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Research Article

Process Optimization of Biodiesel Production Using the Laplacian Harris Hawk Optimization (LHHO) Algorithm

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Continuous power consumption from standard fuel resources is responsible for producing large-scale environmental greenhouse gases. Production of biodiesel fuels from the vegetable oils can be considered an alternative source. Effect of greenhouse gases can also be diminished. The production of biodiesel is done by a chemical process namely transesterification and usually maximized by using the Response Surface Methodology (RSM) tool. This paper presents a new approach to optimize the production of biodiesel by introducing a new variant of recently published metaheuristic Harris Hawk Optimization (HHO). The developed variant is based on the replacement of random numbers of normal distribution at the initialization phase by the random numbers generated from the Laplacian distribution. The proposed variant is named as the Laplacian Harris Hawk Optimization (LHHO) algorithm. The contribution of this paper is in twofold: firstly the performance of the proposed algorithm is verified over a well-known set of benchmark functions, and then, we applied the LHHO to maximize biodiesel production. Comparison of LHHO is carried out with five other recent metaheuristic algorithms. An optimization routine is formulated in the form of a single-objective function with a temperature, methanol to oil ratio, and catalyst concentration as the optimization variables. These parameters are optimized to maximize the production of biodiesel. The results obtained using the proposed LHHO show significant improvement as compared to other algorithms.

1. Introduction

Global warming is a critical issue nowadays with a recognized negative impact on the environment [1, 2]. The issue persists with the growing pollution emitted by industries utilizing fossil fuels. Depletion of fossil fuels and increasing cost of fuels are other issues related to human being as well. To overcome these problems, research is ongoing to search for alternative environmentally friendly fuels or those with less emissions and suitable for internal combustion engines of automobiles. A separate benefit of renewable energy sources is that, besides being sustainable, such sources offer the benefit of not contributing to greenhouse gas (GHG) emissions. Therefore, such energy sources are often referred to as clean energy forms. Clean Air Act Standards (CAAS) [3, 4] and Renewable Fuel Standard (RFS) [5] mandate the two aspects that play a major role in developing renewable fuels from alternative resources. These nonconventional resources have an advantage of unlimited availability and environmental acceptability. In present years, the production of large amount of biodiesel fuel is highly prioritized. In comparison to other fuels, biodiesel is advantageous to the environment because it does not produce any pollution.

The method of transesterification is a chemical reaction used for biodiesel production [6, 7]. The chemical process is shown in Figure 1. A simple flowchart of transesterification process in industries is represented in Figure 2. In this process, an organic group of esters is substituted from an organic group of alcohol. Transesterification is a three-step process in which triglycerides are first converted into diglycerides. Then, as a second step, diglycerides are converted into monoglycerides. In the final step, monoglycerides are converted into glycerol [8]. To convert one mole of triglyceride into diglycerides, three moles of alcohol are needed. However, to drive the process in a forward direction, a higher number of alcohols are needed [9]. By using analogous alkaline catalysts, the transesterification process is catalyzed because in less reaction time, high amount of alkyl ester is produced in this process.

Operating conditions like the reaction temperature, oil/methanol ratio, and quantity of catalyst along with temperature heat remedy used in nature domestic rock, etc. affect the catalyst performance majorly [10]. There are numerous surveys of the alcoholysis of triglycerides using analogous catalysts [11–15]. At temperature of 40 to 65°C, analogous catalysts can easily achieve high conversions in less than an hour of reaction. When used safely for the development of biodiesel, for numerous industrial applications, heterogeneous catalysts can be very effective [16, 17]. When a unique ester is rereacted with liquor in the transesterification process, that is known as alcoholysis as shown in Figure 1. Many parameters are incorporated having impact on transesterification such as temperature, vegetable oil, and type of catalyst. Fungal lipases are used as enzymes which are water soluble and catalyze hydrolysis of long-chain triglycerides. They are used to enhance the production of biodiesel because these enzymes increase the speed of reaction time. Mostly four forms of catalysts are included in the biodiesel synthesis. These are free lipases, immobilized lipases, whole cell, and solid enzymatic preparation/fermented solids (SEP) [18, 19]. There are four important parameters by using fungal lipases in biodiesel productions which are as follows:

(1) Feedstocks

(2) Types of alcohol and alcohol to oil motor ratio

(3) Glycerol effect

(4) Water content

There is lot to be done to maximize biodiesel production while determining significant conditional parameters for chemical reaction; for that, we are using an optimization technique. Over the years, optimization methods have been used for many problems in the planning, operation, and control of power systems. In this work, a modified version of the Harris Hawk Optimization (HHO) algorithm, namely, Laplacian Harris Hawk Optimization (LHHO), is utilized to find out the optimized values of decision parameters for maximization of biodiesel production. The Harris Hawk Optimization is proposed by Heidari et al. recently and provides better efficiency. There are two phases of this algorithm, namely, exploration and exploitation phase.

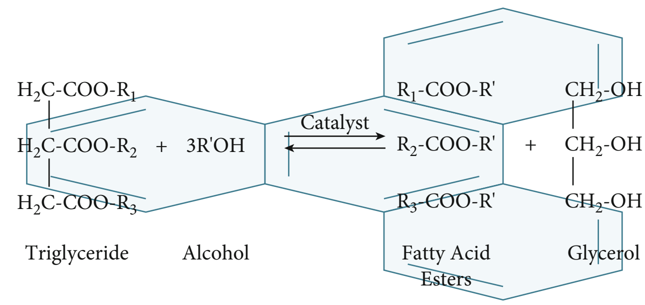

Figure 1: Chemical reaction of transesterification.

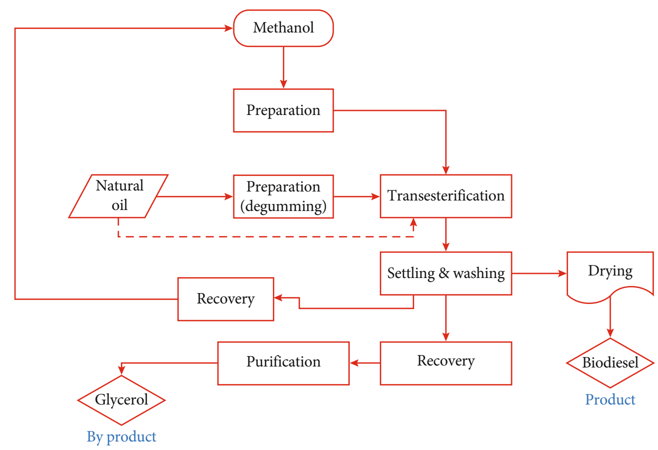

Figure 2: General flowchart of the transesterification process.

