

## Research Article

# A Dual-Channel Sale System in Financially Constrained Supply Chain

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In order to realize the sustainable development of the communication industry, many banks have developed some new projects to provide loan to the mobile phone brand companies (MPBCs). This paper studies a perfect information game among three parties: telecom operator (TO), MPBC, and bank. In the first stage, the bank decides an interest rate and shows it to the TO and the competitive MPBC. In the second stage, the TO and MPBC are engaged in Cournot games: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame. The TO's and the MPBC's decisions and the production/sale quantities are investigated. The impacts of the interest rate and the substitute factor on the TO's and the MPBC's optimal decisions are analyzed. When the substitute factor is high, at a low interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames. However, when the substitute factor is low, at a low enough interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high enough interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames; at a moderate interest rate, the total sales in the TO-as-leader subgame is higher than those in the other two subgames. Besides, the optimal interest rate of the bank is investigated and the impact of the substitute factor on the optimal interest rate is analyzed. The bank sets a higher interest rate in the MPBC-as-leader subgame than those in the other two subgames. Besides, when the substitute factor is low, the bank sets a lower interest rate in the TO-as-leader subgame than that in the simultaneous subgame; however, when the substitute factor is high, the bank sets a higher interest rate in the TO-as-leader subgame than that in the simultaneous subgame.

## 1. Introduction

Recently, in order to increase market share and realize the sustainable development of the TOs, more and more TOs start the customized-phone business and launch a wide range of customized packages, in which telecom operators purchase cell phones from the MPBCs, and the phones can only fit in their own network modes, such as GSM (Global System for Mobile Communications) and CDMA (Code Division Multiple Access). In addition, the customized phones are sold with telecom service provided by the TOs. Usually, the clients sign contracts with the TOs, under which the TOs sell the phones and service to the clients at a discount with a long term, usually from 1 to 3 years. Then, the customized phone is also called contract phone, otherwise

named noncontract phone. In 2016, the mobile phone users in China reached 1,316 billion and the penetration rate was 95.5 mobile phones per 100 persons. (the source is from the web of Ministry of Industry and Information Technology of the People's Republic of China (<http://www.miiit.gov.cn/>)). The mobile phone has become an essential tool in our daily life.

The demand for the better functions and service of mobile phone leads to the cooperation between the TOs and the MPBCs. By selling their cell phones with the service through the TOs, the MPBCs widen their sale channels and motive their existing sale systems, so as to increase their market share and realize the sustainable development of the MPBCs. The TOs obtain market information and feedback timely, which accelerates the phone's upgrading speed.

Recently, because of the increasing usage of the intelligent mobiles, the demanding for the network and connection quality is higher, which makes the TOs earn larger profit and realize the sustainable development of TOs.

There is not only cooperations but also competitions between the TOs and the MPBCs. Specially, on the one hand, the TOs purchase the mobile phones from the MPBCs; on the other hand, both of the TOs and the MPBCs sell mobile phones to customers. The customers' decisions are affected by two main factors: the price of the phone and the service fee of the TOs. Buying the contracted phone, customers can obtain a discount of the mobile phone price, but must pay the additional fee for the service set provided by the TOs. Usually, customers need to pay a minimum consumption for the service set, even if they do not use all of the service in the service set. Many TOs propagate the slogan "take the mobile phone for free." This kind of advertisement really attracts a lot of customers to buy the contract phone.

The TOs realize that the most of the profit is occupied by the MPBCs and then begin to change their operation strategies, even beginning to design and to manufacture their own brand mobile phone actively. The MPBCs also do not consider the TOs as their main sales channel. The MPBCs change their sale strategy from selling cheap phones to accessing high-level market; meanwhile, they start to launch new sale channels like e-commerce. Some of the MPBCs expand the foreign market actively to enhance their market share, such as Huawei. At the same time, they start to focus on the customers' requirements for the mobile phone functions, rather than on reducing the manufacturing cost.

The expansion of either TOs or MPBCs require a large amount of fund. In China, the short-term fund relies on loan from bank and the long-term fund depends mainly on equity financing. However, in Britain and America, companies' external financing relies on issuing bonds instead of stock, such that they are cautious to use bank loans. They choose the bank loans only when they need middle- or long-term fund. The MPBCs need to manufacture all of the phones, including the orders from the TOs. Consequently, the MPBCs need to borrow money from the bank. In order to realize the sustainable development of the communication industry, and many banks have developed some projects to provide loan to the MPBCs, such as "Helidai" is provided by Shanghai Pudong Development Bank.

In order to make the problem trackable, a perfect information game among three parties is designed, in which a TO purchases cell phones from a MPBC, the TO sells the cell phones with its telecom service to customers, and the MPBC manufactures and sells cell phones to both the TO and customers. The TO and MPBC are engaged in three Cournot games (the production quantity is assumed to be the sale quantity), and their products are substitutable for the customers. Besides, the MPBC has the financial constraint and need to borrow money from the bank. The interest rate is set by the bank (the detailed description of the game can be found in Section 3). Based on the analysis of this game, our study makes a number of contributions to the current literature. The contributions are as follows:

- (i) This paper studies a perfect information game among three parties: the bank, the TO, and the competitive MPBC. In the first stage, the bank decides an interest rate and shows it to the TO and the competitive MPBC. In the second stage, this work separates the subgames into three cases: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame. The TO's and the MPBC's optimal decisions and the optimal production/sale quantities are investigated.
- (ii) The impacts of the interest rate and the substitute factor on the TO's and the MPBC's optimal decisions are analyzed. When the substitute factor is high, at a low interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames. However, when the substitute factor is low, at a low interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames; at a moderate interest rate, the total sales in the TO-as-leader subgame is higher than those in the other two subgames. Besides, all the three slopes of the total sales are both negative and the slope of the total sales in the MPBC-as-leader subgame is higher than those in the other subgames, and the slope of the total sales in the TO-as-leader subgame is higher than that in the simultaneous subgames.
- (iii) The bank's decision and the optimal interest rate are investigated, and the impact of the substitute factor on the optimal interest rate is analyzed. The bank can set a higher interest rate in the MPBC-as-leader subgame than those in the other two subgames. Besides, when the substitute factor is low, the bank sets a low interest rate in the TO-as-leader subgame than that in the simultaneous subgame; however, when the substitute factor is high, the bank sets a higher interest rate in the TO-as-leader subgame than that in the simultaneous subgame.

