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Research Article

Traffic Game Model with the Contract Model

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To distribute the right-of-way of intersection reasonably, the game model between each adjacent agent was obtained through agent technology. The Nash equilibrium model to measure the negotiation effects was proposed, and the existence of the Nash equilibrium solution was proved. We obtained the game equilibrium solution between each adjacent agent and thus got the equilibrium price (cost) in the network. According to the equilibrium price, travelers will choose the best path in transportation networks by adopting the path strategy with the minimum cost or fuzzy-comparison strategy. The results indicate that the contract mode algorithm makes signal control of each intersection coordinated and unified and shows subjective initiative of traffic control and management fully. The contract-based algorithm combining the management initiatives with the driver's rational behavior will make the control effectiveness of the road network system increase by 55.58%. Hence, it is an effective measurement for studying coordination between system optimality and user optimality.

1. Introduction

The target of an Intelligent Transportation System is to configure the actual traffic demand rationally in time and space by a certain control means and the induction strategies, so that the distribution equilibrium of traffic flow can be reached in the road network, and the overall resources and the traffic capacity of the road network can be fully utilized. The effectively induced information should consider the traveler's response to the information and choice preferences, which can estimate the expected benefits of both the traveler and transport system. Furthermore, the expected benefits are optimized and integrated. Then, an intelligent transport system will give the inducing advice to accord with the transport system and the driver's choices. At this point, the behavior between managers and travelers is a game.

The first application of game theory in transportation is in the form of Wardrop equilibrium in 1952, where Wardrop analyzed the traffic phenomena comprehensively and then proposed two basic principles of the user optimal equilibrium and the system optimal equilibrium about traffic flow, which was similar to Nash equilibrium of noncooperative game about N agents. After that, the game theory is increasingly applied to the field of transportation.

Game theory is the study of mathematical models of no cooperation and cooperation between rational agents. In these two kinds of game modes, players have two choices: to cooperate or to defect. In terms of traffic control at an intersection, some control strategy using a game-theoretic approach, the basic theory of independent and interdependent decision-making, has been introduced for intelligent traffic light control problem. Dong et al. [1] applied a two-person static game model in multi-intersection coordinate control problem. Technically, they proposed some concepts of game theory such as pure strategy Nash equilibrium, mixed strategy Nash equilibrium, Pareto efficiency solution, and Pareto improvement solution of Nash equilibrium for solving traffic control at multi-intersection coordination. Qi [2] establishes the congestion model of intersection between drivers and traffic administration based on the benefit-tending characteristics of the drivers and traffic administrations, an incorrect inducement for traffic administration to take the strategy that prevents vehicles from crossing the intersection during the amber light, where each intersection is regarded as a noncooperative game.

Regarding the noncooperative game, Dai et al. [3] proposed an algorithm to solve the multi-intersection coordinated control problem, which combines the maximal flow theory with the game theory and considers both the individual interests of one intersection and the interests of the whole traffic network. After that, Bel and Cassir [4], Zhou et al. [5], and Li [6] discussed the game equilibrium about ticket price between the transport operators and passengers. Transport operators made decisions on the ticket price, which is responded by passengers from changing the travel means. It is concluded that transport operators can get a competitive advantage by improving the service quality, and transport operators' strategy may benefit local targets. But it may be harmful to the whole road network target. Then Miyagi et al. [7] and Zhen [8] formulated this traffic game as a stochastic congestion game and proposed a naive user algorithm for finding a pure Nash equilibrium. An analysis of the convergence is based on the Markov chain. Finally, using a single origin-destination network connected by some overlapping paths, the validity of the proposed algorithm is tested.

Recently, in terms of the development of Internet of Things, intersections coordinate controls are regarded as smart agents which can communicate and coordinate with each other. The control of several signalized intersections in a network is decomposed into multiple subnetworks and each subnetwork is considered as a region agent. These studies explored the benefits of reducing problem complexity and improving system performance and learning efficiency when cooperation between agents is enabled. Some scholars have studied the integrated management system on the traffic management system and the traveler information system based on the agent technology, and the information communication platform between them is constructed (Ezzedine et al. [9, 10]; Liu and Wang [11]; Zhang et al. [12]; Zhen [13]; Yuan et al. [14]; Zhou et al. [15]; Zhang and Gao [16]). According to the noncooperative game between two agents, the reliability of transportation network performance was measured by Bel [17] in 2000. Miyagi et al. [18] showed a pure user Nash equilibrium which describes the route-choice behavior of a user in a traffic network comprising several discrete, interactive decision-makers, and the agents use only the utility information of the previous action to get a congestion game. It is a multiagent distributed traffic routing problem with both linear and nonlinear link cost functions. Bui et al. [19] applied a cooperative game-theoretic approach among agents to improve traffic flow with a large network. Thereby, a distributed merge and split algorithm for coalition formation is presented. However, both the timely reflection on the traffic conditions and the actions for adapting to the environment are not considered in these systems.

If traffic management can take proactive actions to adapt to the environment according to the drivers' behavior, the traffic condition will be improved significantly. Thus, we can use agent technology to make signal control system in the intersection abstract and ideological, which is conducive to the implementation of intelligent traffic management, and the analysis of multiagent behavior is more easily transformed into traffic network management.

Regarding the intersection of the road network as an agent, where the agent is the signal control system of an intersection, we adopt bid and bargain between every two adjacent agents and then obtain the equilibrium strategy price for the traffic flow in the road network. According to the latest equilibrium strategy price information provided by the intelligent traffic information system, the drivers further optimize their path selection strategy and get the most favorable path for themselves. At the same time, better system goals are achieved, and the coordination between system optimality and user optimality is retained.

2. Negotiation Model Based on Contract

2.1. Price Game between Adjacent Agents. Let a city traffic network be

$$G = (V, E), \tag{1}$$

where V is the set of intersections in the traffic network and E is the set of directed roads between two adjacent intersections. Assuming that one intersection corresponds to a signal control system, the set of signal control system in the intersection can be also denoted by V. The control system of each intersection in the traffic network represents an agent. For any agent $j \in V$, V_j is the set of the adjacent agents with j.

Denote the phase set of signal control system of intersection j by

$$S_j = \{1, 2, \ldots, s\}, \tag{2}$$

where s is the phase number of a signal control system of intersection j. Each signal phase of signal control system of intersection j gets the green light time alternately. For any $i \in V_j$, let ij be the directed road from agent i to j, and the lane set of road ij controlled by phase k of intersection j can be expressed as

$$R_{ij}^k = R_{ij} \cap R_j^k, \tag{3}$$

where R_{ij} is the lane set of the directed road ij and R_j^k is the import lane set controlled by phase k of intersection j.

Let

$$f_{ij} = \sum_{k \in S_{ji}} f_{ij}^k, \tag{4}$$

be the real-time traffic flow of road ij, where S_{ji} is the phase composition set of S_j that controls lanes of road ij, that is, $S_{ji} \subseteq S_j$, k is the phase of intersection j, and f_{ij}^k is the real-time traffic flow on lane set R_{ij}^k.