This study presents a comparison between HHO and other well-known optimization methods. The major reasons for choosing HHO were its fast convergence rate and the optimality of the 94 outcomes.

With this outset, the paper has contributions in twofold: the first is to propose an efficient variant based on Laplacian probability distribution-based random numbers that are implemented in the initialization phase and verify the performance of the proposed algorithm over conventional 23 minimization problems. The second is to utilize the proposed variant to maximize the production of biodiesel. The research objectives proposed for this work are as follows:

(i) To develop a variant of the recently published algorithm HHO on the basis of replacement of the random numbers of the initialization phase to Laplacian numbers

(ii) To evaluate the impact of this modification with the application of the proposed variant on conventional benchmark functions and their evaluations on certain defined procedure of standard optimization

(iii) To apply a variant on the biodiesel maximization problem and evaluate the performance of the variant in previously reported approaches

1.1. Literature Study. Energy is in high demand from when the industrialization is started. This is because of the development of various machines which always need energy

[20]. Biogenic resources are very useful in a spark ignition and compression ignition engine; in the case of spark ignition, ethanol is useful [21], and for compression ignition of the engine, vegetable oils are very useful [22]. The biodiesel term is first ever used in the first report of Belgian patent 422877 [23, 24]. The currently development to produce esters (biodiesel) basically depends on the use of catalytic reaction which is termed as hydrodeoxygenation from biogenic resources [25, 26]. There are many catalysts, and their responses to chemistry are reviewed in [27, 28]. There are many catalysts such as enzymatic catalysts [7, 29, 30], whole cell catalyst [19], ionic liquids [31], and dolomite [16]. The main objective in [16] is to maximize the biodiesel production by using several parameters in the transesterification of ethanol with soybean oil. Several parameters included are catalyst concentration, vegetable oil molar ratio, and temperature. These all three parameters affect the transesterification process.

Experimental results are available in literature. Hence, for analytical studies, we have taken these results, where a heterogeneous catalyst dolomite is used for the transesterification process [16, 32]. For biodiesel production, this catalyst is ecologically suitable and it has low cost and high basicity.

More recently, a tsunami of applications of metaheuristic algorithms in real-world problems are observed. Some of good examples of these approaches are in every domain such as computer science in evolutionary data clustering [33], protein structure prediction [34, 35], and Model Order Reduction (MOR) [36]. An approach based on grey wolf optimizer and support vector machine has been reported in prediction of ambient air quality [37]. These applications are powerful witnesses of the applicability of the contemporary metaheuristics for solving difficult problems.

1.2. Research Gap and Motivation. As the fuel utility in an optimal manner is the key issue these days, being an alternative to the fossil fuel, production of biodiesel is the requirement of the present scenario. Though the chemical reaction known as transesterification is used to produce biodiesel, there are limitations to determine optimal decision parameters to maximize biodiesel as it requires manual experiments each time for biodiesel production. So, there is lot to be done to maximize biodiesel production while determining significant conditional parameters for chemical reaction. To address this gap, a different optimization approach using the modified version of the Harris Hawk Optimization (HHO) algorithm, namely, Laplacian Harris Hawk Optimization (LHHO), is utilized in this paper to find out the optimized values of decision parameters for maximization of biodiesel production.

The remaining paper is organized as follows: Section 2 represents the problem formulation for biodiesel production. Section 3 depicts this problem as an optimization problem. Section 4 exhibits the conceptual description of the original as well as proposed method of optimization. Section 5 shows the results and analysis of over 23 benchmark functions and biodiesel problem. Section 6 represents conclusions.

Table 1: For biodiesel production range and level coding of design variable.

Independent variables	Lower bound	Average bound	Upper bound
Temperature (x_a)	55	60	65
Methanol/oil molar ratio (x_b)	6 : 1	10.5 : 1	15 : 1
Concentration of catalyst (x_c)	0.6	1.3	2.0

2. Problem Formulation

As per [32], the optimization problem of biodiesel is designed as a nonlinear optimization problem. In this problem, there are three decision variables which are the reaction temperature, the concentration of catalyst, and the methanol/oil molar ratio as described in Table 1. In this table, independent variables are depicted in the first column which are used in the optimization process, and the second column demonstrates their lower bounds. The third and fourth columns depict the average bound and upper bound. A second-order polynomial regression equation is produced by three independent variables, and one response variable is modelled as a function of independent variables to yield methyl ester as the biodiesel fuel. The equation can be defined as [32]

$$X_{\mathrm{me}} = b_o + \sum_{t=1} b_t x_t + \sum_{t=1} b_{tt} x_t^2 + \sum_{t \neq u=1} b_{tu} x_t, \tag{1}$$

where X_{me} shows the yield of methyl ester, b_o is the constant, b_t is the linear coefficients, b_{tt} is the quadratic coefficients, and b_{tu} is the iterative coefficients.

There are three main parameters with a major effect on the efficiency of biodiesel production. These parameters are, namely, temperature (x_a), methanol to oil ratio (x_b), and concentration of catalyst (x_c). The regression equation to form methyl ester from three independent variables is defined as

$$\begin{aligned} X_{\mathrm{me}}(\%) = {} & 37.45 + 0.42x_a + 4.37x_b - 0.16{x_2}^2 + 24.26x_c \\ & - 1.53{x_c}^2 + 0.001x_a x_b - 0.24x_a x_c - 0.44x_2 x_3. \end{aligned} \tag{2}$$

3. Optimization of Biodiesel Production

A proposed nature-inspired algorithm named as the Laplacian Harris Hawk Optimization (LHHO) is used in the optimization of biodiesel production instead of the well-known transesterification process. Three independent variables are optimized by this algorithm such that the biodiesel production is maximized. The objective function which is used in this algorithm is shown as

$$\begin{aligned} \max f(x_a x_b x_c) = {} & 37.45 + 0.42x_a + 4.37x_b - 0.16x_2^2 + 24.26x_c \\ & - 1.53x_c^2 + 0.001x_a x_b - 0.24x_a x_c - 0.44x_2 x_3, \end{aligned} \tag{3}$$

