Hindawi
Security and Communication Networks
Volume 2023, Article ID 9783713, 1 page
https://doi.org/10.1155/2023/9783713

Retraction

Retracted: Traffic Equilibrium Problems with Cross-Boundary Traffic: A Tradable Credit Approach

Security and Communication Networks

Received 26 December 2023; Accepted 26 December 2023; Published 29 December 2023

This article has been retracted by Hindawi, as publisher, following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of systematic manipulation of the publication and peer-review process. We cannot, therefore, vouch for the reliability or integrity of this article.

Please note that this notice is intended solely to alert readers that the peer-review process of this article has been compromised.

Wiley and Hindawi regret that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Q. Liang, A. Sumalee, and R. Zhong, "Traffic Equilibrium Problems with Cross-Boundary Traffic: A Tradable Credit Approach," *Security and Communication Networks*, vol. 2022, Article ID 2431019, 12 pages, 2022.

Hindawi
Security and Communication Networks
Volume 2022, Article ID 2431019, 12 pages
https://doi.org/10.1155/2022/2431019

Research Article

Traffic Equilibrium Problems with Cross-Boundary Traffic: A Tradable Credit Approach

Qingnan Liang[1], **Agachai Sumalee**[2] **and Renxin Zhong**[1]

[1]*School of Intelligent Systems Engineering, Sun Yat-Sen University, Guangzhou, China*
[2]*School of Integrated Innovation, Chulalongkorn University, Bangkok, Thailand*

Correspondence should be addressed to Renxin Zhong; zhrenxin@mail.sysu.edu.cn

Received 1 May 2022; Accepted 18 May 2022; Published 14 June 2022

Academic Editor: Hangjun Che

This study studies the tradable credit scheme design problem considering a mixture of local traffic and cross-boundary traffic. The local traffic refers to the travel demand generated by local residents with O-D pairs inside the network, while the cross-boundary traffic is the traffic with either origin or destination or both be outside the network. As the local authority aims to maximize its local social welfare, it determines the quantity of cross-boundary trips by evaluating the revenue of the cross-boundary traffic. Two credit charging schemes are investigated, i.e., a spatially differentiated credit scheme and an anonymous tradable credit scheme. In the first scheme, due to the different charging prices and the selfishness of the local authority, the travel credits are freely tradable within the local travellers only. The cross-boundary travellers have to buy travel credits from the local authority. In the second scheme, the tradable credit scheme is anonymous. The local authority determines the link-specific number of credits to be charged for using that link, while the travel credits are distributed to local travellers only but are allowed for free trading among both local and cross-boundary travellers. Two standard multi-class traffic equilibrium problems are established with side constraints and a credit restriction constraint. The equilibrium link flow patterns under these credit schemes are then demonstrated with local elastic demand. In both tradable credit schemes, the credit price in the trading market is unique under the equilibrium condition.

1. Introduction

Traffic congestion has caused severe negative effects on productivity, environment, and sustainability of almost all large cities worldwide. Vehicles contribute a major part of the air pollutants in the urban areas, which are apparently higher under congested conditions than under free-flowing traffic conditions. Several traffic management schemes have been proposed to alleviate traffic congestion and emissions, including the market-based approaches (e.g., road pricing and tradable travel permit/credit scheme) and traffic flow control approaches (e.g., signal control and ramp metering). The market-based approaches, as contributed mainly by economists, are demonstrated to be efficient for traffic demand management (see, e.g., Yang et al. [1]; Lindsey [2]; Yang et al. [3]; Zhong et al. [4]; and the references therein). On the other hand, the signal control and ramp metering schemes are efficient for regulating the supply side of a transportation system.

The literature also argues that the traffic congestion and the inherent environmental pollution induced by vehicles in congested urban area should be regarded as external costs that control mechanisms can internalize. The market-based approaches advocated by some engineers and economists, including road pricing [5–7] and tradable travel (or pollution) permits/credits [3, 8, 9], are demonstrated to be efficient in reducing harmful traffic congestion and emissions. Also, the literature in road pricing investigation showed that road pricing methods could internalize the externalities such as traffic congestion, environmental pollution, and noise pollution [10–12]. Nevertheless, though road pricing schemes have been extensively studied by researchers, some fundamental drawbacks exist. From a theoretical point of view, perfect information on both the demand and supply sides of traffic networks is needed for implementation [5, 9]. Besides, though some advances are achieved in electronic tolling collection technology, the pricing scheme is still inequitable and costly [3, 5]. The general political and public resistance to congestion charges is another important reason to prevent road pricing from being practical. Because of this, planners and researchers have turned to quantity control to avoid the general resistance to road pricing. The quantity control schemes aim to restrict the number of private vehicles on the network by managing the travel demand of the traffic system and assigning mobility rights to individual travellers. Simple quantity control schemes include the temporary plate number-based traffic rationing and some long-term implementations of road space rationing, which are practically enforced in China and Latin America [13]. It is reported by Han et al. [13] and Wang et al. [14] that a short-term rationing policy can substantially reduce congestion and improve air quality. However, a long-term traffic rationing policy would implicitly promote undesirable second-car ownership for circumventing the restriction, which may decrease the effectiveness of the rationing policy over time.

The tradable network permit/credit schemes, which have been systematically introduced by Yang et al. [3], ensure the goal that the congestion on the network can be reduced by issuing a certain number of network travel credits and/or imposing capacity constraints on the bottlenecks. Yang et al. [3] further formulate the combined problem as some standard traffic assignment problems subject to a total credit consumption constraint (with/without side constraints). Researchers describe tradable permit/credit schemes as cap-and-trade schemes. In this scheme, the system manager first sets the cap (policy target) in quantity (total number of permits/credits) and distributes a certain number of access tickets (permits/credits) to all eligible users. After this, the users can access the competitive links (or bottlenecks) by paying a link-specific number of tickets (permits/credits) and trade the tickets in a competitive and efficient market. The price for these permits/credits is determined by the trade market. It is claimed by Yang et al. [3] that this kind of cap-and-trade scheme involves at least the same equity as a strict rationing policy. The serious political resistance to road pricing can be avoided as this tradable credit scheme involves no financial transfer from travellers to the government. Some other tradable pollution permit schemes are also investigated; see, e.g., Raux [15]; Raux and Marlot [16]; Perrels [17]; Wadud et al. [18]; and Wadud [19]. Though these categories of traffic management and control schemes are investigated independently, the traffic authority would implement a hybrid of them simultaneously in practice, e.g., road pricing and traffic flow regulation. Thus, evaluating the performance of hybrid control schemes is important for transportation management [20, 21]. In particular, we would investigate the hybrid combination of road pricing and quantity control methods.

In the previous studies of congestion pricing [11, 22, 23] and travel permit/credit [3, 24, 25], the network is assumed to be managed by a central authority that aims to improve the efficiency of the whole transportation network. In this case, the transportation network is regarded as a closed system with boundaries, which may be unrealistic in practice as there would be cross-boundary traffic (also known as through traffic) crossing the local network [26–28]. Generally, there are multiple administrative regions at a federal/state level, with each local region authority aiming to maximize the social welfare of its own region when designing management strategies. For example, special administrative cities such as Macao and Hong Kong that have well-defined geographical limits would maximize the social welfare of their local region. On the other hand, cross-boundary traffic is needed for the local authority to satisfy the requirements of economic activities such as tourism and logistics. In this study, we consider the problem that a local authority intends to maximize the social welfare of the local region by tradable credit schemes considering the cross-boundary traffic. The problem considered in this study is similar to the multi-criteria mixed equilibrium problem for multi-class traffic on networks and the subsequent anonymous link toll design for system optimum [8, 29, 30].

