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Research Article

Optimal Pricing and Ordering Policy for Deteriorating Items with Stock-and-Price Dependent Demand and Presale Rebate

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This paper considers an EOQ inventory model with presale policy for deteriorating items, in which the demand rate depends on both on-hand inventory and selling price. Under the assumption that all the presale orders are fully backlogged with waiting-time dependent rebate, this study develops several propositions and derives optimal pricing and ordering policy by designing an effective algorithm. Two numerical examples are also given to illustrate the effectiveness of the algorithm. Finally, the sensitivity analysis of the main parameters is provided.

1. Introduction

A large number of products have deteriorating property which manifests the decay or devaluation. This kind of products has much shorter life cycle, for example, some seasonal fresh products that cannot be stored for a long time even in modern storage conditions, electronic products with a fast updating speed, and fashion clothing. These products should be sold in a much shorter spot-sale period to reduce deteriorating cost and holding cost. Presale policy is an effective marketing strategy to reduce the spot-sale period, which is that customers are encouraged to order in advance to get a rebate related to the preorder time. All the orders will be satisfied when the products launch into the market. In addition, it is crucial to find out what factors mostly impact the demand of product, so we can decide proper ordering quantity.

It is obvious that the price of product highly influences the demand rate; thus the pricing for the product plays an important role in marketing strategy. Meanwhile, the stock level of product can also affect the demand rate. Many researchers have focused on such topic. Hou and Lin [1] considered an EOQ inventory model for deteriorating items with price-and-stock dependent selling rate, in which the proposed model allowed shortage and complete backorder while the shortage cost is constant. You and Hsieh [2] developed an inventory model for a seasonal item over a finite planning time by determining the optimal ordering quantity and price setting/changing strategy. Chang et al. [3] studied the optimal selling price and order quantity under EOQ model for deteriorating items, and the demand depends on the selling price and stock on display. Dye and Hsieh [4] developed an inventory model for deteriorating items with demand rate depending on selling price and stock, and shortages are allowed with the backlogging rate to be a decreasing function of the waiting time, while Giri and Bardhan [5] studied an integrated single-manufacturer and single-retailer supply chain model for deteriorating items with stock-and-price dependent demand. Relevant references can be found in Teng and Chang [6], Panda et al. [7], and so forth. Furthermore, more inventory models for deteriorating items with a variety of demand rate are summarized in Table 1. The difference between the relevant references and our study can be found in the table.

In our study, presale rebate depending on waiting time is given to encourage customers to order in advance. Although presale rebate cost is similar in form to the traditional shortage cost, it is different in meaning; moreover, the presale rebate cost in this paper is more complex and realistic than the traditional shortage cost which is generally considered to

Table 1: Our paper versus literatures for key assumptions of inventory models with deteriorating items.

Author/authors	Demand rate	Deterioration rate	Shortage or not	Shortage cost	Decision variable*
Skouri et al. [8]	Time-dependent	Weibull distribution	Yes	Constant	t, T
Sicilia et al. [9]	Time-dependent	Constant	Yes	Constant	t, T
Cheng et al. [10]	Time-dependent	Constant	Yes	Constant	t
Zhao [11, 12]	Time-dependent	Weibull distribution	Yes	Constant	t
Begum et al. [13]	Price-dependent	Weibull distribution	No	No	p, T
Dye [14]	Price-dependent	Time-dependent	Yes	Constant	p, t, and T
Yang et al. [15]	Price-dependent	No	Yes	Constant	t, T
B. Sarkar and S. Sarkar [16]	Stock-dependent	Time-dependent	Yes	Constant	t, T
Padmanabhan and Vrat [17]	Stock-dependent	Constant	Yes	Constant	t, T
Avinadav et al. [18]	Price-and-time	Constant	No	No	p, Q, and T
Maihami and Nakhai Kamalabadi [19]	Price-and-time	Noninstantaneous	Yes	Constant	p, T
Hou and Lin [1]	Price-and-stock	Constant	Yes	Constant	t, T
Giri and Bardhan [5]	Price-and-stock	No	No	No	t, n
This paper	Price-and-stock	Constant	Presale	Time-dependent	p, t, and T

* p = selling price, t = replenishment/out of stock time, Q = order quantity, T = inventory cycle, and n = the number of replenishment.

be constant. In addition, we assume the demand is price-and-stock dependent according to the actual situation. Eventually, we derive optimal pricing and ordering policy by designing an effective algorithm and provide the sensitivity analysis of the parameters to assess their effects on the optimal policy.

