

Retraction

Retracted: Some Novel Geometric Aggregation Operators of Spherical Cubic Fuzzy Information with Application

Security and Communication Networks

Received 10 October 2023; Accepted 10 October 2023; Published 11 October 2023

Copyright © 2023 Security and Communication Networks. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] Z. Ullah, H. Bashir, R. Anjum et al., "Some Novel Geometric Aggregation Operators of Spherical Cubic Fuzzy Information with Application," *Security and Communication Networks*, vol. 2021, Article ID 3774353, 14 pages, 2021.

Research Article

Some Novel Geometric Aggregation Operators of Spherical Cubic Fuzzy Information with Application

Zafar Ullah ¹, Huma Bashir ², Rukhshanda Anjum,³ Mabrook Al-Rakhami ⁴,
Suheer Al-Hadhrami ⁵ and Abdul Ghaffar ⁶

¹Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan

²Departemant of Basic Sciences, UCET, Bahauddin Zakariya University, Multan, Pakistan

³Deaprtament of Mathematics and Statistics, University of Lahore, Lahore, Pakistan

⁴STC's Artificial Intelligence Chair, Department of Information Systems, College of Computer and Information Sciences, King Saud University, Riyadh 11543, Saudi Arabia

⁵Computer Engineering Department, Engineering College, Hadhramout University, Hadhramout, Yemen

⁶Department of Mathematics, Ghazi University, DG Khan 32200, Pakistan

Correspondence should be addressed to Mabrook Al-Rakhami; malrakhami@ksu.edu.sa and Suheer Al-Hadhrami; s.alhadhrami1@gmail.com

Received 19 June 2021; Accepted 2 August 2021; Published 13 September 2021

Academic Editor: Kifayat Ullah

Copyright © 2021 Zafar Ullah et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Technology is quickly evolving and becoming part of our lives. Life has become better and easier due to the technologies. Although it has lots of benefits, it also brings serious risks and threats, known as cyberattacks, which are neutralized by cybersecurities. Since spherical fuzzy sets (SFSs) and interval-valued SFS (IVSFS) are an excellent tool in coping with uncertainty and fuzziness, the current study discusses the idea of spherical cubic FSs (SCFSs). These sets are characterized by three mappings known as membership degree, neutral degree, and nonmembership degree. Each of these degrees is spherical cubic fuzzy numbers (SCFNs) such that the summation of their squares does not exceed one. The score function and accuracy function are presented for the comparison of SCFNs. Moreover, the spherical cubic fuzzy weighted geometric (SCFWG) operators and SCF ordered weighted geometric (SCFOWG) operators are established for determining the distance between two SCFNs. Furthermore, some operational rules of the proposed operators are analyzed and multiattribute decision-making (MADM) approach based on these operators is presented. These methods are applied to make the best decision on the basis of risks factors as a numerical illustration. Additionally, the comparison of the proposed method with the existing methods is carried out; since the proposed methods and operators are the generalizations of existing methods, they provide more general, exact, and accurate results. Finally, for the legitimacy, practicality, and usefulness of the decision-making processes, a detailed illustration is given.

1. Introduction

Multiple-attribute decision-making (MADM) means that, from the restricted alternatives set according to multiple attribute, the best alternative is selected that could be called cognitive processing. MADM is a key subdivision of the theory of decision-making (DM), commonly used in human activities [1]. The fuzzy knowledge usually is of two forms: qualitative and quantitative. The quantitative fuzzy knowledge is determined by fuzzy set (FS) [2], intuitionistic FS (IFS) [3], Pythagorean FS (PyFS) [4], and so forth. Zadeh's

FS theory was utilized to characterize fuzzy quantitative knowledge comprising of only membership degree. In light of this, Atanassov proposed IFS, consisting of two degrees, namely, membership and nonmembership. The summation of two grades should be less than or equal to one in an IFS. However, the two degrees often do not fulfill the constraints, but the sum of their squares does. Yager et al. [5] laid down the PyFS, which contains that the square sum of both the degrees is less than or equal to 1. In certain cases, the PyFSs will convey the details more effectively than the IFSs. For instance, if the membership, representing the support of an

expert, is 0.8 and nonmembership representing the opposition, is 0.6, then surely, this information cannot be represented by IFS, but it can be effectively described by PyFS.

Now, the IFS and PyFS do not provide a satisfactory result, since the neutral degree calculates the real-world problems independently. In order to handle this situation, Cuong and Kreinovich [6] originated the notion of a picture fuzzy set (PFS). A PFS is able to use three indexes, namely, membership degree $\check{P}(x)$, neutral degree $\check{I}(x)$, and nonmembership degree $\check{N}(x)$ on condition that $0 \leq \check{P}(x) + \check{I}(x) + \check{N}(x) \leq 1$. Of course, PFSs are more appropriate for managing the fuzziness and ambiguity than IFS or PyFS. Garg [7] presented the picture fuzzy weighted average operator (PFWAO) and picture fuzzy ordered weighted averaging aggregation operators. Since the last few decades, several researchers have investigated PyFS and have successfully applied it to a wide range of fields such as strategic decision-making, decision-making qualities, and design recognition [8–11]. In real life, we often have a lot of problems which cannot be solved with PFS; for instance, $\check{P}(x) + \check{I}(x) + \check{N}(x) > 1$. In these conditions, PFS cannot produce an acceptable result. To make this clear, an example is given: to support and to oppose the extent of an alternative membership, they are, respectively, 1/5, 3/5, and 3/5. This is based on their number exceeding 1 and not being presented for PFS. In light of these conditions, a generalization of PFS is introduced as the concept of spherical fuzzy sets (SFSs). The degrees of membership, neutral, and nonmembership in an SFS has the following condition:

$$0 \leq \check{P}^2(x) + \check{I}^2(x) + \check{N}^2(x) \leq 1. \quad (1)$$

In PyFS, Peng and Yang [12] introduced some new properties that are division, subtraction, and other important characteristics. The authors addressed the superiority and dependence ranking methodologies in the Pythagorean fuzzy setting to clarify the multiattribute decision-making problems. We et al. [13] implemented a maximizing variance protocol in order to clarify decisions based on Pythagorean fuzzy interval conditions. Garg [14] presented the IVPyF average (IVPyFWA) and IVPyF geometric (IVPyFWG) and provided a concept of the new precision function based on PyF's interval-evaluated setting. Liang et al. [10] proposed the concepts of the medium and weighted PyF geometric Bonferroni mean (WPFGBM) operator. Many researchers proposed the idea of IVSF and its applications in DM problems [15–17]. Ayaz et al. [18] introduced the idea of SCFSs and applied it to the problems of multiattribute decision-making.

Many researchers presented the various applications of SFSs, IVSFSs. Kutlu Gündoğdu and Kahraman [19] presented the new approach of SFSs by using the TOPSIS methodology. Gündoğdu [20] presented the principals of SFSs and their applications in DM theory. Zeng et al. [21] gave the new concept by defining the probabilistic interactive aggregation operator of T-SFSs and their applications in solar cell selection. Liu et al. [22] gave the concept of multiattribute decision-making approach for Baiyao's R&D

project selection problem. Mathew et al. [23] presented the novel approach under SFSs for the selection of advanced manufacturing system. Gong et al. [24] gave a new approach of spherical distance for IFSs and its applications in DM. Kutlu Gündoğdu and Kahraman [25] presented the WASPAS extension with spherical fuzzy sets. After that, Gündoğdu and Kahraman [26] gave a new concept and related the TOPSIS methods to IVSFSs. Zeng et al. [27] introduced the TOPSIS methodology with covering-based SF rough set model for MADM. Liu et al. [28] gave a linguistic SFSs approach with applications for evaluation of shared bicycles in China. In 2019, Kutlu Gündoğdu and Kahraman [29] showed a novel approach of VIKOR method using the SFSs and their applications in selection of warehouse site. We et al. [30] presented the similarity measures of SFSs based on cosine function. After that, in 2020, Khan et al. [31] introduced a new approach and related the distance and similarity measures for SFSs and their application in selection of mega project selection. Shishavan et al. [32] extended the idea of similarity measure in the environment of SF information. Recently, Mahmood et al. [33] further enhanced the idea of similarity measure and discussed it applications in pattern recognition and medical diagnosis.

The objectives of this paper include the following: (1) to determine an SCFS, (2) to define SCFNs and the associated operating identity; (3) to propose comparison functions for score, accuracy, or certainty; (4) to propose SCF geometric aggregation operators and some discussion on their properties.

Convincing accretion is one of the commanding tools of decision-making. The values are normalized by the collection operators. Additionally, these operators represent a wide range of data values. The weighted geometric aggregation operators are used for the position values of the required weight. The problems arise whenever the load segments of the weight vectors comprise segments that have a vital difference in parts of the weight vectors. This issue inspired proposing the idea of spherical cubic geometric aggregation operators. Henceforth, we present the notion of spherical cubic geometric aggregation operators. Moreover, the target of the DM techniques is to select the best choice among the available choices. In certain situations, the available choices are arranged in order to select the most suitable choice. These circumstances motivated presenting a method for the ranking of the available choices. We aim to combine the Einstein product and introduce the concepts of SCF Einstein weighted averaging (SCFEWA) operator, SCF Einstein ordered weighted averaging (SCFEOWA) operator, SCF Einstein weighted geometric (SCFEWG) operator, SCF Einstein ordered weighted geometric (SCFEOWG) operator, and some more generalized operators in MADM processes.

This article describes the concept of the SCFS, which is the extension of IVSFS based on the constraints of the fact that the square sum of the supremum of its degrees of membership is less than one. Here, the concept of the SCFS is introduced, that is, the generalization of the IVSFS. We analyze certain SCFS properties. For comparison of SCFNs, the score and degree of deviation are described. The distance

between SCFNs is defined. SCFS states that the square of the supremacy to its membership is less than or equal to 1. Based on this information, aggregation operators are, for example, SCF weighted geometric (SCFWG), SCF ordered weighted geometric (SCFOWG), and SCF hybrid weighted geometric (SCFHWG). In addition, the proposed operators are utilized in problems of decision-making where experts give their preferences in the SCF details to illustrate the practicality of the new method and its efficiency.

The remaining of the paper is organized as follows: Section 2 presents some basic definitions and important properties. Section 3 proposes some geometric aggregation operators for SCFNs and their properties. Section 4 applies the spherical cubic geometric aggregation operators for MADM process. Section 5 presents the numerical illustration for the application and the final section concludes the study.

2. Preliminaries

This section presents a few elementary definitions with their key properties.

Definition 1 (see [2]). Supposing \hat{T} be a universal set, the fuzzy set (FS) \tilde{F} formulates as follows:

$$\tilde{F} = \{ \langle \hat{f}, \tilde{\mu}_{\tilde{F}}(\hat{f}) \rangle | \hat{f} \in \hat{T} \}, \quad (2)$$

where $\tilde{\mu}_{\tilde{F}}: \hat{T} \rightarrow [0, 1]$ is the membership degree of $\hat{f} \in \hat{T}$.