The remainder of this study is organized as follows. In Section 2, we first discuss the system optimum problem of the local traffic network by link-specific optimal tolls, wherein the quantity of the cross-boundary traffic is controlled by the local authority. We then design tradable credit schemes to realize specific desirable network flow patterns (such as local system optimum). To be specific, two credit charging schemes are investigated in Section 3, i.e., a spatially differentiated credit scheme and an anonymous tradable credit scheme. In the first scheme, due to the different charging prices and the selfishness of the local authority, the travel credit is freely tradable within the local travellers only. The cross-boundary travellers have to buy travel credits from the local authority. In the second scheme, the tradable credit scheme is anonymous. The local authority determines the link-specific number of credits to be charged for using that link, while the travel credits are distributed to local travellers only but are allowed for free trading among both local and cross-boundary travellers. The uniqueness of the market price of the travel credits is shown under certain assumptions. Numerical experiments are conducted in Section 4 to demonstrate the theoretical development. Conclusions are then discussed.

2. Local System Optimum Problem Formulation and Optimal Tolling Solution

A strongly connected directed network $G(\mathcal{N}, \mathcal{A})$ is used to describe the urban transportation system, where $\mathcal{N}$ and $\mathcal{A}$ denote the sets of nodes and links, respectively. Let W be the set of OD pairs for local travel demand and P_w be the set of noncyclic paths connecting a specific OD pair $w \in W$. We use $P = \cup_w P_w$ to denote the set of all paths on the network. Let f_p be the flow on path $p \in P$ between OD pair $w \in W$ and v_a be the flow on link $a \in \mathcal{A}$. According to the network topology, $\delta_a^p = 1$ if route p traverses link a and 0 otherwise. For the cross-boundary traffic, we assume the boundary nodes are the origins (if the destinations are inside the local network) or destinations (if the destinations are outside the local network) or both origins and destinations (if the origins and destinations are outside the local network). The cross-boundary traffic activates or ends the local trips on these boundary nodes. For a link a, its link traffic volume v_a is contributed by both the local and cross-boundary traffic flows:

$$v_a = v_a^1 + v_a^2, \tag{1}$$

where v_a^1 and v_a^2 denote the link volume contributed by the local traffic and cross-boundary traffic, respectively. By identifying the boundary nodes as origins or destinations of the cross-boundary trips, we can further define the OD pairs for this type of traffic, which is denoted by $w' \in W'$, with W' as the set of OD pairs for the cross-boundary traffic. By defining the boundary nodes as origins and/or destinations of the cross-boundary trips, the movements of the cross-boundary traffic can be categorized in a subgraph $G'(\mathcal{N}', \mathcal{A}')$ of $G(\mathcal{N}, \mathcal{A})$, wherein the nodes $\mathcal{N}'$ and links $\mathcal{A}'$ used by cross-boundary traffic are subsets of $\mathcal{N}$ and $\mathcal{A}$, respectively. The set of OD pairs is given as $W \cup W'$. The set of paths can be defined as $(\cup_w P_w) \cup (\cup_{w'} P_{w'})$. We denote $t_a(v_a)$ as the nonnegative average cost (in terms of travel time) of link $a \in \mathcal{A}$, which is assumed to be separable and twice continuously differentiable in link flows. The travel time functions $t_a(v_a), a \in \mathcal{A}$ are assumed to be strictly increasing $(dt_a(v_a)/dv_a > 0)$ and convex $(d^2 t_a(v_a)/d(v_a)^2 \geq 0)$. Let $B_w(d)$ be the inverse demand function of the local traffic, which is assumed to be nonnegative and monotonically decreasing. Let $f_{w'}(d^e_{w'})$ be the revenue from a unit trip of the cross-boundary traffic on OD pair $w\prime$ as perceived by the local authority, which is assumed to be monotonically decreasing. The objective function of the local authority is to maximize the social welfare of the local region, which is defined as follows:

$$\begin{aligned}\mathrm{Max}SW = &\sum_{w \in W} \int_0^{d_w} B_w(s)ds + \sum_{w' \in W'} \int_0^{d^e_{w'}} f_{w'}(s)ds \\ &- \sum_{a \in \mathcal{A}} t_a(v_a)v_a^2 - \sum_{a \in \mathcal{A}} t_a(v_a)v_a^1,\end{aligned} \tag{2}$$

subject to

$$\begin{aligned} v_a &= v_a^1 + v_a^2, \forall a \in \mathcal{A}, (\beta_a), \\ v^1 &= \sum_{w \in W} v^{w,1}, (\alpha), \\ v^2 &= \sum_{w' \in W'} v^{w',2}, (\vartheta), \\ Av^{w,1} &= E_w d_w, w \in W, (\rho^w), \end{aligned} \tag{3}$$

$$A^e v^{w\prime,2} = E^e_{w\prime} d^e_{w\prime}, w\prime \in W', \left(\varrho^{w'}\right), \tag{4}$$

$$v_a^2 \leq C_a^e, \forall a \in \mathcal{A}, (\tau_a), \tag{5}$$

$$-v^{w,1} \leq 0, w \in W, (\gamma^w), \tag{6}$$

$$-v^{w\prime,2} \leq 0, w \in W, \left(\lambda^{w\prime}\right), \tag{7}$$

$$-d \leq 0, (\mu), \tag{8}$$

$$-d^e \leq 0, (\eta), \tag{9}$$

where $v^1 = (v_a^1 \colon \forall a \in \mathcal{A})$, $v^2 = (v_a^2 \colon \forall a \in \mathcal{A})$, $d = (d_w \colon \forall w \in W)$, and $d^e = (d_{w'} \colon \forall w' \in W')$. $v^{w,1}$ is the vector of link flows contributed by local traffic for OD pair $w \in W$. $v^{w,2}$ is the vector of link flows contributed by cross-boundary traffic for OD pair $w' \in W'$. (3) and (4) are flow conservation constraints for the local traffic and cross-boundary traffic with respect to certain OD pairs, respectively. The matrix A is the node-link incidence matrix of G. The column incidence vector $E_w = e_p - e_q$ is used to indicate the OD pairs in W, with e_p and e_q be unit vectors. Similarly, the matrix A^e and $E^e_{w'}$ can be defined for the cross-boundary traffic. Formula (5) is the link volume restriction for the cross-boundary traffic, which may also refer to an acceptable network condition for local traffic. Formulae (6)–(9) are the standard nonnegative flow constraints. The variables in brackets are the Lagrange multipliers associated with the corresponding constraints.

As we have explained, the local authority concerns its local welfare only, so the quantity of the cross-boundary traffic is determined by the authority rather than the actual demand for the cross-boundary traffic. In other words, the local authority decides to admit how many cross-boundary trips according to the revenue function they perceived for each unit cross-boundary trip. Under this setting, the actual demand for cross-boundary traffic is implicitly assumed to be not less than the supply of the local authority. On the other hand, the existence of cross-boundary traffic does affect the link travel times of the network and the local marginal benefit function (demand function). The problem is a traffic assignment problem that the local authority aims to pursue system optimum with a side constraint on the cross-boundary traffic. The side constrained traffic assignment approach is commonly used to obtain suboptimal tolls (e.g., Yang et al. [11]; Zhong et al. [4]; and the references therein). We write the augmented Lagrangian function as follows:

$$\begin{aligned} L = &\sum_{a \in \mathcal{A}} t_a(v_a)v_a^1 + \sum_{a \in \mathcal{A}} t_a(v_a)v_a^2 - \sum_{w \in W} \int_0^{d_w} B_w(s)ds \\ &- \sum_{w' \in W'} \int_0^{d^e_{w'}} f_{w'}(s)ds - \sum_{a \in \mathcal{A}} \beta_a\left(v_a - v_a^1 - v_a^2\right) \\ &- \alpha^T\left(v^1 - \sum_{w \in W} v^{w,1}\right) - \vartheta^T\left(v^2 - \sum_{w' \in W'} v^{w',2}\right) \\ &+ \sum_{w \in W} (\rho^w)^T\left(Av^{w,1} - E_w d_w\right) \\ &+ \sum_{w' \in W'} \left(\varrho^{w'}\right)^T\left(A^e v^{w',2} - E^e_{w'} d^e_{w'}\right) + \sum_{a \in \mathcal{A}} \tau_a\left(v_a^2 - C_a^e\right) \\ &- \sum_{w \in W} (\gamma^w)^T v^{w,1} - \sum_{w' \in W'} \left(\lambda^{w'}\right)^T v^{w',2} - \mu^T d - \eta^T d^e. \end{aligned} \tag{10}$$

