

Research Article

A Grey Target Group Decision Method with Dual Hesitant Fuzzy Information considering Decision-Maker's Loss Aversion

Yufeng Zhou ^{1,2}, Yufeng Li,³ and Zhi Li¹

¹Chongqing Engineering Technology Research Center for Information Management in Development, Chongqing Technology and Business University, Chongqing, China

²Postdoctoral Research Station of Management Science and Engineering, Nanjing University of Aeronautics & Astronautics, Nanjing 211106, China

³Research Center for Economy of Upper Researches of the Yangtze River, Chongqing Technology and Business University, Chongqing, China

Correspondence should be addressed to Yufeng Zhou; xtuzyf@qq.com

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The uncertainty, complexity, and behavioral preference are widely existing in real-world decision-making problems. In this paper, different from the previous grey target decision method, we propose a novel grey target group decision method considering decision-maker's loss aversion with positive and negative clouts under the dual hesitant fuzzy environment. Firstly, defining the dual hesitant fuzzy ideal optimization scheme as the positive clout and the ideal inferior scheme as the negative clout, positive and negative target-eye distances are measured by the normalized Hamming distances from the DHFEs to the positive clout and the negative clout. Then, a new comprehensive target-eye distance is proposed to evaluate alternatives between the positive and the negative clout. A nonlinear optimization model is established to obtain the optimal initial attribute weights with the goal of minimizing the comprehensive target-eye distance. Then, a grey target group decision method with dual hesitant fuzzy information considering decision-maker's loss aversion and variable weights is proposed. Finally, a numeral example is given to verify the effectiveness and practicality of the proposed model and method.

1. Introduction

Multiattribute decision-making (MADM) is an important issue of modern decision-making science. Recently, MADM has become a hotspot in the decision-making area, and a lot of academic achievements have been made in the related domain [1–3]. Multiattribute group decision-making (MAGDM) can use the wisdom of the group to promote scientific decision-making. Pattern matching optimization is an effective method, which is applied to many areas in MAGDM, such as tracking and detection [4, 5], computer engineering [6], physical sciences [7], health-related issues [8], natural sciences, and industrial academic areas [9]. In addition, other methods, such as TOPSIS [10, 11], probabilistic linguistic term sets [12], data-capturing devices [13–17], the state-of-the-art features

[18–22], the hidden Markov model (HMM), modified HMM, embedded HMM, Gaussian mixture modal (GMM), and the support vector machine (SVM) are also widely used in the field of MAGDM [23–26]. Grey target decision is one of the main methods using grey system theory to solve the problem of uncertain MADM and MAGDM under incomplete information condition. The main idea of the grey target decision is to achieve a uniform dimension of the Euclidean space through measuring and transforming index sets in the absence of the standard mode, i.e., the grey target. All the alternatives are distributed on the grey target. A target-eye of the grey target is found as the standard mode. Then, all the decision points in the grey target are compared with the target-eye to obtain relative target-eye distances, and the dominant position of each decision point is sorted by the target-eye distances.

In recent years, many scholars have carried out theoretical research on the classic grey target decision methods from different perspectives. Gupta et al. [27] presented an extended TOPSIS method under the interval-valued intuitionistic fuzzy environment. Luo and Wang [28] studied the multiattribute grey target decision method for the attribute value within the three-parameter interval grey number. Wang et al. [29] studied the multiattribute grey target decision method based on soft set theory. Qian et al. [30] built a grey target decision model based on interval grey number panel data for multiindex dynamic problems. Ye [31] proposed an extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection. Fu et al. [32] presented a kind of multiattribute grey target decision model based on the positive-negative target center aimed at the complexity and uncertainty of the actual decision environment. Zeng et al. [33] proposed a grey target decision-making model based on the cobweb area and gave an application of choosing the software development pattern.

Fuzzy sets (FSs) are also an effective method to deal with the uncertain problem of decision-making. FSs are used to deal with vagueness and fuzziness in real decision-making problems, and several extensions have been developed. Rodriguez et al. [34] studied the hesitant fuzzy linguistic term set on MADM. Traditional hesitant fuzzy decision-making methods just provide the membership degrees and without considering the importance of the nonmembership degrees. In fact, the nonmembership plays the same important role in describing the vague decision-making information, which indicates the possible degrees of one element does not belong to a fixed set. To assess the attribute values more precisely, Zhu et al. [35] developed the dual hesitant fuzzy set (DHFS), taking into account much more information given by decision-makers, in which the membership degree and the nonmembership degree are in the form of sets of values in $[0, 1]$. DHFS can avoid information distortion and losing effectively in describing the vague decision-making information. Then, the aggregation operators for aggregating dual hesitant fuzzy elements (DHFES) have attracted the attention from researchers [36–40]. Regarding the MADM problems with dual hesitant fuzzy information, many methods have been proposed. Ren and Wei [41] developed a prioritized multiattribute decision-making method to solve dual hesitant fuzzy decision problems. Then, they [42] further proposed a dual hesitant fuzzy VIKOR method for multicriteria group decision-making based on the fuzzy measure and the new comparison method. Wei et al. [43] investigated the MADM problem based on the geometric aggregation operators with interval-valued dual hesitant fuzzy linguistic information. Lu and Wei [44] investigated the MADM problem based on the arithmetic and geometric aggregation operators with dual hesitant fuzzy uncertain linguistic information.