Definition 2 (see [3]). Supposing \hat{T} be a universal set, the fuzzy set (FS) \tilde{I} formulates as follows:

$$\tilde{I} = \{ \langle \hat{f}, \tilde{\mu}_{\tilde{I}}(\hat{f}), \tilde{\vartheta}_{\tilde{I}}(\hat{f}) \rangle | \hat{f} \in \hat{T} \}, \quad (3)$$

where $0 \leq \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \leq \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \leq 1$, $0 \leq \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \leq \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \leq 1$, $0 \leq \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \leq \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \leq 1$, and $0 \leq (\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}))^2 + (\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}))^2 + (\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}))^2 \leq 1$. If $\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) = \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})$, $\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) = \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})$, $\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) = \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})$, then IVSFS reduces to SFS.

Definition 6 (see 27). Let $\tilde{A}_{\tilde{Q}} = ([\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})], [\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})], [\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})])$ be IVSFS, the score function of $\tilde{A}_{\tilde{Q}}$ can be described in the following way:

where $\tilde{\mu}_{\tilde{T}}: \hat{T} \rightarrow [0, 1]$ is the membership degree and $\tilde{\vartheta}_{\tilde{T}}: \hat{T} \rightarrow [0, 1]$ is the nonmembership degree of $\hat{f} \in \hat{T}$, under the specified condition

$$0 \leq \tilde{\mu}_{\tilde{T}} + \tilde{\vartheta}_{\tilde{T}} \leq 1. \quad (4)$$

Definition 3 (see [18]). Suppose \hat{T} be a universal set. A cubic set (CS) \tilde{C} formulates as follows:

$$\tilde{C} = \{ \langle \hat{f}, \tilde{\mu}_{\tilde{C}}(\hat{f}), \tilde{\vartheta}_{\tilde{C}}(\hat{f}) \rangle | \hat{f} \in \hat{T} \}, \quad (5)$$

where $\tilde{\mu}_{\tilde{C}}$ is an IVFS in \hat{T} and $\tilde{\vartheta}_{\tilde{C}}$ is a simple FS.

Definition 4 (see [18]). Supposing \hat{T} be a universal set, then a spherical fuzzy set (SFS) \tilde{Q} is of the following form:

$$\tilde{Q} = \{ \langle \hat{f}, \tilde{\mu}_{\tilde{Q}}(\hat{f}), \tilde{\eta}_{\tilde{Q}}(\hat{f}), \tilde{\vartheta}_{\tilde{Q}}(\hat{f}) \rangle | \hat{f} \in \hat{T} \}, \quad (6)$$

where $\tilde{\mu}_{\tilde{Q}}: \hat{T} \rightarrow [0, 1]$ is the membership degree, $\tilde{\eta}_{\tilde{Q}}: \hat{T} \rightarrow [0, 1]$ is the neutral degree, and $\tilde{\vartheta}_{\tilde{Q}}: \hat{T} \rightarrow [0, 1]$ is the nonmembership degree of $\hat{f} \in \hat{T}$, under the specified condition:

$$0 \leq (\tilde{\mu}_{\tilde{Q}})^2 + (\tilde{\eta}_{\tilde{Q}})^2 + (\tilde{\vartheta}_{\tilde{Q}})^2 \leq 1. \quad (7)$$

The indeterminacy degree for SFS \tilde{Q} is defined as follows:

$$\tilde{\pi}_{\tilde{Q}} = \sqrt{1 - (\tilde{\mu}_{\tilde{Q}})^2 - (\tilde{\eta}_{\tilde{Q}})^2 - (\tilde{\vartheta}_{\tilde{Q}})^2}. \quad (8)$$

For uncomplicatedness, we represent the SFN as $\tilde{Q} = \Gamma_{\tilde{Q}}, \Upsilon_{\tilde{Q}}, \Lambda_{\tilde{Q}}$.

Definition 5 (see [26]). Supposing \hat{T} be a universal set, an interval-valued SFS (IVSFS) $\tilde{A}_{\tilde{Q}}$ is defined as follows:

$$\tilde{A}_{\tilde{Q}} = \left\{ \left\langle \hat{f}, \left(\left[\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right], \left[\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right], \left[\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right] \right) \right\rangle | \hat{f} \in \hat{T} \right\}, \quad (9)$$

$$\text{Score}(\tilde{A}_{\tilde{Q}}) = \frac{1}{2} \left(\left(\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \right)^2 + \left(\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right)^2 - \left(\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \right)^2 - \left(\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right)^2 - \left(\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}) \right)^2 - \left(\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f}) \right)^2 \right), \quad (10)$$

where $\text{score}(\tilde{A}_{\tilde{Q}}) \in [-1, 1]$.

Definition 7 (see 27). Let $\tilde{A}_{\tilde{Q}} = ([\tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\mu}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})], [\tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\eta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})], [\tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^L(\hat{f}), \tilde{\vartheta}_{\tilde{A}_{\tilde{Q}}}^U(\hat{f})])$ be IVSFS, the accuracy function of $\tilde{A}_{\tilde{Q}}$ can be described in the following way:

$$\text{accuracy}(\tilde{A}_{\tilde{Q}}) = \frac{1}{2} \left(\left(\tilde{\mu}_{A_{\tilde{Q}}}^L(\hat{t}) \right)^2 + \left(\tilde{\mu}_{A_{\tilde{Q}}}^U(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{\tilde{Q}}}^L(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{\tilde{Q}}}^U(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{\tilde{Q}}}^L(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{\tilde{Q}}}^U(\hat{t}) \right)^2 \right), \quad (11)$$

where accuracy $(\tilde{A}_{\tilde{Q}}) \in [0, 1]$.

Definition 8 (see [27]). Let $\tilde{A}_{Q_1} = ([\tilde{\mu}_{A_{Q_1}}^L(\hat{t}), \tilde{\mu}_{A_{Q_1}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_1}}^L(\hat{t}), \tilde{\eta}_{A_{Q_1}}^U(\hat{t})], [\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}), \tilde{\vartheta}_{A_{Q_1}}^U(\hat{t})])$ and $\tilde{A}_{Q_2} = ([\tilde{\mu}_{A_{Q_2}}^L(\hat{t}), \tilde{\mu}_{A_{Q_2}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_2}}^L(\hat{t}), \tilde{\eta}_{A_{Q_2}}^U(\hat{t})], [\tilde{\vartheta}_{A_{Q_2}}^L(\hat{t}), \tilde{\vartheta}_{A_{Q_2}}^U(\hat{t})])$ be two IVSFNs; then,

$$\begin{aligned} S(\tilde{A}_{Q_1}) &= \text{Score}(\tilde{A}_{Q_1}) \\ &= \frac{1}{2} \left(\left(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}) \right)^2 + \left(\tilde{\mu}_{A_{Q_1}}^U(\hat{t}) \right)^2 - \left(\tilde{\eta}_{A_{Q_1}}^L(\hat{t}) \right)^2 - \left(\tilde{\eta}_{A_{Q_1}}^U(\hat{t}) \right)^2 - \left(\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}) \right)^2 - \left(\tilde{\vartheta}_{A_{Q_1}}^U(\hat{t}) \right)^2 \right), \\ S(\tilde{A}_{Q_2}) &= \text{Score}(\tilde{A}_{Q_2}) \\ &= \frac{1}{2} \left(\left(\tilde{\mu}_{A_{Q_2}}^L(\hat{t}) \right)^2 + \left(\tilde{\mu}_{A_{Q_2}}^U(\hat{t}) \right)^2 - \left(\tilde{\eta}_{A_{Q_2}}^L(\hat{t}) \right)^2 - \left(\tilde{\eta}_{A_{Q_2}}^U(\hat{t}) \right)^2 - \left(\tilde{\vartheta}_{A_{Q_2}}^L(\hat{t}) \right)^2 - \left(\tilde{\vartheta}_{A_{Q_2}}^U(\hat{t}) \right)^2 \right), \end{aligned} \quad (12)$$

are the score functions of \tilde{A}_{Q_1} and \tilde{A}_{Q_2} , respectively. And

$$\begin{aligned} H(\tilde{A}_{Q_1}) &= \text{accuracy}(\tilde{A}_{Q_1}) \\ &= \frac{1}{2} \left(\left(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}) \right)^2 + \left(\tilde{\mu}_{A_{Q_1}}^U(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{Q_1}}^L(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{Q_1}}^U(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{Q_1}}^U(\hat{t}) \right)^2 \right), \\ H(\tilde{A}_{Q_2}) &= \text{accuracy}(\tilde{A}_{Q_2}) \\ &= \frac{1}{2} \left(\left(\tilde{\mu}_{A_{Q_2}}^L(\hat{t}) \right)^2 + \left(\tilde{\mu}_{A_{Q_2}}^U(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{Q_2}}^L(\hat{t}) \right)^2 + \left(\tilde{\eta}_{A_{Q_2}}^U(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{Q_2}}^L(\hat{t}) \right)^2 + \left(\tilde{\vartheta}_{A_{Q_2}}^U(\hat{t}) \right)^2 \right), \end{aligned} \quad (13)$$

are the accuracy functions of \tilde{A}_{Q_1} and \tilde{A}_{Q_2} , respectively. Then, we have the following properties:

- (1) If $S(\tilde{A}_{Q_1}) < S(\tilde{A}_{Q_2})$, then $\tilde{A}_{Q_1} < \tilde{A}_{Q_2}$,
- (2) If $S(\tilde{A}_{Q_1}) > S(\tilde{A}_{Q_2})$, then $\tilde{A}_{Q_1} > \tilde{A}_{Q_2}$,
- (3) If $S(\tilde{A}_{Q_1}) = S(\tilde{A}_{Q_2})$, then we have
 - (a) If $H(\tilde{A}_{Q_1}) < H(\tilde{A}_{Q_2})$, then $\tilde{A}_{Q_1} < \tilde{A}_{Q_2}$,
 - (b) If $H(\tilde{A}_{Q_1}) > H(\tilde{A}_{Q_2})$, then $\tilde{A}_{Q_1} > \tilde{A}_{Q_2}$,
 - (c) If $H(\tilde{A}_{Q_1}) = H(\tilde{A}_{Q_2})$, then $\tilde{A}_{Q_1} = \tilde{A}_{Q_2}$.

