# A Pedestrian Evacuation Model with Leaders during the Smoke Dispersion Based on a Social Force Model 

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#### Abstract

The pedestrian evacuation process during the propagation of smoke in case with and without guider is investigated. The effects of leaders on the evacuation are considered. The model is based on the social force model for pedestrians' motions. The advectiondiffusion equation is applied for the dispersion of smoke. The movement direction of a guider is guided by the solution of the Eikonal equation. It relies on the desired speed and the smoke density. A pedestrian who is not a guider follows the rule termed 'flow with the stream' and 'following the wall'. We perform different numerical experiments in a room with one and two exits. The results show that the guiders effect on the evacuation time when they are large number of individuals in simulation. It can help to increase the number of evacuees. With small number of individuals in the experiment, the effect of guiders on evacuation time is not obvious. Further, simulation results are found that the domain with two exits provides higher number of outside pedestrians than the domain with a single door. Longer evacuation time period can increase the number of evacuees. The visibility range of a pedestrian is reduced when an additional smoke source is added to the system. It decreases the number of evacuees. The results of the proposed model are discussed and compared with the existing models.


## 1. Introduction

During the recent decades, pedestrian evacuation in emergency cases, such as fire, human stampedes, or overcrowding incidents, has become an important issue. An example of a major incident in Thailand is the Santika Pub fire on January 1, 2009. Sixty-six people were killed, and more than 200 were injured. The deaths were partly caused by the improper design of the buildings and by the disregard for human safety.

Laboratory experiments and real-life data show that smoke can affect the pedestrians in two ways [9]. First, smoke can harm a human's health, since it contains some poisonous substances. It provokes pedestrians to lose their steadiness in a way that they are unable to escape from the gases. Second, the visibility range of a human can be reduced when the smoke concentration increases. In the thick irritant smoke, pedestrians are not able to open their eyes for a long time. Their tears run so heavily that they cannot see the words on signs. Therefore, the study of human behaviours and their motions during the propagation of smoke
is significant. It can be used to reduce the causalities under smoke conditions.

There are numerous simulation methods to model pedestrian dynamic, for example, the social force model [6], the optimal-velocity model [15], the magnetic force model [16], the cellular automata models [12], and the discrete choice model [2].

Evacuations are essential in the process towards inevitable disasters and emergencies. Some experimental works reveal that it is very important to have leaders inside the building in an emergency situation [17, 21]. Leaders are agents who are trained and have complete knowledge about the inside geometry of a building on fire. They can be distinguished easily by pedestrians and help others during the evacuation procedure. The knowledge gained from the model can help designers to plan the building with respect to safety issues. This will reduce the amount of losses of both life and property in an event of emergency.

Wang et al. [21] simulated the pedestrian evacuation in public places using a multi-agent-based congestion evacuation model. The panic behaviour of agents is incorporated


Figure 1: Visibility domain and movement directions [11].
in their model. Their simulations show that the evacuation is more efficient by adding a virtual leader if the exit is partially clogged. Weifeng and Hai [23] applied a cellular automaton model to simulate the human behaviour termed 'flow with the stream' from a large smoke-filled compartment. In their experiments, the effect of leaders is taken into account. The results of their numerical tests show that the effect of leaders is significant to the evacuation. The evacuation in a scenario without guider is slower than that in a scenario with leaders. Other studies that apply the social force model to study pedestrian evacuation processes are as follows: Frank and Dorso [3] adopted the social force model to study pedestrian evacuation under limited visibility. In this model, pedestrians have to find way out under low visibility conditions. The effect of guiders is not regarded. Three kinds of pedestrian behavioral patterns are analyzed: individualistic behaviour, herding-like behaviour, and the walls following. They obtained unexpected results that some low visibility situations may enhance evacuation performance.

In the work of Pelechano and Badler [17], they have developed a multi-agent communication for evacuation simulations (Maces). It combines the local motion driven by the social force model. They simulated crowd behaviour under two conditions: agents communicate building route knowledge on the one hand, and agents take different roles such as trained personnel, leaders, and followers on the other hand. They performed 25 simulations using a crowd size of 100 with $0,25,50,75$, and 100 percent trained agents. The results show that the evacuation time decreases, as the number of trained agents in the environment increases.

In reference [22], they performed the effect of leaders in pedestrian evacuation process based on the modified social


Figure 2: Possible movement directions (blue arrows) when individuals are near wall [11].
force model. Three evacuation strategies are investigated: situations that there is no leader, there is a leader nearby, and situation that individuals follow the leader with a certain probability. Their simulations show that the evacuation rate is no more than $30 \%$ in situation that there is no leader. The effect of herding behaviour has a slightly better evacuation rate than pedestrians moving alone to exit. The effect of an increase in the number of guiders on evacuation time is not obvious in their model.

Zhou et al. [25] proposed a hybrid bi-level model to optimize the number, initial locations, and routes of leaders in evacuation process. The social force model and its modifications are employed to study crowds with leaders in large-scale public places. The initial locations of leaders are generated by the upper level model. The evacuation routes of leaders are defined by a co-simulation heuristic approach in the lower level model. Simulation results show the importance of the initial locations of leaders and the improvement of evacuation by applying a leader coordination mechanism. Their proposed optimal evacuation strategy has demonstrated best evacuation performance.

In our recent published articles, we adopted a cellular automaton and the social force models to study the motions of pedestrians influenced by smoke spreading [11, 12]. In these models, the roles of leaders are not considered. Therefore, we would like to extend our previous studies to consider the case with and without leaders by adopting the social force model. The advection-diffusion equation [20] is applied for the propagation of smoke. The movement of a guider in our model is defined by the solution of the Eikonal equation. It is the traveling cost to reach a destination, which depends on pedestrian and smoke density in his visibility. The human behaviour terms 'following the wall' [3] and 'flow with the stream' [23] are also investigated in our model.

