

## *Retraction*

# **Retracted: A Chaotic-Map-Based Password-Authenticated Key Exchange Protocol for Telecare Medicine Information Systems**

## **Security and Communication Networks**

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This article has been retracted by Hindawi, as publisher, following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of systematic manipulation of the publication and peer-review process. We cannot, therefore, vouch for the reliability or integrity of this article.

Please note that this notice is intended solely to alert readers that the peer-review process of this article has been compromised.

Wiley and Hindawi regret that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

## **References**

- [1] Y. Lu and D. Zhao, "A Chaotic-Map-Based Password-Authenticated Key Exchange Protocol for Telecare Medicine Information Systems," *Security and Communication Networks*, vol. 2021, Article ID 7568538, 8 pages, 2021.

## Research Article

# A Chaotic-Map-Based Password-Authenticated Key Exchange Protocol for Telecare Medicine Information Systems

Yanrong Lu <sup>1</sup> and Dawei Zhao <sup>2</sup>

<sup>1</sup>School of Safety Science and Engineering, Civil Aviation University of China, Tianjin, China

<sup>2</sup>Shandong Provincial Key Laboratory of Computer Networks, Shandong Computer Science Center (National Supercomputer Center in Jinan) Qilu University of Technology (Shandong Academy of Sciences), Jinan, China

Correspondence should be addressed to Yanrong Lu; yr\_lu@cauc.edu.cn and Dawei Zhao; zhaodw@sdas.org

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Telecare medicine information systems (TMISs) provide e-health services such that patients can access medical resources conveniently and doctors can prescribe treatments rapidly. Authentication is an essential security requirement in TMISs. In particular, the growth of password-based remote patient authenticated key exchange combining extended chaotic maps has enhanced the level of secure communications for TMISs. Recently, Lee suggested an improved random-number-based password-authenticated key exchange (PAKE) using extended chaotic maps and synchronized-clock-based PAKE using extended chaotic maps on Guo and Zhang and Xiao et al.'s PAKE. Unfortunately, we found that the nonce-based scheme of Lee is insecure against known session-specific temporary information and server spoofing attacks. To cope with the aforementioned defects, this study aims to provide a new secure PAKE based on extended chaotic maps with more security functionalities for TMISs. Additionally, we show that the proposed scheme for TMISs provides high security along with low communication cost, computational cost, and a variety of security features.

## 1. Introduction

At present, the researches on the cloud assisted e-health are more and more in-depth. It facilitates health condition monitoring and improves efficiency for medical resources [1]. As one of the most popular applications of e-health care service, telecare medical information systems (TMISs) provide the medical or healthcare for those patients who are disabled or cannot attend hospital normally [2, 3]. With the openness of wireless environment, the security of TMISs is highlighted. How to authenticate the communication entities and thus securely transmit sensitive medical data related with patients is an urgent problem that needs to be researched and solved.

Key exchange schemes aim at establishing a shared session key between two or more communicating entities. The shared session key is used in securing subsequent

communication over an insecure channel. Therefore, the key challenge of designing such a scheme is how to securely and efficiently derive a session key that is only known to the communicated entities. Hitherto, a large number of related authenticated key agreement schemes have been presented with different structures such as pure password schemes, password schemes with smart cards, dynamic schemes, and dynamic schemes with smart cards.

With the extremely studied and widely applied Chebyshev polynomials by the cryptographic research community, various password authenticated key exchange (PAKE) based on chaotic maps and related approaches have been developed recently [4–10]. Kocarev-Tasev [11] presented the first chaotic maps-based public key encryption scheme. Unfortunately, according to the periodicity of cosine function, the scheme of Kocarev-Tasev was demonstrated to be insecure by Bergamo et al. [12]. After that, Xiao

et al. [13] suggested an authenticated key agreement scheme using chaotic map. Nevertheless, Alvarez [14] pointed out that the scheme of Xiao et al. could not withstand man-in-the-middle attack. Shortly after, Xiao et al. [15] introduced an enhanced PAKE to prevent the security threats. Guo-Chang [16] raised a smart card based PAKE using chaotic maps. Later, Lin [17] claimed that the Guo-Chang's scheme might easily leak the identity of communicating user by intercepting the transmitted messages. In addition, Lin [17] also pointed that the session key could be derived by an adversary during the communication in the Guo-Chang's scheme [16]. In order to negate these risks, Lin also developed an improved variant without sacrificing the efficiency.