Definition 9 (see [27]). Let $\tilde{A}_{Q_1} = ([\tilde{\mu}_{A_{Q_1}}^L(\hat{t}), \tilde{\mu}_{A_{Q_1}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_1}}^L(\hat{t}), \tilde{\eta}_{A_{Q_1}}^U(\hat{t})], [\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}), \tilde{\vartheta}_{A_{Q_1}}^U(\hat{t})])$ and $\tilde{A}_{Q_2} = ([\tilde{\mu}_{A_{Q_2}}^L(\hat{t}), \tilde{\mu}_{A_{Q_2}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_2}}^L(\hat{t}), \tilde{\eta}_{A_{Q_2}}^U(\hat{t})], [\tilde{\vartheta}_{A_{Q_2}}^L(\hat{t}), \tilde{\vartheta}_{A_{Q_2}}^U(\hat{t})])$ be two IVSFNs; then,

$$\begin{aligned} (1) \tilde{\lambda} \tilde{A}_{Q_1} &= \left(\left[\frac{\sqrt{(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 + (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^U(\hat{t}))^2} \cdot \sqrt{(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 + (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2}}}{|\tilde{\eta}_{A_{Q_1}}^L(\hat{t}) \cdot \tilde{\eta}_{A_{Q_1}}^U(\hat{t}) \cdot \tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}) \cdot \tilde{\vartheta}_{A_{Q_1}}^U(\hat{t})|}, \right. \\ &\quad \left. \left[\frac{\sqrt{(1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 + (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^2 (\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}))^2)}{\sqrt{(1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 + (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^2 (\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}))^2}} \right] \right) \\ (2) \tilde{\mu}_{A_{Q_1}}^L(\hat{t}) \cdot \tilde{\mu}_{A_{Q_1}}^U(\hat{t}) \cdot \tilde{\eta}_{A_{Q_1}}^L(\hat{t}) \cdot \tilde{\eta}_{A_{Q_1}}^U(\hat{t}) &= \left[\frac{\sqrt{(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 + (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^U(\hat{t}))^2} \cdot \sqrt{(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 + (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2}}{\sqrt{(1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2 + (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2) (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^2 (\tilde{\vartheta}_{A_{Q_1}}^L(\hat{t}))^2}} \right] \\ (3) \tilde{\lambda} \tilde{A}_{Q_1} &= \left(\left[\sqrt{1 - (1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2)^{\tilde{\lambda}}}, \sqrt{1 - (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2)^{\tilde{\lambda}}}, |(\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^{\tilde{\lambda}}, (\tilde{\eta}_{A_{Q_1}}^U(\hat{t}))^{\tilde{\lambda}}|, \right. \\ &\quad \left. \left[\frac{\sqrt{(1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2)^{\tilde{\lambda}} - (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2)^{\tilde{\lambda}} - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^{2\tilde{\lambda}}}}{\sqrt{(1 - (\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^2)^{\tilde{\lambda}} - (1 - (\tilde{\mu}_{A_{Q_1}}^U(\hat{t}))^2)^{\tilde{\lambda}} - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^{2\tilde{\lambda}}}} \right] \right) \end{aligned}$$

$$(4) \tilde{A}_{Q_1}^\lambda = \left(\begin{array}{c} \frac{(\tilde{\mu}_{A_{Q_1}}^L(\hat{t}))^\lambda \cdot (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda}{\sqrt{1 - (1 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^\lambda) \cdot \sqrt{1 - (1 - (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda) \cdot (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^\lambda}}}} \\ \frac{\sqrt{(1 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^\lambda)^2 - (1 - (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda)^2 - (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda}}}{\sqrt{(1 - (\tilde{\eta}_{A_{Q_1}}^L(\hat{t}))^\lambda)^2 - (1 - (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda)^2 - (\tilde{\theta}_{A_{Q_1}}^L(\hat{t}))^\lambda}} \end{array} \right).$$

Definition 10 (see [27]). Let $\tilde{A}_{Q_1} = ([\tilde{\mu}_{A_{Q_1}}^L(\hat{t}), \tilde{\mu}_{A_{Q_1}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_1}}^L(\hat{t}), \tilde{\eta}_{A_{Q_1}}^U(\hat{t})], [\tilde{\theta}_{A_{Q_1}}^L(\hat{t}), \tilde{\theta}_{A_{Q_1}}^U(\hat{t})](\hat{t}))$ and $\tilde{A}_{Q_2} = ([\tilde{\mu}_{A_{Q_2}}^L(\hat{t}), \tilde{\mu}_{A_{Q_2}}^U(\hat{t})], [\tilde{\eta}_{A_{Q_2}}^L(\hat{t}), \tilde{\eta}_{A_{Q_2}}^U(\hat{t})], [\tilde{\theta}_{A_{Q_2}}^L(\hat{t}), \tilde{\theta}_{A_{Q_2}}^U(\hat{t})](\hat{t}))$ be the set comprised of IVSFNs; then, IVSF weighted geometric (IVSFWG) operator is defined as follows:

$$\text{IVSFWG}(\tilde{A}_{Q_1}, \tilde{A}_{Q_2}, \dots, \tilde{A}_{Q_N}) = \left(\begin{array}{c} \left[\prod_{i=1}^n \left(\tilde{\mu}_{A_{Q_i}}^L(\hat{t}) \right)^{\omega_i}, \prod_{i=1}^n \left(\tilde{\mu}_{A_{Q_i}}^U(\hat{t}) \right)^{\omega_i} \right], \\ \left[\sqrt{1 - \prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^L(\hat{t}) \right)^2 \right)^{\omega_i}}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^U(\hat{t}) \right)^2 \right)^{\omega_i}} \right], \left[\left(\tilde{\eta}_{A_{Q_i}}^L(\hat{t}) \right)^{\tilde{\lambda}}, \left(\tilde{\eta}_{A_{Q_i}}^U(\hat{t}) \right)^{\tilde{\lambda}} \right], \\ \left[\sqrt{\prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^L(\hat{t}) \right)^2 \right)^{\omega_i} - \prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^L(\hat{t}) \right)^2 - \left(\tilde{\theta}_{A_{Q_i}}^L(\hat{t}) \right)^2 \right)^{\omega_i}}, \right. \\ \left. \sqrt{\prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^U(\hat{t}) \right)^2 \right)^{\omega_i} - \prod_{i=1}^n \left(1 - \left(\tilde{\eta}_{A_{Q_i}}^U(\hat{t}) \right)^2 - \left(\tilde{\theta}_{A_{Q_i}}^U(\hat{t}) \right)^2 \right)^{\omega_i}} \right] \end{array} \right) \quad (14)$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ represent the weight vector of \tilde{A}_{Q_i} ($i = 1, 2, \dots, n$) and $0 \leq \omega_i \leq 1, \sum_{i=1}^n \omega_i = 1$.

Definition 11 (see [18]). Supposing \hat{T} be a universal set, then a spherical cubic FS (SCFS) \tilde{Q}_C is defined as follows:

$$\tilde{Q}_C = \left\{ \left\langle \hat{t}, \Gamma_{\tilde{Q}_C}(\hat{t}), Y_{\tilde{Q}_C}(\hat{t}), \Lambda_{\tilde{Q}_C}(\hat{t}) \right\rangle \mid \hat{t} \in \hat{T} \right\}, \quad (15)$$

where $\Gamma_{\tilde{Q}_C}(\hat{t}) = [a_{\tilde{Q}_C}^-, a_{\tilde{Q}_C}^+]$, $\tilde{\mu}_{\tilde{Q}_C}$ represents the membership function, $Y_{\tilde{Q}_C}(\hat{t}) = [n_{\tilde{Q}_C}^-, n_{\tilde{Q}_C}^+]$, $\tilde{\eta}_{\tilde{Q}_C}$ represents the neutral degree, and $\Lambda_{\tilde{Q}_C}(\hat{t}) = [b_{\tilde{Q}_C}^-, b_{\tilde{Q}_C}^+]$, $\tilde{\theta}_{\tilde{Q}_C}$ represents the non-membership function, in which $\tilde{\mu}_{\tilde{Q}_C}: \hat{T} \rightarrow [0, 1]$, $\tilde{\eta}_{\tilde{Q}_C}: \hat{T} \rightarrow [0, 1]$, and $\tilde{\theta}_{\tilde{Q}_C}: \hat{T} \rightarrow [0, 1]$, $\hat{t} \in \hat{T}$, under the specified condition:

$$0 \leq \left(\sup \left[a_{\tilde{Q}_C}^-, a_{\tilde{Q}_C}^+ \right] \right)^2 + \left(\sup \left[n_{\tilde{Q}_C}^-, n_{\tilde{Q}_C}^+ \right] \right)^2 + \left(\sup \left[b_{\tilde{Q}_C}^-, b_{\tilde{Q}_C}^+ \right] \right)^2 \leq 1, \quad (16)$$

$$0 \leq \left(\tilde{\mu}_{\tilde{Q}_C} \right)^2 + \left(\tilde{\eta}_{\tilde{Q}_C} \right)^2 + \left(\tilde{\theta}_{\tilde{Q}_C} \right)^2 \leq 1.$$

The indeterminacy degree for (SCFS) \tilde{Q}_C is defined as follows:

$$\tilde{\pi}_{\tilde{Q}_C} = \left\langle \sqrt{1 - \left(\sup \left[a_{\tilde{Q}_C}^-, a_{\tilde{Q}_C}^+ \right] \right)^2 - \left(\sup \left[n_{\tilde{Q}_C}^-, n_{\tilde{Q}_C}^+ \right] \right)^2 - \left(\sup \left[b_{\tilde{Q}_C}^-, b_{\tilde{Q}_C}^+ \right] \right)^2}, \sqrt{1 - \left(\tilde{\mu}_{\tilde{Q}_C} \right)^2 - \left(\tilde{\eta}_{\tilde{Q}_C} \right)^2 - \left(\tilde{\theta}_{\tilde{Q}_C} \right)^2} \right\rangle. \quad (17)$$

For simplicity, we represent the spherical number (SCFN) as $\tilde{Q}_C = \Gamma_{\tilde{Q}_C}, Y_{\tilde{Q}_C}, \Lambda_{\tilde{Q}_C}$.

Definition 12 (see [19]). Let $\tilde{Q}_C = \left\{ \left\langle [a_{\tilde{Q}_C}^-, a_{\tilde{Q}_C}^+], \tilde{\mu}_{\tilde{Q}_C} \right\rangle, \left[n_{\tilde{Q}_C}^-, n_{\tilde{Q}_C}^+ \right], \tilde{\eta}_{\tilde{Q}_C}, \left\langle [b_{\tilde{Q}_C}^-, b_{\tilde{Q}_C}^+], \tilde{\theta}_{\tilde{Q}_C} \right\rangle \right\}$ be SCFS; then, the score function of \tilde{Q}_C is defined as follows:

$$\text{Score}(\tilde{Q}_{C_1}) = \frac{1}{9} \left(\left(a_{Q_{C_1}}^- + a_{Q_{C_1}}^+ + \tilde{\mu}_{Q_{C_1}} \right)^2 + \left(n_{Q_{C_1}}^- + n_{Q_{C_1}}^+ + \tilde{\eta}_{Q_{C_1}} \right)^2 - \left(b_{Q_{C_1}}^- + b_{Q_{C_1}}^+ + \tilde{\vartheta}_{Q_{C_1}} \right)^2 \right), \quad (18)$$

where $\text{Score}(\tilde{Q}_{C_1}) \in [-1, 1]$.