The framework of this paper is organized as follows. The social force model with guiders and a way to couple it with the rule of 'flow with the stream', 'following the wall', the advection-diffusion, and the Eikonal equation is

```
input : \(x_{i}(0) ; v_{i}(0) ; C(x, 0) ; T(x, 0) ; \rho_{\max } ; N ; t_{\text {end }}\)
output: \(x_{i}(t) ; v_{i}(t) ; C(x, t) ; T(x, t)\)
for \(t=1\) to \(t_{\text {end }}\) do
    for \(s=1\) to \# grid points do
        Solve the advection-diffusion equation (14);
    Find grid points with high smoke density, i.e. \(C(x) \geq 0.05\);
    for \(i=1\) to \(N\) do
        1. Compute the visibility distance of person \(i\left(R_{v}\right)\);
        2. Compute the pedestrian density \((\rho(x, t))\) around person \(i\) in a ball radius \(R_{v}\);
        3. Compute the desired velocity of person \(i\left(v_{i}^{d}(t)\right)\)
        if \(i\) is a guider or see an exit then
            4. Set \(T(x)=0\), where \(x\) is a grid point on an exit;
            Set \(F(x)=U_{\text {max }}\left(1-\frac{\rho(x, t)}{\rho_{\text {max }}}\right)\) for walkable areas;
            Set \(F(x)=0.01\) for obstacle cells or cells with high smoke density;
            5. Solve the Eikonal equation (5);
            6. Set \(e_{i}^{d}(t)=-\frac{\nabla T\left(x_{i}\right)}{\mid \nabla T\left(x_{i}\right) ;} ;\)
            7. Compute the movement direction of person \(\operatorname{imd}_{i}(t)=\min _{d_{m} \in D} \arccos \left(d_{m} \cdot e_{i}^{d}(t) /\left|d_{m}\right|\right), m=1, \ldots, 8\);
        else
            Follow the rule of 'flow with the stream'
            and 'following the wall';
            Compute the movement direction of person \(i\);
        8. Compute \(f_{i}^{d}(t)\);
        for \(j=1\) to \(N\) do
            Compute \(f_{i j}^{\text {soc }}(t)\);
            Compute \(\left.f_{i j}^{\text {phy }}(t)\right)\);
    Solve the social force model (1) and (2) by the RK2 method;
    Update the positions and the velocities of all persons;
```

Algorithm 1: Main update algorithm.
demonstrated in Section 2. Then, the numerical methods, which are used to approximate the solutions of the social force model, the Eikonal equation, and the advection equation, are displayed in Section 3. Numerical experiments and results are shown in Section 4. In the end, discussions and conclusions are presented in Section 5.

## 2. Model

We study pedestrian evacuation in one and two exit domain with sources of smoke. We assume that the smoke is not harmful to pedestrians' health, but it affects the visibility range. The effect of guiders on evacuation is investigated in our model. Guider is a person who is familiar with the geometry of simulation domain. He knows where the exits are located. He can lead other pedestrians to the exit although his visibility is limited due to smoke. A microscopic social force model [6] is applied to simulate individuals' positions and velocities. It exploits the idea that pedestrians' movements rely on their own desire to reach a certain destination as well as other environment factors. To simplify the model, all pedestrians are assumed to have eight movement directions as in references [11] and [23] (see Figure 1). The desired direction of a guider or a person who can see the exit is defined to follow the solution of the Eikonal equation. It depends on smoke density and pedestrian's desired velocity.

The movement directions of individuals who are not guiders or not see any exit, are followed the psychological human behaviour termed 'flow with the stream' [23] and 'following the wall' [3]. For the dispersion of smoke, the linear advec-tion-diffusion equation is employed. The microscopic social force equations, together with the Eikonal equation, the advection equation, and human behaviour terms 'flow with the stream' and 'following the wall', are prescribed as the following:

$$
\begin{gather*}
\frac{\mathrm{d} x_{i}}{\mathrm{~d} t}=v_{i}(t)  \tag{1}\\
\frac{\mathrm{d} v_{i}}{\mathrm{~d} t}=f_{i}^{d}(t)+\sum_{j \neq i}\left(f_{i j}^{\mathrm{soc}}(t)+f_{i j}^{\mathrm{ph}}(t)\right), \tag{2}
\end{gather*}
$$

with location $x_{i} \in \mathbb{R}^{2}$ and velocity $v_{i} \in \mathbb{R}^{2}, i=1,2, \cdots, N . N$ is the total number of pedestrians.
$f_{i}^{d}(t)$ is the desire force of pedestrian $i$ at time $t$. It represents the own desire of a pedestrian to reach his destination with a certain desired speed $v^{d}$ in a given desired direction $e^{d}$. It is expressed by the following:

$$
\begin{equation*}
f_{i}^{d}(t)=\frac{v_{i}^{d}(t) e_{i}^{d}(t)-v_{i}(t)}{\tau_{i}} \tag{3}
\end{equation*}
$$



Figure 3: Modeling area for the numerical experiments in 2 dimensions.
where $v_{i}(t)$ is the actual velocity and $\tau_{i}$ is the relaxation time in which the pedestrian adapts his actual velocity to the intended velocity. $e_{i}^{d}(t)$ is the unit vector pointing to the desired direction. For a guider or a pedestrian who see the exit, the moving desired direction is assumed to follow the negative gradient of the Eikonal solution, i.e.

$$
\begin{equation*}
e_{i}^{d}(t)=-\frac{\nabla T\left(x_{i}\right)}{\left|\nabla T\left(x_{i}\right)\right|} \tag{4}
\end{equation*}
$$

where $T\left(x_{i}\right)$ is the travel cost of the pedestrians to reach his destination at point $x_{i}$. It is the solution of the Eikonal equation [10]:

$$
\begin{equation*}
|\nabla T(x)|=\frac{1}{F(x)}, \quad x \in \Omega \tag{5}
\end{equation*}
$$

where $\Omega$ is a simulation domain, and $T(x)$ is the arrival time of the front crossing the point $x . T(x)$ is set to 0 for