Recently, Guo and Zhang [18] identified that the drawbacks of Xiao et al.'s scheme [13] and found that Xiao et al.'s scheme failed to satisfy the requirements with the contributory nature of key agreement. Subsequently, Guo and Zhang developed their own improved version of the remote user PAKE. Recently, Lee [19] observed that both Xiao et al. [13] and Guo and Zhang's schemes [18] were unable to free from offline password guessing attack and achieve the session key security. As a counter measure to these sufferings, Lee developed two PAKE; that is, one uses random numbers, while the other uses timestamp. However, this study shows that Lee (the nonce based) fails to resist known session-specific temporary information and server spoofing attacks.

The merits of this paper are as follows.

- (i) Our proposed scheme demonstrates that Lee's scheme has several drawbacks once the private information is leaked.
- (ii) Our proposed scheme for TMISs withstands an unauthorized patient to deceive the service provided by the telecare medical server.
- (iii) Our proposed scheme for TMISs satisfies high security along with a variety of attributes compared with Xiao et al. [13], Guo and Zhang [18], and Lee schemes. Extensive comparisons are conducted with related schemes to verify the performance of our schemes in terms of security and efficiency.

The remainder of this paper is organized as follows. Section 2 introduces preliminary knowledge of some Chebyshev chaotic maps that we use in our system. We describe Lee's scheme in Section 3. In Section 4, we show that Lee's scheme [19] is vulnerable to various attacks. The proposed scheme for TMISs is presented in detail in Section 5, followed by the security analysis in Section 6. In Section 7, we compare the performance and security of our scheme for TMISs with related schemes. Finally, Section 8 concludes the paper.

## 2. Preliminaries

This section lists the used definitions and defines Chebyshev chaotic maps and corresponding chaotic properties that are used in this paper [11, 12, 20–22].

The Chebyshev polynomial  $T_s(x): [-1, 1] \rightarrow [-1, 1]$  is defined as follows:

$$T_s(x) = 2xT_{s-1}(x) - T_{s-2}(x), \quad (1)$$

where  $s$  be an integer with  $s \geq 2$ ,  $x \in [-1, 1]$ ,  $T_0(x) = 1$ ,  $T_1(x) = x$ .

The Chebyshev polynomial satisfies the semigroup property:

$$T_{st}(x) = T_s(T_t(x)) = T_t(T_s(x)). \quad (2)$$

Chaotic property:

When  $a > 1$ , Chebyshev polynomial map  $T_s: [-1, 1] \rightarrow [-1, 1]$  of degree  $s$  is a chaotic map with invariant density  $f^*(x) = 1/(\pi\sqrt{[2]1-x^2})$  for Lyapunov exponent  $\lambda = \ln s > 0$ .

Quadratic residue assumption:

If  $y = x^2 \bmod n$  has a solution, that is,  $\exists$  a square root for  $y$ , then  $y$  is named as a quadratic residue modulo  $n$ . It is computationally unfeasible to derive  $x$  such that  $y = x^2 \bmod n$  under the condition of not knowing the parameters  $p$  and  $q$  because of the factoring problem  $n = pq$  is NP-hard problem.

## 3. Review of Lee's PAKE

Lee [19] presented two authentication schemes, which are based on nonce and timestamp, respectively. Without loss of generality, we briefly review the nonce based PAKE of Lee, which includes system initialization, authentication and key agreement phases.

### 3.1. System Initialization.

Step 1: Server  $B$  chooses two large primes  $p$  and  $q$  as its private keys.

Step 2:  $B$  calculates  $n = pq$ .

Step 3:  $B$  publishes  $h_1: \{0, 1\}^* \rightarrow \{-\infty, +\infty\}$ ,  $h_2: \{0, 1\}^* \rightarrow \{0, 1\}^\tau$  and  $n$ , where  $\tau$  is the fixed size.

### 3.2. Authentication and Key Agreement

Step 1: User  $A$  computes  $x = h_1(y)$ ,  $X_1 = (y, pw)^2 \bmod n$ ,  $TID_A = ID_A \oplus h_2(y, pw)$ , and  $X_2 = h_2(TID_A, (y, pw), T_r(x))$ , where  $r$  and  $y$  are the random numbers.  $A$  then sends the message  $M_1 = \{TID_A, X_1, X_2, T_r(x)\}$  to  $B$ .