Assume that any agent i can get real-time traffic information on road ij by using the Intelligent Transportation System. Each agent i is both a traffic flow sender and a receiver except for it is the origin and destination in the network and hopes to send traffic volume to lane set R_{ij}^k at the moment of t. According to the real-time traffic flow f_{ij}^k of lane set R_{ij}^k, free flow speed, traffic volume and length of road ij, signal cycle T_j of intersection j, and corresponding phase green time of lane set R_{ij}^k, agent i will calculate the real-time traffic impedance of road ij and intersection j and is willing to quote the receiving end j for traffic impedance.

Since, on the one hand, f_{ij}^k, f_{ij}, and the green time of the phase that corresponded to lane set R_{ij}^k are changing over time and, on the other hand, free flow speed, traffic capacity and length of road ij, and signal cycle T_j of intersection j all have fixed values, we can consider the offering price of agent i to be the function of f_{ij}^k, f_{ij}, and the green time of phase k of lane set R_{ij}^k. Hence, the acceptable traffic impedance price that agent i prefers to give to phase k of receiving end agent j can be expressed as

$$p_{ij}^k = p\left(f_{ij}^k, f_{ij}, \lambda_{jk}\right), \tag{5}$$

where λ_{jk} is green time of the $k-$th phase in intersection j and p is the function formula of offering price p_{ij}^k from i to j.

Each agent i hopes to send certain traffic flow to lane set R_{ij}^k at the moment of t. If road ij accepts the traffic flow from i, then the traffic volume of road ij changes to

$$f_{ij}^{k1} = f_{ij}^k + f_0^k, \tag{6}$$

where f_0^k is the traffic volume that agent i sends to lane set R_{ij}^k at the moment of t.

According to the traffic flow f_{ij}^{k1}, where after road ij accepts traffic flow f_0^k, green time λ_{jk}, and static property of road ij, the traffic impedance of road ij and the delay of intersection j are calculated by the receiving end agent j, furthermore, the impedance bargain q_{ji}^k of intersection j which is willing to accept traffic flow f_0^k from i can be obtained as

$$q_{ji}^k = q\left(f_{ij}^{k1}, f_{ij}, \lambda_{jk}\right), \tag{7}$$

where q is the function of that j bargaining q_{ji}^k to i. When agent i makes an offer p_{ij}^k above or equal to bargaining q_{ji}^k of agent j, the two sides will reach an agreement which leads to the successful negotiations.

As set S_{ji} may contain several elements, it means the traffic flow of road ij can be controlled by multiple phases of j intersection signal control system. For any $k \in S_{ji}$, there exists a corresponding bid p_{ij}^k. Hence, there exists one set of bids from i to j, which will form a bid fracture surface, and it is denoted by

$$p_{ij} = \left(\ldots, \left(p_{ij}^k\right)_{k\in S_{ji}}, \ldots\right), \tag{8}$$

where $(p_{ij}^k)_{k\in S_{ji}}$ is the set of bids from i to j. For example, if $S_{ji} = \{1,2,3\}$, then p_{ij} can be got as

$$p_{ij} = \left(p_{ij}^1, p_{ij}^2, p_{ij}^3\right). \tag{9}$$

For any $k \in S_{ji}$, p_{ij}^k is a function of f_{ij}^k, f_{ij}, and λ_{jk}, where f_{ij} is given by

$$f_{ij} = \sum_{k\in S_{ji}} f_{ij}^k, \tag{10}$$

and then a bid fracture surface p_{ij} from i to j is indeed a function of f_{ij}^k and λ_{jk}; that is,

$$p_{ij} = p\left(\ldots, \left(f_{ij}^k\right)_{k\in S_{ji}}, \ldots, \left(\lambda_{jk}\right)_{k\in S_{ji}}, \ldots\right), \tag{11}$$

where $(f_{ij}^k)_{k\in S_{ji}}$ is the traffic flow group of road ij controlled by different phases before road ij accepts f_0^k, and $(\lambda_{jk})_{k\in S_{ji}}$ is the green time group of traffic flow phase on the control road ij. Similarly, the bargain fracture surface and the function of bargain fracture surface from j to i can be deduced as follows:

$$q_{ji} = \left(\ldots, \left(q_{ji}^k\right)_{k\in S_{ji}}, \ldots\right), \tag{12}$$

$$q_{ji} = q\left(\ldots, \left(f_{ji}^{k1}\right)_{k\in S_{ji}}, \ldots, \left(\lambda_{jk}\right)_{k\in S_{ji}}, \ldots\right), \tag{13}$$

where $(q_{ji}^k)_{k\in S_{ji}}$ is the bargain group from j to i and $(f_{ji}^{k1})_{k\in S_{ji}}$ is the traffic flow group by different phases control on road ij after road ij accepts f_0^k.

If the traffic volume of road ij is f_{ij}, the traffic impedance of road ij is thus

$$t_{ij} = \frac{l_{ij}}{v\left(f_{ij}\right)}, \tag{14}$$

where l_{ij} is the length of road ij and $v(f_{ij})$ is the vehicle speed on road ij. Let

$$u_{ji}^k = t_{ij} + d_{ij}^k, \tag{15}$$

where u_{ji}^k is the traffic impedance of vehicle going through road ij and d_{ij}^k is the red light delay of the import lanes of road ij by the $k-$th phase control, and the traffic impedance u_{ji}^k can also be referred to as the status quo point from agent j to i, which is on road ij by the $k-$th phase control. Agent i quotes impedance p_{ij}^k by the $k-$th phase control lane of signal control system of intersection j, which should be higher than or equal to the status quo point u_{ji}^k from agent j to i; that is,

$$p_{ij}^k \geq u_{ji}^k. \tag{16}$$

Otherwise, agent j will refuse to continue negotiations with agent i on the $k-$th phase.