Table 1: Parameters for evacuation simulations.

| Parameter | Symbol | Value | Reference | Equation |
| :--- | :---: | :---: | :---: | :---: |
| Relaxation time | $\tau_{i}$ | 0.5 | $[11,13,14]$ | $(3)$ |
| Maximum speed of pedestrian | $U_{\max }$ | 1.65 | - | $(7)$ |
| Maximum pedestrian density | $\rho^{\max }$ | 10 | $[4,11-14]$ | $(7)$ |
| Human-human interaction strength | $A_{i}$ | 0.1 | $[11,13]$ | $(12)$ |
| Human-human range of repulsive interaction | $B_{i}$ | 0.21 | $[13,14]$ | $(12)$ |
| Contact distance | $r_{i j}$ | 0.5 | $[11,13,14]$ | $(12)$ |
| Anisotropic parameter | $\lambda_{i}$ | 0.61 | $[11,13,14]$ | $(12)$ |
| Body force coefficient | $k_{n}$ | 0.1 | $[11,13,14]$ | $(13)$ |
| Friction force coefficient | $k_{t}$ | 0.1 | $[11,13,14]$ | $(13)$ |
| Velocity field in $x$-direction | $w_{1}$ | $[-0.5,0.5]$ | $[11,13,14]$ | $(14)$ |
| Velocity field in $y$-direction | $w_{2}$ | $[-0.5,0.5]$ | $[11,13,14]$ | $(14)$ |
| Diffusion constant | $\kappa_{\mathrm{d}}$ | 0.05 | $[11,13,14]$ | $(14)$ |
| Space grid size in $x$ | $\Delta x$ | 0.2 | $[11]$ | $(14)$ |
| Space grid size in $y$ | $\Delta y$ | 0.2 | $[11]$ | $(14)$ |
| Time step size | $\Delta t$ | 0.02 | $(22)$ |  |

Table 2: Average number of evacuees of ten trial runs of 100 individuals with $0 \%$ and $3 \%$ guiders in simulations. Evacuation time period is set to 50 s . The simulation domain is a room with one and two exits as Figure 3. The smoke source is located in the middle of the room.

| Exit | Average number of evacuees (10 rounds) <br> No guider | 3\% guiders |
| :--- | :---: | :---: |
| 1 exit | 38.70 | 44.40 |
| 2 exits | 77.70 | 91.10 |

the destination areas. $F(x)>0$ is the moving speed of the front and relies on the position of $x$. We set it as follows:

$$
F(x)= \begin{cases}0.001, & x \in \Omega_{b}  \tag{6}\\ U(\rho(x)), & x \in \Omega^{\prime} \Omega_{b}\end{cases}
$$

where $\Omega_{b}$ represents the areas that are obstructed by obstacles [10] or areas with high smoke density. $U$ is the speeddensity function. It shows the relationship between the speed and the density of pedestrians. Many speed-density functions are available for use. The following functions are adopted in our simulations [18]:

$$
\begin{gather*}
U(\rho(x, t))=U_{\max }\left(1-\frac{\rho(x, t)}{\rho_{\max }}\right),  \tag{7}\\
\rho(x, t)=\frac{1}{\pi\left(R_{\mathrm{v}}\right)^{2}} \sum_{j,\left|x-x_{j}\right|<R_{\mathrm{v}}} 1 \tag{8}
\end{gather*}
$$

where $U_{\max }$ and $\rho_{\max }$ are the maximum speed and the density of pedestrians, respectively. $R_{\mathrm{v}}$ is the visibility distance
of a pedestrian in smoke area. $\rho(x, t)$ is the pedestrian density in a circle with radius $R_{\mathrm{v}}$. Experiments on human behavior in fire smoke show that the actual visibility distance captured by light reflecting objects can be estimated through the following equation [3, 24]:

$$
\begin{equation*}
R_{\mathrm{v}}=\frac{c V}{K_{m} M_{\mathrm{s}}} \tag{9}
\end{equation*}
$$

where $c$ represents the value that depends on whether the sign is light-emitting or light-reflecting. For the light emitting, its value is 8 . It is 3 for the light-reflecting. $V$ is the volume of the domain where the fire origin is.
$K_{m}=7.6 \mathrm{~m}^{2} / \mathrm{g}$ is applied for soot produced during flaming combustion of wood and plastics, whereas $K_{\mathrm{m}}=4.42$ $\mathrm{m}^{2} / \mathrm{g}$ is used for soot produced during pyrolysis of these materials. $M_{s}$ is the mass of smoke emission and can be calculated through the following equation:

$$
\begin{equation*}
M_{\mathrm{s}}=\epsilon M, \tag{10}
\end{equation*}
$$

where $M$ is the weight of the burning material, and $\epsilon$ is the smoke conversion factor [23]. $v_{i}^{d}(t)$ is the desired speed of pedestrian $i$ at time $t$. It is the speed that pedestrian $i$ adapts his actual velocity $v_{i}(t)$ to the desired speed. In our model, the desired speed of a pedestrian depends on the pedestrian density in the visibility distance. It is calculated through

$$
\begin{equation*}
v_{i}^{d}(t)=U_{\max }\left(1-\frac{\rho\left(x_{i}, t\right)}{\rho_{\max }}\right) \tag{11}
\end{equation*}
$$

where $f_{i j}^{\text {soc }}(t)$ is the repulsive social force. It results in a repulsive effect to avoid getting too close or to keep a


Figure 4: Average number of evacuees of ten trial runs of 100 individuals with $0 \%$ and $3 \%$ guiders in simulations. Evacuation time period is set to 50 s . The simulation domains is a room with one and two exits as Figure 3. The smoke source is placed in the middle of the room.

