

Retraction

Retracted: Distributed Consensus Algorithm for Nonholonomic Wheeled Mobile Robot

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This article has been retracted by Hindawi, as publisher, following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of systematic manipulation of the publication and peer-review process. We cannot, therefore, vouch for the reliability or integrity of this article.

Please note that this notice is intended solely to alert readers that the peer-review process of this article has been compromised.

Wiley and Hindawi regret that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] X. Meng, P. He, X. Xiong, H. Mi, Z. Li, and Z. Wu, "Distributed Consensus Algorithm for Nonholonomic Wheeled Mobile Robot," *Security and Communication Networks*, vol. 2021, Article ID 7617819, 9 pages, 2021.

Research Article

Distributed Consensus Algorithm for Nonholonomic Wheeled Mobile Robot

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This paper focuses on consensus of the nonholonomic wheeled mobile robotic systems whose geometric center and centroid do not coincide. A consensus control algorithm for mobile robots based on the nonstandard chain systems is proposed. Firstly, coordinate transformation is used to transform the nonholonomic robotic systems into the nonstandard chain model. Then, a distributed cooperative control algorithm is designed, and the Lyapunov stability theorem and LaSalle invariance principle are used to prove that each state of the mobile robot is consensus. Finally, the effectiveness of the algorithm is proved through numerical simulation.

1. Introduction

With the coming of information times, there has been great interest in cooperative control of nonholonomic systems, including collective behavior and synchronization phenomenon [1]. The consensus problem is one of the significant control problem in cooperative control [2]. In contrast, most of the systems in reality have nonholonomic constraints, such as wheeled mobile robot [3] and tractor trailer systems [4]. Since the Brockett stability condition [5] is not satisfied, such systems cannot become asymptotically stable by the directly continuous differentiable and time-invariant state feedback control law [6]. Therefore, it is very challenging to achieve the consensus of nonholonomic systems.

Aiming at the consensus problem of nonholonomic systems, many effective methods are proposed [7–10]. A time-varying cooperative control law was applied to the third-order chain system so that the state of the system

converges to the moving point [8]. In addition, by using appropriate coordinate transformation, the homotopy-based method was proposed to solve the problem of matrix inequality [7, 9]. Two kinds of time-invariant continuous state feedback controllers were designed to achieve the consensus of nonholonomic systems [10]. As can be seen from the above, the consensus problem does not require that all agents converge to the same equilibrium point, but to the same variable vector [11]. Moreover, the interconnection graph is used to describe the information flow between agents, and the graph Laplacian is used as the feedback gain. This paper extends the above approach to achieve consensus of nonholonomic systems. Meanwhile, Brockett's stability condition should not be an obstacle to the realization of the consensus problem [5].

Motivated by the consensus of the nonholonomic chain system without pilot [10], this paper designed a decentralized controller, which uses a time-varying and continuous

control scheme to achieve consensus of the nonstandard chain system. Then, combined with undirected communication topology, the stability is proved by the Lyapunov stability theorem and LaSalle invariance principle. Finally, Matlab numerical simulation is carried out to prove the effectiveness of the algorithm.

The main contributions of this paper are summarized in the following two aspects:

- (i) In practice, the geometric center and centroid of most robots do not coincide, which means that this kind of system cannot be transformed into a chain model. Therefore, it is difficult to achieve the consensus of this kind of nonholonomic systems.
- (ii) A distributed algorithm is designed for a class of nonholonomic wheeled mobile robot which cannot be converted into the chain system to achieve the consensus.

The rest of the paper is organized as follows. Section 2 gives the preparatory knowledge, including graph and graph Laplacian, consensus via graph Laplacian and Kronecker product. A distributed cooperative control algorithm is designed for nonstandard chain systems, and the Lyapunov stability theorem and LaSalle invariance principle are used to prove that the state of the mobile robot is consensus in Section 3. In order to verify the effectiveness of the algorithm, the simulation results are obtained through Matlab in Section 4. Conclusions are presented in Section 5.

2. Preparatory Knowledge

2.1. Graph and Graph Laplacian. Algebraic graph theory is a major tool to study the communication between multiple nonlinear systems, which is widely used in consensus of the multiagent system [9]. Graph $G = \{V, E\}$ with the set of node $V = \{1, 2, \dots, N\}$ and edges $E \subset V \times V$. The edge $(i, j) \in E$ means that the information of the j th agent is available for the i th agent. When $(j, i) \in E$ and $(i, j) \in E$, then graph G is called the undirected graph.

