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Research Article

A New Approach to Improve Accuracy of Grey Model GMC$(1, n)$ in Time Series Prediction

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This paper presents a modified grey model GMC$(1, n)$ for use in systems that involve one dependent system behavior and $n - 1$ relative factors. The proposed model was developed from the conventional GMC$(1, n)$ model in order to improve its prediction accuracy by modifying the formula for calculating the background value, the system of parameter estimation, and the model prediction equation. The modified GMC$(1, n)$ model was verified by two cases: the study of forecasting CO_2 emission in Thailand and forecasting electricity consumption in Thailand. The results demonstrated that the modified GMC$(1, n)$ model was able to achieve higher fitting and prediction accuracy compared with the conventional GMC$(1, n)$ and D-GMC$(1, n)$ models.

1. Introduction

Grey model is a useful tool for modeling and forecasting future values of a system based on the information and knowledge obtained from the past and current data. Grey model was developed from the grey system theory introduced by Deng in the early 1980s [1]. It can be used to predict behaviors of systems in the future value with high accuracy without knowing their mathematical models and used in the uncertain coefficients system with small nonnegative data. Grey model has been successfully applied to various systems [1–8]. GM(m, n) denotes a grey model which indicates that n variables are employed in the model and that it is an m-order differential equation. GM$(1, 1)$ is a first-order one-variable grey differential equation, and it is the most widely used grey model in time series prediction [9]. However, it does not find application in multivariable prediction models which are important for real applied works [4, 5, 10–13]. The grey prediction models have been expanded from the original GM$(1, 1)$ to novel prediction types, such as GM$(1, n)$ [14], GMC$(1, n)$ [10, 15], D-GMC$(1, n)$ [14], DGDMC$(1, n)$ [16], and CAGM$(1, n)$ [17]. GM$(1, n)$ model is a grey multivariable model used for estimating the relationship between the system behavior and $n-1$ relative factors [14] and has been used for various applications [4, 11, 18–20]. However, there are some limitations existing in the GM$(1, n)$ which affect the prediction accuracy of GM$(1, n)$ [14, 15]. Then, the grey multivariable model with the convolution integral GMC$(1, n)$, proposed by Tien [10], is developed from GM$(1, n)$ by adding the grey control parameter in the differential equation of GM$(1, n)$ to improve the forecasting accuracy of GM$(1, n)$. The GMC$(1, n)$ model has successfully been applied to many real works [10, 15, 21, 22]. However, the prediction accuracy of the GMC$(1, n)$ model depends on many factors such as smooth condition of the raw data, background value calculation, and model prediction equation. Moreover, there exists a contradiction between discrete equations for parameter estimation and continuous equations for model predictions [5, 23]. Therefore, a high accuracy of prediction cannot be expected of GMC$(1, n)$ for an actual system [8, 16].

In this paper, we proposed a modified grey GMC$(1, n)$ model to improve the prediction accuracy of the conventional GMC$(1, n)$ by modification of the formula for calculating the background value, the system of the parameter estimation, and the model prediction equation. In addition, we presented the case studies with the numerical results for prediction

accuracy for the modified GMC$(1,n)$ model in comparison with the conventional GMC$(1,n)$ and the discrete multivariate grey model D-GMC$(1,n)$ [5].

The paper is organized as follows: Section 2 describes the conventional GMC$(1,n)$, Section 3 proposes a modified grey GMC$(1,n)$ model, Section 4 explains the statistical measure of the forecasting performance, and Section 5 presents the case study with the modified grey GMC$(1,n)$. Finally, the conclusions are drawn in Section 6.

2. GMC$(1,n)$ Model [8, 10]

Suppose that the original multivariate time series $X_i^{(0)} = \{X_i^{(0)}(1), X_i^{(0)}(2), \ldots, X_i^{(0)}(r)\}$, $i = 1,2,\ldots,n$ are a nonnegative series and available at an equispaced interval of time, where the main factor of the system behavior is $X_1^{(0)}$, the relative factors are $X_i^{(0)}$, $i = 2,3,\ldots,n$, and r is the number of data. Then the first-order accumulative generation operation (1-AGO) of $X_i^{(0)}$, $i = 1,2,\ldots,n$, is given by the following equation:

$$X_i^{(1)} = \left\{X_i^{(1)}(1), X_i^{(1)}(2), \ldots, X_i^{(1)}(r)\right\}, \quad i = 1,2,\ldots,n, \tag{1}$$

where $X_i^{(1)}(k) = \sum_{j=1}^{k} X_i^{(0)}(j)$, $i = 1,2,\ldots,n$, $k = 1,2,\ldots,r$.

The prediction procedure of using the conventional GMC$(1,n)$ model is shown as follows.

The grey prediction model based on the 1-AGO data, $X_i^{(1)}$, $i = 1,2,\ldots,n$, is given by the following differential equation:

$$\frac{dX_1^{(1)}(t)}{dt} + b_1 X_1^{(1)}(t) = b_2 X_2^{(1)}(t) + b_3 X_3^{(1)}(t) + \cdots + b_n X_n^{(1)}(t) + u, \quad t = 1,2,\ldots,rf, \tag{2}$$

where $b_1, b_2, \ldots, b_n$ and u are model parameters to be estimated and rf is the number of entries to be predicted.