The remainder of this paper is organized as follows. A literature review is provided in Section 2, and the model and preliminary results is presented in Section 3. In Section 4, the bank's decision is studied and the impact of the substitute factor on the bank's decision is analyzed. The conclusion is shown in Section 5. The proofs of the propositions are relegated to the appendix.

## 2. Literature Review

This work is closely related to the field of supply chain finance. Those works mainly focus on the financially constrained supply chain in which a supplier or/and a retailer need(s) to borrow money from a bank. Some works consider the presence of bankruptcy risks. Kouvelis and Zhao [1]

model a supply chain with a supplier and a retailer. Their strategic interaction is a Stackelberg game with the supplier as the leader. Both of them are capital constrained and in need of short-term financing. Kouvelis and Zhao [1] conclude that, under optimal trade credit contracts, both the supplier's profit and supply chain efficiency improve, and the retailer might improve his profits relative to under bank financing, depending on his current working capital and collateral. Kouvelis and Zhao [2] study a supply chain, in which a supplier sells a wholesale price contract to a financially constrained retailer who faces stochastic demand, so the retailer might need to borrow money from a bank to execute his order. Kouvelis and Zhao [2] find that with the presence of the retailer's bankruptcy risks, the increase of the retailer's wealth leads to the increase of the supplier's wholesale prices, but without the retailer's bankruptcy risks, the supplier's wholesale price stays the same or decreases in retailer's wealth. Xiao et al. [3] consider a financially constrained supply chain in which a supplier (leader) sells products to a retailer (follower) who has no access to bank financing due to her low credit rating. However, the supplier can borrow from a bank and offer trade credit to the retailer to alleviate her financial constraint. Xiao et al. [3] find that the revenue-sharing and buyback contracts can coordinate the supply chain only when the supply chain has a sufficient total working capital. More classical works on this stream can be found in [4–8].

This work is also related to studies of multichannel distribution and dual sales. Chiang et al. [9] build a model in which the manufacturer can open a direct channel to compete with its retailers. They show that the channel can benefit the manufacturer even when no direct sales occur. Tsay and Agrawal [10] study the channel conflict issue between direct sales and existing reseller partners and find that the addition of a direct channel is not necessarily detrimental to the reseller. Arya et al. [11] consider a dual distribution channel in which the manufacturer sells a product to a retailer and also competes with that retailer in the retail market. More works in this area can be found in the survey carried out by Tsay and Agrawal [12]. Recently, Wang et al. [13] build a supply chain, in which a contract manufacturer (CM) acts as both upstream partner and downstream competitor to an original equipment manufacturer (OEM). The two parties can engage in one of three Cournot competition games: a simultaneous game, a sequential game with the OEM as the Stackelberg leader, and a sequential game with the CM as the Stackelberg leader. On the basis of these three basic games, this study investigates the two parties' Stackelberg leadership/followership decisions. Matsui [14] applies an observable delay game framework developed in noncooperative game theory, in which a manufacturer manages dual-channel supply chains consisting of a retail channel and a direct channel. Matsui [14] investigates the timing problem concerning when the manufacturer should post its wholesale price and direct price, and they find that the manufacturer should post the direct price before or upon, but not after, setting the wholesale price for the retailer and this upfront posting of the direct price maximizes the

profits for a manufacturer employing multichannel sales strategies. Niu et al. [15] incorporate the concepts of channel power and fairness concern in a two-stage supply chain comprising a supplier and a retailer. With an online channel, the supplier competes with its retailer in a dual-channel system, but the retailer may shift part of or all orders to another supplier as the counteraction. They analyze the suppliers' decision on whether to open an online direct channel, and they find that the suppliers' fairness concern may effectively reduce its incentives to open an online channel.

Similar with the literature above, we also consider a dual-channel system, in which the MPBC not only sells the TO but also competes with the TO by the direct sales to the customers. Usually, the TO sells the phone with its telecommunication service. The MPBC is assumed to be financially constraint such that a commercial bank as the third party exists in this system. Consequently, we study a game among the three parties. In the first stage, the bank decides an interest rate and shows it to the TO and the competitive MPBC. In the second stage, the TO and MPBC are engaged in Cournot subgames: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame. On the basis of these three basic subgames, we investigate the TO's and the MPBC's optimal decisions and the optimal sales. We also analyze the impacts of the interest rate and the substitute factor on the TO's and the MPBC's optimal decisions.

### 3. Model

We consider that a TO (labeled  $t$ ) purchases phones from a MPBC (labeled  $c$ ). The TO sells the cell phones with its telecom service to customers. The MPBC manufactures and sells its products to both the TO and customers directly. The two parties' products are substitutable for the customers. To simplify the problem, the MPBC incurs the same production cost in producing its own and the TO's products.

The TO and the competitive MPBC engage quantity setting Cournot competition for customers (in the Cournot game, the production quantity is assumed to be the sale quantity). Thus, the market price of their products is jointly determined by their respective production quantities, that is, via inverse demand function. To make the model trackable, we adopt the commonly used inverse demand function for the differentiated product of firm  $c$  and  $t$ :

$$p_c(q_c, q_t) = m - q_c - b_c q_t, \quad (1)$$

where  $p_c$  is firm's market price,  $m$  is the upper bound on market size,  $q_c$  is its production quantity of the MPBC,  $q_t$  is its production quantity of the TO, and  $b_c$  is a parameter that measures the cross-effect of the change in firm  $c$ 's product demand caused by a change in that of firm  $t$  and is interpreted as the substitution rate of firm  $t$ 's product over that of firm  $c$ . Note that the limiting values  $b_c = 0$  and  $b_c = 1$  correspond to the cases of independent products and perfect substitutes, respectively. Similarly, the firm  $t$ 's market price is given as follows:

$$p_t(q_c, q_t) = m - q_t - b_t q_c. \quad (2)$$

The TO's products are always with the telecom service and promotion and are usually regarded as superior to those of the MPBC, and the former are assumed to be perfect substitutes for the latter, but the reverse is not true, that is,  $b_c = 1$ . Furthermore,  $0 \leq b_t \leq 1$ . To omit cases in which no production occurs,  $m$  is assumed to be sufficiently large relative to the wholesale price  $w$ . The MPBC's revenue consists of two parts: one is from customers and the other is from the TO. To simplify the model, the MPBC's marginal production cost is normalized to zero and the unit production cost is  $c_m$ . In this model, we assume that the production cost of the MPBC is borrowed from a bank, and the interest rate is denoted as  $r$ . Then, the MPBC's profit is

$$\pi_c = (m - q_c - q_t)q_c + wq_t - c_m(q_t + q_c)(1 + r). \quad (3)$$

The TO purchases the products (cell phones) from the MPBC and sells the products and its own telecom service to customers, and the service cost per product is denoted as  $c_t$ . Then, the TO's profit is

$$\pi_t = (m - q_t - bq_c)q_t - wq_t - c_t q_t. \quad (4)$$

Both (3) and (4) are concave and differentiable.

We denote  $i$  as the risk-free interest rate, which can be considered as the cost of the money for the bank. Then, the bank's profit is

$$\pi_b = (r - i)c_m(q_t + q_c). \quad (5)$$

We study a perfect information game among three parties: the bank, the TO, and the competitive MPBC. In the first stage, the bank decides an interest rate and shows it to the TO and the competitive MPBC. In the second stage, we separate the subgames into three cases: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame. We mainly focus on the subgames in the second stage in this section and will show the bank's decision in the second stage in the next section.

In this part, given the interest rate, the TO and the competitive MPBC can play three basic subgames: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame.

To explore the Stackelberg leadership preferences of the TO and the competitive MPBC and how those preferences affect the realization of the three aforementioned settings, we study a two-stage extended game called the endogenous timing game (see, e.g., [16, 17]). The extended game features a preplay stage in which the TO and the competitive MPBC simultaneously, though independently, choose either to move first and be the Stackelberg leader (denoted as  $L$ ) or to move second and be the Stackelberg follower (denoted as  $F$ ).

Denote  $\pi_i^S$ ,  $i = t, c$ , as firm  $i$ 's profit when it is engaged in a simultaneous subgame, where  $S$  stands for simultaneous. Consequently,  $q_i^S$ ,  $i = t, c$ , is the resulting production quantity. Denote  $\pi_i^L$  ( $\pi_i^F$ ),  $i = c, t$ , as the firm  $i$ 's profit when it is the Stackelberg leader (follower). Let  $q_i^L$  ( $q_i^F$ ),  $i = c, t$ , represent the corresponding production quantity.

**3.1. Optimal Decisions and Payoffs.** In this part, we compare the equilibrium sale quantity in the simultaneous and Stackelberg settings, in which the TO or the competitive MPBC would prefer Stackelberg leadership. Then, the closed-form expressions for the subgame perfect equilibrium quantities under the three setting are summarized in the following proposition.

**Proposition 1.** *In the simultaneous subgame, the equilibrium sale quantities are*

$$q_c^S = \frac{m + w + c_t - 2c_m(1 + r)}{4 - b}, \quad (6)$$

$$q_t^S = \frac{2(m - w - c_t) - b[m - c_m(1 + r)]}{4 - b}. \quad (7)$$

*In the TO-as-leader subgame, the equilibrium sale quantities are*

$$q_t^L = \frac{m - w - c_t}{2 - b} - \frac{b[m - c_m(1 + r)]}{2(2 - b)}, \quad (8)$$

$$q_c^F = \frac{m + w + c_t}{2(2 - b)} - \frac{bm + (4 - b)c_m(1 + r)}{4(2 - b)}. \quad (9)$$

*In the MPBC-as-leader subgame, the equilibrium sale quantities are*

$$q_c^L = \frac{m + w + c_t}{2(2 - b)} - \frac{bm + (4 - b)c_m(1 + r)}{4(2 - b)} + \frac{b[m - 2w + c_m(1 + r)]}{4(2 - b)}, \quad (10)$$

$$q_t^F = \frac{m - w - c_t}{2 - b} - \frac{b[m - c_m(1 + r)]}{2(2 - b)} - \frac{b[m - w - c_t - bw + bc_m(1 + r)]}{4(2 - b)}. \quad (11)$$

**3.2. The Impact of  $r$  on the Sales Quantity.** The TO's and MPBC's sale quantities decide their market share and their payoffs in the long term, so they are very important for both

the TO and the MPBC. In this section, we will analyze the impact of the interest rate on the MPBC's optimal sale, the TO's optimal sale, and the optimal total sale in the three

subgames. We find a threshold of  $b$  and five thresholds of  $r$  that affect the comparison results. The thresholds are shown as follows:

$$b_t = 1 - \frac{c_t}{m-w}, \quad (12)$$

$$r_1 = \frac{2w-m}{c_m} - 1, \quad (13)$$

$$r_2 = \frac{2w+2c_t+bm-2m}{bc_m} - 1, \quad (14)$$

$$r_3 = \frac{3w-m-c_t-bw}{(2-b)c_m} - 1, \quad (15)$$

$$r_4 = \frac{w+bw+c_t-m}{bc_m} - 1, \quad (16)$$

$$r_5 = \frac{(1-b)w-c_t}{(1-b)c_m} - 1. \quad (17)$$

Then, the MPBC's optimal sales comparison among three different subgames is shown as follows.

