Research Article

Optimal Active Vibration Control of Tensegrity Structures Using Fast Model Predictive Control Strategy

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Abstract

Active vibration control of tensegrity structures is often challenging due to the geometrical nonlinearity, assemblage uncertainties of connections, and actuator saturation of controllers. To tackle these technical difficulties, a fast model predictive control (FMPC) strategy is herein implemented to effectively mitigate the structural vibration. Specifically, based on the explicit expression form of the Newmark-β method, the computation of the matrix exponential is avoided and replaced by one online and two offline transient analyses at each sampling instant on the structure, and the optimal control input is attained from the second-order dynamic equation without forming an expanded state-space equation. Meanwhile, the artificial fish swarm algorithm (AFSA) is embedded to automatically derive optimal arrangement of actuators with the selection of a reasonable objective function. Two illustrative examples, including two standard and clustered tensegrity beams and a clustered tensegrity tower, have been fully investigated. The outcomes from illustrative examples prove the effectiveness and feasibility of the proposed method in optimal active vibration control of tensegrity structures, implying a promising prospect of the investigated approach in analyzing and solving relevant engineering problems.

1. Introduction

The terminology “tensegrity” was originally coined by Fuller as a portmanteau of tensional and integrity [1]. Tensegrity structures are self-balanced flexible networks composed of discrete compressive rods and continuous tensile cables (Figure 1(a)). These networks form compliant, lightweight [2], and load-bearing systems [3] due to the equilibrium between compressive and tensile forces, and the structural configuration is determined by the distribution of internal forces across its components. Actually, the inclusion of rigid rods and elastic cables endows tensegrities with desirable properties such as flexibility, expandability, foldability, and movability, leading to booming applications in fields of civil and architecture [4–10], molecular mechanics and biomechanics [11–16], and aeronautics and aeronautics [17–21]. Meanwhile, the concept of “tensegrity” has also been widely adopted to design functional metamaterials [22–25] and novel robots [26–30]. A fundamental feature of tensegrities is the stress bilateral property: rods and cables must be under compression and tension, respectively. In other words, cables will be slacking when bearing compressive loads, which makes the system flexible and easy to regulate with minor energy input [31, 32]. Another aspect of a tensegrity structure is that its initial structural configuration must be in stable equilibrium in absence of external forces, which to a high degree determines the structural behaviors, i.e., the stiffness to external loading. Additionally, tensegrity systems possess other potential advantages: (i) it is unnecessary to employ complex joints since compressive rods are connected to flexible cables and (ii) the structures are highly promising for active control, and therefore, the
systems can be easily controlled with small amount of energy [4, 33–35]. These properties create the situation that tensegrities are candidates of active and deployable structures.

For a standard tensegrity structure, pins are used to fix the joints so as to assemble the cables and struts and integrate them into a whole system, and this operation is frequently used in practical works and is necessary to simplify the construction work. By contrast, the clustered tensegrity structures are a special category of tensegrities [36], in which several individual cables are substituted by a continuous cable with the installment of several rotating pulleys to replace the corresponding pin joints. Normally, in a clustered tensegrity, cables are classified into two types, namely, the clustered and standard ones, respectively, leading to the fact that multiple solutions of initial prestress may exist to fit the static equilibrium condition of a clustered tensegrity structure that shares identical geometrical configuration with a standard one. Figures 1(b) and 1(c) depict the mechanism of a pulley connection; a clustered cable is connected by nodes 3, 1, and 2 through a rotating pulley. The pulley is assumed to be frictionless; hence, the sliding cables on both sides of pulley share the same tensile forces, while this condition does not hold in the standard cables. Hence, compared to the equivalent standard one, a clustered tensegrity will definitely possess more internal mechanisms. Accordingly, much fewer actuators are required for vibration control of clustered tensegrities, and they indeed have the potential in the regions of shape control and active actuation.

On the other hand, for the sake that tensegrities are flexible systems, the mitigation of structural dynamic responses utilizing relevant active control techniques becomes an ongoing topic in the design of real tensegrities [37–45]. Several works have been reported on numerical simulation of small and simple tensegrity models. Nevertheless, a number of demerits have been revealed for conducting different control policies, resulting in unsatisfied dynamic performance of tensegrities. Generally, there are two aspects that may cause the invalidity: (i) as a typical nonlinear system, the dynamic output of a tensegrity structure is significantly influenced by the initial equilibrium state, i.e., the prestress in components determines the global structural stiffness [46, 47]; (ii) a tensegrity structure may have several vibratory modes; thus, it is a challenge to control its dynamic behavior owing to the structural flexibility, especially in the situation of actuator saturation, namely, some structural modes may be beyond the bandwidth of a specified controller. In practical cases, the omission of actuator saturation can cause a controller designed for structures to lose stability and even fail to work [48, 49]. Fortunately, with the rapid development of computer technology, the model predictive control (MPC) has become a reality by transforming the control saturation problem into a parameter optimization problem [50–54]. The advantage of MPC is that the control saturation can be directly considered and designed in a simple manner, leading to an effective computation for physical constraints and providing satisfactory control performance. However, the application of MPC for large-scale or complicated structures requires expensive computational cost as the future structural states over the prediction horizon are predicted by utilizing the convolution integral on the first-order state equation. Inspired by this, Peng et al. developed a fast model predictive control (FMPC) method based on the standard MPC [55]; the offline computing efficiency of FMPC is several orders of magnitude higher than that of MPC without calculating matrix exponents, leading to the huge reduction of computational complexity for large structural dynamic systems. Nevertheless, to the best of the authors’ knowledge, active vibration control of tensegrities via the MPC/FMPC was rarely seen.

Apart from that, in this article, the maximum and average nodal displacements are constructed as the objective function to automatically derive optimal arrangement of actuators by embedding the artificial fish swarm algorithm (AFSA). The AFSA, which is inspired by the collective movement of fish and their various social behaviors, is one of the best methods of optimization among the swarm intelligence algorithms [56]. Based on a series of instinctive behaviors, the fish always try to maintain their colonies and accordingly demonstrate intelligent behaviors. Searching for food, immigration, and dealing with dangers all happen in a social form, and interactions between all fish in a group will result in an intelligent social behavior. This algorithm has many advantages including high convergence speed, flexibility, fault tolerance, and high accuracy [57–60].

The rest of the paper is organized as follows. Section 2 describes the dynamic models of clustered and standard
2. Dynamic Model of Clustered and Standard Tensegrity Structures

2.1. Difference between Clustered and Standard Tensegrities. A clustered tensegrity structure normally consists of struts, standard cables, and clustered cables, in which the first two parts are the members of a standard tensegrity structure. To analyze a clustered tensegrity structure, several basic assumptions are adopted [39]:

1. Both standard and clustered cables can only bear tensile forces.
2. Struts and standard cables are connected by pin joints.
3. Clustered cables are connected by frictionless pulleys.
4. Both local buckling and global buckling of the structure are neglected.
5. All loads are applied on joints, and the high order of stiffness increment caused by external loads is omitted.

