Research Article

Vibration Mitigation of Sagged Cables Using a Viscous Inertial Mass Damper: An Experimental Investigation

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In this paper, we describe the design and manufacturing of a viscous inertial mass-damper (VIMD) prototype. The damper parameters, the inertial mass, and the viscous damping coefficient can be continuously adjusted. Experimental validations were carried out on the VIMD with various parameters and a scaled sagged cable of 19.47 m long with the VIMD for vibration mitigation, along with a simulation study using the finite element method (FEM). The experimental results show that the modal damping ratio of the scaled cable was remarkably increased by the VIMD. The achieved maximum modal damping ratios for the first two modes were 6.98% and 8.15%, respectively, which are about 10 times the maximum values using a conventional viscous damper. The sag influence on damper-control performance was also studied by changing the cable tension. It showed that the first modal damping ratio provided by the VIMD was significantly reduced by the increased sag, whereas the second modal damping ratio was not. Fourier amplitude spectra of the cable responses subjected to sine sweep excitations show that the cable mode split into two and a pair of fixed points occurred by introducing a VIMD as in the theory of the fixed-point method (FPM) for the optimum design of a tuned mass damper. The performance of the optimum VIMD by the FPM was investigated in comparison with that by the maximum modal damping method.

1. Introduction

Cables are widely used in cable-supported bridges and long-span structures due to their economy, aesthetic effect, and light weight. However, cables are prone to severe vibration because of their low inherent damping and their high flexibility, and this vibration can be detrimental to the serviceability and safety of the pertinent structures.

Recently, the superiority, concerning vibration mitigation, of passive devices possessing properties of negative stiffness, such as negative stiffness damper (NSD) and inerter damper (ID), has been widely studied by many researchers [1–3]. The damper characteristics and control performance have been investigated under random excitations [4, 5], pulse-like excitations [6], and earthquake excitations [7–10]. Furthermore, the effectiveness of the negative stiffness devices has been systematically analyzed for high buildings [11–16], bridges [17–19], cables [20, 21], and ocean structures [22, 23]. Among them, the NSD and ID for cables have attracted strong attention because of their high efficiency. Shi et al. [24] systematically discussed the dynamic behavior of a cable-NSD system including stability, mode shapes, and modal damping ratios. Javanbakht et al. [25] proposed a refined design formula for a cable-NSD system, and the control performances of negative, zero, and positive stiffness dampers were investigated. A novel multimode control approach considering the effect of NSD stiffness was further developed [26]. Sun et al. [27] studied a system of two cables interconnected with a tuned inerter damper (TID), in which the dynamic characteristics of the system were obtained by complex modal analysis, and an approximate method was proposed to optimize the TID parameters. Chen et al. [28] discussed different types of inerter-based vibration absorbers (IVAs) and made comparisons between NSDs and IVAs. Furthermore, Chen proposed a control strategy for multimode cable vibration using NSDs and IVAs, wherein...
the damper parameters were obtained by maximizing the lowest value of the modal damping ratio of the target modes. It was reported that a viscous inertial mass damper (VIMD) may achieve a modal damping ratio of a cable up to 10 times the values reached by a traditional viscous damper [29, 30].

Many experimental studies have been conducted to validate the high efficiency on the structural vibration mitigation by NSD and ID [31–33]. An experimental study on NSD for a taut scaled cable was reported by Shi et al. [34]. The achieved maximum modal damping ratio was 10.31% when the damper was installed at a location of 5% of the cable length away from a cable end. Li et al. [35] showed the results of a full-scaled cable experiment using an electromagnetic shunt damper-inerter damper (EMSD-ID). It was reported that a modal damping ratio of 6% was observed with the damper at the 5% position of the cable, which was one of the highest modal damping ratios obtained in full-scale experiments. Moreover, Li et al. [36] carried out another full-scale experiment using an electromagnetic inertial mass damper (EIMD), wherein the obtained modal damping ratio of the cable-EIMD system was enhanced almost two times the values using a viscous damper (VD). Overall, the high control efficiency of the NSD and ID was predicted from theoretical study in comparison with those of the traditional viscous damper. However, the control performance from the experiments was not as high as the value predicted by the theory, and it still requires validation.

As a traditional design procedure, the fixed-point method (FPM) was recently applied for optimizing inerter damper and negative stiffness devices, which was originally proposed by Den Hartog for a TMD system [37]. Ikago et al. [38] proposed a new tuned viscous mass damper (TVMD) with a ball screw as a mass amplifier, which was used and optimized for an MDOF system with complex modal analysis. The mode vectors of the controlled and uncontrolled system were compared, and a mode-split phenomenon was found. Lazar et al. proposed a control strategy for multistory buildings [39] and cables [40] based on the FPM with an inerter-based device. Jin et al. [41] studied a system of a beam with two IDs and proposed a vibration suppression design using the FPM. It was shown that the target structural mode was split into two modes due to introducing additional inertial mass, which was also found in the cable-ID system [29, 30]. Marian and Giaralis [42] optimized the parameters of the tuned mass-damper inerter (TMDI) by adopting the FPM and showed that the TMDI is more effective than the TMD with the same mass. Gonzalez-Buelga et al. [43] analytically and experimentally assessed a ball-screw-type TID. The dry friction of the TID was considered as a nonlinearity, and the TID parameters were optimized by the FPM. Wang et al. [6] optimized a negative stiffness amplifying damper (NSAD) for an SDOF system, and the performance was simulated and discussed under numerous types of excitations. Aiming at reducing the cable response directly, the present authors extended the FPM for optimizing VIMD parameters for a taut cable and analyzed the cable mode-split phenomenon in detail [44]. Nevertheless, the experimental observations of the mode-split phenomenon and the fixed points on FRF curves are rarely reported, particularly the concerning cables.

The eddy current damper (ECD) was developed and investigated in the past half century because of its advantages of insensitivity to temperature and its noncontact nature. Its feasibility and control performances were studied numerically and experimentally for cantilever beams [45–47], vehicle suspension [48], an SDOF structure [49], and a robotic milling machine [50]. Recently, many novel ECDs combining with the ID have been proposed, because they are effective at emulating viscous dampers and have only small friction. Nakamura et al. [51] studied and proposed an inerter-based ECD named electromagnetic inertial mass damper (EIMD), where its mechanism is similar to a VIMD. Li et al. [36] and Zhu et al. [52] designed an EIMD prototype and investigated its control performance on a 135 m long cable. Gao et al. [53] proposed a novel negative stiffness inerter damper (NSID) consisting of a magnetic negative stiffness device and an inerter-combined ECD. Wang et al. [54] developed an eddy current inertial mass damper (ECIMD) with a rotational ECD and a ball screw, and an experimental investigation for cable vibration mitigation using two ECIMDs was carried out.