Table 3: Average number of evacuees of ten trial runs of 100 individuals with $0 \%$ and $3 \%$ guiders in simulations. Evacuation time periods are set to 20,30 , and 50 s . The simulation domain is a room with two exits as Figure 3(b).

| Time periods | Average number of evacuees (10 |  |
| :--- | :---: | :---: |
|  | No guider |  |
| rounds) | 3\% guiders |  |
| 20 s | 45.00 | 77.90 |
| 30 s | 53.30 | 88.50 |
| 50 s | 77.70 | 91.10 |

certain distance to another person $j$. It is demonstrated through an exponential decaying function as follows:
$f_{i j}^{\mathrm{soc}}(t)=A_{i} \exp \left(\frac{r_{i j}-d_{i j}}{B_{i}}\right) n_{i j}\left(\lambda_{i}+\left(1-\lambda_{i}\right) \frac{1+\cos \left(\varphi_{i j}\right)}{2}\right)$,
where $A_{i}$ and $B_{i}$ are the parameters that show the individual interaction strength and range. $d_{i j}=\left|x_{i}-x_{j}\right|$ is the distance between the centres of mass of the pedestrians $i$ and $j . r_{i j}=$ $r_{i}+r_{j}$ is the sum of the pedestrians' radii $r_{i}$ and $r_{j}$, and $n_{i j}$ $(t)=\left(n_{i j}^{(1)}, n_{i j}^{(2)}\right)=x_{i}(t)-x_{j}(t) / d_{i j}(t)$ is the normalized vector pointing in the direction from pedestrian $j$ to pedestrian
i. $\lambda_{i}$ is a value in the range $[0,1]$, and $\lambda_{i}<1$ reflects an anisotropy effect. It shows that the situation in front of individual $i$ has more impact on its behaviour than the situation behind. $\cos \left(\varphi_{i j}\right)=-n_{i j}(t) \cdot e_{i}(t)$, where $e_{i}(t)=v_{i}(t) /$ $\left|v_{i}(t)\right|$ is the direction of motion of pedestrian $i$, and $\varphi_{i j}(t$ ) denotes the angle between the direction of motion of pedestrian $i\left(e_{i}(t)\right)$ and the direction to pedestrian $j$.
$f_{i j}^{\mathrm{ph}}(t)$ refers the physical interaction force. It is applied to separate two persons when they have physical body contact:

$$
\begin{equation*}
f_{i j}^{\mathrm{ph}}(t)=k_{n} H\left(r_{i j}-d_{i j}\right) n_{i j}+k_{t} H\left(r_{i j}-d_{i j}\right) \Delta v_{j i}^{t} t_{i j}, \tag{13}
\end{equation*}
$$

where $k_{n} H\left(r_{i j}-d_{i j}\right) n_{i j}$ is a body force for body compression and $k_{t} H\left(r_{i j}-d_{i j}\right) \Delta v_{j i}^{t} t_{i j}$ is sliding friction force for relative tangential motion. $H$ is the Heaviside function. Its value is $r_{i j}-d_{i j}$ if $r_{i j} \geq d_{i j}$ (physical body contact), otherwise 0. $t_{i j}=\left(-n_{i j}^{(2)}, n_{i j}^{(1)}\right)$ is the unit tangential vector and orthogonal to $n_{i j}, \Delta v_{j i}^{t}=\left(v_{j}-v_{i}\right) \cdot t_{i j}$ is the tangential velocity difference, and $k_{n}$ and $k_{t}$ are the normal and tangential constants, respectively.

For the dispersion of smoke, the following advection diffusion equation [20] is applied:

$$
\begin{equation*}
\frac{\partial C}{\partial t}+w \cdot \nabla C=\kappa_{\mathrm{d}} \nabla^{2} C+S\left(c_{\mathrm{s}}, t\right) \in \Omega \times \mathbb{R}^{+} \tag{14}
\end{equation*}
$$



Figure 5: Average number of evacuees of ten trial runs of 100 individuals with $0 \%$ and $3 \%$ guiders in simulations. Evacuation time periods are set to 20,30 , and 50 s . The simulation domain is a room with two exits as Figure 3(b).
with the Dirichlet boundary conditions on $\partial \Omega . w=\left(w_{1}, w_{2}\right) \in$ $\mathbb{R}^{2}$ is the velocity field of smoke, and $\kappa_{d}>0$ is the diffusion constant. We suppose that the smoke source emits gas at a constant rate $Q_{c}[\mathrm{~g} / \mathrm{s}]$ from a single source point $c_{\mathrm{s}}=\left(x_{\mathrm{s}}, y_{\mathrm{s}}\right)$. Therefore, the source term is written as follows:

$$
\begin{equation*}
S\left(c_{\mathrm{s}}\right)=Q_{\mathrm{c}} \delta\left(x-x_{\mathrm{s}}\right) \delta\left(y-y_{\mathrm{s}}\right) \tag{15}
\end{equation*}
$$

where $\delta$ is the Dirac delta function given by

$$
\delta(x)=\left\{\begin{array}{ll}
1, & x=0  \tag{16}\\
0, & x \neq 0
\end{array} .\right.
$$

For an individual who is not a guider and does not see any exit, he determines his movement direction by the rule of 'flow with the stream' and 'following the wall' as in references [11] and [23]. It is operated as follows.

1. At time $t$, check whether there is a guider in his visibility. If it is true, he follows the guider. Otherwise, proceed to the next step. If there are more than one guiders in his visibility, he selects randomly one of them to follow.
2. Check whether he see any exit; if it is true, he follows the nearest wall with probability 0.5 to turn left or turn right.
3. Based on the state at time $t-1$, count the number of individuals in his visibility and divide them into groups
according their movement directions. There are eight possible movement directions as defined in Figure 1. Hence, the maximum number of groups is also eight. He follows the leading group, which is the group that contains most pedestrians moving in the same direction. If there are more than one leading group, one of them is chosen randomly to follow.

The procedures for the pedestrian to follow a guider, a wall, or a leading group are as follows.

