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Research Article

Four Equivalent Relations between MCP and CP and Its Implication in Quantum and Information Theory

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(Norraw) modal propositional logic (MCP) tries to be a strict model for quantum and information theory, but it has serious difficulties in syntax, semantics, and metaphysics. According to the guiding definition of minimal hidden variables, this study presents four equivalences between MCP and classical propositional logic (CP). It concludes that MCP is CP containing minimal syntactic hidden variables, modal axiom represents the classification of CP formulas, and possible world is CP formula as assignment background, which establishes a simpler and unified basis for quantum information theory. The black modal of classical propositional logic (BCP), as an assisted discovery method, reveals that modal nature is actually to express the interdependence between things and their environment in a mathematical way, which can express the holism, dialectics, and uncertainty of metaphysics, and use mark hiding (e.g., $f(x)$ is a function of x, $f()$ is the functional expression, functional-e for short, and "□" is, in fact, a cluster of functional-es in which superscript and subscript are hidden) that does not affect the effectiveness of reasoning to express a more simplified, efficient, and consistent artificial intelligence.

1. Introduction

Using modal logic to study information structure and communication and the basis of quantum and information is a long-term topic (modal logic is widely used in quantum and information science and its philosophical research, only listed in references [2–6]). The study of modal logic and philosophy has a long history. Modern research on modal pure syntax has achieved great success, although it has made slow progress by relying on natural reasoning when the semantics is not completely clear. The most powerful achievement in modal semantics is relational semantics, and the most influential is the refined Kripke possible world semantics (standard semantics).

Modal logic mainly includes narrow modal logic, deontic logic, cognitive logic, belief logic, temporal logic, and dynamic logic and has been successfully applied to many fields, such as mathematics, social sciences, computer science, and quantum physics. The application of modal logic in computer science includes programming language, knowledge representation and multi-agent system, model detection, theorem machine proof, and non-monotonic logic. There is a great motivation to combine quantum mechanics and computer science and develop a quantum computer. Quantum logic is the theoretical basis of both sides. Van Fraassen used modal quantum logic with the semantic analysis in 1991. This interpretation opened up a new direction for the development of the philosophy of quantum information. After the development and improvement of Kochen, Diecks, Vermmas, Healey, Clifton, Dickson, Bub, Bacciagaluppi, and others, this idea has formed a very influential and most promising interpretation theory of quantum information. Because of this, the perfect combination of modal logic with computer science and quantum theory, as well as the introduction of possible world theory, has led to new developments in many disciplines in the field of computer science [2, 3].

What kind of mathematical elements is "world" is, however, still unknown, and the exact relationship between modal logic and classical logic is still being sought. From the perspective of metaphysics, modal logic (based on MCP) forms an expansion system with standard notation by adding new primitive symbols (non-classical many-valued (MV) logic, on the other hand, forms a variation system with CP semantics by adding new semantic symbols, based on Łukasiewicz's L_3 [7]). It is contrary to the economic thinking intuition of "reduce the primitive symbols and increase unity," and the same goes for the endogenous route of dialectics (although modal logic has been widely used, it is rarely used to solve the problems of holism, dialectics, and even uncertainty of philosophical basis).

STRF, proposed by the author before, points out that the logic of nature and language should evolve in the most economical way, and MCP (or MV) can also be generated from CP by meeting the "three-in-one" requirements of equivalence, simplification, and reinforcement. This study shows four equivalences between CP and MCP according to STRF.

(1) The truth-valued function system CP is strictly and equivalently transformed into a non-truth-valued function system CPH using *CPH* notation, which ultimately simplifies the standard notation equivalently. At the same time, the "mark hidden" feature of non-truth-valued functional expression shows that most of the "nonequivalent substitutions" is actually ignorance of strictly hidden variables of logic.

(2) The "complete concealability of superscript and subscript" of non-truth-valued functional connectives shows that the cluster "■" (in BCP) of non-truth-valued functional-es as an auxiliary symbol in CPH is syntactically equivalent to the modal "necessary" operator "□" in MCP. BCP can represent the same nature of non-truth-valued function of MCP.

(3) In the definition of relational semantics of any formula $\blacksquare A$ to A in BCP: the unitary entity semantic assignment of the definition item is strictly equivalent to the binary background one; the defined item is the same as Leibniz's naive definition. Therefore, cluster semantics express can not only the different modal "necessary" same as the standard semantics but also the classic.

(4) As long as we think that the world w and w' of standard semantics are A and A' of cluster semantics, their definition items are the same. Therefore, "w" in the defined item of standard semantics can be regarded as redundancy. The standard semantics after de-redundancy is cluster semantics, which leads to some important results: it does not substantially affect the definition of superposing, mixed, or nested modal operators; K axiom is not valid for all models, but it does not affect the relevant syntactic research results; even in the original standard semantics, it is a mistake to make a natural understanding of the necessary rule. The latter two results can further clarify why some first-order formulas are not definable as standard semantic modal or vice versa.

Twelve years ago, after having been following the school of philosophy of quantum mechanics for nearly 20 years, the author began to climb on the invisible front by questioning the nondistribution of Von Neumann's quantum logic and gradually realized that non-classical logic is a basic (foundation 3) to uniformly solve the more basic problems of "holism, dialectics, and uncertainty" (foundation 2), which are double entangled in almost all basic sciences or humanities (foundation 1). However, as mentioned above, the foundation of non-classical logic itself requires a basic mathematical and metaphysical reflection (foundation 4).

In STRF, the author proposed that L_3 system is actually a CP system with "uncertain arrangement of true and false combinations." The reason is so-called that L_3 is incomplete or there is a "third truth" (referred to as 1/2 truth) among them, resulting in "exclusion law" or "(no) contradiction law," which is not a law, because the same true and false combination (corresponding to the truth degree) is not considered as different true and false permutations (corresponding to the truth distribution). This idea has been repeatedly cited by the Łukasiewicz School (see the author's 2014 paper reference [7] and cited reference [8]).

In this study, the standard notation is extremely simplified and CP is equivalently transformed into a non-truth-valued function system CPH, in which the cluster, "■," is group of non-truth-valued functional-es, that is, the modal necessity operator. According to STRF theory, the "three in one" of BCP formed from CP system not only reveals the nature of non-truth-valued function hidden in CP system but also gives a new concise paradigm of modal syntax and semantics.

2. Equivalence between CP and CPH

In recent years, some mathematical problems encountered in the classical understanding in the field of many-valued logic and quantum physics (another possible world, the interpretation group of quantum many worlds, is also the author's first to independently point out the mathematical error of nonconservation of probability in part 3 of reference [9]) have been further solved by the author. From this, the methodological guiding definition of "strict (minimal) hidden variable" existing on the basis of both quantum and non-classical logics is summarized.

Definition 2.0: for any two equivalent forms A and B, if there is a variable x in A and not in B, then x is a hidden variable of B relative to A.

In other words, hidden variables should first be considered in the meaning of the relational semantics of the two equivalent forms, rather than adding a term or variable to the standard form to form super quantum mechanics, as Einstein–Popper–Bohm did. Similarly, when encountering non-classic problems that are difficult to be solved by classical logic, the first consideration is not to add primitive syntactic or semantic symbols outside the standard notation or classical semantics to form an expansion or a variation system of CP, but to consider whether the non-classic problems can be solved by the equivalent transformation of standard notation or classical semantics.

For example, the so-called third truth value "1/2" (without independent ontological status still) in L_3 is a logical constant at the truth degree level, but it is a logical

Table 1: Notations in CP and MCP.

Primitive symbols	Standard notation	MCP notation
[1] Variable	$p_0, p_1, p_2, \dots p_n, \dots, n \in N$	$p_0, p_1, p_2, \dots p_n, \dots, n \in N$
[2] Connectives	$\neg, \wedge;$	$\neg, \wedge; \square$
[3] Structure	(,)	(,)

variable at the truth distribution level; that is, the same 1-0 combination has at least two permutations of "1/0" and "0/1" (they show classic negations each other). Considering this semantically strictly hidden variable, L_3 is still classic and, of course, complete. This study uses such a (syntactic) "hidden variable" to investigate whether MCP can be transformed equivalently from CP system.

2.1. Standard Notation and Its Main Equivalent Transformation Methods. There are other attempts to simplify the CP (Piano–Russell) standard notation: "polish notation" simplifies the structural parentheses with the preposition of connectives; "parenthesis notation" simplifies the connectives with structural parentheses; and even "one variable + comma superscript" is used to simplify the variable of natural number set of the standard notation. However, they only simplify the standard notation equivalently without transforming it into "non-classical" (the "post method" is widely used in computer science to eliminate parentheses. The last step of simplification of the "parenthesis method" created by Qing-Yu Zhang in the 1990s and basically established by Du in 2019 [10] is also completed by the author ([11]; i.e., replacing $\longrightarrow$ to $\wedge$ for less basic information). BNF notation is only an abbreviation of standard notation, not a simplified equivalent transformation). Only the author's STRF theory can meet the "three-in-one" requirements of (extreme) simplification, equivalence (transformation), and increasing (non-classical interpretation and expression) force at the same time. Its core is the *CPH* notation equivalent to the standard notation.

2.2. Non-Truth-Valued Functionality. MCP represents these two non-classics at the same time (Table 1): syntactically, it is a non-truth-valued function system; and semantically, it is a relational semantics that can represent (different) necessaries. CP is a truth-valued function system in syntax and unitary semantics representing entity proposition. How can MCP and CP be equivalent? At first, it looks fantastic. Let us start with the non-truth-valued functionality.

2.2.1. Rough Definition of Non-Truth-Valued Function. Taking mathematics in primary schools (I would like to thank Mr. Kripke, who was an American high school student more than 60 years ago, for his pioneering possible world relation semantics, for giving inspiration to this article, and for paying tribute to him) in China as an example: $z = f(x)$ is a function of x, iff, whenever x (in its domain) takes a certain value, z has always and only has one value corresponding to it and then z is a function of x. Otherwise, z is not a function of x and then z can only be a nonfunction of x.