**Proposition 2.** When  $b < b_t$ ,  $r_2 < r_1 < r_3$ :

$$\text{If } r \leq r_2 < r_1 < r_3, q_c^L < q_c^S \leq q_c^F$$

$$\text{If } r_2 < r \leq r_1 < r_3, q_c^L \leq q_c^F < q_c^S$$

$$\text{If } r_2 < r_1 < r \leq r_3, q_c^F < q_c^L \leq q_c^S$$

$$\text{If } r_2 < r_1 < r_3 < r, q_c^F < q_c^S < q_c^L$$

When  $b > b_t$ ,  $r_3 < r_1 < r_2$ :

$$\text{If } r \leq r_3 < r_1 < r_2, q_c^L \leq q_c^S < q_c^F$$

$$\text{If } r_3 < r \leq r_1 < r_2, q_c^S < q_c^L \leq q_c^F$$

$$\text{If } r_3 < r_1 < r \leq r_2, q_c^S \leq q_c^F < q_c^L$$

$$\text{If } r_3 < r_1 < r_2 < r, q_c^F < q_c^S < q_c^L$$

When  $b = b_t$ ,  $r_2 = r_1 = r_3$ ; if  $r \leq r_1$ ,  $q_c^L \leq q_c^S \leq q_c^F$ ; if  $r > r_1$ ,  $q_c^F < q_c^S < q_c^L$ .

When the substitute factor is high ( $b \geq b_t$ ), only one threshold of the interest rate ( $r_1$ ) affects the strategy of the MPBC. At a low interest rate ( $r \leq r_1$ ), as a follower in the

sequential subgame, the MPBC can sell more products; at a high interest rate ( $r_1 < r$ ), as a leader, the MPBC can sell more products.

When the substitute factor is low ( $b < b_t$ ), there are two thresholds of the interest rate ( $r_2, r_3$ ) that affect the strategy of the MPBC. At a low enough interest rate ( $r \leq r_2$ ), as a follower, the MPBC can sell more products; at a high enough interest rate ( $r_3 < r$ ), as a leader, the MPBC can sell more products; at a moderate interest rate ( $r_2 < r \leq r_3$ ), a simultaneous subgame can help the MPBC sell more products.

The TO's optimal sales comparison among three different subgames is shown as follows.

**Proposition 3.** When  $b < b_t$ ,  $r_2 < r_4 < r_3$ :

$$\text{If } r \leq r_2 < r_4 < r_3, q_t^L \leq q_t^S < q_t^F$$

$$\text{If } r_2 < r \leq r_4 < r_3, q_t^S < q_t^L \leq q_t^F$$

$$\text{If } r_2 < r_4 < r \leq r_3, q_t^S \leq q_t^F < q_t^L$$

$$\text{If } r_2 < r_4 < r_3 < r, q_t^F < q_t^S < q_t^L$$

When  $b > b_t$ ,  $r_3 < r_4 < r_2$ :

$$\text{If } r \leq r_3 < r_4 < r_2, q_t^L < q_t^S \leq q_t^F$$

$$\text{If } r_3 < r \leq r_4 < r_2, q_t^L \leq q_t^F < q_t^S$$

$$\text{If } r_3 < r_4 < r \leq r_2, q_t^F < q_t^L \leq q_t^S$$

$$\text{If } r_3 < r_4 < r_2 < r, q_t^F < q_t^S < q_t^L$$

When  $b = b_t$ ,  $r_2 = r_4 = r_3$ ; if  $r \leq r_4$ ,  $q_t^L \leq q_t^S \leq q_t^F$ ; if  $r > r_4$ ,  $q_t^F < q_t^S < q_t^L$ .

When the substitute factor is low ( $b \leq b_t$ ), only one threshold of the interest rate ( $r_4$ ) affects the strategy of the TO. At a low interest rate ( $r \leq r_4$ ), as a follower, the TO can sell more products; at a high interest rate ( $r_4 < r$ ), as a leader, the TO can sell more products.

When the substitute factor is high ( $b > b_t$ ), there are two thresholds of the interest rate ( $r_2, r_3$ ) that affect the strategy of the TO. At a low enough interest rate ( $r \leq r_3$ ), as a follower, the TO can sell more products; at a high enough interest rate ( $r_2 < r$ ), as a leader, the TO can sell more products; at a moderate interest rate ( $r_3 < r \leq r_2$ ), simultaneous subgame can help the TO sell more products.

The total sales is  $q = q_t + q_c$ . Then, the total sales in the MPBC-as-leader subgame is

$$q^C = q_c^L + q_t^F = \frac{6m-2w-2c_t-3bm-(4-3b)c_m(1+r)}{4(2-b)} + \frac{b[c_t+bw-w+(1-b)c_m(1+r)]}{4(2-b)}, \quad (18)$$

the total sales in the TO-as-leader subgame is

$$q^T = q_c^F + q_t^L = \frac{6m-2w-2c_t-3bm-(4-3b)c_m(1+r)}{4(2-b)}, \quad (19)$$

and the total sales in the simultaneous subgame is

$$q^S = q_c^S + q_t^S = \frac{3m-w-c_t-bm-(2-b)c_m(1+r)}{4-b}. \quad (20)$$

Then, we analyze the impact of  $r$  on the total sales in the three subgames. By comparing the three slopes of the total sales, we have the following proposition.