Figure 1(a) depicts a pin-joint node that connects one strut and two standard cables of a standard tensegrity, where \(l_{0,2}, l_{0,3}, l_{0,4}\) and \(f_{0,2}, f_{0,3}, f_{0,4}\) denote the length and internal force of the adjacent nodes, respectively. If the aforementioned pin-joint node is substituted by a rotating pulley, as displayed in Figure 1(b), the structural configuration is then converted to the so-called clustered tensegrity structure. Equation (1) is given to illustrate the basic features of a clustered tensegrity in conformity with the mechanism properties of the pulley:

\[
\begin{align*}
I_{2,1,3} & = I_{1,2} + I_{1,3}, \\
I_{0,4} & = I_{0,4}, \\
f_{1,3} & = f_{1,2}, \\
I_{1,2,3} & = I_{1,2} + I_{1,3}, \\
I_{0,4} & = I_{0,4}, \\
f_{1,3} & = f_{1,2},
\end{align*}
\]

(1)

where \(l_{1,2}, l_{1,3}, l_{1,4}\) and \(f_{1,2}, f_{1,3}, f_{1,4}\) are the member length and internal force of the corresponding members, respectively. The subscripts describe the linked nodes of cable members, and superscripts 0 and 1 are the initial and current structural configurations, respectively.

2.2. Dynamic Equations for Tensegrity Structures. The dynamic equation of a tensegrity structure without control force can be expressed as

\[
M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = L_1p(t),
\]

(2)

where \(M \in \mathbb{R}^{n \times n}\), \(C \in \mathbb{R}^{n \times n}\), and \(K \in \mathbb{R}^{n \times n}\) refer to the global mass, damping matrix, and stiffness matrix, respectively. \(q \in \mathbb{R}^{n \times 1}\), \(\dot{q} \in \mathbb{R}^{n \times 1}\), and \(\ddot{q} \in \mathbb{R}^{n \times 1}\) represent the vector of nodal displacement, velocity, and acceleration, respectively. \(L_1 \in \mathbb{R}^{m \times 6}\) and \(p \in \mathbb{R}^{m \times 1}\) are the position matrix of external forces and the vector of external forces, respectively.

The global mass and stiffness matrix can be formulated by the following equation [39]:

\[
\begin{align*}
M & = \sum (II M_i II^T), \\
K & = \sum (II K_i II^T),
\end{align*}
\]

(3)

where \(M_i \in \mathbb{R}^{6 \times 6}\) and \(K_i \in \mathbb{R}^{6 \times 6}\) are the elemental mass and stiffness matrix, respectively. II is the connectivity matrix that can be derived from classical finite element method. Table 1 gives the details of constructing structural stiffness matrices of clustered tensegrities.

If a control system is applied onto the structure, the dynamic equations can be written as

\[
M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = L_1p(t) + L_2u(t),
\]

(4)

where \(L_1 \in \mathbb{R}^{m \times n}\) is the position matrix of control forces and \(u \in \mathbb{R}^{m \times 1}\) is the vector of control forces.

In order to achieve a state-space representation of the controlled system, it usually converts the dynamic equation into a state-space formulation, which yields
[\begin{align*}
\Delta_{1,1,3} &= l_{1,3} - l_{1,3}^0 \\
\Delta_{2,1,3} &= l_{2,1,3} + \Delta_{1,1,3} \\
\Delta_{3,1,3} &= l_{3,1,3} - l_{1,3}^0 \\
\end{align*}]

where \( \Delta_{1,2} = l_{1,2} - l_{1,2}^0 \)

\( \Delta_{1,3} = l_{1,3} - l_{1,3}^0 \)

\( \Delta_{2,1} = l_{2,1} - l_{2,1}^0 \)

\( \Delta_{3,2} = l_{3,2} - l_{3,2}^0 \)

\( \epsilon_0 = \frac{q_0}{A_\varepsilon} \)

\( \sigma = E A_\varepsilon \epsilon \)

\( K_i' = \sum_{i=1}^{n_1} K_i + \sum_{n=1}^{n_2} K_i \)

\( K_i = T_i^T K_i T_i \)

\( K_i' = T_i^T K_i T_i \)

where \( K_i = E A_i / l_{i,3}^0 e_i c_i \)

\( c_i = [-1, 1]^T \)

\( T_i = \begin{bmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & c_2 & c_3 \end{bmatrix} \)

\( m = 1; n = [3, 2] \)

\( Y_\varepsilon = Y_\varepsilon Y_\varepsilon \)

\( J_i = l_{i,3}^0 [-c_i, c_i, -c_i^2, c_i^2, -c_i, c_i] \)

\( K_0 = K_1' + K_2' \)

\( \Pi: \text{standard FEM connectivity} \)

\( \Pi: \) Global stiffness matrix

\[ \Pi = \sum (\Pi K_0 \Pi^T) \]

\( \Pi: \text{standard FEM connectivity} \)

The output vector is written as follows:

\[ y = S x, \]

where \( y \in \mathbb{R}^{\xi \times 1} \) is the output vector, \( S \in \mathbb{R}^{\xi \times 2n} \) is the output matrix, and \( \xi \) is the number of output variables.

### 3. Formulation of the FMPC Algorithm

#### 3.1. Explicit Expression Form of the Newmark-\( \beta \) Method

In the standard MPC, the future states of the structural responses are predicted by employing the convolution integral; however, they are predicted by utilizing the explicit expression form of the Newmark-\( \beta \) method in the FMPC algorithm.

The dynamic equation omitting external forces from equation (4) can be rewritten as

\[ \dot{x}(t) = Ax(t) + Bu(t) + Gp(t), \]

where \( x \) is the state vector, \( A, B, G, 0_n \), and \( I_n \) are the state matrix, input matrix, environmental disturbance position matrix, zero matrix, and unit matrix, respectively.
\[
M \ddot{q}(t) + C \dot{q}(t) + K q(t) = L u(t).
\]  

(8)

In the Newmark-$\beta$ method, concerning the velocity and displacement, the relationship between the adjacent time steps is as follows:

\[
\dot{q}_{k+1} = \dot{q}_k + [(1 - \delta)\ddot{q}_k + \delta \ddot{q}_{k+1}] \Delta t,
\]

(9)

\[
\ddot{q}_{k+1} = \ddot{q}_k + \dot{q}_k \Delta t + \left[ \left( \frac{1}{2} - \alpha \right) \ddot{q}_k + \alpha \ddot{q}_{k+1} \right] \Delta t^2,
\]