The adjacency matrix A of graph G is defined as

$$a_{ii} = 0, \quad (1)$$

$$a_{ij} = \begin{cases} 1, & \text{if } (j, i) \in E, \\ 0, & \text{otherwise.} \end{cases}$$

If a_{ij} of adjacency matrix A is the nonnegative real number, then the adjacency matrix A is called as the weighted adjacency matrix, and the corresponding graph is the weighted graph [12]. The Laplacian matrix of graph G is defined as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1, k \neq i}^N a_{ik}, & i = j. \end{cases} \quad (2)$$

2.2. Consensus via Graph Laplacian. The algorithms based on graph Laplacian are considered, and take the first-order integrator model as an example:

$$\dot{x}_i = u_i, \quad (3)$$

where x_i is the state variable and u_i is the control input. The control inputs [13] were proposed as

$$u_i = \sum_{j=1}^N a_{ij}(x_j - x_i), \quad i \in N. \quad (4)$$

Then, the matrix form of multiagents with N is

$$\dot{X} = -(L \otimes I_N)X. \quad (5)$$

The basic properties of the Laplacian matrix are as follows:

- (1) Laplacian matrix is a positive semidefinite matrix
- (2) The minimum eigenvalue is zero because the sum of every row of the Laplacian matrix is zero

2.3. Kronecker Product. Kronecker product is an operation between two matrices of arbitrary size. It is defined as follows. If $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{p \times q}$, then the following block matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}. \quad (6)$$

It is called Kronecker product, which is defined as $A \otimes B = [a_{ij}B]$.

The basic properties of Kronecker product are as follows:

- (1) If A is positive semidefinite matrix and B is positive definite matrix, then $A \otimes B$ is positive semidefinite matrix
- (2) If A, B, C , and D are four matrices and AC and $B D$ exist, then $(A \otimes B)(C \otimes D) = AC \otimes B D$
- (3) If $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{p \times q}$, then the transpose operation conforms to the distributive law $(A \otimes B)^T = A^T \otimes B^T$

3. Problem Description

3.1. Nonholonomic Wheeled Mobile Robot. Consider a group of N robots, whose centroid and geometric center do not coincide. The geometric model of mobile robots is shown in Figure 1.

In Figure 1, C is centroid of two driving wheels, M is the geometric center of the body of the robot, d is the distance point C and M , θ_i is the angle between the robot's motion direction and X -axis, and (x_c, y_c) is the coordinate of the geometric center C of the mobile robot in the XOY coordinate system.

Under the condition of pure rolling without slipping, we get to meet the following nonholonomic constraints of the wheeled mobile robot:

$$\dot{x}_c \sin \theta - \dot{y}_c \cos \theta - d\dot{\theta} = 0. \quad (7)$$

The equation (7) can be rewritten as follows:

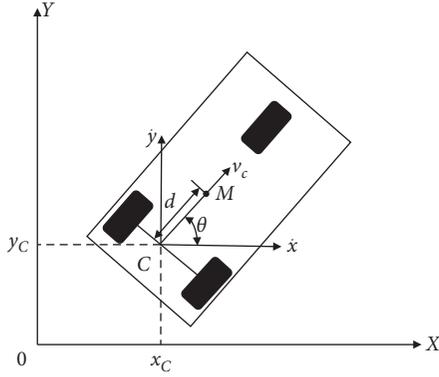


FIGURE 1: Structure diagram of the nonholonomic wheeled mobile robot.

$$A(q)\dot{q} = 0, \quad (8)$$

where $A(q) = [\sin \theta, -\cos \theta, -d]$ is the nonholonomic constraint matrix and $\dot{q} = [\dot{x}_c, \dot{y}_c, \dot{\theta}]$.

Let $S(q)$ spans the null space of $A(q)$ and a full rank matrix formed by a set of smooth and linearly independent vector fields

$$A(q)S(q) = 0. \quad (9)$$

In this case from (8), the matrix $S(q)$ of (9) is

$$S(q) = \begin{bmatrix} \cos \theta & d \sin \theta \\ \sin \theta & -d \cos \theta \\ 0 & 1 \end{bmatrix}. \quad (10)$$

Define

$$M(t) = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix}, \quad (11)$$

where v_c and ω_c are linear velocity and angular velocity of the center of geometric, respectively.