1. The target can be a guider, a wall, or a leading group.
2. A probability of $\alpha$ is set for him to give up following the target. He moves along a direction selected randomly. He follows the target as a result of (1- $\alpha$ ) probability.
3. If he decides to follow the target, a probability of $\beta$ is set for him to move toward the target. A probability of $(1-\beta)$ is set for him to move along movement direction of the target. From a qualitative study, as stated in [23], $\alpha$ is set as 0.2 and $\beta$ as 0.3 .

In the case that a pedestrian is near a wall and his movement direction would lead him to move into the wall in the next time step, he changes his direction randomly to avoid encountering the wall, as shown in Figure 2. Red arrows refer to movement directions that lead into the wall, whereas blue arrows refer to possible movement directions leading away from this boarder.


Figure 6: Continued.


Figure 6: Movements of 100 pedestrians with $3 \%$ guiders during smoke spreading where there are one and two sources of smoke. For a source of smoke, it is located in the middle of the room. Two sources of smoke are placed in the middle of the room and in front of exit 1. The simulation domain is a room with two exits as Figure 3(b). The entire time period of a simulation is 50 s. (a) 1 smoke source, $t=$ 1 s . (b) 2 smoke sources, $t=1 \mathrm{~s}$. (c) 1 smoke source, $t=5 \mathrm{~s}$. (d) 2 smoke sources, $t=5 \mathrm{~s}$.

As mentioned before, each individual is assumed to have eight movement directions numbered from $d_{1}$ to $d_{8}$ at each time step (see Figure 1). To find his movement direction, we write eight movement directions in Figure 1 as vectors: $d_{1}=(0,1), d_{2}=(1,1), d_{3}=(1,0), d_{4}=(1,-1)$,
$d_{5}=(0,-1), d_{6}=(-1,-1), d_{7}=(-1,0)$, and $d_{8}=(-1,1) . D$ $=\left\{d_{1}, \ldots, d_{8}\right\}$. The movement direction of a pedestrian is the direction $d_{m} \in D$, which yields the minimum regarding the angle between the movement direction $\left(d_{m} \in D\right)$ and his desired direction $e_{i}^{d}(t)$. The angle between the two

Table 4: Average number of evacuees of ten trial runs of 100 individuals with $3 \%$ guiders in simulations. There are one and two smoke sources. For a source of smoke, it is located in the middle of the room. Two sources of smoke are placed in the middle of the room and in front of exit 1 . The simulation domain is a room with two exits as Figure 3(b). The entire time period of a simulation is 50 s .

| Smoke sources | Average number of evacuees (10 rounds) |
| :--- | :---: |
| 1 smoke source | 90.70 |
| 2 smoke sources | 64.00 |



Figure 7: Average number of evacuees of ten trial runs of 100 individuals with $3 \%$ guiders in simulations. There are one and two smoke sources. For a source of smoke, it is located in the middle of the room. Two sources of smoke are placed in the middle of the room and in front of exit 1 . The simulation domain is a room with two exits as Figure 3(b). The entire time period of a simulation is 50 s .
vectors is attained by the use of the dot product. Suppose that, at time $t$, pedestrian $i$ is moving in direction $e_{i}^{d}(t)$, the movement direction of pedestrian $i$ can be written as:

$$
\begin{equation*}
m d_{i}(t)=\min _{d_{m} \in D} \arccos \left(\frac{d_{m} \cdot e_{i}^{d}(t)}{\left|d_{m}\right|}\right), \quad m=1, \ldots, 8 \tag{17}
\end{equation*}
$$

If there is more than one movement direction that gives the minimum angle, one of them is chosen randomly. Algorithm 1 is used to update an individuals' position and velocity in each time step.

## 3. Numerical Methods

In this section, we present the numerical methods that are adopted to approximate the solution of the social force models (1) and (2), the Eikonal equation (5), and the advec-tion-diffusion equation (14).
3.1. Solving the Social Force Model. We apply the two-stage second-order Runge-Kutta method to approximate the solution of the social force model. To apply this method, first, we write equations (1) and (2) as follows:

$$
\begin{equation*}
u^{\prime}(t)=f(t, u(t)), \quad u\left(t_{0}\right)=u_{0} \tag{18}
\end{equation*}
$$

where $u\left(t_{0}\right)=u_{0}$ is the initial condition. We generate the equidistant grid $\Omega_{t}$ with respect to time $t$ as

$$
\begin{equation*}
\Omega_{t}=\left\{t_{k}, t_{k}=k \Delta t, k=0,1,2, \ldots, M \text { and } \Delta t=\frac{1}{M}\right\} \tag{19}
\end{equation*}
$$

The two-stage second-order Runge-Kutta method is as follows:

$$
\begin{equation*}
k_{1}=f\left(t_{k}, u_{k}\right) \tag{20}
\end{equation*}
$$



FIgure 8: Contour plots of the solution of the Eikonal equation in simulations of 100 pedestrians during propagation of smoke at $t=5 \mathrm{~s}$. For a source of smoke, it is located in the middle of the room. Two sources of smoke are placed in the middle of the room and in front of exit 1. The simulation domain is a room with two exits as Figure 3(b). (a) 1 smoke source, $t=5 \mathrm{~s}$. (b) 2 smoke sources, $t=5 \mathrm{~s}$.

$$
\begin{align*}
& k_{2}=f\left(t_{k}+\frac{2}{3} \Delta t, y_{k}+\frac{2}{3} \Delta t k_{1}\right),  \tag{21}\\
& u_{k+1}=u_{k}+\Delta t\left(\frac{1}{4} k_{1}+\frac{3}{4} k_{2}\right), \tag{22}
\end{align*}
$$

where $u_{k}=u\left(t_{k}\right), u_{k+1}=u\left(t_{k+1}\right)$, and $t_{k+1}=t_{k}+\Delta t$. The solution of $u$ in the next time step is obtained from equation (22).
3.2. Solving the Eikonal Equation. To solve the Eikonal equation, there are a quite number of numerical methods existing to approximate the solution of the Eikonal equation, for example, the fast marching method [19], the fast marching level set method [19], the fast sweeping method [5], and the fast iterative method [8]. In our experiments, the fast marching method is applied in all simulations. Details of this method can be reviewed in reference [19].