Special attention should be paid to the foreshadowing, such as the two "f" in both $f(x)$ and $f(y)$: if "f" is a functional-e, the two "f" are exactly the same and will not change due to the connection of different variables. However, if "f" is nonfunctional-e, it will be shown later that it is interdependent with its associated variables. That is, at this time, two seemingly identical "f" hide different superscripts and subscripts actually.

Similarly, in CP, $p \wedge q$ is a (two-variable) function formed by variables p and q. Of course, $p \wedge q$ is also two truth-valued functions such as p or q, so the truth-valued function of p and q can be defined.

$p \wedge q = f_{12}(p, q)$, where f_{12} is a truth-valued functional-e. For subscript 12, see 2.3 Table 2.

So, is $p \wedge q$ an (univariable) function of p?

Obviously not! Because when p has a definite truth value "1," $1 \wedge q$ does not always only a truth value corresponding to"1," and $p \wedge q$ does not conform to the definition of the (truth-valued) function of p. So, $p \wedge q$ can only be non-truth-valued function of p.

Further, since the non-truth-valued function of $f_{12}(p, q)$ relative to p is caused by p and the relationship between p and $f_{12}(p, q)$, we can roughly write the non-truth-valued function as follows: $H_{12}p = f_{12}(p, q) = p \wedge q$.

H_{12} represents non-truth-valued function relationship, a non-truth-valued functional-e.

Some scholars may feel confused here.

Since the original definition of truth-valued function has been simple, clear, rigorous, and reliable, why the "specious" definition of non-truth-valued function is introduced? Moreover, consider the definition of assignment; that is, the assignment of truth value is given to each variable in any formula. Then, when p takes a certain truth value, p and q in the corresponding $p \wedge q$ also take a certain truth value each, and $p \wedge q$ also takes a certain truth value. The real non-truth-valued function is that when each variable in the formula is assigned a truth value, the truth value of the formula is still uncertain.

The author's answer is as follows:

(1) As long as $p \wedge q$ does not conform to the definition of a (truth-valued) function of p, it can only be a non-truth-valued function of p, which is determined by the nature of logical dichotomy and has nothing to do with whether to consider the definition of assignment.

(2) From the perspective of Definition 2.0 of narrow hidden variables, how do you know that the hidden element "world" in standard semantics cannot be some (semantically inaccurate) sub-formulas composed of variables and classical connectives?

(3) If you do not know that $p \wedge q$ is $H_{12}p$, then assigning a value to $H_{12}p$ is assigning a value to variable p on $H_{12}p$ only, right? Strictly speaking, it is to assign a value to p on the non-truth-valued function $H_{12}p$ in which q is hidden (ignorance).

2.2.2. Native Definition of Non-Truth-Valued Function. Further, because of the symmetry between p and q in $p \wedge q$, it seems the following: $H_{12}(q) = f_{12}(p, q) = p \wedge q$, $H_{12}(p) = f_{12}(p, r) = p \wedge r$?

Table 2: 16 kinds of truth-valued functional-e in CP and corresponding 16 non-truth-valued functional-es in CPH.

p	q	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
1	1	1	1	1	1	0	1	1	1	0	0	0	1	0	0	0	0
1	0	1	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0
0	1	1	1	0	1	1	0	1	0	1	0	1	0	0	1	0	0
0	0	1	0	1	1	1	0	0	1	0	1	1	0	0	0	1	0
			$p \vee q$			$\neg p \vee q$	p						$p \wedge q$			$\neg p \wedge q$	
p	$H_7{}^q p$	$H_1{}^q$	$H_2{}^q$	$H_3{}^q$	$H_4{}^q$	$H_5{}^q$	$H_6{}^q$	$H_7{}^q$	$H_8{}^q$	$H_9{}^q$	$H_{10}{}^q$	$H_{11}{}^q$	$H_{12}{}^q$	$H_{13}{}^q$	$H_{14}{}^q$	$H_{15}{}^q$	$H_{16}{}^q$

To avoid this confusion, the sure way is to add different superscripts and subscripts to each use of non-truth-valued functional-e:

$$H_{12}{}^{q}{}_{p}p = f_{12}(p,\ q) = p \wedge q,$$
$$H_{12}{}^{p}{}_{q}q = f_{12}(q,\ p) = q \wedge p,$$
$$H_{12}{}^{r}{}_{p}p = f_{12}(p,\ r) = p \wedge r,$$

......

$H_{12}{}^{q}{}_{p}$, $H_{12}{}^{p}{}_{q}$, and $H_{12}{}^{r}{}_{p}$ are complete (native) non-truth-valued functional-es. This also shows that non-truth-value functional-e actually depends on its connected (sub)-formula. So, does the non-truth-valued functional-e, such as "□," also depend on its connected sub-formula? First, a foreshadowing is laid.

As for the classical negation "¬," of course, it can be regarded as a special non-truth-valued functional-e. For example, you can call a bungalow a (first-floor) building, but you cannot call a (second-floor) building a bungalow in turn. It can also be written as $\neg p = H_{11\mathrm{p}}p$.

p is the omission of truth-valued function $f_6(p)$ or non-truth-valued function $H_{6\mathrm{p}}p$, similarly, q, r, . . . Of course, variables are always (completely independent of each other) non-truth-valued functions.

In this way, each truth-valued function in CP is defined (equivalent transformation) as a non-truth-valued function.

Definition 1. The (native) definition of non-truth-valued function is as follows.

Any truth-valued function in CP can be equivalently defined as a non-truth-valued function:

(1) p is its own non-truth-valued function, written as p or H_6p. It can be recursive to general formula A.

(2) For $\neg A$, it is regarded as a (special) non-truth-valued function, which can be written as $H_{11}A$.

(3) For $A \wedge B$, it is defined as $H_{12}{}^{B}{}_{A}A$.

A summary is as follows:

(1) Whether it is truth-valued function or non-truth-valued function, it is actually defined through "relational semantics." The same logical formula can be defined as a mutually equivalent truth-valued function or non-truth-valued function from the semantics of logical relations from different points of view.

(2) The truth function system CP hides the non-truth-valued function (ignorant hidden variables).

Table 3: Core definition of CPH notation system.

	Primitive syntactic symbol set of CPH equivalent to CP standard one
[1]	p, H
[2]	¬, ∧
[3]	(,)

(3) In this way, every truth-valued function in CP can be defined as a non-truth-valued function, but the cost is not only increasing an infinite number of non-truth-valued functional-es but also becoming more complicated because each one has different superscripts and subscripts.

In short, although this method does not introduce a new primitive symbol (let alone open an additional connotative logic route) and transform the non-truth-valued function equivalently, it seems far from the simplicity of the modal non-truth-valued functional connective that only adds a new primitive symbol. Can this method really express the "necessary"? Some smart scholars will immediately point out that this "non-truth-valued function" that reproduces all the information of a truth-valued function with superscript and subscript has no substantive significance except increasing complication! OK, Leibniz also encountered similar doubts in his early stage of binary system.

2.2.3. To Simplify the Set of CP Variables to a Non-Truth-Valued Function of p. Using only two primitive symbols $\{p, {}'\}$ can indeed simplify the variable set of the standard notation equivalently: that is, superscript "′" and its repetition are used to replace the subscript of the standard token variable set. However, this method cannot meet the third requirement of "three in one": increasing the explanatory power or expressive power of non-classical logic because "′," has no independent logical significance.

We only use the two primitive symbols "p and H" (Table 3) to simplify the standard symbolic variable set, where "H" is a non-truth-valued functional-e.

The basic Definition 2.2.3 of H is as follows: $Hp_0 \overset{\mathrm{def}}{\leftrightarrow} p_1$; the recursive definition is as follows: $Hp_n \overset{\mathrm{def}}{\leftrightarrow} p_{n+1}$, $n \in N$.

"H" and its superposition just reshape the variable set of CP one by one

CP: p_0, p_1, p_2., . . ., p_n, . . .; $n \in N$

CPH: p_0, Hp, HHp., . . ., $H. . .Hp$, . . .; $n \in N$; ($H. . .\ H$: n H)

We use "H" instead of "H" first because there are some questions:

(a) Some peers will question that "there is a loophole in the above method of replacing p_0, p_1, $p_{2,}$ . . ., with H and its superposition: according to the result of the previous purpose, different H actually has different superscripts and subscripts, such as $p_1 = H^{p1}{}_{p0}\, p$, $p_2 = H^{p2}{}_{p1}p$. . .. Therefore, this method violates the principle of mathematical monosemy and produces ambiguity."

The author's answer is as follows: (a) logic is the ultimate simplification of mathematics and reflects the strictness. Here, the superscript and subscript can be simplified (hidden) by distinguishing the different positions of H superposition in the formula, but there will be no ambiguity. The seemingly strict method of "adhering to superscript and subscript" is not the real logic method.

(b) Other peers asked, "even so, how to express them when three variables p, q, r are arbitrarily taken from the set? Can they be expressed ad hoc as: p, Hp, HHp?. If not, how to achieve the purpose of simplification?"

The author replied that it is precisely because the logical relationship between variables is "completely independent of each other." No matter what the "multiple mutual independence" is between any two variables or the "sequence of mutual independence" among three variables is, the "complex mutual independence" is still just a kind of mutual independence, as long as the mutual independence of multiple arguments is not collapsed. The superposition method can completely ensure this non-collapse (some readers must associate the superposition operator in the possible world semantics).

Definition 2.2.3-2: for the finite order of variables p, q, r, . . . arbitrarily selected from [7] of Table 3, we can directly use p and H and their superposition as p, Hp, HHp, . . ., $H. . .Hp$, . . .; $n \in N$; ($H. . .$ H: n h).

(c) Different from the new primitive symbols in other CP extension or variation systems, "H" is defined by the primitive symbols of CP, so the *CPH* is equivalent to the standard notation.