The kinematics of the i th robot ($i \in N$) can be obtained:

$$Q_i = S(q_i)M_i, \quad (12)$$

where

$$Q_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix}, \quad (13)$$

$$M_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}.$$

And,

$$S(q_i) = \begin{bmatrix} \cos \theta_i & d \sin \theta_i \\ \sin \theta_i & -d \cos \theta_i \\ 0 & 1 \end{bmatrix}, \quad (14)$$

where x_i and y_i are the coordinates of the i th mobile robot in Cartesian coordinates. v_i and ω_i are the linear and angular velocity of the i th robot, respectively.

It can be obtained that after the following two transformations [14],

$$\begin{cases} z_{i1} = \theta_i, \\ z_{i2} = x_i \cos \theta_i + y_i \sin \theta_i, \\ z_{i3} = x_i \sin \theta_i - y_i \cos \theta_i. \end{cases} \quad (15)$$

$$\begin{cases} u_{i1} = \omega_i, \\ u_{i2} = v_i - \omega_i z_{i3}. \end{cases} \quad (16)$$

It is easy that the above transformation is regular.

Substitute (16) into the derivative of (15), we can obtain

$$\begin{aligned} \dot{z}_{i1} &= \dot{\theta}_i \\ &= \omega_i \\ &= u_{i1}, \\ \dot{z}_{i2} &= \dot{x}_i \cos \theta_i - x_i \sin \theta_i \dot{\theta}_i + \dot{y}_i \sin \theta_i + y_i \cos \theta_i \dot{\theta}_i \\ &= (v_i \cos \theta_i + d\omega_i \sin \theta_i) \cos \theta_i - x_i \sin \theta_i \omega_i \\ &\quad + (v_i \sin \theta_i - d\omega_i \cos \theta_i) \sin \theta_i + y_i \cos \theta_i \omega_i \\ &= u_{i2}, \\ \dot{z}_{i3} &= \dot{x}_i \sin \theta_i + x_i \cos \theta_i \dot{\theta}_i - \dot{y}_i \cos \theta_i + y_i \sin \theta_i \dot{\theta}_i \\ &= (v_i \cos \theta_i + d\omega_i \sin \theta_i) \sin \theta_i + x_i \cos \theta_i \omega_i \\ &\quad - (v_i \sin \theta_i - d\omega_i \cos \theta_i) \cos \theta_i + y_i \sin \theta_i \omega_i \\ &= z_{i2} u_{i1} + d u_{i1}. \end{aligned} \quad (17)$$

The following nonstandard chain model is sorted out:

$$\begin{cases} \dot{z}_{i1} = u_{i1}, \\ \dot{z}_{i2} = u_{i2}, \\ \dot{z}_{i3} = z_{i2} u_{i1} + d u_{i1}, \end{cases} \quad (18)$$

where $Z_i = [z_{i1}, z_{i2}, z_{i3}]^T$ is the state of the system. $U_i = [u_{i1}, u_{i2}]^T$ is the control input. Therefore, the above system can be simplified to the following matrix form:

$$\dot{Z} = RU, \quad (19)$$

where

$$\begin{aligned} Z &= [Z_1^T, Z_2^T, \dots, Z_N^T]^T, \\ R &= \text{diag}[R_1, R_2, \dots, R_N], \\ U &= [U_1^T, U_2^T, \dots, U_N^T]^T, \\ R_i &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ z_{i2} + d & 0 \end{bmatrix}. \end{aligned} \quad (20)$$

3.2. Control Objectives

Assumption 1. It is assumed that the topological structure diagram composed of multirobots is connected and there is no isolated system.

Assumption 2. It is assumed that multirobot networks communicate with each other and communication topology is undirected. Therefore, its Laplacian matrix is positive semidefinite and symmetric.

Remark 1. Assumption 1 is the premise of the consensus problem. Assumption 2 is the premise of the controller

design in this paper. The consensus problem of all states of the system was considered.