Step 2: Once receiving  $M_1$ ,  $B$  retrieves the four solutions  $(y_i, pw_i) (0 \leq i \leq 3)$  from  $X_1$  by using the Chinese remainder theorem (CRT) and checks whether  $h_2(TID_A, (y_i, pw_i), T_r(x)) \stackrel{?}{=} X_2$ . If successful, it means  $B$  has gotten the correct  $y'$  and  $pw'$ . After that,  $B$  checks if  $pw' \stackrel{?}{=} pw$ . If the equation is true,  $B$  derives  $ID_A$  by computing  $TID_A \oplus h_2(y', pw')$  and computes  $x = h_1(y')$ ,  $sk = h_2(T_r(x), T_s(x), T_s(T_r(x)))$  and  $Auth_B = h_2(sk, pw, ID_B, ID_A)$ . Next,  $B$  sends back  $M_2 = \{ID_B, T_s(x), Auth_B\}$  to  $A$ .

Step 3: When receiving  $M_2$ ,  $A$  computes  $sk = h_2(T_r(x), T_s(x), T_r(T_s(x)))$  and  $Auth_B' = h_2(sk, pw, ID_B, ID_A)$ . Next,  $A$  checks whether  $Auth_B' = Auth_B$ . If it holds,  $A$  computes  $Auth_A = h_2(sk, ID_A, ID_B)$  and sends  $M_3 = \{Auth_A\}$  to  $B$ .

Step 4: After receiving  $M_3$ ,  $B$  verifies whether  $h_2(sk, ID_A, ID_B)$  is equal to the received  $Auth_A$ . If it does not hold,  $B$  terminates the session; otherwise,  $A$  and  $B$  have a common session key  $sk = h_2(T_r(x), T_s(x), T_{rs}(x))$ .

#### 4. Security Analysis on Lee's Scheme

Lee [19] found some severe security pitfalls in Xiao et al. [13] and Guo and Zhang's schemes [18, 21] and proposed new chaotic-based authenticated key schemes. It is claimed that their new scheme achieves many security attributes while being secure against general threats. In this part, however, we will demonstrate that Lee's nonce based scheme [19] is actually vulnerable to known session-specific temporary information attack [23], which is one of the most important security properties that most of schemes shall attain. In addition, as the result of overlooking the server is a semi-trusted party, this scheme is subject to server spoofing attack.

**4.1. Known Session-Specific Temporary Information Attack.** Assume the user's session random number  $y$  is corrupted by an adversary  $\mathbb{U}$ . The scheme will suffer the following attack.

Step 1:  $\mathbb{U}$  guesses a candidate password  $pw^*$  and checks whether  $h_2(TID_A, (y, pw^*), T_r(x)) = X_2$ , where  $TID_A$  and  $T_r(x)$  are intercepted information through the public channel by  $\mathbb{U}$ . If the result is true, the correct password has been gotten. Otherwise,  $\mathbb{U}$  continues to execute the aforementioned procedure until he succeeds.

Step 2: Once  $\mathbb{U}$  successfully owns the user  $A$ 's password  $pw$ . There can be no real defense against attacks from  $\mathbb{U}$ . First,  $\mathbb{U}$  derives  $ID_A$  by computing  $TID_A \oplus (y, pw)$  and computes  $X_1 = (y, pw)^2 \bmod n$ . Next,  $\mathbb{U}$  sends the counterfeited message  $M_1 = \{TID_A, X_1, X_2, T_r^*(h_1(y))\}$  to the server  $B$ , where  $r^*$  is the random number chosen by  $\mathbb{U}$ .

Step 3: When the server  $B$  receives  $M_1$ , it performs the scheme without any detection since all the verification information derived from the user "A." Finally, the server  $B$  sends back the message  $M_2 = \{ID_B, T_s(h_1(y)), Auth_B\}$  to  $\mathbb{U}$  who masquerades as a legal user  $A$ , where  $Auth_B = h_2(ID_B, ID_A, pw, sk)$  and  $sk = h_2(T_r^*(h_1(y)), T_s(h_1(y)), T_s(T_r^*(h_1(y))))$ .

Step 4: After receiving the message  $M_2$ ,  $\mathbb{U}$  computes  $sk = h_2(T_r^*(h_1(y)), T_s(h_1(y)), T_r^*(T_s(h_1(y))))$  and verifies whether the equation  $h(ID_B, ID_A, pw, sk)$  is equal to the received  $Auth_B$ . If the result is correct,  $\mathbb{U}$  computes  $Auth_A = h_2(ID_B, ID_A, sk)$  and returns  $M_3 = \{Auth_A\}$  to the server  $B$ .