The following first-order optimality conditions can be derived by evaluating the derivative of the Lagrangian:

$$\begin{aligned}
\frac{\partial L}{\partial v_a} &= v_a^{1*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} + v_a^{2*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} - \beta_a = 0, \forall a \in \mathscr{A},\\
\frac{\partial L}{\partial v_a^1} &= t_a\left(v_a^*\right) + \beta_a - \alpha_a = 0, \forall a \in \mathscr{A},\\
\frac{\partial L}{\partial v_a^2} &= t_a\left(v_a^*\right) + \tau_a + \beta_a - \vartheta_a = 0, \forall a \in \mathscr{A},\\
\frac{\partial L}{\partial v_a^{w,1}} &= \alpha_a + \left(\rho_i^w - \rho_j^w\right) - \gamma_a^w = 0, \forall w \in W, \forall a = (i,j) \in \mathscr{A},\\
\frac{\partial L}{\partial v_a^{w',2}} &= \vartheta_a - \lambda_a^{w'} + \left(\varrho_k^{w'} - \varrho_l^{w'}\right) = 0, \forall w' \in W', \forall a = (k,l) \in \mathscr{A},\\
\frac{\partial L}{\partial d_w} &= -B_w\left(d_w^*\right) + \left(\rho_q^w - \rho_p^w\right) - \mu_w = 0, \forall w = (p,q) \in W,\\
\frac{\partial L}{\partial d_{w'}^e} &= -f_{w'}\left(d_{w'}^{e*}\right) + \left(\varrho_n^{w'} - \varrho_m^{w'}\right) - \eta_{w'} = 0, \forall w' = (m,n) \in W',
\end{aligned} \tag{11}$$

and a set of slackness conditions:

$$\tau_a\left(v_a^{2*} - C_a^e\right) = 0, \tau_a > 0, v_a^{2*} - C_a^e \le 0, \forall a \in \mathscr{A}, \tag{12}$$

$$\left(\gamma^w\right)^T v^{w,1*} = 0, \gamma^w \ge 0, v^{w,1*} \ge 0, \forall w \in W, \tag{13}$$

$$\left(\lambda^{w'}\right)^T v^{w,2*} = 0, \lambda^{w'} \ge 0, v^{w,2*} \ge 0, \forall w' \in W', \tag{14}$$

$$\mu^T d^* = 0, \mu \ge 0, d^* \ge 0, \tag{15}$$

$$\eta^T d^{e*} = 0, \eta \ge 0, d^{e*} \ge 0. \tag{16}$$

To further explore the first-order optimality, we aggregate the stationary and complementary conditions as follows:

$$\begin{aligned}
\sum_{a\in\mathscr{A}} \frac{\partial L}{\partial v_a} v_a^* &= \sum_{a\in\mathscr{A}} \left(v_a^{1*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} + v_a^{2*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} - \beta_a \right) v_a^*\\
&= \sum_{a\in\mathscr{A}} \left(v_a^{1*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} \right) v_a^* + \sum_{a\in\mathscr{A}} \left(v_a^{2*}\frac{\partial t_a\left(v_a^*\right)}{\partial v_a} \right) v_a^* - \sum_{a\in\mathscr{A}} \beta_a \left(v_a^{1*} + v_a^{2*} \right) = 0,\\
\sum_{a\in\mathscr{A}} \beta_a v_a^{1*} &= -\sum_{a\in\mathscr{A}} t_a\left(v_a^*\right) v_a^{1*} + \sum_{a\in\mathscr{A}} \alpha_a v_a^{1*},\\
\sum_{a\in\mathscr{A}} \beta_a v_a^{2*} &= -\sum_{a\in\mathscr{A}} \left(t_a\left(v_a^*\right) + \tau_a \right) v_a^{2*} + \sum_{a\in\mathscr{A}} \vartheta_a v_a^{2*},\\
\sum_{w\in W} \sum_{a\in\mathscr{A}} \alpha_a v_a^{w,1*} &= -\sum_{w\in W} \sum_{a\in\mathscr{A}} \left(\rho_i^w - \rho_j^w \right) v_a^{w,1*}\\
&= \sum_{w\in W} \left(v^{w,1*} \right)^T A^T \rho^w,\\
\sum_{w'\in W'} \sum_{a\in\mathscr{A}} \vartheta_a v_a^{w,2*} &= -\sum_{w'\in W'} \sum_{a\in\mathscr{A}} \left(\varrho_k^{w'} - \varrho_l^{w'} \right) v_a^{w,2*}\\
&= \sum_{w'\in W'} \left(v^{w,2*} \right)^T \left(A^e\right)^T \varrho^{w'},\\
\sum_{w\in W} \mu_w d_w^* &= -\sum_{w\in W} B_w\left(d_w^*\right) d_w^* + \sum_{w\in W} d_w^* E_w^T \rho^w\\
&= \sum_{w\in W} -B_w\left(d_w^*\right) d_w^* + \sum_{w\in W} \left(v^{w,1*} \right)^T A^T \rho^w = 0,\\
\sum_{w'\in W'} \eta_{w'} d_{w'}^{e*} &= -\sum_{w'\in W'} f_{w'}\left(d_{w'}^{e*}\right) d_{w'}^{e*} + \sum_{w'\in W'} \left(v^{w',2*} \right)^T \left(A^e\right)^T \varrho^{w'} = 0.
\end{aligned} \tag{17}$$

Summing up these equations, we obtain the following:

$$\sum_{a\in\mathcal{A}}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a}v_a^*\right)v_a^{1*}+\sum_{a\in\mathcal{A}}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a}v_a^*+\tau_a\right)v_a^{2*}=\sum_{w\in W}B_w(d_w^*)d_w^*+\sum_{w'\in W'}f_{w'}(d_{w'}^{e*})d_{w'}^{e*}. \tag{18}$$

Before summarizing the results in detail, let us look into some special cases.

Remark 1. If the side constraints (5) are inactive, according to the slackness conditions (12), $\tau=0$ can be guaranteed. In this case, one may write the equilibrium condition (18) according to different sets of OD pairs as follows:

$$\sum_{a\in\mathcal{A}}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a}v_a^*\right)v_a^{1*}=\sum_{w\in W}B_w(d_w^*)d_w^*, \tag{19}$$

$$\sum_{a\in\mathcal{A}}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a}v_a^*\right)v_a^{2*}=\sum_{w'\in W'}f_{w'}(d_{w'}^{e*})d_{w'}^{e*}. \tag{20}$$

The above two conditions are similar to those of multi-class mixed equilibrium conditions for Cournot–Nash (CN) players proposed by Yang and Zhang [29] and Han and Yuan [31], wherein a CN player aims to minimize the total travel time of the users under this specific player based on the routing strategies of other users. In other words, the player designs its flow patterns by taking the flow patterns of other players as fixed (or constant). The mixed traffic case is taken as an example, in which either the local traffic v^1 or cross-boundary traffic v^2 is a decision variable in the CN game. However, in our case, both of them are decision variables simultaneously.