Definition 13 (see [19]). Supposing $\tilde{Q}_{C_1} = \left\{ \left\langle \left[a_{Q_{C_1}}^-, a_{Q_{C_1}}^+ \right], \tilde{\mu}_{Q_{C_1}} \right\rangle, \left\langle \left[n_{Q_{C_1}}^-, n_{Q_{C_1}}^+ \right], \tilde{\eta}_{Q_{C_1}} \right\rangle, \left\langle \left[b_{Q_{C_1}}^-, b_{Q_{C_1}}^+ \right], \tilde{\vartheta}_{Q_{C_1}} \right\rangle \right\}$ be SCFSs, then the accuracy function of \tilde{Q}_{C_1} is defined as follows:

$$\text{accuracy}(\tilde{Q}_{C_1}) = \frac{1}{9} \left(\left(a_{Q_{C_1}}^- + a_{Q_{C_1}}^+ + \tilde{\mu}_{Q_{C_1}} \right)^2 + \left(n_{Q_{C_1}}^- + n_{Q_{C_1}}^+ + \tilde{\eta}_{Q_{C_1}} \right)^2 + \left(b_{Q_{C_1}}^- + b_{Q_{C_1}}^+ + \tilde{\vartheta}_{Q_{C_1}} \right)^2 \right), \quad (19)$$

where $\text{accuracy}(\tilde{Q}_{C_1}) \in [0, 1]$.

Definition 14 (see [19]). Let $\tilde{Q}_{C_1} = \Gamma_{Q_{C_1}}^-, Y_{Q_{C_1}}^-, \Lambda_{Q_{C_1}}^-$ and $\tilde{Q}_{C_2} = \Gamma_{Q_{C_2}}^-, Y_{Q_{C_2}}^-, \Lambda_{Q_{C_2}}^-$ be two SCFNs; then, we have the following:

- (1) If $\text{score}(\tilde{Q}_{C_1}) < \text{score}(\tilde{Q}_{C_2})$, then $\tilde{Q}_{C_1} < \tilde{Q}_{C_2}$,
- (2) If $\text{score}(\tilde{Q}_{C_1}) > \text{score}(\tilde{Q}_{C_2})$, then $\tilde{Q}_{C_1} > \tilde{Q}_{C_2}$,
- (3) If $\text{score}(\tilde{Q}_{C_1}) = \text{score}(\tilde{Q}_{C_2})$, then
 - (a) If $\text{accuracy}(\tilde{Q}_{C_1}) < \text{accuracy}(\tilde{Q}_{C_2})$, then
 - (b) If $\text{accuracy}(\tilde{Q}_{C_1}) > \text{accuracy}(\tilde{Q}_{C_2})$, then i $\tilde{Q}_{C_1} > \tilde{Q}_{C_2}$,
 - (c) If $\text{accuracy}(\tilde{Q}_{C_1}) = \text{accuracy}(\tilde{Q}_{C_2})$, then $\tilde{Q}_{C_1} = \tilde{Q}_{C_2}$.

Definition 15 (see [19]). Let $\tilde{Q}_{C_1} = \Gamma_{Q_{C_1}}^-, Y_{Q_{C_1}}^-, \Lambda_{Q_{C_1}}^-$ and $\tilde{Q}_{C_2} = \Gamma_{Q_{C_2}}^-, Y_{Q_{C_2}}^-, \Lambda_{Q_{C_2}}^-$ be two SCFNs; then, the distance function among \tilde{Q}_{C_1} and \tilde{Q}_{C_2} is defined as follows:

$$\bar{D}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}) = \frac{1}{9} \left[\left| \left(a_{Q_{C_1}}^- \right)^2 - \left(a_{Q_{C_2}}^- \right)^2 \right| + \left| \left(a_{Q_{C_1}}^+ \right)^2 - \left(a_{Q_{C_2}}^+ \right)^2 \right| + \left| \left(\tilde{\mu}_{Q_{C_1}} \right)^2 - \left(\tilde{\mu}_{Q_{C_2}} \right)^2 \right| + \left| \left(n_{Q_{C_1}}^- \right)^2 - \left(n_{Q_{C_2}}^- \right)^2 \right| + \left| \left(n_{Q_{C_1}}^+ \right)^2 - \left(n_{Q_{C_2}}^+ \right)^2 \right| + \left| \left(\tilde{\eta}_{Q_{C_1}} \right)^2 - \left(\tilde{\eta}_{Q_{C_2}} \right)^2 \right| + \left| \left(b_{Q_{C_1}}^- \right)^2 - \left(b_{Q_{C_2}}^- \right)^2 \right| + \left| \left(b_{Q_{C_1}}^+ \right)^2 - \left(b_{Q_{C_2}}^+ \right)^2 \right| + \left| \left(\tilde{\vartheta}_{Q_{C_1}} \right)^2 - \left(\tilde{\vartheta}_{Q_{C_2}} \right)^2 \right| \right]. \quad (20)$$

Definition 16 (see [19]). Let $\tilde{Q}_{C_1} = \Gamma_{Q_{C_1}}^-, Y_{Q_{C_1}}^-, \Lambda_{Q_{C_1}}^-$ and $\tilde{Q}_{C_2} = \Gamma_{Q_{C_2}}^-, Y_{Q_{C_2}}^-, \Lambda_{Q_{C_2}}^-$ be two spherical cubic fuzzy numbers; then, the below operational laws hold:

$$(1) \tilde{Q}_{C_1} \oplus \tilde{Q}_{C_2} = \left\{ \begin{array}{l} \left(\left[\sqrt{\left(a_{Q_{C_1}}^- \right)^2 + \left(a_{Q_{C_2}}^- \right)^2 - \left(a_{Q_{C_1}}^- \right)^2 \left(a_{Q_{C_2}}^- \right)^2}, \right. \right. \\ \left. \sqrt{\left(a_{Q_{C_1}}^+ \right)^2 + \left(a_{Q_{C_2}}^+ \right)^2 - \left(a_{Q_{C_1}}^+ \right)^2 \left(a_{Q_{C_2}}^+ \right)^2} \right], \\ \sqrt{\left(\tilde{\mu}_{Q_{C_1}} \right)^2 + \left(\tilde{\mu}_{Q_{C_2}} \right)^2 - \left(\tilde{\mu}_{Q_{C_1}} \right)^2 \left(\tilde{\mu}_{Q_{C_2}} \right)^2}, \\ \left(\left[n_{Q_{C_1}}^-, n_{Q_{C_2}}^- \right], n_{Q_{C_1}}^+, n_{Q_{C_2}}^+ \right), \tilde{\eta}_{Q_{C_1}}^-, \tilde{\eta}_{Q_{C_2}}^-, \\ \left(\left[b_{Q_{C_1}}^-, b_{Q_{C_2}}^- \right], b_{Q_{C_1}}^+, b_{Q_{C_2}}^+ \right), \tilde{\vartheta}_{Q_{C_1}}^-, \tilde{\vartheta}_{Q_{C_2}}^- \end{array} \right\};$$

$$(2) \tilde{Q}_{C_1} \otimes \tilde{Q}_{C_2} = \left\{ \begin{array}{l} \left(\left[a_{Q_{C_1}}^-, a_{Q_{C_2}}^- \right], a_{Q_{C_1}}^+, a_{Q_{C_2}}^+ \right), \left(\left[\tilde{\mu}_{Q_{C_1}}^-, \tilde{\mu}_{Q_{C_2}}^- \right], \tilde{\mu}_{Q_{C_1}}^+, \tilde{\mu}_{Q_{C_2}}^+ \right), \left(\left[n_{Q_{C_1}}^-, n_{Q_{C_2}}^- \right], n_{Q_{C_1}}^+, n_{Q_{C_2}}^+ \right), \tilde{\eta}_{Q_{C_1}}^-, \tilde{\eta}_{Q_{C_2}}^-, \\ \left(\left[\sqrt{\left(b_{Q_{C_1}}^- \right)^2 + \left(b_{Q_{C_2}}^- \right)^2 - \left(b_{Q_{C_1}}^- \right)^2 \left(b_{Q_{C_2}}^- \right)^2}, \right. \right. \\ \left. \sqrt{\left(b_{Q_{C_1}}^+ \right)^2 + \left(b_{Q_{C_2}}^+ \right)^2 - \left(b_{Q_{C_1}}^+ \right)^2 \left(b_{Q_{C_2}}^+ \right)^2} \right], \\ \left. \sqrt{\left(\tilde{\vartheta}_{Q_{C_1}}^- \right)^2 + \left(\tilde{\vartheta}_{Q_{C_2}}^- \right)^2 - \left(\tilde{\vartheta}_{Q_{C_1}}^- \right)^2 \left(\tilde{\vartheta}_{Q_{C_2}}^- \right)^2} \right) \end{array} \right\};$$

$$(3) \tau \cdot \tilde{Q}_{C_1} = \left\{ \begin{array}{l} \left(\left[\sqrt{1 - \left(1 - \left(a_{Q_{C_1}}^- \right)^2} \right)^\tau}, \sqrt{1 - \left(1 - \left(a_{Q_{C_1}}^+ \right)^2} \right)^\tau} \right], \\ \sqrt{1 - \left(1 - \left(\tilde{\mu}_{Q_{C_1}}^- \right)^2 \right)^\tau}, \left(\left(n_{Q_{C_1}}^- \right)^\tau, \left(n_{Q_{C_1}}^+ \right)^\tau, \left(\tilde{\eta}_{Q_{C_1}}^- \right)^\tau \right), \\ \left(\left(b_{Q_{C_1}}^- \right)^\tau, \left(b_{Q_{C_1}}^+ \right)^\tau, \left(\tilde{\vartheta}_{Q_{C_1}}^- \right)^\tau \right) \end{array} \right\};$$

$$(4) \tilde{Q}_{C_1}^\tau = \left\{ \begin{array}{l} \left(\left[\left(a_{Q_{C_1}}^- \right)^\tau, \left(a_{Q_{C_1}}^+ \right)^\tau \right], \left(\tilde{\mu}_{Q_{C_1}}^- \right)^\tau, \right. \\ \left. \left(\left[n_{Q_{C_1}}^- \right]^\tau, \left(n_{Q_{C_1}}^+ \right)^\tau, \left(\tilde{\eta}_{Q_{C_1}}^- \right)^\tau \right) \right) \\ \left(\left[\sqrt{1 - \left(1 - \left(b_{Q_{C_1}}^- \right)^2} \right)^\tau}, \sqrt{1 - \left(1 - \left(b_{Q_{C_1}}^+ \right)^2} \right)^\tau} \right], \right. \\ \left. \sqrt{1 - \left(1 - \left(\tilde{\vartheta}_{Q_{C_1}}^- \right)^2 \right)^\tau} \right) \end{array} \right\}.$$

3. Spherical Cubic Fuzzy Geometric Aggregation Operators

This section proposes a few geometric aggregation operators for spherical cubic fuzzy numbers and discusses their properties.

Definition 17. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a collection of SCFNs; then, 'SCF' weighted geometric (SCFWG) operator is a mapping SCFWG: $SCFN^n \rightarrow SCFN$ defined as follows:

$$SCFWG(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) = \left(\tilde{Q}_{C_1} \right)^{\omega_1} \otimes \left(\tilde{Q}_{C_2} \right)^{\omega_2} \otimes \dots \otimes \left(\tilde{Q}_{C_n} \right)^{\omega_n}, \quad (21)$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$.