Based on the above analysis, we can conclude that the current studies on the DHFS are mainly based on some aggregation operators which are limited to tackle the complex MAGDM problem under the dual hesitant fuzzy

environment. Therefore, other classical uncertain decision methods such as the grey target decision method should be further studied for real decision-making problems under dual hesitant fuzzy information. Therefore, the grey target decision method based on positive and negative clouts proposed is extended to solve the MAGDM problems with dual hesitant fuzzy information. It is the major motivation of our study. To the best of our knowledge, the existing grey target decision-making method has not yet been accommodated to deal with the dual hesitant fuzzy information provided by decision-makers. In addition, the current research on the grey target decision method has also not considered the decision-maker's loss aversion. With the development of behavioral decision theory, the MAGDM method has been widely used on the basis of prospect theory. Therefore, the proposed model considers decision-maker's loss aversion and variable weights of attributes based on the prospect theory. The contributions of this paper can be summarized as follows. (i) We proposed a novel grey target group decision method with positive and negative clouts under the dual hesitant fuzzy environment. (ii) We introduce the prospect theory to the grey target group decision.

The remainder of this paper is structured as follows. Section 2 presents the preliminaries of DHFS and DHFEs. Section 3 is the process of establishing the mathematical models. A numerical example is presented in Section 4 followed by conclusions in Section 5.

2. Preliminaries

In this section, some essential concepts are reviewed as follows [36].

Definition 1. Let X be a given set, DHFS; D is a function mapping to X :

$$D = \{ \langle x, h(x), g(x) \rangle \mid x \in X \}. \quad (1)$$

In expression (1), $h(x)$ and $g(x)$ are two sets of numbers within the interval $[0, 1]$. $h(x)$ and $g(x)$, respectively, denote the possible membership degrees and nonmembership degrees under the following conditions:

$$0 \leq r, \eta \leq 1, 0 \leq r^+ + \eta^+ \leq 1, \quad (2)$$

where $r \in h(x)$, $\eta \in g(x)$, $r^+ \in h^+(x) = \cup_{r \in h(x)} \max\{r\}$, and $\eta^+ \in g^+(x) = \cup_{\eta \in g(x)} \max\{\eta\}$ for all $x \in X$. The pair $d(x) = \langle h(x), g(x) \rangle$ is called the DHFE, which can be denoted simply by $d = \langle h, g \rangle$.

Obviously, if there is only one element in both $h(x)$ and $g(x)$, the DHFE reduces to an intuitionistic fuzzy number.

Definition 2 (see [34]). Let $d_1 = \langle h_1, g_1 \rangle$ and $d_2 = \langle h_2, g_2 \rangle$, respectively, denote two DHFEs; then, the operational laws can be defined as

$$\begin{aligned}
 d_1 \oplus d_2 &= \langle U_{r_1 \in h_1, r_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \{ \{r_1 + r_2 - r_1 r_2\}, \{\eta_1 \eta_2\} \} \rangle, \\
 d_1 \otimes d_2 &= \langle U_{r_1 \in h_1, r_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \{ \{r_1 r_2\}, \{\eta_1 + \eta_2 - \eta_1 \eta_2\} \} \rangle, \\
 \lambda d_1 &= \langle U_{r_1 \in h_1, \eta_1 \in g_1} \{ \{1 - (1 - r_1)^\lambda\}, \{(\eta_1)^\lambda\} \} \rangle, \quad \lambda > 0, \\
 d_1^\lambda &= \langle U_{r_1 \in h_1, \eta_1 \in g_1} \{ \{(r_1)^\lambda\}, \{1 - (1 - \eta_1)^\lambda\} \} \rangle, \quad \lambda > 0.
 \end{aligned}
 \tag{3}$$

Definition 3 (see [43]). Let $d_i = \langle h_i, g_i \rangle$ ($i = 1, 2, \dots, n$) denote a collection of DHFEs; then, the score function $S(d_i)$ and the accuracy function $P(d_i)$ of d_i ($i = 1, 2, \dots, n$) can be defined as

$$\begin{aligned}
 S(d_i) &= \frac{1}{\#h} \sum_{r \in h} r - \frac{1}{\#g} \sum_{\eta \in g} \eta, \\
 P(d_i) &= \frac{1}{\#h} \sum_{r \in h} r + \frac{1}{\#g} \sum_{\eta \in g} \eta,
 \end{aligned}
 \tag{4}$$

where $\#h$ and $\#g$ are the number of values in $h(di)$ and $g(di)$, respectively.

Theorem 1 (see [45]). Let $d_1 = \langle h_1, g_1 \rangle$ and $d_2 = \langle h_2, g_2 \rangle$ denote two DHFEs, and they can be compared according to the following rules following rules:

- if $S(d_1) > S(d_2)$, then, $d_1 \succ d_2$,
- if $S(d_1) = S(d_2)$, then :
- if $P(d_1) > P(d_2)$, then $d_1 \succ d_2$,
- if $P(d_1) = P(d_2)$, then $d_1 = d_2$.