After j receives traffic flow f_0^k from i, we denote M_{ij}^k to be the status quo point from agent i to agent j on road ij controlled by the $k-$th phase, and it holds

$$M_{ij}^k = t_{ij}^1 + d_{ij}^{k1}, \tag{17}$$

where M_{ij}^k is the traffic impedance that vehicles pass the directed road ij after j received traffic flow f_0^k from i, t_{ij}^1 is the traffic impedance of road ij after j receives traffic flow f_0^k from i, and d_{ij}^{k1} is the red light delay on the import lanes of road ij by the $k-$th phase control after j receives traffic flow f_0^k from i. Agent j counteroffers q_{ji}^k to agent i which should not be higher than the status quo point M_{ij}^k from agent i to j, and agent i will refuse negotiation with j if not; that is,

$$q_{ji}^{k} \leq M_{ij}^{k}. \tag{18}$$

As S_{ji} may have several elements and $k \in S_{ji}$, the status quo points from j to i and from i to j will form an individual fracture surface, respectively, which are expressed as

$$u_{ji} = \left(\ldots, \left(u_{ji}^{k}\right)_{k\in S_{ji}}, \ldots\right), \tag{19}$$

$$M_{ij} = \left(\ldots, \left(M_{ij}^{k}\right)_{k\in S_{ji}}, \ldots\right), \tag{20}$$

where $(u_{ji}^{k})_{k\in S_{ji}}$ is the status quo point group from j to i, $(M_{ij}^{k})_{k\in S_{ji}}$ is the status quo point group from i to j, u_{ji} is the status quo point fracture surface from j to i, and M_{ij} is the status quo point fracture surface from i to j. Quotes fracture surface p_{ij} should be preferable to u_{ji}; that is to say,

$$p_{ij} \succ u_{ji}, \tag{21}$$

where $\succ$ means it holds that, for any $k \in S_{ji}$,

$$p_{ij}^{k} \geq u_{ji}^{k}. \tag{22}$$

Furthermore, the bargain fracture surface from j to i should meet the demand of

$$q_{ji} \prec M_{ij}. \tag{23}$$

2.2. Nash Equilibrium of the Negotiation between Agents. Assume that all the agents in V are individually rational; that is to say their acquisition is as good as before after knowing the final results of the negotiation. Agent i quotes each fracture surface p_{ij} which will compare own status quo point fracture surface M_{ij} to j, and agent j counteroffers each fracture surface q_{ji} which will compare own status quo point fracture surface u_{ji} to i as well. For any $k \in S_{ji}$, the utilities obtained by i and j are $M_{ij}^{k} - p_{ij}^{k}$ and $q_{ji}^{k} - u_{ji}^{k}$, respectively, under each bid and bargain. If the negotiation reaches an agreement, the arithmetic product of utility with agents i and j in phase k can be got as

$$U_{ij}^{k} = \left(M_{ij}^{k} - p_{ij}^{k}\right)\left(q_{ji}^{k} - u_{ji}^{k}\right), \tag{24}$$

where it must meet

$$p_{ij}^{k} \geq q_{ji}^{k}. \tag{25}$$

Thus, we have

$$\begin{gathered} U_{ij} = \sum_{k\in S_{ji}} U_{ij}^{k}, \\ p_{ij}^{k} \geq q_{ji}^{k}, \\ j \in V, \\ i \in V_{j}, \end{gathered} \tag{26}$$

where U_{ij} can be regarded as the measurement of negotiation price agreements effect between i and j. The negotiators are all individually rational, and they hope the final price agreement is preferable to that of their status quo points, and the parties have the will to reach an agreement. Now, let us talk about the final agreement.

The bid price strategy between every two adjacent agents in the traffic network forms a fracture surface, which can be expressed as

$$R = \left(\ldots, \left(p_{ij}, q_{ji}\right)_{j\in V, i\in V_{j}}, \ldots\right), \tag{27}$$

where $(p_{ij}, q_{ji})_{j\in V, i\in V_{j}}$ is a bid and bargain strategy for any two adjacent agents in the traffic network. We denote $\mathfrak{R}$ to be the set of all strategy fracture surfaces R and B^{+} to be the set of all possible traffic distributions. For any traffic distribution $B \in B^{+}$ and $R \in \mathfrak{R}$, if there does not exist $B^{/} \in B^{+}$ and $B^{/} \succ B$ is made, we conclude that traffic distribution B is valid to strategy fracture R. Let $P(R)$ be the valid set (Pareto set) of all traffic distributions to R.

Definition 1 (validity). The validity means all agents' strategy fracture surface R, of which the results of traffic distribution are $B \in P(R)$; that is to say, all the agents always choose the most advantageous strategy.

Theorem 1. *If any two adjacent agents in the network are individually rational and follow the validity, the following hold: ① the negotiation between them can reach an agreement; ② the final agreement price reaches Nash equilibrium; that is,*

$$p_{ij}^{k^{*}} = p_{ij}^{k} = q_{ji}^{k} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}, \quad k \in S_{ji}. \tag{28}$$

Proof. ① According to individual rationality, let the initial bid price of agent i be the status quo point u_{ji}^{k} from j to i for any $k \in S_{ji}$. Then, agent i increases his bid price according to the bargain price from agent j. On the other hand, the initial bargain price of agent j is the status quo point M_{ij}^{k} from i to j, and agent j then decreases his bargain price according to the increasing value size of bid price of agent i. Since both sides' bid prices are preferable to their status quo points, respectively, they both wish to reach an agreement. In order to do this, the bid price of i will be higher as long as the bargain price of j will be lower. When the n-th bid and bargain price of them is

$$p_{ij}^{kn} \geq q_{ji}^{kn}, \tag{29}$$

they finally achieve an agreement with the price:

$$p_{ij}^{k^{*}} = p_{ij}^{kn}. \tag{30}$$

Thus, their negotiation can achieve an agreement.

② Assume that both sides achieve an agreement at the l-th bid p_{ij}^{kl} and bargain q_{ji}^{kl}; that is,

$$p_{ij}^{kl} \geq q_{ji}^{kl}. \tag{31}$$

If

$$p_{ij}^{kl} > q_{ji}^{kl}, \tag{32}$$

according to formula (26), the effect of the price agreement of their negotiation is

$$U_{ij}^{k} = \left(M_{ij}^{k} - p_{ij}^{kl}\right)\left(q_{ji}^{kl} - u_{ji}^{k}\right). \tag{33}$$

As

$$p_{ij}^{kl} > q_{ji}^{kl}, \tag{34}$$

we can get that

$$U_{ij}^{k} < \left(M_{ij}^{k} - p_{ij}^{kl}\right)\left(p_{ji}^{kl} - u_{ji}^{k}\right). \tag{35}$$

If the agreed price of both sides is

$$p_{ij}^{k^*} = p_{ij}^{k} = q_{ji}^{k} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}, \tag{36}$$

the arithmetic product of effectiveness for their negotiation is

$$U_{ij}^{k^*} = \left(M_{ij}^{k} - p_{ij}^{k^*}\right)\left(p_{ij}^{k^*} - u_{ji}^{k}\right) = \left(\frac{M_{ij}^{k} - u_{ji}^{k}}{2}\right)^2. \tag{37}$$