Taking the integral of both sides of (2) in the interval $[k-1, k]$ yields

$$X_1^{(0)}(k) + b_1 z_1^{(1)}(k) = b_2 z_2^{(1)}(k) + b_3 z_3^{(1)}(k) + \cdots + b_n z_n^{(1)}(k) + u, \tag{3}$$

where $z_i^{(1)}(k) = \int_{k-1}^{k} X_i^{(1)}(t)dt$ for $i = 1,2,\ldots,n$, which are called the background values [9].

By using the trapezoidal rule, the values of $\int_{k-1}^{k} X_i^{(1)}(t)dt$ for $i = 1,2,\ldots,n$ are approximated by the following equation:

$$z_i^{(1)}(k) \approx \frac{X_i^{(1)}(k) + X_i^{(1)}(k-1)}{2} t, \quad i = 1,2,\ldots,n. \tag{4}$$

Substituting all the data values in (3) gives the system of linear equations that can be written as a matrix equation in the form

$$Ab = Y, \tag{5}$$

where

$$A = \begin{bmatrix} -z_1^{(1)}(2) & z_2^{(1)}(2) & \cdots & z_n^{(1)}(2) & 1 \\ -z_1^{(1)}(3) & z_2^{(1)}(3) & \cdots & z_n^{(1)}(3) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -z_1^{(1)}(r) & z_2^{(1)}(r) & \cdots & z_n^{(1)}(r) & 1 \end{bmatrix},$$
$$Y = \begin{bmatrix} X_1^{(0)}(2) \\ X_1^{(0)}(3) \\ \vdots \\ X_1^{(0)}(r) \end{bmatrix}, \tag{6}$$
$$b = \begin{bmatrix} b_1 & b_2 & b_3 & \cdots & b_n & u \end{bmatrix}^T.$$

Applying the least square method to this system, we obtain

$$b = \left(A^T A\right)^{-1} A^T Y. \tag{7}$$

Solving (2) with the initial condition $\widehat{X}_1^{(1)}(1) = X_1^{(1)}(1)$, the predicted 1-AGO series is obtained as follows:

$$\begin{aligned} \widehat{X}_1^{(1)}(k+1) &= X_1^{(0)}(1) e^{-b_1 k} \\ &\quad + \theta(k-1) \sum_{i=2}^{k+1} \left[e^{-b_1(k+3/2-i)} \frac{1}{2} \left(f(i) + f(i-1) \right) \right], \\ f(i) &= b_2 X_2^{(1)}(i) + b_3 X_3^{(1)}(i) + \cdots + b_n X_n^{(1)}(i) + u, \end{aligned} \tag{8}$$

where $\theta(k-1)$ is the unit step function and $k = 2,3,\ldots,rf$ and $\widehat{X}_1^{(1)}(k)$ denotes the predicted value of $X_1^{(1)}(k)$.

Therefore, the predict series of $X_1^{(0)}(k)$, $k = 2,3,\ldots,rf$ is given by

$$\widehat{X}_1^{(0)}(k) = \widehat{X}_1^{(1)}(k) - \widehat{X}_1^{(1)}(k-1). \tag{9}$$

3. Modified GMC$(1,n)$ Model

The proposed model is modified from GMC$(1,n)$ by using two advanced improvements, as follows:

(1) The equation of the predicted 1-AGO series is obtained accurately from the exactly integrated form of $X_i^{(1)}$.

(2) The system of parameter estimation is derived by using the model prediction equation in order to eliminate the problem of a contradiction between the system of discrete equations for parameter estimation and the system of continuous equations for model predictions.

The grey model can perform well as regards predictions if the raw data satisfies the quasi-smooth and quasi-exponential conditions [17, 24]. However, if the raw data does not satisfy these conditions, function transformation methods are applied in original data sequences [17]. Suppose that

the original data $X_i^{(0)}$, $i = 1, 2, \ldots, n$, satisfy the quasi-smooth and quasi-exponential condition. Then, the 1-AGO data $X_i^{(1)}$, $i = 1, 2, 3, \ldots, n$, are fitted by the exponential function [9, 17], which can be written as

$$X_i^{(1)}(k) = E_i e^{a_i k} + F_i, \quad i = 1, 2, 3, \ldots, n, \; k = 1, 2, 3, \ldots, r. \tag{10}$$

Next, parameters E_i, a_i, and F_i are determined using the least square method, which is as follows:

From (9), the raw data can be expressed by the 1-AGO data as

$$\begin{aligned} X_i^{(0)}(k-1) &= X_i^{(1)}(k-1) - X_i^{(1)}(k-2) \\ &= E_i e^{a_i(k-2)}\left(e^{a_i} - 1\right), \end{aligned} \tag{11}$$

$$\begin{aligned} X_i^{(0)}(k) &= X_i^{(1)}(k) - X_i^{(1)}(k-1) \\ &= E_i e^{a_i(k-1)}\left(e^{a_i} - 1\right). \end{aligned} \tag{12}$$

Dividing (12) by (11), we have

$$X_i^{(0)}(k) = e^{a_i} X_i^{(0)}(k-1) = M_i X_i^{(0)}(k-1), \quad \text{where } M_i = e^{a_i}. \tag{13}$$

Let $H(M_i) = \sum_{k=1}^{r}(1/2)(X_i^{(0)}(k) - M_i X_i^{(0)}(k-1))^2$ be the objective function. By using the least square method, H can be made minimum using the parameter M_i, which should satisfy

$$\begin{aligned} \frac{\partial H}{\partial M_i} &= -\sum_{k=1}^{r}\left(X_i^{(0)}(k) - M_i X_i^{(0)}(k-1)\right) X_i^{(0)}(k-1) \\ &= 0. \end{aligned} \tag{14}$$

By solving this equation, we can obtain

$$M_i = e^{a_i} = \frac{\sum_{k=1}^{r} X_i^{(0)}(k) X_i^{(0)}(k-1)}{\sum_{k=1}^{r}\left(X_i^{(0)}(k-1)\right)^2}, \tag{15}$$

$$a_i = \ln\left(\frac{\sum_{k=1}^{r} X_i^{(0)}(k) X_i^{(0)}(k-1)}{\sum_{k=1}^{r}\left(X_i^{(0)}(k-1)\right)^2}\right). \tag{16}$$

Substituting (15) into (10) gives

$$X_i^{(1)}(k) = E_i W_i(k) + F_i, \tag{17}$$

where $W_i(k) = M_i^k = e^{a_i k}$.