In this paper, we propose a VIMD prototype with adjustable damper parameters (inertial mass and damping). In it, an ECD is used to emulate the mechanical behavior of the viscous damper. The characteristics of the VIMD prototype were analyzed by a series of performance tests. An experimental study was carried out on a sagged scaled cable for various cases with the proposed VIMD for vibration mitigation. A series of vibration tests were conducted with different damper parameters. Numerical simulation analysis was also conducted for a cable-VIMD system, and the results were compared with the experimental results. We analyzed the characteristics of the cable-VIMD system, such as the cable responses and modal damping ratios, and we investigated the influence of sag on the control performance. The frequency-response functions (FRFs) obtained experimentally from the responses of the cable-VIMD system were analyzed in depth, and they revealed a mode-split phenomenon and two fixed points as in the theory of the FPM for the optimum design of a TMD. The performance of the optimum VIMD by the FPM was also investigated in comparison with that by the maximum modal damping method [29].

This paper is organized as follows. In Section 2, the VIMD prototype and the performance test are introduced. Section 3 establishes the model of the cable-VIMD system and briefly analyzes its modal characteristics. Performance of the two optimum VIMD designs is briefly discussed. The scaled-cable experiment is presented in Section 4, where the experimental results are analyzed with respect to the modal damping ratio, sag effect, and frequency-response function curve in detail. Section 5 summarizes the main conclusions of the work.
2. VIMD Prototype and Performance Tests

2.1. Configuration of VIMD. In this study, a parameter-adjustable VIMD was designed and manufactured. The configuration of the damper prototype is shown in Figure 1. The VIMD consists of a viscous damping part and an inertial mass part. The damping part is essentially a planar eddy current damper that is comprised of a copper plate and two electromagnets with opposite magnetic poles, and the inertial mass part comprises a ball screw and a flywheel with four weights. The VIMD has two terminals as shown in the figure. The connecting terminal is used to connect with cables, and the damper frame, as a fixed terminal, is usually connected to a support frame that is fixed to the bridge deck or ground. The cable vibration leads a linear reciprocating movement of the conductor plate in the magnetic field, which generates an electromagnetic viscous damping force. Meanwhile, the linear movement of the screw rod is converted to a rotational movement of the flywheel through the ball nut, which can provide a large inertial mass.

According to Bae et al. [45], the electromagnetic viscous damping force \( f_{\text{ed}} \) can be expressed as the following equation:

\[
f_{\text{ed}} = c_e v, \quad c_e = \alpha S \sigma B^2 \tag{1}
\]

where \( v \) is the moving velocity of the copper plate, \( c_e \) is the electromagnetic viscous damping coefficient, \( \alpha \) is a geometric factor, \( S \) is the projection area of the copper plate in the magnetic field, \( \sigma \) is the conductivity of copper, and \( B \) is the magnetic flux density. The magnetic flux density can be adjusted by giving the electromagnets different current inputs \( I \), which are approximately obtained according to Sodano and Inman with the Biot–Savart Law [55]. In this study, we simulated and analyzed the planar eddy current damper by an FE model using the ANSYS-EMAG module. The electromagnetic viscous damping coefficient was experimentally obtained and verified using a performance test.

Based on the mechanism of a ball screw, the amplified inertial mass \( m_e \) can be calculated by the following equation:

\[
m_e = m_l + r^2 I, \quad I = \frac{m [3(r_1^2 + r_2^2) + h^2]}{12} + m(R + d)^2, \tag{2}
\]

where \( m_l \) is a constant value comprised of the initial mass of the sensors, screw rod, and copper plate and the inertial mass of the ball nuts, drum, and spokes; \( r \) is a converting factor determined by the screw pitch; \( I \) is the moment of inertia of the flywheel; and \( r_1, r_2, h, \) and \( m \) are the inner radius, outer radius, height, and mass of the weight, respectively. \( R \) is the drum radius and \( d \) determines the weight position. The inertial mass \( m_e \) can be adjusted by changing the weight position \( d \).

A support spring with a constant stiffness \( k_n \) is designed to avoid an initial cable deformation due to \( m_l \) as shown in Figure 1. Hence, the theoretical damper force of the prototype can be expressed as follows:

\[
f_d = m_e \ddot{x} + c_e \dot{x} + k_n x, \tag{3}
\]

where \( x, \dot{x}, \) and \( \ddot{x} \) are, respectively, the damper displacement, velocity, and acceleration.

2.2. Performance Tests on the VIMD. Initially, a performance test is conducted on the VIMD as shown in Figure 2(a). One terminal of the VIMD was connected to an actuator and the other to the support frame. One force sensor and one position sensor were employed to measure the damper force \( f_d \) and damper displacement \( x \). A sinusoidal excitation was given to the prototype by the actuator, and thus the velocity and acceleration can be obtained numerically by taking the derivatives of \( x \). Based on the measured \( x \) and the calculated \( \dot{x} \) and \( \ddot{x} \), the unknown parameters in equation (3) \( \{m_e, c_e\} \) can be identified using a curve fitting method, to make the damper force calculated by the equation (3) consistent with the measured experimental \( f_d \). However, the friction between the ball nut and the screw rod will affect the accuracy on the identification of parameters due to the small size of the prototype. Therefore, a modified Bingham model [56] with considering the friction was used to identify the damper parameters from the measured results as follows:

\[
f_d(t) = m_e \ddot{x}(t) + c_e \dot{x}(t) + k_n x(t) + f_{f_r}(t), \tag{4}
\]

\[
f_{f_r}(t) = f_c \left[ 1 - \exp \left( -\text{sgn}[\dot{x}(t)] \frac{\dot{x}(t)}{x_0} \right) \right] \text{sgn}[\dot{x}(t)], \tag{4a}
\]

where \( f_{f_r}(t) \) is the friction term; \( f_c \) means the friction; \( x_0 \) is a regularization parameter that has a velocity dimension and controls the exponential growth of the damping force. \( k_n \) is zero because no support springs were used in the performance test. A least-square method was utilized, and the unknown parameters \( \{m_e, c_e, f_{f_r}, x_0\} \) can be calculated by fitting the damper force calculated by the equation (4) to the measured results of \( f_d(t) \), wherein a nonlinear least-squares function of lscurvefit in MATLAB was used. Then, the friction term \( f_{f_r}(t) \) in equation (4a) can be linearized as follows:

\[
f_{f_r}(t) = c_f \dot{x}(t) + e(t), \tag{4b}
\]

where \( c_f \) is the linearized friction damping coefficient and \( e(t) \) is the linearized error.