Theorem 1. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a collection of SCFNs, and let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, the aggregation of SCFWG operator is also an SCFN:

$$SCFWG(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \dots, \tilde{Q}_{C_n}) = \left\{ \begin{array}{l} \left(\left[\prod_{i=1}^n \left(a_{\tilde{Q}_{C_i}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(a_{\tilde{Q}_{C_i}}^+ \right)^{\omega_i} \right], \prod_{i=1}^n \left(\tilde{\mu}_{\tilde{Q}_{C_i}} \right)^{\omega_i} \right), \\ \left(\left[\prod_{i=1}^n \left(n_{\tilde{Q}_{C_i}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(n_{\tilde{Q}_{C_i}}^+ \right)^{\omega_i} \right], \prod_{i=1}^n \left(\tilde{\eta}_{\tilde{Q}_{C_i}} \right)^{\omega_i} \right), \\ \left(\left[\sqrt{\left[1 - \prod_{i=1}^n \left(1 - \left(b_{\tilde{Q}_{C_i}}^- \right)^2 \right)^{\omega_i}} \right]}, \sqrt{\left[1 - \prod_{i=1}^n \left(1 - \left(b_{\tilde{Q}_{C_i}}^+ \right)^2 \right)^{\omega_i} \right]}, \sqrt{\left[1 - \prod_{i=1}^n \left(1 - \left(\tilde{\vartheta}_{\tilde{Q}_{C_i}} \right)^2 \right)^{\omega_i} \right]} \right) \end{array} \right\}. \quad (22)$$

Proof. Proof of the theorem follows from theorem [7] of [18].

where

$$\begin{aligned} \tilde{Q}_{C(\min)} &= \left(\min \Gamma_{\tilde{Q}_{C_i}}, \min Y_{\tilde{Q}_{C_i}}, \max \Lambda_{\tilde{Q}_{C_i}} \right), \\ \tilde{Q}_{C(\max)} &= \left(\max \Gamma_{\tilde{Q}_{C_i}}, \min Y_{\tilde{Q}_{C_i}}, \min \Lambda_{\tilde{Q}_{C_i}} \right). \end{aligned} \quad (25)$$

3.1. Properties. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a set comprising of SCFNs, and the weight vector of \tilde{Q}_{C_i} is $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, there are some properties that the SCFWG operator has clearly fulfilled.

3.1.3. Monotonicity. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ and $\tilde{Q}_{C_i}^* = \Gamma_{\tilde{Q}_{C_i}}^*, Y_{\tilde{Q}_{C_i}}^*, \Lambda_{\tilde{Q}_{C_i}}^*$ ($i = 1, 2, \dots, n$) be collections of SCFNs; if $\tilde{Q}_{C_i} \subseteq \tilde{Q}_{C_i}^*$ for all, then

3.1.1. Idempotency. If all \tilde{Q}_{C_i} are identical, that is, $\tilde{Q}_{C_i} = \tilde{Q}_{C_i}$, then

$$SCFWG(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) = \tilde{Q}_{C_i}. \quad (23)$$

$$\begin{aligned} &SCFWG(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \\ &\subseteq SCFWG(\tilde{Q}_{C_1}^*, \tilde{Q}_{C_2}^*, \tilde{Q}_{C_3}^*, \dots, \tilde{Q}_{C_n}^*). \end{aligned} \quad (26)$$

3.1.2. Boundary.

$$\tilde{Q}_{C(\min)} \leq SCFWG(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \leq \tilde{Q}_{C(\max)}, \quad (24)$$

Definition 18. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a set comprising of SCFNs; then, SCF ordered weighted

geometric (SCFOWG) operator is a mapping SCFOWG: $\text{SCFN}^n \rightarrow \text{SCFN}$ defined as follows:

$$\begin{aligned} \text{SCFOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \\ = (\tilde{Q}_{C_1})^{\omega_1} \otimes (\tilde{Q}_{C_2})^{\omega_2} \otimes \dots \otimes (\tilde{Q}_{C_n})^{\omega_n}, \end{aligned} \quad (27)$$

where the i^{th} largest weighted value is $\tilde{Q}_{\sigma(C_i)}$ by the total order $\tilde{Q}_{\sigma(C_1)} \geq \tilde{Q}_{\sigma(C_2)} \geq \dots \geq \tilde{Q}_{\sigma(C_n)}$ and $\tilde{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$.

Theorem 2. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a set comprising of SCFNs, and let $\tilde{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, the aggregation of the SCFOWG operator is also a SCFN:

$$\text{SCFOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \dots, \tilde{Q}_{C_n}) = \left\{ \begin{array}{l} \left(\left[\prod_{i=1}^n \left(a_{\tilde{Q}_{\sigma(C_i)}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(a_{\tilde{Q}_{\sigma(C_i)}}^+ \right)^{\omega_i} \right], \left[\prod_{i=1}^n \left(\tilde{\mu}_{\tilde{Q}_{\sigma(C_i)}} \right)^{\omega_i} \right], \right. \\ \left. \left(\left[\prod_{i=1}^n \left(n_{\tilde{Q}_{\sigma(C_i)}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(n_{\tilde{Q}_{\sigma(C_i)}}^+ \right)^{\omega_i} \right], \left[\prod_{i=1}^n \left(\tilde{\eta}_{\tilde{Q}_{\sigma(C_i)}} \right)^{\omega_i} \right], \right. \\ \left. \left(\left[\sqrt{1 - \prod_{i=1}^n \left(1 - \left(b_{\tilde{Q}_{\sigma(C_i)}}^- \right)^2 \right)^{\omega_i}}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(b_{\tilde{Q}_{\sigma(C_i)}}^+ \right)^2 \right)^{\omega_i}}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(\tilde{\vartheta}_{\tilde{Q}_{\sigma(C_i)}} \right)^2 \right)^{\omega_i}} \right] \right), \right\}. \quad (28)$$

Proof. Proof of the theorem follows from theorem [8] of [18].

3.2. Properties. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a set comprising of SCFNs, and the weight vector of \tilde{Q}_{C_i} is $\tilde{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, there are some properties that the SCFOWG operator has clearly fulfilled.

3.2.1. Idempotency. If all \tilde{Q}_{C_i} are identical, that is, $\tilde{Q}_{C_i} = \tilde{Q}_{C_j}$, then

$$\text{SCFOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) = \tilde{Q}_{C_i}. \quad (29)$$

3.2.2. Boundary.

$$\tilde{Q}_{C(\min)} \leq \text{SCFOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \leq \tilde{Q}_{C(\max)}, \quad (30)$$

where

$$\begin{aligned} \tilde{Q}_{C(\min)} &= \left(\min \sigma_{\tilde{Q}_{C_i}}, \min \beta_{\tilde{Q}_{C_i}}, \max \gamma_{\tilde{Q}_{C_i}} \right), \\ \tilde{Q}_{C(\max)} &= \left(\max \sigma_{\tilde{Q}_{C_i}}, \min \beta_{\tilde{Q}_{C_i}}, \min \gamma_{\tilde{Q}_{C_i}} \right). \end{aligned} \quad (31)$$

3.2.3. Monotonicity. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ and $\tilde{Q}_{C_i}^* = \Gamma_{\tilde{Q}_{C_i}^*}, Y_{\tilde{Q}_{C_i}^*}, \Lambda_{\tilde{Q}_{C_i}^*}$ ($i = 1, 2, \dots, n$) be collections of SCFNs; if $\tilde{Q}_{C_i} \leq \tilde{Q}_{C_i}^*$ for all i , then

$$\begin{aligned} \text{SCFOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \\ \subseteq \text{SCFOWG}(\tilde{Q}_{C_1}^*, \tilde{Q}_{C_2}^*, \tilde{Q}_{C_3}^*, \dots, \tilde{Q}_{C_n}^*). \end{aligned} \quad (32)$$

Definition 19. Let i be a set comprised of SCFNs; then, SCF hybrid ordered weighted geometric (SCFHOWG) operator is a mapping SCFHOWG: $\text{SCFN}^n \rightarrow \text{SCFN}$ defined as follows:

$$\begin{aligned} \text{SCFHOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \\ = (\tilde{Q}_{C_1})^{\omega_1} \otimes (\tilde{Q}_{C_2})^{\omega_2} \otimes \dots \otimes (\tilde{Q}_{C_n})^{\omega_n}, \end{aligned} \quad (33)$$

where the i^{th} largest weighted value is $\tilde{Q}_{\sigma(C_i)}$ and $\tilde{Q}_{C_i}^L$ ($\tilde{Q}_{C_i}^L = n\omega_i\tilde{Q}_{C_i}$) and $\tilde{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Also, $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \dots, \tilde{w}_n)$ is the associated weight vector such that $0 \leq \tilde{w}_j \leq 1$ and $\sum_{j=1}^n \tilde{w}_j = 1$.

Theorem 4. Let $\tilde{Q}_{C_i} = \Gamma_{\tilde{Q}_{C_i}}, Y_{\tilde{Q}_{C_i}}, \Lambda_{\tilde{Q}_{C_i}}$ ($i = 1, 2, 3, \dots, n$) be a set comprised of SCFNs, and let $\tilde{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of \tilde{Q}_{C_i} , with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, the aggregation of SCFHOWG operator is an SCFN as well, and

$$\text{SCFHOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \dots, \tilde{Q}_{C_n}) = \left\{ \begin{array}{l} \left(\left[\prod_{i=1}^n \left(a_{Q_{C_i}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(a_{Q_{C_i}}^+ \right)^{\omega_i} \right], \prod_{i=1}^n \left(\tilde{\mu}_{Q_{C_i}} \right)^{\omega_i} \right), \\ \left(\left[\prod_{i=1}^n \left(n_{Q_{C_i}}^- \right)^{\omega_i}, \prod_{i=1}^n \left(n_{Q_{C_i}}^+ \right)^{\omega_i} \right], \prod_{i=1}^n \left(\tilde{\eta}_{Q_{C_i}} \right)^{\omega_i} \right), \\ \left(\left[\sqrt{1 - \prod_{i=1}^n \left(1 - \left(b_{Q_{C_i}}^- \right)^2 \right)^{\omega_i}}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(b_{Q_{C_i}}^+ \right)^2 \right)^{\omega_i}}, \sqrt{1 - \prod_{i=1}^n \left(1 - \left(\tilde{\vartheta}_{Q_{C_i}} \right)^2 \right)^{\omega_i}} \right] \right) \end{array} \right\}. \quad (34)$$

Proof. Proof of the theorem follows from theorem [9] of [18].

3.3. Properties. Let $\tilde{Q}_{C_i} = \Gamma_{Q_{C_i}}^-, Y_{Q_{C_i}}^-, \Lambda_{Q_{C_i}}^-$ ($i = 1, 2, 3, \dots, n$) be a set comprised of SCFNs, and the weight vector of \tilde{Q}_{C_i} is $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^n \omega_i = 1$. Then, there are some properties that the SCFHOWG operator has clearly fulfilled.

