

Research Article

Diffraction Theory Study of Twisted Light's Evolution during Phase-Only OAM Manipulations

Zhi-Yuan Zhou ^{1,2}, Zhi-Han Zhu,³ and Bao-Sen Shi^{1,2}

¹CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Anhui, Hefei 230026, China

²CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China

³Da-Heng Collaborative Innovation Center for Science of Quantum Manipulation & Control, Harbin University of Science and Technology, Harbin 150080, China

Correspondence should be addressed to Zhi-Yuan Zhou; zyzhouphy@ustc.edu.cn

Received 5 March 2023; Revised 5 May 2023; Accepted 26 May 2023; Published 8 June 2023

Academic Editor: YuBo Sheng

Copyright © 2023 Zhi-Yuan Zhou et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Twisted light, or orbital angular momentum (OAM) carrying beams or photons, is a pillar of structured photonics, a booming subfield of modern optics. The generation and transformation of twisted light, which lie with coupling topological charges into target light's wavefunctions, are usually performed by phase-only OAM manipulations. The practical twisted lights, however, obtained by this method are actually noneigen modes of free space, leading to an evolution through the diffractive propagation. Here, we present a general theoretical frame for accurately describing this evolution. Based on the theory, the whole diffractive propagation history in a typical OAM-manipulation loop, converting Gaussian light into a twisted one and then reconverting back, is demonstrated via a set of exact analytical solutions. These results provide a powerful tool for analyzing the transverse profiles of twisted light through the diffractive evolution, as a guideline for studying the science of structured light, which would provide guidelines for research studies in nonlinear optics and high-dimensional quantum information technology based on OAM-degree of freedom.

1. Introduction

Structured photonics, i.e., generation and application of light fields with custom transverse phase and polarization profiles, has gained so rapid progress over the past decade that it has gradually become an important subfield of modern optics [1]. In this new subject, orbital angular momentum (OAM) carrying beams or photons received the most attention from the optical community, ranging from high-capacity optical communications [2, 3], light trapping and manipulation [4], high-precision optical metrology [5–7], nonlinear optics [8–12], to high-dimension quantum information [13–18]. All these research studies originate from the factor that twisted light (or vortex beam) carries a spiral phase term $e^{-i\ell\theta}$ or to say carries $\ell\hbar$ orbital angular momentum each photon [19], leading to two special properties: unbounded

dimension of OAM degree-of-freedom and tailorable light-matter interface owing to its unique phase and intensity distributions. In physics layer, the subfield (OAM photonics) comes down to manipulating and controlling the transverse wavefunctions of the twisted light, or rather, coupling a helical phase $e^{-i\ell\theta}$ (or topological charge) into the target light's wavefunction. We note that these helical phase transition processes not only occur in linear interaction domain but also broadly refer to nonlinear interactions among coherent fields or photons [8–18].

Laguerre–Gaussian (LG) modes, as eigenstates of OAM degree-of-freedom and standard solutions of free-space paraxial wave equation (PWE), hence, are a natural choice for describing twisted light elegantly. For practical twisted light people used, usually converted from Gaussian light via phase-only OAM manipulation, however, is not

a natural solution of the PWE but a hypergeometric Gaussian mode which can be expressed as a linear superposition of different LG modes with the same topological charge ℓ at the waist plane [20]. We note that noneigen solution of free-space PWE means that its transverse wavefunction will not be constant in LG space upon propagation. In the general research studies, most attention is mainly focused on the initial and final states of twisted light's wavefunctions, yet the wavefunctions' evolution during propagation is also pretty important. To date, the practical twisted light and its evolution have been broadly studied via PWE's solutions [21–25]. However, in many scenarios, such as shaping 3D polarization structures [26–28], tailoring nonlinear optical interactions [29–31], and studying fundamental physics [32], to master accurate evolution history of the wavefunctions during multiple/ various manipulations is prerequisite. To address this issue, here, based on diffractive optical theory, we present a universal theoretical frame for describing the accurate dynamical evolution of the twisted light's wavefunction. The theoretical and experimental results indicate that the theoretical model can predict the whole evolution of the wavefunction during cascading phase-only OAM manipulations accurately. This framework provides a feasible way to analyse custom light fields' specific parameters with high accuracy during the whole evolution, such as polarization distributions and wavevector's axial components [26, 27, 32, 33].