Step 5: Once receiving the message  $M_3$ , server  $B$  validates the correctness of the value  $Auth_A$ . Then, server  $B$  accepts the communication request from user "A" and agrees on the session key  $sk$  as a "confidential" session key for concealing the following messages. In this way, the subsequent communication messages seem like plain text such that  $\mathbb{U}$  could do whatever he wants. This shows that, in Lee's scheme,  $\mathbb{U}$  can use the unexpectedly disclosed session random number to successfully complete mutual authentication. This concludes that their scheme lacks strongly the SK-security, which is very essential in the security critical applications.

**4.2. Server Spoofing Attack.** In Lee's scheme, server  $B$  masters the sensitive information  $pw$  of user  $A$ , which leads to a malicious spoofing attack because the legal but malicious server  $B$  could monitor the authentication process of user  $A$  and gather information related to user  $A$  and thus become an adversary. The malicious server  $\mathbb{B}$  can forge the valid request message by performing the following procedures.

Step 1: The malicious server  $\mathbb{B}$  can eavesdrop the message  $M_1 = \{TID_A, X_1, X_2, T_r^*(h_1(y))\}$  during authentication and key agreement phase corresponding to the legitimate user  $A$ . Then,  $\mathbb{B}$  generates two random numbers  $r^*$ ,  $y^*$  and calculates  $x = h_1(y^*)$ ,  $T_r^*(x)$ ,  $z = (y^*, pw)$ ,  $X_1 = z^2 \bmod n$ ,  $TID_A = ID_A^* \oplus h_2(z)$  and  $X_2 = h_2(TID_A, z, T_r^*(x))$ . Next,  $\mathbb{B}$  sends an imitative message  $M_1 = \{TID_A, X_1, X_2, T_r^*(x)\}$  to server  $B$ .

Step 2: When receiving  $M_1$ , server  $B$  derives  $(y', pw')$  from  $X_1$  by using the Chinese remainder theorem (CRT) and examines whether  $h_2(TID_A, (y', pw'), T_r^*(x))$ . Because the computed result equals the received  $X_2$ ,  $B$  will accept  $\mathbb{B}$ 's request. Next,  $B$  derives  $ID_A^*$  by computing  $TID_A \oplus h_2(y', pw')$  and checks whether  $pw' = pw$ . If it holds, server  $B$  computes  $x = h_1(y')$ ,  $T_s(x)$ ,  $sk = h_2(T_r^*(x), T_s(x), T_s(T_r^*(x)))$  and  $Auth_B = h(sk, pw, ID_B, ID_A^*)$ . At last,  $B$  sends the message  $M_2 = \{ID_B, T_s(x), Auth_B\}$  to  $\mathbb{B}$  who is masquerading as user  $A$ .

Step 3: Once receiving  $M_2$ ,  $\mathbb{B}$  computes  $h_2(sk, pw, ID_B, ID_A^*)$  and checks it with  $Auth_B$ , where  $sk = h_2(T_r^*(x), T_s(x), T_s(T_r^*(x)))$ . If they are equivalent,  $\mathbb{B}$  computes  $Auth_A = h_2(sk, ID_A^*, ID_B)$  and sends back  $M_3 = \{Auth_A\}$  to server  $B$ .

Step 4: When receiving  $M_3$ , server  $B$  computes  $h_2(sk, ID_A^*, ID_B)$  and compares it with  $M_3$ . If they are equal,  $B$  authenticates  $\mathbb{B}$ . In this regard,  $\mathbb{B}$  and  $B$  share a common session key  $sk = h_2(T_r^*(x), T_s(x), T_s(T_r^*(x)))$  for securing communication. Therefore, a legal but malicious server can masquerade as a legal user to log into a remote server.

The same flaw can be applied to the timestamp based scheme of Lee. Since they work on the same principle, only nonce-based scheme is analyzed above.

## 5. The Proposed PAKE Scheme for TMISs

To overcome the security pitfalls found in Lee's scheme, we present efficient and secure PAKE using chaotic maps for TMISs. To achieve the patient anonymity and reduce the computation overhead at the patient's side who may take mobile device, the proposed scheme leverages the encryption function to find a trade-off between the security and the cost. The proposed scheme has the following phases: system initialization phase, patient registration phase (Algorithm 1), and authentication and key agreement phase (Algorithm 2).

