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Retraction

Retracted: Hybrid Structures Applied to Subalgebras of BCH-Algebras

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] G. Muhiuddin, D. Al-Kadi, W. Khan, and C. Jana, "Hybrid Structures Applied to Subalgebras of BCH-Algebras," *Security and Communication Networks*, vol. 2021, Article ID 8960437, 8 pages, 2021.

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Research Article

Hybrid Structures Applied to Subalgebras of BCH-Algebras

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The algebraic structures have many applications in coding theory, cryptography, and security networks. In this paper, the notion of hybrid subalgebras of BCH-algebras is introduced and related properties are investigated. Moreover, some characterizations of hybrid subalgebras of BCH-algebras are given. Furthermore, we state and prove some theorems in hybrid subalgebras of BCH-algebras. The homomorphic images and inverse images of fuzzy BCH-subalgebras are studied and discussed.

1. Introduction

The notions of BCK/BCI-algebras were initiated by Imai and Iséki in 1966. A number of research papers have been produced on the theory of BCK/BCI-algebras. Hu and Li [1, 2] introduced the notion of a BCH-algebra as a generalization of BCK/BCI-algebras and subsequently gave examples of proper BCH-algebras and studied some properties. Certain other properties of BCH-algebras have been studied by Ahmad [3], Dudek and Thomys [4], Chaudhry [5], Roh et al. [6, 7], Chaudhry et al. [8], and Dar et al. [9], and Smarandache structure has been applied to BCH-algebra [10].

Fuzzy sets, which were introduced in the 1960s by Zadeh [11], have been developed considerably by many research studies. Molodtsov introduced the concept of soft set [12] and pointed out several directions for its applications (for more details, see [12–15]). This concept was applied to BCH-algebras introducing soft BCH-algebras which were studied in [16]. Moreover, the fuzzy set theoretical approach to BCH-algebras was extensively investigated by many researchers on different aspects. For example, fuzzy n-fold ideals [17], fuzzy closed ideals and fuzzy filters [18], filters based on bipolar-valued fuzzy sets [19], and cubic subalgebras [20].

Jun et al. [21] combined the concepts of fuzzy sets and soft sets, introduced the notion of hybrid structure in a set of parameters over an initial universe set, and investigated several properties. They also introduced the concepts of hybrid linear space, hybrid subalgebra, and hybrid field. Moreover, hybrid structure applications have been studied in semigroups (see [22–25] and references there in), and recently, hybrid ideals of BCK = BCI-algebras were studied in [26–29]. For more important terminologies, the readers are referred to [30–36].

In the present paper, we present an application of fuzzy set theory to an algebraic structure called, BCH-algebra. As we know it algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. This provides sufficient motivation to researchers to review various concepts and results from the realm of abstract algebra in the broader framework of fuzzy setting. The objective of this study is to introduce the concept of hybrid subalgebras of BCH-algebras. The notion of hybrid subalgebras of BCH-algebras is defined, and related properties are investigated. This paper is organized as follows: in Section 2, we recall some definitions related to the subject. In Section 3, the concepts and operations of hybrid subalgebras of BCH-algebras are introduced and their properties are discussed in detail. Furthermore, some properties of hybrid subalgebras of BCH-algebras under homomorphisms are explored.

2. Preliminaries

This section begins with the following definitions and properties that will be needed in the sequel.

An algebra $(L, *, 0)$ of type $(2, 0)$ is called a BCH-algebra [1] if it satisfies the following axioms, for all $q, l, n \in L$:

(1) $q * q = 0$,
(2) $q * l = 0$ and $l * q = 0$ imply $q = l$,
(3) $(q * l) * n = (q * n) * l$.

Any BCH-algebra L satisfies the following axioms:

(i) $q * 0 = q$,
(ii) $(q * (q * l)) * l = 0$,
(iii) $0 * (q * l) = (0 * q) * (0 * l)$,
(iv) $0 * (0 * (0 * q)) = 0 * q$,
(v) $q \leq l$ implies $0 * q = 0 * l$,

for all $q, l, n \in L$ [8].

A nonempty subset S of a BCH-algebra L is called a subalgebra of L if $q * l \in S$, for all $q, l \in S$.

We now review some fuzzy logic concepts as follows.

Let L be the collection of objects denoted generally by q. Then, a fuzzy set [11] A in L is defined as $A = \{\langle q, \mu_A \infty (q)\rangle : q \in L\}$ where $\mu_A(q)$ is called the membership degree of q in A and $0 \leq \mu_A(q) \leq 1$.

Furthermore, we collect some basic notions and results on hybrid structures due to Jun et al. [21]. Let I be the unit interval, L be a set of parameters, and $\mathbb{P}(U)$ be the power set of an initial universe set U.

Definition 1 (see [21]). A hybrid structure in L over U is a mapping:

$$\widetilde{h}_\eta \coloneqq (\widetilde{h}; \eta): L \longrightarrow \mathbb{P}(U) \times I; q \mapsto (\widetilde{h}(q); \eta(q)), \tag{1}$$

where $\widetilde{h}: L \longrightarrow \mathbb{P}(U)$ and $\eta: L \longrightarrow I$ are mappings.