3.3.1. Idempotency. If all \tilde{Q}_{C_i} are identical, that is, $\tilde{Q}_{C_i} = \tilde{Q}_{C_j}$, then

$$\text{SCFHOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) = \tilde{Q}_{C_i}. \quad (35)$$

3.3.2. Boundary.

$$\tilde{Q}_{C_{(\min)}} \leq \text{SCFHOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \leq \tilde{Q}_{C_{(\max)}}, \quad (36)$$

where

$$\begin{aligned} \tilde{Q}_{C_{(\min)}} &= \left(\min \sigma_{Q_{C_i}}^-, \min \beta_{Q_{C_i}}^-, \max \gamma_{Q_{C_i}}^- \right), \\ \tilde{Q}_{C_{(\max)}} &= \left(\max \sigma_{Q_{C_i}}^-, \min \beta_{Q_{C_i}}^-, \min \gamma_{Q_{C_i}}^- \right). \end{aligned} \quad (37)$$

3.3.3. Monotonicity. Let $\tilde{Q}_{C_i} = \Gamma_{Q_{C_i}}^-, Y_{Q_{C_i}}^-, \Lambda_{Q_{C_i}}^-$ and $\tilde{Q}_{C_i}^* = \Gamma_{Q_{C_i}}^*, Y_{Q_{C_i}}^*, \Lambda_{Q_{C_i}}^*$ ($i = 1, 2, \dots, n$) be collections of SCFNs; if $\tilde{Q}_{C_i} \subseteq \tilde{Q}_{C_i}^*$ for all i , then

$$\begin{aligned} &\text{SCFHOWG}(\tilde{Q}_{C_1}, \tilde{Q}_{C_2}, \tilde{Q}_{C_3}, \dots, \tilde{Q}_{C_n}) \\ &\subseteq \text{SCFHOWG}(\tilde{Q}_{C_1}^*, \tilde{Q}_{C_2}^*, \tilde{Q}_{C_3}^*, \dots, \tilde{Q}_{C_n}^*). \end{aligned} \quad (38)$$

4. Application for MADM Based on Spherical Cubic Fuzzy Geometric Aggregation Operators

This section utilizes the SCF geometric aggregation operators for the MADM process. Suppose we have n choices $\hat{Y} = \{\hat{y}_1, \hat{y}_2, \dots, \hat{y}_n\}$ and m criteria $\hat{A} = \{\hat{A}_1, \hat{A}_2, \hat{A}_3, \dots, \hat{A}_m\}$ to be determined with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^n \omega_j = 1$. In order to assess the proficiency of the alternative \hat{y}_i according to \hat{A}_j criteria, the decision-maker must not simply provide the details where \hat{y}_i fulfills the criteria \hat{A}_j , but the alternative \hat{y}_i does not fulfill the criteria \hat{A}_j or the alternative \hat{y}_i remains unchanged. These three components can be defined by $\Gamma_{Q_{C_i}}^-, Y_{Q_{C_i}}^-, \Lambda_{Q_{C_i}}^-$ where $\Gamma_{Q_{C_i}}^- = [a_{Q_{C_i}}^-, a_{Q_{C_i}}^+]$, $\tilde{\mu}_{Q_{C_i}}^-$ represent the degree where the alternative \hat{y}_i fulfills the criteria \hat{A}_j and $Y = [n_{Q_{C_i}}^-, n_{Q_{C_i}}^+]$, $\tilde{\eta}_{Q_{C_i}}^-$ represents the degree where the alternative \hat{y}_i remains unchanged and $\Lambda_{Q_{C_i}}^- = [b_{Q_{C_i}}^-, b_{Q_{C_i}}^+]$, $\tilde{\vartheta}_{Q_{C_i}}^-$ represents the degree where the alternative \hat{y}_i does not fulfill the criteria \hat{A}_j ; then, the efficiency of \hat{y}_i under the criteria \hat{A}_j can be defined by SCFN, $\tilde{Q}_{C_i} = ([a_{Q_{C_i}}^-, a_{Q_{C_i}}^+], \tilde{\mu}_{Q_{C_i}}^-, [n_{Q_{C_i}}^-, n_{Q_{C_i}}^+], \tilde{\eta}_{Q_{C_i}}^-, [b_{Q_{C_i}}^-, b_{Q_{C_i}}^+], \tilde{\vartheta}_{Q_{C_i}}^-)$ with the specified condition $0 \leq (\sup[a_{Q_{C_i}}^-, a_{Q_{C_i}}^+])^2 + (\sup[n_{Q_{C_i}}^-, n_{Q_{C_i}}^+])^2 + (\sup[b_{Q_{C_i}}^-, b_{Q_{C_i}}^+])^2 \leq 1$ and $0 \leq (\tilde{\mu}_{Q_{C_i}}^-)^2 + (\tilde{\eta}_{Q_{C_i}}^-)^2 \leq 1$. The following steps are specified to obtain the ranking of the alternatives.

TABLE 1: First spherical cubic fuzzy decision matrix.

	\hat{A}_1	\hat{A}_2	\hat{A}_3	\hat{A}_4
\hat{y}_1	$\begin{pmatrix} \langle [1, 2], 3 \rangle \\ \langle [4, 5], 4 \rangle \\ \langle [1, 3], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 3 \rangle \\ \langle [1, 4], 3 \rangle \\ \langle [1, 3], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [3, 5], 3 \rangle \\ \langle [1, 5], 5 \rangle \\ \langle [1, 3], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 4], 4 \rangle \\ \langle [1, 4], 2 \rangle \\ \langle [3, 5], 5 \rangle \end{pmatrix}$
\hat{y}_2	$\begin{pmatrix} \langle [2, 4], 1 \rangle \\ \langle [1, 4], 3 \rangle \\ \langle [1, 5], 6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [3, 6], 5 \rangle \\ \langle [1, 2], 2 \rangle \\ \langle [1, 4], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 1 \rangle \\ \langle [1, 4], 2 \rangle \\ \langle [1, 3], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [3, 5], 5 \rangle \\ \langle [1, 4], 5 \rangle \\ \langle [1, 4], 4 \rangle \end{pmatrix}$
\hat{y}_3	$\begin{pmatrix} \langle [1, 2], 1 \rangle \\ \langle [1, 4], 2 \rangle \\ \langle [3, 6], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [2, 5], 5 \rangle \\ \langle [4, 5], 2 \rangle \\ \langle [2, 3], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [2, 4], 1 \rangle \\ \langle [1, 4], 3 \rangle \\ \langle [2, 4], 7 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 2 \rangle \\ \langle [1, 3], 4 \rangle \\ \langle [3, 4], 4 \rangle \end{pmatrix}$
\hat{y}_4	$\begin{pmatrix} \langle [1, 2], 3 \rangle \\ \langle [1, 3], 7 \rangle \\ \langle [1, 5], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 1], 3 \rangle \\ \langle [2, 4], 6 \rangle \\ \langle [3, 7], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [4, 5], 8 \rangle \\ \langle [1, 2], 2 \rangle \\ \langle [1, 2], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 1 \rangle \\ \langle [1, 1], 3 \rangle \\ \langle [2, 3], 6 \rangle \end{pmatrix}$

TABLE 2: Second spherical cubic fuzzy decision matrix.

	\hat{A}_1	\hat{A}_2	\hat{A}_3	\hat{A}_4
\hat{y}_1	$\begin{pmatrix} \langle [1, 2], 5 \rangle \\ \langle [4, 4], 2 \rangle \\ \langle [2, 3], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 4], 1 \rangle \\ \langle [1, 4], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 5], 6 \rangle \\ \langle [2, 3], 1 \rangle \\ \langle [1, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 5 \rangle \\ \langle [3, 6], 5 \rangle \\ \langle [1, 4], 2 \rangle \end{pmatrix}$
\hat{y}_2	$\begin{pmatrix} \langle [1, 4], 3 \rangle \\ \langle [1, 3], 4 \rangle \\ \langle [1, 7], 6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 4], 2 \rangle \\ \langle [1, 2], 1 \rangle \\ \langle [3, 7], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [2, 4], 5 \rangle \\ \langle [1, 5], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 6], 2 \rangle \\ \langle [1, 2], 6 \rangle \\ \langle [2, 5], 2 \rangle \end{pmatrix}$
\hat{y}_3	$\begin{pmatrix} \langle [2, 1], 3 \rangle \\ \langle [1, 5], 5 \rangle \\ \langle [1, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [2, 4], 5 \rangle \\ \langle [6, 7], 1 \rangle \\ \langle [1, 2], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 5], 5 \rangle \\ \langle [1, 2], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 4], 5 \rangle \\ \langle [1, 4], 5 \rangle \end{pmatrix}$
\hat{y}_4	$\begin{pmatrix} \langle [2, 4], 3 \rangle \\ \langle [4, 5], 3 \rangle \\ \langle [2, 4], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [6, 7], 5 \rangle \\ \langle [1, 1], 2 \rangle \\ \langle [1, 4], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 4], 2 \rangle \\ \langle [3, 2], 7 \rangle \\ \langle [2, 6], 4 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 2], 3 \rangle \\ \langle [3, 7], 2 \rangle \end{pmatrix}$

TABLE 3: Third spherical cubic fuzzy decision matrix.

	\hat{A}_1	\hat{A}_2	\hat{A}_3	\hat{A}_4
\hat{y}_1	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 2], 3 \rangle \\ \langle [2, 1], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 1 \rangle \\ \langle [2, 4], 2 \rangle \\ \langle [2, 3], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [3, 5], 6 \rangle \\ \langle [1, 3], 1 \rangle \\ \langle [1, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 5 \rangle \\ \langle [2, 4], 2 \rangle \\ \langle [3, 6], 5 \rangle \end{pmatrix}$
\hat{y}_2	$\begin{pmatrix} \langle [1, 2], 3 \rangle \\ \langle [1, 2], 3 \rangle \\ \langle [2, 4], 1 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 4], 3 \rangle \\ \langle [3, 6], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 1 \rangle \\ \langle [1, 2], 1 \rangle \\ \langle [4, 5], 3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [2, 3], 1 \rangle \\ \langle [3, 4], 5 \rangle \\ \langle [1, 2], 1 \rangle \end{pmatrix}$
\hat{y}_3	$\begin{pmatrix} \langle [1, 5], 3 \rangle \\ \langle [2, 3], 1 \rangle \\ \langle [1, 2], 5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [4, 3], 1 \rangle \\ \langle [2, 4], 5 \rangle \\ \langle [2, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [4, 3], 6 \rangle \\ \langle [1, 4], 1 \rangle \\ \langle [5, 6], 4 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 4], 2 \rangle \\ \langle [3, 4], 1 \rangle \\ \langle [1, 3], 2 \rangle \end{pmatrix}$
\hat{y}_4	$\begin{pmatrix} \langle [1, 2], 4 \rangle \\ \langle [4, 5], 2 \rangle \\ \langle [2, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 2], 4 \rangle \\ \langle [5, 7], 1 \rangle \\ \langle [1, 3], 2 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 5], 2 \rangle \\ \langle [2, 3], 5 \rangle \\ \langle [1, 2], 4 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [1, 3], 2 \rangle \\ \langle [1, 2], 3 \rangle \\ \langle [1, 2], 3 \rangle \end{pmatrix}$

Step 1: Firstly, we make a spherical cubic fuzzy matrix:

$$D_N = \left(\left\langle \left[a_{Q_c}^-, a_{Q_c}^+ \right], \tilde{\mu}_{Q_c} \right\rangle, \left\langle \left[n_{Q_c}^-, n_{Q_c}^+ \right], \tilde{\eta}_{Q_c} \right\rangle, \left\langle \left[b_{Q_c}^-, b_{Q_c}^+ \right], \tilde{\vartheta}_{Q_c} \right\rangle \right)_{m \times n} \quad (39)$$

TABLE 4: Spherical cubic fuzzy weighted geometric aggregation information of decision-makers.