Let $Q(E_i, F_i) = \sum_{k=1}^{r}(1/2)(X_i^{(1)}(k) - E_i W_i(k) - F_i)^2$ be the objective function. The E_i and F_i should satisfy

$$\begin{aligned} \frac{\partial Q}{\partial E_i} &= -\sum_{k=1}^{r}\left(X_i^{(1)}(k) - E_i W_i(k) - F_i\right) W_i(k) = 0, \\ \frac{\partial Q}{\partial F_i} &= -\sum_{k=1}^{r}\left(X_i^{(1)}(k) - E_i W_i(k) - F_i\right) = 0. \end{aligned} \tag{18}$$

Solving this equation yields

$$\begin{aligned} E_i &= \frac{r\sum_{k=1}^{r} W_i(k) X_i^{(1)}(k) - \sum_{k=1}^{r} W_i(k) \sum_{k=1}^{r} X_i^{(1)}(k)}{r\sum_{k=1}^{r}\left(W_i(k)\right)^2 - \left(\sum_{k=1}^{r} W_i(k)\right)^2} \\ &= \frac{r\sum_{k=1}^{r} e^{a_i k} X_i^{(1)}(k) - \sum_{k=1}^{r} e^{a_i k} \sum_{k=1}^{r} X_i^{(1)}(k)}{r\sum_{k=1}^{r}\left(e^{a_i k}\right)^2 - \left(\sum_{k=1}^{r} e^{a_i k}\right)^2}, \\ F_i &= \frac{\sum_{k=1}^{r} X_i^{(1)}(k)}{r} - E_i \frac{\sum_{k=1}^{r} e^{a_i k}}{r}. \end{aligned} \tag{19}$$

From (10), the background value $z_i^{(1)}(k)$ can be written as follows:

$$\begin{aligned} z_i^{(1)}(k) &= \int_{k-1}^{k} X_i^{(1)}(t)\,dt = \int_{k-1}^{k}\left(E_i e^{a_i t} + F_i\right) dt \\ &= \left[\frac{E_i e^{a_i t}}{a_i} + F_i t\right]_{k-1}^{k} \\ &= \frac{E_i e^{a_i k}}{a_i} - \frac{E_i e^{a_i(k-1)}}{a_i} + F_i(k - (k-1)) \\ &= \frac{1}{a_i}\left(E_i e^{a_i(k)} - E_i e^{a_i(k-1)}\right) + F_i(k - (k-1)) \\ &= \frac{1}{a_i}\left(X_i^{(1)}(k) - X_i^{(1)}(k-1)\right) + F_i \\ &= \frac{X_i^{(0)}(k)}{a_i} + F_i, \quad i = 1, 2, 3, \ldots, n. \end{aligned} \tag{20}$$

Substituting (20) into (3), we have

$$\begin{aligned} &X_1^{(0)}(k) + b_1\left(\frac{X_1^{(0)}(k)}{a_1} + F_1\right) \\ &\quad = \sum_{i=2}^{n} b_i\left(\frac{X_i^{(0)}(k)}{a_i} + F_i\right) + u, \\ &\left(\frac{a_1 + b_1}{a_1}\right) X_1^{(0)}(k) \\ &\quad = \sum_{i=2}^{n} b_i\left(\frac{X_i^{(0)}(k)}{a_i} + F_i\right) + u - b_1 F_1, \end{aligned} \tag{21}$$

$$\begin{aligned} &X_1^{(0)}(k) \\ &\quad = \sum_{i=2}^{n}\left(\frac{a_1 b_i}{a_1 + b_1}\right)\left(\frac{X_i^{(0)}(k)}{a_i} + F_i\right) \\ &\qquad + a_1\left(\frac{u - b_1 F_1}{a_1 + b_1}\right), \end{aligned}$$

$$X_1^{(0)}(k) = \sum_{i=2}^{n} b_i^*\left(\frac{X_i^{(0)}(k)}{a_i} + F_i\right) + u^*, \tag{22}$$

where

$$b_i^* = \frac{a_1 b_i}{a_1 + b_1},$$
$$u^* = a_1 \left(\frac{u - b_1 F_1}{a_1 + b_1} \right). \tag{23}$$