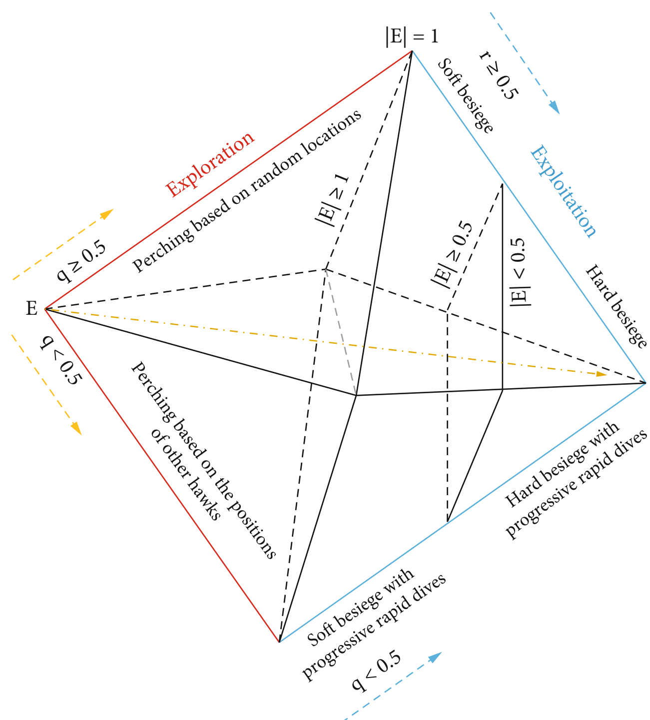

FIGURE 3: Different phases of the Harris Hawk Optimization.

and the constraints are

$$55 \leq x_a \leq 65, \tag{4}$$

$$6 \leq x_b \leq 15, \tag{5}$$

$$0.6 \leq x_c \leq 2.0. \tag{6}$$

4. Method of Optimization: Harris Hawk Optimization (HHO)

The HHO algorithm is proposed by Heidari et al. [38] which is a nature-inspired algorithm. Harris hawks' cooperative behaviour, surprise pounce, and pursuing technique served as inspiration for this algorithm. Two phases are demonstrated in this algorithm in Figure 3, which are exploration and exploitation phases. Some recent applications of HHO and its variant in estimating solar panel parameters can be seen in References.

4.1. Exploration Phase. This section explains the exploration mechanism. Generally, Harris hawks have far-sighted eyes, by that, they detect and spoor the prey. But normally, it is very tough to see prey easily. For that, hawks hold back, observe, and monitor after that attack on the prey. Here, in this algorithm, candidate solutions are Harris hawks, and the optimal solution is considered in each

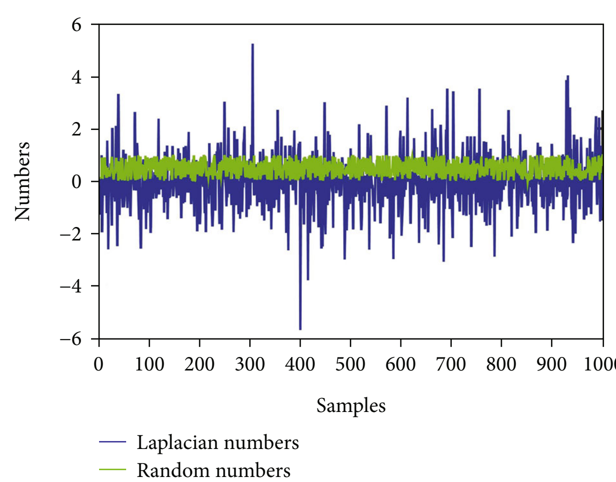

FIGURE 4: Comparison of random number generation by using the uniform distribution and Laplace distribution.

step near to the optimum position. The mathematical representation of the exploration phase is shown as

$$Z(t+1) = \begin{cases} Z_{\text{rand}}(p) - r_1, \\ |Z_{\text{rand}}(p) - 2r_2 Z(p)|, & h \geq 0.5, \\ (Z(p) - Z(p)), \\ r_3(lb + r_4(ub - lb)), & h < 0.5, \end{cases} \tag{7}$$

where p is the upcoming iteration, $Z(t+1)$ is the position of hawks, and Z_{rabbit} is the position of the rabbit. The present position of hawks is Z_p, and r_1, r_2, r_3, r_4, and h

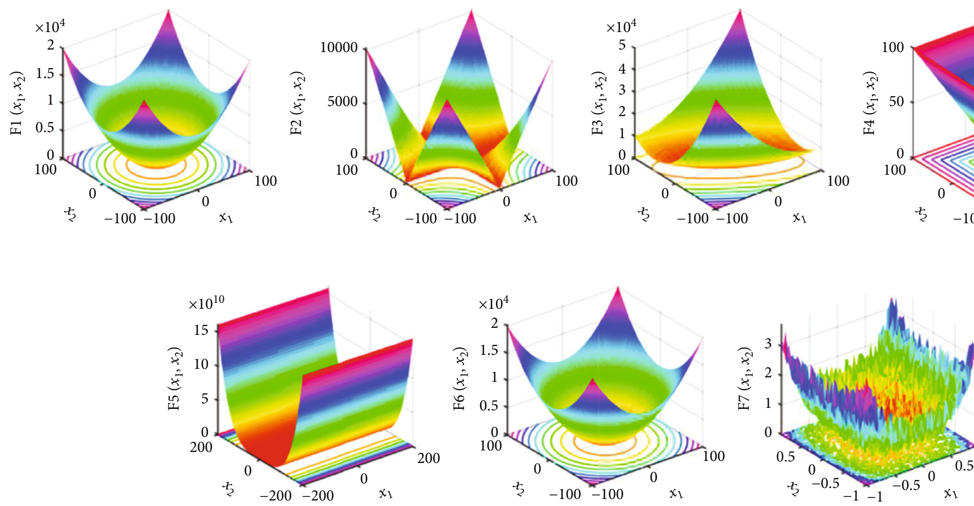

Figure 5: 2-D versions of uni-modal benchmark functions.