	\hat{A}_1	\hat{A}_2	\hat{A}_3	\hat{A}_4
\hat{y}_1	$\left(\begin{array}{l} \langle [.10, .22], .33, \rangle \\ \langle [.16, .36], .28, \rangle \\ \langle [.17, .27], .42, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .27], .19, \rangle \\ \langle [.12, .40], .17, \rangle \\ \langle [.13, .34], .33, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.19, .50], .47, \rangle \\ \langle [.13, .36], .18, \rangle \\ \langle [.10, .30], .17, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .25], .46, \rangle \\ \langle [.18, .47], .29, \rangle \\ \langle [.24, .50], .41, \rangle \end{array} \right)$
\hat{y}_2	$\left(\begin{array}{l} \langle [.13, .34], .20, \rangle \\ \langle [.10, .30], .34, \rangle \\ \langle [.13, .58], .54, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.15, .43], .28, \rangle \\ \langle [.10, .24], .17, \rangle \\ \langle [.25, .60], .20, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .24], .13, \rangle \\ \langle [.13, .34], .24, \rangle \\ \langle [.22, .44], .30, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.17, .47], .23, \rangle \\ \langle [.13, .30], .45, \rangle \\ \langle [.15, .41], .28, \rangle \end{array} \right)$
\hat{y}_3	$\left(\begin{array}{l} \langle [.13, .19], .20, \rangle \\ \langle [.12, .41], .24, \rangle \\ \langle [.20, .43], .41, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.24, .40], .33, \rangle \\ \langle [.40, .54], .19, \rangle \\ \langle [.17, .27], .35, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.18, .33], .21, \rangle \\ \langle [.10, .44], .28, \rangle \\ \langle [.29, .41], .52, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .28], .20, \rangle \\ \langle [.13, .36], .31, \rangle \\ \langle [.20, .38], .41, \rangle \end{array} \right)$
\hat{y}_4	$\left(\begin{array}{l} \langle [.13, .26], .32, \rangle \\ \langle [.25, .42], .36, \rangle \\ \langle [.17, .42], .20, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.20, .26], .40, \rangle \\ \langle [.19, .26], .25, \rangle \\ \langle [.20, .53], .25, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.16, .46], .32, \rangle \\ \langle [.18, .22], .42, \rangle \\ \langle [.15, .43], .33, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .26], .16, \rangle \\ \langle [.10, .16], .30, \rangle \\ \langle [.23, .52], .42, \rangle \end{array} \right)$

TABLE 5: SCF ordered weighted geometric aggregation information of decision-makers.

	\hat{A}_1	\hat{A}_2	\hat{A}_3	\hat{A}_4
\hat{y}_1	$\left(\begin{array}{l} \langle [.19, .45], .62, \rangle \\ \langle [.26, .58], .46, \rangle \\ \langle [.20, .44], .30, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.17, .40], .42, \rangle \\ \langle [.28, .49], .31, \rangle \\ \langle [.23, .40], .39, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.15, .36], .33, \rangle \\ \langle [.13, .44], .36, \rangle \\ \langle [.20, .38], .51, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.13, .40], .43, \rangle \\ \langle [.16, .50], .26, \rangle \\ \langle [.32, .60], .54, \rangle \end{array} \right)$
\hat{y}_2	$\left(\begin{array}{l} \langle [.25, .51], .33, \rangle \\ \langle [.14, .34], .50, \rangle \\ \langle [.15, .46], .27, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.16, .49], .38, \rangle \\ \langle [.16, .50], .50, \rangle \\ \langle [.29, .56], .39, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .40], .26, \rangle \\ \langle [.17, .36], .40, \rangle \\ \langle [.20, .58], .48, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.23, .47], .24, \rangle \\ \langle [.13, .32], .21, \rangle \\ \langle [.34, .64], .51, \rangle \end{array} \right)$
\hat{y}_3	$\left(\begin{array}{l} \langle [.30, .50], .46, \rangle \\ \langle [.50, .64], .30, \rangle \\ \langle [.21, .36], .51, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.26, .50], .26, \rangle \\ \langle [.20, .51], .40, \rangle \\ \langle [.21, .39], .62, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.13, .29], .27, \rangle \\ \langle [.24, .50], .43, \rangle \\ \langle [.26, .47], .44, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .28], .20, \rangle \\ \langle [.13, .36], .31, \rangle \\ \langle [.20, .38], .41, \rangle \end{array} \right)$
\hat{y}_4	$\left(\begin{array}{l} \langle [.13, .26], .32, \rangle \\ \langle [.25, .42], .36, \rangle \\ \langle [.17, .42], .20, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.20, .26], .40, \rangle \\ \langle [.19, .26], .25, \rangle \\ \langle [.20, .53], .25, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.10, .32], .30, \rangle \\ \langle [.29, .48], .60, \rangle \\ \langle [.34, .70], .42, \rangle \end{array} \right)$	$\left(\begin{array}{l} \langle [.24, .36], .19, \rangle \\ \langle [.10, .24], .42, \rangle \\ \langle [.35, .61], .52, \rangle \end{array} \right)$

TABLE 6: Rowwise aggregated (SCFDWG) decision-makers.

\hat{y}_1	$\langle [.16, .11], .25, \rangle, \langle [.20, .29], .31, \rangle, \langle [.19, .41], .35, \rangle$
\hat{y}_2	$\langle [.21, .38], .31, \rangle, \langle [.17, .20], .33, \rangle, \langle [.19, .49], .31, \rangle$
\hat{y}_3	$\langle [.18, .19], .26, \rangle, \langle [.24, .21], .28, \rangle, \langle [.22, .38], .43, \rangle$
\hat{y}_4	$\langle [.19, .13], .25, \rangle, \langle [.19, .18], .18, \rangle, \langle [.20, .49], .35, \rangle$

If there are two types of criteria, for instance, cost-benefit criteria, the SC decision matrix could be transformed into a normalized one:

$$\tilde{\alpha}_{\tilde{Q}_{C_{ij}}} = \left\{ \begin{array}{l} \tilde{Q}_{C_{ij}} \text{ if the benefit - type attribute} \\ \tilde{Q}_{C_{ij}}^c \text{ if the cost - type attribute} \end{array} \right\}, \quad (40)$$

where $\tilde{Q}_{C_{ij}}^c$ represents the complement of $\tilde{Q}_{C_{ij}}$ if every parameter is of the same form as they are not necessary to normalize the decision matrix.

Step 2: To obtain an SCFN $\tilde{Q}_{C_i}^-$ ($i = 1, 2, \dots, n$), use the suggested aggregation operators for the alternatives \hat{y}_i . To be precise, the established operators stem the

$\tilde{Q}_{C_i}^-$ ($i = 1, 2, 3, \dots, n$) of alternative \hat{y}_i where the weighted vector for the criteria is $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)^T$.

Step 3: Determine the scores, score $(\tilde{Q}_{C_i}^-)$ ($i = 1, 2, 3, \dots, n$), and the degree of accuracy $(\tilde{Q}_{C_i}^-)$ of each $\tilde{Q}_{C_i}^-$.

Step 4: The best alternative is selected by the ranking of all the alternatives.

5. Numerical Illustration

Suppose an investment group wants to spend money on the best option (alternative). A board with four possibilities options (alternative) for investing money: $\{\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4\}$.

TABLE 7: Rowwise aggregated (SCFDOWG) decision-makers.

\hat{y}_1	$(\langle [.16, .11], .25 \rangle, \langle [.20, .29], .31 \rangle, \langle [.19, .41], .35 \rangle)$
\hat{y}_2	$(\langle [.21, .38], .31 \rangle, \langle [.17, .20], .33 \rangle, \langle [.19, .49], .31 \rangle)$
\hat{y}_3	$(\langle [.18, .19], .26 \rangle, \langle [.24, .21], .28 \rangle, \langle [.22, .38], .43 \rangle)$
\hat{y}_4	$(\langle [.19, .13], .25 \rangle, \langle [.19, .18], .18 \rangle, \langle [.20, .49], .35 \rangle)$

TABLE 8: Ranking criteria of alternatives.

Operator	$\tilde{Q}_{C_i}(\hat{A}_1)$	$\tilde{Q}_{C_i}(\hat{A}_2)$	$\tilde{Q}_{C_i}(\hat{A}_3)$	$\tilde{Q}_{C_i}(\hat{A}_4)$	Ranking
SCFWG	0.06	0.02	0.04	0.01	$\hat{A}_1 > \hat{A}_3 > \hat{A}_2 > \hat{A}_4$
SCFOWG	0.11	0.07	0.08	0.03	$\hat{A}_1 > \hat{A}_3 > \hat{A}_2 > \hat{A}_4$
SCFHWG	0.05	0.02	0.04	0.01	$\hat{A}_1 > \hat{A}_3 > \hat{A}_2 > \hat{A}_4$

TABLE 9: Comparison analysis using spherical fuzzy sets.

Operator	$\tilde{Q}_{C_i}(\hat{A}_1)$	$\tilde{Q}_{C_i}(\hat{A}_2)$	$\tilde{Q}_{C_i}(\hat{A}_3)$	$\tilde{Q}_{C_i}(\hat{A}_4)$	Ranking
SFWG	0.08	0.07	0.02	0.05	$\hat{A}_1 > \hat{A}_2 > \hat{A}_4 > \hat{A}_3$
SFOWG	0.02	0.03	0.05	0.01	$\hat{A}_3 > \hat{A}_2 > \hat{A}_1 > \hat{A}_4$
SFHWG	0.06	0.09	0.03	0.02	$\hat{A}_2 > \hat{A}_1 > \hat{A}_3 > \hat{A}_4$

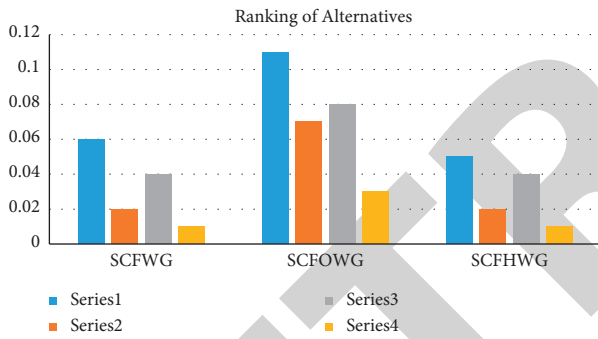


FIGURE 1: Ranking of alternatives using SCFSs.

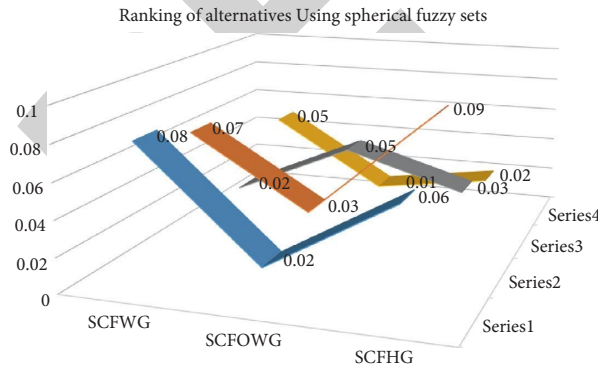


FIGURE 2: Ranking of alternatives using spherical fuzzy sets.