Table 5: Average number of outside pedestrians of ten trial runs. 50, 100 , and 200 individuals with $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders are considered in simulations. The simulation domain is a room with two exits as Figure 3(b). The entire time period of a simulation is 50 s .

| Number of <br> guiders | Average number of evacuees in 50 s (10 rounds) <br> 50 pedestrians |  |  |
| :--- | :---: | :---: | :---: |
|  | 100 pedestrians | 200 pedestrians |  |
| No guider | 41.10 | 77.70 | 119.20 |
| $1 \%$ | 41.80 | 90.30 | 149.00 |
| $3 \%$ | 39.70 | 91.10 | 191.50 |
| $5 \%$ | 41.30 | 88.70 | 193.70 |

3.3. Solving the Advection-Diffusion Equation. The operator splitting method is applied to approximate the solution of the advection-diffusion equation (14). This method is performed on the two-dimensional advection-diffusion equation in the $x$-direction and the $y$-direction separately over two time steps. For details, we refer to references [11], [12], and [12]. The convergence of this method is shown in reference [11].

## 4. Numerical Experiments and Results

We perform numerical experiments of pedestrian evacuation during the smoke dispersion in the case with and without guiders in a room of size $16 \mathrm{~m} \times 20 \mathrm{~m}$. We consider the simulation domain with one or two exits. The width of exits are set to 2 m , which is enough to allow pedestrians to escape simultaneously. The exit is located on the right side of the room for one exit room (see Figure 3(a)). For two exit room, the exits are placed at the bottom and on the right side of the room. They are labeled as Exit 1 and Exit 2, respectively (see Figure 3(b)). A crowd of size 50, 100, and 200 individuals and $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders are considered in our study. The process starts with randomly distributed pedestrians throughout the room at initial time. Each individual are assumed to has eight movement directions numbered from $d_{1}$ to $d_{8}$ at each time step as in reference [11] (see Figure 1). Initial velocity of an individual is selected randomly from the eight movement directions.

For the studied examples, we assume that there is 1 kg of polystyrene burned in the flame inside the experiment room. The smoke conversion factor of polystyrene is assigned to 0.15 as in reference [23]. By equation (10), we attain the mass of smoke emission in the room as follows:

$$
\begin{equation*}
M_{\mathrm{s}}=\epsilon M=0.15 \times 1000=150 \mathrm{~g} . \tag{23}
\end{equation*}
$$

The pedestrian's visibility distance during the smoke dispersion is calculated through equation (9):

$$
\begin{equation*}
R_{\mathrm{v}}=\frac{K_{\mathrm{s}} V}{K_{m} M_{\mathrm{s}}}=\frac{3 \times 20 \times 16 \times 4}{7.6 \times 150}=3.37 \mathrm{~m} \tag{24}
\end{equation*}
$$

In reality, the visibility range of individual is not constant. It changes all the time dependent on the burning rate of material. Therefore, we assume that the visibility range of
an individual is decreased linearly from 3.37 to 2 m in a given time of simulation for a source of smoke.

For the smoke dispersion, the smoke density at the source point is relatively high at the initial time and emits a constant smoke density subsequently, i.e.

$$
Q_{\mathrm{c}}=\left\{\begin{array}{ll}
10 \mathrm{~g} / \mathrm{s}, & t=0  \tag{25}\\
0.1 \mathrm{~g} / \mathrm{s}, & t>0
\end{array} .\right.
$$

At each time step, the velocity field $\left(w_{1}, w_{2}\right)$ of the convection-diffusion in equation (14) is assumed to vary on the interval $[-0.5,0.5]$. Ten trial runs are executed for each example, and their average is applied. The computations are conducted on a HP Intel(R) Core(TM) i7-7700CPU, 3.6 GHz . We implement all programs in MATLAB R2023a. Parameters that are used in all simulations are displayed in Table 1.
4.1. Experiment 1. In the first experiment, we consider evacuation process of 100 pedestrians with $0 \%$ and $3 \%$ guiders in simulations. The simulation domain is a room with one and two exits. A room with one exit is set as Figure 3(a), whereas a room with two exits is as Figure 3(b). The entire evacuation time period of a simulation is set to 50 s . The results of the first experiment are displayed in Table 2. The pedestrian evacuation process in a room with two exits provides higher average number of evacuees than the process in a room with one exit. This coincides well with real situations that individuals have more options to evacuate out of the room. To accelerate evacuation process, it is better to have a room with two exits than a room with one exit. Our results are consistent with results of Aik and Choon [1]. Then, we consider the situations with $0 \%$ and $3 \%$ guiders in experiments. Both one and two exit rooms give similar results that the average number of evacuees is higher in case with $3 \%$ guiders than without guider in simulations. In the presence of guiders, we attain that the average number of evacuees is rather high in the domain with two exits compared with the domain with one exit. The plot of results in first experiment is shown in Figure 4.
4.2. Experiment 2. We investigate numerical experiments of 100 individuals with $0 \%$ and $3 \%$ guiders in a room with two exits (see Figure 3(b)). Different evacuation time periods are regarded. They are set to $20 \mathrm{~s}, 30 \mathrm{~s}$, and 50 s . The results of these experiments are presented in Table 3. Apparently, the average number of evacuees increases when simulation time period increases both with $3 \%$ guiders and without guider in simulations. More evacuees can leave the room with more time period. In all setting evacuation time periods, the average number of evacuees in case with $3 \%$ guiders in simulations is higher than in case without guiders. When simulation time period is lower, guiders are still important on evacuation process. The average number of evacuees is increased in presence of guiders. The comparison plot of the average number of evacuees with different simulation time periods is demonstrated in Figure 5.


Figure 9: Continued.

(c)

Figure 9: Comparisons of the average number of outside pedestrians in period of time 50 s of $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders situations. (a) 50 , (b) 100 and (c) 200 individuals are considered in simulations. The simulation domain is a room with two exits as Figure 3(b).