(10)

where $\Delta t$ is the length of time step and parameters $\alpha$ and $\delta (\delta \geq 0.5, \alpha \geq 0.25 (0.5 + \delta)^2)$ are the coefficients that determine the stability and accuracy of the algorithm. At time step $t_{k+1} = t_k + \Delta t$, the dynamic responses (displacement, velocity, and acceleration) must satisfy the dynamic equilibrium condition, given by

\[
M \ddot{q}_{k+1} + C \dot{q}_{k+1} + K q_{k+1} = L_2 u_{k+1}.
\]

(11)

The dynamic responses at time step $t_{k+1}$ can be obtained by combining equations (9) and (10), given as

\[
\ddot{q}_{k+1} = \dot{q}_k + [\ddot{q}_k]_z \Delta t + \left[ \left( \frac{1}{2} - \alpha \right) \ddot{q}_k + \alpha \ddot{q}_{k+1} \right] \Delta t^2,
\]

(12)

\[
\dot{q}_{k+1} = b_1 \ddot{q}_{k+1} \Delta t + \Theta_1 q_k + \Theta_2 \dot{q}_k + \Theta_3 \ddot{q}_k + \Theta_4 \dot{\ddot{q}}_k + \Theta_5 \dddot{q}_k,
\]

(13)

\[
\ddot{q}_{k+1} = b_4 \dddot{q}_{k+1} \Delta t + \Theta_3 q_k + \Theta_4 \dot{\ddot{q}}_k + \Theta_5 \dddot{q}_k,
\]

(14)

where $\Theta$ is the equivalent stiffness matrix.

\[
\hat{K} = b_3 M + b_4 C + K.
\]

(15)

The detailed expressions of coefficients $\Theta_{\mu v}$ ($\mu, \nu = 1, 2, 3$) and $b_z$ ($z = 1, 2, 3, 4, 5, 6$) in calculating equations (12)–(15) can be found in Appendix.

Normally, the dynamic response at time steps $t_k$ and $t_{k+1}$ can be expressed as

\[
v_k = \begin{bmatrix}
q_k \\
\dot{q}_k \\
\ddot{q}_k
\end{bmatrix},
\]

(16)

\[
v_{k+1} = \begin{bmatrix}
q_{k+1} \\
\dot{q}_{k+1} \\
\ddot{q}_{k+1}
\end{bmatrix}.
\]

Hence, substitute equations (12)–(14) into (16), which yields

\[
v_{k+1} = hv_k + wu_{k+1},
\]

(17)

where

\[
h = \begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} \\
\Theta_{21} & \Theta_{22} & \Theta_{23} \\
\Theta_{31} & \Theta_{32} & \Theta_{33}
\end{bmatrix},
\]

(18)

\[
w = \begin{bmatrix}
b_1 \ddot{q}_{k+1} \Delta t \\
b_3 \dddot{q}_{k+1} \Delta t + \Theta_3 q_k + \Theta_4 \dot{\ddot{q}}_k + \Theta_5 \dddot{q}_k
\end{bmatrix}.
\]

(19)

The initial dynamic response $v_0 = [q_0^T, \dot{q}_0^T, \ddot{q}_0^T]^T$ can be obtained by substituting the initial state $x_0 = [q_0^T, \dot{q}_0^T]^T$ into Equation (11). at time step $t_0$, which yields

\[
v_0 = f_1 x_0 + f_2 u_0,
\]

(20)

By performing an iterative computation of (17), the future dynamic responses for all time steps can be described as

\[
v_k = \hat{H} x_0 + \sum_{j=0}^{k-1} \hat{W}_{kj} u_j, \quad 1 \leq k \leq N,
\]

(21)

where $N$ is the total number of time steps.

\[
\hat{H}_k = h^k f_1,
\]

(22)

\[
\hat{W}_{kj} = \begin{bmatrix}
h^k f_2, j = 0, \\
h^k f_2, 1 \leq j \leq k
\end{bmatrix}.
\]

Equation (21) can be rewritten in matrix form as follows:

\[
V = \hat{H} X_0 + \hat{W} u = \begin{bmatrix}
h f_1 \\
h^2 f_1 \\
h^3 f_1 \\
\vdots \\
h^N f_1
\end{bmatrix} X_0 + \begin{bmatrix}
h f_2 & w & 0 & \cdots & 0 \\
h^2 f_2 & hw & w & \cdots & 0 \\
h^3 f_2 & h^2 w & hw & w & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
h^N f_2 & h^{N-1} w & h^{N-2} w & \cdots & w
\end{bmatrix} u_N,
\]

(23)

where $V$ is the aggregation of all future dynamic responses.
According to explicit expression form in (21) and the output (7), the future outputs $y_k$ for all prediction points are given as follows:

$$y_k = \hat{S}H_kx_0 + \sum_{j=0}^{k} \hat{S}W_{k,j}u_j, \quad 1 \leq k \leq N,$$

(24)

where $\hat{S} = [\bar{S} \ 0]$ is a $3 \times 3n$ matrix.

Equation (24) can also be expressed in matrix form:

$$Y = Fx_0 + GU = \hat{S}Hx_0 + \hat{S}WU.$$

(25)

The symbol $\cdot$ denotes that if $A$ is a $m \times p$ dimensional matrix and $B$ is a $k \times l$ block matrix with $p \times n$ dimensional submatrices, then $A \cdot B$ is a $k \times l$ block matrix with $m \times n$ dimensional submatrices:

$$A \cdot B = \begin{bmatrix} AB_{11} & \cdots & AB_{1l} \\ \vdots & \ddots & \vdots \\ AB_{k1} & \cdots & AB_{kl} \end{bmatrix}.$$

(26)

Hence, the performance index $J$ is given as

$$J = \frac{1}{2}Y^TQY + \frac{1}{2}U^TRU,$$

(27)

where $Q \in \mathbb{R}^{N \times N\xi}$ is a nonnegative definite symmetric weighting matrix and $R \in \mathbb{R}^{Nm \times Nm}$ is a positive definite symmetric weighting matrix.

The optimal control input sequence $U$ is subsequently achieved by minimizing the performance index $J$ given in equation (27), which yields

$$U = -G^TQF \bar{x}_0 = -\bar{K}x_0.$$

(28)

3.2. Fast Computation for FMPC. The optimal control input sequence $U$ can be divided into two parts, namely, the offline part $\bar{K}_1$ and the online part $\bar{K}_2$:

$$U = -\bar{K}_1\bar{K}_2,$$

(29)

where

$$\bar{K}_1 = \left( (\hat{S} \cdot \hat{W})^T Q (\hat{S} \cdot \hat{W}) + R \right)^{-1} (\hat{S} \cdot \hat{W})^T Q,$$

(30)

$$\bar{K}_2 = \hat{S} \cdot Hx_0.$$

(31)

According to the above equations, the key for computation of $\bar{K}_1$ and $\bar{K}_2$ is to obtain matrix $W$ and $Hx_0$, respectively. Therefore, the fast computations of these two matrices are taken into account.