According to the least squares estimation, parameters $b_2^*, b_3^*, \ldots, b_n^*$ and u^* of (22) are obtained as

$$\left[b_2^* \;\; b_3^* \;\; b_4^* \;\; \cdots \;\; b_n^* \;\; u^* \right]^T = \left(B^T B \right)^{-1} B^T Z, \tag{24}$$

where

$$B = \begin{bmatrix} \frac{X_2^{(0)}(1)}{a_2} + F_1 & \frac{X_3^{(0)}(1)}{a_3} + F_3 & \cdots & \frac{X_n^{(0)}(1)}{a_n} + F_n & 1 \\ \frac{X_2^{(0)}(2)}{a_2} + F_2 & \frac{X_3^{(0)}(2)}{a_3} + F_3 & \cdots & \frac{X_n^{(0)}(2)}{a_n} + F_n & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{X_2^{(0)}(r)}{a_2} + F_2 & \frac{X_3^{(0)}(r)}{a_3} + F_3 & \cdots & \frac{X_n^{(0)}(r)}{a_n} + F_n & 1 \end{bmatrix}, \tag{25}$$

$$Z = \begin{bmatrix} X_1^{(0)}(1) \\ X_1^{(0)}(2) \\ \vdots \\ X_1^{(0)}(r) \end{bmatrix}.$$

By solving (2) using the integrating factor method with the initial condition $\widehat{X}_1^{(1)}(1) = X_1^{(1)}(1)$, the model prediction equation for the predicted 1-AGO series can be obtained as follows.

From (2), the integrating factor is $e^{\int b_1 dt} = e^{b_1 t}$. Multiplying (2) by $e^{b_1 t}$ obtains

$$\frac{d}{dt}\left(e^{b_1 t} X_1^{(1)}(t) \right) = \sum_{i=2}^{n} e^{b_1 t} b_i X_i^{(1)}(t)\, dt + e^{b_1 t} u. \tag{26}$$

Taking the integral of both sides of the above equation in the interval $[1, k]$, we get

$$\int_1^k \frac{d}{dt}\left(e^{b_1 t} X_1^{(1)}(t) \right) dt = \int_1^k \sum_{i=2}^{n} e^{b_1 t} b_i X_i^{(1)}(t)\, dt + \int_1^k e^{b_1 t} u\, dt,$$

$$X_1^{(1)}(k) e^{b_1 k} - X_1^{(1)}(1) e^{b_1} = \int_1^k \sum_{i=2}^{n} e^{b_1 t} b_i \left[E_i e^{a_i t} + F_i \right] dt + \int_1^k e^{b_1 t} u\, dt = \int_1^k \sum_{i=2}^{n} b_i \left[E_i e^{(b_1 + a_i)t} + e^{b_1 t} F_i \right] dt + \int_1^k e^{b_1 t} u\, dt,$$

$$X_1^{(1)}(k) e^{b_1 k} = X_1^{(1)}(1) e^{b_1} + \sum_{i=2}^{n} \left[b_i \left(\frac{E_i \left(e^{(b_1 + a_i)k} - e^{(b_1 + a_i)} \right)}{b_1 + a_i} + \frac{F_i \left(e^{b_1 k} - e^{b_1} \right)}{b_1} \right) \right] + \frac{u e^{b_1 k}}{b_1} - \frac{u e^{b_1}}{b_1}, \tag{27}$$

$$X_1^{(1)}(k) = X_1^{(1)}(1) e^{-b_1 (k-1)} + \sum_{i=2}^{n} b_i \left[\frac{E_i e^{a_i k}}{b_1 + a_i} + \frac{F_i}{b_1} \right] - \sum_{i=2}^{n} b_i \left[\frac{E_i e^{-b_1 (k-1) + a_i}}{b_1 + a_i} + \frac{F_i e^{-b_1 (k-1)}}{b_1} \right] + \frac{u}{b_1} - \frac{u e^{-b_1 (k-1)}}{b_1}. \tag{28}$$

Substituting $k = k + 1$ into (28) gives

$$X_1^{(1)}(k+1) = X_1^{(1)}(1) e^{-b_1 k} + \sum_{i=2}^{n} b_i \left[\frac{E_i e^{a_i (k+1)}}{b_1 + a_i} + \frac{F_i}{b_1} \right] - \sum_{i=2}^{n} b_i \left[\frac{E_i e^{-b_1 k + a_i}}{b_1 + a_i} + \frac{F_i e^{-b_1 k}}{b_1} \right] + \frac{u}{b_1} - \frac{u e^{-b_1 k}}{b_1}. \tag{29}$$

Multiplying both sides of (28) by e^{-b_1}, we get

$$e^{-b_1} X_1^{(1)}(k) = X_1^{(1)}(1) e^{-b_1 k} + \sum_{i=2}^{n} b_i \left[\frac{E_j e^{a_i k - b_1}}{b_1 + a_i} + \frac{F_i e^{-b_1}}{b_1} \right] - \sum_{i=2}^{n} b_i \left[\frac{E_j e^{-b_1 k + a_i}}{b_1 + a_i} + \frac{F_i e^{-b_1 k}}{b_1} \right] + \frac{u e^{-b_1}}{b_1} - \frac{u e^{-b_1 k}}{b_1}. \tag{30}$$