Since

$$\begin{aligned}
U_{ij}^{k^*} - U_{ij}^{k} &> \left(\frac{M_{ij}^{k} - u_{ji}^{k}}{2}\right)^2 - \left(M_{ij}^{k} - p_{ij}^{kl}\right)\left(p_{ij}^{kl} - u_{ji}^{k}\right) \\
&= \frac{\left(M_{ij}^{k}\right)^2 + \left(u_{ji}^{k}\right)^2 - 2M_{ij}^{k}u_{ji}^{k}}{4} - \left[M_{ij}^{k}p_{ij}^{kl} - M_{ij}^{k}u_{ji}^{k} - \left(p_{ij}^{kl}\right)^2 + p_{ij}^{kl}u_{ji}^{k}\right] \\
&= \frac{\left(M_{ij}^{k}\right)^2 + \left(u_{ji}^{k}\right)^2 + 2M_{ij}^{k}u_{ji}^{k}}{4} - \left(M_{ij}^{k} + u_{ji}^{k}\right)p_{ij}^{kl} + \left(p_{ij}^{kl}\right)^2 \\
&= \left(\frac{M_{ij}^{k} + u_{ji}^{k}}{2}\right)^2 - \left(M_{ij}^{k} + u_{ji}^{k}\right)p_{ij}^{kl} + \left(p_{ij}^{kl}\right)^2 \\
&= \left(\frac{M_{ij}^{k} + u_{ji}^{k}}{2} - p_{ij}^{kl}\right)^2 \geq 0,
\end{aligned} \tag{38}$$

that is,

$$U_{ij}^{k^*} > U_{ij}^{k}, \tag{39}$$

the Nash equilibrium price of both sides is thus

$$p_{ij}^{k^*} = p_{ij}^{k} = q_{ji}^{k} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}. \tag{40}$$

Hence, for any $k \in S_{ji}$, if

$$p_{ij}^{k^*} = p_{ij}^{k} = q_{ji}^{k} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}, \tag{41}$$

it holds that

$$U_{ij} = \sum_{k \in S_{ji}} U_{ij}^{k}, \tag{42}$$

reaches maximum, and then $p_{ij}^{k^*}$ is the Nash equilibrium price of both agents i and j. According to the validity, bid price of both sides will be selected in the Pareto set, which yields that the final agreement price must be the Nash equilibrium price.

For agent j at any receiving side, it will receive many adjacent agents' quotation. Whose quotation will be prioritized is directly related to the utility obtained by j. So the priority will be given to the specific traffic route, which obtains the largest system utility. Thus, agent j has to balance all agents' transportation requests in V_j and use the available resources (such as adjusting the green signal ratio of imported lanes, etc.) to meet the prior request. □

Theorem 2. *If the traffic flow receiver agent j and the traffic flow sender agent i in V_j are all individually rational and follow the validity, we can get the following: ① each agent in V_j can reach an agreement with agent j, respectively; ② the Nash equilibrium of all agents utility arithmetic product,*

$$U_j = \prod_{i \in V_j} U_{ij}, \tag{43}$$

is the final agreement price, which is expressed as

$$p_{ij}^{k^*} = p_{ij}^{k} = q_{ji}^{k} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}, \quad k \in S_{ji}, i \in V_j. \tag{44}$$

Proof. (1) According to Theorem 1, for any agent i and agent j, they can achieve a final agreement. Hence, each agent in V_j can reach an agreement with agent j, respectively.

(2) According to Theorem 1, for any agent $i \in V_j, k \in S_{ji}$, if the final agreement price between two agents is

$$p_{ij}^{k^*}=p_{ij}^k=q_{ji}^k=\frac{M_{ij}^k+u_{ji}^k}{2}, \tag{45}$$

we can obtain

$$\left(M_{ij}^k-p_{ij}^{k^*}\right)\left(p_{ij}^{k^*}-u_{ji}^k\right)>\left(M_{ji}^k-p_{ji}^k\right)\left(q_{ji}^k-u_{ji}^k\right), \tag{46}$$

where

$$p_{ij}^k>q_{ji}^k, \tag{47}$$

that is,

$$U_{ij}^*=\sum_{k\in S_{ji}}\left(M_{ij}^k-p_{ij}^{k^*}\right)\left(p_{ij}^{k^*}-u_{ji}^k\right)>\sum_{k\in S_{ji}}\left(M_{ij}^k-p_{ij}^k\right)\left(q_{ij}^k-u_{ji}^k\right) = U_{ij}. \tag{48}$$

This leads to

$$U_j^*=\prod_{i\in V_j}U_{ij}^*>\prod_{i\in V_j}U_{ij}=U_j, \tag{49}$$

that is,

$$U_j^*>U_j. \tag{50}$$

Thus, the Nash equilibrium of

$$U_j=\prod_{i\in V_j}U_{ij}, \tag{51}$$

is

$$p_{ij}^{k^*}=p_{ij}^k=q_{ji}^k=\frac{M_{ij}^k+u_{ji}^k}{2}, \tag{52}$$

where $k\in S_{ji}$ and $i\in V_j$. According to the validity, the bid prices of all agents in V_j and agent j will be selected in the Pareto set, and the final agreement price between j and his adjacent agents must be the Nash equilibrium price:

$$p_{ij}^{k^*}=p_{ij}^k=q_{ji}^k=\frac{M_{ij}^k+u_{ji}^k}{2}, \quad k\in S_{ji}, i\in V_j. \tag{53}$$

Agent j can change the price by adjusting the split ratio to achieve the purpose of regulating road traffic and control the traffic volume of the up-down stream. For example, according to the existing number of vehicles in the upstream road and the traffic volume that prepare to enter the road, the agent decreases the traffic volume that is prepared to enter the road by increasing the bargain price to the adjacent agent in the upper stream road and increase outflow by improving the quotation to the downstream adjacent agents. This may guarantee the entire road network traffic flow to be balanced. □

Theorem 3. *If every two adjacent agents in V have a price negotiation with each other and they are all individually rational and follow the validity, we can get the following: ① all adjacent agents of both sides will reach an agreement; ② the entire road network has Nash equilibrium price, and the final price agreement is the Nash equilibrium solution to*

$$U=\prod_{j\in V}U_j, \tag{54}$$

which is expressed as

$$p_{ij}^{k^*}=p_{ij}^k=q_{ji}^k=\frac{M_{ij}^k+u_{ji}^k}{2}, \quad k\in S_{ji}, j\in V, i\in V_j. \tag{55}$$

Proof. (1) According to Theorem 2, for any $j\in V$ and all the agents in V_j, they can reach an agreement. By the arbitrariness of j, all adjacent negotiations of both sides can reach an agreement in the road network.