The rest of the paper is organized as follows: we introduce some basic notations and assumptions in Section 2. Section 3 establishes an inventory model under presale policy and provides an effective procedure to find the optimal pricing and presale strategy. In Section 4, we use several numerical examples to illustrate the procedure of the optimal strategies and analyze the sensitivity of parameters involved. Finally, we provide a summary of the paper.

2. Notation and Assumptions

The fundamental notation and assumptions used in this paper are given as below.

2.1. Notation

c_p: the purchase price of unit item;

A_0: the fixed cost per order;

S: the maximum inventory level;

Q: the ordering quantity;

p: the selling price of unit item

θ: the constant deteriorating rate, $0 \le \theta \ll 1$;

t_1: presale period;

t_2: spot-sale period;

$I(t)$: the level of inventory at time t, $0 \le t \le T$, where $T = t_1 + t_2$;

c_d: the cost of each deteriorated item;

c_h: the inventory holding cost per unit per time;

$D(t, p)$: the demand rate;

$c_s(x)$: the presale rebate cost;

$\prod(t_1, t_2, p)$: the average total profit per unit time.

2.2. Assumptions

(i) The replenishment rate is infinite; that is, replenishment is instantaneous.

(ii) Assume the customers are loyal, and all the presale orders will not be canceled until the products are launched into the market.

(iii) The demand rate, $D(t, p)$: we assumed it to be

$$D(t,p) = \begin{cases} g(p), & 0 \le t \le t_1 \\ g(p) + \alpha I(t), & t_1 \le t \le T, \end{cases} \quad (1)$$

where $g(p)$ is a function of p, $g'(p) < 0$, and $g''(p) \ge 0$, where α denotes the stock-dependent consumption rate.

(iv) $c_s(x)$ is the presale rebate determined by the waiting time x; that is, $c_s(x) = c_0(e^{\lambda x} - 1)$, where c_0 is positive constant and $\lambda \in (0, 1)$.

(v) The marginal gross profit with respect to price is decreasing; that is, $(p - c_p)g(p)$ is a strictly concave function of p.

3. Inventory Model with Presale Policy

In this section, we consider an inventory model with presale policy for deteriorating items, and the behavior of the system is depicted in Figure 1. From $t = 0$ to t_1, the total presale quantity accumulates on account of demand which is fully backlogged and achieves its maximum at time $t = t_1$. During the time interval $[t_1, t_1 + t_2)$, due to customers' demand and deterioration, on-hand inventory level gradually decreases to

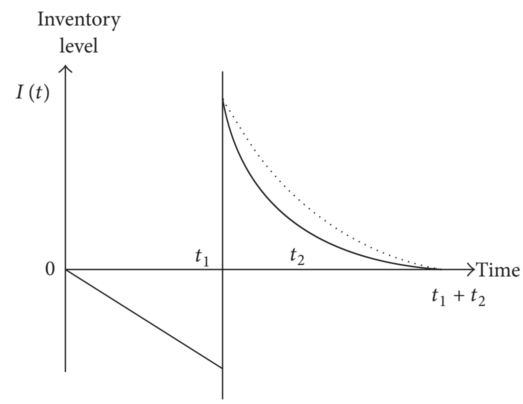

Figure 1: Graphical representation of inventory level over the cycle.

zero. According to the above notations and assumptions, such inventory system can be described by following differential equations:

$$\frac{dI(t)}{dt} = -g(p) \quad 0 \le t \le t_1;$$
$$\frac{dI(t)}{dt} = -\theta I(t) - g(p) - \alpha I(t) \quad t_1 \le t \le t_1 + t_2. \tag{2}$$

With boundary condition $I(0) = 0$, $I(t_1 + t_2) = 0$.