Subtracting (30) from (29), we have

$$\begin{aligned}
&X_1^{(1)}(k+1) - e^{-b_1}\widehat{X}_1^{(1)}(k) \\
&= X_1^{(1)}(1)e^{-b_1 k} + \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i(k+1)}}{b_1+a_i} + \frac{F_i}{b_1}\right] \\
&\quad - \sum_{i=2}^{n} b_i\left[\frac{E_i e^{-b_1 k + a_i}}{b_1+a_i} + \frac{F_i e^{-b_1 k}}{b_1}\right] + \frac{u}{b_1} - \frac{u e^{-b_1 k}}{b_1} \\
&\quad - X_1^{(1)}(1)e^{-b_1 k} - \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i k - b_1}}{b_1+a_i} + \frac{F_i e^{-b_1}}{b_1}\right] \\
&\quad + \sum_{i=2}^{n} b_i\left[\frac{E_i e^{-b_1 k + a_i}}{b_1+a_i} + \frac{F_i e^{-b_1 k}}{b_1}\right] - \frac{u e^{-b_1}}{b_1} + \frac{u e^{-b_1 k}}{b_1} \\
&= \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i(k+1)}}{b_1+a_i} + \frac{F_i}{b_1}\right] \\
&\quad - \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i k - b_1}}{b_1+a_i} + \frac{F_i e^{-b_1}}{b_1}\right] + \frac{u}{b_1} - \frac{u e^{-b_1}}{b_1} \\
&= \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i(k+1)} + F_i}{b_1+a_i} - \frac{F_i}{b_1+a_i} + \frac{F_i}{b_1}\right] \\
&\quad - \sum_{i=2}^{n} b_i\left[\frac{E_i e^{a_i k - b_1} + F_i}{b_1+a_i} - \frac{F_i}{b_1+a_i} + \frac{F_i e^{-b_1}}{b_1}\right] + \frac{u}{b_1} \\
&\quad - \frac{u e^{-b_1}}{b_1} \\
&= \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k+1)}{b_1+a_i} + \frac{F_i}{b_1}\right] \\
&\quad - \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k)e^{-b_1}}{b_1+a_i} + \frac{F_i e^{-b_1}}{b_1}\right] + \frac{u}{b_1} - \frac{u e^{-b_1}}{b_1} \\
&= \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k+1)}{b_1+a_i}\right] - \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k)e^{-b_1}}{b_1+a_i}\right] \\
&\quad + \sum_{i=2}^{n} b_i\left[\frac{F_i}{b_1} - \frac{F_i e^{-b_1}}{b_1}\right] + \frac{u}{b_1} - \frac{u e^{-b_1}}{b_1} \\
&= e^{-b_1}\widehat{X}_1^{(1)}(k) + \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k+1)}{b_1+a_i}\right] \\
&\quad - \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k)e^{-b_1}}{b_1+a_i}\right] + \sum_{i=2}^{n} b_i\left[\frac{F_i}{b_1} - \frac{F_i e^{-b_1}}{b_1}\right] \\
&\quad + \frac{u}{b_1} - \frac{u e^{-b_1}}{b_1}.
\end{aligned} \tag{31}$$

Consequently, the predicted 1-AGO series of $X_1^{(1)}(k)$, $k = 2, 3, \ldots, rf$, is given by

$$\begin{aligned}
\widehat{X}_1^{(1)}(k+1) &= c_1\widehat{X}_1^{(1)}(k) + \sum_{i=2}^{n} c_i X_i^{(1)}(k+1) \\
&\quad + \sum_{i=2}^{n} d_i X_i^{(1)}(k) + v,
\end{aligned} \tag{32}$$

$$\widehat{X}_1^{(0)}(k+1) = \widehat{X}_1^{(1)}(k+1) - \widehat{X}_1^{(1)}(k), \tag{33}$$

where $c_1 = e^{-b_1}$, $c_i = b_i/(b_1 + a_i)$, $d_i = b_i c_1/(b_1 + a_i)$, and $v = \sum_{i=2}^{n} b_i[F_i/b_1 - F_i c_1/b_1] + u/b_1 - u c_1/b_1$ for $i = 2, 3, \ldots, n$.

In the traditional grey model, the parameters are evaluated by (23), whereas the model predictions are given by (32). However, a paradox between these two equations would lead to high levels of error of prediction [5]. We can infer that (23) for parameter estimation is equivalent to (32) for model prediction if parameter b_1 of (2) is zero, which is shown as follows.

Setting $b_1 = 0$ and taking the integral of both sides of (2) in the interval $[1, k]$, we have

$$\begin{aligned}
&\int_1^k \frac{d}{dt}\left(X_1^{(1)}(t)\right)dt = \int_1^k \sum_{i=2}^{n} b_i X_i^{(1)}(t)\,dt + \int_1^k u\,dt, \\
&X_1^{(1)}(k) - X_1^{(1)}(1) = \sum_{i=2}^{n} b_i \int_1^k \left[E_i e^{a_i t} + F_i\right]dt + \int_1^k u\,dt \\
&\quad = \sum_{i=2}^{n} b_i\left[\frac{E_i}{a_i}\left(e^{a_i k} - e^{a_i}\right) + F_i(k-1)\right] + u(k-1) \\
&\quad = \sum_{i=2}^{n} b_i\left[\frac{X_i^{(1)}(k) - X_i^{(1)}(1)}{a_i} + F_i(k-1)\right] \\
&\quad\quad + u(k-1) \\
&\quad = \sum_{i=2}^{n} \frac{b_i}{a_i}\left(X_i^{(1)}(k) - X_i^{(1)}(1)\right) + \sum_{i=2}^{n} b_i F_i(k-1) \\
&\quad\quad + u(k-1),
\end{aligned} \tag{34}$$

$$\begin{aligned}
&X_1^{(1)}(k-1) - X_1^{(1)}(1) \\
&\quad = \sum_{i=2}^{n} \frac{b_i}{a_i}\left(X_i^{(1)}(k-1) - X_i^{(1)}(1)\right) + \sum_{i=2}^{n} b_i F_i(k-2) \\
&\quad\quad + u(k-2).
\end{aligned} \tag{35}$$