### 5.1. System Initialization.

Step 1: The telecare medical server  $B$  chooses two large primes  $p$  and  $q$  as its private keys.

Step 2:  $B$  calculates  $n = pq$ .

Step 3:  $B$  publishes  $h: \{0, 1\}^* \rightarrow \{0, 1\}^\tau$  and  $n$ , where  $\tau$  is the fixed size.

### 5.2. Patient Registration

Step 1: Patient  $A$  computes  $h(pw, ra)$  and sends  $\{ID_A, h(pw, ra)\}$  to the telecare medical server  $B$  over a private channel, where  $ra$  is a random nonce and  $pw$  is  $A$ 's password.

Step 2: When  $B$  receives the message,  $B$  computes  $A_{pw} = E_{K_B}[h(ID_A, h(pw, ra))]$  and stores  $A_{pw}$  in its database, where  $K_B$  is the secret key of  $B$ .

### 5.3. Authentication and Key Agreement

Step 1:  $A$  computes  $x = h(ID_A, h(pw, ra))$ ,  $z = (ID_A, r)^2 \bmod n$ ,  $X_1 = h(ID_A, x, r)$  and  $X_2 = E_x[T_r(x), z, X_1]$ , where  $r$  is a random nonce. Next,  $A$  transmits the messages  $M_1 = \{X_2\}$  to  $B$ .

Step 2: Upon receiving the message,  $B$  first derives  $x = h(ID_A, h(pw, ra))$  by decrypting  $A_{pw}$  and then retrieves  $[T_r(x), z, X_1]$  by decrypting  $X_2$ . Subsequently,  $B$  solves  $z$  by CRT and verifies whether  $h(ID_A, r, x) \stackrel{?}{=} X_1$ . If true,  $B$  computes  $X_3 = E_{T_r(x)}[T_s(x), SID_B, h(ID_A, r')]$  and sends  $M_2 = \{X_3\}$  to  $A$ .

Step 3: On receiving the message,  $A$  retrieves  $[T_s(x), SID_B, h(ID_A, r')]$  by decrypting  $X_3$ , computes  $h(ID_A, r)$ , and checks whether  $h(ID_A, r') \stackrel{?}{=} h(ID_A, r)$ . If successful,  $A$  computes  $sk = h(T_u(x), T_s(x), T_{us}(x))$ ,  $X_4 = E_{T_r(x)}[T_u(x)]$ , and  $Auth_A = h(sk, T_u(x), T_s(x))$ . Next,  $A$  sends back  $M_3 = \{Auth_A, X_4\}$  to  $B$ .

Step 4: When receiving the message,  $B$  retrieves  $T_u(x)$  by decrypting  $X_4$  with computed  $T_r(x)$ . After that,  $B$  computes  $sk = h(T_s(x), T_u(x), T_s(T_u(x)))$  and checks whether  $h(T_s(x), T_u(x), T_s(T_u(x))) \stackrel{?}{=} Auth_A$ . If correct,  $B$  computes  $Auth_B = h(sk, T_s(x), T_u(x), 00)$  and sends  $M_4 = \{Auth_B\}$  to  $A$ .

Step 5:  $A$  verifies the correctness of the value  $Auth_B$ . If not,  $A$  aborts the session. Otherwise,  $A$  and  $B$  share a

common session key  $sk = h(T_s(x), T_u(x), T_{su}(x))$  with each other.

## 6. Cryptanalysis of Our Enhancement

In this section, we provide an in-depth analysis on the security features of our enhanced remote user PAKE scheme for TMISs. We will show that the proposed scheme not only provides anonymity and mutual authentication and but could also withstand the aforementioned attacks.

**6.1. Full Protection for Patient's Identity.** Obviously, the proposed scheme for TMISs provides patient anonymity because patient  $A$ 's identity  $ID_A$  is not transmitted in plain-text via any messages traveling over insecure network. For one thing,  $ID_A$  is protected by hash function as a symmetric key only known by the patient  $A$  and the corresponding telecare medical server  $B$ . The telecare medical server could not know the real identity even if it intends to decrypt the stored value or the legal telecare medical server's private key is embezzled by an illegitimate patient or an illegitimate server to derive the hash value. The real identity  $ID_A$  is concealed by the quadratic residue assumption. As we know, the assumption is secure for chosen-plaintext attack and the identity is always a short string which could not be known by the unauthorized third-party unless it is completely learnt. Besides, the random number  $r$  is not an uncertain number which is not easily guessed. All in all, the proposed scheme for TMISs can be categorized as one preserving the patient privacy.