The revenue function of the cross-boundary traffic is in a position closely analogous to the marginal benefit function of the local traffic. The amplitude of this function, i.e., revenue made from a unit external trip, reveals the marginal benefit contributed by the cross-boundary traffic, which serves as a basis for the local authority to determine how many trips should be permitted. From the equilibrium condition (18), we note that the system optimal toll for cross-boundary traffic is generally higher than that for local traffic, where the local traffic is charged by the marginal cost pricing, i.e., $\partial t_a(v_a^*)/\partial v_a v_a^*$. This is because the local authority tries to maximize its own social welfare (selfish planning to protect the interest of local residents) rather than that of both local traffic and cross-boundary traffic, which is similar to the infrastructure investment problem for a region that faces much cross-boundary traffic (or through traffic) [26, 27, 32]. The literature argues that the regional authority generally cares about the welfare of the local users only when making decisions. It is unlikely for the regional authority to consider the utility of the infrastructure for external users. Thus, our model does not include the marginal benefit (demand function) of the cross-boundary traffic. On the other hand, a local authority has limited incentives to invest if it cannot make a sufficiently large revenue from the cross-boundary [32]. The regional authority also has an incentive to raise the user charge above the marginal cost for external users or the so-called tax exporting behaviour [27, 32], which, in our case, is τ.

Remark 2. The cross-boundary trips can contribute to the local economy, e.g., tourism and logistics. The incentive of the local authority to permit the cross-boundary users is to make a revenue that fully compensates the total marginal social cost, i.e., $\sum_{a\in\mathcal{A}}(t_a(v_a^*)+\partial t_a(v_a^*)/\partial v_a v_a^*)v_a^{2*}$, induced by the cross-boundary traffic to the local network. Given a quantity of cross-boundary trips $d_{w'}^{e*}$, the total revenue perceived by the local authority is $\sum_{w'\in W'}f_{w'}(d_{w'}^{e*})d_{w'}^{e*}$. Let us define the following amount:

$$\triangle^e=\sum_{w'\in W'}f_{w'}(d_{w'}^{e*})d_{w'}^{e*}-\sum_{a\in\mathcal{A}}\left(t_a(v_a^*)+\frac{\partial t_a(v_a^*)}{\partial v_a}v_a^*\right)v_a^{2*}. \tag{21}$$

If all the cross-boundary traffic flows on the network are without the side constraints (5), then according to (20), $\triangle^e=0$. That is, the local authority can compensate for the total marginal social cost induced by the cross-boundary traffic to make a traffic state as good as the case without cross-boundary trips [32]. The toll is designed in a revenue-neutral manner. However, as certain road capacity needs to be allocated for cross-boundary traffic, it seems to be unfair to the local residents. For instance, certain links with high revenue may be fully occupied by cross-boundary traffic in extreme cases. To provide an acceptable network condition for the local traffic, side constraints $(C_a^e, a\in\mathcal{A})$ are set for the cross-boundary traffic and additional tolls $(\tau_a, a\in\mathcal{A})$ are enforced to the external users. Under this case, according to (18), we have $\triangle^e\geq 0$. This amount of monopoly revenue can be viewed as compensation for the capacity loss of local travellers caused by the cross-boundary trips.

The argument in the above remark can be read as follows: since the local authority cares about its local social welfare only, it will not permit cross-boundary trips if the revenue function of the cross-boundary trips is not significant (large enough). On the other hand, even though the revenue function is large enough, the local authority would not encourage a large quantity of cross-boundary trips as it needs to protect the travel right of the local residents.

3. Two Tradable Credit Schemes

Having characterized the equilibrium condition with both local traffic and cross-boundary traffic, we investigate two credit charging schemes (local tradable credit scheme and freely tradable credit scheme) to achieve such desired traffic pattern in this section.

3.1. Local Tradable Credit Scheme. As the local authority cares about its own social welfare, free credits would be distributed to the local residents only. The local users are allowed to trade their credits freely in the market. However, due to the spatial differentiability on optimal charges, the external users need to buy credits from the local authority to travel on the local network. As revealed in (18), the external users are charged a link-specific credit according to the system optimal tolls and the additional tolls induced by the side constraints, i.e., $\partial t_a(v_a^*)/\partial v_a v_a^* + \tau_a, \forall a \in \mathscr{A}$. The local authority can then issue travel credits for these two groups of users separately and independently. Since only the local travel credits are tradable, we consider the tradable credit market and credit price for local traffic. In line with Yang et al. [3], we use κ_a^1 to denote the credit charge for local travellers using link $a \in \mathscr{A}$ and use $\kappa = \{\kappa_a^1, \forall a \in \mathscr{A}\}$ to denote the credit charge for the whole network. The notation (K^1, κ^1) is adopted to represent a credit charging scheme, where K^1 is the total number of credits issued for all users. Due to the credit scheme, we have the following constraint:

$$\sum_{a\in\mathscr{A}} \kappa_a^1 v_a^1 \leq K^1. \tag{22}$$

In this tradable credit scheme, if the authority sets the credit charges according to the system optimal toll scheme, i.e., $\kappa_a^{1*} = \partial t_a(v_a^*)/\partial v_a v_a^*, \forall a \in \mathscr{A}$, and an initial allocation of $K^{1*} = \sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*}$, then the optimality conditions can be restated as follows:

$$\begin{gathered}
\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + p^1\kappa_a^{1*}\right) v_a^{1*} + \sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \frac{\partial t_a(v_a^*)}{\partial v_a} v_a^* + \tau_a\right) v_a^{2*} = \sum_{w\in W} B_w(d_w^*) d_w^* + \sum_{w'\in W'} f_{w'}(d_{w'}^{e*}) d_{w'}^{e*}, \\
\left(K^{1*} - \sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*}\right) p^1 = 0, \\
K^{1*} - \sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*} \geq 0, \; p^1 \geq 0.
\end{gathered} \tag{23}$$

By referring to Remark 1 on the equilibrium condition of the local authority, for the tradable credit market of the local users, we have the following:

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + p^1\kappa_a^{1*}\right) \delta_a^p \geq B_w(d_w^*), \forall p \in P_w, w \in W, \tag{24}$$

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + p^1\kappa_a^{1*}\right) v_a^{1*} = \sum_{w\in W} B_w(d_w^*) d_w^*, \tag{25}$$

$$\begin{gathered}
\left(K^{1*} - \sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*}\right) p^1 = 0, \\
K^{1*} - \sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*} \geq 0, \; p^1 \geq 0.
\end{gathered} \tag{26}$$

For the fact that $\kappa_a^{1*} = \partial t_a(v_a^*)/\partial v_a v_a^*, \forall a \in \mathscr{A}$, combining (19) and (25), the market price for per unit credit is given as follows:

$$\begin{aligned}
p^{1*} &= \frac{\sum_{w\in W} B_w(d_w^*) d_w^* - \sum_{a\in\mathscr{A}} t_a(v_a^*) v_a^{1*}}{\sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*}} \\
&= \frac{\sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*}}{K^{1*}} = 1.
\end{aligned} \tag{27}$$

Therefore, the equilibrium market price remains equal to unity for any system optimal local credit scheme (K^1, κ^1). Besides, the total market value of all credits is constant at $p^{1*}K^{1*}$. This result is consistent with that of Yang et al. [3], which is due to the fact that the authority allows the local users to trade travel credits within the local market only. Given a credit charging scheme (K^1, κ^1), the market credit price p^{1*} and credit market value for local traffic are unique. We summarize this in the following proposition.