Definition 2 (see [21]). For hybrid structures $\widetilde{h}_\eta$ and $\widetilde{g}_\mu$ in L over U, the hybrid intersection denoted by $\widetilde{h}_\eta \Cap \widetilde{g}_\mu$ is a hybrid structure:

$$\widetilde{h}_\eta \Cap \widetilde{g}_\mu: L \longrightarrow \mathbb{P}(U) \times I, q \mapsto ((\widetilde{h \cap g})(q), (\eta \vee \mu)(q)), \tag{2}$$

where

$$\widetilde{h}\widetilde{\cap}\widetilde{g}: L \longrightarrow \mathbb{P}(U), q \mapsto \widetilde{h}(q) \cap \widetilde{g}(q), \eta \vee \mu: L \longrightarrow I, q \mapsto \vee\{\eta(q), \eta(q)\}. \tag{3}$$

Definition 3 (see [21]). Let L be a BCK/BCI-algebra. For a hybrid structure $\widetilde{h}_\eta$ in L over U, $\widetilde{h}_\eta$ is said to be a hybrid subalgebra of L if the following statements are valid:

$$(\forall q, l \in L)\begin{pmatrix} \widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l), \\ \eta(q * l) \leq \vee\{\eta(q), \eta(l)\} \end{pmatrix}. \tag{4}$$

Lemma 1 (see [21]). *Every hybrid subalgebra $\widetilde{h}_\eta$ of a BCK/BCI-algebra L over U satisfies*

$$(\forall q \in L)(\widetilde{h}(0) \supseteq \widetilde{h}(q), \eta(0) \leq \eta(q)). \tag{5}$$

3. Hybrid Subalgebras of BCH-Algebras

In this section, we obtain our main results. Throughout our discussion, L will denote a BCH-algebra unless otherwise mentioned.

Definition 4. Let L be a BCH-algebra. A hybrid structure $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ in L over U is called a hybrid subalgebra of L over U if the following assertions are valid:

$$(\forall q, l \in L)\begin{pmatrix} \widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l), \\ \eta(q * l) \leq \vee\{\eta(q), \eta(l)\} \end{pmatrix}. \tag{6}$$

Let us illustrate this definition using the following example.

Example 1. Let the initial universe be the set $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $L = \{0, l_1, l_2, l_3, l_4\}$ be a BCH-algebra with the Cayley table (Table 1).

Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure in L over U which is given in Table 2.

It can be easily verified that $\widetilde{h}_\eta$ is a hybrid subalgebra of L over U.

Proposition 1. *Every hybrid subalgebra $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ in L over U satisfies the following assertions:*

$$(\forall q \in L)(\widetilde{h}(q) \subseteq \widetilde{h}(0), \ \eta(q) \geq \eta(0)). \tag{7}$$

Proof. For all $q \in L$, we have $\widetilde{h}(0) = \widetilde{h}(q * q) \supseteq \widetilde{h}(q) \cap \widetilde{h}(q) = \widetilde{h}(q)$ and $\eta(0) = \eta(q * q) \leq \vee\{\eta(q), \eta(q)\} = \eta(q)$. □

Proposition 2. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in L over U. Then, the following assertions are equivalent:*

(1) $(\forall q, l \in L)(\widetilde{h}(q * l) \supseteq \widetilde{h}(l), \eta(q * l) \leq \eta(q))$.
(2) $(\forall q \in L)(\widetilde{h}(0) = \widetilde{h}(q), \eta(0) = \eta(q))$.

Proof. If we take $l = 0$ in (1), then $\widetilde{h}(q) \supseteq \widetilde{h}(0)$ and $\eta(q) \leq \eta(0)$ for all $q \in L$. Combining this and Proposition 1, we have $(\widetilde{h}(0) = \widetilde{h}(q), \eta(0) = \eta(q))$ for all $q \in L$.

Conversely, assume that (2) is valid. Then,

TABLE 1: Cayley table of the binary operation $*$.

$*$	0	l_1	l_2	l_3	l_4
0	0	0	l_4	l_3	l_2
l_1	l_1	0	l_4	l_3	l_2
l_2	l_2	l_2	0	l_4	l_3
l_3	l_3	l_3	l_2	0	l_4
l_4	l_4	l_4	l_3	l_2	0

TABLE 2: Table representation of the hybrid structure $\widetilde{h}_\eta$.

L	$\widetilde{h}$	η
0	U	0.3
l_1	$\{u_1, u_2, u_3, u_4\}$	0.3
l_2	$\{u_3, u_4\}$	0.9
l_3	$\{u_3, u_4\}$	0.9
l_4	$\{u_3, u_4\}$	0.9

$$\begin{aligned} \widetilde{h}(l) &= \widetilde{h}(0) \cap \widetilde{h}(l) = \widetilde{h}(q) \cap \widetilde{h}(l) \subseteq \widetilde{h}(q * l), \\ \eta(l) &= \vee\{\eta(0), \eta(l)\} = \vee\{\eta(q), \eta(l)\} \geq \eta(q * l), \end{aligned} \tag{8}$$

for all $q, l \in L$. □

Proposition 3. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in L over U. Then, for all $q \in L$, $\widetilde{h}(q) \subseteq \widetilde{h}(0 * q)$ and $\eta(q) \geq \eta(0 * q)$.*