To sum up, each agent can only obtain information from its neighboring agents and itself [15]. The objective of this paper is to design a time-invariant continuous state feedback law $U_i = F_i(Z_i, Z_j)$, $j \in N_i$, for

$$\lim_{t \rightarrow \infty} \|Z_i - Z_j\| = 0, \quad \forall i \neq j. \quad (21)$$

3.3. Controller Design. Under the conditions of Assumptions 1 and 2, the following control inputs are designed:

$$\begin{aligned} u_{i1} &= -k_{i1} \left\{ g_1 \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) + g_3(z_{i2} + d) \left[\sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \right] \right\}, \\ u_{i2} &= -k_{i2} \left\{ g_2 \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + g_3 \left[\sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \right] \right\}. \end{aligned} \quad (22)$$

The above equation is equivalent to

$$\begin{aligned} \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix} &= \begin{bmatrix} -k_{i1}g_1 & 0 & -k_{i1}g_3(z_{i2} + d) \\ 0 & -k_{i2}g_2 & -k_{i2}g_3 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) & \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) & \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \end{bmatrix} \\ &= - \begin{bmatrix} k_{i1} & 0 \\ 0 & k_{i2} \end{bmatrix} \begin{bmatrix} g_1 & 0 & g_3(z_{i2} + d) \\ 0 & g_2 & g_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) & \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) & \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \end{bmatrix} \\ &= - \begin{bmatrix} k_{i1} & 0 \\ 0 & k_{i2} \end{bmatrix} \begin{bmatrix} 1 & 0 & (z_{i2} + d) \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) & \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) & \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \end{bmatrix} \\ &= - \begin{bmatrix} k_{i1} & 0 \\ 0 & k_{i2} \end{bmatrix} \begin{bmatrix} 1 & 0 & (z_{i2} + d) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) & \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) & \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \end{bmatrix}. \end{aligned} \quad (23)$$

It is can be simplified to the following matrix form:

$$\begin{aligned} U &= -KR^T(I_N \otimes A^T GA)(L \otimes I_N)Z \\ &= -KR^T(L \otimes A^T GA)Z, \end{aligned} \quad (24)$$

where $K = \text{diag}[k_{11}, k_{12}, k_{21}, \dots, k_{N1}, k_{N2}]$, $U = \text{diag}[u_{11}, u_{12}, u_{21}, \dots, u_{N1}, u_{N2}]$, $G = \text{diag}[g_1, g_2, g_3]$, and g_i , $i = \{1, 2, 3\}$, is a positive number, and

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}. \quad (25)$$

In addition, we can obtain

$$A^TGA = \begin{bmatrix} g_1 & 0 & 0 \\ 0 & g_2 & g_2 \\ 0 & g_2 & g_2 + g_3 \end{bmatrix}, \quad (26)$$

where the value of first-order principal minor of matrix A^TGA is $g_1 > 0$. The value of second order of principal minor is $g_1g_2 > 0$. The determinant of the matrix is $g_1g_2g_3 > 0$. So, A^TGA is positive definite symmetric matrix and symmetric matrix.

Substituting (24) into (19), we can obtain the whole closed-loop system:

$$\dot{Z} = -RKR^T(L \otimes A^TGA)Z. \quad (27)$$

4. Proof of Consensus

Theorem 1. *Decentralized algorithm (22) are used to achieve consensus of system (19). In other words, closed-loop system (27) is globally asymptotically stable.*

$$0 = g_1 \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) + g_3(z_{i2} + d) \left[\sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \right], \quad (31)$$

$$0 = g_2 \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + g_3 \left[\sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij}(z_{i3} - z_{j3}) \right]. \quad (32)$$

Combining (31) and (32), we can obtain

$$g_2(z_{i2} + d) \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) = g_1 \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}). \quad (33)$$

By accumulating up all i of the two sides of (33) and then according to basic properties (2) of the Laplacian matrix, we can get

$$g_2 \sum_{i=1}^N (z_{i2} + d) \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2}) = 0, \quad (34)$$

which is equivalent to

$$g_2 \sum_{j=1}^N (z_{j2} + d) \sum_{i=1}^N a_{ij}(z_{j2} - z_{i2}) = 0. \quad (35)$$

Adding (34) and (35), then

$$\sum_{i=1}^N \sum_{j=1}^N a_{ij}(z_{i2} - z_{j2})^2 = 0. \quad (36)$$

Thus, $z_{12} = z_{22} = \dots = z_{N2}$ holds when $\dot{V} = 0$.