**6.2. Mutual Authentication Thwarting Man-in-the-Middle Attack.** The mutual authentication between correspondents is a basic security features for a remote PAKE. Only on the basis of the trust, two unfamiliar participates, that is, the patient and the server, are able to establish the session key for securing the following communication messages. In the proposed scheme for TMISs, patient  $A$  is authenticated by the telecare medical server by verifying the validity of  $X_1$ . This verification needs two indispensable conditions. One is the private key of the telecare medical server  $B$  to derive the hashed value including the identity  $ID_A$ , the password  $pw$ , and the random number  $r_a$ . Another are the two private values  $p$  and  $q$ , which are used to retrieve the plain-text identity  $ID_A$  and the random number  $r$ . The telecare medical server is not able to examine the received message  $M_1 = \{X_2\}$  without the knowledge of the two secrets. In other words, not anyone could generate the valid message  $\{X_2\}$  unless they know all the private information, such as the identity  $ID_A$ , the password  $pw$ , and even the random number  $r_a$  of the registered patient, only known by the patient itself. Additionally, the message  $M_3 = \{Auth_A, X_4\}$  further consolidates the authenticity of patient  $A$  since only the real patient knows the value  $T_r(x)$ , which is employed to compute the authenticated messages. On the other side, following the previously mentioned discussion, only the legitimate telecare medical server retrieves the plain-text identity  $ID_A$ , the random number  $r$ , and the value  $T_r(x)$ ,

Input: $pw, h, p, q, n$ Output: Store $A_{pw}$ . (1) Select $ra$ , (2) Compute $h(pw, ra)$ , (3) $A_{pw} = E_{K_B}[h(ID_A, h(pw, ra))]$ .
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ALGORITHM 1: Patient registration.

which are used for checking by patient  $A$ . Similarly, message  $M_4 = \{Auth_B\}$  is utilized to further confirm the legitimacy of the telecare medical server. According to the previously mentioned analysis, the man-in-the-middle attack is not launched due to lack of personal information. Any forged messages could be detected by the receivers since they have the symmetric key which is unknown by any third party. This confirms that our PAKE scheme for TMISs achieves the property of mutual authentication and thus resists man-in-the middle attack.

**6.3. Resistance to Known Session-Specific Temporary Information Attack.** From Algorithm 2, patient  $A$  and telecare medical server  $B$  use  $T_r(x)$  to encrypt the session key  $SK = h(T_u(x), T_s(x), T_{us}(x))$ , where  $x = h(ID_A, h(pw, r_a))$ . Clearly, even if an adversary gets the temporary information  $r$  and  $r_a$ , it is incapable of computing  $T_u(x)$  without having the knowledge of either  $ID_A$  or  $pw$  [19]. In this way, the proposed scheme for TMISs overcomes the drawbacks found in Lee's scheme. Moreover, without revealing the identity  $pw$  of  $A$  to  $B$ ,  $B$  authenticates  $A$  through decrypting  $A$ 's registered message  $A_{pw} = E_{K_B}[h(ID_A, h(pw, r_a))]$ . Thus, the proposed PAKE scheme for TMISs can withstand this type of attack.

**6.4. Perfect Forward Session Key Secrecy.** Even if the password  $pw$  of the patient  $A$  is lost, the session key is still secure since the password is not related with the computed session key. Actually, if the important values  $T_u(x)$  and  $T_s(x)$  are compromised, an adversary could derive the correct  $u$  and  $s$  using the approach [12]. However, the adversary has no opportunity to get the two values unless they know user  $A$ 's private information, such as the identity  $ID_A$ , password  $pw$ , and the random number  $r_a$  or the private keys of the telecare medical server  $B$ , such as  $K_B$  and two large primes  $p, q$ . Unfortunately, the patient anonymity has guaranteed that it is impossible for the adversary to obtain the patient's personal information. As we know, the telecare medical server's private keys are not easily exposed. These features along with the patient anonymity confirm forward secrecy and known-key secrecy capability of our PAKE scheme.