Proof. Let $q \in L$. Then,

$$\begin{aligned} \widetilde{h}(0 * q) &\supseteq \widetilde{h}(0) \cap \widetilde{h}(q) \\ &= \widetilde{h}(q * q) \cap \widetilde{h}(q) \\ &\supseteq (\widetilde{h}(q) \cap \widetilde{h}(q)) \cap \widetilde{h}(q) \\ &= \widetilde{h}(q), \\ \eta(0 * q) &\leq \vee\{\eta(0), \eta(q)\} \\ &= \vee\{\eta(q * q), \eta(q)\} \\ &\leq \vee\{\vee\{\eta(q), \eta(q)\}, \eta(q)\} \\ &= \eta(q). \end{aligned} \tag{9}$$

This completes the proof. □

For any hybrid structure $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ in L over U, we consider two level sets:

$$\begin{aligned} \widetilde{h}_\eta(\alpha) &:= \{q \in L | \alpha \subseteq \widetilde{h}(q)\}, \\ \widetilde{h}_\eta(t) &:= \{q \in L | \eta(q) \leq t\}, \end{aligned} \tag{10}$$

where $\alpha \in \mathbb{P}(U)$ and $t \in I$.

Theorem 1. *Let L be a BCH-algebra. For a hybrid structure $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ in L over U, the following are equivalent:*

(1) $\widetilde{h}_\eta$ is a hybrid subalgebra of L over U.

(2) For any $\alpha \in \mathbb{P}(U)$ and $t \in I$, the nonempty sets $\widetilde{h}_\eta(\alpha)$ and $\widetilde{h}_\eta(t)$ are subalgebras of L.

Proof

(1) $\Longrightarrow$ (2). Suppose that $\widetilde{h}_\eta$ is a hybrid subalgebra of L. Let $q, l \in \widetilde{h}_\eta(\alpha)$. Then, $\alpha \subseteq \widetilde{h}(q)$ and $\alpha \subseteq \widetilde{h}(l)$. It follows that $\alpha \subseteq \widetilde{h}(q) \cap \widetilde{h}(l) \subseteq \widetilde{h}(q * l)$ and so $q * l \in \widetilde{h}_\eta(\alpha)$. Hence, $\widetilde{h}_\eta(\alpha)$ is a subalgebra of L. Also, let $q, l \in \widetilde{h}_\eta(t)$. Then, $\eta(q) \leq t$ and $\eta(l) \leq t$. It follows that $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\} \leq t$ and so $q * l \in \widetilde{h}_\eta(t)$. Hence, $h_\eta(t)$ is a subalgebras of L.

(2) $\Longrightarrow$ (1). Let for any $\alpha \in \mathbb{P}(U)$ and $t \in I$, the nonempty sets $\widetilde{h}_\eta(\alpha)$ and $\widetilde{h}_\eta(t)$ are subalgebras of L. For contradiction, let $q_0, l_0 \in L$ such that $\widetilde{h}(q_0 * l_0) \subset \widetilde{h}(q_0) \cap \widetilde{h}(l_0)$. Let $\widetilde{h}(q_0) = \beta_1, \widetilde{h}(l_0) = \beta_2$ and $\widetilde{h}(q_0 * l_0) = \alpha$. Then, $\alpha \subset \beta_1 \cap \beta_2$. Let us consider $\alpha_1 | \widetilde{h}(q_0 * l_0) \subset \alpha_1 \subset \widetilde{h}(q_0) \cap \widetilde{h}(l_0)$. We get that $\widetilde{h}(q_0 * l_0) = \alpha \subset \alpha_1 \subset \beta_1 \cap \beta_2$, and so $q_0 * l_0 \notin \widetilde{h}_\eta(\alpha)$ which is a contradiction. Thus, $\widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l)$ for all $q, l \in L$. Also, let $q_0, l_0 \in L$ such that $\eta(q_0 * l_0) > \vee\{\eta(q_0), \eta(l_0)\}$. Let $\eta(q_0) = \eta_1, \eta(l_0) = \eta_2$ and $\eta(q_0 * l_0) = t$. Then, $t > \vee\{\eta_1, \eta_2\}$. Let us consider $t_1 | \eta(q_0 * l_0) > t_1 > \vee\{\eta(q_0), \eta(l_0)\}$. We get that $t > t_1 > \vee\{\eta_1, \eta_2\}$. Hence, $\vee\{\eta_1, \eta_2\} < t_1 < t = \eta(q_0 * l_0)$, and so $q_0 * l_0 \notin \widetilde{h}_\eta(t)$ which is a contradiction. Thus, $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\}$ for all $q, l \in L$. Hence, $\widetilde{h}_\eta$ is a hybrid subalgebra of L. □

Next, we define $H_{\widetilde{h}} = \{q \in L | \widetilde{h}(q) = \widetilde{h}(0)\}$ and $H_\eta = \{q \in L | \eta(q) = \eta(0)\}$. These two sets are also subalgebras of a BCH-algebra L over U.