3.2.1. Fast Computation for Matrix $\hat{W}$. As is observed from equation (23), as the other block columns can be easily computed from $W_{k,2}$, only the first two block columns ($W_{k,1}$ and $W_{k,2}$) in matrix $W$ need to be determined. According to the physical meaning of matrix $W_{k,1}$, $W_{k,1}$ and $W_{k,2}$ can be derived by setting the initial state $x_0 = 0$ and applying the unit control input $u_0 = I_m$ onto the dynamic system at time steps $t_0$ and $t_1$, respectively. Therefore, the dynamic responses for all time steps can be formulated into following forms:

$$V = \begin{bmatrix} h_f^2 & w & 0 & \cdots & 0 \\ h^2 & hw & w & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h^N & h^{N-1}w & h^{N-2}w & \cdots & w \end{bmatrix} \begin{bmatrix} I_m \\ 0 \\ 0 \\ \vdots \end{bmatrix},$$

(32)

$$V = \begin{bmatrix} w \\ hw \\ \vdots \\ h^{N-1}w \end{bmatrix} = \hat{W}_{k,2}.$$

(33)

Equations (32) and (33) denote that $\hat{W}_{k,1}$ and $\hat{W}_{k,2}$ can be obtained by carrying out one transient analysis on the dynamic system using the Newmark-β method with a zero initial state and a unit control input at time steps $t_0$ and $t_1$.

3.2.2. Fast Computation for Matrix $\hat{H}x_0$. According to the explicit expression form of the Newmark-β method, if only an initial state $x_0$ is applied to the system (without control input), the dynamic responses for all time steps can be rewritten as

$$V = \hat{H}x_0.$$

(34)

Equation (34) shows that $\hat{H}x_0$ can be computed by carrying out one transient analysis on the dynamic system with the setting of initial state $x_0$ without control input.

Once $\hat{K}_1$ and $\hat{K}_2$ are determined, the optimal control input $U$ can be attained according to (29).

4. The Artificial Fish Swarm Algorithm (AFSA)

The AFSA, which was developed by Li et al. [62], is a population-based optimization technique inspired by natural fish swarm schooling behaviors. Due to the efficiency in
solving engineering issues, AFSA has gained vast popularity in the past few decades. As a typical swarm intelligent algorithm, each artificial fish hunts for food in conformity with its own manner, including but not limited to random, foraging, swarming, and following behaviors. Each artificial fish allows mutual information communications until attaining a global optimum. More importantly, the gradient information is not a necessary condition in the process of the optimization. Hence, AFSA is widely adopted in the searching of global optimal solutions due to the fact that it takes full advantage of the concentrated emerging mechanism of the individual intelligence [63].

The concept of AFSA is described as follows: a fish swarm comprised of $N$ artificial fish exists in a $d$-dimensional space. Let $T_i = (T_{i1}, T_{i2}, \cdots, T_{id})$ denote the current position of an artificial fish, namely, the position matrix of control inputs $L_2$ in equation (4); the food consistency (the objective function) that this designated fish at position $T_i$ can recognize is depicted as $O_i = f(T_i)$. Visual, $\delta$, and Step, respectively, reflect the perceiving range, the congestion factor, and the moving step. Detailed definitions of the four behaviors are as follows.

4.1. Random Behavior. This is a default behavior, which describes a phenomenon that in the visual range of a fish, the fish randomly selects a target position and moves towards it.

4.2. Foraging Behavior. Foraging is known as the basic behavior to search for food, which is on the basis of a random forage with a tendency towards food concentration. Let $T_j$ represent the position in the visual range of a fish at the current time. For a mathematical minimization problem, if $O_i < O_j$, the fish will move a Step forward in the direction of $(T_j - T_i)$. If not, randomly select a new state $T_j$ and judge whether it can meet the forward condition. The random behavior is performed if the foraging behavior is invalid after preset try-number times, and therefore, the position $T_i^*$ can be updated as

$$T_i^* = \begin{cases} T_i + \frac{T_j - T_i}{d_{ij}} \cdot \text{step} \cdot \text{rand}, & \text{if } O_j < O_i, \\ \text{random behavior, otherwise,} & \end{cases}$$

where $d_{ij} = \|T_i - T_j\|$ denotes the distance between $T_i$ and $T_j$ and rand is generated uniformly within the range $[0, 1]$.

4.3. Swarming Behavior. Fish are species that are sensitive to the external environment, and they usually gather in several groups to minimize possible threats. In a fish swarm of $N_F$ artificial fish, the central position $T_c$ is explored by each fish $T_i$ in its current neighborhood ($d_{ij} < \text{Visual}$). Meanwhile, fish $T_i$ will step forward to $T_c$ if it satisfies the condition $O_j / N_F < \delta \cdot O_j$.

Theoretically,

$$T_i + \frac{T_c - T_i}{d_{ic}} \cdot \text{step} \cdot \text{rand}, \quad \text{if } \frac{O_i}{N_F} < \delta \cdot O_i,$$

$$\text{foraging behavior, otherwise,}$$

4.4. Following Behavior. If one fish is located in a position with a large food concentration coefficient, other fish will follow their neighbors to feed within their visual range. Suppose $T_{\text{best}(i)}$ is the local best companion in the current neighborhood of $T_i$. If $O_{\text{best}(i)} / N_F < \delta \cdot O_i$, fish $T_i$ then attempts to step forward in the direction $(T_{\text{best}(i)} - T_i)$. The following behavior can be conducted as

$$T_i^* = \begin{cases} T_i + \frac{T_{\text{best}(i)} - T_i}{d_{\text{best}(i)}} \cdot \text{step} \cdot \text{rand}, & \text{if } \frac{O_{\text{best}(i)}}{N_F} < \delta \cdot O_i, \\ \text{foraging behavior, otherwise.} & \end{cases}$$

The aforementioned four behaviors are compared and implemented for each artificial fish. Nonetheless, only the best behavior is chosen to renew the current position. Additionally, in a fish swarm, bulletin is employed to record the optimum state $O_{\text{best}}$. In other words, in each step, the state of each fish is compared with the former one, and better state will be renewed automatically in the bulletin.

Overall, Figure 2 gives the flowchart of achieving optimal control strategy using the proposed method. The proposed control strategy can be decomposed into two segments, namely, the FMPC and the AFSA. The FMPC algorithm works online since the control input sequence $U$ is determined by the structural state at each time. Strictly, AFSA is an offline swarm intelligence algorithm, and the function of conducting the AFSA algorithm in this article is only to acquire the optimal actuator placement. However, it is worth mentioning that during the optimization of AFSA, the reservation of elite fish relies on the comparison of output information (the current structural state determined by the control input sequence $U$) generated from FMPC, and therefore, AFSA also works online in this sense.