One case of the identified results is shown in Figures 2(b)–2(e), wherein no weights were used and the ECD was not activated. The identified inertial mass is \( m_e = 16.07 \text{ kg} \), which is close to the true value: \( m_l = 15.38 \text{ kg} \). The identified \( c_e, f_c \), and corresponding \( c_f \) are 0.01 \text{ Ns/m}, 2.42 \text{ N}, and 56.8 \text{ Ns/m}. Strong agreement can be observed between the experimental results (blue curves) and the fitted damping forces (red curves). The force-velocity relationship shows good applicability of the modified Bingham model to this VIMD, and the negative stiffness property can be observed clearly in Figure 2(c).
Figure 1: Configuration of the VIMD prototype.

Figure 2: Continued.

(a) Position sensor
(b) Dumper force, $f_d$ (N)
(c) Displacement (cm)

Experiment results
Modified Bingham model

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2.2.1. Inertial Mass. The parameters of the weights are shown in Table 1. Two sizes of weight (small and large) were designed. A total of six different weight positions \((d = 2.5, 7.5, 14.5, 17.5, 22.5, \text{and} 27.5 \text{ (mm)})\) were tested. Hence, the corresponding theoretical adjustment ranges of the total \(m_c\) are \((20.6–45.79) \text{ kg}\) and \((34.35–107.62) \text{ kg}\). Two sinusoidal excitations with 1.31 Hz and 1.94 Hz, respectively, were used in the tests. The number of the presented tests was 3–6 for each \(d\).

A normalized root mean-square error (nRMSE) value defined as in the following equation was used for quantifying the test results.

\[
nRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( Y_i - \bar{Y} \right)^2},
\]

where \(Y_i\) is the identified \(m_c\) and \(\bar{Y}\) is the theoretical value.

The test results using the smaller weights are shown in Figure 3. The left and right vertical axes denote the inertial mass value and nRMSE value, respectively, and the horizontal axis is for weight position \(d\). We have found that the test results strongly match the theoretical value in equation (2), which verifies the feasibility of the VIMD for adjusting \(m_c\). In most cases, the nRMSE values are smaller than 5%, which means that the damper provides an accurate \(m_c\).

2.2.2. Viscous Damping Coefficient. The parameters of the planar ECD are shown in Table 2. The FE model for a quarter of the VIMD is shown in Figure 4(a). 3D elements with 20 nodes (Solid236) in ANSYS were employed for simulating the copper plate. The total number of FE elements was 47,240. Both edge-flux degree of freedom (AZ) and electric degree of freedom (VOLT) were considered. By giving a moving velocity to the conductor and different input-current values to the electromagnets, the electromagnetic \(c_e\) was evaluated based on the Lorentz force (using command EMFT). The simulation results of the flux density are shown in Figure 4(b), and the relationship between the equivalent \(c_e\) and the current is obtained by a regression analysis as shown in Figure 4(c), \(c_e = 3.08 \times I^2\).

Four groups of tests were carried out with two inertial mass values (4.88 and 45.98 kg) and two excitation frequencies (1.31 and 1.94 Hz). We note that the ball nut and flywheels were not used in the present cases of \(m_c = 4.88 \text{ kg}\), which contain only the mass of the copper plate, screw rod, and force sensor. The test results are fitted to a parabolic equation as shown in Figure 4(c). We can observe that the test results show good accuracy for small current but are larger than the ANSYS results for larger current. This is mainly due to the two electromagnets attracting each other as the magnetic force increases. The damper frame had a small deformation, and the two electromagnets moved closer to the copper plate. This increases the magnetic flux between the magnets, thus making the test results larger than the simulation results. Two regression relationships between the damping and input current, the blue curve and black curve as shown in Figure 4(c), showed good agreement with difference of 10%, whereas the difference was less than 4% in the low current range. The regression curve, the blue curve, based on the test results is used in the present study with \(c_e = 3.408 \times I^2\). Hence, a theoretical maximum damping coefficient is 218.1 Ns/m with considering the safe

\[
\begin{array}{c|c|c}
\text{Components} & \text{Small weights} & \text{Large weights} \\
\hline
\text{Size (mm)} & 2 \times 10 \times 5 & 2 \times 20 \times 5 \\
\text{Mass (kg)} & 0.011 & 0.032 \\
\text{\(m_c\) with \(d_{\min} = 2.5\) mm (kg)} & 1.304 & 4.742 \\
\text{\(m_c\) with \(d_{\max} = 27.5\) mm (kg)} & 7.601 & 23.06 \\
\end{array}
\]
current-carrying capacity of the coil (max. I = 8 A). In this performance test, the supply current to the electromagnets is amplified by the AE Techron 7224 and set in the range of [0–6 A to avoid an overheat of the coil, and the corresponding adjustment range of damping coefficient is thus [0–122.7] Ns/m. The test results also demonstrate that the varying excitation frequencies have little effect on the damper parameters in low frequency and velocity ranges as indicated by references [57–59].

3. Sagged Cable-VIMD System

3.1. Model of Cable-VIMD System. Referring Figure 5, the current sagged cable-damper system can be modeled by a nondimensional equation of motion as follows [60–62]:

\[
\ddot{w}(x,t) + c \dot{w}(x,t) - \frac{1}{\pi^2} \frac{\lambda^2}{\pi^2} \int_0^1 \frac{w(\zeta,t)}{\pi^2} d\zeta = p(x,t) + F_d(t) \delta(x-x_d),
\]

where \( w(x,t) \) is the nondimensional displacement of the cable perpendicular to the chord in the vertical plane from the initial static sagged state; \( x \) is the nondimensional position along the chord \( (0 \leq x \leq 1) \); \( c \) is the cable inherent damping; \( p(x,t) \) is the external vertical force on the cable; \( F_d(t) \) is the force from the VIMD at \( x = x_d \). The nondimensional quantities are related to their dimensional counterparts, shown with overbars, according to the following relations:

\[
t = \omega_0 \bar{t}, \quad \bar{x} = \frac{x}{L}, \quad \bar{c} = \frac{c}{\rho \omega_0}, \quad \bar{w}(x,t) = \frac{w(x,t)}{L}, \quad \bar{\delta}(x-x_d) = L \delta(x-x_d), \quad \bar{p}(x,t) = \frac{L \bar{p}(x,\bar{t})}{\pi^2 \bar{T}}, \quad \bar{F}_d(t) = \frac{F_d(\bar{t})}{\pi^2 \bar{T}},
\]

and \( F_d(t) \) is the force from the VIMD at \( x = x_d \). The nondimensional parameters are defined as follows [63]:

\[
\bar{\lambda}^2 = \left( \frac{\rho g L \cos \theta}{T} \right) \frac{E A L}{T L_c}, \quad L_c = L \left[ 1 + \frac{1}{8} \left( \frac{\rho g L \cos \theta}{T} \right)^2 \right],
\]

in which \( \rho, L, \) and \( T \) are the cable mass per unit length, cable chord length, and cable tension in the chord direction; \( \omega_0 = \pi/L \sqrt{T/\rho} \) is the first natural frequency of the cable without a VIMD, computed without considering the sag effect.