Proposition 4. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in L over U. Then, the sets $H_{\widetilde{h}}$ and H_η are subalgebras of L over U.*

Proof. Let $q, l \in H_{\widetilde{h}}$. Then, $\widetilde{h}(q) = \widetilde{h}(0) = \widetilde{h}(l)$ and so $\widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l) = \widetilde{h}(0)$. By using Proposition 1, we know that $\widetilde{h}(q * l) = \widetilde{h}(0)$. Consequently, $q * l \in H_{\widetilde{h}}$.

Let $q, l \in I_\eta$. Then, $\eta(q) = \eta(0) = \eta(l)$ and so $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\} = \eta(0)$. Again by Proposition 1, we know that $\eta(q * l) = \eta(0)$ or equivalently $q * l \in H_\eta$.

Hence, the sets $H_{\widetilde{h}}$ and H_η are subalgebras of L over U. □

Proposition 5. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure in L over U where $\widetilde{h}$: $L \longrightarrow \mathbb{P}(U)$ and η: $L \longrightarrow I$ are mappings given by*

$$q \mapsto \begin{cases} \alpha_1, & \text{if } 0 * (0 * q) = q, \\ \alpha_2, & \text{otherwise,} \end{cases} \quad \text{such that } \alpha_1 \subset \alpha_2 \in \mathbb{P}(U)$$

$$q \mapsto \begin{cases} t_1, & \text{if } 0 * (0 * q) = q, \\ t_2, & \text{otherwise,} \end{cases} \quad \text{such that } t_1 < t_2 \in I, \tag{11}$$

for $q \in L$. Then, $\widetilde{h}_\eta$ is a hybrid subalgebra of L.

Proof. Let $q, l \in L$. If $0 * (0 * q) = q$ and $0 * (0 * l) = l$, then $\widetilde{h}(q) = \widetilde{h}(l) = \alpha_1, \eta(q) = \eta(l) = t_1$. Since, $(q * l) * (0 * 0 * (q * l))) = 0$ (using condition (3), property (iv), and property (ii)) and $(0 * 0 * (q * l)) * (q * l) = 0$ (using property (iii) and condition (1)). This implies that $(q * l) = (0 * 0 * (q * l))$, by condition (2). Thus, $\widetilde{h}(q * l) = \alpha_1 = \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) = t_1 = \vee\{\eta(q), \eta(l)\}$. If $0 * (0 * q) \neq q$ or $0 * (0 * l) \neq l$, then $\widetilde{h}(q) = \alpha_2$ or $\widetilde{h}(l) = \alpha_2$ and $\eta(q) = t_2$ or $\eta(l) = t_2$. Then, $\widetilde{h}(q * l) \supseteq \alpha_2 = \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) \leq t_2 = \vee\{\eta(q), \eta(l)\}$. Thus, $\widetilde{h}_\eta$ is a hybrid subalgebra. □

Proposition 6. *Let $\emptyset \neq Q \subseteq L$ and let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure of L over U where $\widetilde{h}: L \longrightarrow \mathbb{P}(U)$ and $\eta: L \longrightarrow I$ are mappings given by*

$$q \mapsto \begin{cases} \alpha_1, & \text{if } q * m = (0 * m) * (0 * q), \\ \alpha_2, & \text{otherwise,} \end{cases} \quad \text{such that } \alpha_1 \subset \alpha_2 \in \mathbb{P}(U),$$

$$q \mapsto \begin{cases} t_1, & \text{if } q * m = (0 * m) * (0 * q), \\ t_2, & \text{otherwise,} \end{cases} \quad \text{such that } t_1 < t_2 \in I, \tag{12}$$

for $q \in L$, $m \in Q$. Then, $\widetilde{h}_\eta$ is a hybrid subalgebra of L.

Proof. Let $q, l \in L$. If there exists $m \in Q$ such that $q * m = (0 * m) * (0 * q)$ and $l * m = (0 * m) * (0 * l)$, then $\widetilde{h}(q) = \widetilde{h}(l) = \alpha_1, \eta(q) = \eta(l) = t_1$. Using condition (3), property (ii), and property (iii), we have $(q * l) * m = (0 * m) * (0 * (q * l))$. If there exists $m \in Q$ such that either $q * m \neq (0 * m) * (0 * q)$ or $l * m \neq (0 * m) * (0 * l)$, then $\widetilde{h}(q) = \alpha_2$ or $\widetilde{h}(l) = \alpha_2$ and $\eta(q) = t_2$ or $\eta(l) = t_2$. It follows that $\widetilde{h}(q * l) \supseteq \alpha_2 = \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) \leq t_2 = \vee\{\eta(q), \eta(l)\}$. Therefore, $\widetilde{h}_\eta$ is a hybrid subalgebra of L. □

Proposition 7. *Let $\widetilde{g}_\mu = (\widetilde{g}; \mu)$ be a nonempty subset in L over U and $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure of L over U defined by*

$$\widetilde{h}(q) := \begin{cases} \alpha, & \text{if } q \in \widetilde{g}_\mu, \\ \beta, & \text{otherwise,} \end{cases}$$
$$\eta(q) := \begin{cases} \gamma, & \text{if } q \in \widetilde{g}_\mu, \\ \delta, & \text{otherwise,} \end{cases} \tag{13}$$

for all $\alpha, \beta \in \mathbb{P}(U)$ and $\gamma, \delta \in [0, 1]$. Then, $\widetilde{h}_\eta$ is a hybrid subalgebra of L over U if and only if $\widetilde{g}_\mu$ is a subalgebra of L over U. Moreover, $H_{\widetilde{h}} = \widetilde{g}_\mu = H_\eta$.