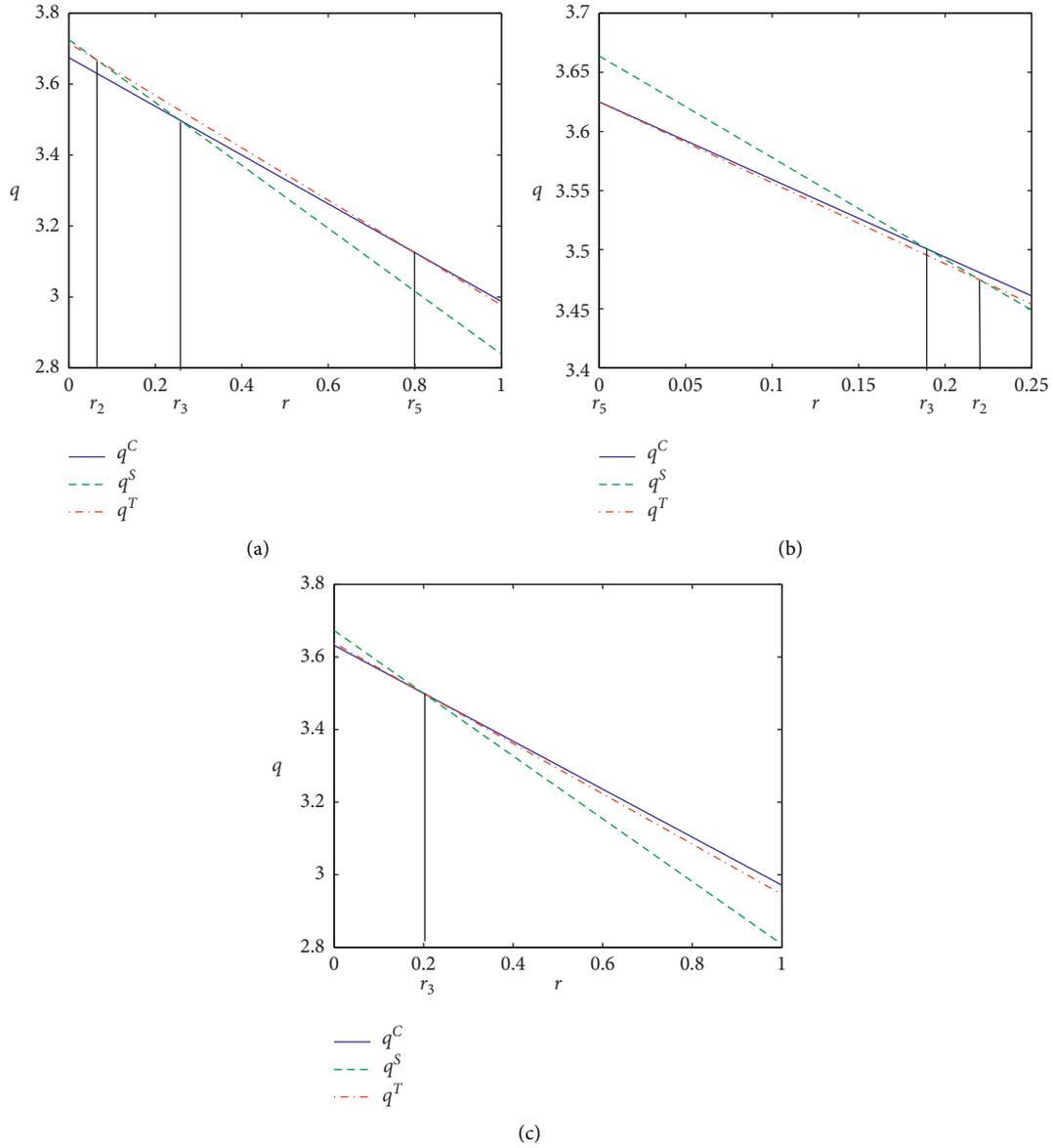


FIGURE 1: Illustration of the effects of  $r$  and  $b$  on the total sales:  $m = 10$ ,  $w = 6.5$ ,  $c_t = 0.2$ ,  $c_m = 2.5$ ,  $b_t = 0.9429$ , and  $b = [0.90, 0.9429, 0.95]$ . (a)  $b < b_t$ . (b)  $b > b_t$ . (c)  $b = b_t$ .

**Proposition 4.**  $\partial q^S / \partial r < \partial q^T / \partial r < \partial q^C / \partial r < 0$ .

$q^S$ ,  $q^C$ , and  $q^T$  linearly decrease in  $r$  and the slope of the total sales in the MPBC-as-leader subgame is lower than those in the other subgames, and the slope of the total sales in the TO-as-leader subgame is lower than that in the simultaneous subgames.

In order to analyze the impact of  $r$ , we also show impact of  $r$  on the total optimal sales (the sum of the TO's optimal sales and the MPBC's optimal sales) comparison among the three different subgames.

**Proposition 5.** When  $b < b_t$ ,  $r_2 < r_3 < r_5$ :

$$\text{If } r \leq r_2 < r_3 < r_5, q^C < q^T \leq q^S$$

$$\text{If } r_2 < r \leq r_3 < r_5, q^C \leq q^S < q^T$$

$$\text{If } r_2 < r_3 < r \leq r_5, q^S < q^C \leq q^T$$

$$\text{If } r_2 < r_3 < r_5 < r, q^S < q^T < q^C$$

When  $b > b_t$ ,  $r_5 < r_3 < r_2$ :

$$\text{If } r \leq r_5 < r_3 < r_2, q^C \leq q^T < q^S$$

$$\text{If } r_5 < r \leq r_3 < r_2, q^T < q^C \leq q^S$$

If  $r_5 < r_3 < r \leq r_2$ ,  $q^T \leq q^S < q^C$

If  $r_5 < r_3 < r_2 < r$ ,  $q^S < q^T < q^C$

When  $b = b_t$ ,  $r_2 = r_3 = r_5$ ; if  $r \leq r_3$ ,  $q^C \leq q^T \leq q^S$ ; if  $r > r_3$ ,  $q^S < q^T < q^C$ .

We summarize the results in Propositions 4 and 5 in Figure 1. As shown in Figure 1, we set  $m = 10$ ,  $w = 6.5$ ,  $c_t = 0.2$ , and  $c_m = 2.5$  and then obtain  $b_t = 0.9429$ . Obviously, all the three figures in Figure 1 show that  $\partial q^S / \partial r < \partial q^T / \partial r < \partial q^C / \partial r < 0$ . When  $b = 0.9 < b_t$ , as shown in Figure 1(a),  $r_2 = 0.0667$ ,  $r_3 = 0.2545$ , and  $r_5 = 0.8$ . If  $r \leq 0.0667$ ,  $q^C < q^T \leq q^S$ ; if  $0.0667 < r \leq 0.2545$ ,  $q^C \leq q^S < q^T$ ; if  $0.2545 < r \leq 0.8$ ,  $q^S < q^C \leq q^T$ ; if  $r > 0.8$ ,  $q^S < q^T < q^C$ . When  $b = 0.95 > b_t$ , as shown in Figure 1(b),  $r_2 = 0.2211$ ,  $r_3 = 0.1905$ , and  $r_5 = 0$ . If  $0 < r \leq 0.1905$ ,  $q^T < q^C \leq q^S$ ; if  $0.1905 < r \leq 0.2211$ ,  $q^T \leq q^S < q^C$ ; if  $r > 0.2211$ ,  $q^S < q^T < q^C$ . When  $b = 0.9429 = b_t$ , as shown in Figure 1(c),  $r_2 = r_3 = r_5 = 0.2$ . If  $r \leq 0.2$ ,  $q^C \leq q^T \leq q^S$ ; in contrast, if  $r > 0.2$ ,  $q^S < q^T < q^C$ .