Proof: To begin with, we choose a candidate Lyapunov function:

$$V = \frac{1}{2}Z^T(L \otimes (A^TGA))Z, \quad (28)$$

where L is the symmetric matrix from Assumption 2 and A^TGA is also the symmetric matrix, so $L \otimes A^TGA$ is the symmetric matrix. According to basic properties (1) of the Laplacian matrix, we get $L \geq 0$. In addition, $A^TGA > 0$, so $L \otimes A^TGA \geq 0$ from basic properties (1) of Kronecker product. Therefore, V is the positive matrix.

Substituting (27) into the derivative of (28), we can obtain

$$\begin{aligned} \dot{V} &= \frac{1}{2} \left\{ \dot{Z}^T(L \otimes (A^TGA))Z + Z^T(L \otimes (A^TGA))\dot{Z} \right\} \\ &= -Z^T(L \otimes (A^TGA))RKR^T(L \otimes (A^TGA))Z \\ &\leq 0. \end{aligned} \quad (29)$$

Therefore, \dot{V} is always nonpositive.

Suppose $\dot{V} = 0$ is true at a certain time, and we can get it from (29):

$$R^T(L \otimes (A^TGA))Z = 0. \quad (30)$$

The above equation expands to

Furthermore, it can be seen from the above conclusion that the left side of (33) is zero. Therefore, it can be obtained from the right side of the following equation:

$$\sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) = 0, \quad (37)$$

which is equivalent to

$$\sum_{i=1}^N z_{i1} \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1}) = 0. \quad (38)$$

Furthermore,

$$\sum_{j=1}^N z_{j1} \sum_{i=1}^N a_{ij}(z_{j1} - z_{i1}) = 0. \quad (39)$$

Adding (38) and (39), then

$$\sum_{i=1}^N \sum_{j=1}^N a_{ij}(z_{i1} - z_{j1})^2 = 0. \quad (40)$$

Therefore, we prove that $z_{11} = z_{21} = \dots = z_{N1}$. Similarly, by sorting out (32), we obtain $z_{13} = z_{23} = \dots = z_{N3}$.

In summary, according to the LaSalle invariance principle, when $\dot{V} = 0$, the consensus of nonholonomic systems is achieved. \square

4.1. Numerical Simulation. In order to verify the effectiveness of the proposed scheme, the wheeled mobile robot (Figure 1) is selected as the controlled object in the simulation model.

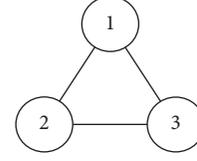


FIGURE 2: Communication graph.

Coordinate transformation is carried out by using (15) and (16) for the nonholonomic mobile robot, and the nonstandard chain model is obtained. The actual control input linear velocity and angular velocity can be expressed as

$$\begin{aligned}
 v_i &= u_{i2} + \omega_i z_{i3} \\
 &= -k_{i2} \left\{ g_2 \sum_{j=1}^N a_{ij} (z_{i2} - z_{j2}) + g_3 \left[\sum_{j=1}^N a_{ij} (z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij} (z_{i3} - z_{j3}) \right] \right\} \\
 &\quad - k_{i1} (x_i \sin \theta_i - y_i \cos \theta_i) \cdot \left\{ g_1 \sum_{j=1}^N a_{ij} (z_{i1} - z_{j1}) + g_3 (z_{i2} + d) \left[\sum_{j=1}^N a_{ij} (z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij} (z_{i3} - z_{j3}) \right] \right\}.
 \end{aligned} \tag{41}$$

$$\begin{aligned}
 \omega_i &= u_{i1} \\
 &= -k_{i1} \left\{ g_1 \sum_{j=1}^N a_{ij} (z_{i1} - z_{j1}) + g_3 (z_{i2} + d) \left[\sum_{j=1}^N a_{ij} (z_{i2} - z_{j2}) + \sum_{j=1}^N a_{ij} (z_{i3} - z_{j3}) \right] \right\}.
 \end{aligned} \tag{42}$$

Suppose that the communication topology of three wheeled mobile robots is shown in Figure 2. Its Laplacian matrix is

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \tag{43}$$

Meanwhile, the initial states of robots are

$$\begin{aligned}
 (x_1, y_1, \theta_1) &= \left(3, 5, -\frac{3\pi}{5} \right), \\
 (x_2, y_2, \theta_2) &= \left(-3, -4, \frac{5\pi}{9} \right), \\
 (x_3, y_3, \theta_3) &= \left(2, 2, -\frac{\pi}{3} \right).
 \end{aligned} \tag{44}$$

The corresponding initial value of the nonstandard chain model is

$$\begin{aligned}
 (z_{11}, z_{12}, z_{13}) &= (-1.88, 2.83, -5.09), \\
 (z_{21}, z_{22}, z_{23}) &= (1.75, -3.15, 3.91), \\
 (z_{31}, z_{32}, z_{33}) &= (-1.05, 1.96, -2.04).
 \end{aligned} \tag{45}$$

Condition 1. Parameter $d = 0$.