(2) According to Theorem 2, for any $j\in V$, $i\in V_j$, and $k\in S_{ji}$, if

$$p_{ij}^k>q_{ji}^k, \tag{56}$$

it holds that

$$U_j^*>U_j, \tag{57}$$

and we thus get that

$$U^*=\prod_{j\in V}U_j^*>\prod_{j\in V}U_j, \tag{58}$$

that is,

$$U^*>U. \tag{59}$$

This yields that the Nash equilibrium solution of U is

$$p_{ij}^{k^*}=p_{ij}^k=q_{ji}^k=\frac{M_{ij}^k+u_{ji}^k}{2}, \quad k\in S_{ji}, j\in V, i\in V_j. \tag{60}$$

According to the validity, every two adjacent agents select their quotation price in the Pareto set, and the final agreement price must be Nash equilibrium solution in the road network. That is, the entire road network forms the Nash equilibrium price. □

2.3. Confirming the Green Time of the Signal Control System in the Intersection. It follows from Theorem 3 that j could obtain the largest utility by adjusting the green time λ_{jk} in the k-th phase when facing any agent i in V_j. Due to the individual rationality, for any $i\in V_j$ and $k\in S_{ji}$, j will increase the green time λ_{jk} in the k-th phase if j can obtain larger utility from i and reduce λ_{jk} conversely if not. Meanwhile, to prevent conflict of green light, we assume that when any phase in an intersection is a green light or yellow one, the others are all red light. Notice that the vehicle may get through the intersection on the yellow time; we fix the green time λ_{jk} here including the yellow time in the k-th phase for discussing conveniently. Therefore, it holds that

$$\sum_{k\in S_j}\lambda_{jk}=T_j, \tag{61}$$

where T_j is the signal circle of the signal control system in intersection j. On the grounds of Theorem 1, together with the status quo point

$$u_{ji}^{k} = t_{ij} + d_{ij}^{k}. \tag{62}$$

from j to i and the status quo point

$$M_{ij}^{k} = t_{ij}^{1} + d_{ij}^{k1}. \tag{63}$$

from i to j, we thus obtain the equilibrium price as follows:

$$p_{ij}^{k^*} = \frac{M_{ij}^{k} + u_{ji}^{k}}{2}, \tag{64}$$

where the traffic impedance of the intersection part is $p_{ij}^{k^*} - (t_{ij} + t_{ij}^{1})/2$. Taking the maximum of intersection stop delay controlled by the k-th phase in different roads as an example, that is, $\vee_{R_{ij}^{k} \in R_{j}^{k}} (p_{ij}^{k^*} - (t_{ij} + t_{ij}^{1})/2)$, the green time of the k-th phase in intersection j can then be denoted by

$$\lambda_{jk} = \min\left\{\lambda_1, \max\left\{\frac{\vee_{R_{ij}^{k} \in R_{j}^{k}}\left(p_{ij}^{k^*} - \left(t_{ij} + t_{ij}^{1}\right)/2\right)}{\sum_{k \in S_j}\left[\vee_{R_{ij}^{k} \in R_{j}^{k}}\left(p_{ij}^{k^*} - \left(t_{ij} + t_{ij}^{1}\right)/2\right)\right]} T_j, \lambda_0\right\}\right\}, \tag{65}$$

where λ_0 is the minimum green time to make sure vehicles and pedestrians get through the intersection safely and λ_1 is the maximum green time of signal phase for intersection j.

3. The Driver Path Selection Rules Based on the Price Information

Let K be the origin set of the OD and let S be the destination set of the OD; it is obvious that $K \subset V$ and $S \subset V$, and all the path set between origin site and the destination one is

$$Z^{\kappa\ell} = \{z_1, z_2, \ldots, z_m\}, \tag{66}$$

where $\kappa \in K$, $\ell \in S$, and z_m is the m-th path from origin κ to destination ℓ. For any path $z_\lambda \in Z^{\kappa\ell}$, it can be expressed as

$$z_\lambda = \left\{\kappa = k_0, k_1, \ldots, k_{m_\lambda} = \ell\right\}, \tag{67}$$

where k_0 is the first intersection on path z_λ from κ to ℓ and k_{m_λ} is the last intersection on path z_λ. The driver hopes to pay the least expenditure for starting from the origin site κ to the destination site; that is to say, he will choose the path with the lowest equilibrium strategy price in the network. Theorem 3 tells that all the adjacent agents in the network can reach an agreement and get an equilibrium price, and thus the intelligent traffic system can continuously provide the driver with the real-time equilibrium price between every two adjacent agents. With the equilibrium strategy price at hand, the driver calculates all path prices from κ to ℓ, which is indeed a price function that can be expressed as follows:

$$U^{\kappa\ell} = \left\{U_{z_\lambda}^{\kappa\ell} \mid U_{z_\lambda}^{\kappa\ell} = \sum_{i \in z_\lambda} p_{ij}^{k^*}, \text{and } i = k_a, j = k_{a+1}, k_a, k_{a+1} \in z_\lambda, z_\lambda \in Z^{\kappa\ell}\right\}, \tag{68}$$

where $p_{ij}^{k^*}$ is the Nash equilibrium price from i to j and $U_{z_\lambda}^{\kappa\ell}$ is the price of path z_λ.

Let the driver set with traffic demand between origin κ and destination ℓ be

$$N = \{1, 2, \ldots, n\}, \tag{69}$$

where n is the total number of the drivers with traffic demand between origin κ and destination ℓ. The price information in the Intelligent Transportation System is updated with a certain period of time which is assumed to be T_0. Assume that the equilibrium price information of Intelligent Transportation Systems is updated once at the moment of t_1. Now, during the period from moment t_1 to $t_1 + T_0$, the equilibrium price of each path between origin κ and destination ℓ has changed with vehicles entering the road network, which is in sharp contrast to the price information in the Intelligent Transportation System that did not update in time. In this case, if the driver selects the path with the least sum of the price which is provided by Intelligent Transportation Systems, the actual price of the path may not be the lowest. As the received price information is not exactly coincided with the actual information, the different path strategies may be selected by different drivers according to their age, gender, education level, and other factors. Assume that the driver has two strategies to choose: the first one is to choose the path with the lowest price offered by the Intelligent Transportation Systems at the moment of t_1; the second one is based on the path price of the Intelligent Transportation System offered at the moment of t_1, and choose the path in $Z^{\kappa\ell}$ by using fuzzy-comparison strategy.

Let

$$Z^{\kappa\ell} = \left\{z_{\pi(1)}, z_{\pi(2)}, \ldots, z_{\pi(m)}\right\}, \tag{70}$$

where π is a rearrangement of all the paths in $Z^{\kappa\ell}$, which makes the price relationship of each path as follows:

$$U_{z_{\pi(1)}}^{\kappa\ell} \le U_{z_{\pi(2)}}^{\kappa\ell} \le \cdots \le U_{z_{\pi(m)}}^{\kappa\ell}. \tag{71}$$

Hence, the equilibrium price for path $z_{\pi(1)}$ is

$$U_{z_{\pi(1)}}^{\kappa\ell} = \min U^{\kappa\ell}. \tag{72}$$

If the driver adopts the first strategy, then path $z_{\pi(1)}$ will be chosen.