When  $b < b_t$ , as shown in Figure 1(a), there are two thresholds of the interest rate ( $r_2, r_5$ ) that affect the total sales. At a low enough interest rate ( $r \leq r_2$ ), the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high enough interest rate ( $r_5 < r$ ), the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames; at a moderate interest rate ( $r_2 < r \leq r_5$ ), the total sales in the TO-as-leader subgame is higher than those in the other two subgames.

When the substitute factor is high ( $b \geq b_t$ ), as shown in Figure 1(b), only one threshold of the interest rate ( $r_3$ ) affects the total sales. At a low interest rate ( $r \leq r_3$ ), the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high interest rate ( $r_3 < r$ ), the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames.

#### 4. The Decision of the Bank

In this part, we consider the bank's decision in the first stage of the game among the three parties and the loan interest rate  $r$ . Recall that the profit of the bank is

$$\pi_b(r) = (r - i)c_m(q_t + q_c). \quad (21)$$

Under the three different subgames in the second stage, we obtain the optimal interest rates and summarize the results as follows.

**Proposition 6.** *In the simultaneous subgame, the equilibrium interest rate is*

$$r^S = \frac{(3 - b)m - w - c_t}{2c_m(2 - b)} - \frac{1 - i}{2}. \quad (22)$$

*In the TO-as-leader subgame, the equilibrium interest rate is*

$$r^T = \frac{3(2 - b)m - 2w - 2c_t}{2c_m(4 - 3b)} - \frac{1 - i}{2}. \quad (23)$$

*In the MPBC-as-leader subgame, the equilibrium interest rate is*

$$r^C = \frac{3m - (1 + b)w - c_t}{2c_m(2 - b)} - \frac{1 - i}{2}. \quad (24)$$

Next, we will make a comparison among the three interest rates:  $r^S$ ,  $r^T$ , and  $r^C$ . Let  $b_1 = c_t + w/m$ . We summarize the comparison result as follows.

**Proposition 7.** *When  $b < b_1$ ,  $r^C > r^S > r^T$ ; when  $b \geq b_1$ ,  $r^C > r^T \geq r^S$ .*

As shown in Proposition 7, the bank sets a higher interest rate in the MPBC-as-leader subgame than those in the other two subgames. Besides, when the value of  $b$  is low ( $b < b_1$ ), the bank sets a low interest rate in the TO-as-leader subgame than that in the simultaneous subgame; however, when the value of  $b$  is high ( $b \geq b_1$ ), the bank sets a higher interest rate in the TO-as-leader subgame than that in the simultaneous subgame.

#### 5. Conclusion

In order to realize the sustainable development of the communication industry, many banks have developed some new projects to provide loan to the mobile phone brand companies (MPBCs). In this work, we study a perfect information game among three parties: the bank, the TO, and the competitive MPBC. In the first stage, the bank decides an interest rate and shows it to the TO and the competitive MPBC. In the second stage, we separate the Cournot subgame into three cases: a simultaneous subgame, a TO-as-leader sequential subgame, and an MPBC-as-leader sequential subgame. We investigate the TO's and the MPBC's optimal decisions and the optimal sales. Besides, we also analyze the impacts of the interest rate and the substitute factor on the TO's and the MPBC's optimal decisions.

We analyze the impacts of substitute factor and interest rate on the MPBC's sale quantity, the TO's sale quantity, and the total sale quantity. For the total sale, when the substitute factor is high, at a low interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames. However, when the substitute factor is low, at a low enough interest rate, the total sales in the simultaneous subgame is higher than those in the other two subgames; at a high enough interest rate, the total sales in the MPBC-as-leader subgame is higher than those in the other two subgames; at a moderate interest rate, the total sales in the TO-as-leader subgame is higher than those in the other two subgames. Besides, all of the three slopes of the total sales are both negative and the slope of the total sales in the MPBC-as-leader subgame is lower than those in the other subgames, and the slope of the total sales in the TO-as-leader subgame is lower than that in the simultaneous subgames.

In addition, we also investigate the bank's decision, the interest rate and analyze the impact of the substitute factor on the interest rate. We find that the bank sets a higher

interest rate in the MPBC-as-leader subgame than those in the other two subgames. Besides, when the substitute factor is low, the bank sets a low interest rate in the TO-as-leader subgame than that in the simultaneous subgame; however, when the substitute factor is high, the bank sets a higher interest rate in the TO-as-leader subgame than that in the simultaneous subgame.

## Appendix

*Proof of Proposition 1.* We consider three different subgames: the simultaneous subgame, the TO-as-leader Stackelberg subgame, and the MPBC-as-leader Stackelberg subgame. And, the proof will be separated into three parts.

In the simultaneous subgame, the two parties make the decisions simultaneously.