The nondimensional parameter \( \lambda^2 \) for the sag effect is defined as follows [63]:

\[
\lambda^2 = \left( \frac{\rho g L \cos \theta}{T} \right) \frac{E A L}{T L_c}, \quad L_c = L \left[ 1 + \frac{1}{8} \left( \frac{\rho g L \cos \theta}{T} \right)^2 \right],
\]

in which \( \theta \) and \( L_c \) are the cable inclination angle and the stretched cable length and \( E, A, \) and \( g \) are Young's modulus of the cable, area of the cable cross section, and gravitational acceleration, respectively.
By introducing a section-wise linear deflection shape of the cable on both sides of the VIMD and a series of sinusoidal shapes on the whole cable section, the initially assumed shape functions of the cable are taken as follows [60]:

\[
\phi_i(x) = \begin{cases} 
\frac{12 + \lambda^2}{12 + \lambda^2 - 3\lambda^2 x_d (1 - x_d)} \left[ \frac{x}{x_d} \sin \left[ \pi x \right] \right] + 1 - \frac{x}{x_d} & i = 1, \\
\sin \left[ (i - 1)\pi x \right] & i \geq 2,
\end{cases}
\]

where \( H(x - x_d) \) is a Heaviside function. In this study, 20 sinusoidal shape functions were used.

By using the shape functions in equation (8), the cable response \( w(x, t) \) can be approximately expressed as follows:

\[
w(x, t) = \sum q_i(t)\phi_i(x) = \Phi^T(x)q(t),
\]

where \( q_i(t) \) is the generalized coordinate.

Then, the equation of motion, equation (6), can be discretized into a matrix form as follows:

\[
M\ddot{q} + C\dot{q} + Kq = f + \phi(x_d)F_d(t),
\]

where \( M, C, \) and \( K \) are the generalized mass, damping, and stiffness matrices, which are related to the assumed shape functions \( \phi_i(x) \) in equation (8). In reference [60], \( f \) is the generalized external load vector:

\[
f = \int_0^1 \phi(x)p(x, t)dx = \left[ \int_0^1 \phi(x)p_0(x)dx \right] f_i(t) = f_i^0 f_i(t),
\]

**Figure 4:** Test and simulation results of electromagnetic viscous damping coefficient. (a) FE model of planar ECD: a quarter model. (b) Simulation results of magnetic flux density. (c) Comparison of test and simulation results.
where the external force on the cable is assumed as $p(x, t) = p_0(x) f_x(t)$.

$$\begin{align*}
M + m_L \phi(x_d) \phi^T(x_d) \dot{q} + [C + c_d \phi(x_d) \phi^T(x_d)] \dot{q} + [K + \bar{K}_n \phi(x_d) \phi^T(x_d)] q &= M_\alpha \ddot{q} + C_\alpha \dot{q} + K_\alpha q = f.
\end{align*}$$

Then, the state-space expression can be obtained as follows [29, 60]:

$$\dot{z} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -M^{-1}_\alpha K & -M^{-1}_\alpha C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}_\alpha \end{bmatrix} f = A_z z + B_z f,$$

where $z$ is the state vector, $A_z$ is the state matrix, and $B_z$ is the input matrix.

### 3.2. Analysis of Modal Damping Ratio

Conjugate pairs of the complex eigenvalues, $\Lambda_j$ and $\Lambda_j^*$, can be obtained from the state matrix $A_z$ and is expressed as follows:

$$\Lambda_j, \Lambda_j^* = -\xi_j \omega_j \pm i \omega_j \sqrt{1 - \xi_j^2}, \quad j = 1, 2, 3, \ldots, n,$$

where $\omega_j$ and $\xi_j$ are the $j$th natural frequency and modal damping ratio and $i$ is the imaginary unit. Thus, the modal damping ratio $\xi_j$ can be obtained as follows:

$$\xi_j = -\frac{\text{Re}(\Lambda_j)}{\text{Abs}(\Lambda_j)}, \quad j = 1, 2, 3, \ldots, n.$$

Figure 6 shows two 3D-plots for $\xi_1$ and $\xi_2$ with respect to the damper parameters ($m_L$ and $c_d$; $\bar{K}_n = 0$ and $x_d = 0.021$) in two different sag conditions ($\lambda^2 = 3.9$ and 6.6). The figures show that, for a specified damper parameter ($m_L$ or $c_d$), $\xi_j$ always increases first and then decreases with the other increasing parameter ($c_d$ or $m_L$). Thus, there exists a theoretical maximum modal damping ratio at a set of damper parameters. The maximum $\xi_1$ is obtained as 8.29% and 7.25% for two $\lambda^2$ values, as shown in Table 3, which are 12 and 14 times than those obtained by optimal viscous dampers (VD) [64]. Moreover, we can see that the maximum $\xi_1$ is significantly reduced with the growing sag parameter. $\xi_1$ decreases from 8.29% to 7.25% as $\lambda^2$ increases.

The damper force of the VIMD at location $x = x_d$ can be expressed as follows:

$$F_d(t) = -\left[ \bar{m}_n \phi^T(x_d) \dot{q} + \bar{r}_n \phi^T(x_d) \ddot{q} + \bar{K}_n \phi^T(x_d) q \right].$$

where $\bar{m}_n$, $\bar{r}_n$, and $\bar{K}_n$ are the nondimensional damper parameters defined as follows:

$$\bar{m}_n = \frac{m_L \omega_n^2}{\pi^2}, \quad \bar{r}_n = \frac{c_d L \omega_n}{\pi^2}, \quad \bar{K}_n = \frac{k_n L}{\pi^2},$$

in which $c_d = c_c + c_f$.

Using equations (10) and (12), it can be rewritten as follows:

$$\begin{align*}
M_\alpha \ddot{q} + C_\alpha \dot{q} + K_\alpha q &= f.
\end{align*}$$

from 3.9 to 6.6, respectively. However, the maximum $\xi_2$ is only slightly affected by the sag effect: 9.36% and 9.35% for two $\lambda^2$. Two 3D graphs of $\xi_2$ are almost the same for two $\lambda^2$ as shown in Figure 6(b).