5. Numerical Case Study

In this section, three illustrative examples including a standard tensegrity beam, a clustered tensegrity beam, and a clustered tensegrity tower are studied utilizing MATLAB (R2021b) platform. The results imply that the presented methods work remarkably well on active vibration control for both standard and clustered tensegrity structures, and reasonable control effect is achieved with optimal arrangement of actuators. In control design, the self-weight of the structure was ignored, i.e., the gravity was not considered, and the weighting matrices are chosen as $Q = 1 \times 10^3 I_1$. 
Initial parameter setting such as Visual, δ and Step

Generate the current position of an artificial fish $T_i, i = 1$ to $\hat{N}$

Calculate the objective function $O_i, i = 1$ to $\hat{N}$

Update the best state $T_{best}$

Check if the maximum number of generations is reached?

Yes

Output the best state $T_{best}$

Output the optimal result $O_{best}$

No

Perform random behavior

Perform foraging behavior in Eq. (35)

Perform swarming behavior in Eq. (36)

Perform following behavior in Eq. (37)

Updated position matrix of control inputs $L_i$ in Eq. (4)

Apply harmonic load $p_{k-1}$

Time integration

\[ \begin{align*}
q_{k-1}, q_\dot{k-1}, q_\ddot{k-1} \\
p_{k-1} \\
u_{k-1}
\end{align*} \]

\[ \Rightarrow q_k, q_\dot{k}, q_\ddot{k} \]

Update $u_{k-1} = u_k$

Compute the on-line part $K_2$ in Eq. (31)

Compute the off-line part $K_1$ in Eq. (30)

Compute the optimal control input sequence $U$ in Eq. (29)

No

Total number of time steps has been reached?

No

Yes

Update $x_{k-1} = x_k$

Record the optimum objective function $O_{best}$ in the bulletin

Figure 2: The flowchart of achieving optimal control strategy using the proposed method.
and \( \mathbf{R} = 1 \times 10^{-2} \mathbf{I}_2 \), where \( \mathbf{I}_1 \in \mathbb{R}^{N \times N} \) and \( \mathbf{I}_2 \in \mathbb{R}^{N \times m} \) are unit matrices.


Two numerical models with identical topological connectivity and geometrical configuration for standard and clustered tensegrity beams, as shown in Figure 3, are herein adopted for comparison purpose. Each structure is composed of 6 quadruple units with 24 struts and 67 cables, among which 32 clustered cables for clustered tensegrity beam are denoted by dashed lines in four colors, and line in the same color represents a clustered cable with identical attribute. For example, the green and brown dashed lines that respectively connects nodes 4, 8, 10, 13, 14, 18, 20, 23, 24, 28, and 30 and nodes 1, 6, 11, 16, 21, 26, and 31 are upper and lower clustered cables. The purple and grey solid lines represent the corresponding struts and standard cables, respectively. Besides, nodes 1, 2, and 3 were completely fixed as the boundary condition to simulate a cantilever beam. Tables 2 and 3 give the geometrical and mechanical parameters of elements, which were determined from our previous studies [64–66].

A harmonic function \( \mathbf{P}(t) \), which was applied in the Z-direction on each node with time step of 0.020 s, was selected as the external loading (N), given as

\[
\mathbf{P}(t) = 10000 \sin(2t + 40), \quad t \in [0 \ 30].
\]

To be clear, the control rate in this article is defined as the ratio of nodal displacement difference (generated by uncontrolled and controlled status) to the uncontrolled nodal displacement. Figure 4 shows the displacement curves of nodes 10, 12, 15, 17, 20, 25, 30, and 32 in the Z-direction for standard and clustered tensegrity cantilever beams with four actuators located at nodes 28, 31, 32, and 33. Table 4 displays the comparative results on maximum displacement and the corresponding control efficiency in mitigating nodal displacement. For the standard tensegrity cantilever beam, the maximum displacement of node 17 situated at midspan was 1.72 m without control, and it diminished to 0.75 m with the intervention of FMPC control (with a reduction rate of 56.26%). More interestingly, with the aid of actuators installed at the end of the cantilever beam, the maximum displacement of the ending node 32 acquired a sharp decline form 2.13 m to 0.71 m after control (with a reduction rate of 66.50%), even smaller than that of node 17. Similar phenomena can be found in the cases of the clustered tensegrity beam, indicating the effectiveness of the FMPC method in active vibration control of both standard and clustered tensegrity structures. Additionally, for a same control policy, i.e., the identical number and placement of actuators, the clustered tensegrity beam vibrates faster to a stable state compared to
the standard one, demonstrating the superiority of clustered tensegrity structures in vibration control due to continuous sliding clustered cables.

Apparently, the FMPC method has good performance in vibration control of tensegrity cantilever beams, and the control effect varies with different control policies, as the number and placement of actuators for each policy are distinctive. Figure 5 exhibits the comparative results on control effect of observed node 33 with different control policies. Interestingly, some significant information can be addressed by taking a deep look at Table 5:

(1) For a standard or clustered tensegrity beam with identical placement of actuators, better control rates can be acquired with the increasing number of actuators. For example, readers can refer to the results of control policies with actuators placed on nodes (6, 11, 16) or (6, 11, 16, 21) for standard tensegrity beam, or control policies with actuators placed on nodes (17, 22, 27) or (17, 22, 27, 32) for clustered tensegrity beam.

(2) For a standard or clustered tensegrity beam with identical number of actuators, the closer

Table 2: Geometrical and mechanical properties of standard and clustered tensegrity beams.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard tensegrity structure</th>
<th>Clustered tensegrity structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross section of struts</td>
<td>$3.2 \times 10^{-2} \text{m}^2$</td>
<td>$3.2 \times 10^{-2} \text{m}^2$</td>
</tr>
<tr>
<td>Cross section of cables</td>
<td>$1 \times 10^{-3} \text{m}^2$</td>
<td>$1 \times 10^{-3} \text{m}^2$</td>
</tr>
<tr>
<td>Young’s modulus of struts</td>
<td>$200 \times 10^3 \text{Pa}$</td>
<td>$200 \times 10^3 \text{Pa}$</td>
</tr>
<tr>
<td>Young’s modulus of cables</td>
<td>$40 \times 10^3 \text{Pa}$</td>
<td>$40 \times 10^3 \text{Pa}$</td>
</tr>
<tr>
<td>Density of cables and struts</td>
<td>$7800 \text{kg/m}^3$</td>
<td>$7800 \text{kg/m}^3$</td>
</tr>
<tr>
<td>Prestress in struts</td>
<td></td>
<td>$-357360.98 \text{N}$</td>
</tr>
<tr>
<td>Prestress in peripheral lower cables</td>
<td></td>
<td>$192500 \text{N}$</td>
</tr>
<tr>
<td>Prestress in internal lower cables</td>
<td></td>
<td>Refer to Table 3</td>
</tr>
<tr>
<td>Prestress in upper cables</td>
<td></td>
<td>$385000 \text{N}$</td>
</tr>
<tr>
<td>Prestress in bracing cables</td>
<td></td>
<td>$385000 \text{N}$</td>
</tr>
</tbody>
</table>

Table 3: Prestress for standard tensegrity structure beam.