(d) "H" is different from any other derived symbols defined by the primitive symbols of standard notation in that "H" and its superposition are alternative equivalent simplifications of form subscripts.

(e) "H" is different from the comma "," that also simplifies the variable set. It has independent logical meaning. It is a non-truth-valued functional-e. Therefore, although the method of "H" and its superposition just reshapes the variable set of CP one by one, the *CPH* notation uses the extreme simplification method to achieve "three in one," especially the third requirement: it increases the expression power of non-classic.

There are several other points to note.

Firstly, *CPH* notation (system) is indeed an equivalent simplification of CP standard notation. Therefore, the logical system formed by *CPH* notation is a system with "special" non-truth-valued function connective. "H" equivalently transformed after simplifying the variable set of classical system. Of course, completeness and consistency need not be proved.

Secondly, the "special" system here is not a general non-truth-valued function one, because the binary truth function connective "∧" has not been strictly "operator symbolized." Whether the method of canceling the (variable) superscript and subscript of "H" by "superposition" is also feasible in "∧" needs to be investigated again.

Moreover, there are different superscripts and subscripts in different positions of "H" superposition which are hidden, it results a very important unexpected result, such as Hp = q, HHp = H(Hp) = Hq = r.

Then, consider Hp and Hq at the same time: when $p = q$, obviously Hp = Hq is not always true.

In other words, there seems to be "equivalence substitution no hold." However, this "equivalent substitution" is not equivalent one! Because the two "H" of Hp and Hq are not the same at all, just to consider the "extreme simplification" nature of logic and omit their different superscripts and subscripts in the basic definition.

The author will show in later papers that the equivalent substitutions no hold in the texts of logic philosophy and language analysis philosophy are, in fact, all because of the "pseudo-equivalent substitution" based on the ignorance of superscript and subscript hidden in the form of non-truth-valued function. This feature is not only the key point for understanding the modal nature, but also the core to understanding the whole basis 3. So "H" is called "the sign of God." That is, "H" hides superscripts and subscripts and embodies extreme simplicity and strictness at the same time, but it makes scientists, logicians, and philosophers have "substitution trouble" when they cannot see God's whole picture.

2.3. *CPH System Which Is Strictly Equivalent to CP*

2.3.1. *One to Many between Truth-Valued Functional-e and Non-Truth-Valued Ones.*

Now consider establishing a general non-truth-valued function system CPH equivalent to CP, that is, calculating the symbolic "∧" on the basis of the above purpose.

The "operator" of binary truth function composed of arbitrary variables p, q, and r is considered: $p \wedge q = H_{12}{}^{\mathrm{q}}{}_{p}(p)$, $q \wedge p = H_{12}{}^{\mathrm{P}}{}_{q}(q)$, $p \wedge r = H_{12}{}^{\mathrm{r}}{}_{\mathrm{p}}(p)$, and $\neg p \wedge q = H_{12\neg p}{}^{q}(\neg p)$.

It can be seen that the subscript of non-truth-valued functional-e (or the rightmost layer of superposition) is always consistent with its directly connected (sub)-formula (at least it can always be consistent through equivalent transformation), so the variable subscript of H_{12} (not the subscript of H_{12}) can be omitted (H_{11} is a unitary connective, and its superscript and subscript can be omitted):

Table 4: Two basic non-truth functional-e H_{11} and H_{12} used in *CPH*.

	CP primitive notation	*CPH* primitive notation 1	*CPH* primitive notation 2	MCP primitive notation
[1]	$p_0, p_1, p_2, \ldots, p_n, \ldots, n \in N$	p, H	p, H	$p_0, p_1, p_2, \ldots p_n, \ldots, n \in N$
[2]	¬, ∧;	¬, ∧;	H_{11}; $H_{12}{}^{p}$, $H_{12}{}^{Hp}$, $H_{12}{}^{HHp}$, ... $H_{12}{}^{H..Hp.}$, .. $n \in N$	¬, ∧; □
[3]	(,)	(,)	(,)	(,)

$p \wedge q = H_{12}{}^{q}(p)$, $q \wedge p = H_{12}{}^{p}(q)$, $p \wedge r = H_{12}{}^{r}(p)$, $\neg p \wedge q = H_{12}{}^{q}(\neg p)$

$p \wedge q \wedge r = H_{12}{}^{r} H_{12}{}^{q}(p)$, $\neg(\neg p \wedge \neg q) \wedge r = H_{12}{}^{r} H_{11} H_{12}{}^{\neg q}(\neg p)$

But

$H_{12}{}^{q}(p) = p \wedge q$, $H_{12}{}^{r}(p) = p \wedge r$, if omitting superscripts

$H_{12}(p) = p \wedge q$, $H_{12}(p) = p \wedge r$

Confused

Therefore, when the binary connective "∧" is strictly operator symbolized, it cannot be distinguished as if the unitary variable set only uses "H" superposition, obviously because the sub-formula after "∧" does not depend on the direct sub-formula before "∧." That is, when the CP system is equivalently transformed into the CPH system, although the truth-valued function and the non-truth-valued function correspond one by one, the truth-valued functional-e and the non-truth-valued functional-e correspond one to many (because the superscripts are different). CPH is extremely trivial due to the explosion of logical quantities (Table 4).

2.3.2. CPH System. Nevertheless, we can use *CPH* notation to establish non-truth-valued function system CPH.

Definition 2. . CHP system.

(1) CPH formula is equivalent to CP formula one by one

  (a) Arbitrarily selected variables $p, q, r, \ldots$, defined as p, Hp, HHp, ...
  (b) $\neg A$, defined as $H_{11}A$
  (c) $A \wedge B$, defined as $H_{12}{}^{B}A$

(2) Formula formation rule: unchanged

(3) Deduction rule: unchanged

(4) Axiom sets: unchanged

Obviously, CPH system is completely equivalent to CP system, but CPH is a non-truth-valued function system and CP is a truth-valued function system. However, up to now, CPH is an extremely trivial system.

A' is a non-truth-valued function of A, V (A, A') denotes assignment to A on A', and V(A') denotes assignment to A'.

How can V (A, A') and V (A') be strictly equivalent in form alone?

Because A' in V (A, A') is a (background) non-truth-valued function of A, and A' in V (A') is an (entity) truth-valued function (of other sub-formula).

A Rough Definition 2.3.1: if AR A', then V $(A, A') = 1$, iff, V $(A') = 1$.

For example, if A and A' are with a non-truth-valued function relation, $A = p$, $A' = \neg p \wedge q = H_{12}H_{11}p$, then $V(\neg p \wedge q) = 1$, iff, $V(H_{12}H_{11}p) = 1$, iff, $V(p, H_{12}H_{11}p) = 1$, iff, $V(p, \neg p \wedge q) = 1$.

In short, it is proper to say that $p \wedge q$ is a non-truth-valued function of p.

p is true on a non-truth-value function, that is, the background, in which p is not required to must be true. That is, in the background, p assignment is true ≠"in the background, p" assignment is true.

2.3.3. Optimization. Through equivalent transformation, for any kind of superposition operator formula, it can not only maintain the consistency between the omitted subscript in the innermost (right) layer of each superposition operator and the connected (direct) sub-formula but also omit its subscript without confusion, to make the direct (sub)-formula connected by each superposition operator always the same.

Optimization Definition 2.3.3: any formula in CP can be defined as the non-truth-valued function of A, whose non-truth function formula is overlapped by H_{11} and H_{12}, and H_{12} must be marked with superscript. Any two formulas can be expressed as two non-truth-valued functions of A, which is abbreviated as $\mathrm{H}^{C}A$ and $\mathrm{H}'^{D}A$.

What is the substantive significance of omitting (hiding) subscripts above? While maintaining strictness, it simplifies the mathematical form and facilitates clustering.

3. "■"as the Cluster of Non-Truth-Valued Functional-es in CPH

As it can be seen from the previous section:

(1) Each truth-valued function in CP can be transformed into a non-truth-valued function in CPH, which is completely equivalent to CP.

(2) Every non-truth-valued function in CPH consists of a non-truth-valued functional-e and its associated (sub)-formula.

(3) The non-truth-valued functional-e has a semi-dependence on the formula connected with it, which leads to the omission of subscripts but not superscripts.

Although (3) leads to the explosion of the number of non-truth-valued functional-es in CPH, formal language ultimately expresses its adaptability to natural language, which is the most economical way of long-term evolution. If a group of family-like non-truth-valued functional-es can be represented by a unified cluster to offset the mediocrity caused by different superscripts, and if recursive logical connectives with new level characteristics different from CP's actual judgment appear in natural language at the same time, this endogenesis is not only consistent with the logical approach of dialectics, but also consistent with "reducing the primitive symbols and increasing unity," and the metaphysical intuition of increasing the explanatory power and expressive power of non-classical problems is consistent.

3.1. Basic Considerations. CPH adds an auxiliary symbol "■," which reads "cluster," and this method is called black modal method "BCP." BCP will provide a helpful discovery method to reveal the nature of modal. Superscript and subscript of general auxiliary symbols are sometimes used: A, B, C; i, j, k; $'$, $''$, $'''$; etc.

Formula Forming Rule: if A is a formula, then ■ A (the auxiliary symbol ■ defined in this way with the general formula A forms ■ A. If ■ A is a well-formed formula of CP or of its equivalent system CPH, then the relevant rules are not really expanded or changed. Here, it is assumed that ■ A is a well-formed formula. If there is a consistent and reliable unified semantics, it is a well-formed formula).

Reasoning Rule: for any formula containing ■, if A and A $\vdash$ B, then B.

The direct meaning of ■ is that it is a convenient mark for a cluster of family-like non-truth-valued functional-es in CPH.

Axioms containing ■ are abbreviated as BCP axiom. ■ is defined in each BCP as follows.

Rough Definition 3.1: A cluster of non-truth-valued functional-es makes the axiom containing ■ A as classical theorems in CPH.