The solution of (2) can be obtained as

$$I(t) = -g(p)t \quad 0 \le t \le t_1,$$
$$I(t) = \frac{g(p)}{\alpha+\theta}\left(e^{(\alpha+\theta)(t_1+t_2-t)} - 1\right) \quad t_1 \le t \le t_1 + t_2. \tag{3}$$

The maximum inventory level per cycle is

$$I(t_1^+) = \frac{g(p)}{\alpha+\theta}\left(e^{(\alpha+\theta)t_2} - 1\right). \tag{4}$$

The presale quantity per cycle is

$$I(t_1^-) = -g(p)t_1. \tag{5}$$

The ordering quantity over the inventory cycle is

$$Q = I(t_1^+) - I(t_1^-) = \frac{g(p)}{\alpha+\theta}\left(e^{(\alpha+\theta)t_2} - 1\right) + g(p)t_1. \tag{6}$$

Based on (3), the total cost per cycle consists of the following elements:

(i) Ordering cost A_0.

(ii) Purchase cost

$$c_p\left[\frac{g(p)}{\alpha+\theta}\left(e^{(\alpha+\theta)t_2} - 1\right) + g(p)t_1\right]. \tag{7}$$

(iii) Sales revenue

$$p\left[g(p)t_1 + \int_{t_1}^{t_1+t_2}(g(p) + \alpha I(t))\,dt\right]. \tag{8}$$

(iv) Deteriorating cost

$$c_d\left[\frac{g(p)}{\alpha+\theta}\left(e^{(\alpha+\theta)t_2} - 1\right) - \int_{t_1}^{t_1+t_2}(g(p) + \alpha I(t))\,dt\right]. \tag{9}$$

(v) Holding cost

$$\frac{c_h g(p)}{\alpha+\theta}\int_{t_1}^{t_1+t_2}\left(e^{(\alpha+\theta)(t_1+t_2-t)} - 1\right)dt. \tag{10}$$

(vi) Presale rebate cost

$$c_0 g(p)\int_0^{t_1}\left(e^{\lambda(t_1-1)} - 1\right)t\,dt. \tag{11}$$

Therefore, the total profit per unit time of the model can be obtained as follows:

$$\begin{aligned}\prod(t_1,t_2,p) &= \frac{1}{t_1+t_2}[\text{sales revenue} - \text{holding cost} \\ &- \text{ordering cost} - \text{presale rebate cost} \\ &- \text{deteriorated items cost} - \text{purchase cost}] \\ &= -\frac{g(p)}{t_1+t_2}\left\{\frac{\Delta}{(\alpha+\theta)^2}\left(e^{(\alpha+\theta)t_2} - 1\right)\right. \\ &+ \frac{c_0}{\lambda^2}\left(e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1 - 1\right) - \left(p - c_p\right)t_1 \\ &\left. - \frac{1}{\alpha+\theta}\left(p\theta + c_h + c_d\theta\right)t_2\right\} - \frac{A_0}{t_1+t_2},\end{aligned} \tag{12}$$

where $\Delta = c_h + (\alpha+\theta)c_p + c_d\theta - p\alpha$.

Taking the first-order partial derivative of $\prod(t_1, t_2, p)$ with respect to t_1, t_2, and p, respectively, we have

$$\begin{aligned}\frac{\partial\prod(t_1,t_2,p)}{\partial t_1} &= -\frac{\prod(t_1,t_2,p)}{t_1+t_2} + \frac{g(p)}{t_1+t_2}\left[\left(p - c_p\right)\right. \\ &\left. - \frac{c_0}{\lambda}\left(e^{\lambda t_1} - \lambda t_1 - 1\right)\right], \\ \frac{\partial\prod(t_1,t_2,p)}{\partial t_2} &= -\frac{\prod(t_1,t_2,p)}{t_1+t_2} \\ &+ \frac{g(p)}{t_1+t_2}\left[\frac{p\theta + c_h + c_d\theta}{\alpha+\theta} - \frac{\Delta}{\alpha+\theta}e^{(\alpha+\theta)t_2}\right], \\ \frac{\partial\prod(t_1,t_2,p)}{\partial p} &= \frac{1}{t_1+t_2}\left\{\frac{g(p)\alpha - g'(p)\Delta}{(\alpha+\theta)^2}\left[e^{(\alpha+\theta)t_2}\right.\right. \\ &\left. - (\alpha+\theta)t_2 - 1\right] - \frac{c_0 g'(p)}{\lambda^2}\left(e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1\right. \\ &\left.\left. - 1\right)\right\} + g(p) + g'(p)\left(p - c_p\right).\end{aligned} \tag{13}$$

For any given selling price p, the optimal solution (t_1^*, t_2^*) is determined by the following equations:

$$\frac{\partial \prod (t_1, t_2, p)}{\partial t_1} = 0,$$
$$\frac{\partial \prod (t_1, t_2, p)}{\partial t_2} = 0. \tag{14}$$