Proof. Let $\widetilde{h}_\eta$ be a hybrid subalgebra of a BCH-algebra L over U. Let $q, l \in L$ such that $q, l \in \widetilde{g}_\mu$. Then, we have $\widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l) = \alpha \cap \alpha = \alpha$ and $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\} = \vee\{\gamma, \gamma\}$. Hence, we have proved that $q * l \in \widetilde{g}_\mu$. Thus, $\widetilde{g}_\mu$ is indeed a subalgebra of L.

Conversely, suppose that $\widetilde{g}_\mu$ is a subalgebra of L. Let $q, l \in L$. Consider the following two cases:

Case (i): if $q, l \in \widetilde{g}_\mu$, then $q * l \in \widetilde{g}_\mu$. Thus, $\widetilde{h}(q * l) = \alpha = \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) = \gamma = \vee\{\eta(q), \eta(l)\}$.

Case (ii): if $q \notin \widetilde{g}_\mu$ or $l \notin \widetilde{g}_\mu$, then $\widetilde{h}(q * l) \supseteq \beta = \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) \leq \delta = \vee\{\eta(q), \eta(l)\}$.

Hence, $\widetilde{h}_\eta$ is a hybrid subalgebra of L.

Also, $H_{\widetilde{h}} = \{q \in L: \widetilde{h}(q) = \widetilde{h}(0)\} = \{q \in L: \widetilde{h}(q) = \alpha\} = \widetilde{g}_\mu$ and $H_\eta = \{q \in L: \eta(q) = \eta(0)\} = \{q \in L: \eta(q) = \gamma\} = \widetilde{g}_\mu$. □

Proposition 8. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in L over U. Then, the set $\Omega := \{q * l \in L | \widetilde{h}(q * l) \cap \alpha \neq \emptyset, \eta(q * l) \leq t\}$ is a subalgebra in L over U, for $\emptyset \neq \alpha \in \mathbb{P}(U)$, $t \in I$.*

Proof. Let $q, l \in L$ such that $q, l \in \Omega$. Thus, $\widetilde{h}(q) \cap \alpha \neq \emptyset$, $\widetilde{h}(l) \cap \alpha \neq \emptyset$ and $\eta(q) \leq t$, $\eta(l) \leq t$. It follows that $(\widetilde{h}(q) \cap \alpha) \cap (\widetilde{h}(l) \cap \alpha) = (\widetilde{h}(q) \cap \widetilde{h}(l)) \cap \alpha \subseteq \widetilde{h}(q * l) \cap \alpha \neq \emptyset$ and $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\} \leq t$. Thus, $q * l \in \Omega$ and so Ω is a subalgebra in L. □

Proposition 9. *Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure in L over U. Then, $\widetilde{h}_\eta$ is a hybrid subalgebra of L over U if and only if for $\alpha \in \mathbb{P}(U)$, $t \in I$, and the sets $\widetilde{h}_\eta(\alpha) := \{q * l \in L | \widetilde{h}(q * l) \supseteq \alpha\}$ and $\widetilde{h}_\eta(t) := \{q * l \in L | \eta(q * l) \leq t\}$ are subalgebras in L.*

Proof. "⇒". Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in a BCH-algebra L over U and consider the sets $\widetilde{h}_\eta(\alpha) := \{q \in L | \widetilde{h}(q) \supseteq \alpha\}$ and $\widetilde{h}_\eta(t) := \{q \in L | \eta(q) \leq t\}$. Now, let $q, l \in L$ such that $q, l \in \widetilde{h}_\eta(\alpha)$ and $q, l \in \widetilde{h}_\eta(t)$. Thus, $\widetilde{h}(q) \supseteq \alpha, \widetilde{h}(l) \supseteq \alpha$ and $\eta(q) \leq t, \eta(l) \leq t$. Then, from (3), we have $\widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l) \supseteq \alpha$ and $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\} \leq t$. That is, $\widetilde{h}(q * l) \supseteq \alpha$ and $\eta(q * l) \leq t$ and so $q * l \in \widetilde{h}_\eta(\alpha)$ and $q * l \in \widetilde{h}_\eta(t)$. Hence, $\widetilde{h}_\eta(\alpha)$ and $\widetilde{h}_\eta(t)$ are subalgebras in L.