Proposition 1. *Assume that the local authority issue travel credits for the local and cross-boundary users separately and independently. Assume further that only local users are allowed to trade their credits freely in the market and the cross-boundary users are restricted to buy credits from the local authority with a link-specific credit charge according to $\partial t_a(v_a^*)/\partial v_a v_a^* + \tau_a$. Then, any tradable credit scheme (K^{1*}, κ_a^{1*}) contained in the following nonempty polyhedron can decentralize a given system optimal flow pattern (v^{1*}, d^*) for the local traffic:*

$$\begin{gathered}
\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \kappa_a^{1*}\right) \delta_a^p \geq B_w(d_w^*), \forall p \in P_w, w \in W, \\
\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \kappa_a^{1*}\right) v_a^{1*} = \sum_{w\in W} B_w(d_w^*) d_w^*, \\
\sum_{a\in\mathscr{A}} \kappa_a^{1*} v_a^{1*} = K^{1*}.
\end{gathered} \tag{28}$$

Remark 3. We did not enforce a market clearing condition for the cross-boundary traffic. There is no credit market for the cross-boundary traffic as the cross-boundary users have to buy credits from the local authority only. The mechanism for cross-boundary traffic is more like that of road pricing. The local authority's incentive for cross-boundary traffic to use its traffic network depends on the revenue it can receive from the cross-boundary trips, as explained in the previous section. The larger revenue the local authority can obtain from a unit cross-boundary trip, the larger quantity of cross-boundary trips is well received. Also, the local authority tends to provide an acceptable network condition for local traffic1, which is captured by the side constraints imposed on the cross-boundary traffic.

3.2. Freely Tradable Credit Scheme. Yang et al. [29] and Han et al. [31] proposed system optimum tolls for networks with multi-class multi-criteria mixed equilibrium behaviours. To internalize their full externality, each UE user is charged with a marginal cost toll equals $\partial t_a(v_a^*)/\partial v_a v_a^*$. In contrast, each user under a specific Cournot–Nash player who has already taken account of his partial externality is charged with a partial marginal cost toll equals $\partial t_a(v_a^*)/\partial v_a(v_a^* - v_a^{k*})$, wherein v_a^{k*} is the volume of link a contributed by vehicles of class k. The partial marginal cost toll can internalize the additional externality that the users under a Cournot–Nash player impose on UE users and users under other players. This renders different system optimum link tolls for users under different players, including the UE player. Furthermore, it was commented by Yang et al. [29] that as all users on networks behave indistinguishably, such link toll differentiation across user classes is hard to be implemented. Also, the nonuniqueness issue arises for the link tolls with different user classes. Nevertheless, observing that the aggregate system optimal link volumes are unique under certain assumptions, it is possible to pursue anonymous link tolls on networks with multi-class multi-criteria mixed equilibrium behaviours to achieve system optimum.

In the credit scheme described in the previous section, we assume only the local residents can trade their travel credits in the credit market. The cross-boundary users have to buy their travel credits from the local authority, which acts like an oligopoly. Now we assume that the cross-boundary traffic users can trade (both buy and sell) travel credits from the credit market. However, no credit is freely issued to them by the authority initially. In the case of road pricing, Yang et al. [29] commented that introducing discriminatory link toll patterns for multiple vehicle classes (in our case, the local traffic and cross-boundary traffic) is unrealistic for the authority. Besides, the credit market is anonymous in the sense that these two classes of vehicles are indistinguishable in a free credit market [3, 14, 25]. Under this situation, it is important for us to seek an anonymous tradable credit scheme that is identical to all users to achieve the system optimum condition. A simple scheme may be for the local authority charges a number of $\partial t_a(v_a^*)/\partial v_a v_a^*$ credits for users on link a, while all the road users can be viewed as anonymous individuals. Again, external users cannot receive free travel credits. It is assumed that all the travel credits are distributed to the local residents 2 (the local authority would not hold any credit), and the external users need to buy travel credits from the credit market to travel on the local traffic network. For this credit scheme, we have the following constraint:

$$\sum_{a\in\mathscr{A}} \kappa_a v_a \le K. \tag{29}$$

As all users can trade the travel credits in a free market, the local authority may not be able to enforce side constraints of the form (12) to control the quantity of cross-boundary traffic to be equal to or less than a prescribed threshold. Instead, the objective of the local authority is to pursue a predetermined system optimum link flow pattern (v^*: $v_a^*, \forall a \in \mathscr{A}$) by an anonymous optimal tolling scheme or a tradable credit scheme. Similar to Yang et al. [29] and Han et al. [31], we can obtain the optimal anonymous link toll pattern by evaluating the Karush–Kuhn–Tucker (KKT) conditions for the following nonlinear programming problem:

$$\begin{aligned}\text{Max } SW = &\sum_{w\in W}\int_0^{d_w} B_w(s)ds + \sum_{w'\in W'}\int_0^{d^e_{w'}} f_{w'}(s)ds\\ &- \sum_{a\in\mathscr{A}} t_a(v_a^*)\left(v_a^1 + v_a^2\right),\end{aligned} \tag{30}$$

subject to

$$v_a^1 + v_a^2 \le v_a^*, \left(\text{or } v_a^* = v_a^1 + v_a^2\right), (\beta_a), \forall a \in \mathscr{A}, \tag{31}$$

$$v^1 = \sum_{w\in W} v^{w,1}, (\alpha), \tag{32}$$

$$v^2 = \sum_{w'\in W'} v^{w',2}, (\vartheta), \tag{33}$$

$$Av^{w,1} = E_w d_w, w \in W, (\rho^w), \tag{34}$$

$$A^e v^{w',2} = E^e_{w'} d^e_{w'}, w' \in W', \left(\varrho^{w'}\right), \tag{35}$$

$$-v^{w,1} \le 0, w \in W, (\gamma^w), \tag{36}$$

$$-v^{w',2} \le 0, w \in W, \left(\lambda^{w'}\right), \tag{37}$$

$$-d \le 0, (\mu), \tag{38}$$

$$-d^e \le 0, (\eta). \tag{39}$$

As we mentioned previously, the design purpose of this program is to pursue the system optimum condition through an anonymous tolling scheme. That is to say, v_a^* and $t_a(v_a^*)$ are given. The augmented Lagrangian function $\mathscr{L}$ of the programs (30)–(39) can be defined similarly to (10). By evaluating the derivative of the Lagrangian with respect to the decision variables v^1, v^2, d, d^e, the first-order optimality conditions can be derived as follows:

$$\frac{\partial \mathscr{L}}{\partial v_a^1} = t_a(v_a^*) + \beta_a - \alpha_a = 0, \forall a \in \mathscr{A},$$
$$\frac{\partial \mathscr{L}}{\partial v_a^2} = t_a(v_a^*) + \beta_a - \vartheta_a = 0, \forall a \in \mathscr{A},$$
$$\frac{\partial \mathscr{L}}{\partial v_a^{w,1}} = \alpha_a + \left(\rho_i^w - \rho_j^w\right) - \gamma_a^w = 0, \forall w \in W, \forall a = (i,j) \in \mathscr{A},$$
$$\frac{\partial \mathscr{L}}{\partial v_a^{w',2}} = \vartheta_a - \lambda_a^{w'} + \left(\varrho_k^{w'} - \varrho_l^{w'}\right) = 0, \forall w' \in W', \forall a = (k,l) \in \mathscr{A},$$
$$\frac{\partial L}{\partial d_w} = -B_w(d_w^*) + \left(\rho_q^w - \rho_p^w\right) - \mu_w = 0, \forall w = (p,q) \in W,$$
$$\frac{\partial L}{\partial d_{w'}^e} = -f_{w'}(d_{w'}^{e*}) + \left(\varrho_n^{w'} - \varrho_m^{w'}\right) - \eta_{w'} = 0, \forall w' = (m,n) \in W', \tag{40}$$

and a set of slackness conditions similar to those defined in (13)–(16). By performing calculations similar to Section 2, we arrive at the following optimality condition:

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \beta_a\right)\left(v_a^{1*} + v_a^{2*}\right) = \sum_{w\in W} B_w(d_w^*)d_w^* + \sum_{w'\in W'} f_{w'}(d_{w'}^{e*})d_{w'}^{e*}. \tag{41}$$

Proposition 2. *Let $\beta_a, a \in \mathscr{A}$ be the Lagrange multipliers associated with the link flow constraint (31) for the programs (30)–(39), and then, $\beta_a, a \in \mathscr{A}$ are the anonymous link tolls.*

Similar to Yang et al. [3], to realize the system optimal traffic pattern by a tradable credit scheme, the authority can set the credit charges according to the anonymous link tolls, i.e., $\kappa_a^* = \beta_a, \forall a \in \mathscr{A}$, and allocate an initial number of credits equals $K^* = \sum_{a\in\mathscr{A}} \kappa_a^* v_a^*$. Then, the equilibrium condition can be restated as follows:

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + p\kappa_a^*\right)v_a^* = \sum_{w\in W} B_w(d_w^*)d_w^* + \sum_{w'\in W'} f_{w'}(d_{w'}^{e*})d_{w'}^{e*}, \tag{42}$$

$$\left(K^* - \sum_{a\in\mathscr{A}} \kappa_a^* v_a^*\right)p = 0,$$
$$K^* - \sum_{a\in\mathscr{A}} \kappa_a^* v_a^* \geq 0, p \geq 0. \tag{43}$$

Combining (41) and (42), the market price for per unit credit is given as follows:

$$p^* = \frac{\sum_{w\in W} B_w(d_w^*)d_w^* + \sum_{w'\in W'} f_{w'}(d_{w'}^{e*})d_{w'}^{e*} - \sum_{a\in\mathscr{A}} t_a(v_a^*)v_a^*}{\sum_{a\in\mathscr{A}} \kappa_a^* v_a^*} = 1. \tag{44}$$

Therefore, the equilibrium market price remains equal to unity for any system optimal credit scheme (K^*, κ^*). We summarize the result as follows.

Proposition 3. *Any tradable credit scheme (K^*, κ_a^*) contained in the following nonempty polyhedron can decentralize a given system optimal flow pattern (v^*, d^*, d^{e*}):*

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \kappa_a^*\right)\delta_a^p \geq B_w(d_w^*), \forall p \in P_w, w \in W, \tag{45}$$

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \kappa_a^*\right)\delta_a^p \geq f_{w'}(d_{w'}^{e*}), \forall p \in P_{w'}, w' \in W', \tag{46}$$

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \kappa_a^*\right)v_a^* = \sum_{w\in W} B_w(d_w^*)d_w^* + \sum_{w'\in W'} f_{w'}(d_{w'}^{e*})d_{w'}^{e*}, \tag{47}$$

$$\sum_{a\in\mathscr{A}} \kappa_a^* v_a^* = K^*. \tag{48}$$

The anonymous tradable credit scheme differs from the local tradable credit scheme in the following aspects:

(1) The objective of the local authority in the freely tradable credit scheme is to pursue a predetermined system optimum link flow pattern $(v^*: v_a^*, \forall a \in \mathscr{A})$ and to design credit charging schemes according to this link flow pattern. Two vehicle classes, v_a^{1*}, v_a^{2*}, $\forall a \in \mathscr{A}$, are anonymous to the local authority.

(2) Travel credits are distributed to local users only. However, any user willing to pay the anonymous link tolls or credit charges can travel on the network.

(3) The local authority does not have the power to control or affect this free credit market. The two groups of users can trade their travel credits according to their needs and favours.

Remark 4. We proceed with the analysis in Remark 2. Under the freely tradable credit scheme, $\triangle^e$ defined by (21) equals zero, as the equilibrium conditions (45)–(48) can be given as follows:

$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \frac{\partial t_a(v_a^*)}{\partial v_a} v_a^*\right) v_a^{1*} = \sum_{w\in W} B_w(d_w^*)d_w^*,$$
$$\sum_{a\in\mathscr{A}} \left(t_a(v_a^*) + \frac{\partial t_a(v_a^*)}{\partial v_a} v_a^*\right) v_a^{2*} = \sum_{w'\in W'} f_{w'}(d_{w'}^{e*})d_{w'}^{e*}. \tag{49}$$

It is clear that the local authority is designing tolls according to the elasticity of the local travel demand and the revenue made by the cross-boundary traffic. This is interesting and important as the local authority aims to maximize its local social welfare, implying that the elasticity of the cross-boundary traffic is not critical. Finally, we would like to mention that this result is consistent with the findings in the urban economics research; see, e.g., De Borger et al. [32]; Ubbels et al. [26]; and De Borger et al. [27]. It is proven in these papers that, for a single corridor with cross-boundary traffic, the regional authority can always stick to the policy that produces at least the same regional welfare as before (i.e., the case without cross-boundary traffic) as long as it can control the pricing and investment decisions. In our case, the local authority designs optimal tolls to compensate for the total marginal social cost induced by the cross-boundary traffic according to the revenue it contributes. On the other hand, according to programs (30)–(39) the local government can choose the set of local system optimal link volumes as to the target flow pattern and set the tolls or credit charges accordingly. Practically, the anonymous tradable credit scheme offers the policy with better political feasibility, as this tradable credit scheme involves no financial transfer from travellers to the government.

4. Numerical Experiments

4.1. Comparison of Two Credit Charging Schemes. We present a simple example to illustrate the proposed schemes. The network shown in Figure 1 is adopted from Yang et al. [29], which consists of 3 nodes and 4 links. The link cost functions are assumed to be linear and given as follows:

$$\begin{aligned} t_1(v_1) &= 6 + 2v_1, \\ t_2(v_2) &= 20 + v_2, \\ t_3(v_3) &= 8 + v_3, \\ t_4(v_4) &= v_4. \end{aligned} \tag{50}$$

For the local traffic on the network, there are two OD pairs ($1 \longrightarrow 3$ and $2 \longrightarrow 3$), whose demands are assumed to be linear and given by $d_{13} = 100 \times (20 - \pi_{13})$ and $d_{23} = 100 \times (20 - \pi_{23})$, where π_{pq} denotes the minimum OD travel cost. The inverse demand function is thus given by $B_{13} = 20 - 1/100 d_{13}$ and $B_{23} = 20 - 1/100 d_{23}$. We assume that nodes 1 and 3 are boundary nodes and node 2 is the center (or internal node) of the network. The cross-boundary vehicles can enter the network from node 1 and exit from node 3 by link 1 only. Therefore, the cross-boundary traffic has an OD pair $1 \longrightarrow 3$. We further assume that the revenue function of the cross-boundary traffic is given by $f_{13} = 20 - 1/150 d^e_{13}$. By solving the system optimal traffic assignment without cross-boundary traffic, the optimal link volume vector is given by $v^{so} = [3.4913\ 0\ 5.9208\ 9.9208]^T$.