### 3.3. Characteristics of a Cable with a VIMD

The present authors proposed an optimal design procedure for a VIMD on a taut cable using the fixed-point method (FPM: Reference [44]). The FPM was originally proposed for optimizing a tuned mass damper (TMD) for ordinary structures by Den Hartog [37], where the mode-split phenomenon and two fixed points on the frequency response function (FRF) curves were analyzed. In this study, the FPM is extended for a sagged cable, and the typical characteristics are briefly discussed. Based on equations (9), (11), (14), and (15), the FRF of the cable response at $x$ under a harmonic excitation, $p(x, t) = p_0(x) f_x(t)$ with $f_x(t) = e^{i \omega t}$, can be expressed using the state-space formulations as follows:

$$H(x, \omega) = \left[ \phi^T(x) 0 \right] \Psi \left[ i \omega I - \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda^* \end{bmatrix} \right]^{-1} \Psi^T B_s f_0,$$

where $f_0$ is the modal force coefficient vector associated with the shape functions $\phi(x)$ and the load distribution $p_0(x)$ as in equation (11); $\Lambda$ and $\Lambda^*$ are the diagonal matrices for conjugate pairs of eigenvalues; and $\Psi$ is the eigen matrix of the state matrix $A_z$, normalized as $\Psi^T \Psi = I$. 

Figure 5: A sagged cable with a VIMD.


Table 3: Theoretical maximum modal damping ratios of VD and VIMD ($\xi_m = 0$).

<table>
<thead>
<tr>
<th>Mode numbers</th>
<th>$\lambda^2$</th>
<th>$x_d$</th>
<th>VD</th>
<th>$\xi$ (%)</th>
<th>$\bar{m}_e$</th>
<th>VIMD</th>
<th>$\xi$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 (Reference [26])</td>
<td>0.02</td>
<td>5.05</td>
<td>1.02</td>
<td>4.81</td>
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<td>3.88</td>
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<td>3.045</td>
<td>1.161</td>
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<td>0.02</td>
<td>2.55</td>
<td>1.02</td>
<td>1.2</td>
<td>1.05</td>
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<tr>
<td></td>
<td>3.9</td>
<td>0.021</td>
<td>2.42</td>
<td>1.08</td>
<td>1.14</td>
<td>1.02</td>
<td>9.36</td>
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<td>2.42</td>
<td>1.08</td>
<td>1.14</td>
<td>1.02</td>
<td>9.35</td>
</tr>
</tbody>
</table>

A figure illustrating the variation of the modal damping ratio with respect to VIMD parameters under $\lambda^2 = 3.9$ and 6.6. (a) First cable mode. (b) Second cable mode.

\[
\Psi = \begin{bmatrix}
\psi \\
\psi^* \end{bmatrix},
\]

where $\psi$ and $\psi^*$ are the corresponding conjugate pairs of eigenvectors, which represent the enhanced complex modes of cable vibration from the assumed shape functions $\phi_i(x)$ in equation (8).

Assuming $\lambda^2 = 3.9$ and $x_d = 0.021$ with a concentrated harmonic actuation $p(x,t) = \delta(x-x_c)e^{i\omega t}$ at $x_c = 0.99$, Figure 7(a) shows the FRF curves of the cable with different VIMDs shown in Section 4, at the cable midpoint obtained for various values of $\bar{m}_e$ and $\bar{z}_d$ using equation (18). In the figure, the blue curves are with zero damping and the red curve is with infinite damping. The blue dashed curve is for the cable without the VIMD, which shows the original first cable frequency ($\omega_1 = 1.148\omega_0$). It is interesting to observe that the blue solid curve ($\bar{m}_e = 3.528$) has two resonant frequency peaks, $\omega_{1-1} = 1.074\omega_0$ and $\omega_{1-2} = 1.263\omega_0$. The dotted curve and the red curve are the same although the damper parameters are different. This shows that the original first cable mode splits into two modes by introducing a proper inertial mass, and that the cable is clamped at the damper location when $\bar{z}_d$ or $\bar{m}_e$ approaches infinity. Figure 7(b) shows four FRF curves at the cable midpoint for various $\bar{z}_d$ values with $\bar{m}_e = 3.528$. It clearly shows that all the four curves pass through two points, $P$ and $Q$, as in the theory of the FPM, which shows that the locations of $P$ and $Q$ are independent of the VIMD damping when $\bar{m}_e$ is fixed. It can be also seen that the responses around the first resonant frequency ($\omega_1$) can be significantly suppressed with a properly selected damping, e.g., $\bar{z}_d = 0.88$, so that the optimum VIMD design may be obtained using the FPM. The main idea for the optimum design in the FPM is to find the VIMD parameters ($\bar{m}_e$ and $\bar{z}_d$) so that the FRF curves may have maxima with the same amplitude at the two fixed points ($P$ and $Q$). The detailed optimization procedures can be found in reference [44].

3.4. Comparison of Two Optimum VIMD Design Methods. Two optimum VIMD design methods were introduced in the previous sections. One is by the maximum modal damping ratio as in Figure 6, and the other is by the FPM as illustrated by VIMD-1 in Figure 7. Figure 8 shows the comparisons of the cable responses computed for different types of excitations, wherein uniformly distributed sinusoidal excitations are used in Figures 8(a) and 8(b), and distributed random excitations are in Figures 8(c) and 8(d). Two kinds of VIMDs with different parameters were considered, where VIMD-1 is the optimum design to the first mode obtained by the FPM ($\bar{m}_e = 3.528$, $\bar{z}_d = 0.88$, and $\xi_1 = 5.32%$) and VIMD-2 is the one by the maximum modal damping ratio ($\bar{m}_e = 3.528$, $\bar{z}_d = 1.426$, and $\xi_1 = 8.29%$) as shown also in Figure 7(b). Three different excitation frequencies were used for the sinusoidal excitation, which were $\Omega = 1.164$, 1.232, and 1.35. For $\Omega = 1.164$, the maximum nondimensional response amplitude, $w$ in equation (6), using VIMD-1 is 0.048 in Figure 8(a), whereas the result using VIMD-2 is 0.075. However, the amplitudes using two
VIMDs become nearly the same for $\Omega = 1.232$ as expected in Figure 7(b). The amplitude using VIMD-1 is slightly larger than the result using VIMD-2 when $\Omega = 1.35$, which are 0.0218 and 0.0206, respectively. Figure 8(b) shows the root mean-square (RMS) values of the nondimensional displacement along the cable. It is obvious that the cable responses using VIMD-2 is considerably larger than those using VIMD-1 when $\Omega = 1.164$, although VIMD-2 provides higher modal damping ratio ($\xi_1 = 8.29\%$) than the one ($\xi_1 = 5.32\%$) by VIMD-1. The vibration amplitudes then become similar with the increasing $\Omega$ as expected by Figure 7(b). The differences of the cable responses at the midpoint are 5.63\% and 5.78\% for $\Omega = 1.232$ and 1.35, respectively. Figure 8(c) shows the nondimensional cable responses subjected to distributed random excitations of white noises. Figure 8(d) shows the mean RMS values of the nondimensional cable responses considering 20 sets of different distributed random excitations, which shows that the control performances of two VIMDs are very similar under random excitations, where the reduction of cable responses using VIMD-1 is slightly better around the cable midpoint. Experimental validations of the abovementioned comparison on the two methods are provided in Section 4. It is to be noted that the optimum design method by the maximum modal damping method can be used more conventionally in practice, because it provides a VIMD that is independent of the response location, whereas the optimum VIMD by the FPM depends on the response location.