"$\Leftarrow$". For $\alpha \in \mathbb{P}(U)$, $t \in I$, let $\widetilde{h}_\eta(\alpha) \coloneqq \{q * l \in L | \widetilde{h}(q * l) \supseteq \alpha\}$ and $\widetilde{h}_\eta(t) \coloneqq \{q * l \in L | \eta(q * l) \leq t\}$ be subalgebras in L. Let $q \in L$ such that $\widetilde{h}(q) = \alpha$ and $\eta(q) = t$. Suppose for contradiction that

$$\begin{aligned} &\widetilde{h}(q * l) \subset \widetilde{h}(q) \cap \widetilde{h}(l), \\ &\eta(q * l) > \vee\{\eta(q), \eta(l)\}. \end{aligned} \tag{14}$$

Take α and t such that

$$\begin{aligned} &\widetilde{h}(q * l) \subset \alpha \subset \widetilde{h}(q) \cap \widetilde{h}(l), \\ &\eta(q * l) > t > \vee\{\eta(q), \eta(l)\}. \end{aligned} \tag{15}$$

This means that both $\widetilde{h}_\eta(\alpha)$ and $\widetilde{h}_\eta(t)$ are not subalgebras which contradicts the assumption. Thus, $\widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l)$ and $\eta(q * l) \leq \vee\{\eta(q), \eta(l)\}$. Hence, $\widetilde{h}_\eta$ is a hybrid subalgebra of L. □

For any hybrid structure $\widetilde{h}_\eta$ in L over U, let $\widetilde{h}_\eta^* \coloneqq (\widetilde{h}^*; \eta^*)$ be a hybrid structure in L over U defined by

$$\begin{aligned} &\widetilde{h}^*: L \longrightarrow \mathbb{P}(U), q \mapsto \begin{cases} \widetilde{h}(q), & \text{if } q \in \widetilde{h}_\eta(\alpha), \\ \beta, & \text{otherwise}, \end{cases} \\ &\eta^*: L \longrightarrow I, q \mapsto \begin{cases} \eta(q), & \text{if } q \in \widetilde{h}_\eta(s), \\ t, & \text{otherwise}, \end{cases} \end{aligned} \tag{16}$$

where $\alpha, \beta \in \mathbb{P}(U)$ and $s, t \in I$ with $\beta \subsetneq \widetilde{h}(q)$ and $t > \eta(q)$.

Proposition 10. *Let L be a BCH-algebra. If $\widetilde{h}_\eta$ is a hybrid subalgebra in L over U, then so is $\widetilde{h}_\eta^*$.*

Proof. Assume that $\widetilde{h}_\eta$ is a hybrid subalgebra of a BCH-algebra L over U. Then, $\widetilde{h}_\eta(\alpha)$ and $\widetilde{h}_\eta(t)$ are subalgebras of L for all $\alpha \in \mathbb{P}(U)$ and $t \in I$ provided that they are nonempty by Proposition 9. Let $q, l \in L$. If $q, l \in \widetilde{h}_\eta(\alpha)$, then $q * l \in \widetilde{h}_\eta(\alpha)$. Thus,

$$\widetilde{h}^*(q * l) = \widetilde{h}(q * l) \supseteq \widetilde{h}(q) \cap \widetilde{h}(l) = \widetilde{h}^*(q) \cap \widetilde{h}^*(l). \tag{17}$$

If $q \notin \widetilde{h}_\eta(\alpha)$ or $l \notin \widetilde{h}_\eta(\alpha)$, then $\widetilde{h}^*(q) = \beta$ or $\widetilde{h}^*(l) = \beta$. Hence,

$$\widetilde{h}^*(q * l) \supseteq \beta = \widetilde{h}^*(q) \cap \widetilde{h}^*(l). \tag{18}$$

Now, if $q, l \in \widetilde{h}_\eta(s)$, then $q * l \in \widetilde{h}_\eta(s)$. Thus,

$$\eta^*(q * l) = \eta(q * l) \leq \vee\{\eta(q), \eta(l)\} = \vee\{\eta^*(q), \eta^*(l)\}. \tag{19}$$

If $q \notin \widetilde{h}_\eta(s)$ or $l \notin \widetilde{h}_\eta(s)$, then $\eta^*(q) = t$ or $\eta^*(l) = t$. Hence,

$$\eta^*(q * l) \leq t = \vee\{\eta^*(q), \eta^*(l)\}. \tag{20}$$

Therefore, $\widetilde{h}_\eta^*$ is a hybrid subalgebra of L over U.

Table 3: Cayley table of the binary operation $*$.

$*$	0	l_1	l_2	l_3
0	0	l_1	l_2	l_3
l_1	l_1	0	l_3	l_2
l_2	l_2	l_3	0	l_1
l_3	l_3	l_2	l_1	0

The converse of Proposition 10 may not true in general.

Example 2. Let $L = \{0, l_1, l_2, l_3\}$ be a *BCH*-algebra with the Cayley table (Table 3) and $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ be an initial universe set.

Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure in L over U which is given in Table 4.Let $\widetilde{h}_\eta(\alpha) = \widetilde{h}_\eta(s) = \{0, l_1\}$, where $\alpha = \{u_1, u_3, u_5, u_7, u_9\}$ and $s = 0.6$. Define the hybrid structure $\widetilde{h}_\eta^* \coloneqq (\widetilde{h}^*; \eta^*)$ by Table 5.

It can be easily verified that $\widetilde{h}_\eta^*$ is a hybrid subalgebra of L. Moreover, $\widetilde{h}_\eta$ is not hybrid subalgebra of L as $\widetilde{h}(l_1 * l_2) = \widetilde{h}(l_3) = \{u_7\} \supseteq \{u_5\} = \widetilde{h}(l_1) \cap \widetilde{h}(l_2) = \{u_1, u_3, u_5, u_7, u_9\} \cap \{u_2, u_5, u_8\}$.

