model can be used to predict the drying kinetics of spherical particles at any

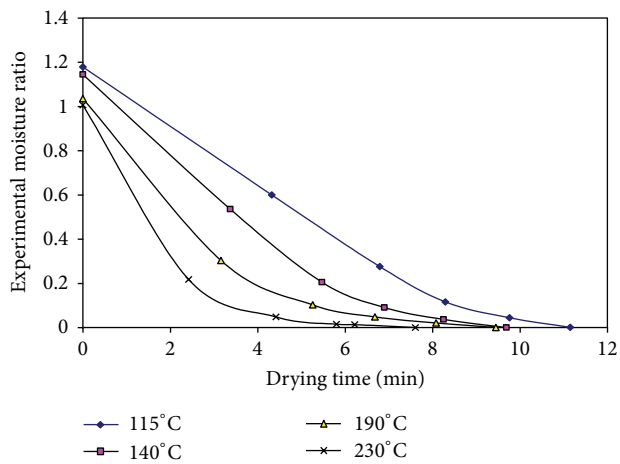

Figure 4: Experimental moisture ratio at different inlet temperature.

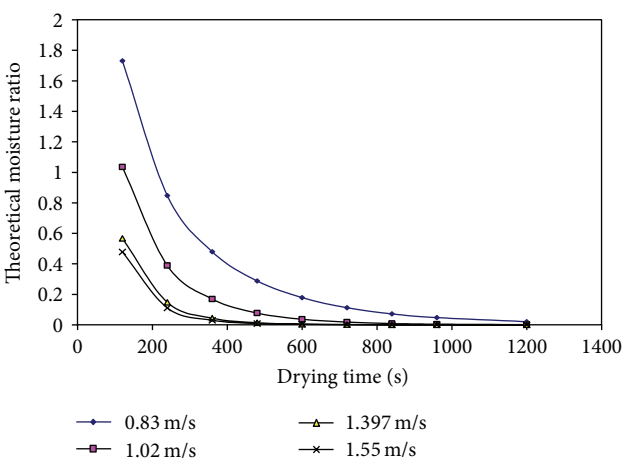

Figure 5: Variation of moisture ratio with time at different inlet air velocity using the theoretical model.

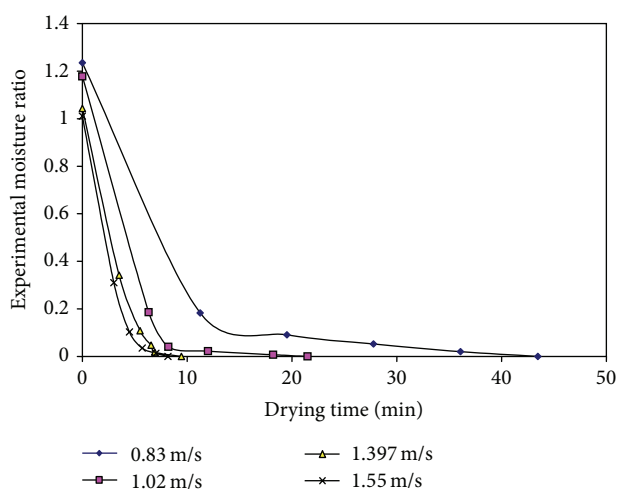

Figure 6: Experimental moisture ratio at different inlet air velocity.

known particle diameter, rotary drum diameter, and dryer length once the diffusion coefficient is known.

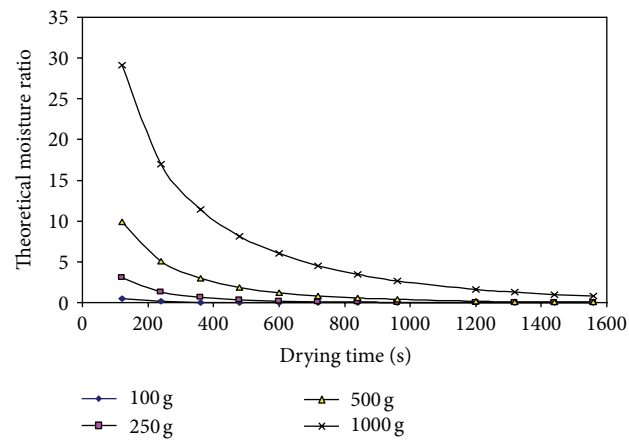

Figure 7: Variation of moisture ratio with time at different mass of feed using the theoretical model.