2. Results and Discussion

In this work, without loss of generality, we focus on a typical case in twisted light's manipulation and control, converting Gaussian light into a twisted one and then reconverting back. Now, we consider the first step of the demonstration shown in Figure 1, as the most common situation in the generation of twisted light, and a collimated Gaussian beam is used as the initial field. The transverse wavefunction of the Gaussian beam at the beam waist w_0 , or rather the transformation plane, can be expressed as $\sqrt{2/\pi} \exp(-r^2/w_0^2)$. Coupling a helical phase $e^{-i\ell\theta}$ into the input light field's wavefunction via applying a corresponding phase mask on the SLM-1, the wavefunction will experience a phase transition and become

$$E_0(r_0, \theta_0) = \sqrt{2/\pi} \exp(-r_0^2/w_0^2) \exp(-i\ell\theta_0), \quad (1)$$

as it is leaving from the 2D transformation plane. The field shown in equation (1) can be expressed as a linear superposition of LG modes with the same topological charge $\sum_p \alpha_p LG_\ell^p$ [18]. One can note that, however, this field is not an eigenstate of free-space traveling wave field, meaning that it will evolve during the propagation and gradually approach a certain mode. Nevertheless, information stored in the wavefunction at the 2D plane (equation (1)) can completely determine the evolution during the following propagation. Thus, the wavefunction diffracting from SLM-1 can be derived from diffractive optical theory with the initial condition of $\{E_0(r_0, \theta_0), z = 0\}$, i.e., Collins Diffraction Integral Equation [8, 24, 27] shown as follows:

$$E_2(r_1, \theta_1, z_1) = \frac{i}{\lambda B} \exp(-ikz_1) \int_0^{2\pi} \int_0^\infty E_1(r_0, \theta_0) \exp \left\{ -\frac{ik}{2B} \times [Ar_0^2 - 2r_1r_0 \cos(\theta_1 - \theta_0) + Dr^2] \right\} r_0 dr_0 d\theta_0, \quad (2)$$

where z_1 is the propagation distance since leaving the transformation plane, and the ABCD matrix for free space propagation is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z_1 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

By plugging equations (1) (3) into (2), we obtain an analytical solution for describing the wavefunction during the diffraction process, a function of the propagation distance z_1 :

$$E_1(r_1, \theta_1, z_1) = \frac{i^{\ell+1}}{2\lambda z_1} \sqrt{\frac{2}{\pi}} \frac{1}{w_0} \exp(-ikz_1) \exp(-i\ell\theta_1) \exp\left(-\frac{ik}{2z_1} r_1^2\right) \frac{b_1^\ell}{\varepsilon_1^{1+\ell/2}} F\left(\frac{\ell}{2}, \ell + 1, -\frac{b_1^2}{\varepsilon_1}\right). \quad (4)$$

Equation (4) represents a hypergeometric Gaussian mode, where $F(\alpha, \beta, z)$ is the hypergeometric function, and $b_1 = kr_1/2z_1$, $\varepsilon_1 = 1/w_0^2 + ik/2z_1$. It can be noted that the helical phase term $e^{-i\ell\theta}$, referring to OAM conservation, is invariant throughout the diffracting evolution process. Figure 2(a) shows the theoretical transverse profiles of the twisted light, simulated via equation (4), upon diffraction. We can see the vortex inlayed at the center of the donut

gradually enlarges upon propagation and, especially, rapidly grows up during the early stage of evolution. More notably, although the topological charge ℓ is constant, the orthogonal radial structure changes through the propagation history. It is complex (multirings) and evolves rapidly at the beginning and, finally, approaches a simplex structure (single-ring) or to say approximates to a certain LG mode in the far field. Figure 2(b) shows the corresponding experimental results,