<table>
<thead>
<tr>
<th>Upper cables</th>
<th>Lower cables</th>
<th>Bracing cables</th>
<th>Struts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node number</td>
<td>Prestress (N)</td>
<td>Node number</td>
<td>Prestress (N)</td>
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<tr>
<td>4</td>
<td>357360.98</td>
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<td>178680.49</td>
</tr>
<tr>
<td>8</td>
<td>226459.55</td>
<td>2</td>
<td>357360.98</td>
</tr>
<tr>
<td>10</td>
<td>320102.26</td>
<td>3</td>
<td>357360.98</td>
</tr>
<tr>
<td>13</td>
<td>316279.94</td>
<td>4</td>
<td>357360.98</td>
</tr>
<tr>
<td>14</td>
<td>325044.97</td>
<td>5</td>
<td>357360.98</td>
</tr>
<tr>
<td>18</td>
<td>307049.18</td>
<td>6</td>
<td>357360.98</td>
</tr>
<tr>
<td>20</td>
<td>322062.87</td>
<td>7</td>
<td>357360.98</td>
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<td>23</td>
<td>326806.93</td>
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<td>357360.98</td>
</tr>
<tr>
<td>28</td>
<td>344480.70</td>
<td>9</td>
<td>357360.98</td>
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<tr>
<td>5</td>
<td>226617.38</td>
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<td>357360.98</td>
</tr>
<tr>
<td>13</td>
<td>445063.92</td>
<td>11</td>
<td>357360.98</td>
</tr>
<tr>
<td>15</td>
<td>495063.92</td>
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<td>357360.98</td>
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<td>600063.92</td>
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</tr>
<tr>
<td>28</td>
<td>720063.92</td>
<td>16</td>
<td>357360.98</td>
</tr>
<tr>
<td>30</td>
<td>780063.92</td>
<td>17</td>
<td>357360.98</td>
</tr>
</tbody>
</table>
Actuators are placed to the observation point, the better the control rate can be achieved. For example, readers can refer to the results of control policies with three actuators placed on nodes (11, 12, 13) or (21, 22, 23) for standard tensegrity beam, or control policies with four actuators placed on nodes (13, 14, 15, 18) or (23, 24, 25, 28) for clustered tensegrity beam.

(3) For a standard or clustered tensegrity beam with same control condition, namely, identical number of actuators and distance to the observation point, the control rates are nearly the same for actuators placed

Figure 4: Nodal displacement responses for standard and clustered tensegrity beams in the $Z$-direction: (a) node 10, (b) node 12, (c) node 15, (d) node 17, (e) node 20, (f) node 25, (g) node 30, and (h) node 32.
on upper or lower nodes. For example, readers can refer to the results of control policies with three actuators placed on nodes (6, 11, 16) or (8, 13, 18) for standard tensegrity beam, or control policies with four actuators placed on nodes (21, 22, 26, 27) or (23, 24, 25, 28) for clustered tensegrity beam.

(4) For most cases, the control rates of the clustered tensegrity beam are significantly higher than those of the standard tensegrity beam with identical control policy, confirming the superiority of clustered tensegrity structures in vibration control.

5.2. A Six-Layer Clustered Tensegrity Tower. A six-layer clustered tensegrity tower (Figure 6), which consists of six quadruple units with 24 struts and 60 cables, is herein utilized to search for optimal control policy. Similarly, 24 clustered cables are denoted by dashed lines in four colors, and line in the same color represents a clustered cable with identical attribute. The solid lines in purple and grey represent the struts and standard cables, respectively. Nodes 3, 17, 10, and 24 are fully constrained. Table 6 shows the information of structural parameters, and the elemental prestress is given in Table 7. The external loadings, given in (40), are applied in the $X$-direction on each node with time step of 0.020 s.

$$P(t) = -10000 \sin(2t + 40), \quad t \in [0 \ 30]. \quad (39)$$

In this example, the aforementioned artificial fish swarm algorithm is carried out to search for optimal control policies with given number of actuators, and the following

<p>| Table 4: The comparative results on maximum displacement and the control efficiency of two tensegrities. |
|---------------------------------------------------------------|---------------------------------|-----------------------------|---------------------------------|---------------------------------|
| Node number | Standard tensegrity beam | Clustered tensegrity beam |</p>
<table>
<thead>
<tr>
<th>Uncontrolled (m)</th>
<th>With FMPC (m)</th>
<th>Control rate (%)</th>
<th>Uncontrolled (m)</th>
<th>With FMPC (m)</th>
<th>Control rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.83</td>
<td>0.35</td>
<td>57.54</td>
<td>0.32</td>
<td>0.09</td>
</tr>
<tr>
<td>12</td>
<td>1.23</td>
<td>0.53</td>
<td>57.05</td>
<td>0.53</td>
<td>0.15</td>
</tr>
<tr>
<td>15</td>
<td>1.59</td>
<td>0.68</td>
<td>56.99</td>
<td>0.71</td>
<td>0.20</td>
</tr>
<tr>
<td>17</td>
<td>1.72</td>
<td>0.75</td>
<td>56.26</td>
<td>0.89</td>
<td>0.25</td>
</tr>
<tr>
<td>20</td>
<td>1.75</td>
<td>0.81</td>
<td>53.84</td>
<td>1.06</td>
<td>0.31</td>
</tr>
<tr>
<td>25</td>
<td>1.26</td>
<td>0.81</td>
<td>35.86</td>
<td>1.33</td>
<td>0.39</td>
</tr>
<tr>
<td>30</td>
<td>1.66</td>
<td>0.67</td>
<td>59.46</td>
<td>1.52</td>
<td>0.44</td>
</tr>
<tr>
<td>32</td>
<td>2.13</td>
<td>0.71</td>
<td>66.50</td>
<td>1.60</td>
<td>0.47</td>
</tr>
</tbody>
</table>
Table 5: The comparative results on control effect of node 33 with different control strategies.