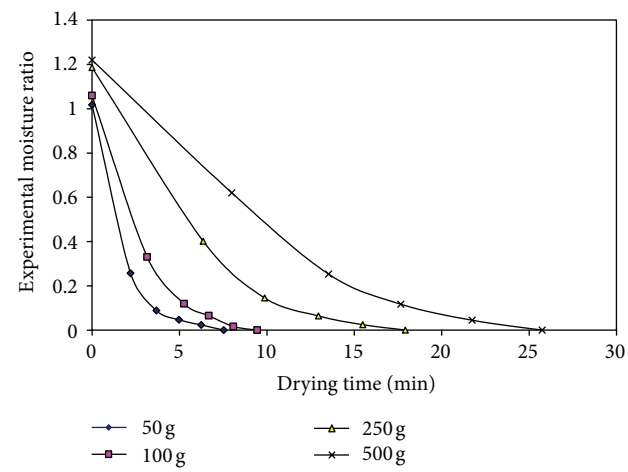

Figure 8: Experimental moisture ratio at different mass of feed.

3.2. Comparison of Theoretical and Experimented Results. The simulated theoretical result compared favorably with those of the experimental results. The similarity is shown from the high value (r close to 1) obtained for the coefficient of multiple determinations R^2 at different inlet air temperature and inlet air velocity as shown in Figures 9 and 10. However, better fit could be obtained if the average density particle is correctly chosen. The theoretical (Abowei-Ademiluyi model) moisture ratio also compared well with experimentally moisture ratio at different mass of feed as shown in Figure 11.

4. Conclusion

The new theoretical, Abowei-Ademiluyi model has been developed for predicting drying kinetics of spherical particles at any known particle diameter, rotary drum diameter, and dryer length. The new model also account for the mass of feed. Model validation was carried out by drying fermented ground cassava particles in a bench scale rotary dryer at inlet air temperatures of 115–230°C, air velocities of 0.83 m/s–1.55 m/s, feed mass of 50–500 g, drum drive speed of 8 rpm,

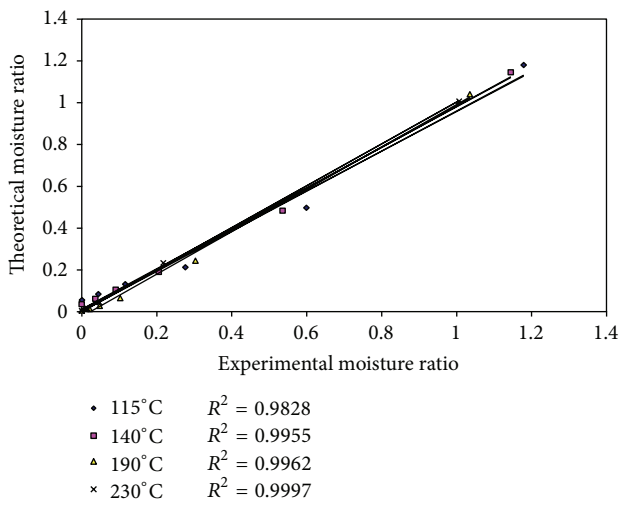

Figure 9: Theoretical (Abowei-Ademiluyi model) and experimental moisture ratios at different inlet air temperature.

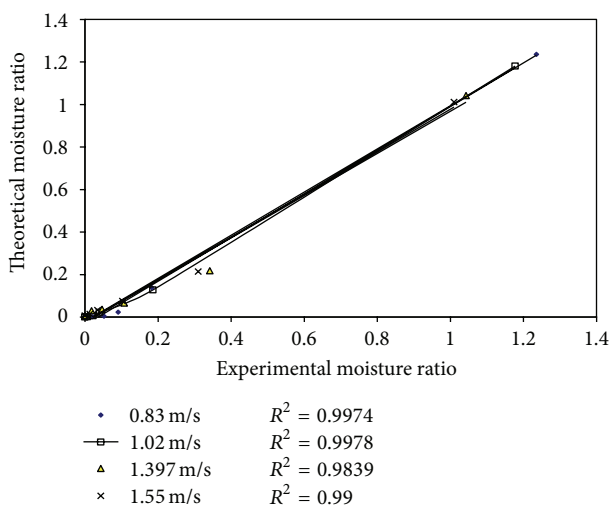

Figure 10: Theoretical (Abowei-Ademiluyi model) and experimental moisture ratios at different inlet air velocity.

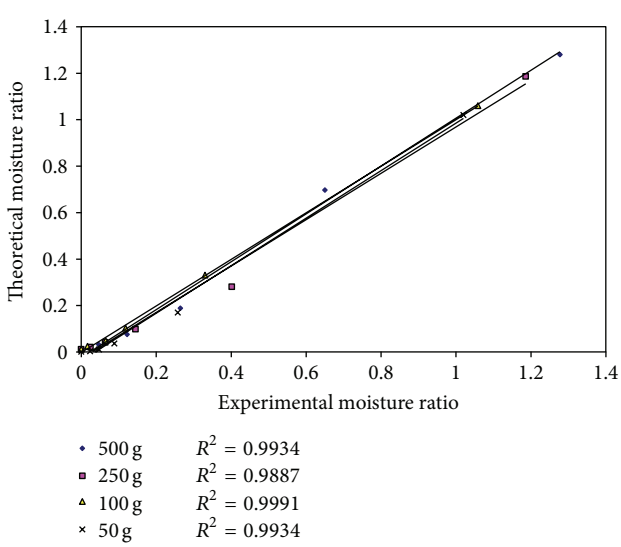

Figure 11: Theoretical (Abowei-Ademiluyi model) and experimental moisture ratios at different mass of feed.

and feed drive speed of 100 rpm. The theoretical moisture ratio calculated from the model compared favorably with experimental moisture ratio.