■A in each axiom is a formula cluster. Such a definition is obviously a direct classification of non-truth-valued functions in CPH or truth-valued functions (propositions, formulas) in equivalent system CP (here, the general cluster mode of non-truth-valued functional-e is represented; that is, as its sub-formula, it can also be or contain formula cluster, but its bottom formula is always CP proposition composed of standard notations only; that is, there is no so-called irreducible "atomic black modal proposition ■ p," which is different from the "atomic modal proposition $\Box p$" understood in the past. There are fundamental differences. In fact, to maintain the extreme classic of the system, two ad hoc definitions can be made: Definition 3/1: ▌, it is only an atomic cluster; that is, the formula contained is CP formula only, which is called dominant BCP; Definition 3/2: ▌A is only a single layer cluster, that is, only a cluster of 16 non-truth-valued function-es formed by the combination of A and C), and each BCP axiom expresses a complete subclass of CP proposition set, and the whole system is actually a classical system.

3.2. The Magic of BCP. From 2.2.3, we can see that the apparent equivalent substitution of Hp and Hq is not tenable due to the different subscripts of Hp and Hq. What about this situation in the BCP?

We first consider the definition 3/2, CPT-1 of two formulas ▌p and ▌q: $\vdash$ ▌p⟶p, only p to q is considered; accordingly, $\vdash$ ▌q⟶q only q to r (so called "four sentences").

(1) ▌ in ▌p is and only is the cluster of the four non-truth-valued functional-es in CPH:

$H_{12}{}^{p}$, $H_{12}{}^{\neg p}$, $H_{12}{}^{q}$, $H_{12}{}^{\neg q}$

(2) ▌p is and only is the cluster of the four non-truth-valued functions in CPH:

$H_{12}{}^{p}p$, $H_{12}{}^{\neg p}p$, $H_{12}{}^{q}p$, and $H_{12}{}^{\neg q}p$

(3) ▌p is and only is the cluster of these four truth-valued functions in CP that are strictly equivalent to CPH:

$p\wedge p$, $p\wedge\neg p$, $p\wedge q$, and $p\wedge\neg q$

(4) The axiom ▌$p\longrightarrow p$ is and only is a cluster of the four theorems in CP:

$\vdash p\longrightarrow p$, $\vdash p\wedge\neg p\longrightarrow p$, $\vdash p\wedge q\longrightarrow p$, and $\vdash p\wedge\neg q\longrightarrow p$

Similarly, ▌ in ▌q is and only is the cluster of the four non-truth-valued functional-es.

Of course, ▌ in ▌p can omit its subscript $_p$. Why can the superscript { p, $\neg p$, q, $\neg q$ } also be omitted? Because it was previously stipulated that the cluster ▌p is only 4 of the 16 formulas of the combination of p and q that conform to the axiom ▌$p\longrightarrow p$. In other words, p and axiom of BCP T-1 completely determine that the superscript can only be {p, $\neg p$, q, $\neg q$}. Similarly, the subscript of ▌ in ▌q can be omitted. However, the superscript cannot be omitted from each of the four non-truth-valued functional-es conforming to the ▌$p\longrightarrow p$, because its superscript is not completely determined by the axiom and p. For example, $p\wedge$q is equivalently transformed into $H_{12}{}^{q}p$, and its superscript is determined by the sub-formula q still (those scholars who question the first equivalence are "complicating the problem" should begin to realize the significance of this study: in CPH equivalent to CP, non-truth-valued functional-es are not combined formulas in isolation; after a single non-truth-valued function connected formula, only subscripts can be omitted, while after the cluster of non-truth-valued functional-es connected formula, both subscripts and superscripts can be omitted).

On the contrary, although the "cluster type" of ▌p and ▌q is the same (the sequence degree of clusters is the same, and the connectives of each pair of truth function in the sequence are the same), in fact, in the two clusters, not only the sub-formulas directly connected by the two "▌" are different but also the "tails" of each pair of truth functions in the corresponding two series (e.g., q in $p\wedge q$ corresponds to r in $q\wedge r$).

In this special case, there is such a feature. Generally, there will be such a feature in the BCP formulas ■A and ■B in the BCP axiom: although the two "■" in ■A and ■B as formula clusters can be regarded as (apparent) the same logical symbol, when considering the expansion of formula clusters, not only the hidden subscripts are different but also the superscripts of the expanded series of formula pairs are different. As will be shown later, this is hiding the extremely important characteristics of real modal logic.

In terms of "apparent" logic, it is considered that any two "■" in a BCP system are indeed the same logical symbol (it does not directly display superscript and subscript), which is actually ignorance (neglect) of the different (subscript) superscripts omitted! In other words, when the two "■" in ■A and ■B are extracted, they are indeed exactly the same symbol, but when they are put into different "contexts," that is, connecting different (sub)-formulas, they will highlight different hidden variables and have different effects (which leads to the failure of apparent equivalent substitution). From the perspective of logical metaphysics, their meaning is "context-dependent," or has the holism that "the meaning of form depends on the content it expresses." This "same and different" that will not lead to ambiguity is a typical embodiment of dialectics. As will be shown later, semantically, it further embodies the entanglement of dialectics, holism, and uncertainty. This fully reflects the nature of logic as reflecting its transcendence through the extreme refinement of philosophy.

Because Definition 3.1 implies that the two "■" in ■A and ■B are the same cluster symbol, ■p represents the non-truth-valued function cluster of p, which can be recursively deduced as ■A represents the non-truth-valued function cluster of A. Although ■ linking with different (sub)-formulas actually causes it to hide different superscripts and subscripts, it does not affect its recursion. ■ simultaneous occurrence of A and B will not cause confusion (which will be further guaranteed by semantics) (nature is secret and great. Simple understanding symbolizes the same natural word into the same logical symbol; "negation of negation" in the form of narrow dialectical logic, the former is a classical negative symbol and the latter is a paraconsistent negative symbol; in modal logic, the same symbol represents a non-truth-valued functional connective, which actually hides the apparent same after different superscripts and subscripts in different contexts. Here, the two same ■ different superscripts in ■A and ■B are "different contexts" depended on ■A or ■B. The general semantic definition of ■A later shows the dependence of ■A on context; that is, things are interdependent with their living environment, while the standard semantics $V(\Box A, w)$ and $V(\Box B, w)$ only show the dependence of the original thing on the environment), and this fully reflects the nature of logic that it embodies its strictness through the extreme simplification of mathematics.

Because CPH is the equivalent system of CP and ■A is the non-truth-valued function cluster in CPH, ■A is the corresponding truth-valued function (proposition) cluster in CP. The recursion shows that ■A is a non-truth function connective not only of CPH but also of CP.

In summary, we find a set of three important characteristics of non-truth-valued function in CPH, which is completely equivalent to CP, Feature 3.2:

(a) When considering the operator simplification of variables p, q, and r, the same H and its different superpositions can be used logically and consistently. The superposition of variables is distinguished and the superscripts and subscripts are omitted, but "H" cannot be recursively used in general formulas A, B, and C.

(b) In the symbolic calculation of general formulas, the (variable) subscripts of specific H_{12} corresponding to different conjunctive formulas can be omitted without affecting logical consistency, but their superscripts cannot be omitted. However, these non-truth-valued functions without subscripts can be semi-recursive to the general formula.

(c) In the computation of general formula clusters, different composite formulas correspond to different H_{12} in which subscripts and superscripts can be omitted. At this time, the non-truth-valued functional-e can be recursively deduced to the general formula, so "■" is a general non-truth-valued functional connective.

The non-truth-valued functionality of "■" (in fact, the hidden truth-valued functionality) is determined not only by "■" itself but also by formula A or B it is connected to. The syntactic nature of "■" appears completely in its pragmatics.

In general, there is Definition 3.2.

When ■A and ■B with the same apparent modal degree appear simultaneously in the same formula or system, as the hidden superscript in natural reasoning of pure syntactic reasoning, the corresponding cluster is expanded into formula series:

■A corresponds to (A, A_0), (A, A_1), (A, A_2), ..., $(A, A_{(n)})$, ...

■B corresponds to (B, B_0), (B, B_1), (B, B_2), ..., $(B, B_{(n)})$, ...

In short, the *magic* of the BCP is as a formula cluster, which can logically and uniformly hide the superscript and subscript, and "pretends" the same "■." In other words, the omitted subscript of ■A is A, and the omitted superscript is the set of formulas corresponding to cluster expansion, which is called the series of $n\ A'$; in ■B, the omitted subscript of B is "B," and the omitted superscript is a series of $n\ B'$.

It can be seen later that the subscript of ■A is an A, and the corresponding superscript is $n\ A'$, which are all related to A in a relationship R determined by the BCP axiom (set) where ■A is located. Then, the outer "■" of ■■A is that each of the $n\ A'$ in the superscript has a R relationship with $n\ A$." That is, ■■A is a cluster of n of n (yes but not only n^2).

3.3. Equivalence among BCP Systems. You can also define a duality auxiliary symbol "◆": ◆$A = \neg$■$\neg A$.

Similarly:

BCPK: ├ ■$(A \longrightarrow B) \longrightarrow ($■$A \longrightarrow$ ■$B)$

BCPD: ├ ■$A \longrightarrow$ ◆A

BCP4: ├ ■$A \longrightarrow$ ■■A

......

Special Mention of BCPN: A and then ■A.

Semantically, the BCPN rule is not a naturally established theorem, but at most a cross-system axiom, which is more suitable to be called normal deduction rule. In the normal system series, the minimum system is C + BCPN + K, and other systems are its extensions. However, semantically, whether each BCP (set) superimposes N will have obviously different characteristics.

For another example, BCPT-1, there are and only are these four formulas (actually formula modes) $A\wedge A$, $A\wedge\neg A$, $A\wedge C$, and $A\wedge\neg C$, which are substituted into cluster ▌A, respectively, making BM T-1 a set of classical theorems: $\vdash A\wedge A\longrightarrow A$, $\vdash A\wedge\neg A\longrightarrow A$, $\vdash A\wedge C\longrightarrow A$, and $\vdash A\wedge\neg C\longrightarrow A$.