Given  $q_t$ , based on (3), taking the first-order derivative of  $\pi_c$  with respect to  $q_c$ , we obtain

$$\frac{\partial \pi_c}{\partial q_c} = m - q_c - q_t - q_c - c_m(1+r). \quad (\text{A.1})$$

Obviously,  $\partial \pi_c / \partial q_c$  decreases in  $q_c$  and  $\pi_c$  is concave in  $q_c$ . Making  $\partial \pi_c / \partial q_c = 0$ , we obtain

$$q_c = \frac{m - q_t - c_m(1+r)}{2}. \quad (\text{A.2})$$

Given  $q_c$ , based on (4), taking the first-order derivative of  $\pi_t$  with respect to  $q_t$ , we obtain

$$\frac{\partial \pi_t}{\partial q_t} = m - q_t - bq_c - q_t - w - c_t. \quad (\text{A.3})$$

Obviously,  $\partial \pi_t / \partial q_t$  decreases in  $q_t$  and  $\pi_t$  is concave in  $q_t$ . Making  $\partial \pi_t / \partial q_t = 0$ , we obtain

$$q_t = \frac{m - bq_c - w - c_t}{2}. \quad (\text{A.4})$$

Combining (A.2) with (A.4), we obtain

$$\begin{aligned} q_c^S &= \frac{m + w + c_t - 2c_m(1+r)}{4-b}, \\ q_t^S &= \frac{2(m - w - c_t) - b[m - c_m(1+r)]}{4-b}. \end{aligned} \quad (\text{A.5})$$

In the TO-as-leader subgame, the TO is the leader and the MPBC is the follower in the Stackelberg subgame. Given  $q_t$ , we have obtained the MPBC's optimal decision as shown in (A.2). Coupled with (A.2) and (4), we obtain

$$\pi_t = \left\{ m - q_t - \frac{b[m - q_t - c_m(1+r)]}{2} \right\} q_t - wq_t - c_tq_t. \quad (\text{A.6})$$

Based on the above equation, taking the first-order derivative of  $\pi_t$  with respect to  $q_t$ , we obtain

$$\frac{\partial \pi_t}{\partial q_t} = m - w - c_t - \frac{b[m - c_m(1+r)]}{2} - 2q_t + bq_t. \quad (\text{A.7})$$

Clearly,  $\partial \pi_t / \partial q_t$  decreases in  $q_t$  and  $\pi_t$  is concave in  $q_t$ . Making  $\partial \pi_t / \partial q_t = 0$ , we obtain

$$q_t^L = \frac{m - w - c_t}{2-b} - \frac{b[m - c_m(1+r)]}{2(2-b)}. \quad (\text{A.8})$$

Coupled with (A.2), we have

$$q_c^F = \frac{m + w + c_t}{2(2-b)} - \frac{bm + (4-b)c_m(1+r)}{4(2-b)}. \quad (\text{A.9})$$

In the MPBC-as-leader subgame, the MPBC is the leader and the TO is the follower in the Stackelberg subgame. Similarly, based on (A.4) and (3), we can obtain

$$\begin{aligned} q_c^L &= \frac{m + w + c_t}{2(2-b)} - \frac{bm + (4-b)c_m(1+r)}{4(2-b)} + \frac{b[m - 2w + c_m(1+r)]}{4(2-b)}, \\ q_t^F &= \frac{m - w - c_t}{2-b} - \frac{b[m - c_m(1+r)]}{2(2-b)} - \frac{b[m - w - c_t - bw + bc_m(1+r)]}{4(2-b)}. \end{aligned} \quad (\text{A.10})$$

*Proof of Proposition 2.* Based on (6), (9), and (10), we obtain

$$q_c^L - q_c^F = \frac{b[m + c_m(1+r) - 2w]}{4(2-b)}, \quad (\text{A.11})$$

$$q_c^S - q_c^F = \frac{b[2m - 2w - 2c_t - bm + bc_m(1+r)]}{4(4-b)(2-b)}, \quad (\text{A.12})$$

$$q_c^L - q_c^S = \frac{b[m + c_t + bw - 3w + (2-b)c_m(1+r)]}{2(4-b)(2-b)}. \quad (\text{A.13})$$

where  $q_c^L > q_c^F$  leads to  $r > 2w - m/c_m - 1$ ,  $q_c^S > q_c^F$  leads to  $r > 2w + 2c_t + bm - 2m/bc_m - 1$ , and  $q_c^L > q_c^S$  leads to  $r > 3w - m - c_t - bw/(2-b)c_m - 1$ .

Let  $r_1 = 2w - m/c_m - 1$ ,  $r_2 = 2w + 2c_t + bm - 2m/bc_m - 1$ , and  $r_3 = 3w - m - c_t - bw/(2-b)c_m - 1$ . Now, we will

compare the values of  $r_i, i = 1, 2, 3$ , with each other, and by doing so, can we compare the values of  $q_c^L, q_c^S$ , and  $q_c^F$ :

$$r_1 - r_2 = \frac{2bw - 2w - 2c_t - 2bm + 2m}{bc_m} = \frac{2[(1-b)(m-w) - c_t]}{bc_m}. \quad (\text{A.14})$$

As a result,  $r_1 > r_2$  leads to  $b < 1 - c_t/m - w$ . Let  $b_t = 1 - c_t/m - w$ . Similarly,  $r_2 > r_3$  leads to  $b > b_t$  and  $r_1 > r_3$  leads to  $b > b_t$ .

Therefore, when  $b < b_t, r_2 < r_1 < r_3$ . If  $r \leq r_2 < r_1 < r_3$  (or  $r \leq r_2, r < r_1$ , and  $r < r_3$ ),  $q_c^L < q_c^S \leq q_c^F$  (or  $q_c^S \geq q_c^F, q_c^L > q_c^F$ , and  $q_c^L > q_c^S$ ).

Similarly, if  $r_2 < r \leq r_1 < r_3, q_c^L \leq q_c^F < q_c^S$ ; if  $r_2 < r_1 < r \leq r_3, q_c^F < q_c^L \leq q_c^S$ ; if  $r_2 < r_1 < r_3 < r, q_c^L < q_c^S < q_c^F$ .

When  $b > b_t, r_3 < r_1 < r_2$ . Similarly, if  $r \leq r_3 < r_1 < r_2, q_c^L \leq q_c^S < q_c^F$ ; if  $r_3 < r \leq r_1 < r_2, q_c^S < q_c^L \leq q_c^F$ ; if  $r_3 < r_1 < r \leq r_2, q_c^S \leq q_c^F < q_c^L$ ; if  $r_3 < r_1 < r_2 < r, q_c^F < q_c^S < q_c^L$ .