\hat{y}_1 is a vehicle manufacturer, \hat{y}_2 a food company, \hat{y}_3 a washing machine company, and \hat{y}_4 a television company. Four criteria, including a risk analysis (\hat{A}_1), a growth and development analysis (\hat{A}_2), a social impact analysis (\hat{A}_3), and an environment impact analysis (\hat{A}_4), are taken into

account. $\omega = (0.1, 0.2, 0.25, 0.45)^T$ is the \hat{A}_j criterion weight vector ($j = 1, 2, \dots, n$). A three-decision-maker committee explores four possible alternatives \hat{y}_i ($i = 1, 2, 3, 4$) in the four \hat{A}_j ($j = 1, 2, 3, 4$) criteria above. Then, the ranking is needed to assess the investment company $\{\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4\}$. The following matrices represent these experts in the form of spherical cubic fuzzy numbers:

Step 1. The decision-makers make their decisions given in Table 3 1–3.

Step 2. Using the SCFWG aggregation operator, we get the collection of SCFN for \hat{y}_i alternative given in Table 4. Table 5 contains the decision-makers' SCF ordered weighted geometric (SCFOWG) aggregation information. And the rowwise aggregated (SCFDWG) decision-makers are given in Table 6. In Table 7, the rowwise aggregation of (SCFOWG) operators is given.

Step 3. Using equation (22) to determine the scores of \tilde{Q}_{C_i} ($i = 1, 2, 3, \dots, n$), the ranking criteria of alternatives using the spherical cubic geometric aggregation operators are given in Table 8.

Step 4. The scores are arranged in a descending order. Choose the highest alternative. Figure 1 represents the graphical representation of alternative by using the spherical cubic geometric aggregation operators. By using the proposed aggregation operator, the ranking of the four possible alternatives achieved is more accurate in comparison with Table 9. The best choice for these operators is \hat{A}_1 . These approaches are ideal for addressing circumstances in which the inputs, viewpoints, and the interaction of experts and criteria may be considered which are more likely to handle such problems. Figure 2 shows the graphical representation

of alternative by using the spherical geometric aggregation operators.

6. Conclusion

In this paper, we have discussed the spherical cubic fuzzy set (SCFS) which is the generalization of the interval-valued SFS (IVSFS). Some operational laws of SCFS were presented. For the comparison of SCFNs, we have established accuracy and score functions. We also described SCF distances between SCFNs. Further, we proposed various aggregation operators in the SCF environment, SCFWG, SCFOWG, and SCFHWG operators.

Some interesting properties such as idempotency, boundary, and monotonicity were also discussed. Furthermore, a relationship was formed among the proposed operators. Additionally, we suggested a MADM to demonstrate the prominence and the strength of the proposed operators. Moreover, by applying the developed aggregation operators, we explained the problems of decision-making. A numerical illustration is presented to indicate an alternative way of addressing the decision-making process more effectively by using the proposed operators. Finally, we compared the practicality, efficiency, and validity of the new approach with existing operators.

We would combine other SCFS approaches such as Einstein product and introduce the concept of SCF Einstein weighted averaging (SCFEWA) operator, SCF Einstein ordered weighted averaging (SCFEOWA) operator, SCF Einstein weighted geometric (SCFEWG) operator, SCF Einstein ordered weighted geometric (SCFEOWG) operator, and some more generalized operators in MADM processes.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare no conflicts of interest about the publication of the research article.

Acknowledgments

The authors are grateful to the Deanship of Scientific Research, King Saud University, for funding through Vice Deanship of Scientific Research Chairs.

References

- [1] S.-M. Chen and J.-M. Tan, "Handling multicriteria fuzzy decision-making problems based on vague set theory," *Fuzzy Sets and Systems*, vol. 67, no. 2, pp. 163–172, 1994.
- [2] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 332–83, 1965.
- [3] K. T. Atanassov, *Intuitionistic Fuzzy Sets*, Physica-Verlag hd, Heidelberg, Germany, pp. 1–137, 1999.
- [4] R. R. Yager, "Properties and applications of pythagorean fuzzy sets," in *Imprecision and Uncertainty in Information Representation and Processing*, pp. 119–136, Springer, Berlin, Germany, 2016.
- [5] R. R. Yager, J. Kacprzyk, and G. Beliakov, Eds., *Recent Developments in the Ordered Weighted Averaging Operators: Theory and Practice*, Springer, Vol. 265, Berlin, Germany, 2011.
- [6] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets—a new concept for computational intelligence problems," in *Proceedings of the 2013 3rd World Congress on Information and Communication Technologies (WICT 2013)*, pp. 1–6, IEEE, Hainan, China, December 2013.
- [7] H. Garg, "Some picture fuzzy aggregation operators and their applications to multicriteria decision-making," *Arabian Journal for Science and Engineering*, vol. 42, no. 12, pp. 5275–5290, 2017.
- [8] H. Garg, "A novel accuracy function under interval-valued pythagorean fuzzy environment for solving multicriteria decision making problem," *Journal of Intelligent and Fuzzy Systems*, vol. 31, no. 1, pp. 529–540, 2016.
- [9] X. Gou, Z. Xu, and P. Ren, "The properties of continuous pythagorean fuzzy information," *International Journal of Intelligent Systems*, vol. 31, no. 5, pp. 401–424, 2016.
- [10] W. Liang, T. Zhang, and M. Liu, "The maximizing deviation method based on interval-valued pythagorean fuzzy weighted aggregating operator for multiple criteria group decision analysis," *Discrete Dynamics in Nature and Society*, vol. 2015, Article ID 746572, 15 pages, 2015.
- [11] D. Liang, Z. Xu, and A. P. Darko, "Projection model for fusing the information of pythagorean fuzzy multicriteria group decision making based on geometric Bonferroni mean," *International Journal of Intelligent Systems*, vol. 32, no. 9, pp. 966–987, 2017.
- [12] X. Peng and Y. Yang, "Some results for pythagorean fuzzy sets," *International Journal of Intelligent Systems*, vol. 30, no. 11, pp. 1133–1160, 2015.
- [13] M. Wei, T. Tang, and Y. Wei, "Pythagorean hesitant fuzzy hamacher aggregation operators and their application to multiple attribute decision making," *International Journal of Intelligent Systems*, vol. 33, no. 6, pp. 1197–1233, 2018.
- [14] H. Garg, *Special Issue on "Pythagorean Fuzzy Set and its Extensions in Decision-Making Process*, Springer, Berlin, Germany, 2019.
- [15] F. K. Gündoğdu and C. Kahraman, "Spherical fuzzy sets and decision making applications," in *Proceedings of the International Conference on Intelligent and Fuzzy Systems*, pp. 979–987, Istanbul, Turkey, July 2019.
- [16] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Computing & Applications*, vol. 31, no. 11, pp. 7041–7053, 2019.
- [17] K. Ullah, H. Garg, T. Mahmood, N. Jan, and Z. Ali, "Correlation coefficients for t-spherical fuzzy sets and their applications in clustering and multi-attribute decision making," *Soft Computing*, vol. 24, no. 3, pp. 1647–1659, 2020.
- [18] T. Ayaz, M. M. Al-Shomrani, S. Abdullah, and A. Hussain, "Evaluation of enterprise production based on spherical cubic hamacher aggregation operators," *Mathematics*, vol. 8, no. 10, p. 1761, 2020.
- [19] F. Kutlu Gündoğdu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 1, pp. 337–352, 2019.
- [20] F. K. Gündoğdu, "Principals of spherical fuzzy sets," in *Proceedings of the International Conference on Intelligent and Fuzzy Systems*, pp. 15–23, Istanbul, Turkey, July 2019.

- [21] S. Zeng, H. Garg, M. Munir, T. Mahmood, and A. Hussain, "A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of t-spherical fuzzy sets and its application in the selection of solar cells," *Energies*, vol. 12, no. 23, p. 4436, 2019.
- [22] P. Liu, B. Zhu, and P. Wang, "A multi-attribute decision-making approach based on spherical fuzzy sets for yunnan baiyao's R&D project selection problem," *International Journal of Fuzzy Systems*, vol. 21, no. 7, pp. 2168–2191, 2019.
- [23] M. Mathew, R. K. Chakraborty, and M. J. Ryan, "A novel approach integrating AHP and TOPSIS under spherical fuzzy sets for advanced manufacturing system selection," *Engineering Applications of Artificial Intelligence*, vol. 96, Article ID 103988, 2020.
- [24] Z. Gong, X. Xu, Y. Yang, Y. Zhou, and H. Zhang, "The spherical distance for intuitionistic fuzzy sets and its application in decision analysis," *Technological and Economic Development of Economy*, vol. 22, no. 3, pp. 393–415, 2016.
- [25] F. Kutlu Gundogdu and C. Kahraman, "Extension of WASPAS with spherical fuzzy sets," *Informatica*, vol. 30, no. 2, pp. 269–292, 2019.
- [26] F. K. Gündoğdu and C. Kahraman, "A novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets," *Engineering Applications of Artificial Intelligence*, vol. 85, pp. 307–323, 2019.
- [27] S. Zeng, A. Hussain, T. Mahmood, M. Irfan Ali, S. Ashraf, and M. Munir, "Covering-based spherical fuzzy rough set model hybrid with TOPSIS for multi-attribute decision-making," *Symmetry*, vol. 11, no. 4, p. 547, 2019.
- [28] P. Liu, B. Zhu, P. Wang, and M. Shen, "An approach based on linguistic spherical fuzzy sets for public evaluation of shared bicycles in China," *Engineering Applications of Artificial Intelligence*, vol. 87, Article ID 103295, 2020.
- [29] F. Kutlu Gündoğdu and C. Kahraman, "A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection," *Journal of Intelligent and Fuzzy Systems*, vol. 37, no. 1, pp. 1197–1211, 2019.
- [30] G. Wei, J. Wang, M. Lu, J. Wu, and C. Wei, "Similarity measures of spherical fuzzy sets based on cosine function and their applications," *IEEE Access*, vol. 7, pp. 159069–159080, 2019.
- [31] M. J. Khan, P. Kumam, W. Deebani, W. Kumam, and Z. Shah, "Distance and similarity measures for spherical fuzzy sets and their applications in selecting mega projects," *Mathematics*, vol. 8, no. 4, p. 519, 2020.
- [32] S. A. S. Shishavan, F. Kutlu Gündoğdu, E. Farrokhizadeh, Y. Donyatalab, and C. Kahraman, "Novel similarity measures in spherical fuzzy environment and their applications," *Engineering Applications of Artificial Intelligence*, vol. 94, Article ID 103837, 2020.
- [33] T. Mahmood, M. Ilyas, Z. Ali, and A. Gumaei, "Spherical fuzzy sets-based cosine similarity and information measures for pattern recognition and medical diagnosis," *IEEE Access*, vol. 9, pp. 25835–25842, 2021.