The theoretic basis of the driver adopting the second strategy is that drivers calculate the price of each path by the Intelligent Traffic Information System at the moment of t_1, and price $U_{z_{\pi(1)}}^{\kappa\ell}$ of path $z_{\pi(1)}$ with the smallest traffic impedance between κ and ℓ was obtained by calculating. With the vehicles entering and exiting the path, the price on the path has been changed in the period of time of $[t_1, t_1 + T_0)$. Indeed, this yields the fuzziness of the price. Next, let the fuzzy price interval of $z_{\pi(1)}$ be

$$\widetilde{U}_{z_{\pi(1)}}^{\kappa\ell} = \left[(1-\eta)U_{z_{\pi(1)}}^{\kappa\ell}, (1+\eta)U_{z_{\pi(1)}}^{\kappa\ell}\right], \tag{73}$$

where $0 \le \eta \le 1$ represents the fuzzy degrees of the path price provided by the Intelligent Transportation Information System. The probability of choosing path z_r for the drivers adopting the second strategy is as follows:

$$\left\{\varphi\left(z_r\right): Z^{\kappa\ell} \longmapsto [0,1], \text{and} \sum_{z_r \in Z^{\kappa\ell}} \varphi\left(z_r\right) = 1\right\}, \tag{74}$$

where $\varphi(z_r)$ is the probability of the driver chosen path z_r.

In general, the driver of the second strategy regards only parts of the paths between κ and ℓ in the period of time of $[t_1, t_1 + T_0)$, but not all of them, as a candidate one. Consequently, the driver should choose the path whose price belongs to $\widetilde{U}^{\kappa\ell}_{z_{\pi(1)}}$ in $Z^{\kappa\ell}$ before entering the network; and we denote these paths to be a set as follows:

$$A = \left\{z_{\pi(e)} \mid z_{\pi(e)} \in Z^{\kappa\ell}, 2 \le e \le h\right\}, \tag{75}$$

where h is a positive integer lesser than or equal to m which satisfies

$$U^{\kappa\ell}_{z_{\pi(h)}} \in \widetilde{U}^{\kappa\ell}_{z_{\pi(1)}}, \tag{76}$$

$$U^{\kappa\ell}_{z_{\pi(h+1)}} \notin \widetilde{U}^{\kappa\ell}_{z_{\pi(1)}}. \tag{77}$$

And m is the total number of paths between κ and ℓ. The driver randomly selects the path in set C by a certain probability, where C is expressed as

$$C = A \cup \left\{z_{\pi(1)}\right\}. \tag{78}$$

If A is empty, the driver chooses path $z_{\pi(1)}$ unconditionally. If A is not empty, the path will be chosen with much higher probability as the gap between the path price in A and $U^{\kappa\ell}_{z_{\pi(1)}}$ is smaller. According to the specificity of $z_{\pi(1)}$ in $Z^{\kappa\ell}$, we define the probability of $z_{\pi(1)}$ to be chosen as

$$\varphi'\left(z_{\pi(1)}\right) = 1 - \varphi'\left(z_{\pi(2)}\right), \tag{79}$$

where $z_{\pi(2)}$ is the lowest-price path in set A and φ' is the probability of $z_{\pi(1)}$ to be chosen. Recalling that

$$U^{\kappa\ell}_{z_{\pi(1)}} \le U^{\kappa\ell}_{z_{\pi(e)}} \le (1+\eta) U^{\kappa\ell}_{z_{\pi(1)}}, \tag{80}$$

it holds that the probability of path $z_{\pi(e)}$ to be chosen is

$$\begin{aligned}\varphi'\left(z_{\pi(e)}\right) &= \frac{(1+\eta)U^{\kappa\ell}_{z_{\pi(1)}} - U^{\kappa\ell}_{z_{\pi(e)}}}{(1+\eta)U^{\kappa\ell}_{z_{\pi(1)}} - U^{\kappa\ell}_{z_{\pi(1)}}} \\ &= \frac{(1+\eta)U^{\kappa\ell}_{z_{\pi(1)}} - U^{\kappa\ell}_{z_{\pi(e)}}}{\eta U^{\kappa\ell}_{z_{\pi(1)}}}.\end{aligned} \tag{81}$$

When $2 \le e \le h$, thus

$$\varphi'\left(z_{\pi(e)}\right) = \begin{cases} 1 - \varphi'\left(z_{\pi(2)}\right), & e = 1, \\ \dfrac{(1+\eta)U^{\kappa\ell}_{z_{\pi(1)}} - U^{\kappa\ell}_{z_{\pi(e)}}}{\eta U^{\kappa\ell}_{z_{\pi(1)}}}, & 2 \le e \le h, \\ 0, & h < e \le m. \end{cases} \tag{82}$$

Furthermore, we obtain after normalizing (82) that the probability of each path chosen by the driver is

$$\varphi\left(z_{\pi(e)}\right) = \frac{\varphi'\left(z_{\pi(e)}\right)}{\sum_{k=1}^{m} \varphi'\left(z_{\pi(k)}\right)}, \quad 1 \le e \le m. \tag{83}$$

This is the resulting path selection rule for the drivers of the second strategy by applying the fuzzy-comparison method. More precisely, the Intelligent Transportation System updates all path prices in the network at the moment of $t_1 + T_0$. Next, the drivers in the network choose the path by the latest information of path price in the period of time of $[t_1 + T_0, t_1 + 2T_0)$. Then, the network price will be updated again at the moment of $t_1 + 2T_0$. The above steps repeat indefinitely to be a circle.

4. Algorithm

According to Theorem 3, the driver obtains the Nash equilibrium price of each path in the network by using an Intelligent Transportation System. Based on the equilibrium price, the algorithm of the dynamic traffic game model can be given as follows:

Step 1: We will give some initial data as follows: the initial value of traffic volume of all the paths in the network is $f_{\pi(e)} = f_0$, the initial value of green time is λ_{jk}, the driver ratio of the first strategy is γ, the traffic volume of entering the network in the period of time of T_0 is $f^{\kappa\ell}$, and the demand of all OD in the network is $d^{\kappa\ell}$; let $d_0^{\kappa\ell} = 0$ and $t_0^{\kappa\ell} = 0$, where $\kappa \in K$ and $\ell \in S$.

Step 2: Calculate t_{ij}, d_{ij}^k, t_{ij}^1, and d_{ij}^{k1}, respectively, and then also

$$u_{ji}^k = t_{ij} + d_{ij}^k, \tag{84}$$

$$M_{ij}^k = t_{ij}^1 + d_{ij}^{k1}. \tag{85}$$

This leads to

$$p_{ij}^{k^*} = \frac{M_{ij}^k + u_{ji}^k}{2}. \tag{86}$$

We will calculate λ_{jk} by formula (65), where $j \in V$, $i \in V_j$, and $k \in S_{ji}$.

Step 3: Calculating $U^{\kappa\ell}$ on the basis of formula (68) and ranking all the elements in $U^{\kappa\ell}$, it holds that

$$\left\{U^{\kappa\ell}_{z_{\pi(1)}}, U^{\kappa\ell}_{z_{\pi(2)}}, \ldots, U^{\kappa\ell}_{z_{\pi(m)}}\right\}. \tag{87}$$

Step 4: If an OD is

$$d_0^{\kappa\ell} \ge d^{\kappa\ell}. \tag{88}$$

We ignore distributing it, and if

$$d_0^{\kappa\ell} < d^{\kappa\ell}. \tag{89}$$

$\varphi'(z_{\pi(e)})$ will be got by using formula (82). Then, $\varphi(z_{\pi(c)})$ can be calculated by formula (83). Here, c is a positive integer satisfying $1 \le c \le m$.