<table>
<thead>
<tr>
<th>Actuation position</th>
<th>Uncontrolled</th>
<th>Standard tensegrity beam 2.13 m</th>
<th>Control rate (%)</th>
<th>Clustered tensegrity beam 1.59 m</th>
<th>Control rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 11 16</td>
<td>1.68</td>
<td>21.30</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>17 22 27</td>
<td>1.35</td>
<td>36.61</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>8 13 18</td>
<td>1.70</td>
<td>20.10</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>18 23 28</td>
<td>1.36</td>
<td>36.03</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>11 12 13</td>
<td>1.67</td>
<td>21.58</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>21 22 23</td>
<td>1.56</td>
<td>26.76</td>
<td>—</td>
<td>—</td>
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</tr>
<tr>
<td>Three actuators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 11 16</td>
<td>—</td>
<td>—</td>
<td>1.27</td>
<td>19.86</td>
<td>—</td>
</tr>
<tr>
<td>17 22 27</td>
<td>—</td>
<td>—</td>
<td>0.70</td>
<td>56.22</td>
<td>—</td>
</tr>
<tr>
<td>8 13 18</td>
<td>—</td>
<td>—</td>
<td>1.26</td>
<td>20.67</td>
<td>—</td>
</tr>
<tr>
<td>18 23 28</td>
<td>—</td>
<td>—</td>
<td>0.68</td>
<td>57.06</td>
<td>—</td>
</tr>
<tr>
<td>11 12 13</td>
<td>—</td>
<td>—</td>
<td>1.34</td>
<td>15.96</td>
<td>—</td>
</tr>
<tr>
<td>21 22 23</td>
<td>—</td>
<td>—</td>
<td>0.69</td>
<td>56.69</td>
<td>—</td>
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<tr>
<td>Four actuators</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>11 12 16 17</td>
<td>1.46</td>
<td>31.31</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>13 14 15 18</td>
<td>1.46</td>
<td>31.31</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>6 11 16 21</td>
<td>1.47</td>
<td>31.15</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>17 22 27 32</td>
<td>1.23</td>
<td>42.21</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>21 22 26 27</td>
<td>1.26</td>
<td>40.84</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>23 24 25 28</td>
<td>1.35</td>
<td>36.63</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Figure 6: A six-layer clustered tensegrity tower: (a) perspective view, (b) top view, and (c) side view.
optimization model is formulated to mitigate structural vibration:

\[
\text{minimize } f(x) = \sum_{i=1}^{24} (\alpha |U_i(x)| + \beta |U_i(x)|_{\text{max}}).
\]

subject to \[
\begin{align*}
0 &< \alpha \leq 1, \\
0 &< \beta \leq 1, \\
1 &\leq x \leq 28, x \neq 3, 10, 17, 24,
\end{align*}
\] where \( x \) is a variable representing the expected placement of actuators, which ranges from 1 to 28 (the tower has 28 nodes with 4 fixed nodes 3, 10, 17, and 24), \( i \) is the node number, \( \alpha \) and \( \beta \) are the weight coefficients designated by designers, \( U_i(x) \) is the average amplitude in \( X \)-direction of node \( i \) within the observation time (30s), and \( |U_i(x)|_{\text{max}} \) is the maximum amplitude in \( X \)-direction of node \( i \) within the observation time (30s).

In the case of control policy with four actuators, the parameters in the AFSA algorithm are set as follows: \( N = 180 \) (number of artificial fish), \( \text{MaxGen} = 150 \) (maximum

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross section of struts</td>
<td>( 3.2 \times 10^{-2} \text{m}^2 )</td>
</tr>
<tr>
<td>Cross section of cables</td>
<td>( 1 \times 10^{-2} \text{m}^2 )</td>
</tr>
<tr>
<td>Young's modulus of struts</td>
<td>( 200 \times 10^9 \text{Pa} )</td>
</tr>
<tr>
<td>Young's modulus of cables</td>
<td>( 40 \times 10^9 \text{Pa} )</td>
</tr>
<tr>
<td>Density of components</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>Prestress in components</td>
<td>Refer to Table 7</td>
</tr>
</tbody>
</table>

**Table 6: Parameters of a six-layer clustered tensegrity tower.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross section of struts</td>
<td>( 3.2 \times 10^{-2} \text{m}^2 )</td>
</tr>
<tr>
<td>Cross section of cables</td>
<td>( 1 \times 10^{-2} \text{m}^2 )</td>
</tr>
<tr>
<td>Young's modulus of struts</td>
<td>( 200 \times 10^9 \text{Pa} )</td>
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<tr>
<td>Young's modulus of cables</td>
<td>( 40 \times 10^9 \text{Pa} )</td>
</tr>
<tr>
<td>Density of components</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>Prestress in components</td>
<td>Refer to Table 7</td>
</tr>
</tbody>
</table>

**Table 7: Prestress for clustered tensegrity tower.**

<table>
<thead>
<tr>
<th>Outer bracing cables</th>
<th>Prestress (N)</th>
<th>Horizontal cables</th>
<th>Prestress (N)</th>
<th>Struts</th>
<th>Prestress (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node number</td>
<td>Prestress (N)</td>
<td>Node number</td>
<td>Prestress (N)</td>
<td>Node number</td>
<td>Prestress (N)</td>
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<tr>
<td>3</td>
<td>2</td>
<td>219567.41</td>
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<td>17</td>
<td>109783.71</td>
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<td>17</td>
<td>10</td>
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<tr>
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<td>7</td>
<td>219567.41</td>
<td>10</td>
<td>24</td>
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</tr>
<tr>
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<td>7</td>
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<td>24</td>
<td>3</td>
<td>109783.71</td>
</tr>
<tr>
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</table>
number of iterations), $\text{Try}_{\text{number}} = 5$ (maximum test number of prey), $\text{Step} = 2$ (moving step), $\Delta = 0.423$ (congestion factor), and Visual = 1.0 (perception of distance). Figure 7 displays the ladder-like convergent curve of the objective function, in which four stages are shown clearly during the optimization process. The results imply that the feasible actuator positions are obtained at the $3^{\text{rd}}$, $10^{\text{th}}$, $20^{\text{th}}$, and $30^{\text{th}}$ generations, with actuators placed on nodes $(6, 11, 12, 18)$, $(4, 5, 18, 19)$, $(4, 6, 11, 18)$, and $(4, 11, 18, 19)$, respectively. The optimal solution is acquired at the $40^{\text{th}}$ generation with the placement of actuators on nodes $(4, 11, 12, 18)$. Nevertheless, if we take a deep look at Figure 8, it can be found that the feasible solutions derived at the $3^{\text{rd}}$, $10^{\text{th}}$, $20^{\text{th}}$, and $30^{\text{th}}$ generations are quite close to the optimal one attained at the $40^{\text{th}}$ generation in mitigating the vibration of the clustered tower as node 25 has the maximum displacement in $X$-direction under external loadings, proving the fast searching ability in optimization using FMPC and AFSA. More importantly, Table 8 depicts the results of maximum displacement for each tower layer with and without control. Apparently, the maximum displacement of the tensegrity tower emerges at the upper layer with an effective control rate of 41.75% under the obtained optimal control policy, indicating the validity of the presented algorithm in active vibration control of clustered tensegrity structures. Additionally, according to the aforementioned optimal actuator placement, Figure 9 gives the displacement-force curvature of actuators with regard to the observation nodes on the 2nd, 4th, and 6th layer, respectively. It can be concluded that for each actuator under the same observation node, the actuator force is relatively uniform and the amplitude will not exceed the saturation value preset on the actuator, which confirms the validity of the proposed control strategy in guaranteeing the stability of actuator performance during work.