Definition 4 (see [46]). Let $d_1 = \langle h_1, g_1 \rangle$ and $d_2 = \langle h_2, g_2 \rangle$ be two DHFEs. The normalized Hamming distance is defined as

$$d(d_1, d_2) = \frac{1}{2} \left(\left| \frac{1}{\#h_1} \sum_{r_1 \in h_1} r_1 - \frac{1}{\#h_2} \sum_{r_2 \in h_2} r_2 \right| + \left| \frac{1}{\#g_1} \sum_{\eta_1 \in g_1} \eta_1 - \frac{1}{\#g_2} \sum_{\eta_2 \in g_2} \eta_2 \right| \right),
 \tag{6}$$

where $\#h_i$ and $\#g_i$ ($i = 1, 2$) are the number of values in h_i and g_i , respectively.

3. A Grey Target Decision Method for MAGDM Problems with Dual Hesitant Fuzzy Information and Decision-Maker's Loss Aversion

In this section, a grey target decision method based on positive and negative clouts is developed to solve the MAGDM problem under the dual hesitant fuzzy environment and uncertain weight information.

Let $A = \{A_1, A_2, \dots, A_m\}$ be a finite set of m alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be the set of n attributes. Let $DM = \{DM_1, DM_2, \dots, DM_k\}$ be the set of decision-makers whose

weight vector is $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_k\}^T$ such that $\lambda_f \in [0, 1]$ and $\sum_{f=1}^k \lambda_f = 1$. Suppose that $Df = (d_{ij}^f)_{m \times n}$ is a dual hesitant fuzzy decision matrix given by DM_f , where $d_{ij}^f = \langle h_{ij}^f, g_{ij}^f \rangle$ denotes the DHFEs for the evaluation of the attribute C_j ($j = 1, 2, \dots, n$) with respect to the alternative A_i ($i = 1, 2, \dots, m$), with $h_{ij}^f = \cup_{r_{ij}^f \in h_{ij}^f} (r_{ij}^f)$ and $g_{ij}^f = \cup_{\eta_{ij}^f \in g_{ij}^f} (\eta_{ij}^f)$. Then, to determine the most desirable alternatives, a novel method is proposed based on the grey target decision method with positive and negative clouts.

Step 1. Aggregate all individual dual hesitant fuzzy decision matrices $Df = (d_{ij}^f)_{m \times n}$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n, f = 1, 2, \dots, k$) into a collective dual hesitant fuzzy decision matrix by the following equation [24]:

$$d_{ij} = \left\langle \bigcup_{r_{ij}^f \in h_{ij}^f, \eta_{ij}^f \in g_{ij}^f} \left\{ \left\{ 1 - \prod_{f=1}^k (1 - r_{ij}^f)^{\lambda_f} \right\}, \left\{ \prod_{f=1}^k (\eta_{ij}^f)^{\lambda_f} \right\} \right\} \right\rangle.
 \tag{7}$$

Step 2. Identifying positive and negative clouts, and defining the dual hesitant fuzzy ideal optimization scheme as the positive clout r^+ and the ideal inferior scheme as the negative clout r^- . Positive and negative clouts in the collective dual hesitant fuzzy decision matrix D can be computed according to equations (8) and (9), respectively:

$$r^+ = (r_{i1}^+, r_{i2}^+, \dots, r_{in}^+), \quad (i = 1, 2, \dots, m),
 \tag{8}$$

$$r^- = (r_{i1}^-, r_{i2}^-, \dots, r_{in}^-), \quad (i = 1, 2, \dots, m).
 \tag{9}$$

For the benefit attributes, $r_{ij}^+ = \max_{1 \leq i \leq m} d_{ij}$ and $r_{ij}^- = \min_{1 \leq i \leq m} d_{ij}$, the attribute values d_{ij} can be compared by the score function $S(d_{ij})$ and the accuracy function $P(d_{ij})$ according to Theorem 1.

Step 3. Calculating positive and negative target-eye distances based on r^+ and r^- , and the positive and negative target-eye distances of each alternative are calculated using the following equations:

$$\zeta_{ij}^+ = \sqrt{\omega_1 d_{i1}^{+2} + \omega_2 d_{i2}^{+2} + \cdots + \omega_n d_{in}^{+2}}, \quad (i = 1, 2, \dots, m), \quad (10)$$

$$\zeta_{ij}^- = \sqrt{\omega_1 d_{i1}^{-2} + \omega_2 d_{i2}^{-2} + \cdots + \omega_n d_{in}^{-2}}, \quad (i = 1, 2, \dots, m), \quad (11)$$

where $\{\omega_1, \omega_2, \dots, \omega_n\}$ denote the attribute weight vector. $d_{ij}^+ = d(d_{ij}, r_{ij}^+)$ and $d_{ij}^- = d(d_{ij}, r_{ij}^-)$ denote the normalized Hamming distances between the DHFEs d_{ij} to r_{ij}^+ and r_{ij}^- , respectively.

Step 4. Calculating comprehensive target-eye distance: the evaluation vector of each alternative is always between the positive and negative clouts. Luo and Wang [28] proved that the optimal solution can be obtained based on r_{io}^* which is on the line between positive and negative clouts. Obviously, the larger r_{io}^* , the better the corresponding scheme. The projection r_{io}^* is the comprehensive target-eye distance, and r_{io}^* can be calculated by the following equation:

$$r_{io}^* = \frac{\left| (\zeta_{ij}^+)^2 + (\zeta^0)^2 - (\zeta_{ij}^-)^2 \right|}{\zeta^0}, \quad (12)$$

where ζ^0 is the distance between positive clout ζ_{ij}^+ and negative clout ζ_{ij}^- , $\zeta^0 = d(\zeta_{ij}^+, \zeta_{ij}^-)$, which can be calculated according to equation (6).