$\vdash$▌$A\longrightarrow A$ is actually to write this group of CP theorems again (in fact, the general formula is also), so the BCP system is actually a CP system; it's too troublesome to write one by one at a time. They are simplified to write a cluster.

Similarly, other BCP axioms can be discussed, such as N, K, D, 4, O, and TR. Each BCP axiom represents a different subclass of classical theorems, so each axiom is not equivalent to other, but all BCP systems are equivalent because they are equivalent to CP. For example, natural number sets are not equivalent to rational number sets, but "real number sets + natural number sets" or "real number sets + rational number sets" are equivalent to "real number sets."

3.4. Correspondence between BCP and MCP. Because both MCP and BCP add only a new symbol outside the standard symbol (although the former is external primitiveness and the latter is endogenous auxiliary), there is no substantive difference between the original languages; in fact, there is no substantial difference between the reasoning rules and formula formation rules in MCP and BCP systems, and natural reasoning is carried out according to the two truth values (the only difference is that there seems to be irreducible atomic modal formula $\Box p$ in MCP); the C axiom set in the two is also exactly the same. Any MCP axiom can be "p-shaped," that is, it collapses into a CP theorem, and any BCP axiom is a CP theorem cluster, so each MCP axiom has a homomorphic BCP axiom.

So does every BCP axiom have a homomorphic MCP axiom? The author hopes to put forward counterexamples. In other words, "□" must be actually "■," only our ignorance thinks they are different? Of course, it also depends on semantics. If the semantics are the same, the syntax will be strictly equivalent. Now, at least it can be said that each BCP system is syntactically corresponding to a MCP system, and "■" almost expresses the same non-truth-valued functionality as "□" does.

4. BCP Semantics

The core of the BCP assignment definition is as follows: what is V (■A) = 1?

More precisely, what is the semantic relationship between ■A and A? How to express "necessity" from it? Intuitively, because ■A is a formula cluster, saying "■A is true" means that every formula in this cluster must be true. Of course, this is only a rough expression.

4.1. K-2 as Another Auxiliary Discovery Method. 1. Back to Example BCP T-1.

Although ▌A is and only is the cluster of the four classical formulas $A\wedge A$, $A\wedge\neg A$, $A\wedge C$, and $A\wedge\neg C$ (which are actually formula patterns) (hereinafter referred to as a cluster A': A^1, A^2, A^3, and A^4; correspondingly, ▌B is B' cluster: B^1, B^2, B^3, and B^4), these four formulas will not be all true: $A\wedge\neg A$ is forever false. In addition, $A\wedge C$ and $A\wedge\neg A$ cannot be true at the same time.

Therefore, "▌A is assigned to true," and A^2 must be excluded firstly. For the remaining three formulas A^1, A^3, and A^4, they cannot be defined by any classical composite proposition assigned by their unified truth value (because they will not be true at the same time, and other composite propositions do not conform to the natural semantics of "all must be true"), nor can they be defined by the assignment of any composite proposition of these three formulas (because ▌A is their cluster rather than the compound proposition of these propositions). Therefore, "each of these three propositions is true" is actually the conjunction of the respective assignments of each proposition as true. For example, when $A\wedge C$ is true and $A\wedge\neg C$ is true, it is obvious that two different truth values of A and C are assigned {1,1} and {1,0} (we are only talking about the unity of assignment at the level of A and A'. A itself is also made up of sub-propositions such as $p\vee q$, which is obviously not unique for assignment of truth values). Thus, for BCPT-1, V (▌A) = 1, iff, $(V(A^1) = 1) \wedge (V(A^3) = 1) \wedge (V(A^4) = 1)$.

2. This is equivalent to saying that among the 16 formulas composed of A and C in Table 5, only the above 4 can be substituted into BCPT-1 to make them become classical theorems, and only 3 of 4 can be assigned true. More interestingly, only 4 of the 16 formulas are reflexive to them by A (A is greater than or equal to A').

Some peers worry that this nonuniform assignment will lead to the collapse of the BCP assignment system and further lead to "■A" not being a well-formed formula?

The author's reply that not at all! The following text shows that their respective assignments that only appear in the calculation ■A are true and do not appear in the defined items of general (sub)-formulas. It is this apparent "respective assignment" that makes us find the third and core equivalent transformation of modal logic.

In fact, we have already established the calculation (see table 5 in reference 11) (although the expression was not strict at that time, the calculation was almost good, and we directly faced Kripke and Hindika at the two international conferences in the summer of 2012). Now, it is revised as follows.

The situation of modal axiom in 16 basic combinations formed by binary (A, C) formulas (which can reflect the basic characteristics of modal axiom. Of course, no matter how many formulas A' is combined with, it can be regarded as C), is shown in Table 5.

The double superposition should be at 8×8 calculated in formulas. There is double superposition at the same time, and the K axiom should be 64×64 calculated in formulas in general, and computer programming can be used. Of course, the specific calculation can be greatly simplified considering various correlations. Although Table 5 is a helpful discovery work only, it greatly enhances the confidence of BCP research.

Table 5: K-2 shows the validity of 10 common (simplified) BCP axioms in the 16 formulas composed of (A, C) (see [7]. The valid formulas of each axiom form a logical relationship revealed by the standard semantics with the original formula A. The relationship among axioms is also the same as the results of previous modal syntax research).

	C	K	D	T	Tr	4	B	E	M	O	V
H_1A	1	1				1	1	1		1	1
H_2A	1					1		1			
H_3A	1					1		1			
H_4A	1						1				
H_5A	1						1				
H_6A	1	1	1	1	1	1	1	1	1	1	
H_7A	1										
H_8A	1	1	1								
H_9A	1	1	1								
$H_{10}A$	1										
$H_{11}A$	1	1	1				1				
$H_{12}A$	1	1	1	1							
$H_{13}A$	1	1	1	1							
$H_{14}A$	1	1	1						1		
$H_{15}A$	1	1	1						1		
$\mathrm{H}_{16}\mathrm{A}$	1	1	1	1		1			1		

Considering BCP4-1 below, the existence of double superposition should be calculated in 64 formulas (e.g., when $H_{12}{}^{C}H_{12}{}^{C}A = H_{12}{}^{C}\mathrm{A}$, substituting $H_{12}{}^{C}A$ into BCP4 is obviously a classical theorem). However, in general consideration, the BCP4 results in Table 5 are also sufficient.

▌A is and only is A, $A\vee\neg A$, $A\vee C$, and $A\vee\neg C$, which are all cluster that can be true formula that has a transitive relationship with A (hereinafter referred to as A' series, i.e., A^1, A^2, A^3, and A^4). The assignment of each of the four formulas that can be true is {1; } {1; 0}, {1, 1; 1, 0; 0, 1}, , {1, 0; 1, 1; 0, 0}. That is, relative to A, only K kinds are true, and ▌A has $1\times 2\times 3\times 3\times K = 18K$ kinds that are true. In practice, there will be more general modal formulas, but they can be bound by universal quantifiers.

4.2. Core Semantics of BCP. According to the relationship theory, Table 5 shows that the valid formulas in BCPT-1 are reflexive for A, continuous in BCPD-1, and transitive in BCP4-1. The relationship R between A and A' in each axiom is the relationship after deducting the 16th line that cannot be true, that is, untruth-able to A.

"Excluding untruth-able propositions" ("■" of Strict Definition 4.1: iff, a cluster of non-truth-valued functional-es of A in Definition 3.1 except those that form untruth-able formulas with A. However, the nature of "■" discussed in Section 3 remains the same) is the same for all axioms. It can be considered that ARA' is an accessible relationship between formulas, so the general semantics is obtained.

Definition 3. $\mathrm{V}\ (\blacksquare A) = 1$, iff, $\forall A'(\mathrm{ARA}' \longrightarrow \mathrm{V}(A') = 1)$.

R: iff, the relationship that A has the same logical one for each formula in a cluster A'. These A' can be assigned to true, and each substitution $\blacksquare A$ makes the modal axiom (set) where $\blacksquare A$ is located a classical theorem.

As mentioned in the definition of non-truth function in Section 1, when people do not know $H_{12}{}^{Hp}p$ (especially when they are ignorant of its superscript), they will only think that a truth assignment is given to p on the non-truth-valued function $H_{12}{}^{Hp}p$, but when it is clear that $H_{12}{}^{Hp}p$ is actually $p\wedge q$, they will think that a set of truth assignments are given to p and q in $p\wedge q$.

However, it should be noted that when p is true, Hp is true in two cases ($q = 1$ and $q = 0$, respectively). That is, the assignment of true to p on the non-truth-valued function Hp is not only related to p but also related to H and the whole non-truth function Hp.

In the BCP4-1 of the discovery aid method K-2, the four formulas, A, $A\vee\neg A$, $A\vee C$, and $A\vee\neg C$, will also be regarded as different non-truth-valued functions of A that are transitive to A. Then, it is natural to think that the combination of assigning truth values to each of the four formulas is the combination of assigning truth values to A on the non-truth-valued functions of the four A in turn (although there are 18K combinations of assigning truth values to each of the four formulas, the corresponding combination of assigning truth values to A on the four non-truth functions is also 18K, that is, assigning truth values to ▌A is also 18K kinds).

According to Definition 2.3.1, generally, Definition 3.2-1, which is equivalent to Definition 3, is as follows:

$\mathrm{V}(\blacksquare A) = 1$, iff, $\forall A'\ (ARA' \longrightarrow \mathrm{V}\ (A, A') = 1)$.

In other words, considering the non-truth-valued functional relationship from A to A′: An assignment of A′ which is a non-truth-valued function of A is true, iff, the assignment of A on the A′ which is assignment background of A is true, as $\mathrm{V}\ (\mathrm{A}, \mathrm{A}') = 1$.