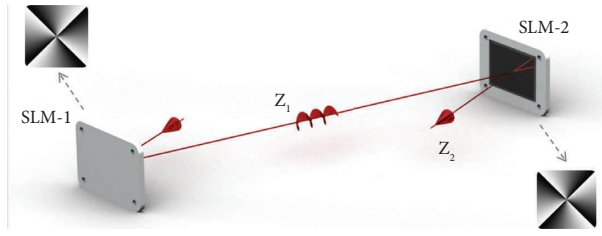


FIGURE 1: Schematic illustration of a typical OAM manipulation and control. A Gaussian light is converted into a twisted one via SLM-1 and then reconverted back.

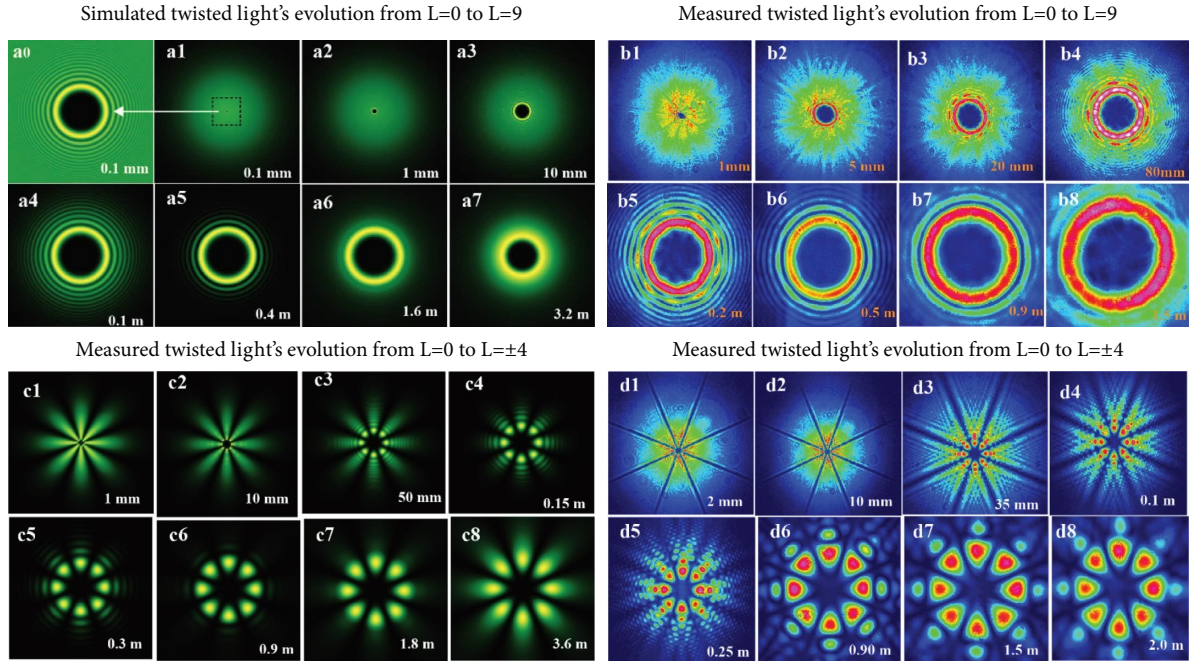


FIGURE 2: Simulated (left) and experimentally measured (right) beam profiles upon propagation distance Z_1 since being generated. A 4f system is adopted in the measurement.

which pretty much agree with the simulation. Besides, an analogue result for a superposition state $\ell = \pm 4$ is shown in Figures 2(c) and 2(d), the simulation and experimental results agree well with each other, again, proving the framework's superiority.