By simplifying (14), we have

$$\prod (t_1, t_2, p) - g(p) \left[(p - c_p) - \frac{c_0}{\lambda} \left(e^{\lambda t_1} - \lambda t_1 - 1 \right) \right] = 0, \tag{15}$$

$$\prod (t_1, t_2, p) - g(p) \left[\frac{p\theta + c_h + c_d\theta}{\alpha + \theta} - \frac{\Delta}{\alpha + \theta} e^{(\alpha+\theta)t_2} \right] = 0. \tag{16}$$

From the analysis of (15) and (16), we have the following propositions.

Proposition 1. *For any given price p,* (i) *if $\Delta \le 0$, then there does not exist any optimal solution (t_1, t_2) so as to maximize the profit $\prod(t_1, t_2, p)$;* (ii) *if $\Delta > 0$, then the optimal solution (t_1^*, t_2^*) which solves (15) and (16) simultaneously not only exists but also is unique.*

Proof. (i) From (15) and (16), we have

$$\frac{\Delta}{\alpha + \theta} \left(e^{(\alpha+\theta)t_2} - 1 \right) = \frac{c_0}{\lambda} \left(e^{\lambda t_1} - \lambda t_1 - 1 \right). \tag{17}$$

If $\Delta < 0$, it is obvious that there does not exist any nonzero feasible solution to satisfy (17), which means that the optimal solution of the maximum profit $\prod(t_1, t_2 \mid p)$ does not exist.

If $\Delta = 0$, by solving (17), we have $t_1 = 0$. Substituting it into (16), we have $\prod(0, t_2, p) = ((p\theta + c_h + c_d\theta)/(\alpha + \theta))g(p)$. Thus, from (12), we obtain $A_0 = 0$, which means a contradictory. Therefore, if $\Delta < 0$, there does not exist any optimal solution (t_1, t_2) to minimize the profit $\prod(t_1, t_2 \mid p)$.

(ii) Let

$$f(x) = \frac{\Delta}{\alpha + \theta} \left(e^{(\alpha+\theta)x} - 1 \right) - \frac{c_0}{\lambda} \left(e^{\lambda t_1} - \lambda t_1 - 1 \right). \tag{18}$$

Taking the first-order derivative of (18) with respect to x, it yields

$$f'(x) = \Delta e^{(\alpha+\theta)x} \ge 0 \tag{19}$$

which means that $f(x)$ is a nondecreasing function in $[0, +\infty)$.

Similarly, we know that the right hand-side of (17) is a strictly increasing function of t_1 and goes to infinite as $t_1 \to \infty$.

On the other hand, from (17), we have $\Delta e^{(\alpha+\theta)t_2}(dt_2/dt_1) = c_0(e^{\lambda t_1} - 1) > 0$, which means $dt_2/dt_1 > 0$, and

$$t_2 = \frac{1}{\alpha + \theta} \ln \left[\frac{(\alpha + \theta) c_0}{\lambda \Delta} \left(e^{\lambda t_1} - \lambda t_1 - 1 \right) + 1 \right]. \tag{20}$$

Therefore, for any given $\tilde{t}_1$, there exists a unique $\tilde{t}_2$ such that $\tilde{t}_2 = (1/(\alpha + \theta))\ln[((\alpha + \theta)c_0/\lambda\Delta)(e^{\lambda\tilde{t}_1} - \lambda\tilde{t}_1 - 1) + 1]$.

To prove the uniqueness of the solution, substitute (12) into (15), and let

$$F(t_1) = -g(p) \left[\frac{\Delta}{(\alpha + \theta)^2} \left(e^{(\alpha+\theta)t_2} - 1 \right) + (p - c_p) t_2 - \frac{t_2}{\alpha + \theta} (p\theta + c_h + c_d\theta) \right] - A_0 - \frac{c_0 g(p)}{\lambda^2} \left[e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1 - 1 - \lambda \left(e^{\lambda t_1} - \lambda t_1 - 1 \right) (t_1 + t_2) \right]. \tag{21}$$