Step 5: Let

$$f^{\kappa\ell}_{\pi(1)} := f^{\kappa\ell}_{\pi(1)} + \gamma f^{\kappa\ell}, \tag{90}$$

$$f_{\pi(e)} := f_{\pi(e)} + (1-\gamma) f^{\kappa\ell} \varphi\left(z_{\pi(e)}\right), \tag{91}$$

where $1 \le e \le m$. Set

$$d_0^{\kappa\ell} := d_0^{\kappa\ell} + f^{\kappa\ell}, \tag{92}$$

$$t_0^{\kappa\ell} := t_0^{\kappa\ell} + T_0. \tag{93}$$

Step 6: If all the OD meet the condition

$$d_0^{\kappa\ell} \ge d^{\kappa\ell}, \tag{94}$$

we thus stop calculating; otherwise, we get back to Step 2.

This algorithm cycles on the basis of cycle time T_0. There is a traffic distribution to the entire network in each cycle and to test the distribution of each OD demand in the network. If the distribution of an OD demand is finished, the allocation of the next cycle will skip the OD demand, until all OD demand allocations come to an end. The price information in the algorithm is constantly updated over time, and it also reflects the human behavior factors in the distribution. Hence, it is much closer to the actual traffic distribution.

5. Example

Figure 1 is a simple road network, which consists of 7 points and 20 directed roads. Each road has 3 lanes, points 2–6 are signalized intersections, where points 2, 3, 5, and 6 are T-intersections, and point 4 is the crossed intersection. All the T-intersections adopt three-phase control scheme, and the signal cycle is 90 s. The crossed intersection adopts four-phase control scheme, and the signal cycle is 120 s. The phase and phase sequence in each intersection can be expressed in Figures 2 and 3.

The initial green times of phases 1, 2, and 3 for the T-intersection are 50 s, 20 s, and 20 s, respectively. The initial green times of phases 1, 2, 3, and 4 for the crossed intersections are 40 s, 20 s, 40 s, and 20 s, respectively. The length, traffic capacity, and free-flow speed of each road are shown in Table 1.

There are two cases in finding the solution of average delay on the lanes of import road ij of the k-th phase control in a time cycle. On the one hand, if the arrival rate is less than signal traffic capacity, it can be calculated by the steady-state delay model. On the other hand, if the arrival rate is higher than signal traffic capacity, it can be calculated by fixed number delay model. The steady-state delay model and fixed-number delay model (Wang and Yan) [20] can be, respectively, expressed as

$$d_{ij}^{k} = \frac{c_{ij}^{m}\left(T_j - \lambda_{jk}\right)^2}{2T_j\left(c_{ij}^{m} - f_{ij}^{k1}\right)}, \tag{95}$$

$$d_{ij}^{k} = \frac{c_{ij}^{m}\left(T_j - \lambda_{jk}\right)}{2 f_{ij}^{k1}} + \frac{\left(f_{ij}^{k1} - c_{ij}^{m}\right)T_j}{2}, \tag{96}$$

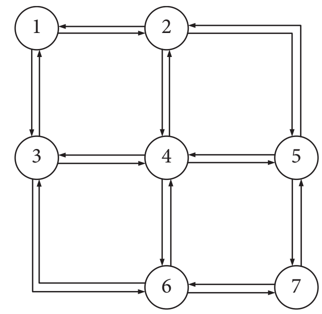

Figure 1: Simple traffic network.

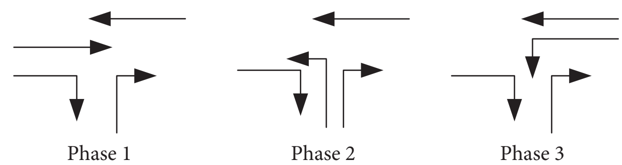

Figure 2: T-intersection design phase and phase sequence diagram.

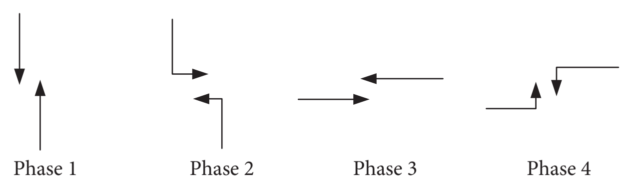

Figure 3: Cross-intersection design phase and phase sequence diagram.

where v_{ij}^{m} is the free flow speed on road ij. The traffic impedance of each road is obtained by the numerical correspondence relationship between (traffic volume/traffic capacity) and (average speed/free flow speed), which is shown in Table 2 (Markose et al.) [21].

There are three OD pairs in the network, that is, (1, 7), (7, 1), and (4, 7), and the OD demands are $d_{17} = 3000\,\text{veh}\cdot\text{h}^{-1}$, $d_{71} = 4000\,\text{veh}\cdot\text{h}^{-1}$, and $d_{47} = 1020\,\text{veh}\cdot\text{h}^{-1}$. There are six paths between OD pair (1, 7), that is, $a_1: 1 \longrightarrow 2 \longrightarrow 5 \longrightarrow 7$, $a_2: 1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 5 \longrightarrow 7$, $a_3: 1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 6 \longrightarrow 7$, $a_4: 1 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 7$, $a_5: 1 \longrightarrow 3 \longrightarrow 4 \longrightarrow 6 \longrightarrow 7$, and $a_6: 1 \longrightarrow 3 \longrightarrow 6 \longrightarrow 7$. There are two paths between OD pair (4, 7); that is, $a_7: 4 \longrightarrow 5 \longrightarrow 7$ and $a_8: 4 \longrightarrow 6 \longrightarrow 7$. There are six paths between OD pair (7, 1), that is, $a_9: 7 \longrightarrow 5 \longrightarrow 2 \longrightarrow 1$, $a_{10}: 7 \longrightarrow 5 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$, $a_{11}: 7 \longrightarrow 6 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$, $a_{12}: 7 \longrightarrow 5 \longrightarrow 4 \longrightarrow 3 \longrightarrow 1$, $a_{13}: 7 \longrightarrow 6 \longrightarrow 4 \longrightarrow 3 \longrightarrow 1$, and $a_{14}: 7 \longrightarrow 6 \longrightarrow 3 \longrightarrow 1$. The traffic flows on these fourteen paths are represented by $f_{a_1}, f_{a_2}, \ldots, f_{a_{14}}$, respectively.

The fuzzy degree of the path price provided by the Intelligent Transportation Information System is $\eta = 0.2$; the ratio of drivers adopting the first strategy is $\gamma = 0.5$. We carry out road network traffic assignment by the dynamic traffic game model based on the contraction algorithm and the shortest path algorithm, respectively. We conduct

Table 1: Static attributes of simple traffic network.