4. Scaled-Cable Experiments and Performance Evaluation

Experiments are carried out on a scaled cable with the parameters shown in Table 4. Figure 9 shows the experimental setups. The actuator and the damper were installed at 1% and 2.1% of cable length $L$ near the two ends. A series of experiments was carried out for two sag conditions, i.e., $\lambda^2 = 3.9$ and 6.6. The first two cable modes were obtained for each sag condition with various damper parameters. The measured natural frequencies ($\omega_1$ and $\omega_2$) for the first two cable modes without a VIMD in two sag conditions are also shown in Table 4, which are found to be very close to the theoretical values. The natural frequencies ($\omega_0$) computed considering the cable tension only without the sag effect are smaller than the frequencies including the sag effect. The maximum modal damping ratios and the corresponding damper parameters theoretically obtained by the state-space formulation [29, 60] are given in Table 5, which indicates that the modal damping ratios can be increased to a range of 6.1–8.5% by introduction of a VIMD. A sinusoidal excitation with $\omega_e$ close to $\omega_1$ or $\omega_2$ was applied to the cable. The electromagnets were then activated after the vibration reached a steady state, and the excitation was removed after reaching another steady state. The cable displacements at the midpoint (for $\omega_e$ near $\omega_1$) and the quarter point near the bottom anchorage (for $\omega_e$ near $\omega_2$) were measured using noncontact measurement with a digital camera. The modal damping ratios were identified from the free vibration part. The damper force and the damper displacement were recorded by a force sensor and a position sensor, respectively.

4.1. Damper Parameter Identification. Figure 10(a) shows the experimental time histories of the damper force and the damper displacement for $\lambda^2 = 6.6$. It is to be noted that the results shown in Figure 10 were measured from a cable-VIMD system, whereas the data of Figures 2(b)–2(e) are for a VIMD without the cable. The cable was excited under a harmonic excitation with $\omega_e = 2.1$ Hz, and the electromagnets were not activated in this case. With the least-square method given in Section 2.2, the damper parameters in equation (4) of this case were identified as $m_e = 21.3$ kg, $c_e = 0$ Ns/m, and $f_e = 3.39$ N, wherein the design values for $m_e$ and $c_e$ considering the second cable mode are 21 kg and 0 Ns/m, respectively. The identified friction term in equation (4b) is linearized by fitting the force-velocity hysteretic loop as

Figure 7: FRF curves for cable-VIMD system at $L/2$: $\lambda^2 = 3.9$ and $\omega_0 = 1.157$ Hz. (a) Different $m_e$ and $c_d$. (b) Different $c_d$ with $m_e = 3.528$. 
shown in Figure 10(b). The blue curve is the friction term with $f_c = 3.39$ N, and the red curve is the equivalent damping with $c_f = 27.16$ Ns/m. Theoretically, the friction should be identical for all experimental cases. However, it was changed to a small degree for different inertial mass values because the weight positions were manually adjusted. Figures 10(c) and 10(d) show comparisons between the experimental results and the fitted results using the identified parameters. The fitted damper force strongly matches the measured counterpart.

4.2. Analysis of Modal Damping Ratio. In this section, we discuss mainly the experimental results of $\lambda^2 = 3.9$, while the results for a different sag ($\lambda^2 = 6.6$) are shown in the next table.
section. Figure 11(a) shows the free vibration response at the midpoint without a VIMD after being excited with \( \omega_c = \omega_1 = 1.333 \text{ Hz} \). The envelope curve gives the cable inherent modal damping ratio: \( \xi = 0.24\% \). It is to be noted that this inherent modal damping ratio includes the influence of the actuator, which is very small compared with that provided by the damper. This inherent modal damping ratio is simulated as a mass proportional damping matrix \( C \) in equations (10) and (14) and included the damping parameters shown in Table 5. Figure 11(b) shows the Fourier-amplitude-spectrum (FAS) results with and without a damper. We can see that the response amplitude is significantly reduced by the VIMD, and the cable frequency is slightly enlarged.

Figure 12(a) shows the vibration response at \( L/2 \) (after being excited at \( \omega_c = 1.414 \text{ Hz} \)) with the damper parameters optimized to the first mode (by maximizing \( \xi_1 \)), where the identified values are \( m_e = 66.4 \text{ kg} \) and \( c_d = 168.1 \text{ Ns/m} \), which are very close to the target values (66.4 kg, 168.1 Ns/m) shown in Table 5. The envelope curve of cable responses \( H(t) \) was obtained using the Hilbert-transform, and the modal damping ratio \( \xi \) was obtained by taking the slope of the logarithm of \( H(t) \) in the free decaying part. Figure 12(b) shows the response at \( L/4 \) (after being excited at \( \omega_c = 2.29 \text{ Hz} \)) with the damper parameters optimized to the second mode (by maximizing \( \xi_2 \)), where the identified values are \( m_e = 21.4 \text{ kg} \) and \( c_d = 114.3 \text{ Ns/m} \), which are also close to the target values (21.4 kg, 122.2 Ns/m). The identified modal damping ratios are 6.98\% and 8.15\% for the two modes, which are very close to the theoretical maximum values (7.23\% and 8.52\%). The identified damping ratios are found to be 10 and 7.5 times of those using a traditional VD shown in Table 3. Figures 12(c) and 12(d) give comparisons between the theoretical damping ratios (by the state-space method) and the experimental values. Strong agreement can be found between the two results. The results show that there exists a pair of \( m_e \) and \( c_d \) that provides a maximum modal damping ratio for each mode.