When $d = 0$, then (18) becomes a third-order chain system [10]:

$$\begin{cases} \dot{z}_{i1} = u_{i1}, \\ \dot{z}_{i2} = u_{i2}, \\ \dot{z}_{i3} = z_{i2} u_{i1}. \end{cases} \tag{46}$$

The controller parameters are $G = \text{diag}[5, 2, 6]$ and $k_i = 35$, $i \in \{1, 2, 3, 4, 5, 6\}$. Three states of the system can be obtained by numerical simulation. Figure 3 shows the trajectory of the first state variable of the chain system. Figure 4 shows the trajectory of the second state variable of the chain system. Figure 5 shows the trajectory of the third state variable of the chain system.

It can be seen from Figures 3–5 that the three states of the mobile robot under the third-order chain system can achieve consensus.

Condition 2. Parameter $d = 10$.

Similarly, in order to verify the correctness of the proposed scheme, we choose the larger structure parameter of the robot system. The controller parameters $d = 10$, $k_i = 65$, $i \in \{1, 2, 3, 4, 5, 6\}$, and $G = \text{diag}[5, 2, 9]$. Through the numerical simulation of the proposed control strategy, three states of the system can be obtained. Figure 6 shows the trajectory of the first state variable after the transformation of three robots. Figure 7 shows the trajectory of the second

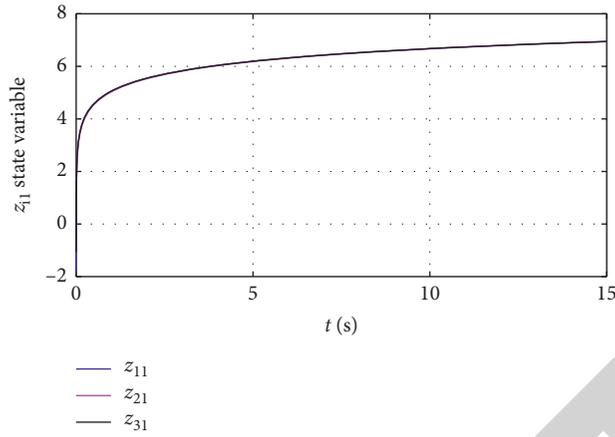


FIGURE 3: Trajectory of z_{i1} in the chain system.

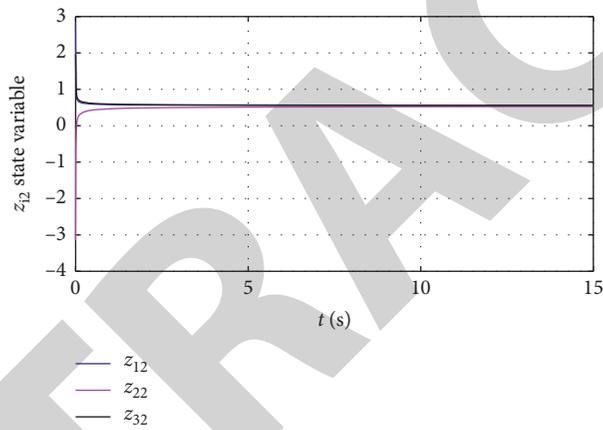


FIGURE 4: Trajectory of z_{i2} in the chain system.