Table 6: CPU time of ten trial runs of 50,100 , and 200 individuals with $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders in simulations. The simulation domain is a room with two exits as Figure 3(b). The entire time period of a simulation is 50 s .

| Number of <br> guiders | CPU time (10 rounds) |  |  |
| :--- | :---: | :---: | :---: |
|  | 50 pedestrians | 100 pedestrians | 200 pedestrians |
| No guider | 35.1688 h | 90.7668 h | 227.8468 h |
| $1 \%$ | 39.605 h | 101.9206 h | 245.4958 h |
| $3 \%$ | 35.7173 h | 106.5785 h | 256.3765 h |
| $5 \%$ | 34.3156 h | 83.4702 h | 235.0324 h |

4.3. Experiment 3. We perform evacuation process of 100 individuals with $3 \%$ guiders in a room with two exits. One and two sources of smoke are considered in this experiment. For a source of smoke, it is placed in the middle of the room. For two smoke sources, they are located in the middle of the room and in front of exit 1 (see Figure 6). Time period for a simulation is set to 50 s . In this experiment, the visibility range of a pedestrian is assumed to be constant both with one source and two sources of smoke. It is calculated through equation (9). We obtain the visibility range 3.37 m for a smoke source, and it is 1.68 m for two sources of smoke. Table 4 displays results of this experiment. A high average number of evacuees is received in the domain with a smoke source compared with when there are two smoke sourced presented. Two sources of smoke can produce more smoke
density that it causes to reduce visibility of individuals. Individuals have less chance to see an exit; see a guider or move with others in a small visibility range. The comparison plot of the average number of evacuees where there are one and two smoke sources is presented in Figure 7. Movements of individuals in one and two smoke sources' situations at time 1 and 5 s are demonstrated in Figure 6. Small groups are formed and observed in the case with two smoke sources.

Figure 8 shows contour plots of the solution of the Eikonal equation in simulations of 100 pedestrians during propagation of smoke at $t=5 \mathrm{~s}$. Figure 8(a) shows a source of smoke. Figure 8(b) shows two sources of smoke. From the plots, we see that the traveling time to reach a destination is absolutely high in regions where the smoke sources or the wall grids are located. Hence, guiders or individuals who see exit or walls will move away from these areas. This is by reason of the moving speed $F(x)$ in equation (6) that it is assigned to small value for areas with high smoke density or wall grid regions.
4.4. Experiment 4. In this experiment, a crowd of size 50 , 100 , and 200 individuals with $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders is considered. The simulation domain is a room with two exits (see Figure 3(b)). The smoke source is located in the middle of the room. The entire evacuation time period is set to 50 s . Table 5 displays the average number of outside pedestrians in case of $0 \%, 1 \%, 3 \%$, and $5 \%$ guiders in experiments. It shows that the average number of outside pedestrians in case of with and without guiders is not very


Figure 10: Computation time of 50,100 , and 200 pedestrians in case of $5 \%$ guiders in simulations. The entire time period of a simulation is 50 s . The simulation room consists of two exits as Figure 3(b).


Figure 11: Traces of five chosen pedestrians out of 200 pedestrians in case of $5 \%$ guiders in simulation. Black, blue, and cyan colors refer to footprints of guiders. Red and green colors are footprints of pedestrians that are not guiders. The entire time period of a simulation is set to 50 s .
different when there are small number of individuals in simulations, i.e., 50 pedestrians in experiment. When there are large number of individuals in experiment, the difference in the average number of outside pedestrians in case of with and without guider is prominent. The average number of
outside pedestrians in the case with guiders is relatively higher than that in the case without guider, i.e., 100 and 200 individuals in simulation. Therefore, the effect of guiders is essential to the evacuation, especially when there are large number of individuals in simulations.

(b)

Figure 12: Continued.


Figure 12: Continued.


Figure 12: Continued.


Figure 12: Movements of 200 pedestrians during smoke spreading in the case that there are no guider and $5 \%$ guiders in simulation. The smoke source is in the middle of the room. The entire time period of a simulation is 50 s . The simulation room consists of two exits as Figure 3(b). (a) no guider, $t=0 \mathrm{~s}$. (b) $5 \%$ guiders, $t=0 \mathrm{~s}$. (c) no guider, $t=5 \mathrm{~s}$. (d) $5 \%$ guiders, $t=5 \mathrm{~s}$. (e) no guider, $t=20 \mathrm{~s}$. (f) $5 \%$ guiders, $t=20 \mathrm{~s}$. (g) no guider, $t=40 \mathrm{~s}$. (h) $5 \%$ guiders, $t=40 \mathrm{~s}$.

The comparison plots of the average number of outside pedestrians of crowd of size 50,100 , and 200 with $0 \%, 1 \%$, $3 \%$, and $5 \%$ guiders are shown in Figure 9. From Figure 9(a), the average number of outside pedestrians in case of with and without guiders is approximately the same in the beginning of simulation, i.e., at time $0-5 \mathrm{~s}$. After that,
the average number of outside pedestrians in case of $5 \%$ guiders provides highest results. At the end of given simulation time, we see that the average number of outside pedestrians is roughly the same in all cases.

In the events of 100 pedestrians in simulations (Figure 9(b)), the results of the average number of outside
pedestrians are similar in case with and without guiders in the beginning of simulation, i.e., at time $0-3 \mathrm{~s}$. After that ,the average number of outside pedestrians in case of with guiders is obviously higher than that in the case without guider. At time from 3 to 25 s , the average number of outside pedestrians in case of $3 \%$ and $5 \%$ guiders is higher than that in case of $1 \%$ guiders in experiment. After the time from 25 s until end of given period time, the average number of outside pedestrians is roughly the same in case of $1 \%$ and $5 \%$ guiders. It is highest when there are $3 \%$ guiders in experiment. This result is agreeable to reference [22]. When the number of leaders is larger than a certain value, the effect of increasing number of guiders on evacuation time is not evident. Increasing the number of guiders does not always increase the number of evacuees.