Next, we use the framework introduced previously to address the second step of the demonstration shown in Figure 1, i.e., reconvert the twisted light back to Gaussian modes. Here, we firstly consider a relatively simple case and the twisted light generated in the previous step has approximately evolved (propagating far enough) into an eigen

LG mode. In this simple case, the new initial condition of the reconverting process at the 2D transformation plane of SLM-2 ($z_2 = 0$) is a LG mode and can be expressed as

$$E_2(r_1, \theta_1)_{z=0} = \sqrt{\frac{2}{\pi \ell!}} \frac{1}{w_0} \left(\frac{\sqrt{2} r_1}{w_0} \right)^\ell \exp(-i\ell\theta_1) \exp(i\ell\theta_1). \quad (5)$$

By using equation (2), we can obtain an analytical solution for describing the transverse structure of the reconverted twisted light after propagating z_2 .

$$E_2(r_2, \theta_2, z_2) = \frac{ik}{2z_2} \sqrt{\frac{2}{\pi}} \frac{1}{w_0} \exp(-ikz_2) \exp\left(-\frac{ik}{2z_2} r_2^2\right) \frac{b_2^\ell}{\varepsilon_2^{1+\ell/2}} F\left(-\frac{\ell}{2}, 1, \frac{b_2^2}{\varepsilon_2}\right), \quad (6)$$

where $b_2 = kr_2/2z_2$, $\varepsilon_2 = 1/w_0^2 + ik/2z_2$. Figures 3(a) and 3(b) show the simulated transverse profile of the reconverted light upon diffraction distance z_2 . The light will lose OAM when coupling a reverse helical phase $\exp(i\ell\theta_1)$ into its

wavefunction; however, it can be seen that the transverse profile is still in a donut shape at the initial stage of diffraction. After diffracting about 0.4 m, a bright point starts to emerge at the beam center and then grows up and becomes

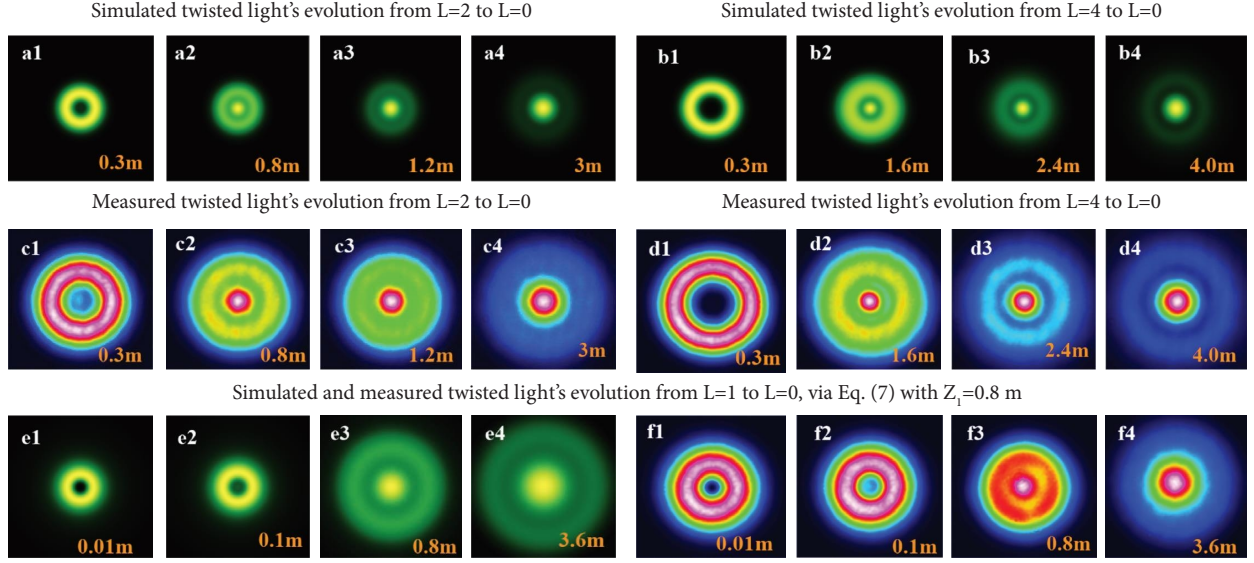


FIGURE 3: Simulated (a1–b4) and experimentally measured (c1–d4) beam profiles upon propagation distance Z_2 since being reconverted. A 4f system is adopted in the measurement. (e1–e4) and (f1–f4) are simulated beam profiles via the accurately analytical solution and the corresponding measured beam profiles.

brighter upon propagation. Finally, more than 95% power of the beam is concentrated at the central regime as propagating far enough. This light, approximately a Gaussian distribution, can be effectively coupled into a single mode fiber. In addition, a radial structure emerges during the reconverting process, accompanied by the emergence of the Gaussian spot, and it becomes more complex upon propagation and gradually faints in finally. This can be attributed to the fact that the transverse wavefunction leaving from SLM-2, as shown in equation (5), is not a free-space eigenmode. Thus, it will evolve during the propagation and gradually approach an eigenmode.