Road	$1 \longrightarrow 2$ $2 \longrightarrow 1$	$2 \longrightarrow 5$ $5 \longrightarrow 2$	$1 \longrightarrow 3$ $3 \longrightarrow 1$	$2 \longrightarrow 4$ $4 \longrightarrow 2$	$3 \longrightarrow 4$ $4 \longrightarrow 3$	$4 \longrightarrow 5$ $5 \longrightarrow 4$	$3 \longrightarrow 6$ $6 \longrightarrow 3$	$4 \longrightarrow 6$ $6 \longrightarrow 4$	$5 \longrightarrow 7$ $7 \longrightarrow 5$	$6 \longrightarrow 7$ $7 \longrightarrow 6$
Length/(km)	2.9	5	2.875	0.65	0.94	1.18	4	0.975	2.72	3.2
Traffic capacity/(veh·h^{-1})	3000	3000	2500	3000	3000	3000	2500	3000	3000	2500
Free flow speed/(km·h^{-1})	50	50	40	40	40	30	40	40	50	40

Table 2: Relationship between vehicle average speed and path flow.

(Traffic volume)/(traffic capacity)	≤0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	≥1.3
(Average speed)/(free flow speed)	1	34/35	31/35	24/35	19/35	14/35	9/35	4/35	3/35	2/35	0

Table 3: Assignment results of link flow/(veh·h^{-1}).

Path flow	f_{a_1}	f_{a_2}	f_{a_3}	f_{a_4}	f_{a_5}	f_{a_6}	f_{a_7}	f_{a_8}	f_{a_9}	$f_{a_{10}}$	$f_{a_{11}}$	$f_{a_{12}}$	$f_{a_{13}}$	$f_{a_{14}}$
Shortest path algorithm	4	2988	0	4	4	0	1016	4	4	3988	0	0	4	4
Contraction model algorithm	4	864	732	736	664	0	688	332	0	1028	1000	1040	932	0

Table 4: The red light total delay of all vehicles at the intersection/(s).

Intersection	2	3	4	5	6	Red light delay	Increased utility (%)
Shortest path algorithm	2.2307×10^8	2.6634×10^2	1.2521×10^8	2.2313×10^8	1.1258×10^2	5.7141×10^8	55.58
Contraction algorithm	5.7744×10^7	2.7525×10^7	6.6767×10^7	5.9933×10^7	4.1850×10^7	2.53819×10^8	

simulation by MATLAB2006a, and the simulation time is 15 minutes. The result of traffic assignment in each path is shown in Table 3.

In Table 3, the traffic assignment result has been given based on the contract model and the shortest path algorithm. On the one hand, the assignment results based on the shortest path algorithm are listed as follows: there is 2988 veh·h^{-1} traffic flow choosing path a_2 among the traffic demand volume of 3000 veh·h^{-1} of OD pair (1, 7); there is 3988 veh·h^{-1} traffic flow choosing path a_{10} among the traffic demand volume of 4000 veh·h^{-1} of OD pair (7, 1); there is 1016 veh·h^{-1} traffic flow choosing path a_7 among the traffic demand volume of 1020 veh·h^{-1} of OD pair (4, 7). On the other hand, the traffic assignment results based on the contract model algorithm are stated as follows: the traffic demands of OD pair (1, 7) distributing the traffic flow to paths a_2, a_3, a_4, and a_5 are 864 veh·h^{-1}, 732 veh·h^{-1}, 736 veh·h^{-1}, and 664 veh·h^{-1}, respectively; the traffic demands of OD pair (7, 1) distributing the traffic flow to paths a_{10}, a_{11}, a_{12}, and a_{13} are 1028 veh·h^{-1}, 1000 veh·h^{-1}, 1040 veh·h^{-1}, and 932 veh·h^{-1}, respectively; the traffic demands of OD pair (4, 7) distributing the traffic flow to paths a_7 and a_8 are 688 veh·h^{-1} and 332 veh·h^{-1}, respectively. Compared with the contract model algorithm, the traffic flow is overconcentrated on the shortest path algorithm and it does not respond to the human behavior factor in transportation route choice. For the contract model algorithm, according to the characteristic of traffic flow, every two adjacent intersections have a game, and the equilibrium price of the road network is obtained. It adjusts the intersection signal phase green time and meets the condition of the equilibrium price for the road network. Finally, it induces by an Intelligent Transportation Information System and considers behavior factors of the driver, which succeed in distributing the traffic flow to the road network equally. Therefore, the contract model algorithm promotes traffic to be distributed more evenly to the road network and makes a more rational distribution of traffic flow in the network and thus avoids the phenomenon of overconcentration of the traffic flow.

It is shown from Table 4 that the red light delays of intersections 3 and 6 for the traffic flow assigned by shortest path method are far below the one assigned by the contract model algorithm. Indeed, since OD traffic demand distribution is overconcentrated on certain paths by the shortest path method, there is little traffic flow passing intersections 3 and 6, and the red light delays of intersections 3 and 6 by using the shortest path algorithm are fewer correspondingly. From the total red light delay of all the intersections in the network, we find that the total red light delay of all the intersections in the network by using the contract model algorithm is 2.53819×10^8 s, which is significantly lower than the total delay of 5.7141×10^8 s by using the shortest path algorithm. Hence, it has improved the total utility of the network signal control by 55.58%. Thus, the contract model algorithm makes the signal control of each intersection coordinated and unified by agent technology. It is a full initiative of traffic management and control and thus improves greatly the system utility in the road network.

The contract model algorithm obtains the least price path by coordinating and unifying the control system of the road network. Furthermore, this combined with the positive inducement by the Intelligent Transportation System, and the uncertainty of driver's choosing the path, the system utility in the network thus plays its role effectively. By the simulation example, we can find that the dynamic traffic game model based on contract considers fully the system optimization of the road network and also the utility maximization of the driver's choosing the path which achieves coordination between system optimization and user optimization and thus improves greatly the effectiveness of the system in the road network.

6. Conclusions

This paper combines the manager's initiatives with the driver's personal rational behavior closely by using agent technology and thus studies intensively the allocation of each intersection traffic right in the road networks. Based on the fact that the Intelligent Transportation Systems provides continually the real-time traffic information of road network, it is much easier for agents to achieve equilibrium strategy price. Next, the equilibrium strategy price will be given to the driver through the information system. Then, the driver takes the corresponding path selection strategy by the latest price information of the road network, which succeeds in making the traffic flow assignment more reasonable in the road network. The example shows that the network utility will be greatly increased by using a dynamic traffic game model based on the contract model. The model gives full consideration to the system optimality and takes into account the user optimality, which conforms to the actual situation.

The next step is based on the traffic conditions of the road network, to which agents (intersection) price control strategy makes the appropriate dynamic response, and pricing strategies influence each other between the various agents and so forth.

Data Availability

The data used to support the study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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