In traditional grey target decision methods, r_{io}^* can be directly calculated when the attribute weights are known. Then, the alternatives can be sorted by r_{io}^* . In the case of unknown attribute weight, the method that how to establish the model to solve the optimal initial weight vector is to be given as follows.

Step 5. Optimization of initial attribute weights ω^0 : if the weight sequence $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is unknown, the sequence is the grey connotative sequence, and the grey entropy can be defined as

$$H_{\otimes}(\omega) = - \sum_{s=1}^n \omega_s \ln \omega_s, \quad (13)$$

According to the maximum entropy principle, ω_s ($1, 2, \dots, n$) should be adjusted to minimize the sequence uncertainty of $\omega^0 = (\omega_1, \omega_2, \dots, \omega_n)$ and maximize $H_{\otimes}(\omega)$. At the same time, considering the proximity between the effect measure of each scheme and the positive clout, the distance between the positive and negative clouts is minimized, and the optimization model is proposed as follows:

$$\begin{cases} \min \sum_{i=1}^m \sum_{j=1}^n r_{io}^*, \\ \max H_{\otimes}(\omega) = - \sum_{s=1}^n \omega_s \ln \omega_s, \\ \sum_{s=1}^n \omega_s = 1, \\ 0 \leq \omega_s \leq 1, \quad \forall s = 1, 2, \dots, n. \end{cases} \quad (14)$$

Let $f_1(\omega) = \sum_{i=1}^m \sum_{j=1}^n r_{io}^*$ and $f_2(\omega) = \sum_{s=1}^n \omega_s \ln \omega_s$; $f_1^{\max}(\omega)$ and $f_1^{\min}(\omega)$ are the maximum and minimum values of $f_1(\omega)$, respectively; $f_2^{\max}(\omega)$ and $f_2^{\min}(\omega)$ are the maximum and minimum values of $f_2(\omega)$, respectively. The two targets are dimensionless. Then, the above multi-objective optimization problems can be transformed into the following single-objective optimization problem:

$$\begin{cases} \min \lambda \left[\frac{f_1(\omega) - f_1^{\min}(\omega)}{f_1^{\max}(\omega) - f_1^{\min}(\omega)} \right] + (1 - \lambda) \left[\frac{f_2(\omega) - f_2^{\min}(\omega)}{f_2^{\max}(\omega) - f_2^{\min}(\omega)} \right], \\ \sum_{s=1}^n \omega_s = 1, \\ 0 \leq \omega_s \leq 1, \quad \forall s = 1, 2, \dots, n, \\ 0 \leq \lambda \leq 1. \end{cases} \quad (15)$$

In view of the fair competition of optimizing objective functions, usually set $\lambda = 0.5$ [28]. The optimal initial weight vector can be obtained by using available mathematical programming software. The optimal solution ω^* is the initial weight ω^0 .

Step 6. Calculating the variable weights: firstly, the relative closeness of alternatives should be calculated by the following equation:

$$z_{ij} = \frac{d(d_{ij}, r_{ij}^+)}{d(d_{ij}, r_{ij}^+) + d(d_{ij}, r_{ij}^-)}. \quad (16)$$

Then, let z_j be the relative closeness of a positive ideal solution and $S(z_j)$ be the state variable weight vector.

$$S_i(z_j) = \frac{\partial B(z_j)}{\partial z_{ij}}. \quad (17)$$

In equation (17), $B(z_j)$ is an equilibrium function.

The variable weights can be calculated by the following equation:

$$w(z_j) = \frac{\omega^0 \otimes S(z_j)}{\sum_i z_{ij}}. \quad (18)$$

Step 7. Calculating the loss values of attribute weights: taking the initial weight as the reference point, the weight loss matrix of the variable weight vector relative to the initial weight is established as F :

$$F = [F(w_i(z_j))]_{m \times n} = \begin{cases} w_i(z_j) - w_i^0, & \text{if } w_i(z_j) \geq w_i^0, \\ -(w_i^0 - w_i(z_j)), & \text{if } w_i(z_j) \leq w_i^0, \end{cases} \quad (19)$$

Step 8. Establishing the foreground theory matrix: considering the decision-maker's different risk attitudes towards the gain and loss of weight, the prospect theory matrix is established as

$$V(w_i(z_j)) = \begin{cases} F(w_i(z_j))^\alpha, & \text{if } w_i(z_j) \geq w_i^0, \\ -\theta(-F(w_i(z_j)))^\beta, & \text{if } w_i(z_j) \leq w_i^0, \end{cases} \quad (20)$$

Step 9. Ranking the alternatives according to comprehensive target-eye distances, R_{io}^* is calculated based on the variable weight vector and the prospect theory matrix. The most desirable alternative which has a higher value of R_{io}^* should be selected:

$$R_{io}^* = \sum_i V(w_i(z_j)) * |r_{io}^*|. \quad (21)$$

In summary, the decision process can be described as Figure 1.