Figures 3(c) and 3(d) show the corresponding experimental results, which agree well with the theoretical prediction once again. The slight distortion of the experimental beam profiles to the theoretical beam profile can be caused by beam misalignment and low surface quality of the spatial light modulator. In experiment, Z_1 was set as 40 cm. The results shown here raise a key question concerned by most researchers dedicating in OAM photonics: how to reduce or avoid the residual outer rings left over by reconverting processes (meaning a low conversion efficiency and energy loss) and how to accomplish the processes more quickly? Here, the key point is although the topological charge ℓ changes instantaneously in mode conversions, it has left an

indelible imprint on the “carrier light” during the first diffracting propagation. To comprehend this point, one can find the parameter ℓ in terms that express as a function of propagation distance in equation (4) or standard LG modes in Gouy phase. In some degree, the question raised previously is equivalent to ask: can light or photons diffracted from a slit completely pass through another slit? Based on the time reversal invariance of wave dynamics, a straightforward proposal for overcoming this point is to erase any imprints recorded in the propagation history, namely back to the initial condition shown in equation (1), before the reconversion. A feasible method for achieving the proposal is time-reversal or phase-conjugation operation, and more specific details are out of the scope of this paper and are covered in a separate article.

At last, we further address the relatively more complex case in reconverting a twisted light back to Gaussian one, i.e., the twisted light does not propagate far enough since being generated by phase-only OAM manipulations. In this case, obviously, equation (6) is not accurate and the analytical solution shown in equation (4) should be used as the initial condition of the reconverting process. We replace it into equation (5), and after a complex calculation using equation (2), we obtain an analytical solution again:

$$E_2^\dagger(r_2, \theta_2, z_1, z_2) = \frac{k^2}{2z_1z_2} \exp[-ik(z_1 + z_2)] \exp\left(-\frac{ik}{2z_2}r_2^2\right) \frac{b_1'^\ell}{a_1^{1+\ell/2}g} g^{-\ell/2} \times \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Gamma(n + \ell/2)}{\Gamma(\ell/2)} \frac{\Gamma(\ell + 1)}{\Gamma(n + \ell + 1)} \Gamma\left(n + \frac{\ell}{2} + 1\right) \left(\frac{\tau}{g}\right)^n F\left(n + \frac{\ell}{2} + 1, 1, -\frac{b_2^2}{g}\right). \quad (7)$$

Here, the definitions of parameters in equation (7) are $b'_2 = k/2z_1$, $a_1 = 1/w_0^2 + ik/2z_1$, $b_2 = kr_2/2z_2$, $g = b_1/a_1 + ik/2z_1 + ik/2z_2$, and $\tau = b_1/a_1$, respectively. This result (not only pointing to equation (7) but broadly face towards similar situations), though it is too complicated for simulation, can be used for accurately analyzing evolution of light's transverse structure during continuous phase-only OAM manipulation, especially performed in a short interval. Figure 3(e1–e4) shows the simulated reconversion process for $L = 1$ and $Z_1 = 0.8$ m via equation (7), and the corresponding measured results are shown in Figure 3(f1–f4).