References

[1] O. Myklestad, "Heat and mass transfer in rotary dryers," *Chemical Engineering Progress Symposium Series*, vol. 59, no. 41, pp. 129–137, 1963.

[2] D. S. Jayas, S. Cenkowski, S. Pabis, and W. E. Muir, "Review of thin-layer drying and wetting equations," *Drying Technology*, vol. 9, no. 3, pp. 551–588, 1991.

[3] H. O. Menges and C. Ertekin, "Mathematical modeling of thin layer drying of Golden apples," *Journal of Food Engineering*, vol. 77, no. 1, pp. 119–125, 2006.

[4] A. Iguaz, A. Esnoz, G. Martínez, A. López, and P. Vírseda, "Mathematical modelling and simulation for the drying process of vegetable wholesale by-products in a rotary dryer," *Journal of Food Engineering*, vol. 59, no. 2-3, pp. 151–160, 2003.

[5] Q. Liu and F. W. Bakker-Arkema, "Stochastic modelling of grain drying: part 2. Model development," *Journal of Agricultural Engineering Research*, vol. 66, no. 4, pp. 275–280, 1997.

[6] Y. C. Agrawal and R. P. Singh, "Thin layer drying studies on short grain rough ice," ASAE Paper 3531, 1977.

[7] Q. Zhang and J. B. Litchfield, "Optimization of intermittent corn drying in a laboratory scale thin layer dryer," *Drying Technology*, vol. 9, no. 1, pp. 233–244, 1991.

[8] G. M. White, T. C. Bridges, O. J. Loewer, and I. J. Ross, "Thin layer drying model for soybeans," *Transactions of the American Society of Agricultural Engineers*, vol. 24, no. 6, pp. 1643–1646, 1981.

[9] M. S. Chhinnan, "Evaluation of selected mathematical models for describing thin-layer drying of in-shell pecans," *Transactions of the American Society of Agricultural Engineers*, vol. 27, no. 2, pp. 610–615, 1984.

[10] A. Yagcıoglu, A. Degirmencioglu, and F. Cagatay, "Drying characteristics of laurel leaves under different drying conditions," in *Proceedings of the 7th International Congress on Agricultural Mechanization and Energy*, pp. 565–569, Adana, Turkey, May 1999.

[11] M. S. Rahman, C. O. Perera, and C. Thebaud, "Desorption isotherm and heat pump drying kinetics of peas," *Food Research International*, vol. 30, no. 7, pp. 485–491, 1997.

[12] C. Y. Wang and R. P. Singh, "Use of variable equilibrium moisture content in modelling rice drying," ASAE Paper 78-6505, ASAE Press, St. Joseph, Mich, USA, 1978.

[13] T. Ademiluyi, K. Oduola, and J. Eke, "Mathematical modeling of drying different Cassava Chips in thin layers," *Journal of Nigerian Society of Chemical Engineers*, vol. 22, p. 148, 2007.

[14] T. Ademiluyi, E. O. Oboho, and M. Owudogu, "Investigation into the thin layer drying models of Nigerian popcorn varieties," *Leonardo Electronic Journal of Practices and Technologies*, vol. 7, no. 13, pp. 047–062, 2008.

[15] T. Ademiluyi, *Development of predictive models for drying fermented ground cassava [Ph.D. thesis]*, River State University of Science and Technology, Port Harcourt, Nigeria, 2009.

[16] W. L. McCabe, J. C. Smith, and P. Harriott, *Unit Operations of Chemical Engineering*, McGraw-Hill Book Company, New York, NY, USA, 4th edition, 1987.

[17] K. A. Stroud, *Advance Engineering Mathematics*, Palgrave Macmilian, Hampshire, UK, 4th edition, 2003.

[18] M. F. N. Abowei, *Prediction model development for simulation of petroleum weathering process in aquatic environment [Ph.D. thesis]*, Department of Chemical Engineering University of Lagos, Lagos, Nigeria, 1991.

[19] T. Ademiluyi, M. F. N. Abowei, Y. T. Puyate, and S. C. Achinewhu, "Effect of variety on the drying and engineering properties of fermented ground cassava," *Journal of Newviews in Engineering analysis and Modelling*, pp. 80–79, 2006.

[20] E. Joy, *physicochemical and pasting properties of tapioca from different cassava cultivars in food science and technology [Ph.D. thesis]*, River State University of Science and Technology, Port Harcourt, Nigeria, 2006.

[21] O. Bozkir, "Thin-layer drying and mathematical modelling for washed dry apricots," *Journal of Food Engineering*, vol. 77, no. 1, pp. 146–151, 2006.