4. An Example and the Analysis

4.1. A Numerical Example. In this section, a numerical example is given to present the application of the MAGDM method and to demonstrate its feasibility and effectiveness in a realistic scenario. An investment firm would like to invest four possible alternatives within the volume of total investment. The problem could be resolved through selecting the best options.

Three decision-makers $\{DM_1, DM_2, DM_3\}$ (the weight vector is $\lambda = (0.25, 0.40, 0.35)^T$) are invited to evaluate the alternatives based on the following three attributes: C_j ($j = 1, 2, 3$), where C_1 denotes the market share analysis, C_2 denotes the market growth analysis, and C_3 denotes the benefit analysis. In this case, all attributes are positive. The attribute weight vector is unknown. The four possible alternatives A_i ($i = 1, 2, 3, 4$) are evaluated by using the DHFEs under the above three attributes; then, the dual hesitant fuzzy decision matrices $Df = (d_{ij}^f)_{4 \times 3}$ ($i = 1, 2, 3, 4$; $j = 1, 2, 3$; and $f = 1, 2, 3$) are constructed as shown in Tables 1–3.

- (1) Aggregating all individual dual hesitant fuzzy decision matrices $Df = (d_{ij}^f)_{4 \times 3}$ ($i = 1, 2, 3, 4$; $j = 1, 2, 3$; and $f = 1, 2, 3$) into the collective dual hesitant fuzzy decision matrix $D = (d_{ij})_{4 \times 3}$ by equation (7):

- $$d_{11} = \{[0.4113, 0.4767, 0.5160, 0.5201, 0.5734, 0.6054, 0.4375, 0.5000, 0.5376, 0.5415, 0.5924, 0.6230, 0.4680, 0.5271, 0.5627, 0.5663, 0.6145, 0.6435], [0.2213, 0.2551, 0.2603, 0.3000, 0.2378, 0.2741, 0.2797, 0.3224]\}.$$
- $$d_{12} = \{[0.4150, 0.4589, 0.4561, 0.4970, 0.5026, 0.5399, 0.4622, 0.5026, 0.5000, 0.5376, 0.5427, 0.5771], [0.1569, 0.2305, 0.1845, 0.2711, 0.1737, 0.2551, 0.2042, 0.3000]\}.$$
- $$d_{13} = \{[0.3568, 0.3966, 0.4419, 0.4267, 0.4622, 0.5026, 0.4671, 0.5000, 0.5376, 0.3917, 0.4293, 0.4722, 0.4578, 0.4914, 0.5296, 0.4960, 0.5271, 0.5627], [0.2280, 0.2521, 0.2558, 0.2828, 0.2711, 0.2998, 0.3041, 0.3364]\}.$$
- $$d_{21} = \{[0.4000, 0.4371, 0.4898, 0.5214, 0.5453, 0.5734, 0.4267, 0.4622, 0.5126, 0.5427, 0.5655, 0.5924], [0.2138, 0.2312, 0.2821, 0.3050, 0.2297, 0.2484, 0.3031, 0.3278]\}.$$
- $$d_{22} = \{[0.4358, 0.5214, 0.5573, 0.4971, 0.5734, 0.6054, 0.4609, 0.5427, 0.5771, 0.5195, 0.5924, 0.6230], [0.1803, 0.2297, 0.2797, 0.3565, 0.1906, 0.2429, 0.2958, 0.3770]\}.$$
- $$d_{23} = \{[0.4006, 0.4799, 0.4518, 0.5243, 0.4671, 0.5376, 0.5126, 0.5771, 0.4960, 0.5627, 0.5390, 0.6000], [0.2305, 0.2549, 0.3041, 0.3364, 0.2741, 0.3031, 0.3617, 0.4000]\}.$$
- $$d_{31} = \{[0.4106, 0.4671, 0.5000, 0.4609, 0.5126, 0.5427, 0.4813, 0.5309, 0.5599, 0.5256, 0.5710, 0.5975], [0.2711, 0.2998, 0.3041, 0.3364, 0.3000, 0.3318, 0.3366, 0.3722]\}.$$
- $$d_{32} = \{[0.3143, 0.3566, 0.4006, 0.4377, 0.5114, 0.5416, 0.3618, 0.4013, 0.4422, 0.4767, 0.5453, 0.5734, 0.3903, 0.4280, 0.4671, 0.5000, 0.5655, 0.5924], [0.1677, 0.1933, 0.2213, 0.2551, 0.1803, 0.2077, 0.2378, 0.2741]\}.$$
- $$d_{33} = \{[0.4013, 0.4993, 0.4767, 0.5624, 0.4280, 0.5216, 0.5000, 0.5819, 0.4965, 0.5790, 0.5599, 0.6320], [0.2000, 0.2305, 0.2352, 0.2711, 0.2213, 0.2551, 0.2603, 0.3000]\}.$$
- $$d_{41} = \{[0.3364, 0.4371, 0.4794, 0.3831, 0.4767, 0.5160, 0.4004, 0.4914, 0.5296, 0.4426, 0.5271, 0.5627, 0.4958, 0.5723, 0.6044, 0.5313, 0.6024, 0.6322], [0.1978, 0.2521, 0.2219, 0.2828, 0.2352, 0.2998, 0.2639, 0.3364]\}.$$
- $$d_{42} = \{[0.4419, 0.4954, 0.5376, 0.5819, 0.5771, 0.6176, 0.4722, 0.5228, 0.5627, 0.6045, 0.6000, 0.6383], [0.1569, 0.2305, 0.2071, 0.3041, 0.1866, 0.2741, 0.2462, 0.3617]\}.$$
- $$d_{43} = \{[0.4578, 0.5296, 0.6309, 0.4960, 0.5627, 0.6569, 0.4955, 0.5622, 0.6565, 0.5309, 0.5930, 0.6807], [0.1741, 0.2219, 0.1904, 0.2426, 0.2291, 0.2921, 0.2505, 0.3193]\}.$$