Although series A' (including or excluding A) in the binary assignment is also a formula, it is different from a formula, which plays an entity role in the assignment, and they exist as the context of entity formula or the background of pattern cluster, which can be called "modal background." To distinguish these two roles, we can still call the formula of entity role A, rewrite the "modal background" (metaphor "World") as w', and then change A (which is also the background) considering the R relationship with A' to w, and then, 4.2-1 is equivalently rewritten to 4.2-1B: $V(\blacksquare A) = 1$, iff, $\forall w'(wRw' \longrightarrow V\ (A, w') = 1)$.

The corresponding standard semantic core assignment definition can be recorded as 4.2-1M: $V(\Box A, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow \mathrm{V}\ (A, w') = 1)$.

That is to say, as long as the modal background A and A' in the cluster semantics are considered to be the worlds w and w' as the standard semantics, the definition items of the two definitions are completely isomorphic. Therefore, the only difference between the two core definitions is that the former does not have a world "w" in the defined items. It is interesting to regard the proposition as "background" in Definition 3 as world is a wonderful metaphor. In the traditional understanding, all variables in a definition can only take the same set of truth values at the same assignment. Now, different branches of the same definition can take different sets of truth values at the same time, so they can only exist in "different worlds."

4.3. Comparison between the Core Semantics of BCP and the Other Two Typical Semantics. Firstly, the core semantics of the BCP method (including defined items and definition items) is to equivalently transform the non-truth-valued function system CPH from CP in strict accordance with mathematics, then take the cluster of non-truth functional formula in CPH as auxiliary syntactic symbols to form the non-truth function cluster $\blacksquare A$ in CPH (i.e., CP), and then directly give the general assignment Definition 3 and its equivalent Definition 3 of $\blacksquare A$. No any other logical or metaphysical conditions are attached (e.g., the "world" is added in standard one). $\blacksquare A$ has unified semantics 4.2 and 4.2-1, so $\blacksquare A$ is a well-formed formula, and it is obvious that $\blacksquare A$ is a non-truth function of A and $\blacksquare$ is a logical connective.

Secondly, even if A and A' are regarded as the world w and w', 4.2-1B is more in line with the original meaning of the history of philosophy (see [12], p89-p97. Naive semantics: Leibniz believes that A necessary true means that A is true in all possible worlds. Standard semantics: A necessary true in world w is, on every world w' that is in some accessible relationship to w, A is true. Cluster: A necessary true is every A' on which it is in some accessible relationship with A is true; A and A' as contexts are the worlds. The defined term of BCP cluster is the same shape as Leibniz's term, and the defined item of cluster is the same shape as standard semantics.) than 4.2-1M. At the same time, it can interpret the modal operator as a variety of different modal concepts like the latter and more tightly express the multimodal language (obviously, whether or not it can be interpreted as a different modal concept is not related to the defined term, but only to the defined term, whereas 4.2-1B is homomorphic to the defined term of 4.2-1M. BCP for multimodal logic, such as dynamic modal logic, does not always have to use different superscripts or subscripts to represent two different modal operators, such as Definition 3.2, the two "$\blacksquare$" in $\blacksquare A$ and $\blacksquare B$ already hide different superscripts and subscripts, while for multiple modals, in principle, it is always possible to transform them into unary modals).

Moreover, the defined item of 4.2-1B has no "w," which leads to the binary semantics appearing only in the definition item of $\blacksquare A$ assignment definition, not when it is used as a general formula. Therefore, the whole BCP assignment definition system is still a unitary semantics, which is actually a classical semantics (at most, it is the classical two-valued semantics, which contains the uncertainty of true-false combination and arrangement based on Łukasiewicz's L_3 we discover).

To summarize,

(1) The third equivalence: unitary entity assignment is equivalent to binary background assignment

$V(\blacksquare A) = 1$, iff, $\forall A'\ (ARA' \longrightarrow V\ (A') = 1)$

$V(\blacksquare A) = 1$, iff, $\forall A'\ (ARA' \longrightarrow V\ (A, A') = 1)$

(2) The combined formula (proposition) as the background (context) of assignment can be regarded as "world"

$V(\blacksquare A) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (w,w') = 1)$

(3) The core semantic similarities and differences among native (Leibniz), standard (Kripke), and BCP cluster (Wan)

L: $V(\Box A) = 1$, iff, $\forall w'\ (V\ (A, w') = 1)$

K: $V(\Box A, w) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (A, w') = 1)$

W: $V(\blacksquare A) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (A, w') = 1)$

(4) Necessity A and necessity B occur at the same time

Standard semantics (because w does not depend on the change in $\Box A$ to $\Box B$, it only expresses that things are dependent on their environment)

$V(\Box A, w) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (A, w') = 1)$

$V(\Box B, w) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (B, w') = 1)$

Accordingly, cluster semantics (things are interdependent with their living environment) are as follows:

$$V(\blacksquare A) = 1, \text{ iff, } \forall w_A'\ (w_A R w_A' \longrightarrow V\ (A, w_A') = 1)$$
$$V(\blacksquare B) = 1, \text{ iff, } \forall w_B'\ (w_B R w_B' \longrightarrow V\ (B, w_B') = 1)$$

Finally, if you have to express the situation that "$\blacksquare A$ is true in two different worlds" in BCP, it is just to add completely redundant world "w_1,w_2" to the defined item.

5. Standard Semantics after De-Redundancy and Its Consequences

5.1. Redundancy Removal. Possible world standard semantics: triple frame structure: $M = <W, R, V>$, where W represents the set of possible worlds; R represents the relationship between possible worlds; wRw' represents the distance from possible world w to possible world w'; and V represents the set of assignment functions. If the propositional variable p is assigned 1 (true) in world w, it is written as $V\ (p, w) = 1$.

(0) Any p, $V\ (p, w) = 1$ or $V\ (p, w) = 0$; it can be recursive to general formula A.

(1) Any formula A, if $V\ (A, w) = 0$, then $V\ (\neg A, w) = 1$; otherwise, $V\ (\neg A, w) = 0$.

For any combination formula A and B, if $V\ (A, w) = 1$ and $V\ (B, w) = 1$, then $V\ (A \wedge B, w) = 1$; otherwise, $V\ (A \wedge B, w) = 0$.

(2) For any formula A, if all $w'(wRw')$ accessible from w have $V\ (A, w') = 1$, then $V\ (\Box A, w) = 1$; otherwise, $V\ (\Box A, w) = 0$.

Definition 3-1-M: $V(\Box A, w) = 1$, iff, $\forall w'\ (wRw' \longrightarrow V\ (A, w') = 1)$.

Analysis: first look (2), the reason why the defined item has a binary of w is that it has to correspond with the w in the binary assignment of the defined item. The reason why the latter part of the definition item entailment has w is to correspond with the relationship wRw'. Only with wRw' different necessaries can be expressed. However, if w and w' are actually formulas A and A', then A in the defined item $\Box A$ can directly correspond to the A in the definition item, and$w$ of the defined item can be redundant.

Looking at (0), the reason why the general formula has duality $V(A, w)$ is that the assignment of $\Box A$ as an example of the general formula has the second element w. Now w of the defined item of (2) is redundant, so w in (0) general formula is redundant (in addition, w does not really participate in the calculation here, which is obviously redundant).

Let's just look at definition (1). For the respective binary calculations of the complete set $(\neg, \wedge)$, there is $V(\neg A, w) = \neg V(A, w)$, rather than $V\neg(A, w) = \neg V(A, w)$, so w is independent of the assignment calculation. Therefore, w here is written in, but no operation is actually added, and the effect is only redundant. The same is true for "$\wedge$."

In short, the w of the defined item in the standard semantic core definition is redundancy, which will produce some subversive consequences. However, there is still a triple framework structure: $M = <\mathbf{A}, R, V>$, where $\mathbf{A}$ is the set of CP formulas.

5.2. Superposition Operator Problem. Intuitively, $\blacksquare\blacksquare A$ is the necessity of necessity A; that is, a corresponds to a cluster of a cluster of worlds (formulas).

Definition 4. $V(\blacksquare\blacksquare A, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow \forall w''$ $(w'Rw'' \longrightarrow V(A, w'') = 1))$.

The duality standard semantics seem to have obvious apparent proof advantages here:

0-1. $V(\blacksquare A, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow V(A, w') = 1)$; basic definition

0-2. $V(\blacksquare\blacksquare A, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow V(\blacksquare A, w') = 1)$; $\blacksquare A$ is substituted into A

0-3. $V(\blacksquare A, w') = 1$, iff, $\forall w''$ $(w'Rw'' \longrightarrow V(A, w'') = 1)$; w deformation is w', corresponding w' deformation is w''

0-4. $V(\blacksquare\blacksquare \mathbf{A}, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow \forall w''$ $(w'\mathbf{R}w'' \longrightarrow \mathbf{V}(\mathbf{A}, w'') = \mathbf{1}))$; 0-3 left substitute 0-2 light

That is all.

Although the semantics of unitary clusters cannot be directly substituted as above, it can also prove the above superposition definition.

1-1. $V(\blacksquare A) = 1$, iff, $\forall A'$ $(ARA' \longrightarrow V(A') = 1)$; basic definition.

1-2. $V(\blacksquare\blacksquare A) = 1$, iff, $\forall(\blacksquare A)'$ $((\blacksquare A)R$ $(\blacksquare A$ $)' \longrightarrow V(\blacksquare A)' = 1)$; $\blacksquare A$ replace a with A.

1-3. $V'(\blacksquare A') = 1$, iff, $\forall A''(A'RA' \longrightarrow V(A'') = 1)$; A deformation is A', corresponding w' deformation is $w.''$

1-4. $\forall A'$ $(\mathbf{A}R\mathbf{A}' \longrightarrow V(\blacksquare A') = 1)$, iff, $\forall A'$ $(\mathbf{A}R\mathbf{A}' \longrightarrow \forall \mathbf{A}''(A'RA'' \longrightarrow V(A'') = 1))$; replace 1-3 right forms with 1-4 left forms.