### 4.3. Analysis of Sag Effect
To study the sag effect on the control performance, experiments were also conducted on a cable with \( \lambda^2 = 6.6 \). The cable tension \( T \) has been changed from 1497.5 N to 1266.7 N, as shown in Table 4. Note that the load cell gives the tension at the bottom anchorage. Figure 13(a) shows an example time history of the cable response at \( L/4 \) with excitation frequency \( \omega_c = 2.33 \text{ Hz} \), where the electromagnets were turned on at \( 3 \text{ s} \) and the excitation was removed at \( 40 \text{ s} \). The identified parameters were \( m_e = 22.6 \text{ kg}, c_f = 47.4 \text{ Ns/m} \) (0–3 s), and \( c_d = 112.5 \text{ Ns/m} \) (3–50 s), respectively. The vibration was suppressed in 2 s after the excitation was removed, and the identified \( \xi_2 \) was 7.34\%, whereas the theoretical maximum value was 8.34\% as in Table 5. Figures 13(b) and 13(c) show comparisons between the theoretical modal damping ratios and the experimental results for the various VIMD parameters \( (m_e, c_d) \), wherein good agreements can be generally found. Compared with the case of \( \lambda^2 = 3.9 \) (Figures 12(c) and 12(d)), it can be observed that both \( m_e \) and \( c_d \), corresponding to the maximum point of the first modal damping ratio \( \xi_1 \), reduced considerably, whereas those of the second mode remained almost the same, as expected by Figure 6. Figure 13(d) shows the theoretical maximum damping ratios \( (\xi_1, \xi_2) \) and the experimental maximum results for two sag conditions. The solid curves represent the theoretical value and the dashed curves with the markers “O” represent the experimental values. The experimental maximum value of \( \xi_1 \) is reduced from 6.98\% \( (\lambda^2 = 3.9) \) to 5.27\% \( (\lambda^2 = 6.6) \), whereas \( \xi_2 \) decreases from 8.15\% to 7.34\%. It can be observed that the vibration-reduction performance of the first cable mode is significantly affected by the sag, whereas the second mode is

### Table 5: Maximum \( \xi \) and corresponding theoretical damper parameters by the state-space formulation.

| Test modes | \( \lambda^2 \) | \( m_e \) (kg) | \( m_c \) (kg) | \( c_d \) (Ns/m) | Max. \( \xi \) (%)
<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>1st</td>
<td>3.9</td>
<td>4.62</td>
<td>66.4</td>
<td>1.61</td>
<td>168.1</td>
</tr>
<tr>
<td></td>
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<td>4.16</td>
<td>59.8</td>
<td>1.34</td>
<td>128.7</td>
</tr>
<tr>
<td>2nd</td>
<td>3.9</td>
<td>1.49</td>
<td>21.4</td>
<td>1.17</td>
<td>122.2</td>
</tr>
<tr>
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<td>6.6</td>
<td>1.56</td>
<td>22.4</td>
<td>1.19</td>
<td>114.3</td>
</tr>
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</table>

The VIMD parameters in Table 5 are different from those in Table 3 due to \( k_e \) of the additional spring in the VIMD prototype.
Figure 10: Identification of damper force; $\lambda^2 = 6.6$ and $c_e = 0$. (a) Recorded experimental time histories. (b) Equivalent damping. (c) Comparisons between the experimental results and the fitted results. (d) Comparisons of the hysteretic loops.

Figure 11: Dynamic cable characteristics without VIMD: $\lambda^2 = 3.9$ and $\omega_e = 1.33 \text{ Hz}$. (a) Free vibration without VIMD. (b) FAS of displacement at the cable midpoint.
Larger differences are observed between the experimental and theoretical results for the maximum modal damping ratios of the second modes in the low range of $c_d$, which might be caused by the identification error in the data process.

4.4. Analysis of Maximum Responses and FRF Curves

4.4.1. Responses to Harmonic Excitations. Figure 14(a) shows the typical time-history responses of the damper force, the damper displacement, and the cable displacement at $L/4$ in the case: $\omega_e = 1.414\text{ Hz}$, $\lambda^2 = 3.9$, $m_e = 66.1\text{ kg}$, and $c_d = 164.2\text{ Ns/m}$, and $\xi = 6.98\%$). (b) Cable vibration response at $L/4$: $\omega_e = 2.29\text{ Hz}$ ($m_e = 21.4\text{ kg}$, $c_d = 113.4\text{ Ns/m}$, and $\xi = 8.15\%$). (c) Comparisons of the first modal damping ratios. (d) Comparisons of the second modal damping ratios.

Figure 12: Experimental results of the modal damping ratio, $\lambda^2 = 3.9$. (a) Cable vibration response at $L/2$: $\omega_e = 1.414\text{ Hz}$ ($m_e = 66.1\text{ kg}$, $c_d = 164.2\text{ Ns/m}$, and $\xi = 6.98\%$). (b) Cable vibration response at $L/4$: $\omega_e = 2.29\text{ Hz}$ ($m_e = 21.4\text{ kg}$, $c_d = 113.4\text{ Ns/m}$, and $\xi = 8.15\%$). (c) Comparisons of the first modal damping ratios. (d) Comparisons of the second modal damping ratios.

not. Larger differences are observed between the experimental and theoretical results for the maximum modal damping ratios of the second modes in the low range of $c_d$, which might be caused by the identification error in the data process.
are also given in Figure 14(b). It can be observed that in all the cases the amplitudes of the damper force and damper displacement decreased with the increasing \( c_d \), whereas the cable displacement (\( \frac{L}{4} \)) increased. However, the changes in the cable responses are not always the same for all the cases. Comparing the cable displacement responses at \( \frac{L}{4} \) in Figures 13(a) and 14(a), different trends in the response amplitude changes (before/after the activation of the electromagnets) can be found in two sag conditions, although the excitation is almost the same. The amplitude increases from 20.1 mm to 42.3 mm for \( \lambda^2 = 3.9 \) after activating the electromagnets but decreases from 44.6 mm to 22.8 mm for \( \lambda^2 = 6.6 \). The identified \( \xi_2 \) is 2.7%, 8.15%, and 3.47% for \( c_d = 62.2, 113.4, \) and 189.2 Ns/m with \( \lambda^2 = 3.9 \) in Figure 14(b). The blue curve with the smallest \( \xi_2 \) gives the smallest cable response amplitude at \( \omega_e = 2.3 \) Hz, whereas the green curve with the largest \( c_d \) gives the largest cable response.