- (2) Identifying the dual hesitant fuzzy positive clout r^+ and negative clout r^- of alternatives in the collective dual hesitant fuzzy decision matrix D by equations (8) and (9), respectively:

$$r^+ = (\langle \{0.4113, 0.4767, 0.5160, 0.5201, 0.5734, 0.6054, 0.4375, 0.5000, 0.5376, 0.5415, 0.5924,$$

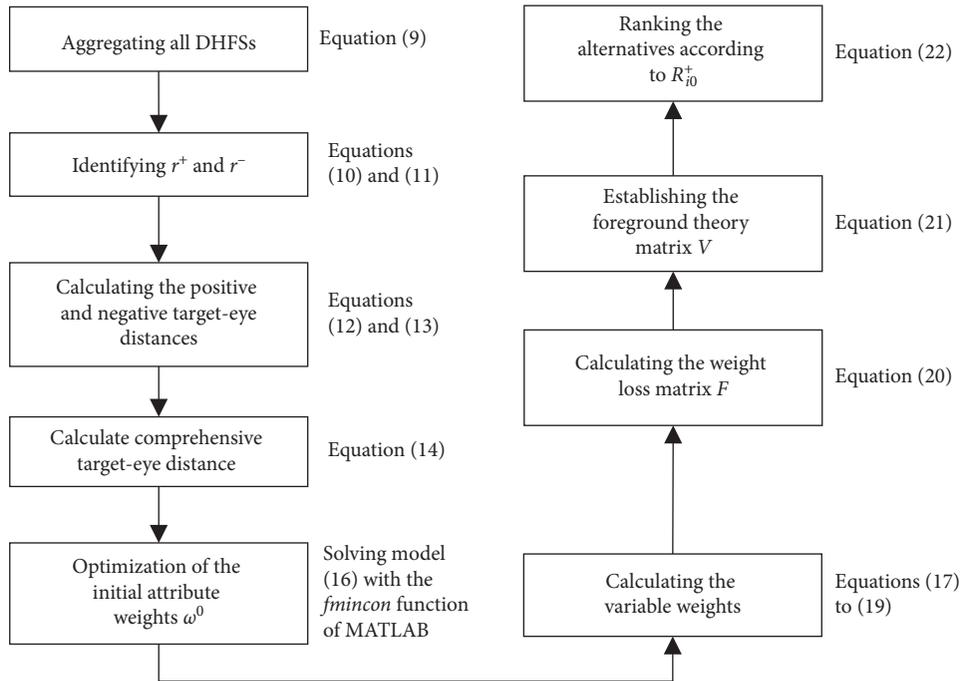


FIGURE 1: Decision flow of the proposed method.

TABLE 1: Dual hesitant fuzzy decision matrix D_1 given by DM_1 .

	C1	C2	C3
A1	A1	{{0.4 0.5 0.6}, [0.3 0.4]}	{{0.3 0.5}, [0.2 0.3]}
A2	A2	{{0.4 0.5}, [0.3 0.4]}	{{0.4 0.5}, [0.4 0.5]}
A3	A3	{{0.5 0.7}, [0.2 0.3]}	{{0.2 0.4 0.5}, [0.3 0.4]}
A4	A4	{{0.4 0.6 0.8}, [0.1 0.2]}	{{0.5 0.6}, [0.2 0.4]}

Data source: Yang and Ju [45].

TABLE 2: Dual hesitant fuzzy decision matrix D_2 given by DM_2 .

	C1	C2	C3
A1	A1	{{0.5 0.7}, [0.2 0.3]}	{{0.4 0.5 0.6}, [0.2 0.3]}
A2	A2	{{0.4 0.6 0.7}, [0.1 0.2]}	{{0.6 0.7}, [0.1 0.3]}
A3	A3	{{0.5 0.6}, [0.3 0.4]}	{{0.3 0.5 0.7}, [0.1 0.2]}
A4	A4	{{0.4 0.5}, [0.3 0.4]}	{{0.2 0.5 0.6}, [0.2 0.4]}

Data source: Yang and Ju [45].

TABLE 3: Dual hesitant fuzzy decision matrix D_3 given by DM_2 .

	C1	C2	C3
A1	{{0.3 0.5 0.6}, [0.2 0.3]}	{{0.5, 0.6}, [0.1 0.2]}	{{0.4 0.5 0.6}, [0.3 0.4]}
A2	{{0.4 0.5}, [0.4 0.5]}	{{0.2 0.5 0.6}, [0.2 0.4]}	{{0.4 0.6}, [0.3 0.4]}
A3	{{0.2 0.4 0.5}, [0.3 0.4]}	{{0.4 0.5}, [0.2 0.3]}	{{0.6 0.7}, [0.2 0.3]}
A4	{{0.2 0.5 0.6}, [0.2 0.4]}	{{0.6 0.7}, [0.1 0.3]}	{{0.4 0.6 0.8}, [0.1 0.2]}

Data source: Yang and Ju [45].