Here, we test the anonymous credit scheme first, in which the local authority initially distributes credits to all eligible local travellers only and allows for free trading among both local and cross-boundary travellers. The optimal link pattern for local traffic is given by $v^{1,so} = [1.3986\ 0\ 5.9208\ 9.9208]^T$. Note that the other OD pairs

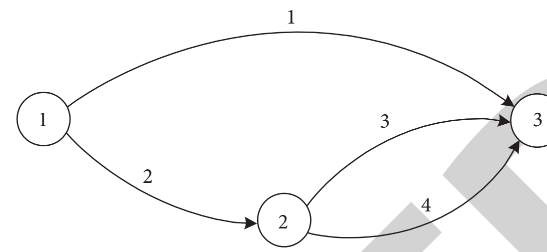

Figure 1: A simple network example.

remain unchanged except for OD pair $1 \longrightarrow 3$ as the cross-boundary traffic does not affect them. The optimal link pattern for cross-boundary traffic is $v^{2,so} = [2.0979\ 0\ 0\ 0]^T$. The credit charges are anonymous and are given by $\kappa_1 = 6.9930$, $\kappa_3 = 5.9208$, and $\kappa_4 = 9.9208$. As can be seen, the cross-boundary traffic volume on link 1 is larger than the local traffic volume, which may be unacceptable to the local residents. However, the anonymous tradable credit scheme cannot set discriminatory tolls for local traffic and cross-boundary traffic. Thus, it cannot provide an acceptable condition for the local traffic.

We then test the local tradable credit scheme, in which only the local users are allowed to trade their credits in the market, and the external users need to buy credits from the local authority to travel on the network. To provide an acceptable condition for the local traffic, the local authority can impose a side constraint $C^e_1 = 1$ on the cross-boundary traffic volume. By solving the credit design program, we have that the side constraint will be violated. Under this case, the link volume of the local traffic is given by $v^{1,so} = [2.4938\ 0\ 5.9208\ 9.9208]^T$ with the cross-boundary traffic given by the upper bound of the side constraint. Now, by the local tradable credit scheme, the credit charges for the two classes of users are $\kappa_1 = 6.9875$, $\kappa^e_1 = 7.0058$, $\kappa_3 = 5.9208$, and $\kappa_4 = 9.9208$. The local authority will distribute a number of credits equals $\sum_{i=1}^{3} \kappa_i v_i$ to the local users, while the cross-boundary users need to buy credits from the authority by $\kappa^e_1 = 7.0058$. The local users then can trade the credits in the local credit market.

4.2. Local Social Welfare under Tradable Credit Schemes. This experiment investigates the local social welfare under tradable credit schemes. A hypothetical network from Yang et al. [33] depicted in Figure 2 is adopted. The network consists of 7 nodes, 11 links, and 4 OD pairs ($1 \longrightarrow 7, 2 \longrightarrow 7, 3 \longrightarrow 7$ and $6 \longrightarrow 7$). The demand functions for the local traffic are given as follows:

$$\begin{aligned} D_{1\longrightarrow 7} &= 600 e^{-0.04\pi_{1\longrightarrow 7}}, \\ D_{2\longrightarrow 7} &= 500 e^{-0.03\pi_{2\longrightarrow 7}}, \\ D_{3\longrightarrow 7} &= 500 e^{-0.05\pi_{3\longrightarrow 7}}, \\ D_{6\longrightarrow 7} &= 400 e^{-0.05\pi_{6\longrightarrow 7}}. \end{aligned} \tag{51}$$

We further assume that node 1 is a boundary node. Cross-boundary trips enter from node 1 and travel to node 5. Under this case, except for the demand functions for local

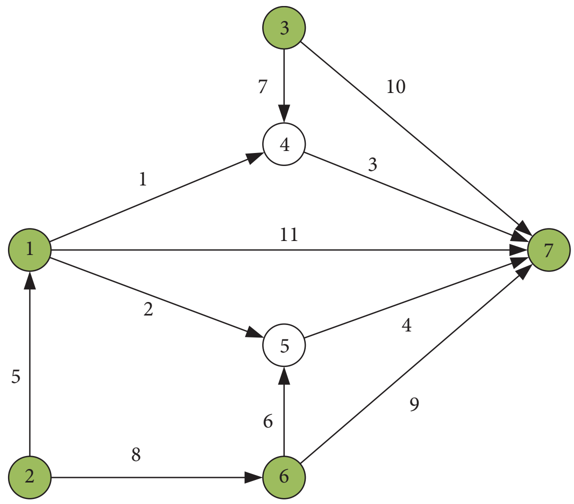

Figure 2: A hypothetical network source: Yang et al. [33].

Table 1: Link parameters for the network.

Link #	t_a^0	C_a
1	6	200
2	5	200
3	6	200
4	7	200
5	6	150
6	1	150
7	5	150
8	10	200
9	11	200
10	11	200
11	15	200

traffic, the demand function for cross-boundary traffic is defined as $D_{1\longrightarrow 5} = 200e^{-0.02\pi_{1\longrightarrow 5}}$. As the cross-boundary trips contribute to the local economy, the revenue function of the cross-boundary traffic is assumed to be $f_{15} = 6 - d_{15}^e/80$. Standard BPR link travel time function (52) is adopted. The link free-flow travel time t_a^0 and the physical link capacity C_a are summarized in Table 1.

$$t_a(v_a) = t_a^0\left(1 + 0.15\left(\frac{v_a}{C_a}\right)^4\right), \forall a \in A. \quad (52)$$

Without tradable credit schemes, both local and external users choose their routes according to the actual travel time and demand elasticity. In this case, the equilibrium condition is the well-known user equilibrium, under which the transportation network would operate at low efficiency. In contrast, the tradable credit schemes can drive the system from user equilibrium to system optimum, under which the transportation network can operate at its highest efficiency. Furthermore, the local authority can maximize its local social welfare by determining the quantity of cross-boundary trips through the revenue function. The equilibrium link flow patterns with or without tradable credit schemes are presented in Table 2, wherein 2^e is the link flow pattern for the cross-boundary traffic. As can be seen, the tradable credit schemes can control the quantity of cross-boundary trips so that the cross-boundary traffic would not damage the social welfare of the local region.

Table 2: Link flow patterns with or without tradable credit schemes.

Patterns/link	1	2	2^e	3	4	5
Without control	224.60	37.62	177.38	257.78	258.13	104.50
With control	161.23	71.35	45.29	206.51	211.63	106.75
Patterns /link	6	7	8	9	10	11
Without control	220.51	33.18	152.94	156.23	221.58	159.73
With control	140.28	45.28	121.29	168.49	177.74	154.59

The social welfare of the local traffic can be defined as $\sum_{w\in W}\int_0^{d_w} B_w(s)ds - \sum_{a\in\mathcal{A}} t_a(v_a)v_a^1$, while the welfare contributed by the cross-boundary traffic is defined in Remark 1 as $\triangle^e$. Without tradable credit schemes, the social welfare of the local traffic is 2.6089×10^4, while the welfare contributed by the cross-boundary traffic is -1.1042×10^3, which means the cross-boundary traffic has caused welfare loss to the local region, which is unwanted for the local authority. After introducing tradable credit schemes, the social welfare of the local traffic becomes 2.7102×10^4, while the welfare contributed by the cross-boundary traffic is 0, which means the revenue of the cross-boundary traffic can compensate for the total marginal social cost induced by it. Meanwhile, the local social welfare increases as the tradable credit schemes can pursue a system optimum flow pattern.

5. Conclusions

This study extended the tradable travel credit schemes to the mixed local and cross-boundary traffic case. The problems were formulated as optimization programs wherein the local authority aims to maximize its local social welfare and pursue a specific set of system optimal link flow patterns. As we assumed that the local authority aims to maximize its local social welfare, the quantity of cross-boundary trips was determined by evaluating the revenue of the cross-boundary traffic. Link-specific optimal tolls were obtained by solving these programs. We then designed two credit charging schemes to achieve a given system optimal flow pattern, i.e., a spatially differentiated credit scheme and an anonymous tradable credit scheme. In the first scheme, the local traffic and cross-boundary traffic are charged with different local system optimal tolls, while the travel credits are freely tradable within the local travellers only. The cross-boundary travellers have to buy travel credits from the local authority. In the second scheme, the tradable credit scheme is anonymous. The local authority determines the link-specific number of credits to be charged for using that link, while the travel credits are distributed to local travellers only but are allowed for free trading among both local and cross-boundary travellers. The equilibrium link flow patterns under these credit schemes are demonstrated with local elastic demand. In both tradable credit schemes, the credit price in the trading market is unique under the equilibrium condition. The first scheme offers more regulatory power to control the volume of the cross-boundary traffic, as the local authority can set side constraints for the cross-boundary traffic. On the other hand, the second scheme is more practical and feasible, as this tradable credit scheme involves no financial transfer from travellers to the government. Numerical results show that the tradable credit schemes can significantly improve the local social welfare while resolving the welfare loss caused by the cross-boundary traffic. In particular, the spatially differentiated credit scheme can provide an acceptable network condition for the local traffic by setting side constraints for the cross-boundary traffic and enforcing additional tolls to external users.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study gratefully acknowledged financial support from the National Natural Science Foundation of China (Project 72071214).