3. Conclusion

In summary, we have presented a universal theoretical framework for describing the practical twisted light during phase-only OAM manipulation and control. The transverse wavefunction, derived from this universal diffractive optical model, can accurately predict the evolution of the twisted light through a typical OAM manipulation loop, including generation, diffracting propagation, and the reverse transformation. Our results provide a helpful and powerful theoretical tool for researching and applying structured light. Moreover, by changing the initial conditions (input light) and transformation conditions (optical system parameters) in equation (2), one can easily extend this flexible framework to describe other experimental configurations, for instance, predicting OAM-multiplexed beam properties since emit from the optical system.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Key Research and Development Program of China (2022YFB3607700 and 2022YFB3900027), National Natural Science Foundation of China (NSFC) (11934013 and 92065101), Anhui Initiative in Quantum Information Technologies (AHY020200), Innovation Program for Quantum Science and Technology (2021ZD0301100), and the Space Debris Research Project of China (no. KJSP2020020202).

References

- [1] J. P. Torres and L. Torner, *Twisted Photons: Applications of Light with Orbital Angular Momentum*, John Wiley and Sons, Hoboken, NJ, USA, 2011.
- [2] J. Wang, J.-Y. Yang, I. M. Fazal et al., "Terabit free-space data transmission employing orbital angular momentum multiplexing," *Nature Photonics*, vol. 6, no. 7, pp. 488–496, 2012.
- [3] N. Bozinovic, Y. Yue, Y. Ren et al., "Terabit-scale orbital angular momentum mode division multiplexing in fibers," *Science*, vol. 340, no. 6140, pp. 1545–1548, 2013.
- [4] M. P. MacDonald, L. Paterson, K. Volke-Sepulveda, J. Arlt, W. Sibbett, and K. Dholakia, "Creation and manipulation of three-dimensional optically trapped structures," *Science*, vol. 296, no. 5570, pp. 1101–1103, 2002.
- [5] V. D'Ambrosio, N. Spagnolo, L. Del Re et al., "Photonic polarization gears for ultra-sensitive angular measurements," *Nature Communications*, vol. 4, no. 1, p. 2432, 2013.
- [6] M. P. J. Lavery, F. C. Speirits, S. M. Barnett, and M. J. Padgett, "Detection of a spinning object using light's orbital angular momentum," *Science*, vol. 341, no. 6145, pp. 537–540, 2013.
- [7] Z.-Y. Zhou, Y. Li, D.-S. Ding et al., "Optical vortex beam based optical fan for high-precision optical measurements and optical switching," *Optics Letters*, vol. 39, no. 17, Article ID 5098, 2014.
- [8] Z.-Y. Zhou, D.-S. Ding, Y.-K. Jiang et al., "Orbital angular momentum light frequency conversion and interference with quasi-phase matching crystals," *Optics Express*, vol. 22, no. 17, Article ID 20298, 2014.
- [9] G. Walker, A. S. Arnold, and S. Franke-Arnold, "Trans-spectral orbital angular momentum transfer via four-wave mixing in Rb vapor," *Physical Review Letters*, vol. 108, no. 24, Article ID 243601, 2012.
- [10] Y. Li, Z. Y. Zhou, D. S. Ding, and B. S. Shi, "Sum frequency generation with two orbital angular momentum carrying laser beams," *Journal of the Optical Society of America B*, vol. 32, no. 3, p. 407, 2015.
- [11] Y. Li, W. Z. Li, Z. Y. Zhang, K. Miller, R. Shori, and E. G. Johnson, "Concentric vortex beam amplification: experiment and simulation," *Optics Express*, vol. 24, no. 2, p. 1658, 2016.
- [12] Z.-H. Zhu, W. Gao, C.-Y. Mu, and H.-W. Li, "Reversible orbital angular momentum photon-phonon conversion," *Optica*, vol. 3, no. 2, p. 212, 2016.
- [13] G. Molina-Terriza, J. P. Torres, and L. Torner, "Management of the angular momentum of light: preparation of photons in multidimensional vector states of angular momentum," *Physical Review Letters*, vol. 88, no. 1, Article ID 013601, 2001.
- [14] R. Fickler, R. Lapkiewicz, W. N. Plick et al., "Quantum entanglement of high angular momenta," *Science*, vol. 338, no. 6107, pp. 640–643, 2012.
- [15] D.-S. Ding, W. Zhang, Z.-Y. Zhou et al., "Quantum storage of orbital angular momentum entanglement in an atomic ensemble," *Physical Review Letters*, vol. 114, no. 5, Article ID 050502, 2015.
- [16] Z.-Y. Zhou, Y. Li, D.-S. Ding et al., "Orbital angular momentum photonic quantum interface," *Light: Science & Applications*, vol. 5, no. 1, Article ID e16019, 2016.
- [17] Z.-Y. Zhou, S.-L. Liu, Y. Li et al., "Orbital angular momentum-entanglement frequency transducer," *Physical Review Letters*, vol. 117, no. 10, Article ID 103601, 2016.
- [18] A. Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein, and J. Laurat, "A quantum memory for orbital angular momentum photonic qubits," *Nature Photonics*, vol. 8, no. 3, pp. 234–238, 2014.
- [19] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Physical Review*, vol. 45, no. 11, pp. 8185–8189, 1992.
- [20] E. Karimi, G. Zito, B. Piccirillo, L. Marrucci, and E. Santamato, "Hypergeometric-Gaussian modes," *Optics Letters*, vol. 32, no. 21, p. 3053, 2007.