0.6230, 0.4680, 0.5271, 0.5627, 0.5663, 0.6145, 0.6435}, {0.2213, 0.2551, 0.2603, 0.3000, 0.2378, 0.2741, 0.2797, 0.3224}>, <{0.4419, 0.4954, 0.5376, 0.5819, 0.5771, 0.6176, 0.4722, 0.5228, 0.5627, 0.6045, 0.6000, 0.6383}, {0.1569, 0.2305, 0.2071, 0.3041, 0.1866, 0.2741, 0.2462, 0.3617}>, <{0.4578, 0.5296, 0.6309, 0.4960, 0.5627, 0.6569, 0.4955, 0.5622, 0.6565, 0.5309, 0.5930, 0.6807}, {0.1741, 0.2219, 0.1904, 0.2426, 0.2291, 0.2921, 0.2505, 0.3193}>).

$r^- = (\langle \{0.4106, 0.4671, 0.5000, 0.4609, 0.5126, 0.5427, 0.4813, 0.5309, 0.5599, 0.5256, 0.5710, 0.5975\}, \{0.2711, 0.2998, 0.3041, 0.3364, 0.3000, 0.3318, 0.3366, 0.3722\} \rangle, \langle \{0.3143, 0.3566, 0.4006, 0.4377, 0.5114, 0.5416, 0.3618, 0.4013, 0.4422, 0.4767, 0.5453, 0.5734, 0.3903, 0.4280, 0.4671, 0.5000, 0.5655, 0.5924\}, \{0.1677, 0.1933, 0.2213, 0.2551, 0.1803, 0.2077, 0.2378, 0.2741\} \rangle, \langle \{0.3568, 0.3966, 0.4419, 0.4267, 0.4622, 0.5026, 0.4671,$

$0.5000, 0.5376, 0.3917, 0.4293, 0.4722, 0.4578, 0.4914, 0.5296, 0.4960, 0.5271, 0.5627\}, \{0.2280, 0.2521, 0.2558, 0.2828, 0.2711, 0.2998, 0.3041, 0.3364\} \rangle).$

- (3) Calculating the positive and negative target-eye distances of each alternative from r^+ and r^- by equations (10) and (11), respectively:

$$\zeta_{ij}^+ = \left(\frac{\sqrt{0 * \omega_1 + 0.0016\omega_2 + 0.0049\omega_3}, \sqrt{3.1108e - 04\omega_1 + 3.1204e - 04\omega_2 + 0.0040\omega_3},}{\sqrt{0.0015\omega_1 + 0.0037\omega_2 + 8.3702e - 04\omega_3}, \sqrt{5.3528e - 04\omega_1 + 0 * \omega_2 + 0 * \omega_3}}, \right), \tag{22}$$

$$\zeta_{ij}^- = \left(\frac{\sqrt{0.0383\omega_1 + 0.0213\omega_2 + 0 * \omega_3}, \sqrt{0.0295\omega_1 + 0.0663\omega_2 + 0.0362\omega_3},}{\sqrt{0 * \omega_1 + 0 * \omega_2 + 0.0413\omega_3}, \sqrt{0.0350\omega_1 + 0.0608\omega_2 + 0.0702\omega_3}} \right).$$

- (4) Calculate comprehensive target-eye distance r_{io}^* by equation (12):

$$r_{io}^* = \frac{(\zeta_{ij}^+)^2 + (\zeta^0)^2 - (\zeta_{ij}^-)^2}{\zeta^0}, \tag{23}$$

where ζ^0 can be calculated by equation (6), $\zeta^0 = \sqrt{0.0015 * \omega_1 + 0.0037\omega_2 + 0.0049\omega_3}$.

- (5) Optimization of the initial attribute weights ω^0 : the *fmincon* function of MATLAB is used to solve the model, and the optimal weight can be obtained, $\omega^0 = (\omega_1, \omega_2, \omega_3) = [0.4823 \ 0.3699 \ 0.1478]$.

- (6) Calculating the variable weights: the relative closeness of alternatives can be calculated by equation (16):

$$z_{ij} = \begin{bmatrix} 0 & 0.6490 & 1.0000 \\ 0.3744 & 0.2104 & 0.6367 \\ 1.0000 & 1.0000 & 0.4121 \\ 0.3982 & 0 & 0 \end{bmatrix}. \tag{24}$$

With reference to [46, 47], as well as taking p as the reference point, the S-shaped utility curve combined with the preference of decision-makers is constructed as follows:

$$u(t) = \frac{1}{6}t^3 - \frac{1}{2}pt^2 + \left(\frac{5}{6} + \frac{1}{2}p\right)t. \tag{25}$$

Chinese people's decision-making preference is mostly an S-shaped utility curve [44] if $p = 0.5$, which is positioned as the following:

$$u(t) = \frac{1}{6}t^3 - \frac{1}{4}t^2 + \frac{13}{12}t, \tag{26}$$

that is, $\omega_i(z_j) = \frac{\omega_i^0((1/2)z_{ij}^2 - (1/2)z_{ij} + (13/12))}{\sum_{i=1}^n \omega_i^0((1/2)z_{ij}^2 - (1/2)z_{ij} + (13/12))}$. (27)