Proposition 11. *If $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ and $\widetilde{g}_\mu = (\widetilde{g}; \mu)$ are two hybrid subalgebras in L over U, then the hybrid intersection $\widetilde{h}_\eta \Cap \widetilde{g}_\mu$ is also a hybrid subalgebra of L.*

Proof. Let $q, l \in L$. Then,

$$\begin{aligned} (\widetilde{h} \tilde{\cap} \widetilde{g})(q * l) &= \widetilde{h}(q * l) \cap \widetilde{g}(q * l) \\ &\supseteq (\widetilde{h}(q) \cap \widetilde{h}(l)) \cap (\widetilde{g}(q) \cap \widetilde{g}(l)) \\ &= (\widetilde{h}(q) \cap \widetilde{g}(q)) \cap (\widetilde{h}(l) \cap \widetilde{g}(l)) \\ &= (\widetilde{h} \tilde{\cap} \widetilde{g})(q) \cap (\widetilde{h} \tilde{\cap} \widetilde{g})(l), \\ (\eta \vee \mu)(q * l) &= \vee\{\eta(q * l), \mu(q * l)\} \\ &\leq \vee\{\vee\{\eta(q), \eta(l)\}, \vee\{\mu(q), \mu(l)\}\} \\ &= \vee\{\vee\{\eta(q), \mu(q)\}, \vee\{\eta(q), \mu(l)\}\} \\ &= \vee\{(\eta \vee \mu)(q), (\eta \vee \mu)(l)\}. \end{aligned} \tag{21}$$

Consequently, $\widetilde{h}_\eta \Cap \widetilde{g}_\mu$ is a hybrid subalgebra of L. □

Definition 5. Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid structure of a BCH-algebra L over U. Then, the "power-m" operation on a hybrid structure of a BCH-algebra L over U is defined as follows:

$$\widetilde{h}_\eta^m \coloneqq \left(\widetilde{h}^m; \eta^m\right), \tag{22}$$

where m is any nonnegative integer.

Proposition 12. *If $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ is a hybrid subalgebra in L over U, then $\widetilde{h}_\eta^m$ is a hybrid subalgebra in L over U.*

Proof. Let $\widetilde{h}_\eta = (\widetilde{h}; \eta)$ be a hybrid subalgebra in a BCH-algebra L over U and let $q \in L$. Then,

Table 4: Table representation of the hybrid structure $\tilde{h}_\eta$.

L	$\tilde{h}$	η
0	U	0.5
l_1	$\{u_1, u_3, u_5, u_7, u_9\}$	0.6
l_2	$\{u_2, u_5, u_8\}$	0.8
l_3	$\{u_7\}$	0.8

Table 5: Table representation of the hybrid structure $\tilde{h}^*_\eta$.

L	$\tilde{h}^*$	η^*
0	U	0.5
l_1	$\{u_1, u_3, u_5, u_7, u_9\}$	0.6
l_2	$\varnothing$	0.9
l_3	$\varnothing$	0.9

$$\begin{aligned}
\tilde{h}^m(0) &= [\tilde{h}(0)]^m \\
&= [\tilde{h}(q*q)]^m \\
&\supseteq [\tilde{h}(q) \cap \tilde{h}(q)]^m \\
&= \tilde{h}(q)^m \cap \tilde{h}(q)^m \\
&= \tilde{h}^m(q) \cap \tilde{h}^m(q) \\
&= \tilde{h}^m(q), \\
\eta^m(0) &= [\eta(0)]^m \\
&= [\eta(q*q)]^m \\
&\leq \vee\{\eta(q), \eta(q)\}^m \\
&= \vee\{\eta(q)^m, \eta(q)^m\} \\
&= \vee\{\eta^m(q), \eta^m(q)\} \\
&= \eta^m(q).
\end{aligned} \tag{23}$$

Let $q, l \in L$. Then,

$$\begin{aligned}
\tilde{h}^m(q*l) &= [\tilde{h}(q*l)]^m \\
&\supseteq \{\tilde{h}(q) \cap \tilde{h}(l)\}^m \\
&= \tilde{h}(q)^m \cap \tilde{h}(l)^m \\
&= \tilde{h}^m(q) \cap \tilde{h}^m(l) \\
\eta^m(q*l) &= [\eta(q*l)]^m \\
&\leq \vee\{\eta(q), \eta(l)\}^m \\
&= \vee\{\eta(q)^m, \eta(l)^m\} \\
&= \vee\{\eta^m(q), \eta^m(l)\}.
\end{aligned} \tag{24}$$

Hence, $\tilde{h}^m_\eta$ is a hybrid subalgebra of L. □

Definition 6. Let $\tilde{h}_\eta = (\tilde{h}; \eta)$ be a hybrid structure of a BCH-algebra L over U. Then, the "ρ-multiply" operation on a hybrid structure of a *BCH*-algebra L over U is defined as

$$\rho\tilde{h}_\eta \coloneqq (\rho\tilde{h}, \rho\eta), \tag{25}$$

where ρ is any nonnegative integer.