To further investigate the factors that may influence the searching quality in the optimization procedure, two crucial indicators are herein considered, namely, the number of iterations/generations and artificial fish. A total of twelve cases are fully discussed, i.e., Case I (30 fish with 30 generations), Case II (30 fish with 60 generations), Case III (30 fish with 90 generations), Case IV (30 fish with 120 generations), Case V (30 fish with 150 generations), Case

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**Table 8:** The maximum displace-ments and the corresponding control rates for each layer.

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Node number</th>
<th>Max. displacement (m)</th>
<th>Control rate (%)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Uncontrolled</td>
<td>With FMPC</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>0.45</td>
<td>0.26</td>
</tr>
<tr>
<td>15</td>
<td>28</td>
<td>0.84</td>
<td>0.48</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>1.24</td>
<td>0.72</td>
</tr>
<tr>
<td>25</td>
<td>12</td>
<td>1.61</td>
<td>0.93</td>
</tr>
<tr>
<td>30</td>
<td>18</td>
<td>1.94</td>
<td>1.13</td>
</tr>
</tbody>
</table>
Figure 9: The relationship between the nodal displacement in X-direction and the actuator forces on the obtained optimal position of actuators: (a) node 15 on the 2nd layer; (b) node 13 on the 4th layer; (c) node 18 on the 6th layer.

Figure 10: Optimization process of the objective function with 30 artificial fish under different number of iterations: (a) 30 generations; (b) 60 generations; (c) 90 generations; (d) 120 generations; (e) 150 generations; (f) 180 generations.
VI (30 fish with 180 generations), Case VII (60 fish with 150 generations), Case VIII (80 fish with 150 generations), Case IX (100 fish with 150 generations), Case X (120 fish with 150 generations), Case XI (140 fish with 150 generations), and Case XII (160 fish with 150 generations).

Figure 10 gives the convergence curves of different generations in the optimization process of the objective function with 30 artificial fish. The value of the objective function reaches 22.9259, 22.3675, 22.3675, 22.3675, 22.3675, and 22.3675 for generation of 30, 60, 90, 120, 150, and 180, respectively, implying that reasonable selection of generation is critical as small iterations will lead to divergence of the optimization problem. Another index is the fish number, as is shown in Figures 11(a)–11(c), multiple platforms are emerged during the optimization process of the objective function. However, as is displayed in Figures 11(d)–11(f), these platforms diminish with the raising of the fish number. Moreover, the searching speed is also accelerated with the configuration of more artificial fish, and this can be explained by the fact that the probability in finding excellent individuals (the optimal actuator positions) is consequentially improved with increasing number of artificial fish.

Table 9 gives the detailed information regarding the convergence and the iterations for twelve cases. Each item in this table represents the difference between the values of objective function corresponding to the current iteration and the first iteration. As can be seen, under the condition of same fish number, designers can obtain better value of objective function with fewer iterations to save computational time. Moreover, the results from Table 10 imply that with the same condition of total iteration number, designers can accelerate the optimization speed of the algorithm by selecting a reasonable total fish number (30 or 180 in this illustrative example).
Finally, the optimal control policies concerning various actuators for the 2nd (node 15), 4th (node 13), and 6th (node 18) layers are investigated in detail with the setting of 50 artificial fish and 100 generations (Table 11). It is obvious that the number of actuators plays a positive effect on the enhancement of control efficiency; nevertheless, with the increase of the actuator number, the lifting effect of control rate becomes slower. Moreover, the control rates are close to each other for different layers with identical optimal control policy found by the proposed algorithm, and the structural vibration is successfully mitigated compared to uncontrolled case, demonstrating the validity of the presented method.

6. Conclusions

As a kind of flexible space structure, tensegrity structure is easy to cause structural deformation under external load, so it needs to be controlled efficiently and safely. Among them, cluster tensegrity as a relatively new flexible space structure is also very much needed. In this paper, a novel fast model predictive control method is proposed for standard tensegrity mechanisms and cluster tensegrity structures, and the artificial fish swarm intelligence algorithm is optimized for actuator placement. The results show the following. (1) Fast model predictive control is suitable for standard tensegrity structure and cluster tensegrity structure and has good control effect. (2) The application of artificial fish swarm algorithm in the optimization of actuator layout has obvious effect and achieves the global optimization objective. (3) The study of examples proves the prospect of control method and artificial fish swarm algorithm in solving practical engineering problems.

Therefore, the results of the study are worthy of further research, such as the development of a more efficient control algorithm or a theoretical basis more in line with practical engineering (e.g., time lag problems). In the actuator arrangement optimization, a multi-objective function is set to achieve a more economical and efficient actuator arrangement. This is the next stage to be worked on.

<table>
<thead>
<tr>
<th>Number of actuators</th>
<th>Optimal placement of actuators</th>
<th>Node number</th>
<th>Displacement (m)</th>
<th>Control rate (%)</th>
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<tr>
<td>0</td>
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<td>0.45</td>
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<tr>
<td>3</td>
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<td></td>
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<tr>
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Appendix

\[
\begin{align*}
\Theta_{11} &= b_1\mathbf{K} M + b_1\mathbf{K} C, \\
\Theta_{12} &= -\left(b_1\mathbf{K} M + b_1\mathbf{K} C\right), \\
\Theta_{13} &= -\left(b_1\mathbf{K} M + b_1\mathbf{K} C\right), \\
\Theta_{21} &= b_2b_1\mathbf{K} M + b_1b_2\mathbf{K} C - b_1I_1, \\
\Theta_{22} &= -\left(b_2b_1\mathbf{K} M + b_1b_2\mathbf{K} C - b_2I_1\right), \\
\Theta_{23} &= -\left(b_2b_1\mathbf{K} M + b_1b_2\mathbf{K} C - b_2I_1\right), \\
\Theta_{31} &= b_1b_3\mathbf{K} M + b_1b_3\mathbf{K} C - b_1I_1, \\
\Theta_{32} &= -\left(b_1b_3\mathbf{K} M + b_1b_3\mathbf{K} C - b_3I_1\right), \\
\Theta_{33} &= -\left(b_1b_3\mathbf{K} M + b_1b_3\mathbf{K} C - b_3I_1\right), \\
\end{align*}
\]

\[\begin{aligned}
b_1 &= \frac{\delta}{a\Delta t}, \\
b_2 &= 1 - \frac{\delta}{a}, \\
b_3 &= \left(1 - \frac{\delta}{2a}\right) \Delta t, \\
b_4 &= \frac{1}{a(\Delta t)^2}, \\
b_5 &= -\frac{1}{a\Delta t}, \\
b_6 &= 1 - \frac{1}{2a}.
\end{aligned}\]

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References