The abovementioned phenomena can be explained through the analysis of the FRF curves. Figure 15 shows FRF curves at \( \frac{L}{4} \) obtained for various parameters using equation (18). The solid curve is without a VIMD, the rest of the curves are with a VIMD using different values of \( m_e \) and \( c_d \) considered in the experimental cases in Figure 14. It can be seen that the peak frequencies without a VIMD are almost the same as those from the experiments on the scaled cable shown in Table 4. The original second mode (the solid curve with \( \omega_e = 2.336 \) Hz) has split into two, as in the dashed curve with \( \omega_e = 2.194 \) and 2.549 Hz shown in Figure 15, after introducing a VIMD with \( m_e = 21.4 \) kg and
These two modes move closer to each other by increasing $c_d$ as shown by the dotted and dash-dot curves, wherein the two split frequencies are 2.285 and 2.449 Hz for $c_d = 113.4$ Ns/m and 2.364 and 2.37 Hz for $c_d = 189.2$ Ns/m, respectively. Then, they converge to one frequency, i.e., $f_s \rightarrow 2.364$ Hz, when $c_d$ becomes infinity. From the FRF curves, it can be clearly seen that the response amplitude increases from Point A to Point C at $\omega_e = 2.3$ Hz as $c_d$ increases from 62.2 to 189.2 Ns/m, which confirms the trends of the experimental results in Figure 14(b). In addition, two fixed points (P and Q) can be observed in Figure 15 denoted with the markers “$\times$.” It is evident that all the three FRF curves with the same $m_e = 21.4$ kg pass through the fixed points “$\times$” regardless of the $c_d$ values, as expected. It is to be noted that the case with $m_e = 21.4$ kg and $c_d = 113.4$ Ns/m represents the optimum VIMD with the maximum modal damping ratio ($\xi$) of 8.15%. Besides, the red curve is for the optimum VIMD obtained by the FPM, which provides a flat “region” of the FRF curve around the original second frequency.

Figure 16(a) shows three experimental displacement time histories at 0.5 $L$ with $m_e = 66.1$ kg, $\omega_e = 1.414$ Hz, and $\lambda^2 = 3.9$. The total damping coefficient ($c_d = c_e + c_f$) is 82.7, 121.1, and 164.2 Ns/m, for those cases, which are the first, second, and fourth yellow points in Figure 12(c). The corresponding modal damping ratio $\xi$ increases with the increasing damping as in Figure 12(c). However, the steady state amplitudes of those three cases are almost the same as in Figure 16(a).

Figure 16(b) shows three other experimental results at 0.25 $L$ with the same $m_e = 22.6$ kg, $\omega_e = 2.33$ Hz, and $\lambda^2 = 6.6$, but different $c_d = 47.4$, 112.5, and 152.9 Ns/m, which are the first, highest, and last yellow points in Figure 13(c). The
Figure 16: Experimental responses for various VIMD parameters and corresponding FRF curves by equation (18). (a) Vibration responses at $L/2$ with $\omega_1 = 1.414$ Hz, $\lambda^2 = 3.9$, and $m_e = 66.1$ kg. (b) Vibration responses at $L/4$ with $\omega_1 = 2.33$ Hz, $\lambda^2 = 6.6$, and $m_e = 22.6$ kg. (c) FRF curves at $L/2$ with $\lambda^2 = 3.9$ and $m_e = 66.1$ kg. (d) FRF curves at $L/4$ with $\lambda^2 = 6.6$ and $m_e = 22.6$ kg.

4.4.2. Responses to Sinusoidal Sweep Excitations. Figure 17(a) shows the measured cable responses at $L/4$ of the cable with $m_e$ of a VIMD optimized to the second mode (by maximizing $\xi_2$) subjected to a 60 s long sinusoidal sweep excitation, wherein the sweep frequency range is from 0.5 to 3.5 Hz. Four time-history curves are shown with the same sag and inertial mass ($\lambda^2 = 6.6$, $m_e = 22.6$ kg) but with different damping. The corresponding FAS curves are shown in Figure 17(b). It is to be noted that the general shapes of the FAS curves of the responses for the sinusoidal sweeping excitation are similar to the FRF curves in the corresponding frequency range. It can be seen that the original frequency ($\omega_2 = 2.13$ Hz) has split into two (1.963 and 2.307 Hz) in the case of $c_d = 25.3$ Ns/m. As $c_d$ increases, the amplitudes of the two split modes are first reduced, and the two peaks move closer to each other. Then, the two split modes become one mode, and the amplitude keeps growing with the increasing $c_d$. The peak frequency for $c_d = 144.1$ Ns/m is obtained as...
2.153 Hz, which is slightly higher than original frequency. It can be also observed that all the experimental FAS curves pass through two fixed points (P and Q), regardless of the \( c_d \) value, as expected.

Figure 18 shows the FAS curves of the responses at \( L/2 \) under the same sine sweep excitation for the case of \( \lambda^2 = 3.9 \) and \( m_e = 66.1 \) kg. The same phenomenon of mode splitting and two fixed points can be clearly observed.

5. Conclusions

In this study, a VIMD prototype with adjustable damper parameters was developed. Performance tests were carried out on the VIMD and a scaled cable with the damper for the vibration-mitigation effect. The experimental results were systematically analyzed along with the simulated results by FEM, and the conclusions are summarized as follows:

1. The damper parameters of the proposed VIMD prototype are continuously and easily adjustable. The inertial mass can be adjusted by moving the weights, and the electromagnetic viscous damping coefficient is adjusted by changing the input current to the electromagnets. The results of the performance tests show strong agreement with those from the theory and the simulation. Good feasibility and accuracy were found on the VIMD parameter adjustment.

2. The modal damping ratios of the first two cable modes are remarkably enhanced with the installing of the VIMD. The identified maximum modal damping ratios of the first two modes are 6.89% and 8.15%, which are almost 10 and 8 times the maximum values of those using a viscous damper. Those results are one of the highest damping ratios obtained in experiments with a damper location at 2.1% so far. Besides, we find that there is one maximum modal damping ratio for each mode with a set of the damper parameters (\( m_e \) and \( c_d \)), as in the theory.

3. The first modal damping ratio is significantly reduced due to the larger sag effect. For example, the identified maximum first modal damping ratio decreases by 24.5% when \( \lambda^2 \) increases from 3.9 to 6.6. However, the maximum second modal damping ratio is reduced by only 9.9%. The VIMD parameters corresponding to the maximum first modal damping ratio also reduce, but those for the second mode do not.

4. The cable mode is split into two modes by introducing a VIMD with a proper inertial mass. For
a selected inertial mass value, two split frequencies move closer to each other with the increasing damping coefficient. The two frequencies converge to one frequency, as the damping approaches infinite. Besides, two fixed points are observed on the FAS results of the cable responses subjected to sinusoidal sweep excitations. For a selected inertial mass value, all the FRF curves pass through the two fixed points regardless of damping, as in the fixed-point method (FPM). This can be used to obtain the optimum VIMD design.

(5) The optimum VIMD by the FPM may provide considerably smaller modal damping ratio than the one by the maximum modal damping method. But the FPM may give significantly smaller cable responses particularly for harmonic excitations near the resonant frequency, though it is expected to give very similar RMS responses for wide-banded random excitations.

Further studies are suggested for the full-scale experimental validation with considering higher modes and the multimode cable vibration control using multiple VIMDs.

**Data Availability**

The experimental data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