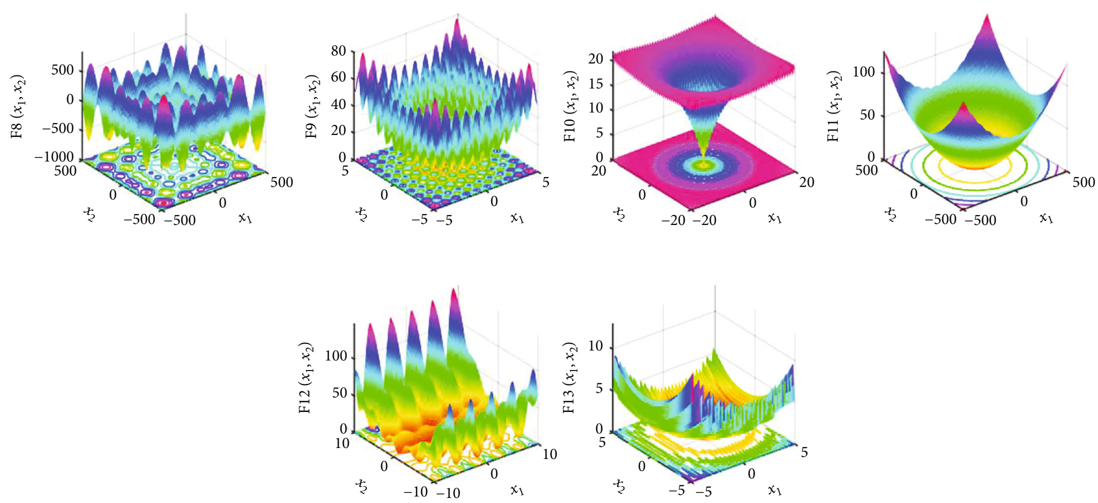

Figure 6: 2-D versions of multi-modal benchmark functions.

are the random numbers in the range of 0 to 1. The average position of hawks is described as

$$Z_m(p) = \frac{1}{S}\sum_{p=1} Z_i(p), \tag{8}$$

where $Z_i(p)$ is the location of each hawk in iteration p and the total number of hawks is denoted by S.

4.2. Exploration to Exploitation Transition. On the basis of prey's escaping energy working, the exploratory phase can be converted to the exploitative phase and that can switch between varieties of exploitative behaviour. The energy of prey decreases significantly during the fleeing behaviour. To demonstrate this, the energy of prey is calculated as follows:

$$\epsilon = 2\epsilon_O\left(1 - \frac{p}{T}\right), \tag{9}$$

where ϵ indicates the prey's escaping energy, while the preliminary condition of energy is shown by ϵ_O and the maximum number of iterations are denoted as T.

4.3. Exploitation Phase. This section described the surprise methods to attack on the prey in actual situations. In the exploration phase, prey tries to run away from the perilous location. Hence, this part shows the various chasing techniques of prey in an actual scenario. There are four methods of the exploitation phase as follows:

(1) Soft besiege

(2) Hard besiege

(3) Soft besiege with progressive rapid dives

(4) Hard besiege with progressive rapid dives

4.4. Laplacian Harris Hawk Optimization (LHHO). In this section, we propose a new variant of HHO that is known as the Laplacian Harris Hawk Optimizer (LHHO). In

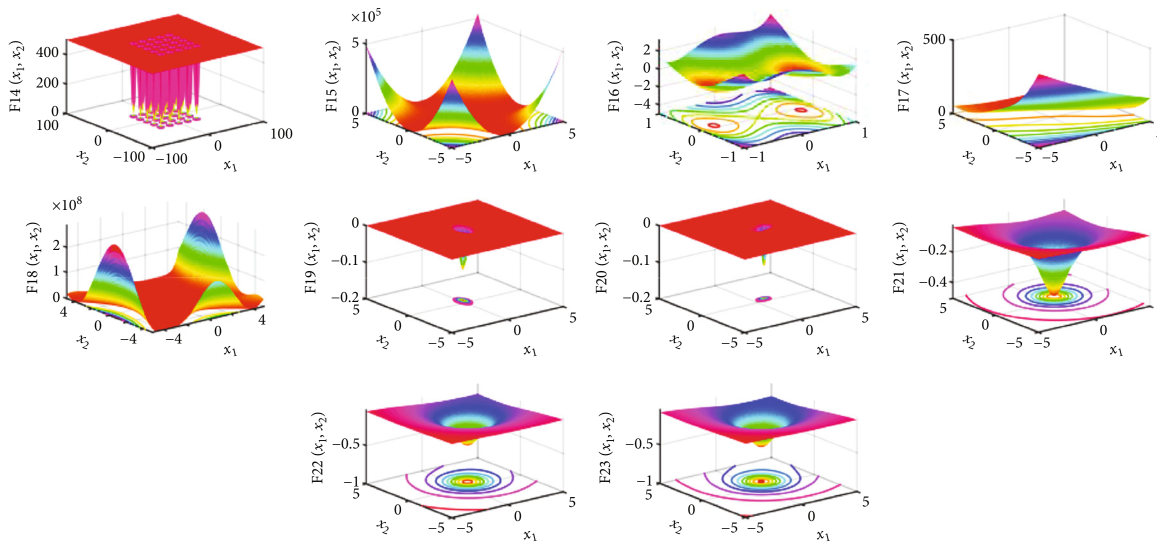

Figure 7: 2-D version of fixed-dimension multi-modal benchmark functions.