Taking the first-order derivative of $F(t_1)$ with respect to t_1, we have

$$\frac{dF(t_1)}{dt_1} = -\frac{\Delta g(p)}{(\alpha + \theta)} \left(e^{(\alpha+\theta)t_2} - 1 \right) \frac{dt_2}{dt_1} + \frac{c_0 g(p)}{\lambda} \left[\left(e^{\lambda t_1} - \lambda t_1 - 1 \right) \frac{dt_2}{dt_1} + \lambda \left(e^{\lambda t_1} - 1 \right) (t_1 + t_2) \right] = c_0 g(p) \left(e^{\lambda t_1} - 1 \right) (t_1 + t_2) \ge 0 \tag{22}$$

which means that $F(t_1)$ is an increasing function in $[0, +\infty)$. Using (17), it deduces $t_2 = 0$ as $t_1 = 0$; then we have $F(0) = -A_0 < 0$ and $\lim_{t_1 \to +\infty} F(t_1) = +\infty$. With the Intermediate Value Theorem, we can find a unique root $t_1^* \in [0, +\infty)$ such that $F(t_1^*) = 0$.

From the above analysis, it is concluded that if $\Delta > 0$, the optimal solution (t_1^*, t_2^*) which solves (15) and (16) simultaneously not only exists but also is unique. □

Proposition 2. *For any given p, the solution (t_1^*, t_2^*) of (15) and (16) simultaneously is the global maximum of the profit per unit time.*

Proof. Taking the second-order partial derivative of $\prod(t_1, t_2 \mid p)$ with respect to t_1 and t_2, respectively, we have

$$\left. \frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_1^2} \right|_{(t_1,t_2)=(t_1^*,t_2^*)} = -\frac{c_0 g(p)}{t_1^* + t_2^*} \left(e^{\lambda t_1^*} - 1 \right) < 0, \tag{23}$$

$$\left. \frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_2^2} \right|_{(t_1,t_2)=(t_1^*,t_2^*)} = -\frac{\Delta g(p)}{t_1^* + t_2^*} e^{(\alpha+\theta)t_2^*} < 0, \tag{24}$$

$$\left. \frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_1 \partial t_2} \right|_{(t_1,t_2)=(t_1^*,t_2^*)} = 0. \tag{25}$$

Then, the Hessian matrix satisfies

$$|H| = \left.\frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_1^2}\right|_{(t_1,t_2)=(t_1^*,t_2^*)} \times \left.\frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_2^2}\right|_{(t_1,t_2)=(t_1^*,t_2^*)} - \left[\left.\frac{\partial^2 \prod (t_1, t_2 \mid p)}{\partial t_1 \partial t_2}\right|_{(t_1,t_2)=(t_1^*,t_2^*)} \right]^2 > 0. \quad (26)$$

Therefore, we conclude that the stationary point (t_1^*, t_2^*) is a global optimal solution for the considered problem. □

In the following, we will show that the optimal selling price also exists and is unique for the problem.

Proposition 3. *For any given* (t_1, t_2)*, there exists a unique optimal selling price such that* $\prod(p \mid t_1, t_2)$ *is maximized.*

Proof. For any given t_1 and t_2, the first-order derivation of $\prod(p \mid t_1, t_2)$ with respect to p is

$$\frac{d \prod (p \mid t_1, t_2)}{dp} = \frac{1}{t_1 + t_2} \left\{ \frac{g(p)\alpha - g'(p)\Delta}{(\alpha+\theta)^2} \left[e^{(\alpha+\theta)t_2} - (\alpha+\theta)t_2 - 1 \right] - \frac{c_0 g'(p)}{\lambda^2} \left(e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1 - 1 \right) \right\} + g(p) + g'(p)(p - c_p). \quad (27)$$

Because $g'(p) < 0$, we have $g(p)\alpha - g'(p)\Delta > 0$. Thus, $d\prod(p \mid t_1, t_2)/dp = 0$ provides a solution only if $g(p) + (p - c_p)g'(p) < 0$.