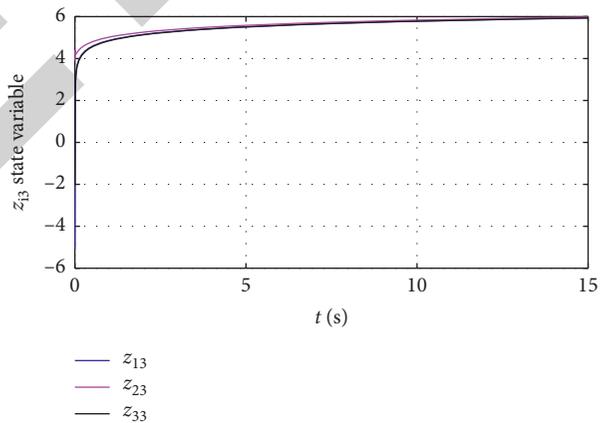
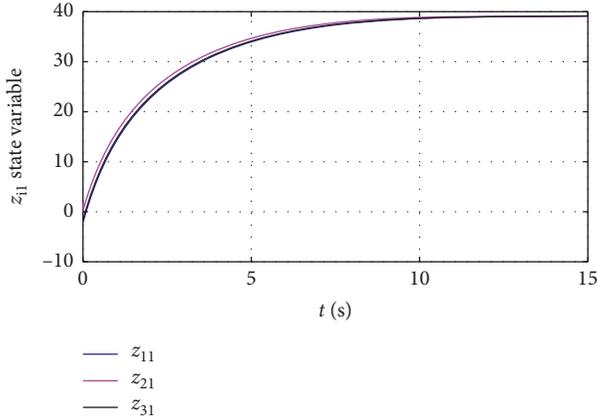
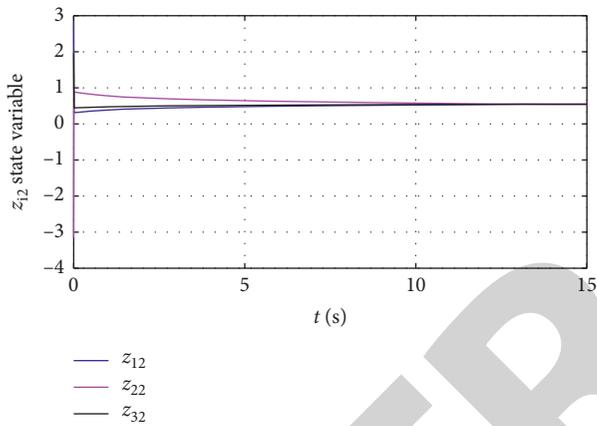
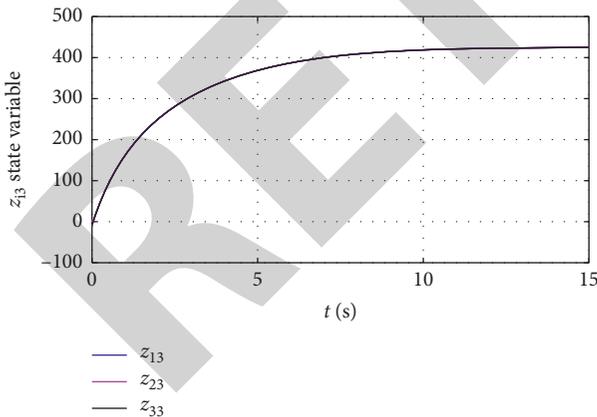


FIGURE 5: Trajectory of z_{i3} in the chain system.

state variable after the transformation of three robots. Figure 8 shows the trajectory of the third state variable after the transformation of three robots.

As can be seen from Figures 6–8, the first state variable can achieve consensus in about 10 seconds. The second state

variable can achieve consensus in about 13 seconds. The third state variable can achieve consensus in about 2 seconds. Although the convergence rates of the three states are different, they all have good convergence performance. The convergence rate of three state variables can also be achieved

FIGURE 6: Trajectory of z_{i1} .FIGURE 7: Trajectory of z_{i2} .FIGURE 8: Trajectory of z_{i3} .

by adjusting the corresponding control parameters. In conclusion, the control scheme mentioned above is effective.

5. Conclusion

In this paper, the consensus of nonholonomic wheeled mobile robotic systems whose geometric center and centroid

do not coincide was studied. The time-invariant continuous state feedback control law is proposed by changing the structure of the nonholonomic systems. Compared with the existing chain form, the nonstandard chain form is more in line with the actual motion state of the robot. At the same time, the controllers can set free parameters to adjust when dealing with more complex control requirement.

In the future, the relationship between the consensus value of state and initial states of all agents will be studied. In addition to this, the consensus of nonholonomic systems, such as communication delay and switching topology [16] are further analyzed.

Data Availability

The data used to support the findings of this study are included within this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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