When there are 200 individuals in experiments (Figure 9(c)), the significance of having guiders in simulations is evidently seen. The difference in the average number of outside pedestrians in case of with and without guiders is clearly observed from the plot. After the time from 5 s until end of simulation period, the average number of outside pedestrians is increased when the number of guiders is increased. It is highest when there are $5 \%$ guiders in testing.

From Figure 9, we can conclude that all case studies provide similar results of the average number of outside pedestrians in the beginning of simulations. This is because the visibility distance of an individual is large in the beginning. Pedestrians who near or see the exit can evacuate out of the room without difficulty. Therefore, guiders have no effect on evacuation in the beginning of simulation. As time is increased, the visibility range of an individual is reduced. Pedestrians who cannot see any exit cannot evacuate out of the room easily. If they are near a guider, they will follow him. Guider can lead them to the exit. This can increase the number of outside pedestrians. When there are large number of individuals in experiments, the effect of having guiders on evacuation is clearly dominant.

Table 6 shows computation time of ten trial runs of 50, 100 , and 200 pedestrians in simulations. The plot of computation times of 50,100 , and 200 pedestrians in case of $5 \%$ guiders in simulations is displayed in Figure 10. It is seen that as the number of pedestrians is increased, the computation time is increased exponentially.

Figure 11 shows the footprints of five chosen pedestrians out of 200 pedestrians in case of $5 \%$ guiders in simulation. Black, blue, and cyan colors are footprints of guiders, whereas red and green colors are footprints of individuals that are not guider. The initial positions of black, blue, cyan, red, and green color individuals are at (13.5292, 8.4356), (12.2952, 4.1928), ( $0.8082,14.9861$ ), ( $18.2861,1.5468$ ), and (15.9470, 3.9792), respectively. It can be seen that the footprints of guiders point directly to the exit since they know well the geometry of the room and know where the exits are located. Hence, they can move directly to the exit. On account of the limited visibility, the red and green pedestrians cannot find the exit directly. They follow others by the rule of 'flow with the stream' and 'following the wall'. The traces of them are overlapped, and they fail to evacuate out of the room in a given period of time.

Movements of 200 pedestrians during smoke spreading at time $0,5,20$, and 40 s in the case that there is no guider in the simulation are demonstrated in Figures 12(a), 12(c), 12(e), and 12(g). At initial time, pedestrians are randomly distributed through out the room. The initial movement direction of an individual is chosen randomly from the eight movement directions. At time 5 s , several groups of individuals are observed in this stage. Individuals who cannot see the exit move in the direction determined by the rule of 'flow with the stream'. The movement directions of pedestrians in each group point about in the same direction. When the crowds are near or see walls, they would follow the wall. The human behaviour term 'the wall following' is also observed in our model (see Figure 12(e)). The crowds move by the rule of 'flow with the stream' until they see an exit and move out. Because of the limited visibility, one group of individuals cannot find the exit directly and fails to move out of the room in a given period of time.

Figures 12(b), 12(d), 12(f), and 12(h) display movements of 200 pedestrians during smoke dispersion at time $0,5,20$, and 40 s in the case that there are $5 \%$ guiders in the experiment. Guiders are marked with red circles and placed randomly in the room at initial time. Guiders point their movement directions to the exit since they are familiar with the simulation domain and know well where the exits are located. Their movement directions are determined through the solution of the Eikonal equation, which depends on pedestrian and smoke density. Pedestrians who are near see the guider point their movement directions to the guider or along the movement directions of the guider (see Figure 12(d)). Guiders lead other individuals around them to the exit. At time 20 s, there are no guiders in the room, and one small group cannot find the exit. They move together by the rule of 'the flow with the stream' and 'following the wall' (see Figures 12(f) and 12(h)). They cannot evacuate out of the room in the given period of simulation time.

## 5. Discussion and Conclusions

In this research, we consider individuals' movements during smoke dispersion in case with and without guiders. The human behaviour terms 'the flow with the stream' and 'following the wall' are regarded in our model. Our model is based on the social force model, which is applied for pedestrians' motions. It is coupled with the Eikonal equation and the advection-diffusion equation. The Eikonal equation is used to guide the movement direction of a guider or a pedestrian who can see the exit. The advection-diffusion equation is employed for the propagation of smoke. In our experiments, it shows that guiders are important for evacuation, especially when there are large number of pedestrians in simulation. They can lead others around them to exit. Therefore, it increases the number of evacuees. This result is consistent with that in references [17], [22], and [23] in case that there are large number of individuals in simulation. The average number of outside pedestrians in case with guiders is higher than that in case without guider. For a small number of pedestrians, the impact of guiders on
evacuation time is not obvious in our model. The effect of increasing the number of guiders leading to an increase in the number of evacuee is obtained in the event that there are 200 pedestrians in experiment. For 100 individuals in simulation, this effect is not clear. This gives similar result as reported in reference [22]. Considering the simulation domain with one and two exits, the number of evacuees in a room with two exits is absolutely higher than the number of evacuees in a room with one exit. This result is agreeable with the work by Aik and Choon [1]. Regarding safety, it is preferable to build a room with two exits than a room with one exit. This experiment confirm that guiders are important on evacuation both in the domain with one and two exits. On the study of different simulation time periods, the average number of evacuees increases in the presence of guiders. With an additional smoke source, the visibility range of the individual is reduced. It leads to a decrease in the number of evacuees. Human behaviours terms, the clogging [6, 7], 'the flow with the stream' [23] and 'following the wall' [3] effects are also observed in our model. For further study, we can consider the effect of the initial locations of guiders on evacuation process.

## Data Availability

Previously, reported data were used to support this study and are cited at relevant places within the text as references [1], [6], [7], [17], [22], and [23].

## Conflicts of Interest

The author(s) declare that they have no conflicts of interest.

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