Proposition 13. *If $\tilde{h}_\eta = (\tilde{h}; \eta)$ is a hybrid subalgebra in L over U, then $\rho\tilde{h}_\eta$ is a hybrid subalgebra of L over U.*

Proof. Let $\tilde{h}_\eta = (\tilde{h}; \eta)$ be a hybrid subalgebra of a BCH-algebra L over U and let ρ be any nonnegative integer. Then, for any $q \in L$, we have

$$\begin{aligned}
\rho\tilde{h}(0) &= \rho\tilde{h}(q*q) \\
&\supseteq \rho[\tilde{h}(q) \cap \tilde{h}(q)] \\
&= \rho\tilde{h}(q) \cap \rho\tilde{h}(q) \\
&= \rho\tilde{h}(q), \\
\rho\eta(0) &= \rho\eta(q*q) \\
&\leq \rho\vee\{\eta(q), \eta(q)\} \\
&= \vee\{\rho\eta(q), \rho\eta(q)\} \\
&= \rho\eta(q).
\end{aligned} \tag{26}$$

Let $q, l \in L$. Then,

$$\begin{aligned}
\rho\tilde{h}(q*l) &\supseteq \rho[\tilde{h}(q) \cap \tilde{h}(l)] \\
&= \rho\tilde{h}(q) \cap \rho\tilde{h}(l), \\
\rho\eta(q*l) &\leq \rho\vee\{\eta(q), \eta(l)\} \\
&= \vee\{\rho\eta(q), \rho\eta(l)\}.
\end{aligned} \tag{27}$$

Hence, $\rho\tilde{h}_\eta$ is a hybrid subalgebra of L over U. □

Let φ be a mapping from the set L into the set Q. Let $\tilde{g}_\mu$ be a hybrid structure of a BCH-algebra L over U. Then, the preimage of $\tilde{g}_\mu$ is defined as $\varphi^{-1}(\tilde{g}_\mu) = (\varphi^{-1}(\tilde{g}), \varphi^{-1}(\mu))$ in L with the membership function and nonmembership function given by $\varphi^{-1}(\tilde{g})(q) = \tilde{g}(\varphi(q))$ and $\varphi^{-1}(\mu)(q) = \mu(\varphi(q))$. It can be shown that $\varphi^{-1}(\tilde{g}_\mu)$ is a hybrid structure of a BCH-algebra L over U.

Definition 7. A mapping $\varphi\colon L \longrightarrow Q$ is called a homomorphism of a BCH-algebra if $\varphi(q*l) = \varphi(q)*\varphi(l)$, for all $q, l \in L$. Note that if $\varphi\colon L \longrightarrow Q$ is a homomorphism of a *BCH*-algebra, then $\varphi(0) = 0$.

Proposition 14. *Let $\varphi\colon L \longrightarrow Q$ be a homomorphism of BCH-algebras. If $\tilde{g}_\mu = (\tilde{g}, \mu)$ is a hybrid subalgebra of a BCH-algebra Q over U, then the preimage $\varphi^{-1}(\tilde{g}_\mu) = (\varphi^{-1}(\tilde{g}), \varphi^{-1}(\mu))$ of $\tilde{g}_\mu$ under φ is a hybrid subalgebra of a BCH-algebra L over U.*

Proof. Assume that $\widetilde{g}_{\mu} = (\widetilde{g}; \mu)$ is a hybrid subalgebra of a BCH-algebra Q over U and let $q, l \in L$. Then,

$$\begin{aligned}
\varphi^{-1}(\widetilde{g})(q * l) &= \widetilde{g}(\varphi(q * l)) \\
&= \widetilde{g}(\varphi(q) * \varphi(l)) \\
&\supseteq \widetilde{g}(\varphi(q)) \cap \widetilde{g}(\varphi(l)) \\
&= \varphi^{-1}(\widetilde{g})(q) \cap \varphi^{-1}(\widetilde{g})(l), \\
\varphi^{-1}(\mu)(q * l) &= \mu(\varphi(q * l)) \\
&= \mu(\varphi(q) * \varphi(l)) \\
&\leq \vee\{\mu(\varphi(q)), \mu(\varphi(l))\} \\
&= \vee\left\{\varphi^{-1}(\mu)(q), \varphi^{-1}(\mu)(l)\right\}.
\end{aligned} \tag{28}$$

Therefore, $\varphi^{-1}(\widetilde{g}_{\mu}) = (\varphi^{-1}(\widetilde{g}), \varphi^{-1}(\mu))$ is a hybrid subalgebra of L. □

4. Conclusion

The present work is devoted to the study of hybrid subalgebras of BCH-algebras introduced, and related properties are investigated. Furthermore, some characterizations of hybrid subalgebras of BCH-algebras are given. Also, we stated and proved some theorems in hybrid subalgebras of BCH-algebras. Finally, the homomorphic images and inverse images of fuzzy BCH-subalgebras are studied and discussed. To extend these results, one can further study these notions on different algebras such as rings, hemirings, BL-algebras, MTL-algebras, R0-algebras, MV-algebras, EQ-algebras, d-algebras, Q-algebras, and lattice implication algebras. Some important issues for future work are as follows: (1) to develop strategies for obtaining more valuable results and (2) to apply these notions and results for studying related notions in other algebraic (hybrid) structures.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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