- [21] M. A. Bandres and J. C. Gutiérrez-Vega, "Circular beams," *Optics Letters*, vol. 33, no. 2, p. 177, 2008.
- [22] G. Vallone, "On the properties of circular beams: normalization, Laguerre Gauss expansion, and free-space divergence," *Optics Letters*, vol. 40, no. 8, p. 1717, 2015.
- [23] G. Vallone, A. Sponselli, V. D'Ambrosio, L. Marrucci, F. Sciarrino, and P. Villoresi, "Birth and evolution of an optical vortex," *Optics Express*, vol. 24, no. 15, Article ID 16390, 2016.
- [24] B. Sephton, A. Dudley, and A. Forbes, "Revealing the radial modes in vortex beams," *Applied Optics*, vol. 55, no. 28, p. 7830, 2016.
- [25] Y. Yang, Y. Dong, C. Zhao, and Y. Cai, "Generation and propagation of an anomalous vortex beam," *Optics Letters*, vol. 38, no. 24, p. 5418, 2013.
- [26] X. C. Lu, Z. W. Wu, W. H. Zhang, and L. X. Chen, "Polarization singularities and orbital angular momentum sidebands from rotational symmetry broken by the Pockels effect," *Scientific Reports*, vol. 4, no. 1, p. 4865, 2014.
- [27] T. Bauer, P. Banzer, E. Karimi et al., "Observation of optical polarization Möbius strips," *Science*, vol. 347, no. 6225, pp. 964–966, 2015.
- [28] Y. Li, Z.-Y. Zhou, D.-S. Ding, and B.-S. Shi, "Dynamic mode evolution and phase transition of twisted light in nonlinear process," *Journal of Modern Optics*, vol. 63, no. 21, pp. 2271–2278, 2016.
- [29] S. Slussarenko, A. Alberucci, C. P. Jisha et al., "Guiding light via geometric phases," *Nature Photonics*, vol. 10, no. 9, pp. 571–575, 2016.
- [30] F. Bouchard, H. Larocque, A. M. Yao et al., "Polarization shaping for control of nonlinear propagation," *Physical Review Letters*, vol. 117, no. 23, Article ID 233903, 2016.
- [31] Z.-H. Zhu, L.-W. Sheng, Z.-W. Lv, W.-M. He, and W. Gao, "Orbital angular momentum mode division filtering for photon-phonon coupling," *Scientific Reports*, vol. 7, no. 1, Article ID 40526, 2017.
- [32] Z.-Y. Zhou, Z.-H. Zhu, S.-L. Liu et al., "Quantum twisted double-slits experiments: confirming wavefunctions' physical reality," *Science Bulletin*, vol. 62, no. 17, pp. 1185–1192, 2017.
- [33] F. Bouchard, J. Harris, H. Mand, R. W. Boyd, and E. Karimi, "Observation of subluminal twisted light in vacuum," *Optica*, vol. 3, no. 4, pp. 351–354, 2016.