Let

$$G(p) = \frac{1}{t_1 + t_2} \left\{ \frac{g(p)\alpha - g'(p)\Delta}{(\alpha+\theta)^2} \left[e^{(\alpha+\theta)t_2} - (\alpha+\theta)t_2 - 1 \right] - \frac{c_0 g'(p)}{\lambda^2} \left(e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1 - 1 \right) \right\} + g(p) + g'(p)(p - c_p). \quad (28)$$

Since the gross profit $g(p)(p - c_p)$ is a strictly concave function of p, which means $2g'(p) + g''(p)(p - c_p) < 0$, we have

$$G'(p) = \frac{1}{t_1 + t_2} \left\{ \frac{g'(p)\alpha - g''(p)\Delta}{(\alpha+\theta)^2} \left[e^{(\alpha+\theta)t_2} - (\alpha+\theta)t_2 - 1 \right] - \frac{c_0 g''(p)}{\lambda^2} \left(e^{\lambda t_1} - \frac{1}{2}\lambda^2 t_1^2 - \lambda t_1 - 1 \right) \right\} + 2g'(p) + g''(p)(p - c_p) < 0. \quad (29)$$

Therefore, there exists a unique optimal selling price p^* which maximizes $\prod(p \mid t_1^*, t_2^*)$ and completes the proof. □

From the above analysis, we know that the solution of $g(p) + (p - c_p)g'(p) = 0$ is the lower bound for the optimal selling price p such that $d\prod(p \mid t_1, t_2)/dp = 0$. Combining Propositions 2 and 3, we establish an algorithm to obtain the optimal policy of the considered model as follows.

Algorithm 4.

Step 1. Start with $i = 0$ and let p_i be a solution of $g(p) + (p - c_p)g'(p) = 0$.

Step 2. Put p_i into (15) and (16) to obtain the corresponding values of (t_1^*, t_2^*); then substitute them into (27) and determine the optimal p_{i+1}.

Step 3. If $|p_{i+1} - p_i| \le \varepsilon$, where ε is any small positive number, then set $p^* = p_{i+1}$ and (t_1^*, t_2^*, p^*) is the optimal solution to minimize $\prod(t_1, t_2, p)$; otherwise, set $i = i + 1$ and go back to Step 2.

4. Numerical Examples and Sensitivity Analysis

To demonstrate our theoretical results, we study several numerical examples to explain the algorithm proposed in the above section.

Example 1. Consider an inventory system with the following data: $A_0 = 25$\$/order, $\alpha = 0.03$\$, $c_p = 10.0$\$/unit, $c_d = 1.2$\$/unit, $c_h = 1.0$\$/unit, $c_0 = 0.5$\$/unit, $\theta = 0.2$, $\lambda = 0.3$, and $g(p) = 50e^{-0.04p}$, where $p \in [10, +\infty)$.

For the inventory model with presale policy, by using Algorithm 4, we have $t_1^* = 2.090$, $t_2^* = 0.552$, $p^* = 37.709$\$, $Q^* = 32.104$\$, and $\prod(t_1^*, t_2^*, p^*) = 209.707$\$.

Example 2. Consider an inventory system with the following data: $A_0 = 25$\$/order, $\alpha = 0.05$\$, $c_p = 10.0$\$/unit, $c_d = 1.2$\$/unit, $c_h = 1.0$\$/unit, $c_0 = 0.5$\$/unit, $\theta = 0.02$, $\lambda = 0.6$, and $g(p) = 100 - 5p$, where $p \in [10, 20]$.

For the inventory model with presale policy, by using Algorithm 4, we have $t_1^* = 1.739$, $t_2^* = 0.669$, $p^* = 15.130$\$, $Q^* = 59.014$\$, and $\prod(t_1^*, t_2^*, p^*) = 108.783$\$. The three-dimensional total profit per unit time graph for any given p is shown in Figure 2 and the numerical results indicate that $\prod(p \mid t_1^*, t_2^*)$ is strictly concave in p, as shown in Figure 3.

In order to illustrate the effect of the parameters $(\lambda, \theta, \alpha)$ on the optimal policy, that is, optimal pricing, the optimal ordering quantity, and the optimal total profit for the inventory models, the sensitivity analysis is performed on the base of Example 2 by changing the value of only one parameter at a time and keeping the rest of the parameters at their initial values. The results are shown in Table 2.

From Table 2, we can observe the following:

(1) As λ increases, t_1^*, Q^*, and $\prod(t_1^*, t_2^*, p^*)$ will decrease, while t_2^* and p^* will increase simultaneously, which

Table 2: The effect of parameters on the optimal policy for presale model.