In equation (25), $i = 1, 2, \dots, 3$ and $j = 1, 2, \dots, 4$. Then, the variable weights W can be calculated by equation (18):

$$W = \begin{bmatrix} 0.5018 & 0.3444 & 0.1538 \\ 0.4760 & 0.3779 & 0.1461 \\ 0.4904 & 0.3761 & 0.1335 \\ 0.4531 & 0.3907 & 0.1561 \end{bmatrix}. \tag{28}$$

- (7) Calculating the weight loss matrix F by equation (19):

$$F = \begin{bmatrix} 0.0195 & -0.0255 & 0.0060 \\ -0.0063 & 0.0080 & -0.0017 \\ 0.0081 & 0.0062 & -0.0143 \\ -0.0292 & 0.0208 & 0.0083 \end{bmatrix}. \tag{29}$$

- (8) Establishing the foreground theory matrix V by equation (20):

$$V = \begin{bmatrix} 0.0085 & -0.0533 & 0.0020 \\ -0.0128 & 0.0029 & -0.0034 \\ 0.0029 & 0.0021 & -0.0296 \\ -0.0612 & 0.0092 & 0.0030 \end{bmatrix}. \tag{30}$$

TABLE 4: Decision results of different reference points.

Utility functions	Reference points	Alternatives				Ranking of alternatives
		ξ_1	ξ_2	ξ_3	ξ_4	
Risky utility function	$p = 0$	-0.0179	-0.0101	-0.0005	-0.0430	$A_3 > A_2 > A_1 > A_4$
	$p = 0.25$	-0.0505	-0.0265	-0.0014	-0.0321	$A_3 > A_2 > A_4 > A_1$
S-shaped utility functions	$p = 0.5$	-0.0427	-0.0515	-0.0406	-0.1100	$A_3 > A_1 > A_2 > A_4$
	$p = 0.75$	-0.0406	-0.0254	-0.0001	-0.0720	$A_3 > A_2 > A_1 > A_4$
Conservative utility function	$p = 1$	-0.0652	-0.0392	-0.0008	-0.0967	$A_3 > A_2 > A_1 > A_4$

TABLE 5: Decision results of different methods.

Methods	Ranking of alternatives
Gupta et al. [27]	$A_4 > A_2 > A_1 > A_3$
Wei et al. [43]	$A_4 > A_2 > A_1 > A_3$
This article: not considering decision-maker's loss aversion	$A_4 > A_2 > A_1 > A_3$
This article: considering decision-maker's loss aversion ($p = 0.5$)	$A_3 > A_1 > A_2 > A_4$
This article: considering decision-maker's loss aversion ($p = 0.75$)	$A_3 > A_2 > A_1 > A_4$

(9) Ranking the alternatives according to R_{io}^* .

R_{io}^* is calculated by equation (21), and $R_{io}^* = [-0.0427 - 0.0515 - 0.0406 - 0.1100]$.

The alternatives can be ranked according to R_{io}^* ; thus, $A_3 > A_1 > A_2 > A_4$. Therefore, the most desirable alternative is A_3 .

4.2. The Sensitivity Analysis. In order to illustrate the influence of reference points on decision results, the sensitivity analysis results are shown in Table 4 by changing the values of p and repeating Steps (6) to (9). As can be seen from Table 4, in all the utility functions, the optimal alternative is the same, but the ranking of evaluation results at different reference points is different. Therefore, this method fully reflects the flexibility of the process and the limited rationality of the decision-maker.

4.3. Comparison and Analysis with Previous Methods. Compared with the grey target decision method proposed in [27] and the grey relational analysis (GRA) method based on DHFS proposed in [43], the comparison results are shown in Table 5. At the same time, an ablation study is given to analyze the impact of decision-maker's loss aversion on results. We calculate the results of the grey target decision method with positive and negative clouts without considering the loss aversion factor of decision-makers. The tackle method is described as follows. Calculating Step 1 to Step 5 in Section 3, and then substituting the weight calculated in Step 5 into Step 4 to get r_{io}^* ; the alternatives can be sorted by r_{io}^* . The results show that, without considering the decision-maker's loss aversion, the decision results obtained by the three methods are the same. The reliability of the grey target decision method with positive and negative clouts and the DHFS proposed in this article is proved. After considering the decision-maker's loss aversion, the sorting results changed. Then, the decision results reflect the limited rationality and preference of decision-makers.

5. Conclusions

In this paper, a novel MAGDM problem based on the grey target decision method with positive and negative clouts under the dual hesitant fuzzy environment is proposed. An optimization model is established, which is employed to determine the initial weight vector of attributes. The optimization model is constructed with considering the proximity between the effect measures of alternatives, the positive ideal value, and the uncertainty of the attribute weight. The alternatives are sorted according to the comprehensive target-eye distance. Finally, a numerical example is presented to illustrate the application of the proposed method. The method provides a scientific and practical decision support to solve the MAGDM problem with double hesitation fuzzy numbers and decision-maker's loss aversion. This study is limited in that it cannot solve some new DHFSs, such as the probabilistic dual hesitant fuzzy set (PDHFS). Further research can introduce multiple new DHFSs to study the grey target group decision method.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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