References

[1] H. Yang and H. J. Huang, *Mathematical and Economic Theory of Road Pricing*, Elsevier, Amsterdam, Netherlands, 2005.
[2] R. Lindsey, "Do economists reach a conclusion?" *Econ Journal Watch*, vol. 3, no. 2, pp. 292–379, 2006.
[3] H. Yang and X. Wang, "Managing network mobility with tradable credits," *Transportation Research Part B: Methodological*, vol. 45, no. 3, pp. 580–594, 2011.
[4] R. Zhong, H. Cai, D. Xu, C. Chen, A. Sumalee, and T. Pan, "Dynamic feedback control of day-to-day traffic disequilibrium process," *Transportation Research Part C: Emerging Technologies*, vol. 114, pp. 297–321, 2020a.
[5] T. Tsekeris and S. Voß, "Design and evaluation of road pricing: state-of-the-art and methodological advances," *Netnomics: Economic Research and Electronic Networking*, vol. 10, no. 1, pp. 5–52, 2009.
[6] Z. Tan, H. Yang, and R. Y. Guo, "Dynamic congestion pricing with day-to-day flow evolution and user heterogeneity," *Transportation Research Part C: Emerging Technologies*, vol. 61, pp. 87–105, 2015.
[7] B. Zhou, M. Bliemer, H. Yang, and J. He, "A trial-and-error congestion pricing scheme for networks with elastic demand and link capacity constraints," *Transportation Research Part B: Methodological*, vol. 72, pp. 77–92, 2015.
[8] X. Wang, H. Yang, D. Zhu, and C. Li, "Tradable travel credits for congestion management with heterogeneous users," *Transportation Research Part E: Logistics and Transportation Review*, vol. 48, no. 2, pp. 426–437, 2012.
[9] T. Akamatsu and K. Wada, "Tradable network permits: a new scheme for the most efficient use of network capacity," *Transportation Research Part C: Emerging Technologies*, vol. 79, pp. 178–195, 2017.
[10] R. Lindsey, "Reforming road user charges: a research challenge for regional science," *Journal of Regional Science*, vol. 50, no. 1, pp. 471–492, 2010.
[11] H. Yang, W. Xu, B.-s. He, and Q. Meng, "Road pricing for congestion control with unknown demand and cost functions," *Transportation Research Part C: Emerging Technologies*, vol. 18, no. 2, pp. 157–175, 2010.
[12] R. Zhong, R. Xu, A. Sumalee, S. Ou, and Z. Chen, "Pricing environmental externality in traffic networks mixed with fuel vehicles and electric vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, 2020.
[13] D. Han, H. Yang, and X. Wang, "Efficiency of the plate-number-based traffic rationing in general networks," *Transportation Research Part E: Logistics and Transportation Review*, vol. 46, no. 6, pp. 1095–1110, 2010.
[14] X. Wang, H. Yang, and D. Han, "Traffic rationing and short-term and long-term equilibrium," *Transportation Research Record*, vol. 2196, no. 1, pp. 131–141, 2010.
[15] C. Raux, "The use of transferable permits in transport policy," *Transportation Research Part D: Transport and Environment*, vol. 9, no. 3, pp. 185–197, 2004.
[16] C. Raux and G. Marlot, "A system of tradable co2 permits applied to fuel consumption by motorists," *Transport Policy*, vol. 12, no. 3, pp. 255–265, 2005.
[17] A. Perrels, "User response and equity considerations regarding emission cap-and-trade schemes for travel," *Energy Efficiency*, vol. 3, no. 2, pp. 149–165, 2010.
[18] Z. Wadud, R. B. Noland, and D. J. Graham, "Equity analysis of personal tradable carbon permits for the road transport sector," *Environmental Science & Policy*, vol. 11, no. 6, pp. 533–544, 2008.
[19] Z. Wadud, "Personal tradable carbon permits for road transport: why, why not and who wins?" *Transportation Research Part A: Policy and Practice*, vol. 45, no. 10, pp. 1052–1065, 2011.
[20] P. Varaiya, "Congestion, ramp metering and tolls," *Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences*, vol. 366, no. 1872, pp. 1921–1930, 2008.
[21] H. Wang and X. Zhang, "Joint implementation of tradable credit and road pricing in public-private partnership networks considering mixed equilibrium behaviors," *Transportation Research Part E: Logistics and Transportation Review*, vol. 94, pp. 158–170, 2016.
[22] K. A. Small, E. T. Verhoef, and R. Lindsey, *The Economics of Urban Transportation*, Routledge, England, UK, 2007.
[23] A. De Palma and R. Lindsey, "Traffic congestion pricing methodologies and technologies," *Transportation Research Part C: Emerging Technologies*, vol. 19, no. 6, pp. 1377–1399, 2011.
[24] X. Wang and H. Yang, "Bisection-based trial-and-error implementation of marginal cost pricing and tradable credit scheme," *Transportation Research Part B: Methodological*, vol. 46, no. 9, pp. 1085–1096, 2012.
[25] D.-L. Zhu, H. Yang, C.-M. Li, and X.-L. Wang, "Properties of the multiclass traffic network equilibria under a tradable credit scheme," *Transportation Science*, vol. 49, no. 3, pp. 519–534, 2015.
[26] B. Ubbels and E. T. Verhoef, "Governmental competition in road charging and capacity choice," *Regional Science and Urban Economics*, vol. 38, no. 2, pp. 174–190, 2008.
[27] B. De Borger, F. Dunkerley, and S. Proost, "Capacity cost structure, welfare and cost recovery: are transport infrastructures with high fixed costs a handicap?" *Transportation Research Part B: Methodological*, vol. 43, no. 5, pp. 506–521, 2009.

[28] M. H. Lin, "Congestion pricing and capacity for internationally interlinked airports," *Transportation Research Part B: Methodological*, vol. 142, pp. 126–142, 2020.
[29] H. Yang and X. Zhang, "Existence of anonymous link tolls for system optimum on networks with mixed equilibrium behaviors," *Transportation Research Part B: Methodological*, vol. 42, no. 2, pp. 99–112, 2008.
[30] F. Zhang, J. Lu, and X. Hu, "Tradable credit scheme design with transaction cost and equity constraint," *Transportation Research Part E: Logistics and Transportation Review*, vol. 145, Article ID 102133, 2021.
[31] D. Han and X. Yuan, "Existence of anonymous link tolls for decentralizing an oligopolistic game and the efficiency analysis," *Journal of Industrial and Management Optimization*, vol. 7, no. 2, p. 347, 2011.
[32] B. De Borger, F. Dunkerley, and S. Proost, "Strategic investment and pricing decisions in a congested transport corridor," *Journal of Urban Economics*, vol. 62, no. 2, pp. 294–316, 2007.
[33] H. Yang, Q. Meng, and D. H. Lee, "Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions," *Transportation Research Part B: Methodological*, vol. 38, no. 6, pp. 477–493, 2004.