**6.5. Resistance to Patient Impersonation Attack.** Evidently, the most essential goal of a secure PAKE scheme is to withstand impersonation attack, which means an interception of the transmitted messages from both sides will not lead to the serious threats on the system. In the proposed PAKE scheme for TMISs, no adversaries are able to

impersonate patient  $A$  by eavesdropping the communication messages, since the secret parameters including  $A$ 's identity  $ID_A$ , password  $pw$ , and random number  $r_a$  are unknown to the adversary. Additionally, it is computationally infeasible to find  $ID_A$  and  $r$  from  $z = (ID_A, r)^2 \bmod n$  without the knowledge of  $p$  and  $q$ , where  $n = p * q$ . Therefore, the proposed PAKE scheme for TMISs provides the resilience against patient impersonation attack.

**6.6. Resistance to Telecare Medical Server Spoofing Attack.** Suppose an adversary plans to impersonate the telecare medical server  $B$  by eavesdropping the communication message:  $M_1 = X_2 = E_x[T_r(x), z, X_1]$ , where  $x = h(ID_A, h(pw, r_a))$ ,  $z = (r, ID_A)^2 \bmod n$  and  $X_1 = h(x, ID_A, r)$ . They could not pass the authentication by patient  $A$  without knowing the telecare medical server  $B$ 's secret key  $K_B$ . How easy will it be to get hold of patient  $A$ 's identity  $ID_A$  and  $r$  without the help of the correct value  $x$ ? Hence, the proposed PAKE scheme for TMISs can withstand telecare medical server spoofing attack.

**6.7. Resistance to Bergamo et al.'s Attack.** The implementation of the Bergamo et al.'s attack [12] is based on the following facts: (i) Chebyshev polynomials can be alternatively defined as the cosine function, which leads to the same value due to the periodicity of the cosine function; (ii)  $T_u(x), T_s(x)$ , and  $x$  as the public keys are transmitted in an open channel, which can be intercepted by an adversary. However, in the proposed scheme for TMISs, patient  $A$  and the telecare medical server  $B$  transmitted the encrypted messages  $Auth_A$  and  $Auth_B$  over a public channel, where  $Auth_A = h(sk, T_u(x), T_s(x))$ ,  $Auth_B = h(sk, T_s(x), 00, T_u(x))$ , respectively. Without knowing  $T_r(x)$ , no adversaries can decrypt the message  $X_4$  and thus they cannot recover  $T_u(x)$ . Additionally, the value  $x = h(ID_A, h(pw, r_a))$  is related with the patient  $A$ 's sensitive information, and adversaries are incapable of getting such sensitive information. Therefore, the proposed PAKE scheme for TMISs is free from Bergamo et al.'s attack [12].

**6.8. Resistance to Replay Attack.** With the purposing of free from replay attack, we use a random number  $r$  which is only recovered by the telecare medical server  $S$ . If an adversary attempts to masquerade  $A$  by immediately replaying the previous authentication messages  $M_1 = \{X_2\}$  after eavesdropping, the telecare medical server  $S$  would obviously refuse the request because the invalid random number  $r$  will be detected by checking  $h(x, ID_A, r) \stackrel{?}{=} X_1$ . Moreover, the