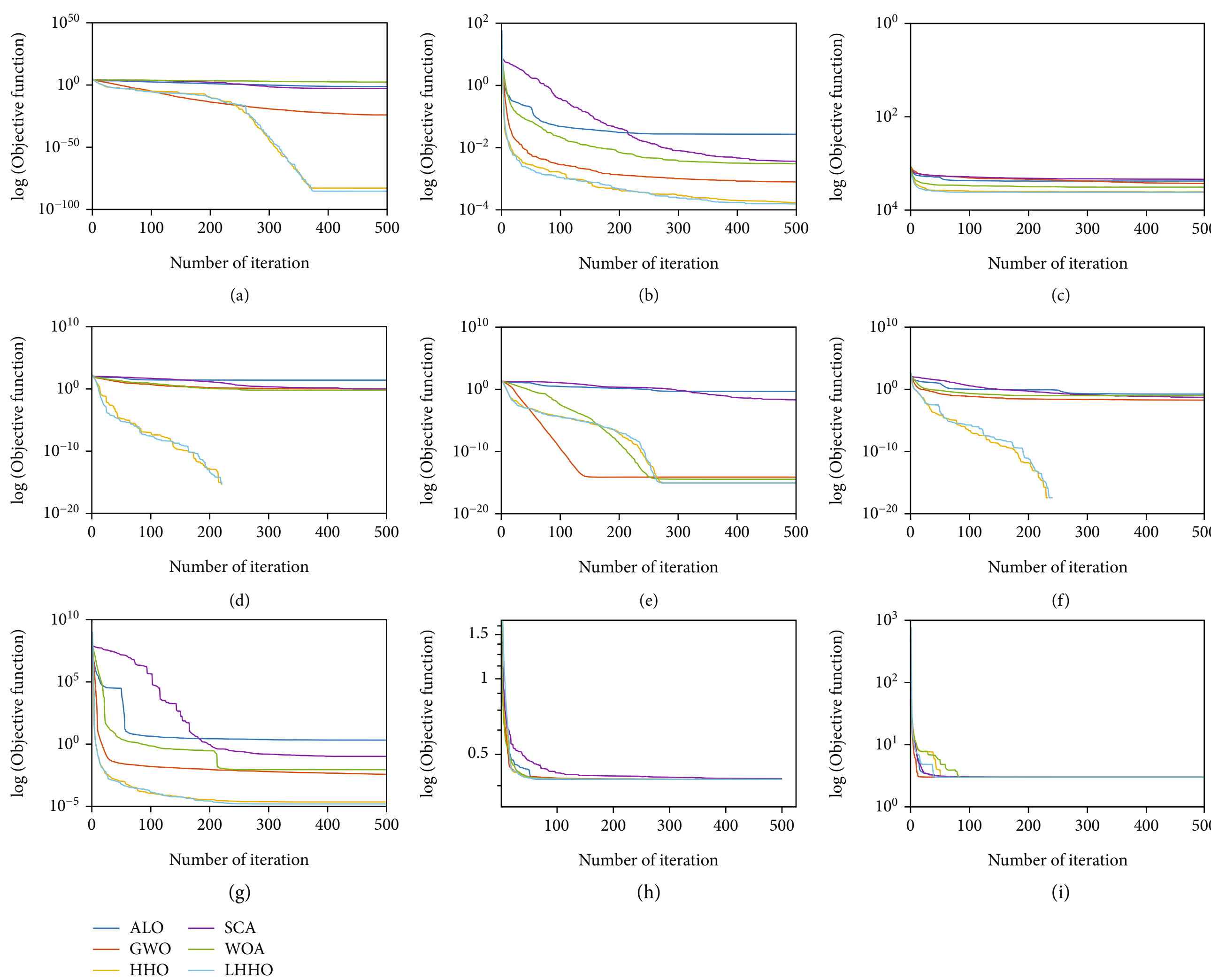

Figure 8: Convergence Curves of a few Benchmark Functions.

Table 2: Results of conventional benchmark functions (F1-11).

		ALO [40]	GWO [41]	HHO [38]	SCA [42]	WOA [43]	LHHO
F1	Min	$2.10E-09$	$1.00E-61$	$1.05E-110$	$2.26E-18$	$6.80E-89$	$1.07E-115$
	Max	$2.08E-08$	$6.84E-56$	$1.53E-98$	$2.24E-11$	$1.47E-76$	$1.65E-92$
	Mean	$8.28E-09$	$4.46E-57$	$7.03E-100$	$1.10E-12$	$5.22E-78$	$5.89E-94$
	SD	$4.46E-09$	$1.50E-56$	$2.90E-99$	$4.10E-12$	$2.67E-77$	$3.01E-93$
F2	Min	$1.86E-05$	$4.18E-35$	$4.92E-61$	$8.20E-13$	$1.97E-59$	$3.92E-60$
	Max	$7.10E+00$	$1.80E-31$	$1.30E-48$	$1.45E-08$	$3.98E-52$	$2.22E-49$
	Mean	$7.40E-01$	$1.32E-32$	$5.71E-50$	$1.76E-09$	$1.69E-53$	$9.01E-51$
	SD	$1.54E+00$	$3.48E-32$	$2.47E-49$	$3.29E-09$	$7.24E-53$	$4.06E-50$
F3	Min	$4.75E-05$	$3.79E-31$	$4.14E-104$	$5.65E-10$	$8.08E-01$	$5.67E-101$
	Max	$4.28E-01$	$1.75E-23$	$4.17E-82$	$2.75E-02$	$1.42E+03$	$1.31E-84$
	Mean	$5.81E-02$	$9.15E-25$	$1.39E-83$	$1.74E-03$	$2.51E+02$	$4.71E-86$
	SD	$1.02E-01$	$3.28E-24$	$7.62E-83$	$6.31E-03$	$3.44E+02$	$2.38E-85$
F4	Min	$1.32E-04$	$8.58E-21$	$4.71E-58$	$1.03E-06$	$1.01E-04$	$5.96E-58$
	Max	$6.77E-02$	$2.47E-17$	$6.59E-48$	$1.79E-03$	$5.83E+01$	$7.65E-47$
	Mean	$7.06E-03$	$2.64E-18$	$2.89E-49$	$3.15E-04$	$5.14E+00$	$2.61E-48$
	SD	$1.58E-02$	$4.97E-18$	$1.24E-48$	$4.21E-04$	$1.24E+01$	$1.40E-47$
F5	Min	$1.66E-03$	$5.94E+00$	$1.20E-05$	$6.62E+00$	$6.38E+00$	$4.10E-05$
	Max	$2.67E+03$	$7.21E+00$	$1.65E-02$	$8.73E+00$	$8.08E+00$	$2.24E-02$
	Mean	$2.07E+02$	$6.64E+00$	$2.96E-03$	$7.56E+00$	$7.11E+00$	$3.67E-03$
	SD	$5.67E+02$	$4.81E-01$	$4.50E-03$	$4.91E-01$	$4.84E-01$	$5.57E-03$
F6	Min	$2.34E-09$	$1.22E-06$	$8.04E-08$	$1.83E-01$	$1.69E-04$	$3.30E-08$
	Max	$5.94E-08$	$7.50E-02$	$2.96E-04$	$9.21E-01$	$1.01E-01$	$1.95E-04$
	Mean	$9.90E-09$	$2.50E-03$	$6.13E-05$	$4.86E-01$	$5.28E-03$	$3.27E-05$
	SD	$1.06E-08$	$1.37E-02$	$8.99E-05$	$1.98E-01$	$1.86E-02$	$4.97E-05$
F7	Min	$5.83E-03$	$7.66E-05$	$1.12E-05$	$5.02E-04$	$7.79E-05$	$9.22E-06$
	Max	$9.28E-02$	$2.25E-03$	$7.98E-04$	$9.61E-03$	$1.73E-02$	$7.50E-04$
	Mean	$2.70E-02$	$7.88E-04$	$1.71E-04$	$3.51E-03$	$3.04E-03$	$1.55E-04$
	SD	$1.72E-02$	$5.95E-04$	$1.80E-04$	$2.75E-03$	$3.98E-03$	$1.45E-04$
F8	Min	$-3.68E+03$	-3.45E+03	$-4.19E+03$	$-2.67E+03$	$-4.19E+03$	$-4.19E+03$
	Max	$-1.93E+03$	$-2.07E+03$	$-2.59E+03$	$-1.91E+03$	$-2.10E+03$	$-3.48E+03$
	Mean	$-2.38E+03$	$-2.69E+03$	$-4.06E+03$	$-2.19E+03$	$-3.22E+03$	$-4.17E+03$
	SD	$4.38E+02$	$3.18E+02$	$4.00E+02$	$1.67E+02$	$6.80E+02$	$1.30E+02$
F9	Min	$1.19E+01$	$0.00E+00$	$0.00E+00$	$0.00E+00$	$0.00E+00$	$0.00E+00$
	Max	$4.97E+01$	$5.40E+00$	$0.00E+00$	$1.87E+01$	$1.92E+01$	$0.00E+00$
	Mean	$2.56E+01$	$9.98E-01$	$0.00E+00$	$9.39E-01$	$6.41E-01$	$0.00E+00$
	SD	$9.56E+00$	$1.91E+00$	$0.00E+00$	$3.62E+00$	$3.51E+00$	$0.00E+00$
F10	Min	$2.58E-05$	$4.44E-15$	$8.88E-16$	$2.74E-10$	$8.88E-16$	$8.88E-16$
	Max	$2.58E+00$	$1.15E-14$	$8.88E-16$	$3.35E-01$	$7.99E-15$	$8.88E-16$
	Mean	$4.58E-01$	$7.64E-15$	$8.88E-16$	$2.06E-02$	$3.49E-15$	$8.88E-16$
	SD	$8.15E-01$	$1.71E-15$	$0.00E+00$	$7.77E-02$	$2.63E-15$	$0.00E+00$