When  $b = b_t, r_2 = r_1 = r_3$ ; if  $r \leq r_1, q_c^L \leq q_c^S \leq q_c^F$ ; if  $r > r_1, q_c^F < q_c^S < q_c^L$ .  $\square$

*Proof of Proposition 3.* The proof is similar to the proof of Proposition 2.  $\square$

*Proof of Proposition 4.* Taking the first-order derivative of  $q$  with respect to  $r$  in the three subgames, we have

$$\frac{\partial q^C}{\partial r} = -c_m \frac{(2-b)}{4}, \quad (\text{A.15})$$

$$\frac{\partial q^T}{\partial r} = -c_m \frac{(4-3b)}{(8-2b)}, \quad (\text{A.16})$$

$$\frac{\partial q^S}{\partial r} = -c_m \frac{(2-b)}{(4-b)}, \quad (\text{A.17})$$

$$\begin{aligned} \frac{\partial q^T}{\partial r} - \frac{\partial q^C}{\partial r} &= \frac{c_m b(b-1)}{8-4b} < 0, \\ \frac{\partial q^S}{\partial r} - \frac{\partial q^C}{\partial r} &= \frac{-b(2-b)}{4(4-b)} < 0, \\ \frac{\partial q^T}{\partial r} - \frac{\partial q^S}{\partial r} &= \frac{b^2}{(4-b)(8-4b)} > 0. \end{aligned} \quad (\text{A.18})$$

*Proof of Proposition 5.* The Proof is similar to the proof of Proposition 2.  $\square$

*Proof of Proposition 6.* We also separate the proof into three parts: the simultaneous subgame, the TO-as-leader Stackelberg subgame, and the MPBC-as-leader Stackelberg subgame.

In the simultaneous subgame, based on (21), coupled with (6) and (7), we obtain

$$\begin{aligned} \pi_b^S &= (r-i)c_m(q_t^S + q_c^S) \\ &= (r-i)c_m \left\{ \frac{m+w+c_t-2c_m(1+r)}{4-b} \right. \\ &\quad \left. + \frac{2(m-w-c_t)-b[m-c_m(1+r)]}{4-b} \right\} \\ &= \frac{c_m}{4-b} [(3-b)m-w-c_t-(2-b)c_m(1+r)](r-i). \end{aligned} \quad (\text{A.19})$$

Based on (A.19), taking the first-order derivative of  $\pi_b^S$  with respect to  $r$ , we obtain

$$\frac{\partial \pi_b^S}{\partial r} = \frac{c_m}{4-b} \{(3-b)m-w-c_t-(2-b)c_m(1-i+2r)\}. \quad (\text{A.20})$$

Since  $0 \leq b \leq 1, \partial \pi_b^S / \partial r$  decreases in  $r$ . Making  $\partial \pi_b^S / \partial r = 0$ , we obtain

$$r^S = \frac{(3-b)m-w-c_t}{2c_m(2-b)} - \frac{1-i}{2}. \quad (\text{A.21})$$

In the TO-as-leader subgame, based on (21), coupled with (8) and (9), we obtain

$$\pi_b^T = \frac{c_m}{4(2-b)} [6m-2w-2c_t-3bm-(4-3b)c_m(1+r)](r-i). \quad (\text{A.22})$$

Similarly, taking the first-order derivative of  $\pi_b^T$  with respect to  $r$  and making  $\partial \pi_b^T / \partial r = 0$ , we obtain

$$r^T = \frac{3(2-b)m-2w-2c_t}{2c_m(4-3b)} - \frac{1-i}{2}. \quad (\text{A.23})$$

In the MPBC-as-leader subgame, based on (21), coupled with (10) and (11), we obtain

$$\pi_b^C = \frac{c_m}{4} [3m-w-c_t-bw-(2-b)c_m(1+r)](r-i). \quad (\text{A.24})$$

Similarly, taking the first-order derivative of  $\pi_b^C$  with respect to  $r$  and making  $\partial \pi_b^C / \partial r = 0$ , we obtain

$$r^C = \frac{3m-(1+b)w-c_t}{2c_m(2-b)} - \frac{1-i}{2}. \quad (\text{A.25})$$

*Proof of Proposition 7.* In this part, we will make an comparison among  $r^S, r^T$ , and  $r^C$ . Based on (22) and (23),

$$\begin{aligned} r^S - r^T &= \frac{(3-b)m-w-c_t}{2c_m(2-b)} - \frac{3(2-b)m-2w-2c_t}{2c_m(4-3b)} \\ &= \frac{w+c_t-bm}{2c_m(2-b)(4-3b)}. \end{aligned} \quad (\text{A.26})$$

As a result,  $r^S > r^T$  when  $b < c_t + w/m$ ;  $r^S \leq r^T$  when  $b \geq c_t + w/m$ . Let  $b_1 = c_t + w/m$ . Based on (22) and (24),

$$r^C - r^S = \frac{3m - (1+b)w - c_t}{2c_m(2-b)} - \frac{(3-b)m - w - c_t}{2c_m(2-b)} = m - w. \quad (\text{A.27})$$

where  $m > w$ ; then,  $r^C > r^S$ .

Based on (23) and (24),

$$\begin{aligned} r^C - r^T &= \frac{3m - (1+b)w - c_t}{2c_m(2-b)} - \frac{3(2-b)m - 2w - 2c_t}{2c_m(4-3b)} \\ &= \frac{3b(1-b)[m - w + c_t/3(1-b)]}{2c_m(2-b)(4-3b)}. \end{aligned} \quad (\text{A.28})$$

Because  $b < 1$  and  $m > w$ ,  $m - w + c_t/3(1-b) > 0$ . As a result,  $r^C > r^T$ .

Combining the results above, we summarize the conclusion as follows.

When  $b < b_1$ ,  $r^C > r^S > r^T$ ; when  $b \geq b_1$ ,  $r^C > r^T \geq r^S$ .  $\square$

## Data Availability

The data used to support the findings of this study are included within the article.

## Disclosure

The part of the manuscript was presented in 2017 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM).

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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