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Input:  $ID_A, pw, ra$ 
Output: true: success; false: failure
1: Generate  $r$ ,
2: Compute  $x = h(ID_A, h(pw.ra)), y = (ID_A, r)$ 
3:  $Z = y^2 \text{ mod } n, X_1 = h(x, ID_A, r), X_2 = E_X[T_r(x), z, X_1]$ .
4: Transmit  $M_1 = \{X_2\}$  to  $B$ .
5: Decrypt  $A_{pw} \rightarrow x = h(ID_A, h(pw.ra)), X_2 \rightarrow [T_r(x), z, X_1]$ .
6: Solve  $z$  by CRT, determinate  $(ID'_A, r')$ 
7: if  $h(x, ID'_A, r') \doteq X_1$  then
8: Generate  $s, X_3 = E_{T_r(x)}[T_s(x), SID_B, h(ID_A, r)]$ 
9: Transmit  $X_3$  to Patient  $A$ .
10: Decrypt  $X_3 \rightarrow [T_s(x), SID_B, h(ID_A, r)]$ 
11: if  $h(ID_A, r) \doteq h(ID'_A, r')$  then
12: Generate  $u, sk = h(T_u(x), T_s(x), T_{us}(x))$ 
13:  $X_4 = E_{T_r(x)}[T_u(x), ID_A], Auth_A = h(sk, T_u(x), T_s(x))$ .
14: Transmit  $Auth_A$  and  $X_4$  to  $B$ .
15: Decrypt  $X_4 \rightarrow [T_u(x), ID_A]$ .
16:  $sk = h(T_s(x), T_u(x), T_s(T_u(x)))$ .
17: if  $h(sk, T_s(x), T_u(x)) \doteq Auth_A$  then
18:  $Auth_B = h(sk, T_s(x), T_u(x), 00)$ 
19: Transmit  $Auth_B$  to  $M_4 = \{Auth_B\}$ .
20: if  $Auth_B \doteq h(sk, T_s(x), T_u(x), 00)$  then
21: return true
22: else
23: return false
24: else
25: return false
26: else
27: return false
28: else
29: return false
30: end if

```

ALGORITHM 2: Authentication and Key agreement.

patient also checks the random number which is sent from the telecare medical server to prevent the replay attack.

## 7. Security Attributes and Performance Comparison

In the following section, we analyze the security attributes and the computational efficiency of the proposed PAKE scheme for TMISs and compare to Xiao et al. [15], Guo and Zhang [18], and Lee [19] since they are all based on chaotic-maps PAKE schemes. Table 1 shows the security attributes comparison among our presented scheme and other schemes [15, 18, 19]. Compared with other schemes, both Guo and Zhang and Xiao et al.'s schemes cannot achieve user anonymity and perfect forward session key secrecy. Furthermore, both of their schemes cannot withstand

patient impersonation attack. In addition, Lee's scheme fails to prevent known session-specific temporary information and server spoofing attacks.

Table 2 lists the computational complexity comparison of our proposed PAKE scheme with other schemes, where  $T_c$  denotes the time of executing a Chebyshev polynomial computing;  $T_h$  denotes the time of executing a hash operation;  $T_s$  denotes the time of executing a symmetric key encryption/decryption;  $T_{sq}$  denotes the time of executing a squaring;  $T_{sr}$  denotes the time of executing a squaring root solving. According to [15], the execution time for  $T_s$  is about 70 times than  $T_c$ , and  $T_s$  is almost equal to  $T_h$  in software. Therefore, our proposed PAKE scheme consumes a slightly higher computation cost than others. We think it is worth slightly sacrificing the efficiency in the hope of guaranteeing a high level security for TMISs.

TABLE 1: Comparison of security attributes.

	Ours	Lee [19]	Guo and Zhang [18]	Xiao et al. [15]
Provide anonymity	Yes	Yes	No	No
Provide perfect forward session key secrecy	Yes	Yes	No	No
Provide mutual authentication	Yes	Yes	Yes	Yes
Resist man-in-the-middle attack	Yes	Yes	Yes	Yes
Resist replay attack	Yes	Yes	Yes	Yes
Resist known session-specific temporary information attack	Yes	No	—	—
Resist Bergamo et al.'s attack	Yes	Yes	No	Yes
Resist patient impersonation attack	Yes	Yes	No	No
Resist server spoofing attack	Yes	No	Yes	No

TABLE 2: Comparison of computational cost.

	Ours	Lee [19]	Guo and Zhang [18]	Xiao et al. [15]
User	$3T_c + 7T_h + 3T_s + 1T_{sq}$	$2T_c + 6T_h + 1T_{sq}$	$2T_c + 8T_h$	$2T_c + 2T_h$
Server	$2T_c + 8T_h + 4T_s + 1T_{sr}$	$2T_c + 9T_h + 1T_{sr}$	$2T_c + 10T_h$	$2T_c + 2T_h$
No. of message communications	4	3	6	5

## 8. Conclusion

In this paper, we first reviewed Lee's scheme and then demonstrated that Lee's scheme is vulnerable to the known session-specific temporary information and server spoofing attacks. With the purpose of remedy of these security loopholes, we presented an improved PAKE scheme using extended chaotic maps for TMISs. We showed that our design is secure and provides more functionalities compared with the related schemes. Performance analysis showed the proposed PAKE scheme for TMISs is secure and efficient. In the future, we will further optimize the proposed scheme regarding security and performance using encryption and machine learning in order to apply to network structure to improve its availability.

## Data Availability

The data are included in the manuscript.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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