Para.	Value of para.	t_1^*	t_2^*	p^*	Q^*	$\prod(t_1^*, t_2^*, p^*)$
λ	0.2	2.792	0.480	15.086	80.609	113.323
	0.4	2.087	0.594	15.111	65.851	110.583
	0.6	1.739	0.669	15.130	59.014	108.783
	0.8	1.517	0.725	15.246	54.882	107.431
θ	0.01	1.725	0.744	15.131	60.532	109.017
	0.02	1.739	0.669	15.130	59.014	108.783
	0.03	1.749	0.608	15.128	57.236	108.542
	0.04	1.758	0.557	15.127	56.752	108.337
α	0.04	1.749	0.608	15.128	57.685	108.542
	0.05	1.739	0.669	15.130	59.014	108.783
	0.07	1.728	0.731	15.136	60.396	109.128
	0.10	1.709	0.851	15.152	63.142	109.486

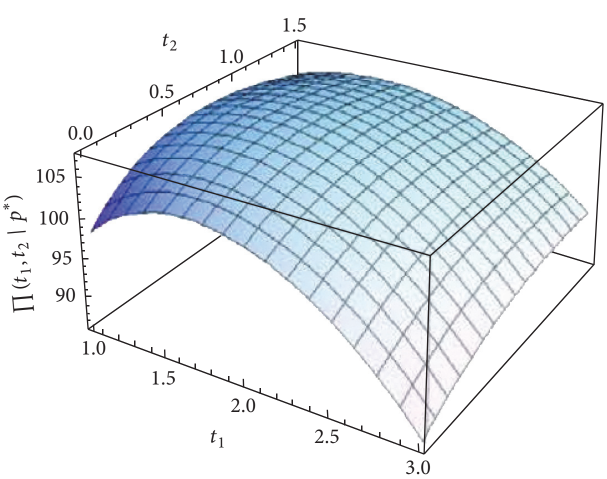

Figure 2: Total profit per unit time $\prod(t_1, t_2 \mid p^*)$.

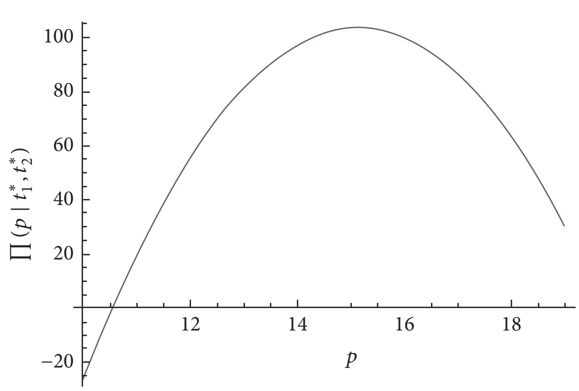

Figure 3: Total profit per unit time $\prod(p \mid t_1^*, t_2^*)$.

means that if the presale rebate increases, the presale period will get shorter to reduce the rebate cost. Moreover, the optimal order quantity and profit will decrease, while the selling price and spot-sale period will increase.

(2) As θ increases, t_2^*, p^*, Q^*, and $\prod(t_1^*, t_2^*, p^*)$ will decrease, and t_1^* will increase. It implies that the increase in deterioration cost can lengthen the presale period and then shorten the spot-sale period. As a result, the selling price will decrease for sales promotion; moreover, the optimal order quantity and profit will decrease.

(3) As α increases, t_1^* will decrease, and t_2^*, p^*, Q^*, and $\prod(t_1^*, t_2^*, p^*)$ will increase. It means that higher demand rate in spot-sale period will lead to shortening the presale period, while the selling price, the optimal order quantity, and profit will increase.

(4) In general, the fluctuation of λ has more effect on t_1^*, $\prod(t_1^*, t_2^*, p^*)$ than that of θ and α, and the fluctuation of λ, θ, and α each has more effect on t_2^*, Q^* than that of p^*.

5. Conclusion

In this paper, we study an EOQ inventory model with presale policy for deteriorating items in which the demand rate depends on both on-hand inventory and the price of items. By analyzing the inventory model, an optimal pricing and inventory policy is proposed. We also use several numerical examples to illustrate the solution procedure. Moreover, the sensitivity analysis of the parameters is provided to assess their effects on the optimal policy of the studied problem. From the results of numerical experiments, we find that λ and θ have a negative effect on the profit of inventory system, while α has a positive effect on the profit of inventory system. In addition, this paper provides an interesting topic for further study of inventory models. It also can be extended in other ways, that is, considering the nonconstant or noninstantaneous deterioration rate and others.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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