1-5. $\forall(\blacksquare A)'$ $((\blacksquare A)R(\blacksquare A)$ $' \longrightarrow$ $V(\blacksquare A)' = 1) = \forall A'(\mathbf{A}R\mathbf{A}' \longrightarrow V(\blacksquare A') = 1)$; key (why is 1-5 true? Note: $V(\blacksquare\blacksquare A) = 1$ means that the assignment of the cluster of the formula, which is R related to the formula, which is R related to A. 1-5 Left: the conjunction of the assignment values of each cluster in R relation to A; $m \times (A \Rightarrow m$ $A' \Rightarrow m$ $A'') = m \times m$ A." So, the left $\equiv$ the right. Take BCPT-2 (p, q, r). Left: $(V(p) = 1 \wedge V(p \wedge q) = 1 \wedge V$ $(p \wedge \neg q) = 1) \wedge (V(p \wedge r) = 1 \wedge V(p \wedge q \wedge r) = 1 \wedge V(p \wedge \neg q \wedge r) = 1) \wedge (V(p \wedge \neg r) = 1 \wedge V(p \wedge q \wedge \neg r) = 1 \wedge V(p \wedge \neg q \wedge \neg r) = 1)$. Right: $(V(p) = 1 \wedge V(p \wedge r) = 1 \wedge V(p \wedge \neg r) = 1) \wedge (V(p \wedge q) = 1 \wedge V$ $(p \wedge q \wedge r) = 1 \wedge V$ $(p \wedge q \wedge \neg r) = 1)$ $\wedge (V$ $(p \wedge \neg q) = 1 \wedge V(p \wedge \neg q \wedge r) = 1 \wedge V(p \wedge \neg q \wedge \neg r) = 1)$. That is all.).

1-6. $\forall(\blacksquare A)$ $'((\blacksquare A)R$ $(\blacksquare A$ $)$ $' \longrightarrow V(\blacksquare A)' = 1)$, iff, $\forall A'(\mathbf{A}R\mathbf{A}' \longrightarrow \forall \mathbf{A}''(A'RA' \longrightarrow V(A'') = 1))$; 1-5 left = 1-4 right.

1-7. $V(\blacksquare\blacksquare A) = 1$, iff, $\forall A'(ARA' \longrightarrow \forall A'(A'RA' \longrightarrow V$ $(A'') = 1))$; 1.2 right = 1-6 left.

1-8. $V(\blacksquare\blacksquare A) = 1$, iff, $\forall A'(\mathbf{A}R\mathbf{A}$ $' \longrightarrow \forall \mathbf{A}''(A'RA' \longrightarrow V$ $(A, A'') = 1))$; third equivalence.

That is all.

5.3. Influence on Axiomatic Valid Model Set. It is the binary assignment of the standard semantics to the general formula that gives the following:

$V(\Box A, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow V(A, w') = 1$

$V(\Box B, w) = 1$, iff, $\forall w'$ $(wRw' \longrightarrow V(B, w') = 1$

Just because the world w is the same when $\Box A$ and $\Box B$ appear at the same time, it can be semantically proved that the K axiom is valid for all models.

Now, the de-redundant standard semantics is equivalent to the cluster semantics, and the K axiom is obviously not valid for all models (in fact, it was originally ad hoc, taking the respective "tails" of the superscript $\Box A$ and $\Box B$ as the same. See the author's 2012 article, i.e., tables K-1 and K-2 in the reference, and the effective formula sets of some other axioms have also changed). So, will this affect the syntactic research? For example, will K system no longer be a minimum normal system?

No! Although 4 axioms are not an extension of K axiom (they are cross-relationships). However, in normal modal logic series, "$S4$ system is an extension of K system" means "the model set valid not only for K and T axioms, but also for S axioms and also for N." Therefore, it is always correct to say that other normal systems are extensions of K systems.

Syntactically, de-redundancy standard semantics will not change any recognized results of modal syntactic research result (of course, it will be more accurate to understand some results). On the contrary, knowing the "cards" will help to verify the newly discovered modal axioms and laws and speed up syntactic research.

5.4. Recognition of Necessary Rules. Obviously, if the semantics of assignment is not duality, it is impossible to realize the natural understanding of the rule of necessary.

Naturalized Understanding: if A is a theorem, then $\Box A$ must be a theorem.

Now, according to the cluster semantics, this becomes when A is a theorem, A is a theorem in all contexts.

This is obviously absurd. For example, even if A is a theorem, $A \wedge C$ in BCPT will not be a theorem naturally.

Analysis: in fact, no matter whether to eliminate redundancy or not, it is a mistake to understand the necessary rule as naturalization, and eliminating redundancy is just a further explanation.

(1) According to the classical or cluster semantics, the value of p is 1 or 0. The same is true for general formula A.

According to the definition of the standard semantic assignment of $\Box p$, we can know that p can be assigned either true or false in a world (of course, it does not mean that p can assign both true and false in this world at the same time). That is, it does not mean that p can only have one assignment in a world, either $V(p, w)$ true or $V(p, w)$ false. The same is true for A.

(2) Therefore, according to the standard semantics, A is a theorem, which of course means that it is a theorem in a certain world; that is, every assignment in this world is true, and it cannot be deduced that every assignment in each world of a cluster can be true, let alone that every assignment in each world of this cluster is true. Therefore, it is possible that the founders of world semantics are actually confused "in one world, A' is a theorem" and "in one world, "A is a theorem."

(3) To say the least, even if we think that p is true means that p assigns only a value to a world; that is, p assigns values to the elements of the world set W and divides these worlds into two halves, true in half the number of worlds and false in the other half. However, from the superposition definition (let A be $\Box p$ and $\Box\Box p$ is $\Box A$), we can still find that each set of w and the set of w' and set of w'' are not at the same level.

(4) For example, p is true on a world w_1. The corresponding $\Box p$ is true on w_1, iff, p is true in all of $m\ w'$, which are R related to w_1 in a total of O worlds in the W' set, assuming that 3 of the 16 are w'_1, w'_2, and w'_3.

$\Box\Box p$ is true on w_1; that is, there is $m \times m$ in W'' set in total $O \times O$ worlds; e.g., 3×3 is true in 16×16 worlds: w'_1 to w''_{11},w''_{12},w''_{13}, w'_2 to w''_{21},w''_{22},w''_{23};w'_3 to w''_{31}, w''_{32}, andw''_{33}.

(5) Even the level of truth is different, so the level of eternal truth is also different. A eternal truth means that there are O worlds in the W' set; that is, every w' is true.

$\Box A$ eternal truth refers to $O \times O$ worlds in the W'' set every world is true.

Unless W'' collapses to W', W' generally does not include W'. W'' does not contain W' in the D axiom.

Since A is true in every w' in the W' set, of course, it cannot be naturally deduced that $\Box A$ is true in every w'' in the W'' set. From A is all true in 16 w' in the W' set, we cannot naturally deduce that $\Box A$ is all true in 9 w'' in the W'' set still.

(6) In short, "A, $\Box A$" cannot be deduced naturally. It is just an additional constraint, which is equivalent to the characteristic axiom of cross-modal system.

Of course, the result of this detailed semantic analysis does not affect the modal syntactic results produced by N.

In the de-redundant standard semantics, it is much clearer that A is always true; that is, all its internal truth assignments are true. $\Box A$ is always true or A' that has an R relationship with A is always true. It is obvious that $\Box A$ always true cannot be naturally deduced from A always true.

There are countless similar results in the history of human cognition: the inventor of a theory thought he said I, but in fact he had to say J. For example, Lao Tzu thought that the "Tao" was an unfathomable "origin." In fact, what he said was just "the form of analogical reasoning." See [13].

Based on the above points, it is found that the following is observed.

The standard semantic person mistakenly regards the N rule as the naturalized one; that is, when A is a theorem, it is wrongly exaggerated $\Box A$ as a theorem; the standard semantics makes $\Box A$ and $\Box B$ appear at the same time with the same w. It is unknown whether the world is actually formula A or B. These confusions are not the sufficient results of duality w in the defined items of the standard semantic core assignment definition. This further shows that the second element w in the defined item of the standard semantic with general formula assignment definition is indeed redundant.

However, the former exaggeration mistakes some non-classical formulas for theorems, which may result in some modal formulas (based on the standard semantics of wrong understanding) not being first-order definable; the latter mistake that K axiom is valid for all models is, in fact, some classical formulas are artificially cut off, which may result in some first-order formulas no modal definable.

In short, in the past, it was generally believed that the standard semantics were non-classical binary semantics, which further strengthened the belief that modal logic was non-classical. In fact, it is because neither the founder of standard semantics nor have its followers figured out what it really says. The truth is that the modal semantic duality only appears in the semantic assignment definition of the relationship between $\Box A$ and A, and any modal formula as a general formula as a whole (defined item) is still a unitary semantics. Therefore, modal semantics is still unitary and, of course, classic (at most including uncertainty of 1-0 combination and permutation as L_3).

From the redundancy of "world" in the definition of basic assignment of standard semantics to the collapse of "world" levels in different modal formulas, mathematically speaking, it is because the operator "$\Box$" in modal formulas hides the ignorance of superscript and subscript to adapt to the logical nature of extreme simplification and mathematical strictness (see only the forest, not the tree). Metaphysically speaking, it is still due to the wrong path of plug-in paradigm revolution (adding new primitive syntactic symbols or semantics to form an expansion or variation system). On the way of this paradigm, which is contrary to both Occam's razor economic thinking and the endogeneity of dialectics, even with more mathematical or logical knowledge and skills, it is impossible to really understand the nature of non-truth function (and the holistic dialectical uncertainty of philosophy), let alone the modal logic.