Table 2: Continued.

		ALO [40]	GWO [41]	HHO [38]	SCA [42]	WOA [43]	LHHO
F11	Min	$4.43E-02$	$0.00E+00$	$0.00E+00$	$0.00E+00$	$0.00E+00$	$0.00E+00$
	Max	$5.10E-01$	$6.65E-02$	$0.00E+00$	$3.96E-01$	$6.67E-01$	$0.00E+00$
	Mean	$1.71E-01$	$1.83E-02$	$0.00E+00$	$5.20E-02$	$9.77E-02$	$0.00E+00$
	SD	$1.01E-01$	$2.02E-02$	$0.00E+00$	$8.84E-02$	$1.65E-01$	$0.00E+00$

LHHO, variant random numbers are replaced by Laplacian numbers. In recent approaches, the reported Laplacian random number generator is applied by that output of the function which is a matrix with a mean value of 0 and standard deviation of 1.

By using normal distribution and Laplace distribution, random numbers are generated for 1000 iterations as shown in Figure 4. It can be seen from this figure that the Laplacian distributed random numbers explore more region of the search space as compared to normally distributed random numbers guaranteeing that these random numbers are more likely to determine new candidate solutions not explored yet. So, this procedure assures an enhanced diversity of search space and is less prone to be stagnant at local optima and will in general move towards the global optima. So, the improved random numbers using the Laplace distribution as utilized in [39] can be defined as shown in

$$R\left(S_{A,n}^{d}\right) = [\mathrm{cumsum}(2\times \mathrm{Lap}(t_1)-1), \\ \mathrm{cumsum}(2\times \mathrm{Lap}(t_2)-1), \cdots, \\ \mathrm{cumsum}(2\times \mathrm{Lap}(t_{\max})-1). \tag{10}$$

The probability density function (PDF) is defined as the conventional Laplace probability distribution (LPD), and LPD is denoted by $\mathrm{Lap}(S, T)$.

$$f(x;S,T) = \frac{1}{2T}\ \exp - \frac{|x-S|}{T}, \quad -\infty < x < \infty. \tag{11}$$

The Laplace distribution function is given as

$$F(x) = \begin{cases} \dfrac{1}{2}\ \exp - \dfrac{|x-S|}{T}, & x \le S, \\ 1 - \dfrac{1}{2}\ \exp - \dfrac{|x-S|}{T}, & x \ge S, \end{cases} \tag{12}$$

where $S \in (-\infty, \infty)$ shows the location and T determines the scale parameters. In between the range of $[0, S]$, function f is increasing, and between the range of $[S, \infty]$, it is decreasing with mode $x = S$.

5. Result and Analysis

In this section, the maximization process is solved by six different recently published algorithms that are Ant Lion Optimizer (ALO) [40], grey wolf optimizer (GWO) [41], Harris Hawk Optimization (HHO) [38], sine cosine algorithm (SCA) [42], whale optimization algorithm (WOA) [43], and Laplacian Harris Hawk Optimization (LHHO). For application of any metaheuristic algorithm on a real problem, it requires performance analysis of the algorithm on some known conventional mathematical functions. This process is known as benchmarking of the algorithm. The process is inevitable due to the fact that real problems do not contain any relevant information about minima. On the other hand, standard conventional benchmark functions possess the characteristics that are prior known to the designer. This information can be in the form of location of global minima, number of local minima, and shape. To deal with this, uncertainty exists in real applications; it is necessary to check the proposed variant on known functions; in other words, prior benchmarking is extremely important and required. With this outset, in the next section, the results of benchmarking of LHHO are presented.

5.1. Results on Conventional Benchmark Functions. It is empirical to state that often modifications are suggested in algorithms to support real applications. As per no free lunch theorem, one cannot categorize any particular algorithm for all real applications [44]. On the other hand, performance of algorithms is sometimes problem specific.

To test the proposed variant, we choose the bench of 23 standard benchmark functions that are already defined in HHO [38, 45]. The shapes of these functions are simulated and given in Figures 5–7 for unimodal, multimodal, and fixed dimension functions, respectively. The following are key observations when analyzing the results of applying LHHO:

(i) Comparing all algorithms, LHHO obtained 12 optimal mean values and 14 optimal standard values out of 23 standard functions. The comparison of mean and standard deviation values is presented in Tables 2 and 3, respectively, for all 23 benchmark functions. From these tables, it is evident that the proposed variant shows competitive performance as compared to contemporary bioinspired optimizers

(ii) The convergence curves of benchmark functions are shown in Figure 8. From this figure, it can also be judged that LHHO shows a superior convergence property over other algorithms. Functions 3, 7, 8, 9, 10, 11, 12, 17, and 18 are chosen to showcase the superiority of LHHO over the competitor algorithms

(iii) To show the efficacy of the proposed modification, a trajectory curve analysis is also simulated. The

Table 3: Results of conventional benchmark functions (F12-23).