Substituting $k = k + 1$ into (35) gives

$$\begin{aligned}
X_1^{(1)}(k) - X_1^{(1)}(1) &= \sum_{i=2}^{n} \frac{b_i}{a_i}\left(X_i^{(1)}(k) - X_i^{(1)}(1)\right) \\
&\quad + \sum_{i=2}^{n} b_i F_i(k-1) + u(k-1).
\end{aligned} \tag{36}$$

Subtracting (36) from (35), we have

$$\begin{aligned} X_1^{(1)}(k) - X_1^{(1)}(k-1) &= \sum_{i=2}^{n} \frac{b_i}{a_i}\left(X_i^{(1)}(k) - X_i^{(1)}(k-1)\right) + \sum_{i=2}^{n} b_i F_i + u, \\ X_1^{(0)}(k) &= \sum_{i=2}^{n} b_i \left(\frac{X_i^{(0)}(k)}{a_i} + F\right) + u. \end{aligned} \tag{37}$$

Therefore, (32) is the same as (23).

The different forms of these two equations affect the prediction accuracy of the model. Therefore, in this study, (32) is used for parameter estimation and model prediction in order to overcome this problem.

Firstly, we substitute all the data values into (32). Then, the linear system is derived as follows:

$$Dc = P, \tag{38}$$

where

$$D = \begin{bmatrix} X_1^{(1)}(1) & X_2^{(1)}(2) & X_3^{(1)}(2) & \cdots & X_n^{(1)}(2) & X_2^{(1)}(1) & X_3^{(1)}(1) & \cdots & X_n^{(1)}(1) & 1 \\ X_1^{(1)}(2) & X_2^{(1)}(3) & X_3^{(1)}(3) & \cdots & X_n^{(1)}(3) & X_2^{(1)}(2) & X_3^{(1)}(2) & \cdots & X_n^{(1)}(2) & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ X_1^{(1)}(r-1) & X_2^{(1)}(r) & X_3^{(1)}(r) & \cdots & X_n^{(1)}(r) & X_2^{(1)}(r-1) & X_3^{(1)}(r-1) & \cdots & X_n^{(1)}(r-1) & 1 \end{bmatrix},$$

$$P = \begin{bmatrix} X_1^{(1)}(2) \\ X_1^{(1)}(3) \\ \vdots \\ X_1^{(1)}(r) \end{bmatrix}, \tag{39}$$

$$c = \left[c_1 \;\; c_2 \;\; c_3 \;\; \cdots \;\; c_n \;\; d_2 \;\; d_3 \;\; \cdots \;\; d_n \;\; v\right]^T.$$

The parameters $c_1, c_2, c_3, \ldots, c_n, d_2, d_3, \ldots, d_n$ and v are determined using the least squares estimation, as follows.

If $D^T D$ is a nonsingular matrix, then the solution of (38) can be obtained using the following equation:

$$\begin{aligned} c &= \left[c_1 \;\; c_2 \;\; c_3 \;\; \cdots \;\; c_n \;\; d_2 \;\; d_3 \;\; \cdots \;\; d_n \;\; v\right]^T \\ &= \left(D^T D\right)^{-1} D^T P. \end{aligned} \tag{40}$$

However, if $D^T D$ is a singular matrix, then the solution of (28) can be determined using the equation

$$c = \left[c_1 \;\; c_2 \;\; c_3 \;\; \cdots \;\; c_n \;\; d_2 \;\; d_3 \;\; \cdots \;\; d_n \;\; v\right]^T = D^+ P, \tag{41}$$

where D^+ is Moore-Penrose pseudoinverse of matrix D [25, 26].

Finally, the predicted series of $X_1^{(0)}(k)$, $k = 2, 3, \ldots, rf$ is calculated by using (32) and (9).

4. Statistical Measure of the Forecasting Performance

To evaluate the performance of model simulation and prediction, two criteria, namely, the mean absolute percentage error (MAPE) and the root mean square percentage error (RMSPE), are applied for this study. MAPE and RMSPE are the most commonly used accuracy measures in prediction models [27–30]. Generally, MAPE and RMSPE are defined, respectively, as

$$\text{MAPE} = \frac{1}{r-1} \sum_{k=2}^{r} \left| \frac{X^{(0)}(k) - \widehat{X}^{(0)}(k)}{X^{(0)}(k)} \right| \times 100\%, \tag{42}$$

$$\text{RMSPE} = \sqrt{\frac{\sum_{k=2}^{r} \left(\left(X^{(0)}(k) - \widehat{X}^{(0)}(k)\right) / X^{(0)}(k)\right)^2}{r-1}} \times 100\%, \tag{43}$$

where $X^{(0)}(k)$ is the actual value at time k, $\widehat{X}^{(0)}(k)$ is its model value, and r is the number of data used for prediction. The values of MAPE and RMSPE sufficiently describe the goodness of prediction effect. The lower the values of MAPE and RMSPE, the more accurate the prediction. The criteria of MAPE and RMSPE are presented in Table 1 [29, 30]. In addition, absolute percentage error (APE) is used to evaluate the accuracy of the model for each data point, which is defined as

$$\text{APE} = \left| \frac{X^{(0)} - \widehat{X}^{(0)}}{X^{(0)}} \right| \times 100\%. \tag{44}$$

Table 1: Criteria of MAPE and RMSPE.

MAPE and RMSPE (%)	Forecasting power
<10	Highly accurate forecasting
10–20	Good forecasting
20–50	Reasonable forecasting
>50	Inaccurate forecasting

5. Application of Modified GMC$(1, n)$ Model

In order to verify the performance of the modified GMC$(1, n)$ prediction model, two different real cases of prediction problems are considered. In addition, we compare prediction accuracy with the conventional GMC$(1, n)$ and the discrete multivariate grey model (D-GMC$(1, n)$).