5.5. De-Redundant Standard Semantic System. The standard semantics after de-redundancy is cluster semantics, that is, the classical semantics containing substructure clusters.

Definition 5. De-redundant standard semantic integrity assignment definition: triplet frame structure: $M = <A, R$, and $V>$, where **A** represents the set of CP formulas; R represents the relationship between formulas, and ARA' represents that formula A' can be accessible from formula A; V represents the set of assignment functions. If the variable p is assigned 1 (true) in form A, it is written as $V(p) = 1$.

(0) Assignment of any variable p, $V(p) = 1$ or $V(p) = 0$.

It can be recursive to the general formula A, $V(A) = 1$ or $V(A) = 0$, whether A contains MCP operator □ or not.

(1) Any formula A, if $V(A) = 1$, then $V(\neg A) = 0$, vice versa.

Any formula $A \wedge B$, $V(A \wedge B) = V(A) \wedge V(B)$, vice versa.

(2) $\Box A$ definition of assignment of to A (R is some relationship between accessible formulas) $V(\Box A) = 1$, iff, $\forall A'(ARA' \longrightarrow \forall A' V(A') = 1)$.

It can also be equivalently written as follows: $V(\Box A) = 1$, iff, $\forall A'$ $(ARA' \longrightarrow V(A, A') = 1)$.

(3) Simultaneous occurrence of $\Box A$ and $\Box B$ in the same modal system is as follows: $V(\Box A) = 1$, iff, $\forall A'$ $(ARA' \longrightarrow V(A, A') = 1)$; $V(\Box B) = 1$, iff, $\forall B'$ $(BRB' \longrightarrow V(B, B') = 1)$.

Only (0)-(4) above.

Note: the above (0) and (1) show that the de-redundancy standard semantics is the classical unitary semantics. (2) shows that the de-redundant standard semantics is not only the classical but also has a finer structure (this is like Guoping Du's (2007) philosophical logic revealed, the author (2018) interpretation: the basic dialectical negation of each process is consistent with deductive logic effective reasoning, but each dialectical negation process is composed of two subprocesses of negation of negation. Neither of these subprocesses is efficient reasoning, but they reflect the finer structure of the human mind) with binary relationship of cluster semantics in the modal formula. (3) It is derived completely according to the basic definition of modal formula and cluster semantics. (4) It is derived from the basic characteristics of the concealment of non-truth-valued function formula (pattern) revealed by MCP.

Confirmatory □ is ■ with Definition 4.1; MCP is BCP exactly in both syntax and semantics.

The de-redundant standard semantics cannot prove that the K axiom is valid for all models (see also the second line of Table 5 of the supplementary study, which is invalid at least when $\Box A$ on $A \vee C$ and $\Box B$ on $B \vee D$, and A, C, B, and D are assigned 0, 1, 0, and 0 in turn).

As for the assignment definition of superposition operator $\Box\Box A$, the standard semantic redundancy only changes its giving method, but does not change its definition, which is the same as that of BCP. There are similarities in the semantics of nested and mixed modal formulas. A correct understanding of the scope of modal semantics will help to better understand the corresponding relationship between modal formula and first-order formula. The problem of "equivalent substitution no hold," which has been puzzling in analytical philosophy for a long time, will also be solved quickly. In particular, it is helpful to understand dialectical logic and holism. The author will describe them in detail in later papers.

6. Effect on Information Science and Quantum Information Philosophy

At least six aspects are conducive to information science and quantum information.

First, unified information theory especially needs the underlying logic of non-classical information theory. For example, the current superpower of quantum computing cannot achieve the general effect. It is now known in principle that all kinds of non-classical logic are actually different non-classical substructures under the unified classical framework, rather than being satisfied with the appeasement of the dichotomy of object language and meta-language.

Second, with regard to the information structure, the current problem is that even if a machine as complex as the human brain is built, its scale and energy consumption are astronomical. STRF theory not only simplifies the standard natation but also hides the marks to the greatest extent and reveals that the more common form depends on the content of its representation more, so as more to simplify the software and hardware programming and integration structure level.

Third, for the strict equivalence transformation between truth function and non-truth function, the indirect reality (strictly hidden variable) and self-representation (equivalence) of information nature are expressed by strict mathematics, and the negative pseudo-equivalence substitution trouble is clarified and more efficient.

Fourth, artificial intelligence faces the intelligent problem of expressing holism, dialectics, and uncertainty. It is now known that the holistic whole is a non-(truth-valued) function of its part and is equivalent to a (truth-valued) function containing strictly hidden variables after emergency. In principle, the holism method can be conventionally mathematicized. $\Box A$ and $\Box B$ appear at the same time. Omitting two □ with different superscripts and subscripts is both the same and different, which is a typical embodiment of dialectics and also reveals the deep entanglement of information uncertainty and robustness.

Fifth, for quantum physics, it is clear that modal logic is actually the classical logic with strict syntactic hidden variables (many-valued logic is the classical logic with strict semantic hidden variables); that is, the essence of "cluster" semantics of condensed matter physics and "non-classic" of simplified standard form with von Neumann quantum logic can be discussed consistently from the perspective of modal or many-valued and can provide methodology and even directly useful research results to clarify the basic quantum problems.

Sixth, the philosophy of quantum physics must be rigorous and reliable in three aspects: decisive experiment, standard mathematical form, and philosophical logic. According to the deconstruction of the two "many-world" approaches of modal logic and quantum mechanics, the standard form of quantum mechanics is likely to hide the minimal hidden variables similar to the depths of non-classical logic. A certain ring of the whole quantum information theory which constructed from quantum entanglement and Bell inequality to quantum bit transmission and quantum computing may produce redundancy like standard semantics; this is what the author plans to overcome in the next few years.

The adaptive natural evolution process of logically expressing information, the STRF called function of special theory of relativity, did not deliberately rely on Einstein. Before the establishment of theory of special relativity, the difficult problems both in experiment (Michelson zero interference) and in mathematics (Lorentz transformation) had been solved. Based on the about ten-year philosophical thinking of simultaneity and relativity, Einstein put forward the principle of constant speed of light and the principle of special relativity, which are more innovative thought than a plenty of mathematical and physical knowledge, and derived the most universal mass-energy relationship in the material world. STRF does not rely on unfathomable mathematical or deep philosophical knowledge. Based on the common sense intuition of "three in one" reflected about ten years, STRF puts forward two basic principles: for the same thing from different perspectives, syntactically, truth function is equivalent to non-truth function; semantically, binary assignment is equivalent to unitary assignment and derives the most universal cluster semantics in the form world that adopts the semantic strength of two great men. The difference is that today's innovation, which requires the penetration of different thinking, requires more growth time for talents.

The most basic level laws often have the extreme simplicity and efficiency evolved, and there is the isomorphism of the unity of truth, goodness, and beauty in many fields, not logic just the extension of simple understanding of mechanical procedures. Mathematics is the mother of science, and philosophy is the representative of humanities. Modern logic is extremely simplified, mathematics reflects rigor, and philosophy reflects transcendence, thus building a bridge between science and humanities, analysis, and speculation, classic and non-classic, and even mind and body. The "three in one" approach not only reflects the consistent simplification of thought since the birth of modern science but also reflects that contemporary single basic science is approaching the interactive application limit of mathematics and experiment, but the knowledge explosion leads to the limited bonus of interdisciplinary research. It is also immersed in the cultural background of oriental wisdom (e.g., to see a world in a flower). In the era of fierce competition between intelligent information from the perspective of carbon neutralization and digital paradigm, more urgent needs are also entirely possible.

Unified logic and unified information is unified picture of cognitive world.

7. Conclusion

Based on the STRF theory initially created in 2014 and the "three-in-one" idea derived in 2019, this study finds four equivalences. In these discussions, the characteristics of non-truth function (formula) are gradually clarified, and the nature of modal logic is also clarified.

The obvious feature of non-truth-valued function is the concealability of its elements. If we correctly understand the characteristics of non-truth function, we can naturally understand that the standard notation system is extremely simplified, but it can also transform the modal (and many-valued) logic endogenously and equivalently; it is the first "three in one" to transform truth-valued function system into non-truth-valued function system, which has reached the halfway of apparent and complex syntactic optimization. It also reveals that many "equivalent substitutions no hold" in non-truth function are actually non-equivalent substitutions ignorant of hidden variables. Modal necessary connectives come from the second "three in one" of a cluster of large number of non-truth-valued functional expressions, which can completely optimize the syntax by hiding their own superscripts and subscripts. While optimizing syntax, it is also optimizing semantics. The third "three in one" is to smooth the ignorance of the assignment of explicit variables on non-truth-valued functions. In fact, it is the ignorance of the assignment of hidden variables in equivalent truth-valued functions. The modal formula is not a unified unitary assignment, so the equivalence transformation is expressed as a unified binary assignment; thanks to the pioneering work of modal logicians in syntax and semantics, the author can make the fourth "three in one" today, clarify the level of "world set" while removing the redundancy of standard semantics, and finally express modal logic as the classical logic with the internal substructure of the minimum formula cluster, which has the highest explanatory power.

There are more open questions: correspondence, duality, algebra; generalized, multivariate, and multimodal; and holism, dialectics, and uncertainty. They will be further explained in future papers.

BCP method is a method of facilitating discovery. Its philosophical significance is that it is not the lack of expression power of classical logic, but the lack of understanding of classical logic. Quantum puzzle and modal puzzle are the two most representative difficulties in the contemporary rigorous academic field. After the modal puzzle is solved, the author will first solve the quantum problem from the condensed matter physics in which it has another "cluster" also.

Although the mainstream modal logic (represented by standard semantics) in the 20th century has been published for 60 years (the mainstream many-valued logic has been published for 100 years) and has solved many partial problems, it claims to establish an extension or variation system of CP when it is unclear about the dependence between the new so-called primitive syntactic symbols or semantic symbols and standard notation, and it is still too hasty to refer to the history of strict cognition by the whole mankind.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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