		ALO [40]	GWO [41]	HHO [38]	SCA [42]	WOA [43]	LHHO
F12	Min	$2.01E-09$	$2.94E-07$	$2.55E-08$	$4.93E-02$	$1.83E-04$	$1.29E-07$
	Max	$7.93E+00$	$2.00E-02$	$1.31E-04$	$2.06E-01$	$4.66E-02$	$1.19E-04$
	Mean	$2.12E+00$	$3.79E-03$	$2.25E-05$	$1.04E-01$	$8.87E-03$	$1.55E-05$
	SD	$2.09E+00$	$7.74E-03$	$3.30E-05$	$3.87E-02$	1.18E-02	$2.41E-05$
F13	Min	$2.81E-09$	$1.19E-06$	$8.51E-08$	$1.20E-01$	$1.38E-03$	$9.28E-08$
	Max	$2.10E-02$	$2.04E-01$	$4.56E-04$	$5.05E-01$	$1.85E-01$	$7.18E-04$
	Mean	$5.40E-03$	$1.94E-02$	$8.24E-05$	$3.02E-01$	$4.28E-02$	$1.16E-04$
	SD	$7.28E-03$	$5.48E-02$	$1.13E-04$	$9.13E-02$	$4.58E-02$	$1.81E-04$
F14	Min	$9.98E-01$	$9.98E-01$	$9.98E-01$	$9.98E-01$	$9.98E-01$	$9.98E-01$
	Max	$6.90E+00$	$1.27E+01$	$5.93E+00$	$2.98E+00$	$1.08E+01$	$5.93E+00$
	Mean	$2.62E+00$	$5.46E+00$	$1.23E+00$	$1.86E+00$	$2.83E+00$	$1.59E+00$
	SD	$1.52E+00$	$4.73E+00$	$9.23E-01$	$9.99E-01$	$3.33E+00$	$1.50E+00$
F15	Min	$3.07E-04$	$3.08E-04$	$3.09E-04$	$3.79E-04$	$3.08E-04$	$3.10E-04$
	Max	$2.04E-02$	$2.04E-02$	$1.92E-03$	$1.57E-03$	$2.24E-03$	$1.57E-03$
	Mean	$2.71E-03$	$2.42E-03$	$5.03E-04$	$9.92E-04$	$6.62E-04$	$5.30E-04$
	SD	$5.43E-03$	$6.09E-03$	$4.26E-04$	$3.42E-04$	$4.72E-04$	$4.10E-04$
F16	Min	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$
	Max	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$
	Mean	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$	$-1.03E+00$
	SD	$1.48E-13$	$1.58E-08$	$5.07E-10$	$3.62E-05$	$3.16E-10$	$7.51E-10$
F17	Min	$3.98E-01$	$3.98E-01$	$3.98E-01$	$3.98E-01$	$3.98E-01$	$3.98E-01$
	Max	$3.98E-01$	$4.00E-01$	$3.98E-01$	$4.07E-01$	$3.98E-01$	$3.98E-01$
	Mean	$3.98E-01$	$3.98E-01$	$3.98E-01$	$4.00E-01$	$3.98E-01$	$3.98E-01$
	SD	$9.70E-14$	$3.19E-04$	$1.55E-05$	$2.34E-03$	$3.74E-05$	$2.02E-05$
F18	Min	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$
	Max	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$
	Mean	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$	$3.00E+00$
	SD	$7.07E-13$	$5.18E-05$	$9.59E-07$	$1.04E-04$	$6.83E-05$	$1.92E-06$
F19	Min	$-3.86E+00$	$-3.86E+00$	$-3.86E+00$	$-3.86E+00$	$-3.86E+00$	$-3.86E+00$
	Max	$-3.86E+00$	$-3.85E+00$	$-3.85E+00$	$-3.85E+00$	$-3.84E+00$	$-3.86E+00$
	Mean	$-3.86E+00$	$-3.86E+00$	$-3.86E+00$	$-3.85E+00$	$-3.86E+00$	$-3.86E+00$
	SD	$6.67E-12$	$2.34E-03$	$2.94E-03$	$2.66E-03$	$5.95E-03$	$2.11E-03$
F20	Min	$-3.32E+00$	$-3.32E+00$	$-3.26E+00$	$-3.25E+00$	$-3.32E+00$	$-3.25E+00$
	Max	$-3.20E+00$	$-3.08E+00$	$-2.67E+00$	$-1.45E+00$	$-3.00E+00$	$-2.81E+00$
	Mean	$-3.28E+00$	$-3.27E+00$	$-3.08E+00$	$-2.85E+00$	$-3.25E+00$	$-3.07E+00$
	SD	$5.85E-02$	$7.57E-02$	$1.37E-01$	$4.04E-01$	$9.95E-02$	$1.11E-01$
F21	Min	$-1.02E+01$	$-1.02E+01$	$-1.01E+01$	$-6.38E+00$	$-1.02E+01$	$-5.06E+00$
	Max	$-2.63E+00$	$-5.06E+00$	$-5.05E+00$	$-3.51E-01$	$-8.81E-01$	$-5.04E+00$
	Mean	$-5.78E+00$	$-9.47E+00$	$-5.39E+00$	$-2.38E+00$	$-7.30E+00$	$-5.05E+00$
	SD	$2.87E+00$	$1.75E+00$	$1.27E+00$	$1.95E+00$	$3.23E+00$	$3.48E-03$

Table 3: Continued.

		ALO [40]	GWO [41]	HHO [38]	SCA [42]	WOA [43]	LHHO
F22	Min	$-1.04E+01$	$-1.04E+01$	$-1.04E+01$	$-7.16E+00$	$-1.04E+01$	$-5.09E+00$
	Max	$-1.84E+00$	$-6.03E+00$	$-5.05E+00$	$-5.21E-01$	$-2.76E+00$	$-5.05E+00$
	Mean	$-7.12E+00$	$-1.03E+01$	$-5.42E+00$	$-3.21E+00$	$-8.09E+00$	$-5.08E+00$
	SD	$3.44E+00$	$7.99E-01$	$1.28E+00$	$2.03E+00$	$2.80E+00$	$9.01E-03$
F23	Min	$-1.05E+01$	$-1.05E+01$	$-1.04E+01$	$-6.59E+00$	$-1.05E+01$	$-5.13E+00$
	Max	$-2.42E+00$	$-5.17E+00$	$-5.11E+00$	$-9.42E-01$	$-1.67E+00$	$-5.11E+00$
	Mean	$-7.29E+00$	$-1.04E+01$	$-5.30E+00$	$-3.75E+00$	$-5.71E+00$	$-5.12E+00$
	SD	$3.62E+00$	$9.79E-01$	$9.69E-01$	$1.52E+00$	$3.34E+00$	$4.24E-03$