Case 1 (forecasting CO_2 emission levels in Thailand). From a previous study [31], it has been established that the main factors that affect the amount of CO_2 released into the atmosphere are natural gas, solid fuel, liquid fuel, population, and GDP. Therefore, these factors are used to predict the CO_2 emission levels in Thailand for this study. All the data, from 1994 to 2013, were obtained from the Energy Policy and Planning Office (EPPO) of Thailand (data source: http://www.eppo.go.th/info/index-statistics.html).

These factors along with the factor of CO_2 emission are defined as the variables in the modified grey model GMC$(1, 6)$, which are written as follows:

$X_1^{(0)}(t)$ is the time series of CO_2 emission (1,000 tons).

$X_2^{(0)}(t)$ is the time series of natural gas consumption (MMSCFD).

$X_3^{(0)}(t)$ is the time series of solid fuel consumption (1,000 tons).

$X_4^{(0)}(t)$ is the time series of liquid fuel consumption (barrels/day).

$X_5^{(0)}(t)$ is the time series of population (persons).

$X_6^{(0)}(t)$ is the time series of GDP (billion bahts).

t is an order of the time series: $t = 1$ refers to the year 1994 and $t = 20$ refers to the year 2013.

The actual data are separated into two parts: the first part (1994–2010) is used to construct the models, while the second part (2011–2013) is employed for model prediction. All the data satisfy the quasi-smooth and quasi-exponential conditions. The result of the modified GMC$(1, 6)$ model, obtained from (32), has the form

$$\begin{aligned}\widehat{X}_1^{(1)}(k+1) &= 0.248\widehat{X}_1^{(1)}(k) - 1.133X_2^{(1)}(k+1) \\ &\quad - 1.359X_2^{(1)}(k) + 0.326X_3^{(1)}(k+1) \\ &\quad + 0.227X_3^{(1)}(k) + 0.487X_4^{(1)}(k+1) \\ &\quad + 0.300X_4^{(1)}(k) + 0.031X_5^{(1)}(k+1) \\ &\quad + 0.015X_5^{(1)}(k) + 0.064X_6^{(1)}(k+1) \\ &\quad - 0.095X_6^{(1)}(k) + 1.285, \\ &\qquad k = 1, 2, \ldots, 19.\end{aligned} \tag{45}$$

The actual values, the model values, the predictive values, and the APE of D-GMC$(1, 6)$ and the modified GMC$(1, 6)$ models are presented in Table 2. Table 3 shows the performance of the model simulation and prediction of D-GMC$(1, 6)$ as well as the modified GMC$(1, 6)$. As demonstrated in these tables, both the models have highly accurate forecasting abilities. However, the modified GMC$(1, 6)$ has lower MAPE and RMSPE than D-GMC$(1, 6)$, which indicates that the modified GMC$(1, 6)$ has higher accuracy of prediction than D-GMC$(1, 6)$. As far as the conventional GMC$(1, 6)$ is concerned, large errors were obtained for the model simulation and prediction (results not shown).

Case 2 (forecasting electricity consumption in Thailand). From a previous study [32], it has been established that population, gross domestic product (GDP), stock index (SET index), and total revenue from exporting industrial products (export) are the main factors that influence the electricity consumption (measured in GWh) of Thailand. Therefore, we used these factors to construct the grey model. All data were obtained from the Energy Policy and Planning Office and Stock Exchange of Thailand. The data set was collected annually from 2002 to 2014 and is as presented in Table 4. These factors and electricity consumption are defined as the variables in the modified grey model GMC$(1, 5)$, which are written as follows:

$X_1^{(0)}(t)$ is the time series of electricity consumption (GWh).

$X_2^{(0)}(t)$ is the time series of population (persons).

$X_3^{(0)}(t)$ is the time series of gross domestic product (billion bahts).

$X_4^{(0)}(t)$ is the time series of stock index (point).

$X_5^{(0)}(t)$ is the time series of the total revenue from exporting industrial products (billion bahts).

t is an order of the time series: $t = 1$ refers to the year 2002 and $t = 13$ refers to the year 2014.

We will use the steps of the modified grey GMC$(1, 5)$ model for finding the parameters from 2002 to 2011 for the prediction of electricity consumption (measured in GWh) from 2012 to 2014. All the data satisfy the quasi-smooth

Table 2: Comparison of actual values, model values, and APEs of D-GMC(1, 6) and modified GMC(1, 6) models.

Years	t	CO_2 emission data	D-GMC(1, 6)		Modified GMC(1, 6)	
			Model value	APE	Model value	APE
1994	1	122,876.13	122,876.13	0.00	122,876.13	0.00
1995	2	137,691.67	142,056.72	3.17	138,089.13	0.29
1996	3	155,553.15	148,750.34	4.37	154,325.20	0.79
1997	4	161,057.54	154,933.62	3.80	162,115.22	0.66
1998	5	145,451.52	161,292.65	10.89	146,188.49	0.51
1999	6	149,881.53	140,012.17	6.58	148,098.26	1.19
2000	7	150,832.00	150,968.83	0.09	150,578.46	0.17
2001	8	153,545.22	157,968.41	2.88	156,157.34	1.70
2002	9	163,558.08	164,645.66	0.66	161,506.52	1.25
2003	10	172,053.83	176,232.82	2.43	172,557.16	0.29
2004	11	187,418.22	176,180.65	6.00	189,201.76	0.95
2005	12	192,574.90	193,410.05	0.43	188,058.52	2.35
2006	13	193,233.99	197,621.97	2.27	196,939.39	1.92
2007	14	200,494.35	194,241.16	3.12	199,340.27	0.58
2008	15	203,261.89	208,243.34	2.45	204,008.47	0.37
2009	16	208,267.86	211,387.03	1.50	207,577.79	0.33
2010	17	220,467.16	217,488.33	1.35	220,488.97	0.01
2011*	18	224,465.07	236,377.49	5.31	222,859.77	0.72
2012*	19	240,583.97	234,631.65	2.47	240,026.56	0.23
2013*	20	240,802.07	248,902.18	3.36	241,530.87	0.30

*Prediction.