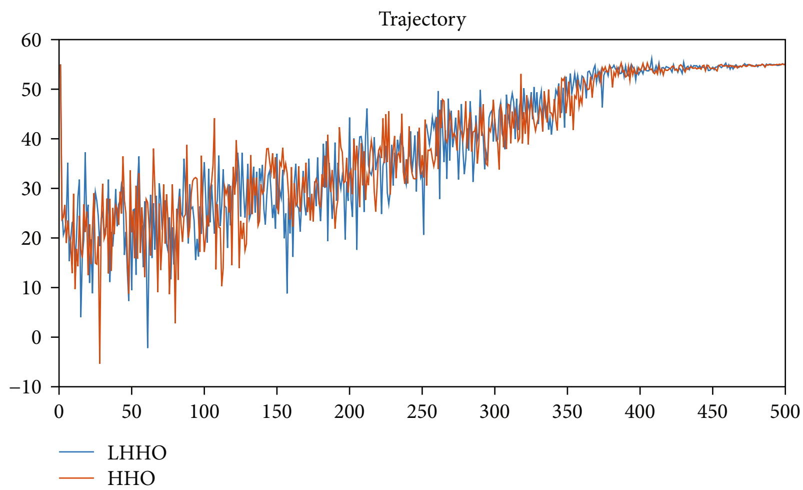

Figure 9: Trajectory of LHHO and HHO for biodiesel function.

Table 4: Statistical attributes of objective function values in the maximization process.

	ALO [40]	GWO [41]	HHO [38]	SCA [42]	WOA [43]	LHHO
Min	$8.67E+01$	$9.66E+01$	$9.68E+01$	$8.58E+01$	$9.68E+01$	$9.68E+01$
Max	$9.19E+01$	$9.68E+01$	$9.68E+01$	$8.58E+01$	$9.68E+01$	$9.68E+01$
Mean	$8.95E+01$	$9.67E+01$	$9.68E+01$	$8.58E+01$	$9.68E+01$	$9.68E+01$
SD	$1.44E+00$	$8.38E-02$	$8.65E-11$	$0.00E+00$	$1.56E-06$	$1.79E-11$

trajectory curve for the first variable of the biodiesel problem is depicted in Figure 9. From the figure, it can be concluded that the trajectory curve is more stable for LHHO over the parent algorithm. That is the indicator of a balanced exploration and exploitation process. Hence, it can be concluded that the proposed modification establishes a fair balance between exploration and exploitation processes of HHO

5.2. Results of Biodiesel Process Optimization. In the second stage of optimization, all algorithms are applied for solving the maximization process of biodiesel production. For making a fair comparison of all algorithms, authors run all the algorithms autonomously for 30 times with the population size of 40, and the maximum number of iterations is 500. All algorithms are initialized in such a manner that three independent variables x_a, x_b, and x_c are corresponding with the three-dimensional vector, respectively. Equations (4), (5), and (6) show the lower bound and upper bound values of all three parameters.

It is evident from Figure 9 that better convergence rate is exhibited by LHHO. In Table 4, the values of min, max, mean, and standard deviation parameters for the methyl ester yield are presented. Mostly for the maximization process of biodiesel, mean values of HHO, WOA, and LHHO are the same, but standard deviation values of LHHO are minimum which demonstrates that the maximum number of data is clustered closely around the mean values for LHHO. Other algorithms have higher standard values than

Table 5: Comparative result analysis of biodiesel output using participating algorithms.

Algorithm	Independent variables x_a	x_b	x_c	Optimum value of the methyl ester yield
ALO [40]	59.3159	6.5414	0.8083	89.20%
GWO [41]	64.9511	12.0453	0.9070	96.70%
HHO [38]	65	12.4245	1.0435	96.81%
SCA [42]	55	6	0.6000	85.84%
WOA [43]	65	12.4232	1.0438	96.81%
LHHO	65	12.4245	1.0435	96.81%

LHHO, so their data are not close to mean values. From this observation, it is clear that optimization properties of LHHO are substantially enhanced with the proposed modification.

Statistical significance of this problem is conducted as per [16, 32]; in accordance with the F-test, LHHO has 95% of significance level. With the comparison of HHO and WOA, the value of F is 100.00. With ALO, SCA, and GWO, this F-value is 118.12, 127.21, and 100.24, respectively, with the comparison of LHHO. The p value is $0.30142E-10$ which is significantly very low for LHHO. p value shows the marginal significance, representing the level of appearance probability, of a given event in a statistical hypothesis test. Table 5 shows the values of all three variables and the optimum values of the methyl ester yield of different optimization techniques.

6. Conclusion

Maximization of biodiesel production problem has been addressed in this paper. The problem contains three independent variables: temperature, catalyst, and methanol/oil ratio that affect the production of biodiesel. To solve this optimization problem, a new variant of HHO has been proposed. First, the variant is tested over standard benchmark problems. After the validation of the improved performance, it is applied on biodiesel production. A fair comparison of different contemporary optimizers has been carried out on standard and biodiesel problems. The following conclusions can be drawn from this work:

(i) A variant based on the Laplacian random number initialization has been proposed and named as LHHO. Random numbers with normal probability had been replaced by Laplacian distribution probability in the initialization phase. To judge the implications of this modification, several standard benchmark functions are solved

(ii) LHHO had been tested over conventional benchmark functions. Different statistical results along with convergence property analysis have been presented. These analyses revealed that the proposed variant outperforms the HHO and some of the contemporary optimizers

(iii) After benchmarking of the proposed LHHO, a real application of LHHO in maximization of biodiesel production has been reported. The regression algorithm depicted relationship between the three variables: temperature (x_a), methanol/vegetable oil ratio (x_b), and catalyst concentration (x_c). This relationship has been successfully optimized by the LHHO algorithm. The optimal output of biodiesel obtained by this algorithm is 96.8199% at parameter values of 65°C temperature, 12.4245 methanol to oil ratio, and 1.0435% (w/v) catalyst concentration which is higher than that of other algorithms. From these results, it can be concluded that LHHO can be a preferred choice of a biodiesel manufacturer for obtaining higher profits

(iv) The statistical significance with the F-test showed the predictability of the model, and the less p values are advocate the feasibility of the proposed version in comparison to other algorithms

Development of new variants for optimizing the process of obtaining biodiesel from nonconventional sources lays in the future scope of this work. As per the optimization capacity of LHHO, it will be very interesting to judge the performance of LHHO on many challenging problems of numerical and stochastic optimization.

Data Availability

No data collection was used. We proposed an algorithm, and that will be available on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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