Table 3: Performance evaluation for fitting and Prediction accuracies of D-GMC(1, 6) and modified GMC(1, 6) models.

Criteria	D-GMC(1, 6)		Modified GMC(1, 6)	
	Model value	Predictive value	Model value	Predictive value
MAPE (%)	3.25	3.71	0.83	0.42
RMSPE (%)	4.19	3.90	1.06	0.47

and quasi-exponential conditions. The result of the modified GMC(1, 5) model, obtained from (32), has the form

$$\begin{aligned}\widehat{X}_1^{(1)}(k+1) = {} & 0.417\widehat{X}_1^{(1)}(k) + 0.398X_2^{(1)}(k+1) \\ & - 0.025X_2^{(1)}(k) + 0.910X_3^{(1)}(k+1) \\ & - 0.491X_3^{(1)}(k) + 0.006X_4^{(1)}(k+1) \\ & - 0.013X_4^{(1)}(k) - 0.139X_5^{(1)}(k+1) \\ & - 0.018X_5^{(1)}(k) - 0.226, \\ & k = 1, 2, \ldots, 12.\end{aligned} \tag{46}$$

The actual values, the model values, the predictive values, and the APE of D-GMC(1, 5) and the modified GMC(1, 5) models are presented in Table 4. Table 5 shows the MAPE and RMSPE values of D-GMC(1, 5) and the modified GMC(1, 5) models, which are used to measure the accuracy of prediction of the models. These results indicate that the modified GMC(1, 5) has lower MAPE and RMSPE values than D-GMC(1, 5). For the conventional GMC(1, 5), large errors were obtained for the model simulation and prediction (results not shown). Therefore, it can be safely concluded that the modified GMC(1, 5) has a higher degree of accuracy of prediction than the conventional GMC(1, 5) and D-GMC(1, 5).

6. Conclusion

This study presents a modified GMC(1, n) model for improving the prediction accuracy of conventional GMC(1, n) by modification of the formula for calculating the background value, the system of parameter estimation, and the model prediction equation. The prediction accuracy can be compared between the conventional GMC(1, n), D-GMC(1, n), and the modified GMC(1, n) models in terms of the MAPE and RMSPE values. The empirical results obtained using the modified GMC(1, n) reveal the least error for MAPE and RMSPE in comparison with conventional GMC(1, n) and D-GMC(1, n) for two real cases of prediction problems. This success indicates that the modified GMC(1, n) improves the accuracy of the simulation and prediction of the conventional GMC(1, n) models. In addition, using the criteria in Table 1, it can be substantiated that the modified GMC(1, n)

Table 4: Comparison of actual values, model values, and APE of D-GMC(1, 5) and modified GMC(1, 5) models.

Years	t	Electricity consumption data	D-GMC(1, 5)		Modified GMC(1, 5)	
			Model value	APE	Model value	APE
2002	1	100,091	100,091.00	0.000	100091.00	0
2003	2	106,987	105,968.82	0.952	106987.17	4.11×10^{-12}
2004	3	115,101	117,863.18	2.400	115101.15	1.29×10^{-11}
2005	4	121,240	119,605.49	1.348	121240.03	1.28×10^{-11}
2006	5	127,879	127,053.81	0.645	127879.26	1.25×10^{-11}
2007	6	133,113	132,870.66	0.182	133112.82	1.24×10^{-11}
2008	7	135,520	137,355.47	1.354	135520.31	1.24×10^{-11}
2009	8	135,181	134,748.54	0.320	135180.80	1.22×10^{-11}
2010	9	149,301	148,048.60	0.839	149301.14	1.15×10^{-11}
2011	10	148,855	149,991.01	0.763	148855.54	1.28×10^{-11}
2012*	11	161,779	169,923.11	5.034	159807.95	1.22
2013*	12	164,341	161,959.15	1.449	164233.19	0.07
2014*	13	168,620	165,305.30	1.966	163396.00	3.10

*Prediction.

Table 5: Performance evaluation for fitting and prediction accuracies of D-GMC(1, 5), and modified GMC(1, 5) models.

Error	D-GMC(1, 5)		Modified GMC(1, 5)	
	Model value	Predictive value	Model value	Predictive value
MAPE (%)	0.98	2.82	1.15×10^{-11}	1.46
RMSPE (%)	1.16	3.23	1.17×10^{-11}	1.92

model demonstrates highly accurate forecasting. However, the modified GMC(1, n) model needs to be validated with more real problems and cannot be directly written in the form of the differential equation which is given as (2). For practical applications, the proposed model can be applied to many real systems, especially the systems which have the similarity in trend between the predicted variables and their influencing factors [8]. In addition, the proposed techniques can be applied to the real studies [10, 15, 21, 22] which have successfully used the conventional GMC(1, n).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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